# Submission Number: PET11-11-00072 

## Games within borders: are geographically differentiated taxes optimal?

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#### Abstract

Traditional theoretical models combined with arguments of administrative simplicity suggest that the optimal commodity tax should be levied at a uniform rate within a state. However, when tax differentials at state borders exist, uniform taxation may not be optimal. For example, Mexico's Value Added Tax features a twenty kilometer preferential tax zone near the lower tax United States border. In such a context, this paper addresses the following question: Is it optimal for states to levy geographically differentiated commodity taxes - where tax rates are lower or higher the closer to a low- or high-tax state border? In a model where states in a federation maximize social welfare, I show that a state's optimal tax schedule is geographically differentiated. The optimal pattern of geographic differentiation critically depends on fundamental parameters as well as whether a state has a preference for high or low tax rates. The model generalizes to cross-country tax differentials at international borders, so long as the border is open.


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# Games Within Borders: 

# Are Geographically Differentiated Taxes Optimal? 

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February 2011


#### Abstract

The discontinuous tax treatment of sales at borders creates incentives for individuals to cross-border shop. This paper addresses whether it is optimal for states to levy geographically differentiated commodity taxes or preferential tax rates near borders. I show that in a model where states in a federation maximize social welfare, a state's optimal commodity tax system is geographically differentiated. The optimal pattern of geographic differentiation critically depends on fundamental parameters as well as whether a state has a preference for high or low tax rates. The model's results generalize to international borders, so long as the border is open.


JEL: H21, H25, H73, H77, R12
Keywords: Commodity Taxation, Cross-border Shopping, Tax Competition, Preferential Tax Rates

[^1]
## 1 Introduction

"Jeux Sans Frontières" - the title of a classic commodity tax competition paper (Kanbur and Keen 1993) - translates from French to "Games Without Borders." But, tax differentials at borders are an essential consideration in determining the optimal tax system given the incentives these "lines" create. What happens if countries play games within borders by setting geographically differentiated tax rates?

In a federal system such as the United States, the retail sales tax differs based on where the taxed good is purchased. Although the location of state borders are not chosen as a matter of policy, geographic borders between different states create a discontinuous tax policy. A state border and the discontinuity in tax rates induced is an example of a "line" in the tax system. The line creates a "notch" or a discontinuous jump in tax liability as it relates to the tax base - in characteristic space as in Kleven and Slemrod (2009) - where the characteristic of the good is the location of purchase. International borders potentially create the same type of notches.

Given that the discontinuous treatment of sales creates incentives for individuals to cross borders, it is important to know if a uniform tax policy is socially optimal compared to a policy that differentiates tax rates based on geographic location. Consider the example of high tax Massachusetts and low tax New Hampshire. In this example, given that New Hampshire has an exogenously lower preference for taxes (or public goods), a differentiated policy would be one where Massachusetts sets a lower tax rate closer to the New Hampshire border. In the standard tax competition literature, differences in tax rates at state borders arise because of differences in size or public good preferences. However, in all these models, the state or country can compete over only one sales tax rate. In this paper, acknowledging that tax competition will imply differentials in tax rates at borders, I consider what the optimal state policy is when the state can select two sales tax rates within its borders - a rate near the state border and a rate away from the border.

Forty-five American states impose a sales tax ranging between $2.9 \%$ and $7 \%$. Among

European Union member states, the Value Added Tax (VAT) ranges from $15 \%$ to $25 \%$. Given these disparities in tax rates, consumers have possibly large incentives to engage in cross-border shopping.

An example of geographic differentiation designed to mitigate such incentives is analyzed in Davis (2010). The Mexican VAT features a geographically differentiated tax rate depending on distance to the United States border. The standard tax rate in Mexico is $16 \%$, but goods purchased within twenty kilometers of the lower tax United States border are assessed a rate of $11 \%$. Although the reduced tax rate discourages Mexico's residents from crossing the United States border to purchase goods, Davis (2010) finds that there is also a modest but statistically significant distortion that encourages Mexicans living in the high tax zone to shop or locate in the preferred tax zone. It can be inferred that if the size of the distortion induced within Mexico is small relative to the decrease in the size of the distortion at the international border, then such a policy may be optimal. Furthermore, several European countries have historically experimented with discriminatory tax regimes.

In this paper, I consider whether under the assumption that neighboring states do not respond to geographically differentiated taxes, a state would adopt a tax policy that is different at the state's interior versus at the state's border. In theory, states may respond to cross-border shopping a in a variety of ways depending on the benevolent or Leviathan nature of the government (Edwards and Keen 1996; Kotsogiannis and Lopez-Garcia 2007). A possibility suggested by this paper is that a state social welfare planner will want to set multiple sales tax rates that are geographically differentiated depending on proximity to the border.

The paper makes several contributions to the literature. In the context of a Haufler (1996) model that combines elements from both Mintz and Tulkens (1986) and Kanbur and Keen (1993), I demonstrate that from a welfare maximizing state planner's perspective, the optimal tax system in the presence of borders is almost always a geographically differentiated tax. The existing tax competition literature, has shown that country size, other characteristics,
and preferences of the public good are determinants of whether a state assesses relatively high or low tax rates. The evidence in this paper suggests that in addition to these factors, the spatial composition of towns or regions within a state is an essential factor to consider when setting tax rates. As such, a broad uniform tax rate within the state may not be optimal. The results are broadly applicable to other types of state policies that vary at the border and distort consumption or firm location decisions (for example, business regulations that may distort the price of the production good relative to the neighboring state).

I proceed as follows. First, I review the literature on commodity taxation and crossborder shopping. Second, I develop a model where state governments choose the regional tax rates and the levels of public good provision in order to maximize social welfare of the state's residents. I relate this to the case of a uniform tax rate within the state. Next, I discuss the optimal tax rate with local public goods, revenue maximizing governments, and horizontal equity considerations. Finally, I conclude and discuss the welfare consequences of the policy.

## 2 Tax Competition and Cross-Border Shopping

The literature on tax competition has developed with an emphasis on trying to explain asymmetries in tax rates among competing jurisdictions when each jurisdiction chooses a uniform tax rate within its boundaries. The approach to solving this problem has varied substantially. For example, Mintz and Tulkens (1986) has a model with multiple goods, general equilibrium principles, and governments as welfare maximizers. Kanbur and Keen (1993) develops a model with a single good, partial equilibrium principles, and governments as revenue maximizers. Nielsen (2001) extends Kanbur and Keen (1993) to welfare maximizing governments, but relies on an additively separable relationship between consumer surplus and revenue. Haufler (1996) combines elements from both types of models and provides conditions for existence of a Nash equilibrium when the asymmetry in tax policy results from
different preferences of a public good.
The conclusions from these different models highlights how states (or countries) may reach an equilibrium with different tax rates. Haufler (1996) implies that a state like Massachusetts may have higher tax rates than New Hampshire because of stronger preferences for public good provision. This is in contrast to models such as Bucovetsky (1991), Kanbur and Keen (1993), Nielsen (2001), and Trandel (1994), which focus on country size or population as an explanation for variation in tax rates. However, why differences exist at state borders is not the question of this paper and is irrelevant to its outcomes. Rather, this paper uses the result that tax differentials will exist at state borders as a starting point. Certainly, tax differentials may come with benefits such as allowing for Tiebout sorting. Higher or lower taxes may be optimal depending on demographics and industry mix across states. But, because previous models only allow for one tax rate within a state, the natural question to ask is what happens if the state can choose multiple rates within its borders. Can multiple tax rates be used to mitigate the inefficiency losses (and perhaps revenue leakages) resulting from cross-border shopping?

In that spirit, the responsiveness of cross-border shopping plays a key role. Mikesell (1970), Fox (1986), Walsh and Jones (1988), and Tosun and Skidmore (2007) all provide estimates for how responsive individuals are to cross-border shopping. In a theoretical model where states maximize resident's welfare, Arnott and Grieson (1981) shows several results relating to that elasticity. (1) The level at which a good is taxed or subsidized depends on the proportion of the good purchased by residents. (2) The efficient level of the public good is higher, the larger the proportion of tax revenue raised from non-residents. (3) After accounting for equity concerns, the level of taxation on a particular good depends both on its income elasticity and the proportion of the good purchased by residents. As the theoretical and empirical literature implies, the responsiveness of cross-border shopping is important to determine the optimal rate of taxation and whether the state will optimally want to geographically differentiate its tax rate based on distance to the border.

Recently, several studies have focused precisely on the role of distance to a competing jurisdiction as a key variable of interest. Lovenheim (2008) studies how distance to the nearest low tax cigarette state relates to the demand elasticity of home state consumption. He finds that cigarette demand becomes more elastic to the home state price the farther individuals live from a low price cigarette border. Merriman (2010) collects a random sample of discarded cigarette packages in Chicago and using the tax stamp placed on cigarette cartons, he shows that many of the cigarette packages collected in Chicago come from outside of the city limits. Further, the likelihood of having an Indiana stamp (the low-tax neighboring state to Illinois) is decreasing in the distance from the Indiana border. Lovenheim and Slemrod (2010) studies the impact of having a neighboring county with a lower minimum legal drinking age on accident fatalities and find that only in locations further than 25 miles from such a jurisdiction see a reduction in drunk driving fatalities. The results from these papers clearly indicate that distance to the border shapes the responsiveness of individuals. These results together with the prediction of the theoretical results above provide a powerful argument for geographic differentiation of tax rates.

Although discriminatory taxation has been studied in the context of multiple tax bases (Janeba and Peters 1999; Keen 2001), relatively few papers have studied the geographic differentiation of taxes. One known exception is Nielsen (2010), which studies geographically differentiated taxes in the context of a Kanbur and Keen (1993) model. Nielsen (2010) differs from this paper by considering the problem in the context of revenue maximizing governments and only for high tax states. Some results are also stated for welfare maximizing governments, but these results require a separable relationship between revenue and private consumption. The simplifying assumption of revenue maximizing governments allows Nielsen (2010) to find a Stackelberg equilibrium with the foreign country as the leader.

In summary, the existing literature indicates that competition over one tax instrument will naturally result in differences in tax rates at borders. Given that these differences arise, the empirical and theoretical literature indicates that cross-border shopping is an
important factor to consider. However, the responsiveness of cross-border shopping arising from differentials at state borders is empirically not uniform in a state, resulting in large responses the closer the proximity to the border. In light of this, geographically differentiated taxes or tax-preferred zones may provide a mechanism for achieving a more efficient tax regime than a uniform rate. Although preferential tax regimes have been discussed in the literature, geographic preferential tax regimes are not as commonly discussed. This paper builds upon the model in Haufler (1996) because differences in public goods provision is a natural starting point for explaining how differences in tax rates arise across borders and, more importantly, because a model of welfare maximizing governments (without separable utility functions) will allow for the interaction of both public good and private consumption externalities within a federation. ${ }^{1}$

## 3 Model

In this section, I develop a model to evaluate the optimal commodity tax rate when the state government can differentiate the tax rate across local jurisdictions or regions. The goal of this model is to show that from a social welfare planner's perspective, the optimal sales tax in a state with multiple regions is not uniform.

This paper modifies the basic setup of Haufler (1996), which considers a two state model of cross-border shopping with public goods. Horizontal fiscal externalities play a large role in the model. When a state chooses a regional tax rate, cross-border shopping may occur and has the potential to change the revenue raised in another jurisdiction - creating a horizontal fiscal externality.

[^2]Figure 3.1: Geographic Layout of the Model

| State Left (L) |  | State Right (R) |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Town <br> Away <br> (A) | Town <br> Border <br> (B) | Town <br> Next <br> (N) | Town <br> Outer <br> (O) |

### 3.1 Setup of the Model

The model features two states located on a line segment and are indexed by $k=L, R$. Each state has two identical towns ${ }^{2}$ indexed $i=A, B, N, O$, where Town Away (A) and Town Border (B) are in State L. The geographic setting is depicted in Figure 3.1.

The production side of the economy is straightforward. Firms providing one private good are assumed to be located exogenously at any point where consumers purchase goods. The model assumes that firms are perfectly competitive and set price equal to marginal cost. The implication of perfect competition is that increases in the demand of a good in a particular town resulting from cross-border shopping will not alter the pre-tax price relative to the pre-tax price in another town. ${ }^{3}$ Therefore, the pre-tax price is normalized to one.

All four towns are identical in their geographic scope. A representative consumer who lives at the center of each town inhabits each town. She cannot migrate. Each representative consumer is endowed with $M$ dollars of income. The consumer has the choice to purchase a quantity of the consumption good, $c$, in her home town or in a neighboring town. Let superscripts on $c$ index the town that the person lives in and subscripts index the town that

[^3]the good is purchased in. Thus $c_{j}^{i}$ denotes the quantity of the consumption good that the resident of town $i$ purchases in town $j$. Let $c^{i}$ denote the total consumption of a resident of town $i$. It must be the case that $c^{i}=\sum_{j} c_{j}^{i}$ for all $i$ and that $c^{i}=c_{i}^{i}$ if $c_{j}^{i}=0$ for all $j .{ }^{4}$

The consumer has preferences over aggregate consumption and a publicly provided good, $G$. The functional form of the utility function is identical across all individuals. Preferences are given by the utility function $U^{i}\left(c^{i}, G\right)$, which is strictly quasi-concave in $c$ and $G$ and satisfies diminishing marginal utility.

### 3.2 The State Planner's Problem

In the model, taxes are levied according to the origin principle, which implies that if an individual crosses a border he will pay the tax to the jurisdiction of purchase. ${ }^{5}$ Each state government provides a public good from a tax on private good expenditures. There are two possibilities for the state - to provide the public good at the state or local level.

The paper assumes that states provide the public good at the state level, meaning that all the revenue raised in the state is aggregated to provide one public good that is uniform across all of the towns within the state. The state government sets a tax, $t_{i}$, on goods purchased in town $i$ to fund the public good. ${ }^{6}$ Under this scenario, define $G$ as the level of the state provided public good. Letting $R_{i}$ denote the total revenue raised in jurisdiction $i$ and letting $0 \leq \rho \leq 1$ denote the rate of transformation between revenues and expenditures, it will be that $G=\sum_{i=A, B} \rho R_{i}\left(t_{i}\right)$ for $k=L$. When the public good is provided at the state level, the planner is deciding the optimal public good provision for the entire state, thus

[^4]giving the optimal tax system for a centralized tax and spending problem. ${ }^{7}$ I assume that the production technology is unity, so then $\rho=1$.

Governments are assumed to be operating on the side of the Laffer curve such that marginal revenue is positive. The incidence of the tax is assumed to be fully passed through to the consumer.

State governments select the optimal tax rates for its two localities by maximizing the social welfare of its two residents. The government of State L chooses tax rates for Towns A and B, while the government of State R chooses the tax rates for Towns N and O. In the scenarios to follow, the tax rates in State R will be exogenously fixed, so I present the welfare function for one state only. ${ }^{8}$ Subject to government and individual constraints, State L sets tax rates by maximizing the utilitarian social welfare function $W_{k}$ :

$$
\begin{equation*}
W_{k}=\sum_{i=A, B} U^{i}\left(c^{i}, G\right) \text { for } k=L \tag{3.1}
\end{equation*}
$$

where $G$ is a state public good.
Note that because the utility function has diminishing marginal utility, satisfying $U^{\prime \prime}<0$ in $c$ and $G$, the social welfare function will also capture equity concerns. In the subsequent analysis, I assume that the social welfare functions are continuous and strictly quasi-concave in the strategies.

[^5]
### 3.3 The Consumer's Problem

The consumer has the choice to purchase $c$ in her home town or in a neighboring town. Transportation costs constrain consumers from purchasing goods in another jurisdiction. If the individual decides to purchase in the home town, she simply goes to the store at the point of the line corresponding to where she lives and no transportation costs are incurred. If shopping abroad, the individual faces a transportation cost function denoted $D_{i}\left(c_{j}^{i}\right)$ for $j$ not equal to $i .{ }^{9}$ I assume that the marginal cost of purchasing the first unit abroad is zero for residents of each town. This will guarantee that if taxes are not equal, the individual in the high tax town will immediately begin to purchase some of the goods from the low tax town. In addition, I assume that $D_{i}^{\prime}\left(c_{j}^{i}\right)>0$ and $D_{i}^{\prime \prime}\left(c_{j}^{i}\right)>0$ for $c_{j}^{i}>0$. Additionally, $D_{i}(0)=0$, $D_{i}^{\prime}(0)=0$ and $D_{i}^{\prime \prime}(0)>0$. These (strictly convex) assumptions on the transportation cost function guarantee that as the tax differential between the towns increase, the individual will purchase more of his consumption from the low tax town.

A convex transportation cost function can be justified by a composite consumption good that represents consumption goods that are heterogeneous in terms of the ease of their transportability. More importantly, a convex transportation cost makes the representative agent problem behave like a Hotelling-style model where consumers are aligned along a continuum. Convex transportation cost functions implicitly underlie models of cross-border shopping where agents live along a continuum and are heterogeneous in their distance to the border. The reason to use a convex transportation cost function is a modeling technique for mimicking a Kanbur and Keen (1993) model while also allowing for welfare maximizing governments with a representative agent. Although the literal implication of the function is that the marginal cost is increasing as the amount of cross-border shopping increases, the more intuitive way to think of this function is that it implicitly makes the representative

[^6]agent proxy for heterogeneous consumers located at different differences from the border who face increasing costs the further they live from the border.

Individuals must optimally choose how much of the consumption good to purchase at home and abroad. Because producer prices are equal in all jurisdictions, a resident of a high tax town will cross-border shop until the marginal benefit of doing so is equal to the marginal cost. In other words, cross-border shopping will equalize the marginal transportation cost and the marginal tax savings (benefit). As in Haufler (1996), it must be the case that in any equilibrium, the following consumer arbitrage condition holds:

$$
D_{i}^{\prime}\left(c_{j}^{i}\right)= \begin{cases}t_{i}-t_{j} & \text { for } t_{i}>t_{j}  \tag{3.2}\\ 0 & \text { for } t_{i} \leq t_{j}\end{cases}
$$

The function in Equation (3.2) implicitly defines the level of cross-border shopping $c_{j}^{i}$ :

$$
c_{j}^{i}\left(t_{i}-t_{j}\right)= \begin{cases}\left(D_{i}^{\prime}\right)^{-1} & \text { for } t_{i}>t_{j}  \tag{3.3}\\ 0 & \text { for } t_{i} \leq t_{j}\end{cases}
$$

Given the transportation cost function and an individual budget constraint, consumers will select how much of the consumption good to purchase at home and how much to purchase abroad given the differences in tax rates.

I must make assumptions that guarantee the second order conditions of the state's constrained maximization problem are fulfilled. Note that the first order conditions of the government's constrained maximization problem must be continuous when a jurisdiction switches from being the high-tax jurisdiction to a low tax jurisdiction. This will be the case if the transportation cost function, described below, is identical across all towns. ${ }^{10}$ Further note that the first order conditions must be concave in the strategies. This will be the case

[^7]if $D^{\prime \prime \prime}=0$ (the transportation cost function is quadratic) and if governments are on the left side of the Laffer Curve such that marginal revenue is positive. ${ }^{11}$ Continuous and concave first order conditions and quasi-concavity of the objective function will guarantee an interior solution.

## 4 Results

With the essential framework of the model established, I can now proceed to the predictions of the model. In the subsequent sections, I will detail what the optimal tax should be in a variety of scenarios.

The first scenario considers the case of all closed borders as a benchmark case. This case is presented in order to demonstrate that no horizontal fiscal externalities exist when an individual is constrained from purchasing goods abroad. This will be the case if the consumption tax is levied according to the designation principle rather than the origin principle. ${ }^{12}$

The second scenario considers what the optimal tax policy is when individuals can crossborder shop within a state, but cannot cross the state border. This case is provided for several reasons, most especially because some borders are effectively closed because of added costs to crossing those borders.

The final scenario considers what can happen when all borders are open. This is likely to be the case among member states in the European Union or in the United States. When crossing state borders is as easy as crossing town borders, the optimal tax policy is detailed for high and low tax states.

[^8]
### 4.1 The Case of Closed Borders

Consider a state that chooses tax rates for its towns but does not allow for residents to crossborder shop - either across the state border or the town border. This scenario is important because the borders are effectively closed to cross-border shopping if the the tax system is levied (and effectively enforced) under the destination principle.

Both states are identical so the optimum will be symmetric and I can consider the problem of one state below. Noting that because the border is closed, $c^{i}=c_{i}^{i}$, the individual budget constraints are given by the following equation for all $i$ :

$$
\begin{equation*}
\left(1+t_{i}\right) c^{i}=M \tag{4.1}
\end{equation*}
$$

The government budget constraint for the state provided public good is given by:

$$
\begin{equation*}
G=\sum_{i=A, B} t_{i} c^{i} \tag{4.2}
\end{equation*}
$$

The state government selects $t_{A}$ and $t_{B}$ to maximize (3.1) subject to the two constraints above. Denoting the marginal utility of consumption for an individual in town $i$ as $U_{C}^{i}$ and the marginal utility from public good provision as $U_{G}^{i}$, the first order conditions for this problem immediately imply that for all $i$ :

$$
\begin{equation*}
\sum_{i=A, B} U_{G}^{i}=U_{C}^{i} \tag{4.3}
\end{equation*}
$$

Proposition 1. If taxes are credibly levied according to the destination principle or if all borders are closed, the optimal tax system features uniform tax rates $\left(t_{A}=t_{B}\right)$ within a state.

Proof. Equation 4.3 will hold for $i$ equal to A and B. The implication of these two first order conditions is that $U_{C}^{A}=U_{C}^{B}$ across both towns, which in turn implies that total consumption is equal in both towns. Individuals in both towns are identical in terms of income, so it
immediately follows that the tax rates must be identical across all jurisdictions to equalize consumption.

Intuitively, the borders are closed and there is no reason for the government to differentiate the tax rate if all towns are identical. Differentiated tax rates would simply induce wasteful cross-border shopping as additional transportation costs are incurred. From a consumer choice perspective, the level of the public good is the same no matter the town of residence. The individual's only choice is over how much of the consumption good to purchase at home. The social welfare planner wants identical consumers to choose identical consumption bundles, and therefore, the welfare maximizing choice is is to equate the marginal utilities of consumption across all individuals.

If the public good provision is done locally, then the tax rates will also be uniform at an optimum. ${ }^{13}$

### 4.2 The Case of Open Borders Within a State

Now I consider the case where individuals can cross-border shop within the state but are prohibited from crossing the state border. Because the state border is closed, the optimal tax rates will be symmetric in both states and I can consider the case of only one state below. This case is considered for several reasons. First, international borders are much less easily crossed than state borders. For example, crossing the United States-Mexico border comes

[^9]for all towns within the state. State L will solve
\[

$$
\begin{equation*}
\max \sum_{i=A, B} U^{i}\left(c^{i}, g_{i}\right) \quad \text { for } k=L \tag{A.4.2}
\end{equation*}
$$

\]

The first order conditions for the maximization problem will imply

$$
\begin{equation*}
\frac{U_{g}^{i}}{U_{C}^{i}}=1 \tag{A.4.3}
\end{equation*}
$$

which is no different than the standard Samuelson Rule for the optimal provision of public goods in a locality. Individuals in both towns are identical; it immediately follows that the tax rates should be identical across all jurisdictions.
with added time costs along with some probability that customs agents will search for items being brought across the border. Exchange rates are often uncertain and conversion fees must be paid. ${ }^{14}$ The added costs and uncertainty may make crossing the border much more costly, perhaps effectively closing it for cross-border shopping. If this is case, the optimal tax system at international borders may be different compared to the solution when the border is open. Second, even within a federation, this scenario is highly relevant in terms of policy making because some state borders are effectively closed as a result of geographic barriers along borders. ${ }^{15}$ Third, it is also important to consider this case in order to build intuition for the case of completely open borders. In this case, the trade-offs that a state faces with a geographically differentiated tax rate begin to emerge.

Solving this problem requires specifying possible directions of cross-border shopping within a state. Because both towns within the state are identical, I can consider the case of cross-border shopping in one direction without loss of generality. For this purpose, assume $t_{A} \geq t_{B}$. Any cross-border shopping will occur by residents of Town A in the direction of Town B. From the budget constraints written below, one can observe that $D_{i}\left(c_{j}^{i}\right)$ is a deadweight cost.

The individual budget constraints are given by:

$$
\begin{array}{lc}
\text { town A: } & \left(1+t_{A}\right) c_{A}^{A}+\left(1+t_{B}\right) c_{B}^{A}+D_{A}\left(c_{B}^{A}\right)=M  \tag{4.4}\\
\text { town B: } & \left(1+t_{B}\right) c^{B}=M
\end{array}
$$

Recalling that I have assumed that Town A is the high tax town, the state budget constraint is given by:

$$
\begin{equation*}
G=t_{A} c_{A}^{A}+t_{B} c_{B}^{A}+t_{B} c^{B} \tag{4.5}
\end{equation*}
$$

Equation (3.2) will hold in equilibrium. Differentiating equation (3.3) using the inverse

[^10]function theorem and totally differentiating the individual budget constraints yields several derivatives necessary to derive the first order conditions. ${ }^{16}$

In this problem, because I have assumed the direction of cross-border shopping to originate from Town A, Equation (A.4.4) can be used to differentiate variables for Town A. Equation (A.4.5) can be used to differentiate variables for Town B.

Substituting in for the government constraint and accounting for the individual constraints when differentiating, ${ }^{17}$ the government of State L selects $t_{A}$ and $t_{B}$ to solve (3.1).

The first order conditions of this problem imply

$$
\begin{gather*}
t_{A}: U_{G}^{A}+U_{G}^{B}=\frac{U_{C}^{A} \frac{c_{A}^{A}}{1+t_{A}}}{M R_{A}}  \tag{4.6}\\
t_{B}: U_{G}^{A}+U_{G}^{B}=\frac{U_{C}^{B} \frac{c^{B}}{1+t_{B}}}{M R_{B}}+\frac{U_{C}^{A} \frac{c_{B}^{A}}{1+t_{A}}}{M R_{B}} \tag{4.7}
\end{gather*}
$$

where $M R_{A}=\frac{c_{A}^{A}}{1+t_{A}}-\frac{t_{A}}{D_{A}^{\prime \prime}}+\frac{t_{B}}{D_{A}^{\prime \prime}}$ and $M R_{B}=\frac{c^{B}}{1+t_{B}}+\frac{t_{A} c_{B}^{A}}{1+t_{A}}-\frac{t_{B}}{D_{A}^{\prime \prime}}+\frac{t_{A}}{D_{A}^{\prime \prime}}+c_{B}^{A}$ denote the marginal revenue from a change in the tax rate. ${ }^{18}$
${ }^{16}$ The derivatives used to solve the problem are presented below.
For the high tax town $\left(t_{i} \geq t_{j}\right)$ :

$$
\begin{array}{ccc}
\frac{\partial c_{j}^{i}}{\partial t_{i}}=\frac{1}{D_{i}^{\prime \prime}}>0 & \frac{\partial c_{i}^{i}}{\partial t_{i}}=-\frac{c_{i}^{i}}{1+t_{i}}-\frac{1}{D_{i}^{\prime \prime}}<0 & \frac{\partial c^{i}}{\partial t_{i}}=-\frac{c_{i}^{i}}{1+t_{i}}<0 \\
\frac{\partial c_{j}^{i}}{\partial t_{j}}=-\frac{1}{D_{i}^{\prime \prime}}<0 & \frac{\partial c_{i}^{i}}{\partial t_{j}}=-\frac{c_{j}^{i}}{1+t_{i}}+\frac{1}{D_{i}^{\prime \prime}} \gtreqless 0 & \frac{\partial c^{i}}{\partial t_{j}}=-\frac{c_{j}^{i}}{1+t_{i}}<0 \tag{A.4.4}
\end{array}
$$

For the low tax town $\left(t_{i}<t_{j}\right)$ :

$$
\begin{equation*}
\frac{\partial c^{i}}{\partial t_{i}}=-\frac{c^{i}}{1+t_{i}}<0 \quad \frac{\partial c^{i}}{\partial t_{j}}=0 \tag{A.4.5}
\end{equation*}
$$

${ }^{17}$ An alternative approach would be to use the indirect utility function rather than the direct utility function. I use the direct utility function because it allows me to see the direct effects on revenue and individual utility (public and private consumption effects) as discussed in the tax competition literature, following Haufler (1996) as a benchmark. Of course, both approaches yield the same result.
${ }^{18}$ If the public good is provided locally, 4.4 will still hold. However, the government budget constrain must now obey

$$
\begin{array}{cl}
g_{i}=t_{i} c_{i}^{i} & \text { if } t_{i} \geq t_{j} \\
g_{i}=t_{i} c^{i}+t_{i} c_{j}^{i} & \text { if } t_{i}<t_{j} \tag{A.4.6}
\end{array}
$$

for each respective town within the state. The state will solve A.4.2 and the first order conditions for the maximization problem will imply

$$
\begin{equation*}
t_{A}: \frac{U_{g}^{A}}{U_{C}^{A}}=\frac{\frac{c_{A}^{A}}{1+t_{A}}}{M R_{A}}-\frac{U_{g}^{B}\left(\frac{t_{B}}{D_{A}^{\prime \prime}}\right)}{U_{C}^{A} M R_{A}} \tag{A.4.7}
\end{equation*}
$$

Equations (4.6) and (4.7) imply optimization rules where the left hand side is the marginal benefit and the right hand side of the equation is the marginal cost of funds (or MCF). ${ }^{19}$ As in Dahlby and Wilson (1994), in a federation it must be the case that the marginal cost of funds is equal in all jurisdictions at an optimum. ${ }^{20}$ Here, the equality of the $M C F$ in a federation follows immediately from the first order conditions, which indicate that the sum of the marginal benefits of $G$ (the left hand side) are equal in both (4.6) and (4.7). This leads to Proposition 2.

Proposition 2. If the state border is closed or if the transportation cost function effectively closes the border, equal tax rates in Town $A$ and Town $B\left(t_{A}=t_{B}\right)$ is the socially optimal solution.

Proof. The first order conditions imply that the right hand side of (4.6) and (4.7) must be equal at an optimum. If $t_{A}=t_{B}, c_{B}^{A}=0$ because no cross-border shopping will occur. This implies that $c_{A}^{A}=c^{A}=c^{B}$ because incomes are identical. Given identical preferences, $U_{C}^{A}=U_{C}^{B}$. After substituting these equalities in the first order conditions, it is easy to see the right hand sides of the first order conditions above are equal if $t_{A}=t_{B}$.

In the case of closed state borders, the optimal tax rate from a state planner's perspective is characterized by equal tax rates. If the tax rates are not equal, the $M C F$ is different in each town and this is inefficient. Intuitively, the towns are identical and the state cares equally about both within-state towns. Differentiating the tax rate within a state will in-

$$
\begin{equation*}
t_{B}: \frac{U_{g}^{B}}{U_{C}^{B}}=\frac{\frac{c^{B}}{1+t_{B}}}{M R_{B}}-\frac{U_{g}^{A}\left(\frac{t_{A}}{D_{A}^{\prime \prime}}-\frac{t_{A} c_{B}^{A}}{1+t_{A}}\right)}{U_{C}^{B} M R_{B}}+\frac{U_{C}^{A} \frac{c_{B}^{A}}{1+t_{A}}}{U_{C}^{B} M R_{B}} \tag{A.4.8}
\end{equation*}
$$

where $M R_{A}=\frac{c_{A}^{A}}{1+t_{A}}-\frac{t_{A}}{D_{A}^{\prime \prime}}$ and $M R_{B}=\frac{c^{B}}{1+t_{B}}-\frac{t_{B}}{D_{A}^{\prime \prime}}+c_{B}^{A}$ denote the marginal revenue from a change in the tax rate. The only differences of these equations with the state public good first order conditions is the second term in both of the equations suggest the existence of differential effects on the public good provision in the neighboring town. Equal tax rates will still satisfy the conditions for an optimum.
${ }^{19}$ A large literature on the marginal cost of funds has emerged including Slemrod and Yitzhaki (1996) and Slemrod and Yitzhak (2001). For a complete overview of the literature, see Dahlby (2008).
${ }^{20}$ The intuition for this can be seen in an example. If the MCF is 1.5 in Town A but is 1.1 in Town B, then raising an additional dollar of revenue in Town B is less costly than raising the additional dollar in Town A. Raising additional revenue from Town B is welfare enhancing for the state even though it is not a Pareto improvement for Town B.
cur only wasted resources through the deadweight transportation cost, meaning that the transportation cost provides no utility or profits while the taxes are distorting consumption.

In order to gain intuition of the mechanisms at work, I will explain each of the terms in the first order conditions separately. Two components of the $M C F$ are already explained in the tax competition literature. I explain the intuitive meaning of these terms before proceeding to explain the externality terms that are not frequently discussed in the literature. All these effects become especially important in the following section.

As revenue is equal to the product of the tax rates and the tax bases, there are two distinct ways for a government to increase its revenue. One way is to expand the base, which in this model requires attracting additional cross-border shoppers by lowering the rate. The second way is the increase the tax rate, thereby increasing the revenue raised from those shoppers who continue to shop within the jurisdiction.

Letting $i$ denote the relatively high tax jurisdiction and $j$ denote the relatively low tax jurisdiction, terms $M R_{i}$ and $M R_{j}$ contain a $-\frac{t_{i}}{D_{i}^{\prime \prime}}$ and $-\frac{t_{j}}{D_{i}^{\prime \prime}}$ respectively. I will refer to this at the "tax base effect" or TBE. ${ }^{21}$ This is the change in revenue that jurisdiction $i$ can raise resulting from changes in the amount of cross-border shopping due to a tax rate change. The relationship between changes in the base and changes in the tax rate is negative because increases in the tax rate decrease the amount of consumption purchased within the town through reductions in cross border shopping. For the high tax jurisdiction, the effect depends on the transportation cost function for its own residents. For the low tax region, the effect depends on the transport function for residents of the adjacent high tax region.

A second effect is that term $M R_{j}$ contains a $c_{j}^{i}$ term. I will call this the "tax exporting effect" or TEE. ${ }^{22}$ The interpretation of this effect is that as more residents of $i$ cross the border to shop in $j$, the larger are the incentives of $j$ to raise its tax rate to extract additional revenue from non-residents. Thus, this effect moves in the opposite direction of the "tax base effect." Intuitively, this enters into the marginal revenue term because as the tax rate rises,

[^11]all else equal, this will help to generate additional revenue from the cross-border shoppers. The "tax exporting effect" is only present in the low tax town. The reason for this is that marginal changes in the tax rate in the high tax town do not export any of the tax burden because no cross-border shoppers purchase goods in the high tax town.

The TBE and TEE focus on the marginal revenues of the two equations. I will now explain the meaning of each additive term in the $M C F$. The first term of the $M C F$ in equations (4.6) and (4.7) with respect to $t_{A}$ and $t_{B}$ is standard in the tax competition literature - and would be the $M C F$ without any externalities. It represents the cost to jurisdiction $i$ of increasing revenue by an additional dollar through an increase in the tax rate of jurisdiction $i$. Taxpayers are altering their decisions as the tax rate changes, so this is a cost to society. This first term specifically measures the direct cost in jurisdiction $i$ of changing tax rate $i$. For this reason, I will call this the "within cost of funds" or $W C F$ for short.

The remaining terms are the results of the federalism present in the model. Equation (4.7) contains a second term in the $M C F$, which I call the "private consumption externality" or PCE. This term captures how changes in the low tax town's rate distort the consumption decisions of individuals in the high tax town. As the tax differential between the two towns becomes larger, the high tax town will have a larger fraction of its goods purchased abroad resulting in relative distortions to the consumption profile as the deadweight transportation cost adjusts. The social planner needs to account for this effect on consumption when choosing the tax rate in the low tax town. Notice that the choice of the high tax town's rate never affects the consumption mix of the low tax town because the residents of the low tax town always purchase consumption at home. Therefore, the $P C E$ is always zero in a high tax town. Note that as the tax rate changes smoothly, the $P C E$ also changes smoothly and has no discontinuous jumps.

If the public good is financed as a state public good, there is no explicit public good externality because a change in the tax rate of one town effects the public good provision of
the other town in a uniform manner. However, if the public good were a local public good, the effect on the revenue of the two towns will be different. The second term of Equations (A.4.7) and (A.4.8) capture the social welfare planner's concern for how changes in the tax rate of jurisdiction $i$ affects the revenue raising capacity of town $j$. The numerator represents the gain to jurisdiction $j$ from a change in the public good, while the denominator represents the cost to what town $i$ is giving up. Because this term has a negative sign in front, it implies an overall benefit to society, thus reducing the cost of raising funds in the jurisdiction. I refer to this term in these equations as the "public good effect" or $P G E$ because they demonstrate the planner's need to consider changes in the public good provision resulting from tax changes of within-state neighbors.

With respect to the $P G E$ in (A.4.7), changes in the high tax town's rate will directly affect the low town's ability to raise revenue through the "tax base effect" because the level of cross-border shopping responds to the difference between the two tax rates. Because this equation is for the town that was assumed to be the high tax town, there is no "tax exporting effect" from a change in the high tax rate in the numerator. With respect to the second term in (A.4.8), the social planner accounts for how changes in the low tax town's rate influences revenue in the high tax town. Here both the tax exporting and tax base effects influence how much the high tax town's revenue changes. Depending on the relative magnitudes, the $P G E$ may either raise or lower the $M C F$. The denominator of both equations contains $M R_{i}$ so that it represents the cost of raising an additional dollar in $i$.

The discussion in the last few paragraphs is designed to provide intuition for the pressures that equalize the tax rate in this example. The clarity of the effects will help to demonstrate the trade-offs facing the government. These effects will play a major role in the next section when I allow for the state border to open.

### 4.3 The Case of All Open Borders

Next, I consider the case where individuals can cross-border shop within the state and across state lines. All borders are open. In such a scenario, a state's decision will be affected by the level of the tax rate in the other state. I wish to show that the optimal tax rate is a function of the town's location. I derive the optimal tax rates assuming that State $R$ sets a fixed and uniform tax rate in Towns N and $\mathrm{O}\left(t_{N}=t_{O}=\bar{t}\right)$ and that this state does not respond competitively to geographic differentiation in the neighboring state. ${ }^{23}$ This assumption also allows for a more complete model of welfare maximizing governments to be considered. It is important to stress that the states are not simultaneously selecting local rates and State R does not react to State L's differentiated rates. Therefore, the equations that follow characterize the optimal response to a fixed tax rate rather than a Nash equilibrium.

To solve this problem, I introduce additional notation. Let $D_{i}$ still denote the transportation cost of crossing a border within a state. Let $S_{i}$ denote the transportation cost function for crossing a state border. For now, I operate under the assumption that $S_{i}=D_{i}$, but it is conceivable that crossing a state border may have additional costs (such as the presence of a toll at the border).

I assume that individuals are willing to travel at most one town to make cross-border purchases. This means that residents of Town A would be willing to shop either in their home town or the adjacent town, but they would never be willing to shop in Town N or $\mathrm{O} .{ }^{24}$

To solve this problem, it is important I specify the direction of cross border shopping in order to account for where goods are purchased. The cases to consider are whether State L is a high or low tax state. The first case is when State L is a relatively low tax state (and

[^12]Figure 4.1: Summary of Possible Cases

wants its tax rate at the border strictly less than State R). The second case to consider is when State L is a relatively high tax state (and wants its tax rate at the border strictly greater than State R). I assume that the states have these exogenously different tax rates, perhaps because of different preferences for public goods.

There are two sub-cases to consider. The sub-cases to consider are whether there is an optimum when the away town of State L sets higher rates than the border town and viceversa. Sub-case 1 will denote where State L's extremity town sets a higher rate than the interior town, while Sub-case 2 will denote the reverse of this. ${ }^{25}$

Given that a disparity exists in the tax rates across states, the problem considered here seeks to answer what the optimal solution is when the state can choose two tax rates. The four possible scenarios are presented in Figure 4.1.

This section considers the optimal tax with a uniformly provided state public good assuming no response of the neighboring state. The solutions to the cases are presented below.

Case Low: $t_{B} \leq t_{N}=t_{O}=\bar{t} \quad$ In this scenario, State R is a high tax state (Massachusetts), while State L (New Hampshire) has a preference for lower taxes. The problem facing New Hampshire's social planner is that she wishes determine the optimal combination of local tax rates given Massachusetts has selected a high fixed rate of $\bar{t}$. There are now two sub-cases.

Sub-Case 1: $t_{A} \geq t_{B}$. Note that sub-case 1 implies that tax rates are lower at the State border.

To solve this problem, the individual budget constraints for Town A and B and the government budget constraint are as follows:

$$
\begin{gather*}
\left(1+t_{A}\right) c_{A}^{A}+\left(1+t_{B}\right) c_{B}^{A}+D_{A}\left(c_{B}^{A}\right)=M \\
\left(1+t_{B}\right) c^{B}=M  \tag{4.8}\\
G=t_{A} c_{A}^{A}+t_{B} c_{B}^{A}+t_{B} c^{B}+t_{B} c_{B}^{N}
\end{gather*}
$$

State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (3.1)

[^13]subject to the constraints above. I can use Equations (A.4.4) and (A.4.5) to solve this problem. As in the case of a closed state border, the first order conditions can be arranged so that the marginal benefit $\left(U_{G}^{A}+U_{G}^{B}\right)$ is equal to the $M C F$, which immediately implies that the $M C F$ must be equal across all $i$. The following condition $\left(M C F_{A}=M C F_{B}\right)$ must hold at an optimum:
\[

$$
\begin{equation*}
\frac{U_{C}^{A} \frac{c_{A}^{A}}{1+t_{A}}}{M R_{A}}=\frac{U_{C}^{B} \frac{c^{B}}{1+t_{B}}}{M R_{B}}+\frac{U_{C}^{A} \frac{c_{B}^{A}}{1+t_{A}}}{M R_{B}} \tag{4.9}
\end{equation*}
$$

\]

where the marginal revenue from a change in the tax rate is denoted:

$$
\begin{gather*}
M R_{A}=\frac{c_{A}^{A}}{1+t_{A}}-\frac{t_{A}-t_{B}}{D_{A}^{\prime \prime}}  \tag{4.10}\\
M R_{B}=\frac{c^{B}}{1+t_{B}}+\frac{c_{B}^{A}}{1+t_{A}}-\frac{t_{B}-t_{A}}{D_{A}^{\prime \prime}}-\frac{t_{B}}{S_{N}^{\prime \prime}}+c_{B}^{N}
\end{gather*}
$$

Sub-Case 2: $t_{B} \geq t_{A}$. This sub-case implies that tax rates are higher at the border of the state.

The individual budget constraints for Town A and B and the government budget constraint are as follows:

$$
\begin{gather*}
\left(1+t_{A}\right) c^{A}=M \\
\left(1+t_{A}\right) c_{A}^{B}+\left(1+t_{B}\right) c_{B}^{B}+D_{B}\left(c_{A}^{B}\right)=M  \tag{4.11}\\
G=t_{A} c^{A}+t_{A} c_{A}^{B}+t_{B} c_{B}^{B}+t_{B} c_{B}^{N}
\end{gather*}
$$

Again, State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (3.1) subject to the constraints above. The first order conditions immediately imply that $M C F_{A}=M C F_{B}$ must hold at an optimum:

$$
\begin{equation*}
\frac{U_{C}^{A} \frac{c^{A}}{1+t_{A}}}{M R_{A}}+\frac{U_{C}^{B} \frac{c_{A}^{B}}{1+t_{B}}}{M R_{A}}=\frac{U_{C}^{B} \frac{c_{B}^{B}}{1+t_{B}}}{M R_{B}} \tag{4.12}
\end{equation*}
$$

where the marginal revenue from a change in the tax rate is denoted:

$$
\begin{gather*}
M R_{A}=\frac{c^{A}}{1+t_{A}}+\frac{c_{A}^{B}}{1+t_{A}}-\frac{t_{A}-t_{B}}{D_{B}^{\prime \prime}}  \tag{4.13}\\
M R_{B}=\frac{c_{B}^{B}}{1+t_{B}}-\frac{t_{B}-t_{A}}{D_{B}^{\prime \prime}}-\frac{t_{B}}{S_{N}^{\prime \prime}}+c_{B}^{N}
\end{gather*}
$$

Given that the marginal cost of funds must be equal in a federation, ${ }^{26}$ it is intuitive that the tax rate will be lower in the border zone if the right hand side of (4.9) is larger than the left hand side when the tax rates are equal in both towns. If this is true, it immediately implies that the social planner should raise the tax rate in Town A relative to Town B. If the right side of (4.12) is smaller than the left side when the tax rates are equal, the tax rate should be higher in the border region.

Case High: $t_{B} \geq t_{N}=t_{O}=\bar{t}$ This case implies that tax rates in State L are relatively high. This case guarantees that State L has a preference for higher taxes. Such a scenario asks what Massachusetts should do in response to New Hampshire's low tax rate of $\bar{t} .{ }^{27}$ The same two sub-cases as above must be considered. Because they were detailed above, I immediately set up the optimization problem.

## Sub-Case 1: $t_{A} \geq t_{B}$

The individual budget and government constraints for this model are as follows:

$$
\begin{gather*}
\left(1+t_{A}\right) c_{A}^{A}+\left(1+t_{B}\right) c_{B}^{A}+D_{A}\left(c_{B}^{A}\right)=M \\
\left(1+t_{B}\right) c_{B}^{B}+\left(1+t_{N}\right) c_{N}^{B}+S_{B}\left(c_{N}^{B}\right)=M  \tag{4.14}\\
G=t_{A} c_{A}^{A}+t_{B} c_{B}^{A}+t_{B} c_{B}^{B}
\end{gather*}
$$

State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (3.1) subject to the above constraints. The first order conditions are rewritten so that the following equation equating the marginal cost of funds across towns characterizes an optimum:

[^14]\[

$$
\begin{equation*}
\frac{U_{C}^{A} \frac{c_{A}^{A}}{1+t_{A}}}{M R_{A}}=\frac{U_{C}^{B} \frac{c_{B}^{B}}{1+t_{B}}}{M R_{B}}+\frac{U_{C}^{A} \frac{c_{B}^{A}}{1+t_{A}}}{M R_{B}} \tag{4.15}
\end{equation*}
$$

\]

where the marginal revenue from a change in the tax rate is denoted:

$$
\begin{gather*}
M R_{A}=\frac{c_{A}^{A}}{1+t_{A}}-\frac{t_{A}-t_{B}}{D_{A}^{\prime \prime}}  \tag{4.16}\\
M R_{B}=\frac{c_{B}^{B}}{1+t_{B}}+\frac{c_{B}^{A}}{1+t_{A}}-\frac{t_{B}-t_{A}}{D_{A}^{\prime \prime}}-\frac{t_{B}}{S_{B}^{\prime \prime}}
\end{gather*}
$$

Sub-Case 2: $t_{B} \geq t_{A}$
The individual budget and government constraints for this model are as follows:

$$
\begin{gather*}
\left(1+t_{A}\right) c^{A}=M \\
\left(1+t_{A}\right) c_{A}^{B}+\left(1+t_{B}\right) c_{B}^{B}+\left(1+t_{N}\right) c_{N}^{B}+D_{B}\left(c_{A}^{B}\right)+S_{B}\left(c_{N}^{B}\right)=M  \tag{4.17}\\
G=t_{A} c^{A}+t_{A} c_{A}^{B}+t_{B} c_{B}^{B}
\end{gather*}
$$

State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (3.1) subject to the above constraints. I can again use (A.4.4) and (A.4.5) to solve this problem. ${ }^{28}$ The first order conditions are rewritten so that the following equation equating the marginal cost of funds across towns characterizes an optimum:

$$
\begin{equation*}
\frac{U_{C}^{A} \frac{c^{A}}{1+t_{A}}}{M R_{A}}+\frac{U_{C}^{B} \frac{c_{A}^{B}}{1+t_{B}}}{M R_{A}}=\frac{U_{C}^{B} \frac{c_{B}^{B}}{1+t_{B}}}{M R_{B}} \tag{4.18}
\end{equation*}
$$

where the marginal revenue from a change in the tax rate is denoted:

$$
\begin{gather*}
M R_{A}=\frac{c^{A}}{1+t_{A}}+\frac{c_{A}^{B}}{1+t_{A}}-\frac{t_{A}-t_{B}}{D_{B}^{\prime \prime}}  \tag{4.19}\\
M R_{B}=\frac{c_{B}^{B}}{1+t_{B}}-\frac{t_{B}-t_{A}}{D_{B}^{\prime \prime}}-\frac{t_{B}}{S_{B}^{\prime \prime}}
\end{gather*}
$$

The optimal tax rate will be geographically differentiated if $M C F_{A} \neq M C F_{B}$ when the tax rates are equal. The exact conditions for geographic differentiation will be specified below.

[^15]The four possible cases above can be analyzed together, yielding Proposition 3 and 4 .

Proposition 3. For a low tax state with open borders, the optimal tax system depends on the relative size of the tax base effect and tax exporting effect. If, at the state border, the tax base effect is larger in absolute value than the tax exporting effect, then $t_{A}>t_{B}$ is the optimal response to a neighboring state's high tax rate. If the reverse is true, then $t_{A}<t_{B}$ is optimal. Only if the tax base effect is equal to the tax exporting effect is uniform taxation is optimal.

Proof. It must be the case that equation (4.9) or (4.12) holds at an optimum. Note that the limit of the left hand side as $t_{A} \rightarrow t_{B}$ of (4.9) converges to the left side of (4.12). The same is true for the right hand side, implying that the first order conditions are continuous when a state changes regimes from sub-case 1 to sub-case 2 . Starting from a point where all jurisdictions in State $L$ have equal tax rates $\left(t_{A}=t_{B}=t\right.$, I wish to show conditions under which $M C F_{A}-M C F_{B}=0$. Because the tax rates are equal at this starting point then $c_{j}^{i}=0$ for all $i \neq j$. Because no one from Town B crosses the state border, this immediately implies that $c^{A}=c^{B}=c_{A}^{A}=c_{B}^{B}=c$ and the marginal utilities of consumption are equal across both towns. Recall that residents of State R will cross-border shop so that $c_{B}^{N}>0$. Using the simplifications above, $M C F_{A}\left(t_{A}=t_{B}\right)-M C F_{B}\left(t_{A}=t_{B}\right)=\frac{\frac{c}{1+t}}{\left(\frac{c}{1+t}\right)}-\frac{\frac{c}{1+t}}{\left(\frac{c}{1+t}-\frac{t}{S_{N}^{\prime}}+c_{B}^{N}\right)}$. A little algebra will show that this expression is equal to zero if $c_{B}^{N}=\frac{t}{S_{N}^{\prime \prime}}$, implying equal tax rates. The tax system will be differentiated if $M C F_{A}\left(t_{A}=t_{B}\right) \neq M C F_{B}\left(t_{A}=t_{B}\right)$. Using the limit of equation (4.9) or (4.12) and the above substitutions, $M C F_{A}\left(t_{A}=t_{B}\right)-M C F_{B}\left(t_{A}=t_{B}\right)<$ 0 if $c_{B}^{N}<\frac{t}{S_{N}^{\prime \prime}}$. If the tax base effect is larger than the tax exporting effect, $M C F_{A}<M C F_{B}$ and the government must lower the tax rate in Town B relative to Town A to equalize the marginal cost of funds in the two jurisdictions. Therefore, $c_{B}^{N}<\frac{t}{S_{N}^{\prime \prime}}$ guarantees that $t_{A}>t_{B}$ is welfare improving from the case of equal tax rates. Similarly, it can be shown that $M C F_{A}>M C F_{B}$ if $c_{B}^{N}>\frac{t}{S_{N}^{\prime \prime}}$, which implies $t_{A}<t_{B}$ is optimal.

Intuitively, because the towns are located in the low-tax state, its residents do not have
any opportunity to shop in the other state. The consumption profiles are equal in both towns when the tax rates are equal. For this reason, differences in the marginal cost of funds will be determined solely by the relative magnitudes from any efficiency losses to the public good, which are driven entirely by the differences in marginal revenue. These differences in the marginal revenue result from the presence of the tax base and tax exporting effect in the border town.

Corollary 1. The tax exporting effect will equal the tax rate effect if the elasticity ( $\varepsilon$ ) of cross-state shopping $\left(c_{B}^{N}\right)$ with respect to the border town's tax rate $\left(t_{B}\right)$ is unit elastic. If $\varepsilon>1$ in absolute value, the tax base effect will dominate the tax exporting effect. The tax exporting effect dominates if $\varepsilon<1$ in absolute value.

Proof. The term of interest in the marginal revenue terms is $-\frac{t_{B}}{S_{N}^{N}}+c_{B}^{N}$. This can be rewritten as $t_{B} \frac{\partial c_{B}^{N}}{\partial t_{B}} \frac{c_{B}^{N}}{c_{B}^{N}}+c_{B}^{N}=(\varepsilon+1) c_{B}^{N}$, where $\varepsilon$ is the elasticity of $c_{B}^{N}$ with respect to $t_{B}$. $\varepsilon$ is negative.

Intuitively, if the cross-border shoppers are very price responsive, then small deviations down in the border town's tax rate will result in large quantities of cross-border shoppers and additional revenue gains from the other state. On the other hand, if consumers are inelastic with respect to the neighboring jurisdiction's tax rate, then the government can increase revenue by raising the tax rate and exporting the tax to foreign residents.

Therefore, on the low tax side, the optimal tax is geographically differentiated so long as the demand of the other state's resident's is not unit elastic. The pattern of geographic differentiation within a low tax state will hinge critically on the how responsive these crossborder shoppers are to changes in the price.

Now I consider the high tax side. On the high tax side, cross-border shopping occurs over the state line. This implies that the proof defining the conditions under which a geographically differentiated tax will have additional complications.

Proposition 4. For a high tax state with open borders, the optimal tax system depends on the
relative size of the tax base effect and the relative size of the marginal utility of consumption across the two towns. If the tax base effect is sufficiently large and differences in the marginal utility of consumption are sufficiently small $\left(U_{C}^{A} \approx U_{C}^{B}\right)$, then $t_{A}>t_{B}$ is the optimal response to a neighboring state's low tax rate. If the tax base effect is sufficiently small and differences in the marginal utility of consumption are sufficiently large ( $U_{C}^{A} \ggg U_{C}^{B}$ ), then $t_{A}<t_{B}$ is optimal. If the tax base effect exactly offsets the inequality in the differences in the marginal utility of consumption, then uniform taxation is optimal.

Proof. It must be the case that equation (4.15) or (4.18) holds at an optimum. Note that the limit of the left hand side as $t_{A} \rightarrow t_{B}$ of (4.15) converges to the left side of (4.18). The same is true for the right hand side, implying that these first order conditions are also continuous when a state changes regimes from sub-case 1 to sub-case 2. Starting from a point where all jurisdictions in State L have equal tax rates $\left(t_{A}=t_{B}=t\right.$, I wish to show conditions under which $M C F_{A}\left(t_{A}=t_{B}\right)-M C F_{B}\left(t_{A}=t_{B}\right)=0$. Because the tax rates are equal then $c_{j}^{i}=0$ for all $i \neq j$ in State L. However, some consumption from Town B is purchased across the state border so that $c_{N}^{B}>0$. This implies that $c^{A} \leq c^{B}, c_{A}^{A} \geq c_{B}^{B}$ and $U_{C}^{A} \geq U_{C}^{B}$. Simplifying, $t_{A}=t_{B}$ is optimal if $M C F_{A}\left(t_{A}=t_{B}\right)-M C F_{B}\left(t_{A}=t_{B}\right)=U_{C}^{A}-\frac{U_{C}^{B} \frac{c_{B}^{B}}{1+t}}{\left(\frac{c_{B}}{1+t}-\frac{t}{S_{B}^{\prime \prime}}\right)}=0$. The tax system will be differentiated if $M C F_{A}\left(t_{A}=t_{B}\right) \neq M C F_{B}\left(t_{A}=t_{B}\right)$. Using the limit of equation (4.15) or (4.18), a similar logic shows $M C F_{A}\left(t_{A}=t_{B}\right)-M C F_{B}\left(t_{A}=t_{B}\right)<0$ if $U_{C}^{A}<\frac{U_{B}^{B} \frac{c_{B}^{B}}{1+t}}{\left(\frac{c_{B}}{1+t}-\frac{t}{S_{B}^{\prime \prime}}\right)}$ and the government must lower the tax rate in Town B relative to Town A to equalize the marginal cost of funds in the two jurisdictions. Similarly, it can be shown that $U_{C}^{A}>\frac{U_{B}^{B} \frac{C_{B}^{B}}{1+t}}{\left(\frac{c_{B}}{1+t}-\frac{t}{S_{B}^{\prime \prime}}\right)}$ implies $t_{A}<t_{B}$. The conditions stated in the proposition above relate what is sufficient to make these inequalities hold true.

Intuitively, when a state sets higher taxes than its neighbors, the consumer in the border town will purchase some of his consumption abroad. This is bad for the state because of revenue leakage. But, when the tax rate in the two towns within a state are equal, residents of the border town may have more consumption than residents of the away town because of
the arbitrage opportunity that exists. The state planner may want lower taxes in the border region to decrease the revenue leakage from cross-border shopping. However, this will also raise consumption in the border town relative to the exterior town, furthering inequality. For this reason, the direction of the inequality for:

$$
\begin{equation*}
-\frac{t}{S_{B}^{\prime \prime}} \gtreqless \frac{\left(U_{C}^{B}-U_{C}^{A}\right)}{U_{C}^{A}} \frac{c_{B}^{B}}{1+t} \tag{4.20}
\end{equation*}
$$

will determine the relative pattern of geographic differentiation. ${ }^{29}$
Lowering taxes at the border will reduce the revenue leakage. This will be optimal if the right hand side of (4.20) is larger, keeping in mind that both sides are negative numbers. If the tax base effect is large in absolute value, then the probability that it is optimal to lower taxes in the border region is higher because individuals are very responsive. Inequalities in the marginal utilities of consumption - the right side of (4.20) - may be small depending on how concave the utility function is as well as how much of the gains to consumption are offset by the total transportation cost. If the tax base effect is significantly responsive and the consumption profiles in the two towns are similar (when the tax rates are equal) or if the utility function is not very concave so that even large differences in consumption do not concern the social planner, then a preferential tax rate near the border will be optimal.

However, if the resident of Town B has a lot more total consumption than the resident of Town A or if the utility function is very concave so that even small differences in consumption concern the social planner, it may be possible for tax rates to be higher in the border region. This will be especially true if the tax base effect is sufficiently small in absolute value. Intuitively, this scenario arises because the social planner cares about the equality of consumption between the two towns due to the concavity of the social welfare function. Thus if the tax base effect is sufficiently small, but the discrepancies in the marginal utility of consumption are large, the social planner will want to raise taxes in the border region.

[^16]Corollary 2. If the utility function is linear in consumption, i.e. has a marginal utility that is constant, and tax rates are non-zero, then $t_{A} \leq t_{B}$ is never optimal. Preferential tax rates in the border region will always be optimal.

Proof. If the marginal utility is constant, then $U_{C}^{A}=U_{C}^{B}$ because the utility functions are equivalent across all individuals even though the consumption profiles are different. To have $t_{A} \leq t_{B}$, then $M C F_{A}\left(t_{A}=t_{B}\right) \geq M C F_{B}\left(t_{A}=t_{B}\right)$ implies $1 \geq \frac{\frac{c_{B}^{B}}{1+t}}{\left(\frac{c_{B}}{1+t}-\frac{t}{s_{B}^{\prime \prime}}\right)}$. But, this requires $-\frac{t}{S_{B}^{\prime \prime}} \geq 0$, which is never true. However, $M C F_{A}\left(t_{A}=t_{B}\right)<M C F_{B}\left(t_{A}=t_{B}\right)$ is always true because $-\frac{t}{S_{B}^{\prime \prime}}<0$. Following the proof to Proposition 4, this immediately implies $t_{A}>t_{B}$ is optimal.

If the individual has a utility function that is linear in consumption (for example utility that is quasi-linear with respect to the consumption good), then the social planner does not care about how consumption is allocated across all individuals and the problem is similar to revenue maximizing governments. Because utility is no longer concave with respect to consumption, equity concerns vanish from the planner's problem. With no equity concerns, raising the tax rate near the border never is optimal because it will only increase the revenue leakage across state lines while also inducing a distortion within the state.

## 5 Extensions and Discussion

In this section, I illustrate how the state planner's solution to a decentralized problem (decentralized tax rates and pubic goods) will incorporate a public good externality. I also discuss revenue maximizing governments, the presence of income taxation, horizontal equity, and the possibility that the incidence is not fully passed through to the consumer.

### 5.1 Locally Provided (Town Differentiated) Public Goods

As a robustness check, this section considers the optimal tax with a locally provided state public good assuming no response of the neighboring state. The solution to this problem approximates the wishes of the state planner if the problem of taxing and public good decisions were decentralized to the towns. ${ }^{30}$

The main difference compared to the uniform public good case is that the PGE's must

[^17] constraint for each town in the optimization problem:
\[

$$
\begin{gather*}
g_{A}=t c_{A}^{A}+t_{A} c_{A}^{B} \\
g_{B}=t_{B} c_{B}^{A}+t_{B} c_{B}^{B}+t_{B} c_{B}^{N} \tag{5.1}
\end{gather*}
$$
\]

where $c_{A}^{B}=0$ for Sub-Case 1 and $c_{B}^{A}=0$ for Sub-Case 2. State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (4.2) subject to the the individual budget constraints in the previous section and the government budget constraints above. The first order conditions are rewritten such that they imply modified Samuelson rules:

$$
\begin{align*}
& t_{A}: \frac{U_{A}^{A}}{U_{C}^{A}}=\frac{\frac{c}{A}{ }^{\frac{1}{T} t_{A}}}{M R_{A}}-\frac{U_{G}^{B}\left(\frac{t_{B}}{S_{A}^{N}}-\frac{t_{B} C_{A}^{B}}{1+t_{B}}\right)}{U_{C}^{A} M R_{A}}+\frac{U_{C}^{B} \frac{C}{B}}{U_{C}^{A} M t_{B}}  \tag{5.2}\\
& t_{B}: \frac{U_{B}^{B}}{U_{C}^{B}}=\frac{\frac{C^{B}}{1+t_{B}}}{M R_{B}}-\frac{U_{G}^{A}\left(\frac{t_{A}}{D_{A}^{A}}-\frac{t_{A} C_{B}^{A}}{1+t_{A}}\right)}{U_{C}^{A} M R_{B}}+\frac{U_{C}^{A} \frac{C A}{1} \frac{A}{1}}{U_{C}^{B} M R_{B}}
\end{align*}
$$

where the marginal revenues from a change in the tax rate is denoted $M R_{A}=\frac{c^{A}}{1+t_{A}}-\frac{t_{A}^{\prime \prime}}{D_{B}^{\prime \prime}}+c_{A}^{B}$ and $M R_{B}=\frac{c^{B}}{1+t_{B}}-\frac{t_{B}}{D_{A}^{\prime \prime}}-\frac{t_{B}}{S_{N}^{\prime \prime}}+c_{B}^{N}+c_{B}^{A}$ and $c_{A}^{B}=0$ for Sub-Case 1 and $c_{B}^{A}=0$ for Sub-Case 2 in the above first order conditions.

Case High: $t_{B} \geq t_{N}=t_{O}=\bar{t}$
The individual budget constraints are unchanged from the uniform public good case, but the government now faces two constraints in each optimization problem:

$$
\begin{align*}
& g_{A}=t_{A} c_{A}^{A}+t_{A} c_{A}^{B}  \tag{5.3}\\
& g_{B}=t_{B} c_{B}^{A}+t_{B} c_{B}^{B}
\end{align*}
$$

where $c_{A}^{B}=0$ for Sub-Case 1 and $c_{B}^{A}=0$ for Sub-Case 2. Again, State L selects $t_{A}$ and $t_{B}$ taking as given and fixed $t_{N}=t_{O}=\bar{t}$ by maximizing (4.2) subject to the above constraints and the individual budget constraints. The first order conditions are rewritten such that they implied modified Samuelson rules:

$$
\begin{align*}
& t_{B}: \frac{U_{B}^{B}}{U_{C}^{B}}=\frac{\frac{c B}{B+t_{B}}}{M R_{B}}-\frac{U_{G}^{A}\left(\frac{t_{A}}{D_{A}^{A}}-\frac{t_{A} A_{B}^{A}}{1+t_{A}}\right)}{U_{C}^{B} M R_{B}}+\frac{U_{C}^{A} \frac{C A}{A}}{U_{C}^{B} M t_{A}} \tag{5.4}
\end{align*}
$$

where the marginal revenues from a change in the tax rate is denoted $M R_{A}=\frac{c^{A}}{1+t_{A}}-\frac{t_{A}}{D_{B}^{\prime \prime}}+c_{A}^{B}$ and $M R_{B}=$ $\frac{c_{B}^{B}}{1+t_{B}}-\frac{t_{B}}{D_{A}^{\prime \prime}}-\frac{t_{B}}{S_{B}^{\prime \prime}}+c_{B}^{A}$ and $c_{A}^{B}=0$ for Sub-Case 1 and $c_{B}^{A}=0$ for Sub-Case 2 in the above first order conditions.
now be considered when the state government equalizes the marginal cost of funds. The numerator of each $P G E$ (the second additive term in both first order conditions) represents the marginal benefit from the additional revenue that the town $j$ receives from a change in the tax rate of town $i$. The denominator represents the marginal revenue for the town whose tax rate is changing. In the case of a state public good, the effect of a tax change has the same effect on the revenue of both towns within the state because the level of public good provision was common to both states. However, when the public goods are different, the $P G E$ represents how much benefit is provided to town $j$ when the state planner wants town $i$ to raise additional revenue.

Therefore, when the state social planners solves the decentralized problem, the solution must consider the relative magnitude of the public good externality in addition to the sizes of the tax base and tax exporting effects. In this case, the conditions outlined in the propositions above will need to be modified.

### 5.2 Other Possibilities

In this section, I discuss how the results are similar or different under revenue maximizing governments, in the presence of multiple taxing instruments, and if the the full incidence of the tax is not passed through to consumers. I also discuss horizontal equity considerations briefly.

First, many of the results from the section on state public goods are applicable to revenue maximizing governments. Although the above results are derived under welfare maximizing governments, a large portion of the tax competition literature has relied on revenue maximizing governments. In the context of this model, revenue maximizing governments are characterized by the marginal utility of consumption being zero. Therefore, a model with governments as revenue maximizers is nested within the model presented above. To determine the optimum, the relevant terms to compare would be the marginal revenue terms. As such, the social planner would want to equalize the marginal revenue across both of the
towns, increasing the tax rates where the marginal revenue was largest. The results for the low tax side of the border would hold. The results for the high tax side of the border would simplify such that the state government will always want an unambiguously lower tax rate near the border. Thus, welfare maximizing governments accounts for additional considerations facing high tax states.

Second, many states utilize multiple tax instruments such as income taxes and sales taxes. ${ }^{31}$ This raises the question as to whether geographically differentiated sales taxes are optimal if a state can pick the sales tax rates along with an income tax rate. Intuitively, a state with a high preference for public goods could equalize its sales tax rate to the rate of the neighboring state and then assess a higher income tax. In the absence of migration, such a solution would eliminate any possibility of cross-border shopping while obtaining the desired level of public services. However, uniform sales tax rates across the states is rare. One reason for this is that sales tax revenues and income tax revenues are imperfect substitutes as states often seek to rely on multiple sources of income for reasons such as stabilizing revenues. Taking as given that these differences in the sales tax rate will exist and assuming that the state cannot geographically differentiate its income tax rate, then all of the above results will remain applicable.

Third, I assume that producers fully pass forward the tax to the consumer. This may be the case for the incidence of both local and state taxes. Furthermore, firms may adjust their prices depending on how far they are located from the border. Harding, Leibtag and Lovenheim (2010) find that the incidence of taxation varies depending on a firm's distance to the nearest low tax border. As such, firms may be behaving in a way similar to the social planner's geographic differentiation of tax rates in this paper. If the individual firms' revenues are affected in the same manner that government revenues are affected by the tax base and tax exporting effect, then geographic differentiation of tax rates will still be optimal. The degree of geographic differentiation in the tax rates will be different than if firms fully

[^18]passed the tax forward. This is because the firms are making some of the price adjustments that the social planner would do through the geographic differentiation of the tax rate. ${ }^{32}$

Finally, geographic differentiation of tax rates presents issues relating to horizontal equity. Although consumers in both towns have equal incomes, the tax burden that they face will vary depending on their residence. Such a violation of horizontal equity is a concern, however, it would also be present if the tax system were uniform within a state. Under uniform taxation, residents of equal incomes have heterogeneous opportunities for cross-border shopping in the neighboring state. The implication is that the uniform tax system will be horizontally unequal on the basis of some residents cross-border shopping while other residents will be unable to cross-border shop because of how far they reside from the border. So, although horizontal equity issues arise with geographic taxation, it is unclear as to which tax system is more horizontally equitable.

### 5.3 Discussion

The results in this paper very clearly demonstrate the role of the tax base effect and the tax exporting effect as well as possible externalities in this two region model. Also highlighted are the relative magnitudes the marginal costs of funds. Each of these effects are important to determining the direction of geographic differentiation.

If a state has a preference for high taxes (Massachusetts), it can never capture crossborder shoppers from the neighboring low tax state (New Hampshire). One option is to lower taxes at the border to reduce the distortion of the notch at the state border; the Massachusetts town closer to the border may set a lower tax rate. Intuitively, the only way New Hampshire affects Massachusetts is through a tax base effect and through a distortion of the consumption profile in the border town. Although differentiated taxation will cause some distortion between towns within the state, this process may help to equalize the $M C F$ across the two towns. Alternatively, a high tax state will want higher taxes at the border

[^19]than at its exterior if the differences in the consumption profiles resulting from unequal arbitrage opportunities are so significant that this inequality in consumption outweighs the distortions to revenue. The reason for this is that lowering taxes at the exterior relative to the border will introduce an additional distortion, but will help to smooth the consumption profiles of the two towns.

If a state has a preference for low tax rates (New Hampshire), the optimal response of the state depends on the relative size of the tax base and tax exporting effects at the border. Assuming the tax rate in New Hampshire is non-zero, then the state faces a trade-off. If it raises its rates at the border, it will lose some cross-border shoppers, while if it lowers its rates it will gain more shoppers but at a lower tax rate. When the tax base effect at the border is large, the expansion of the tax base from lowering the tax rate outweighs changes in the rate. If the tax exporting effect is large relative to the tax base effect, then this means that many residents from the high tax state cross the border. Thus, the government can "export" a large portion of its revenue raising capabilities to non-residents of the state by increasing the tax rate at the expense of losing relatively few consumers. When the tax base effect is large relative to the tax exporting effect, the optimal tax rate is lower in the border town. The relative magnitudes of the TBE and TEE depend critically on the elasticity of cross-border shopping. The tax base effect will be larger than the tax exporting effect if the elasticity of cross state shoppers with respect the the border town's tax rate is inelastic. If this elasticity is relatively elastic, the reverse is true.

The results above suggest that under most circumstances and utility functions, equal taxes within a state are likely to be a knife's edge choice. Therefore, although the model predicts conditions under which uniform taxation is optimal (for example, the tax base effect equaling the tax exporting effect in low tax states), the likelihood of this equality holding is unlikely and geographic differentiation will be optimal in most situations.

The comparison of the results with open and closed borders suggest that geographic differentiation need not be the only policy remedy to tax differentials at borders. Effectively
enforcing the tax system on the basis of the destination principle within a federation would be another way of eliminating the inefficiency from tax differentials at state borders. Such border inefficiencies may naturally be eliminated at international borders if border enforcement, exchange rate uncertainty and added time costs effectively close the border. Therefore, if country borders are effectively closed to cross-border shopping, uniform taxation will be optimal. For this reason, the optimal tax system may depend not only on the existence of tax differentials, but also on the precise nature of the border - and the ability of the planner to enforce taxes on the basis of the destination principle.

## 6 Conclusion

The model presented here suggests that when the tax system is characterized by a line resulting from geographic borders, uniform within-state taxation is not an optimal policy under most conditions. When a state government wishes to have higher or lower state tax rates than its neighbors - perhaps because of preferences for public good provisions - it is not optimal to levy a single rate. The reason is that the discontinuity in tax rates arising at the border encourage cross-border shopping. The welfare maximizing policy would be to allow for a geographically differentiated sales tax. This result is mostly consistent with the findings presented in Kleven and Slemrod (2009). Those two papers demonstrate that the closer goods are in their characteristics, the smaller the optimal tax rate differential should be. Goods purchased in New Hampshire and Massachusetts are unlikely to have any meaningful difference in the physical characteristics of the good. Instead the sole differentiating characteristic of the good is whether it originated from a high or a low tax jurisdiction. Here, such a result breaks down under certain circumstances and differences in the tax rates of similar goods may be magnified depending on the responsiveness of the tax base. The reason a state may want to increase the tax differential at a state border is to equalize consumption within a state or to export the tax burden to out of state residents.

The line in the tax system created at state borders induces welfare distortions with respect to consumption as some residents cross the border to purchase cheaper goods. This distortion is indicative of deadweight loss. It is also likely to create horizontal inequities if some residents cross the border while others do not. Even a uniform tax system will have horizontal inequities if some individuals purchase goods abroad. Nonetheless, the distortion on the consumer side is not the sole distortion resulting from line drawing. Tax-driven product innovation will also occur. Such innovations require no technological innovations. These innovations are a distortion of the characteristics of the goods to avoid tax payments. In the context of the retails sales tax, this innovation arises as firms distort their locational characteristic to the tax favorable side of the notch in order to capture cross-border shoppers. ${ }^{33}$ Absent the notch, the firm may have decided to locate just a small distance on the other side of the border. Instead, the innovation in the firm's location results in a socially distorted good (in its locational characteristics) that is provided particularly for private tax benefits.

The model presented above presents an administratively feasible tax system, where discrete changes in the tax system based on a jurisdiction's location is likely to be welfare enhancing. The discrete steps will induce additional notches within the tax system, but the state planner can utilize these differences to increase revenue or smooth consumption. Although shopping in a different town within the state incurs inefficiencies in transportation, it does not create revenue loss within the state, while also having the possibility of obtaining additional revenues from residents outside of the state. The result is a first order gain with a second order loss.

The results raise additional questions for future research. One, the role of added administrative complexity and enforcement underlies this model. How the optimal solution here varies if the government faces additional administrative costs from geographic differentiation is an important question to answer. Two, this paper assumes that there is no response of the neighboring jurisdiction. Although allowing for geographic differentiation on both sides

[^20]of the border results in a significant number of cases to consider, it would be an important area of continued research. Introducing tax competition with geographic differentiation in both states (perhaps with revenue maximizing governments), would provide important policy insights as to whether preferential tax treatment heightens or weakens tax competition incentives.

The simple results presented in this paper suggests that as states seek ways to increase revenue and as the European Union continues its process of integration, additional research on geographically differentiated taxes should be an important part of the research agenda for the future. The central implication of this study is that uniform taxation within a state border is sub-optimal when tax differentials exist and neighbors do not respond strategically. In the presence of these differentials, a state can improve the social welfare of residents through geographic differentiation of the tax rate or by successfully enforcing use taxes. The optimal tax system may depend not only on the existence of tax differentials, but also on the precise nature of the border. The optimal pattern of geographic differentiation applies to international borders so long as the cost of crossing these borders is not prohibitive of cross-border shopping. If the borders are prohibitive of cross-border shopping or if the use tax is effectively enforced, then uniform taxation is optimal. The principle of geographic differentiation within a state is likely to apply to other types of non-tax policies where similar distortions result from policy differentials at the border.

## References

Arnott, Richard, and Ronald E. Grieson. 1981. "Optimal Fiscal Policy for a State or Local Government." Journal of Urban Economics, 9(1): 23-48.

Atkinson, Anthony B., and Joesph E. Stiglitz. 1976. "The Design of Tax Structure: Direct versus Indirect Taxation." Journal of Public Economics, 6(1-2): 55-75.

Bucovetsky, Sam. 1991. "Asymmetric Tax Competition." Journal of Urban Economics, $30(2): 167-181$.

Dahlby, Bev. 2008. The Marginal Cost of Public Funds. Cambridge, MA:The MIT Press.

Dahlby, Bev, and Leonard S. Wilson. 1994. "Fiscal Capacity, Tax Effort, and Optimal Equalization Grants." Canadian Journal of Economics, 27: 657-672.

Davis, Lucas. 2010. "The Effects of Preferential VAT Rates Near International Borders: Evidence from Mexico." University of California, Berkeley Working Paper.

Edwards, Jeremy, and Michael Keen. 1996. "Tax Competition and Leviathan." European Economic Review, 40(1): 113-134.

Fox, William F. 1986. "Tax Structure and the Location of Economic Activity Along State Borders." National Tax Journal, 39(4): 387-401.

Harding, Matthew, Ephraim Leibtag, and Michael Lovenheim. 2010. "The Heterogeneous Geographic and Socioeconomic Incidence of Cigarette Taxes: Evidence from Nielsen Homescan Data." Working Paper.

Haufler, Andreas. 1996. "Tax Coordination with Different Preferences for Public Goods: Conflict or Harmony of Interest." International Tax and Public Finance, 3(1): 5-28.

Janeba, Eckhard, and Wolfgang Peters. 1999. "Tax Evasion, Tax Competition and the Gains From Nondiscrimination: the Case of Interest Taxation in Europe." The Economic Journal, 109(452): 93-101.

Kanbur, Ravi, and Michael Keen. 1993. "Jeux Sans Frontières: Tax Competition and Tax Coordination When Countries Differ in Size." American Economic Review, 83(4): 877892.

Kaplow, Louis. 2006. "On the Undesirability of Commodity Taxation Even When Income Taxation Is Not Optimal." Journal of Public Economics, 90(6-7): 1235-1250.

Keen, Michael. 2001. "Preferential Regimes Can Make Tax Competition Less Harmful." National Tax Journal, 54(4): 757-762.

Kleven, Henrik Jacobsen, and Joel Slemrod. 2009. "A Characteristics Approach to Optimal Taxation and Tax-Driven Product Innovation." London School of Economics and Political Science Working Paper.

Kotsogiannis, Christos, and Miguel-Angel Lopez-Garcia. 2007. "Imperfect competition, indirect tax harmonization and public goods." International Tax and Public Finance, 14(2): 135-149.

Lovenheim, Michael F. 2008. "How Far to the Border?: The Extent and Impact of CrossBorder Casual Cigarette Smuggling." National Tax Journal, 61(1): 7-33.

Lovenheim, Michael F., and Joel Slemrod. 2010. "Fatal Toll of Driving to Drink: The Effect of Minimum Legal Drinking Age Evasion on Traffic Fatalities." Journal of Health Economics, 29(1): 62-77.

Merriman, David. 2010. "The Micro-geography of Tax Avoidance: Evidence from Littered Cigarette Packs in Chicago." American Economic Journal: Economic Policy, 2(2): 61-84.

Mikesell, John L. 1970. "Central Cities and Sales Tax Rate Differentials: The Border City Problem." National Tax Journal, 23(2): 206-213.

Mintz, Jack, and Henry Tulkens. 1986. "Commodity Tax Competition Between Member States of a Federation: Equilibrium and Efficiency." Journal of Public Economics, 29(2): 132-172.

Nielsen, Søren Bo. 2001. "A Simple Model of Commodity Taxation and Cross-Border Shopping." The Scandinavian Journal of Economics, 103(4): 599-623.

Nielsen, Søren Bo. 2010. "Reduced Border-Zone Commodity Tax?" Working Paper.

Slemrod, Joel, and Shlamo Yitzhak. 2001. "Integrating Expenditure and Tax Decisions: The Marginal Cost of Funds and the Marginal Benefit of Projects." National Tax Journal, 54(2): 189-202.

Slemrod, Joel, and Shlamo Yitzhaki. 1996. "The Cost of Taxation and the Marginal Efficiency Cost of Funds." IMF Staff Papers, 43(1): 172-198.

Tosun, Mehmet S., and Mark L. Skidmore. 2007. "Cross-Border Shopping and the Sales Tax: An Examination of Food Purchases in West Virginia." B.E. Journal of Economic Analysis and Policy, 7(1): 1-18.

Trandel, Gregory. 1994. "Interstate Commodity Tax Differentials and the Distribution of Residents." Journal of Public Economics, 53(3): 435-457.

Walsh, Michael, and Jonathan Jones. 1988. "More Evidence on the Border Tax Effect: The Case of West Virginia, 1979-84." National Tax Journal, 41(2): 261-265.


[^0]:    I thank Joel Slemrod for encouraging me to pursue this project and for always providing me with excellent advice. I also wish to thank David Albouy, Lucas Davis, Dhammika Dharmapala, Michael Gideon, Makoto Hasegawa, James Hines, Ravi Kanbur, Michael Lovenheim (discussant), Ben Niu, Stephen Salant, and Caroline Weber for helpful suggestions and discussions. Suggestions from conference participants at the Michigan Tax Invitational and seminar participants in the Michigan Summer Seminar greatly improved the paper. Any remaining errors are my own.
    Submitted: February 24, 2011.

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[^2]:    ${ }^{1}$ If governments maximize revenue, then the problem presented below is simpler, but will not fully characterize the optimal solution because it will fail to account for consumption levels and consumption inequality across residents. Further, a model of revenue maximizing governments will be nested in the model I present.

[^3]:    ${ }^{2}$ The towns could be two equal sized regions. The word "town" need not imply a governing entity. Rather, the problem can be viewed as the state picking tax rates for two regions - one being the preferred tax region.
    ${ }^{3}$ The reason for this is that as consumer demand increases at a particular point along the line, free entry of firms will guarantee that price is equal to the marginal cost of production.

[^4]:    ${ }^{4}$ Thus, if no one purchases goods in another town, I could write either $c^{i}$ or $c_{i}^{i}$ as the town's consumption purchased within the jurisdiction. When all consumption goods are purchased at home, I will use $c^{i}$ to denote the consumption profile of the resident.
    ${ }^{5}$ The sales tax in the United States is levied de facto according to the origin principle. Technically speaking, the retail sales tax system is levied according to the destination principle. However, the use tax is notoriously under-enforced. Because the use tax is often evaded, taxes are implicitly paid based on the location of purchase rather than the destination of the sale.
    ${ }^{6}$ When the public good is provided at the state level, this means that the size of the tax base in one town is independent of the level of public good provision that the town receives. It is as if the state is aggregating all the revenue and distributing it equally to the two towns.

[^5]:    ${ }^{7}$ In a series of footnotes, I characterize the optimal solution if the public goods are provided at the local level and I show that the results are similar in spirit. If the public good is provided at the local level, the revenue raised in jurisdiction $i$ funds a public good for jurisdiction $i$ and has no benefit for people in other jurisdictions. Under state provided local public goods, the state government sets a tax, $t_{i}$, on goods purchased in town $i$ to fund the public good. If the revenue raised in one town is higher than the other, the state will provide that town with more of the public good. Letting $g_{i}$ denote the local public good and $R_{i}$ denote the total revenue raised in jurisdiction $i$, it will be that $g_{i}=\rho R_{i}\left(t_{i}\right)$. If this is the case, the social welfare function below will replace $G$ with $g_{i}$. When the public good is provided at the local level, the solution describes the state optimum to a decentralized problem (as if the state planner were choosing the rates and public good that localities would choose). The results for this type of public good will be discussed as a robustness check to the model presented. These results are provided in footnotes, but could be removed entirely or moved to an appendix to shorten the paper.
    ${ }^{8}$ The reason that the neighboring state is two towns rather than one is so that the jurisdictions are symmetric and differ only in the tax preferences. This way size does not play any role in the model.

[^6]:    ${ }^{9}$ The transportation cost is independent of distance because each individual lives at the center of the town. If they wish to purchase the good abroad, then they cross the border and shop at the first point across the line. Because cross-border shopping will be restricted to adjacent towns only, this is the same distance for all individuals.

[^7]:    ${ }^{10}$ Taking the limit as $t_{A} \rightarrow t_{B}$ of the first order conditions will show that the first order conditions are indeed continuous when switching from a high to low tax region.

[^8]:    ${ }^{11}$ These conditions can be derived by differentiating the first order conditions with respect to the strategies.
    ${ }^{12}$ Again, in the United States, the use tax is notoriously under-enforced. As a result, taxes are assessed implicitly under the origin principle.

[^9]:    ${ }^{13}$ If the public good is provided locally, 4.1 will still hold. However, the government budget constraint must now obey

    $$
    \begin{equation*}
    g_{i}=t_{i} c^{i} \tag{A.4.1}
    \end{equation*}
    $$

[^10]:    ${ }^{14}$ On the other hand, state borders in the United States and European Union are mostly crossed without such added costs.
    ${ }^{15}$ Many state borders are delineated by the presence of rivers or mountains, which effectively limit border crossings to areas where bridge-ways or major roadways exist. For policymakers in these states, the towns close to such a border essentially face a closed state border.

[^11]:    ${ }^{21}$ In the language of Mintz and Tulkens (1986) these are "public consumption effects."
    ${ }^{22}$ Mintz and Tulkens (1986) refer to this as a "private consumption effect."

[^12]:    ${ }^{23}$ Tax competition, especially with geographically differentiated rates on both sides of the border would add additional complexities and cases. The goal here is to derive the optimal tax rate rather than the equilibrium rate. Introducing tax competition would require additional simplifications that will eliminate the interaction of the effects presented below.
    ${ }^{24}$ This is a non-trivial assumption, which if relaxed would complicate the problem. Intuitively, if the assumption were relaxed, the degree of geographic differentiation would be changed. The solution would also depend on whether some residents begin to cross the state border as soon as a tax differential exists or whether they would face a particular cutoff benefit before cross-border shopping.

[^13]:    ${ }^{25}$ Both sub-cases include equal tax rates as a knife's edge case.

[^14]:    ${ }^{26}$ This follows from the first order conditions of the problem.
    ${ }^{27}$ Or alternatively this is interpreted as what New Hampshire should do if it switched to becoming the high tax state.

[^15]:    ${ }^{28}$ Note I need to add one more derivative to Equation (A.4.4) to solve this problem. This derivative can be obtained by totally differentiating Town B's individual budget constraint.

[^16]:    ${ }^{29}$ Both sides of the equation in the text are negative because $U_{C}^{A} \geq U_{C}^{B}$. The left hand side illustrates the revenue leakage through the tax base effect. The right hand side illustrates the social planner's concern for equalizing consumption across all residents.

[^17]:    ${ }^{30}$ Because of space considerations, I present the technical results in this footnote and discuss them in the text.

    Case Low: $t_{B} \leq t_{N}=t_{O}=\bar{t}$
    The individual budget constraints are unchanged, but the government now faces a government budget

[^18]:    ${ }^{31}$ The optimal tax schedule in the presence of multiple instruments is the subject of Atkinson and Stiglitz (1976) and Kaplow (2006).

[^19]:    ${ }^{32} \mathrm{~A}$ similar logic is true if some fraction of consumers purchase goods from the Internet, but some consumers cross-border shop.

[^20]:    ${ }^{33}$ The model presented in this paper did not allow for firms to choose their locational decisions. However, shopping patterns must be linked with retail locations.

