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A little bit nasty, some of the time: mixed strategy equilibria in political campaigns with continuous negativity

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Abstract

Despite the importance of negative campaigning in elections, few formal models have been developed to help understand its use. We develop a model in which candidates must decide what fraction of their campaigns will be negative. The candidates' initial reputations are then altered by the type of campaign each runs. Negative campaigning increases a candidate's probability of winning by lowering the opponent's reputation, but also decreases the candidate's own reputation. The winner is the candidate who ends the campaign with the higher reputation. Candidates care both about winning and about their reputations. Typically, no pure strategy equilibrium exists. Mixed strategies are over a continuous space of pure strategies, the degree of campaign negativity. One necessary condition in a mixed strategy equilibrium is that the only possible discontinuities in either candidate's distribution is at completely positive campaigns. Another is that the continuous part must be a connected interval over which the cdf is strictly increasing. Only sometimes are these conditions compatible; otherwise, the game has no equilibrium in pure or mixed strategies. When an equilibrium does exist, we present some comparative statics. When there is no equilibrium, we consider what happens in a discrete game which approximates the continuous strategic variable, fraction negative. In the finite game, an additional atom can exist at a point of significantly negative campaigning.

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A LITTLE BIT NASTY, SOME OF THE TIME: MIXED STRATEGY EQUILIBRIA IN POLITICAL CAMPAIGNS WITH CONTINUOUS NEGATIVITY

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Despite the importance of negative campaigning in elections, few formal models have been developed to help understand when and to what extent it will be utilized. In this paper, we develop a model in which candidates must decide what fraction of their campaigns will be negative. The candidates start with initial reputations which are then altered by the type of campaign each runs. Negative campaigning is effective in increasing a candidate's probability of winning the election by lowering the reputation of the opponent. However, a candidate who does more negative advertising also suffers a loss in reputation. This may be because of the opportunity cost of not enhancing his or her own reputation through positive advertising or because of a boomerang effect in which voters dislike candidates whose advertising is heavily negative. The winner of the election is the candidate who ends the campaign having the higher reputation. Candidates care both about winning and about their reputations.

Under reasonable parameter restrictions, no pure strategy equilibrium exists in this model. In the absence of pure strategy equilibria, the mixed strategies that candidates may utilize are specified over a continuous space of pure strategies, the degree to which each runs a negative campaign. These mixed strategies can be described by the cumulative distribution function over this variable. The cdfs may have a continuous part and points of discontinuities, which are equivalent to atoms in the pdf. We derive necessary conditions on these cdfs that must be satisfied in any equilibrium. These include that the only possible discontinuities in either candidate's distribution is at completely positive campaigns, and that the continuous part must be a connected interval over which the cdf is strictly increasing. In some circumstances, these conditions are compatible and an equilibrium does exist. In other cases, however, they are inconsistent and the game has no equilibrium, either in pure or mixed strategies.

When an equilibrium does exist, we derive some comparative statics of how the equilibrium varies with relevant parameters. First, if one candidate cares more about winning, the other runs a more positive campaign, and second, if voters are made very angry by negative campaigns, candidates will engage in more negative campaigning. Third, as the difference in the candidates' initial reputations becomes larger, both candidates will tend to run more positive campaigns. Finally, an increase in a candidate's initial reputation can reduce the candidate's expected equilibrium utility.

When there is no equilibrium, we consider what happens in a discrete game which approximates the continuous strategic variable by having a discrete set of the fraction of campaign negativity. In this finite game there is always an equilibrium, and we use Gambit to numerically solve for these equilibria. In the finite game, an additional atom can exist at a point of significantly negative campaigning. Finally, we use data on political advertising in the 2002 elections for U.S. congress to compare the equilibrium predictions from either the continuous model or the discrete approximation to actual campaigns.

1. Introduction

Negative campaigning is widely perceived to be an important aspect of political contests and to be growing in significance. Despite this, only a small number of models have been developed that analyze the decision by candidates to run positive or negative campaigns. These include Skaperdas and Grofman (1995), Harrington and Hess (1996), and Fletcher and Slutsky (2010), each of which focuses on a different aspect of negative campaigning.

Skaperdas and Grofman (1995) assume that there are three types of voters: undecideds and those initially favoring each of the two candidates. Undecided individuals shift to supporting one or the other of the two candidates with the division based upon how much positive campaigning each candidate does. Negative campaigning has no direct effect on undecideds but is undertaken to shift some of the opponent's supporters into being undecided. Due to a boomerang effect, it also loses some of the candidate's own supporters to the undecided status. They assume that these effects vary continuously with the actions taken, that there are diminishing returns to positive campaigning, and that candidates seek to maximize their net votes. They show that there is a pure strategy Nash equilibrium under these assumptions and derive properties of that equilibrium. Among these properties are that the front-runner (the candidate who initially has more supporters) does more positive and less negative campaigning than the opponent and that a candidate does more negative campaigning if the opponent has more initial supporters.

While Skaperdas and Grofman do not specify what campaigning (positive or negative) relates to, Harrington and Hess (1996) impose more structure on individual preferences and are more specific on what campaigning seeks to do. They assume that voters have preferences over both the issue positions of the candidates and over candidate attributes. The voters differ in their ideal points in issue space but have common perceptions of where each of the candidates is located in that issue space. Candidate attributes are valence issues such as character or competence which all voters value identically. The purpose of campaigning is to shift voters' perceptions of where candidates are in the issue space. Positive campaigning seeks to move the perceptions of the candidate's own position whereas negative campaigning seeks to move perceptions of the opponent's issue position. Campaigning of either type moves the issue positions continuously and with non-increasing returns. The candidates choose their mix of campaigning to maximize their vote share. They provide conditions for the existence of a pure strategy Nash equilibrium and show that the candidate who is stronger with respect to valence issues runs a more positive campaign than the opponent.

While candidates do campaign about policy issues as in Harrington and Hess, they also campaign about valence traits, for example, in a negative campaign, attacking an opponent's honesty or competence. Fletcher and Slutsky (2010) consider positive and negative campaigns in a context with only valence issues. They assume that candidates have reputations entering the campaign and that positive campaigns raise the candidates own reputation while negative campaigns lower the opponent's reputation. Unlike both Skaperdas and Grofman and Harrington and Hess, they assume that the decision of whether to run a positive or negative campaign is discrete and not continuous. That is, a campaign is either entirely positive or entirely negative with nothing in between. In addition, they assume that the winner is the candidate with the higher post-campaign reputation. Thus, again in contrast to the previous studies, they assume that campaign activities have a discontinuous effect on outcomes given that the candidates maximize the sum of their post-campaign reputations and a reputation bonus from winning. The combination of the discreteness and the discontinuous effects yields Nash equilibria in mixed strategies for a range of parameter values. They show that the candidates' expected payoffs are not monotonically increasing in pre-campaign reputations. When the campaign game is embedded as the second stage in a two stage model where pre-campaign reputations are chosen in the first stage, this non-monotonicity means that will often not choose the maximum possible reputation in the first stage.

This paper considers a model essentially the same as Fletcher and Slutsky (2010) but assumes that the decisions of how negative a campaign to run are continuous instead of being discrete. In some ways, this is clearly more realistic. Candidate ads are typically not all positive or all negative. Even with a continuous choice variable, the discontinuity in the election outcome as a function of their relative reputations implies that pure strategy Nash equilibria will still not exist for a wide range of parameters. This arises for similar reasons to the non-existence of pure strategy equilibria in a Bertrand pricing game where firms benefit by just undercutting their competitor's price. Here, a candidate wants to be just negative enough so that there is not a tie in post-campaign reputations. A characterization of the mixed strategy equilibrium in terms of properties of the candidates' cumulative distribution functions over degrees is derived. For a specific example, the mixed strategy equilibrium cdf's are explicitly derived.

DISCUSS NATURE OF RESULTS

The model is specified in Section 2. Pure strategy equilibria are considered in Section 3. The characterization of mixed strategy equilibria is presented in Section 4. An explicit example is derived in Section 5. The consistency of the results with campaign data from congressional and gubernatorial elections in the United States in 2002 is discussed in Section 6. Conclusions are given in Section 7.

2. The model

Consider an election between candidates F and T who enter the campaign with initial reputations X_F and X_T, which are based upon exogenous traits or pre-campaign actions. The candidates are labeled so that $X_F \geq X_T$, with F the initial frontrunner and T the initially trailing candidate. The candidates affect these reputations by the campaigns they run, which can vary in the degree to which they are positive or negative in orientation. Positive campaigning raises the reputation of the candidate running the campaign, while negative campaigning lowers the reputation of the opponent. Let f and t be the fractions of the campaigns of F and T, respectively, that are negative. The effects of the campaigns on the candidates' reputations are given by functions $\theta^I(i, j)$ where i, j = f, t, $i \neq j$, so that the post-campaign reputations are $X_F + \theta^F(f, t)$ for the frontrunner and $X_T + \theta^T(t, f)$ for the trailing candidate.

Assume that these functions are continuously differentiable in i and j and that:

$$\partial\theta^J/\partial i < \partial\theta^I/\partial i < 0 \tag{1}$$

Running a more negative campaign lowers the candidate's own reputation --- that is, $\partial\theta^I/\partial i < 0$ holds --- either because of the opportunity cost of reducing the positive campaigning that would have raised the candidate's own reputation, or because there is a "boomerang" effect where voters feel less positive about candidates who attack their opponents. That this effect is smaller in magnitude than the reduction in the opponent's reputation --- that is, $\partial\theta^J/\partial I$ --- means that negative campaigning is effective on the margin in raising the relative reputation of the candidate doing it.

In addition, we make the following boundary assumptions about $\theta^I(i, j)$:

$$\theta^I(0, 0) > 0, \;\; \theta^I(1, 1) < \theta^I(0, 1) < 0, \;\; \theta^I(0, 0) > \theta^I(1, 0) > \theta^I(0, 1) \qquad (2)$$

When both candidates run entirely positive campaigns, each candidate's reputation is greater than its pre-campaign level ($\theta^I(0, 0) > 0$). A candidate who runs an entirely negative campaign is successful in lowering the opponent's reputation whether the opponent's campaign is entirely positive or entirely negative ($\theta^I(0, 1) < 0$ and $\theta^I(1, 1) < 0$). The effect on a candidate's own reputation of running an entirely negative campaign against the opponent's entirely positive is ambiguous; $\theta^I(1, 0)$ could be positive or negative. The presence of a strong boomerang effect would tend to make this term negative.

The outcome of the election depends on the relative post-campaign reputations of the candidates $\Delta(f, t) = X_F + \theta^F(f, t) - X_T - \theta^T(t, f)$ where candidate F wins if $\Delta > 0$, T wins if $\Delta < 0$, and the candidates have an equal chance of winning if $\Delta = 0$. This is summarized by a function $h(\Delta)$ giving the probability that F wins, where $h(\Delta) = 1$ if $\Delta > 0$, $h(\Delta) = 0$ if $\Delta < 0$, and $h(0) = ½$. The candidate who wins the election receives a bonus to reputation, B_F or B_T; these are assumed to be strictly positive. Their goals are to maximize their expected post-election reputations $U^F(f, t) = X_F + \theta^F(f, t) + h(\Delta)B_F$ and $U^T(t, f) = X_T + \theta^T(t, f) + (1 - h(\Delta))B_T$.

We assume that there exist levels of f and t such that $\Delta(f, t) = 0$, as shown in Figure 1. Sufficient conditions for this are that any difference between X_F and X_T is sufficiently small and that θ^F and θ^T are sufficiently responsive to f and t and are sufficiently similar in their values. Given monotonicity of θ^I in f and t, the values of f and t for which $\Delta(f, t) = 0$ form a thin curve between points (0, t*) and (f*, 0). Denote the locus of such points as $i = m^I(j)$. That is, $\Delta(m^F(t), t) = 0$ for $t^* \leq t \leq 1$ or, equivalently, $\Delta(f, m^T(f)) = 0$ for $0 \leq f \leq f^*$.

To analyze the game, it is useful to convert it to the mixed extension in terms of the cumulative distribution functions of the players' mixed strategies. Let $\Gamma^I(i)$ be the cumulative distribution function for each player's mixed strategy. As a cdf, Γ^I is right-hand continuous with at most a countable number of discontinuities or atoms. At any point at which there is an atom, denote its probability magnitude as $\gamma^I(i)$. The expected payoff to candidate I against candidate J's mixed strategy would be:

$$EU^I(i, \Gamma^J) = X_I + E_j[\theta^I(i, j)] + B_I[\Gamma^J(m^J(i)) - \gamma^J(m^J(i)/2] \qquad (3)$$

A Nash equilibrium is a pair of cdfs such that neither candidate, given the other's cdf, can increase his own expected utility by changing his own cdf. In the next two sections, we derive some necessary conditions for such a Nash equilibrium.

3. Pure strategy equilibria

Lemma 1 shows that the set of possible pure strategy equilibria is limited.

Lemma 1: The only possible pure strategy equilibria are (a) $f = t = 0$ when $\Delta(0, 0) \neq 0$ and (b) $f = t = 1$ when $\Delta(1, 1) = 0$.

Proof: Under assumptions (1) and (2), a pair (f, t) is not a pure strategy equilibrium either if (i) $\Delta(f, t) \neq 0$ and either $f > 0$ or $t > 0$ or (ii) $\Delta(f, t) = 0$ and either $f < 1$ or $t < 1$. If (i) holds and $f > 0$, then f could be reduced slightly, increasing θ^F but not changing $h(\Delta)$, thereby raising F's post-election reputation. A similar argument holds if $t > 0$. If (ii) holds and $f < 1$, an arbitrarily small increase in f would cause an infinitesimal drop in θ^F but would raise $h(\Delta)$ from ½ to 1, causing a discrete increase in F's post-election reputation. A similar argument holds in this case when $t < 1$. Conditions (a) and (b) are the only remaining possibilities. Q.E.D.

Additional parameter restrictions rule out the only two possible pure strategy equilibria. First, assume that the winning bonus for T is sufficiently large relative to the direct reputation effect of running a negative campaign:

$$B_T > \theta^T(0, 0) - \theta^T(0, 1) \quad (4)$$

Starting from (0, 0), candidate T loses the election but would instead win it by running a sufficiently negative campaign (at least t*). The gain from doing this would be the extra winning bonus B_T, while the reputation loss from running a more negative campaign would be $\theta^T(0, 0) - \theta^T(0, t^*) < \theta^T(0, 0) - \theta^T(0, 1)$. Hence, under condition (4), regardless of the exact value of t*, candidate T would gain by raising t from 0 to above t*, ruling out the possible equilibrium in Lemma 1(a).

The second possible pure strategy equilibrium is ruled out if the candidates are not tied in reputation when they both run entirely negative campaigns or if the winning bonuses are not too big:

$$\Delta(1, 1) \neq 0 \text{ or } \theta^I(0, 1) - \theta^I(1, 1) > \tfrac{1}{2}B_I, \text{ for I either F or T} \quad (5)$$

Clearly, $\Delta(1, 1) \neq 0$ rules out the equilibrium in Lemma 1(b). This holds unless $X_F = X_T$ and $\theta^F(1, 1) = \theta^T(1, 1)$ or $X_F > X_T$ and asymmetries between $\theta^F(1, 1)$ and $\theta^T(1, 1)$ exactly balance out the difference in initial reputations. If $\Delta(1, 1) = 0$ does hold, then each candidate's post-election reputation would be $X_I + \theta^I(1, 1) + \tfrac{1}{2}B_I$. Either candidate could concede the election but gain in reputation by running a completely positive campaign. The post-election reputation of that candidate would then be $X_I + \theta^I(0, 1)$. When $\theta^I(0, 1) - \theta^I(1, 1) > \tfrac{1}{2}B_I$, the equilibrium in Lemma 1(b) is ruled out.

Note that conditions (4) and (5) are consistent with each other even though one imposes a lower bound on B_T while the other, in some circumstances, imposes an upper bound on that bonus. These bounds are consistent if, in response to an entirely negative campaign by F, there would be a large difference between the reputation T would achieve from an entirely negative campaign and an entirely positive one. Assuming both (4) and (5) rules out all pure strategy equilibria, leaving only mixed strategy equilibria possible.

4. Properties of equilibrium mixed strategies

The following seven lemmas specify some important properties that characterize any mixed strategy equilibrium. First, in Lemma 2, we show that the frontrunner puts no probability weight on very negative campaigns. The trailing candidate puts no probability weight on very positive campaigns, except for possibly having a mass of weight on being entirely positive.

Lemma 2: $\Gamma^F(1) = \Gamma^F(f^*) = 1$ and $\Gamma^T(t^* - \varepsilon) = \Gamma^T(0)$ for any $0 < \varepsilon < t^*$

Proof: For any t with $0 \leq t < t^*$ and any f, F wins the election so that $\Delta(f, t) = 1$ and $U^T(t, f) = X_T + \theta^T(t, f)$. Then, for any $\Gamma^F(f)$, $EU^T(t, \Gamma^F) = X_T + E_f[\theta^T(t, f)]$ which is strictly decreasing in t. Hence, $EU^T(0, \Gamma^F) > EU^T(t, \Gamma^F)$, any t, $0 < t < t^*$. This means that candidate T will not put any probability weight in the interval $(0, t^*)$ making $\Gamma^T(t^* - \varepsilon) = \Gamma^T(0)$, for any $\varepsilon > 0$ as asserted in the Lemma. Similarly, for any f with $f^* < f \leq 1$ and any t, F wins so that $\Delta(f, t) = 1$ and $U^F(f, t) = X_F + \theta^F(f, t) + B_F$. For any $\Gamma^T(t)$, $EU^F(f, \Gamma^T) = X_F + B_F + E_t[\theta^F(f, t)]$ which is strictly decreasing in f. Hence, $EU^F(f^*, \Gamma^T) > EU^F(f, \Gamma^T)$

so that F will put no probability weight on the interval (f*, 1] with $\Gamma^F(1) = \Gamma^F(f^*) = 1$ as asserted. Q.E.D.

Second, as shown in Lemma 3, at any point on the $\Delta = 0$ locus, at least one candidate does not have an atom.

Lemma 3: $\gamma^T(t)\, \gamma^F(m^F(t)) = 0$, any $t^* \leq t \leq 1$.

Proof: Assume that there exist a t' and an f' with $t' = m^T(f')$ and $\gamma^T(t') > 0$. Then $\Delta(f', t) = 1$ if $t < t'$, $\Delta(f', t') = ½$, and $\Delta(f', t) = 0$ if $t > t^*$. Hence, the expected payoff to F when playing f' is:

$$EU^F(f', \Gamma^T) = X_F + E_t[\theta^F(f', t)] + B_F(\Gamma^T(t') - \gamma^T(t')/2).$$

Consider F playing a strategy slightly more negative, $f' + \varepsilon$. Let $\delta = m^T(f' + \varepsilon) - t'$. Because a cdf has only a countable number of atoms, a sequence of ε going to 0 can always be chosen such that $\gamma^T(t' + \delta) = 0$ for every δ in the sequence. Then, F's expected utility at $f' + \varepsilon$ is:

$$EU^F(f'+ \varepsilon, \Gamma^T) = X_F + E_t[\theta^F(f' + \varepsilon, t)] + B_F\Gamma^T(t' + \delta).$$

Since Γ^T is right hand continuous and θ^F is continuous, $\lim_{\varepsilon\to 0}[\Gamma^T(t' + \delta) - \Gamma^T(t')] = 0$ and $\lim_{\varepsilon\to 0}[E_t[\theta^F(f' + \varepsilon, t)] - E_t[\theta^F(f', t)]] = 0$. Hence, $\lim_{\varepsilon\to 0}[EU^F(f'+ \varepsilon, \Gamma^T) - EU^F(f', \Gamma^T)] = B_F\gamma^T(t')/2 > 0$. Thus, F's expected payoff at f' is strictly less than at nearby f so that gF(f') = 0 must hold, yielding $\gamma^T(t')\gamma^F(f') = 0$ as asserted. Q.E.D.

Third, as shown in Lemma 4, if one candidate's cdf has an atom at some strategy i', then the other candidate must put no probability weight in some interval just below the corresponding strategy on the $\Delta = 0$ locus, $m^J(i')$.

Lemma 4:

(a) If $\gamma^F(f') > 0$, any f' with $0 < f' \leq f^*$, then there exists some $\delta' > 0$ such that $\Gamma^T(m^T(f')) = \Gamma^T(m^T(f') - \delta')$

(b) If $\gamma^T(t') > 0$, any t' with $t^* < t' \leq 1$, then there exists some $\delta' > 0$ such that $\Gamma^F(m^F(t')) = \Gamma^F(m^F(t') - \delta')$

Proof:

(a) T's expected payoff at $t' = m^T(f')$ is:

$$EU^T(t', \Gamma^F) = X_T + E_f[\theta^T(t', f)] + B_F(\Gamma^F(f') - \gamma^F(f')/2).$$

Consider values of $t = t' - \delta$, for values of $\delta > 0$. At such values, T's expected payoff is:

$$\gamma^F(m^F(t' - \delta))\text{where } f' - \varepsilon = m^T(t' - \delta).$$

Since Γ^F has at most a countable number of atoms, if $\gamma^F(f' - \varepsilon) > 0$ for some ε, then there are arbitrarily close values of $t > t' - \delta$ at which $\gamma^F(m^T(t)) = 0$. T's expected payoff at those values of t is strictly greater than at $t' - \delta$. Hence, T puts no probability weight at any $t' - \delta$ at which $\gamma^F(m^F(t' - \delta)) > 0$. Then consider a $t' - \delta$ at which $\gamma^F(m^T(t' - \delta))= 0$:

$$EU^T(t', \Gamma^F) - EU^T(t' - \delta, \Gamma^F) = E_f[\theta^T(t', f) - \theta^T(t' - \delta, f)] + B_T[\Gamma^F(m^T(t')) - \Gamma^F(m^T(t' - \delta)) - \gamma^F(m^T(t'))/2].$$

For any sequence of δ going to 0, $\theta^T(t' - \delta, f)$ converges to $\theta^T(t', f)$ since $\theta^T(t', f)$ is continuous. Since Γ^F is right hand continuous but has an atom at $f' = m^T(t')$, as δ converges to 0, $\Gamma^F(m^T(t')) - \Gamma^F(m^T(t' - \delta))$ converges to $\gamma^F(m^T(t'))$. Hence, $\lim_{\delta\to 0}[EU^T(t', \Gamma^F) - EU^T(t' - \delta, \Gamma^F)] = B_T\gamma^F(f')/2 > 0$. There must then exist some δ' such that $EU^T(t', \Gamma^F) > EU^T(t' - \delta, \Gamma^F)$, all $0 < \delta < \delta'$. T will put no probability weight in the interval $[t' - \delta', t']$. Since $\gamma^T(t') = 0$ from Lemma 3, $\Gamma^T(t') = \Gamma^T(t' - \delta')$ as asserted.

(b) The proof follows identically to that in (a), switching the roles of F and T. Q.E.D.

Fourth, as shown in Lemma 5, if one candidate has an interval in which no probability weight is placed, then the other candidate places no probability weight in the corresponding interval on the $\Delta = 0$ locus. **See Figure 2 for an example of this.**

Lemma 5: If there exists an interval (a, b) with $0 < a < b < f^*$, and $\Gamma^F(b - \varepsilon) = \Gamma^F(a)$ for all $0 < \varepsilon < b - a$, then $\Gamma^T(m^T(b - \varepsilon)) = \Gamma^T(m^T(a))$.

Proof: Consider any t with $m^T(b) > t > m^T(a)$. Then $EU^T(t, \Gamma^F) = X_T + E_f[\theta^T(t, f)] + B_T\Gamma^F(a)$. From (i), this is decreasing in t. Hence, T will put no probability weight on t in the given interval. Q.E.D.

Fifth, as shown in Lemma 6, neither candidate has an atom on the interior of the intervals in which they may place probability weight, $0 < f < f^*$ for F and $t^* < t < 1$ for T.

Lemma 6: $\gamma^F(f) = 0$ for all f with $0 < f < f^*$ and $\gamma^T(t) = 0$ for all t with $t^* < t < 1$.

Proof: Assume that the Lemma is not true and that there exists an f', $0 < f' < f^*$ with $\gamma^F(f') > 0$. Then from Lemma 3, $\gamma^T(m^T(f')) = 0$ must hold. From Lemma 4, $\Gamma^T(m^T(f') = \Gamma^T(m^T(f' - \delta))$ for some $\delta > 0$. Then $EU^F(f', \Gamma^T) = X_F + E_t[\theta^F(f', t)] + B_T\Gamma^T(m^T(f'))$ and $EU^F(f' - \varepsilon, \Gamma^T) = X_F + E_t[\theta^F(f' - \varepsilon, t)] + B_T\Gamma^T(m^T(f'))$ and $EU^F(f' - \varepsilon, \Gamma^T) - EU^F(f', \Gamma^T) = E_t(\theta^F(f' - \varepsilon, t) - \theta^F(f', t)] > 0$. This contradicts F being willing to put probability weight at f', so $\gamma^F(f') = 0$ must hold as asserted. A similar contradiction follows if $\gamma^T(t') > 0$, some $t^* < t' < 1$ is assumed. Q.E.D.

Next, as shown in Lemma 7, neither candidate has an interval with no probability weight in the relevant range, that is, each cdf is strictly increasing over that range.

Lemma 7: There exist f' and t', with $0 < f' < f^*$, $t^* < t' < 1$, and $t' = m^T(f')$ such that:

(i) $\Gamma^F(f') = 1$ and $\Gamma^F(b) > \Gamma^F(a)$ for any $0 < a < b < f^*$,and

(ii) $\Gamma^T(t') = 1$ and $\Gamma^T(b) > \Gamma^T(a)$ for any $t^* < a < b < 1$.

Proof: Since there does not exist a pure strategy equilibrium, not all probability weight can be at an atom at 0. From Lemma 2, F must have probability weight in the interval $(0, f^*)$ and from Lemma 6, none of this is at an atom. Assume that there is an interval $(a, b) \in (0, f^*)$ with $\Gamma^F(b) = \Gamma^F(a)$ but $\Gamma^F(b + \delta) > \Gamma^F(b)$, for any small $\delta > 0$. From Lemma 5, $\Gamma^T(m^T(b)) = \Gamma^T(m^T(a))$ must also hold. Then:

$$EU^F(b + \delta, \Gamma^T) = X_F + E_t[G^F(b + \delta, t)] + B_F\Gamma^T(m^T(b + \delta)),$$ while at a,

$$EU^F(a, \Gamma^T) = X_F + E_t[G^F(a, t)] + B_F\Gamma^T(m^T(a)).$$

Since $\lim \Gamma^T(m^T(b + \delta)) = \Gamma^T(m^T(b)) = \Gamma^T(m^T(a))$, then, given (1):

$$\lim_{\delta\to 0}[EU^F(b + \delta, \Gamma^T) - EU^F(a, \Gamma^T)] = E_t[G^F(b, t)] - E_t[G^F(a, t)] < 0.$$

This contradicts F placing probability weight above b. There must thus exist an f', $0 < f' \leq f^*$, with $\Gamma^F(f)$ strictly increasing between 0 and f'. Q.E.D.

Finally, as shown in Lemma 8, the only possible atoms for each candidate are at completely positive campaigns.

Lemma 8: The only atom for F is at $f = 0$ with $\gamma^F(0) \geq [E_{\tilde{f}}[\theta^T(0, \tilde{f})] - E_{\tilde{f}}[\theta^T(t^*, \tilde{f})]] / B > 0$. The only possible atom for T is at $t = 0$.

Proof: From Lemmas 6 and 7, $\Gamma^F(f)$ is continuous and increasing over $(0, f')$ and $\Gamma^T(t)$ is continuous and increasing over (t^*, t'). Combining this with Lemma 4, F cannot have an atom at f' and T cannot have an atom at t'. Thus, the only possible atom for F is at $f = 0$ and the only possible atoms for T are at $t = 0$ or $t = t^*$. The expected utility for T must be at least as high at $t^* + \varepsilon$ as at 0 since T puts probability weight near t^*, from Lemma 7. Hence, for all $\varepsilon > 0$, $X_T + [E_{\tilde{f}}[\theta^T(t^* + \varepsilon, \tilde{f})] + B^T\Gamma^F(m^F(t^* + \varepsilon)) \geq X_T + E_{\tilde{f}}[\theta^T(0, \tilde{f})]$. Since $\lim_{\delta\to 0} \Gamma^F(m^F(t^* + \varepsilon)) = \gamma^F(0)$, this yields the given bound on $\gamma^F(0)$. Since F has an atom at 0 then T cannot have one at t^* from Lemma 3. Q.E.D.

Building on these results, a more complete characterization of the cdfs for the two candidates is given in Theorem 1. As pointed out below, this is not, however, an explicit solution for them.

Theorem 1: There exist f' and t' with $\Delta(f', t') = 0$ such that the equilibrium cdfs satisfy the following:

$$\Gamma^F(f) = 1 + [E_{\tilde{f}}[\theta^T(t', \tilde{f})] - E_{\tilde{f}}[\theta^T(m^T(f), \tilde{f})]]/B_T, \; 0 \le f \le f'$$

$$\Gamma^F(f) = 1, \; f' \le f \le 1$$

$$\gamma^F(0) = 1 + \left[E_{\tilde{f}}[\theta^T(t', \tilde{f})] - E_{\tilde{f}}[\theta^T(t^*, \tilde{f})]\right]/B_T$$

$$\Gamma^T(t) = \gamma^T(0), \; 0 \le t \le t^*$$

$$\Gamma^T(t) = 1 + [E_{\tilde{t}}[\theta^F(f', \tilde{t})] - E_{\tilde{t}}[\theta^F(m^F(t), \tilde{t})]]/B_F, \; t^* \le t \le t'$$

$$\Gamma^T(t) = 1, \; t' \le t \le 1$$

$$\gamma^T(0) = 1 + \left[E_{\tilde{t}}[\theta^F(f', \tilde{t}) - E_{\tilde{t}}[\theta^F(0, \tilde{t})]\right]/B_F$$

In addition, one of the following must hold:

(a) $E_{\tilde{f}}[\theta^T(t', \tilde{f})] = E_{\tilde{f}}[\theta^T(0, \tilde{f})] - B_T$ and $E_{\tilde{t}}[\theta^F(m^F(t'), \tilde{t})] \ge E_{\tilde{t}}[\theta^F(0, \tilde{t})] - B_F$,

or

(b) $E_{\tilde{t}}[\theta^F(m^F(t'), \tilde{t})] = E_{\tilde{t}}[\theta^F(0, \tilde{t})] - B_F$ and $E_{\tilde{f}}[\theta^T(t', \tilde{f})] \ge E_{\tilde{f}}[\theta^T(0, \tilde{f})] - B_T$

Proof: For candidate F to be willing to put probability weight in the interval [0, f'] consistent with Lemmas 7 and 8, EU^F must be constant on that interval. That is:

$$EU^F(f, \Gamma^T) = X_F + E_{\tilde{t}}[\theta^F(f, \tilde{t})] + B_F\Gamma^T(m^T(f)) = C_F,\ 0 \le f \le f'.$$

Solving yields:

$$\Gamma^T(m^T(f)) = [C_F - X_F - E_{\tilde{t}}[\theta^F(f, \tilde{t})]] / B_F,\ 0 \le f \le f'.$$

Substituting $t = m^T(f)$ and $f = m^F(t)$ yields:

$$\Gamma^T(t) = [C_F - X_F - E_{\tilde{t}}[\theta^F(m^F(t), \tilde{t})]] / B_F,\ t^* \le t \le t'.$$

Since $\Gamma^T(t') = 1$ from Lemma 7, $C_F = B_F + X_F + E_{\tilde{t}}[\theta^F(m^F(t'), \tilde{t})]$, which gives:

$$\Gamma^T(t) = 1 + [E_{\tilde{t}}[\theta^F(m^F(t'), \tilde{t})] - E_{\tilde{t}}[\theta^F(m^F(t), \tilde{t})]] / B_F,\ t^* \le t \le t'.$$

From Lemma 2, $\Gamma^T(t^* - \varepsilon) = \Gamma^T(0)$, all $0 < \varepsilon < t^*$. Since T can only have an atom at 0, $\gamma^T(0) = \Gamma^T(t^*)$.

For T to be willing to put probability everywhere in the interval [t*, t'], EU^T must be constant on that interval with:

$$EU^T(t, \Gamma^F) = X_T + E_{\tilde{f}}[\theta^T(t, \tilde{f})] + B_T\Gamma^F(m^F(t)) = C_T,\ t^* \le t \le t'.$$

Solving yields:

$$\Gamma^F(m^F(t)) = [C_T - X_T - E_{\tilde{f}}[\theta^T(t, \tilde{f})]] / B_T,\ t^* \le t \le t'.$$

Since Γ^T is constant on [t', 1], then from Lemma 5, Γ^F is constant on [f', 1] with $f' = m^F(t')$. Then $\Gamma^F(f') = 1$ must hold. Using this to solve for C_T and substituting into the expression for $\Gamma^F(m^F(t))$ yields:

$$\Gamma^F(m^F(t)) = 1 + [E_{\tilde{f}}[\theta^T(t', \tilde{f})] - E_{\tilde{f}}[\theta^T(t, \tilde{f})]] / B_T,\ t^* \le t \le t'.$$

To tie down the value of f' and t', three possibilities exist: T has an atom at 0, T has a higher expected utility anywhere in the range (t', t*) than at 0, or T is indifferent between 0 and points in the range (t', t*) but has no atom at 0. If T has an atom, then $\gamma^T(0) = \Gamma^T(t^*) = 1 + E_{\tilde{t}}[\theta^F(f', \tilde{t})] - E_{\tilde{t}}[\theta^F(0, \tilde{t})]] / B_F > 0$ or:

$$E_{\tilde{t}}[\theta^F(m^T(t', \tilde{t})] > E_{\tilde{t}}[\theta^F(0, \tilde{t})]] - B_F.$$

T must then be indifferent between t = 0 and any t with $t^* < t \leq t'$. Since T always wins when playing t' but never wins when playing 0, this implies that $E_{\tilde{f}}[\theta^T(t', \tilde{f})] = E_{\tilde{f}}[\theta^T(0, \tilde{f})] - B_T$. If T is not willing to have an atom at t = 0, $E_{\tilde{f}}[\theta^T(t', \tilde{f})] > E_{\tilde{f}}[\theta^T(0, \tilde{f})] - B_T$. Since t* is the lower bound on the support of T's distribution, and there is no atom there from Lemma 8, then $\Gamma^T(t^*) = 0$ must hold. From the formula for $\Gamma^T(t)$, $B_F + E_{\tilde{t}}[\theta^F(m^F(t'), \tilde{t})] - E_{\tilde{t}}[\theta^F(m^F(t^*), \tilde{t})] = 0$ or $E_{\tilde{t}}[\theta^F(f', \tilde{t})] = E_{\tilde{t}}[\theta^F(0, \tilde{t})] - B_F$. In the third case where T is indifferent but has no atom, these conditions, when combined with those in each of the first two cases, yield (a) and (b). Q.E.D.

Although the $\Gamma^I(i)$ functions given in Theorem 1 characterize the equilibrium mixed strategies, they are not explicit solutions. $\Gamma^F(f)$ depends on an expectation taken with respect to probabilities specified by the derivative of $\Gamma^F(f)$. To actually find the equilibrium cdfs, a fixed point is needed. Assuming a $\Gamma^F(f)$ and substituting it into the expectations in Theorem 1 yields a new $\hat{\Gamma}^F(f)$. To be an equilibrium, this $\hat{\Gamma}^F(f)$ must be the same as the function initially posited. Such a fixed point may or may not exist. When it does not, the game has no equilibrium. While it is difficult in general to determine when there is no equilibrium or to specify it when there is, for some specific θ^I functions, it is possible to explicitly solve for the equilibrium cdfs, as in the example given in the next section.

5. An explicit solution

Consider the following functional form for the effect of campaigning on reputation:

$$\theta^I(i, j) = G(1 - i) - jL + (L - D)(ij) \quad (6)$$

Then $\theta^I(0, 0) = G$, $\theta^I(1, 1) = -D$, $\theta^I(1, 0) = 0$, $\theta^I(0, 1) = G - L$.[1] For this function, the conditions (1), (2), (4), and (5) under which there is no pure strategy equilibrium are straightforward.

$$\max\{0, L - D\} < G < L \quad (7)$$

$$G < B \quad (8)$$

$$0 < X_F - X_T < L - G \quad (9)$$

(7) ensures that conditions (1) and (2) are satisfied and (8) implies that (4) is satisfied. For this θ^I, $\Delta(f, t) = X_F - X_T + (f - t)(L - G)$. Since $\Delta(1, 1) = X_F - X_T$, the lower inequality in (9) ensures that (5) is satisfied. Setting $\Delta = 0$ and solving yields $t = f + (X_f - X_t)/(L - G) = m^T(f)$. Then $t^* = 1 - f^* = (X_f - X_t)/(L - G)$. The inequalities in (9) ensure that t^* and f^* are between 0 and 1.

Expressions for the means of the distributions of f and t, $\mu_i \equiv E_{\tilde{\imath}}[\tilde{\imath}]$, are given in Lemma 9.

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[1] This $\theta^I(i, j)$ function is consistent with the assumptions in the discrete decision model in Fletcher and Slutsky (2010). Negative campaign wars may cause either voter anger or voter apathy here, as D may be greater or less than L.

Lemma 9: In an equilibrium, the following must hold:

$$\mu_F = \frac{G(f')^2}{2B + (L-D)(f')^2}$$

$$\mu_T = \frac{G((t')^2 - (t^*)^2)}{2B + (L-D)((t')^2 - (t^*)^2)}$$

Proof: Substituting $\theta^I(i, j)$, $m^T(f)$, and t* into Theorem 1 yields $\Gamma^F(f) = 1 + [(t - t')(G - (L - D)\mu_F)]/B$ and $\gamma^F(0) = 1 + [(t^* - t')(G - (L - D)\mu_F)]/B$. For any assumed value of μ_f, a cdf is determined whose mean can be computed by taking the derivative of $\Gamma^f(f)$ found after substituting t = f + t*. Then the mean of this cdf is $\hat{\mu}_F = \gamma^F(0) \cdot 0 + \int_0^{f'} f(G - \frac{(L-D)\mu_F}{B})df = \frac{[G-(L-D)\mu_F]}{2B}(f')^2$. Setting $\hat{\mu}_F = \mu_F$ and solving yields the expression given for μ_F. Similarly, μ_T can be found from substituting into $\Gamma^T(t)$. Q.E.D.

Since $f' = t' - t^*$, both means, and hence, both cdf's are determined by t'. Thus there exist a family of pairs of cdf's that depend on only one variable. A Nash equilibrium exists if in this family there exists a t' between t* and 1 which also satisfies either (a) or (b) of Theorem 1. Theorem 2 specifies the parameter values at which such a t'exists, gives its unique value in those cases, and gives the parameter values under which no such t' exists.

Theorem 2: Assume that campaigns effect reputation according to the function in (6) under the parameter restrictions in (7) – (9).

(I) The unique Nash equilibrium cdf's for the two candidates are:

$$\Gamma^{F}(f) = 1 + \frac{2(f - f')G}{2B + (L-D)(f')^2}, \; 0 \le f \le f'$$

$$\Gamma^{F}(f) = 1, \; f' \le f \le 1$$

$$\Gamma^{T}(t) = 1 + \frac{2(t^{*} - t')G}{2B + (L-D)((t')^2 - (t^{*})^2)}, \; 0 \le t \le t^{*}$$

$$\Gamma^{T}(t) = 1 + \frac{2(t - t')G}{2B + (L-D)((t')^2 - (t^{*})^2)}, \; t^{*} \le t \le t'$$

$$\Gamma^{T}(t) = 1, \quad t' \le t \le 1$$

This Nash equilibrium exists in either of the following circumstances:

(i) $D - L > 0$ and $B \le \min\{(3/2)G^2/(D - L) + t^*G, (1/2)(D - L)(1 - t^*)^2 + G\}$

with $t' = f' + t^* = t^* - G/(D - L) + [(G/(D - L))^2 + 2(B - t^*G)/(D - L)]^{1/2}$

(ii) $D - L > 0$ and $(3/2)G^2/(D - L) + t^*G \le B \le (1/2)(D - L)(1 - (t^*)^2) + (1 - t^*)G$

with $t' = f' + t^* = - G/(D - L) + [((G/(D - L)) + t^*)^2 + 2B/(D - L)]^{1/2}$

(II) No Nash equilibrium exists in any of the following ranges of parameter values:

(iii) $D - L \le 0$

(iv) $(1/2)(D - L)(1 - t^*)^2 + G < B < (3/2)G^2/(D - L) + t^*G$

(v) $\max\{\min\{(3/2)G^2/(D - L) + t^*G, (1/2)(D - L)(1 - t^*)^2 + G\}, (1/2)(D - L)(1 - (t^*)^2) + (1 - t^*)G\} < B$

Proof: Substituting the values of μ_i given in Lemma 9 into the cdf's of Theorem 1 after having substituted the specific functions of the example yields the $\Gamma^{F}(f)$ and $\Gamma^{T}(t)$ given in (I) of the Theorem. For these to be valid either (a) or (b) of Theorem 1 must hold.

In (a), T is willing to place probability weight on 0 so must receive the same expected payoff at 0 as at t'. Substituting $\theta^T(t, f)$ into the expected payoffs for T at these two values yields :

$$-t'G + (L - D)t'\mu_F + B = 0$$

Since $B > t'G$ from (8), this can only hold if $L - D < 0$. Substituting the value of μ_F from Lemma 9 and simplifying yields the following quadratic equation in t':

$$(t')^2 + 2(G/(D - L) - t^*)t' + ((t^*)^2 - 2(G/(D - L)) = 0$$

Solving this using the quadratic formula yields:

$$t' = t^* - G/(D - L) + ((G/(D - L))^2 + 2(B - t^*G)/(D - L))^{1/2}$$

Of the two roots in the quadratic formula, only the + root is valid since $t' > t^*$ must hold. For this to be valid, $t' \leq 1$ must hold. This is true if and only if the parameter restriction $B \leq (1/2)(D - L)(1 - t^*)^2 + G$. In addition, at this t', $0 \leq \gamma^T(0)$ must hold. Substituting this t' into $\gamma^T(0) = \Gamma^T(t^*)$ and manipulating yields:

$$B \leq (3/2)G^2/(D - L) + t^*G$$

Thus, the three conditions in (i) are necessary and sufficient for there to exist a t' with $t^* < t' \leq 1$ with $EU^T(0, \Gamma^F) = EU^T(t', \Gamma^F)$ and $0 \leq \gamma^T(0)$. Since $f' = t' - t^*$, then f' lies between 0 and 1 making Γ^F a valid cdf.

In (b) of Theorem 1, $\gamma^T(0) = 0$ holds. Since $\gamma^T(0) = \Gamma^T(t^*)$, this is true if

$$2B + (L - D)[(t')^2 - (t^*)^2] + 2G(t^* - t') = 0$$

As above, from (8), this is possible only if $D > L$. Solving this quadratic equation yields:

$$t' = - G/(D - L) + ((G/(D - L) + t^*)^2 + 2B/(D - L))^{1/2}$$

where again the positive root is taken to ensure that $t > t^*$. Imposing $t' \leq 1$ yields the following parameter restriction:

$$B \leq (1/2)(D - L)(1 - (t^*)^2) + (1 - t^*)G$$

T is willing to have $\gamma^T(0) = 0$ iff $EU^T(0, \Gamma^F) \leq EU^T(t', \Gamma^F)$. This holds iff:

$$0 \leq 2B + (L - D)(t' - t^*)^2 - 2Gt'$$

Substituting the value of t, this holds under the following parameter restriction:

$$(3/2)G^2/(D - L) + t^*G \leq B$$

Thus, the parameter restrictions of (ii) are necessary and sufficient for (b) of Theorem 1 to hold for this specific case.

Since an equilibrium exists iff the conditions in (I) hold, no equilibrium exists if both (i) and (ii) are violated given (7) – (9). The conditions in (iii), (iv), and (v) hold iff both (i) and (ii) fail. Q.E.D.

Γ^F and Γ^T are shown in Figure 3 for case (i) which corresponds to (a) in Theorem 1 where T has an atom at 0. Case (ii) corresponds to Theorem 1(b) in which T has no atoms. In either case, since the cdf of Γ^F is linear on the range (0, f'], the pdf is uniform on that range. Similarly, the cdf of Γ^T is linear on the range [t*, t'].

The regions for the three cases in which no equilibrium exists have straightforward descriptions. The region in (iii) arises when voters either are not adversely affected by a negative campaign war or do have meltdown in their attitudes but meltdown is in the form of apathy, D < L. That is, voters do not like negative campaign wars but their response to one is to tune out and not pay attention to campaigns, lessening the effectiveness of such things as negative ads. For an equilibrium to exist, a negative campaign war must cause meltdown in the form of anger. Each candidate’s reputation must fall more in a negative campaign war than from just facing an opponent’s attacks, D

> L. However, if D – L is positive but small, the condition in (iv) is satisfied and no equilibrium exists. D – L must be sufficiently large that at least one of the inequalities in (iv) is violated for an equilibrium to exist.

Nonexistence can also arise if B is sufficiently large as given in region (v). In this case, candidates, especially candidate T, must be Lombardians, carrying most heavily about winning relative to other reputational effects. Since assuming that the goal of candidates is to maximize their probability of winning, this possibility of nonexistence is significant. How candidates would behave in such cases is considered below.

The frontrunner appears to put more weight on positive campaigning than the trailing candidate. This is consistent with the results from Skaperdas and Grofman () and Herrington and Hess ().

Theorem 3: Under the parameter values at which a mixed strategy equilibrium exists, then $\gamma^T(0) < \gamma^F(0)$ and $\mu_F < \mu_T$.

Proof: Using $\gamma^T(0) = \Gamma^T(t^*)$ and $\gamma^F(0) = \Gamma^F(0)$, the sign of $\gamma^T(0) - \gamma^F(0)$ is the same as the sign of (t* - t')(D – L). Since D > L for an equilibrium to exist and t* < t', then $\gamma^T(0) < \gamma^F(0)$ must hold. Using the expressions for μ_F and μ_T from Lemma 9, the sign of $\mu_F - \mu_T$ is the same as the sign of t* - t', which must be negative. Q.E.D.

Given the explicit solutions for the equilibrium mixed strategies, the comparative statics with respect to the various parameters can be determined. These are given in Theorem 3.

Theorem 4:

(a) An increase in B_I causes a first order stochastic dominating increase in positive campaigning by J.

(b) An increase in D causes a first order stochastic dominating increase in the amount of negative campaigning by both candidates.

(c) A larger difference in the candidates' initial reputations (X_F - X_T larger) causes a first order stochastic dominating increase in the amount of positive campaigning by both candidates.

(d) A candidate's expected utility can decline in their initial reputation.

Proof:

In the next section we will examine evidence from the 2002 elections for governor, Senate and House to see if it is consistent with the theory. This is complicated because most of the parameters of the model are unobservable. Thus, we cannot directly know which candidate enters the race with the higher reputation. We can observe the outcome of the election, but a trailing candidate might win the election with a sufficiently negative campaign. Given the distributions given in Theorem 2, we can compute the pdfs of the winners and losers after taking this strategic interaction into account. Theorem 5 derives the pdfs for the winners and losers of the election; these can be directly compared to the data.

Theorem 5:

Proof:

6. Some empirical evidence

Most of the variables in the model are not observable. For example, we cannot directly observe how positive or negative advertising affects the candidates' reputations, or the value the candidates place on winning the election. Complete polling data would give us an idea of the candidates' relative pre-election reputations, but most polls are conducted during ---- not before --- the campaigns, and polling questions and samples are not consistent across races.

However, the theory does make some predictions that can be tested with available data. The Campaign Media Analysis Group (CMAG) collects real-time data on political advertisements at the individual ad level, including such information as ad content, length, and time and station on which the ad was aired. The data are sold to candidates and news organizations, as well as the Wisconsin Advertising Project, which releases the data for academic use after a lag of several years, and after adding a significant amount of content analysis. This includes some characterizations that can be used to determine whether ads are positive or negative. One of these characterizations is "Is the favored candidate's opponent mentioned in the ad?" As candidates are unlikely to mention their opponents unless their purpose is to put them in a negative light, we consider any ad where the opponent is mentioned to be a negative ad, and all others to be positive.[2] From this, we construct the fraction of each candidate's advertising occurrences that are negative ("negativity").

Recall that the theory makes the following predictions:

1. The only pure strategy equilibria have either both candidates going completely negative or both candidates going completely positive.
2. The frontrunner has an atom at 0 negativity and puts no weight near negativity of 1.
3. The trailing candidate may have an atom at 0 negativity, but puts no weight above but near zero. The trailing candidate may have weight at or near complete negativity.

We use CMAG data from the 2002 races for governor, Senate and House of Representatives to construct the negativity measure for each candidate, and determine whether the candidates' choices are consistent with the mixed strategy equilibria predicted by the theory.[3] We drop from our sample races with more than two serious candidates, as our model addresses only two-candidate contests, and races that are effectively uncontested. Our sample includes 280 candidates from the 140 races with two serious candidates who both purchased campaign advertising.

First, consider that if the equilibria were in pure strategies, the negativity between the winner and the loser of the race would show a strong correlation. The simple

[2] Other characterizations that can be used as measures of negativity are also available, but fewer ads could be characterized than with our favored measure. However, the results using alternative measures are qualitatively similar to those presented here, and are available on request from the authors.

[3] We include only advertisements shown after the primaries in the relevant state, as a candidate --- particularly an incumbent --- might not have a clear sense of her opponent's identity until after the primary. Election outcomes are from cnn.com.

correlation between the negativity of the winners and losers is a rather weak 0.23, suggesting that most candidates are not employing pure strategies. Figure 3 shows the histograms of negativity for all winning candidates, and separately for the losing candidates.

Figure 1
Winner of the election as a function of campaign negativity