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Employment Protection, Labor Market Turnover and the Effects of Globalization

Philip Schuster [†]

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Abstract

A parsimonious search and matching model of the labor market with endogenous separation is embedded in a North-North intermediate goods trade framework. International product market integration leads to redistribution of market shares from less to more productive firms within an industry, implying chances and threats for firms at the same time, as firms are ex-ante unaware of their productivities. Opening the economy will therefore increase the dispersion of potential revenues and consequently lead to higher labor market turnover, higher welfare and increased wage inequality, while the effect on employment is ambiguous. Ceteris paribus, the effects are qualitatively similar to decreasing employment protection in form of costly firing restrictions which prevent the economy from reaching a first best allocation. The positive welfare effects of opening to trade are decreasing in the level of firing costs. This can therefore lead to a substantial failure in reaping the benefits that could result from economic integration, by preventing labor reallocation. The main results are robust to the introduction of risk-averse workers.

Keywords: employment protection, firing margin, trade in intermediate goods, labor market turnover, revenue risk

JEL Classification: F15, F16, J63, J65

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1 Introduction

Recent work has emphasized the need for understanding the role of the welfare state and social protection in a world of increasing economic integration. So far these papers have predominantly focused on unemployment insurance (see e.g. Keuschnigg and Ribi, 2009). This work extends the analysis by looking at the role of employment protection (EP) in an open economy. A parsimonious static version of the Diamond-Mortensen-Pissarides (DMP) model with one-shot worker-firm matching, bilateral wage bargaining and endogenous separation is enriched to capture effects of trade through changes in revenues which are usually assumed to be exogenous and constant in the canonical DMP model. While most studies that integrate modern trade models (e.g. Melitz, 2003) with theories on frictional labor markets focus on unemployment levels, the developed model is especially equipped to analyze labor market turnover as an important channel connecting EP and trade. This is important as the same level of unemployment can be generated by different compositions of job creation and destruction. The specific question the paper seeks to answer is how the effects of EP are altered after arrival of a trade liberalization shock in a North-North trade setting. In addition the optimal implementation of EP is discussed.

In principle, the causal relationship of labor market institutions and trade patterns can be two-fold. Papers, like Cuñat and Melitz (2007) and Helpman and Itskhoki (2010) describe how a flexible labor market implies a comparative advantage in producing in high-volatility sectors. This paper is related to another stream of the literature that analyses how trade patterns shape labor market institutions and social protection. Most attention concerning the role of social protection in an open economy was put on unemployment insurance (UI). EP, as another pillar of the welfare state, and its interaction with globalization shocks has received comparably little attention so far. But what is a potential role of EP on top of UI? Blanchard and Tirole (2008) argue that existing UI creates a firing externality that can be undone by introducing EP in form of a firing tax. It is well understood that EP reduces both job destruction as well as job creation (see for example Messina and Vallanti (2007) for empirical support). While the effect on employment of locally increasing EP from a small level is ambiguous it is clear that ever increasing EP will eventually lead to a reduction in employment, which characterizes the downside of EP. Hence, an optimally chosen EP efficiently trades off the positive effect of correcting the firing externality and the negative effect of a reduction in the level of employment. This trade-off will be picked up in the normative part of the paper. This study also features a positive part dealing with the effects on welfare for a given level of firing restrictions.

That increased international integration should lead to more volatility in employment has been widely argued in the literature (see for example Rodrik, 1997, or Bhagwati and Dehejia, 1994, for theoretical argumentation and Beaulieu et al., 2004, for empirical evidence) although there are hardly any formalizations of this idea. A typical argument is that increased opportunities to trade in intermediate goods make labor demand of domestic final goods producers more elastic as they can more easily switch suppliers and source from abroad. I explicitly model a channel which implies that whenever opening to trade leads to an increase in 'chances' and 'threats' for domestic firms, job creation and destruction will rise. Chances can be thought of as new export markets while threats can come from import competition. I will make use of a production technology similar to the idea of trade in tasks by Grossman and Rossi-Hansberg (2008). While there is no substitution between domestic tasks or intermediate goods, a domestic variety[1] could be perfectly replaced by a foreign one from the same industry. Imagine assembling a car and how a German car engine cannot be substituted by a German car wheel but by a French car engine. In contrast to their framework I do not apply the technology assumption in an off-shoring- or North-South-context, where one country has a persistent cost advantage in which case, given the additional option of cheap sourcing from the South, Northern firms unambiguously profit and generate a clear positive productivity effect. As already hinted by the car example I will focus on a perfectly symmetric North-North set-up where the cost advantage is stochastic and driven by idiosyncratic shocks to productivity that are uncorrelated between sectors. Hence, in one sector a firm can 'steal business' from the corresponding firm in the other country, while this pattern could be reversed in another sector. As a domestic firm is ex-ante unaware of whether it can gain revenues from the foreign firm, or if it will lose market shares, the distribution of potential revenues increases which will be the key determinant for a trade-induced rise in labor turnover. This kind of spread is also present in new trade theory models à la Melitz (2003) that predict a reallocation of market shares from low- to high-productivity firms[2] while the link to job flows is absent in those frameworks. The idea that openness to trade can amplify the 'winner-loser'-pattern within an industry is well established empirically (see for example Pavcnik, 2002 and Bernard, 2004).

In the parsimonious model developed in this paper job creation is directly linked to expected profits of firms. If expected profits increase more firms will enter the labor market

[1]The terms 'task', 'intermediate good', and 'variety' have the same meaning in this context and are used interchangeably.

[2]Note that the mechanism in Melitz (2003) works quite differently. In contrast to my framework, less productive firms do not suffer directly through increased competition as this channel is not present in the Melitz set-up because of the monopolistic competition assumption. Instead, less productive firms are hurt by the increased labor demand of new entrants that bid up the wage on the competitive labor market, which is not present in this framework.

pushing up the probability of a worker to find a job. Job destruction is driven by match-specific shocks to productivity and consequently revenues. Hence, there is a cut-off revenue level below which a firm will destroy a job and lay off the involved worker. If trade liberalization lowers the potential revenues in low-productivity matches as argued above, it is clear that the mass of revenues below the cut-off increases, which consequently implies a rise in the job destruction rate. At the same time trade liberalization is supposed to increase potential revenues in high-productivity matches, boosting expected profits and therefore job creation because firms just care about potential revenues above the cut-off, i.e. revenues that are actually realized. This argument makes clear how openness to trade affects job flows and labor turnover, while the effect on the level of employment, which is at the heart of the analysis in many other studies, might be much less accentuated. The interaction of trade and EP follows directly as EP, as argued before, is a policy instrument that works exactly at the labor turnover margin.

The model delivers the following results. Ceteris paribus, EP and openness to trade have opposing effects along many dimensions. EP unambiguously decreases job creation and destruction, while the opposite is true for a trade openness shock. While both entail ambiguous effects on the employment level, the output effects are clear cut. EP will always lead to less total and average net output per worker. An openness to trade shock will imply the opposite. Both 'shocks' make the wage distribution unambiguously more disperse which increases worker inequality. But the spreads in the wage distribution are of completely different nature. In case of EP the wage distribution increases on both tails as 1) all wages are pushed up by a constant fraction generated in the process of wage bargaining and 2) more low paying jobs are operated because of decreased job destruction. Hence, the effect on the average wage is ambiguous. In case of a trade shock, the wage distribution it widened on the right tail as a direct consequence of increased volatility in potential revenues, while the cut-off on the left side is unaffected. Consequently, the average wage increases unambiguously. In the welfare analysis I first consider a benchmark environment with risk neutral workers such that welfare and net output coincide. The normative question on optimal EP is therefore trivialized as there are no externalities present[3] that justify firing restrictions. Concerning the positive part where I look at the effects of given firing costs, the parsimonious model set-up allows to derive a simple solution for the second best. It is shown that, for a reasonable parameterization, the welfare loss due to firing restrictions is increasing in job turnover, i.e. openness to trade. That means that while an open economy will always enjoy higher welfare, the distance to a possible first best is also increasing. Hence, EP can lead to a substantial failure in reaping possible

[3]The typical search externalities in the labor market are assumed to be balanced (Hosios (1990)-condition) throughout the paper.

welfare gains from economic integration by preventing necessary labor reallocation. As an extension, I introduce risk aversion of the workers to create a firing externality in the spirit of Blanchard and Tirole (2008). Workers demand unemployment insurance which creates a fiscal externality as firms do not take into account that an UI-system has to be financed when they decide to lay off a worker. Blanchard and Tirole (2008) find that an optimal firing tax has to be positive in that case. My results differ because I explicitly model endogenous job creation. It is still true that firms do not internalize the social costs of firing, but they also do not internalize the social benefit from hiring. Therefore whether an optimal firing tax should be positive or not is tightly linked to the effect on the level of unemployment[4]. A short numerical example reveals that employment is likely to decrease in the present set-up and that a firing tax should be set to 0, as in the case of risk neutral workers.

The paper further relates to other strands of the literature. Helpman et al. (2010) and Felbermayr et al. (2011) integrate frictional labor markets of DMP style into the Melitz (2003)-framework in order to analyze the effect of trade liberalization on inequality and the level of unemployment. Both develop thorough dynamic treatments incorporating detailed product markets, firm self-selection into exporter/non-exporter-status, multi-worker firms - features that are missing in the present paper. In the other hand they do not allow for trade effects along the job destruction margin which is at the heart of this study. In addition the parsimonious set-up allows for a more comprehensive welfare analysis. Probably the most closely related analysis was done by Jansen and Turrini (2004) which also features endogenous job destruction. They consider two 'globalization scenarios'. First, an economy is allowed to move from autarky to symmetric two-country trade. Given their technology assumptions integration is followed by increased demand for intermediate goods and consequently prices, as supply is fixed. The effect is comparable to a positive shock to total factor productivity and shifts the whole distribution of potential revenues to the right, leading to a reduction in job destruction. In a second scenario, they assume an exogenous increase in the volatility of cost-price markup, as in Mortensen and Pissarides (1994), which results in an increase in job destruction. The contribution of this paper is to bring both scenarios together by showing that using Grossman and Rossi-Hansberg (2008)-technology implies that market integration of two symmetric countries leads to a spread in potential revenues, which is more consistent with the 'winner-loser' pattern present in new trade theory. Further, the increase in job destruction seems to

[4]Empirical evidence on the effect of EP on employment levels - mostly drawn from cross-country analyses - is mixed. Some papers, including the seminal work of Lazear (1990), or Nickell (1997), Heckman and Pagés (2000), and Kahn (2007) find a significant negative effect. Other studies like OECD (1999) and Addison et al. (2000) report no significant effects. Addison and Teixeira (2003) provide an extensive summary of the empirical literature.

have also empirical support. Groizard et al. (2010), for example, find a negative relation of trade costs and job destruction for manufacturing in California.

The remainder of the paper is organized as follows. Section 2 presents a simple labor market model featuring endogenous job creation and separation where the distribution of revenues is taken as given. In section 3 I will discuss the effects of an increase in revenue risk on job flows. Section 4 presents a simple intermediate goods trade model that delivers a microfoundation for a rise in volatility of the form that was assumed in the previous section. The welfare implications of openness to trade and EP are derived in section 5 before section 6 concludes.

2 Labor market model

First, I analyze the labor market assuming that production is just governed by an exogenous distribution of possible production values. This assumption will be motived and interpreted in section 4. The set-up of the labor market model is static and can be summarized by the following sequence of events:

Stage 1. A mass 1 of workers starts out as unemployed.

Stage 2. Firms enter the labor market according to a free entry condition by posting one vacancy each at cost c.

Stage 3. Workers are hired according to a matching technology $\mathcal{M}$.

Stage 4. A value of production y (realization of a given random variable Y) is revealed to each firm leading to firing of the most unprofitable workers.

Stage 5. In case of separation firms have to pay firing costs F. Laid off workers receive $\mu = h + b$ like the workers who were never hired, where h denotes home production and b is unemployment compensation. Production is started with the remaining workers, who receive a bargained wage w.

Before solving the model by backward induction I will specify technology and preferences. The labor market is governed by a typical matching function assumed to fulfilled the following conditions. The matching function $\mathcal{M}(u, v)$ is homogeneous of degree 1 and increasing in its two arguments: number of initially unemployed u and number of vacancies v. Define labor market tightness as the vacancy-unemployment ratio, i.e. $\theta \equiv \frac{v}{u} = \frac{v}{1}$. The firm's probability of matching can be expressed as m^f with $\frac{dm^f}{d\theta} < 0$ where the elasticity of $m^f = \frac{\mathcal{M}}{v}$ is given by $-\eta \in (-1, 0)$ which is assumed to be constant. A worker's probability of being matched is $m = \frac{\mathcal{M}}{u} = \theta m^f$ with $\frac{dm}{d\theta} = (1 - \eta)m^f > 0$. Hence, m is referred to as the job finding or job creation rate[5] and let G be the separation or job destruction rate. Employment[6] is given as the number of workers that are matched and not subsequently laid off

$$e = m(1 - G) \tag{2.1}$$

which evolves as follows

$$de = (1 - G)dm - mdG \tag{2.2}$$

Clearly employment is increasing the job creation rate and decreasing in the job destruction rate. I will now turn to the workers whose utility functions are strictly increasing, $u'(\cdot) > 0$. Expected utility of a worker i is given as

$$V_i = \underbrace{m(1 - G)}_{e} \cdot u(w_i) + \underbrace{(1 - m(1 - G))}_{1-e} \cdot u(\mu) \tag{2.3}$$

Wages will differ for workers because worker-firm pairs may have different values of production y. Integrating this expression over all individuals gives total welfare $\Omega = \int_0^1 V_i di$ as firms make zero profits in equilibrium. For the moment, I will focus on risk neutral[7] workers, i.e. $u(x) = x$.

The last decision stage of the agents is the firing or separation decision. Firms will lay off workers whenever the actual value of production[8] y minus labor costs $w(y)$ is lower than the firing costs F, i.e.

$$y - w(y) < -F \Rightarrow \underline{y} = w(\underline{y}) - F \tag{2.4}$$

where $\underline{y}$ denotes the value of production at which a firm is indifferent between firing and keeping the worker. y is the realization of an i.i.d. draw from the known distribution $G_Y(\cdot)$. Consequently, $G_Y(\underline{y})$, or in short G, gives the probability that a worker-firm pair is insufficiently profitable and will therefore be referred to as firing or job destruction rate.

[5]As the initial unemployment rate is equal to 1, the job finding rate m and the number of matches $\mathcal{M}$ give the same number.

[6]Using mass 1 of workers implies that e also denotes the probability of being employed for a specific worker due to the law of large numbers.

[7]This assumption is relaxed in section 5.3.

[8]Think of the market value of production y as 'before wage profits' that capture productivity as well as the demand structure, i.e. the price and revenue that can be generated, in reduced form. The paper will be more explicit about how y is derived in section 4. As the distribution of match-specific productivity will be invariant all changes in the distribution of Y can be interpreted as changes in the distribution of revenues.

To specify G it is necessary to know the reservation wage $w(\underline{y})$. Wages are determined using a standard bilateral Nash bargaining game

$$w(y) = \text{argmax}\,[u(w) - u(\mu)]^{\omega}\,[y - w + F]^{1-\omega} \tag{2.5}$$

where ω denotes the worker's bargaining power. After using the assumption of risk neutrality the FOC reads

$$\frac{\omega}{w - \mu} = \frac{1 - \omega}{y - w + F} \tag{2.6}$$

Hence, the wage schedule is given by

$$w(y) = (1 - \omega)\mu + \omega\,[y + F] \tag{2.7}$$

Substitute the reservation wage out of the cut-off condition (2.4) to get

$$\underline{y} = \mu - F \tag{2.8}$$

Equation (2.8) is the first central equilibrium condition and will be referred to as the job destruction condition. The intuition is that at the cut-off the surplus of forming a worker-firm pair is 0, hence the wage is pushed down to the outside option μ. Consequently, there will be no inefficient firing[9] that could be prevented by bilateral trade between the firm and the worker. The job destruction rate G is just

$$G = G_Y\,(\mu - F) \tag{2.9}$$

After solving for the firing decision one can analyze the preceding job creation decision which is driven by the profit a firm can anticipate to make. The expected profit conditional on being matched with a worker is denoted

$$\pi^e = [y^e - w^e]\,(1 - G) - GF \tag{2.10}$$

where y^e and w^e denote conditional expectations of y, i.e. $y^e = E(Y|y > \underline{y}) = \frac{\int_{\underline{y}}^{\infty} y dG_Y(y)}{1-G}$, and $w(y)$, i.e. $w(y^e)$. Substitute the wage schedule (2.7) into the expected per worker profits π^e and rearrange to get

$$\pi^e = (1 - \omega)\,\left[y^e - \underline{y}\right]\,(1 - G) - F \tag{2.11}$$

[9]The terms 'firing' and 'separation' can therefore be used interchangeably.

Expected profits are decreasing in the outside option, the bargaining power of the worker and the firing costs. Latter can eventually lead to negative expected profits. Firms are assumed to have an outside option of 0 and will therefore enter the labor market as long as $m^f \pi^e - c \geq 0$ where $c > 0$ are costs of entering. I assume $\pi^e > 0$ to avoid the uninteresting case of zero entry. As more firms enter, the tightness of the market increases which drives down the probability of being matched with a worker. In equilibrium firms will enter up to the point where there is no more gain from doing so. The free entry condition therefore states that $m^f \pi^e = c$ or

$$(1 - \omega) \left[y^e - \mu + F\right] (1 - G) - F = \frac{c}{m^f} \tag{2.12}$$

This pins down the job creation rate which is given as[10]

$$m \equiv m(\theta) = m \left(\left[m^f\right]^{-1} \left(\frac{c}{\pi^e}\right)\right) \tag{2.13}$$

where $\left[m^f\right]^{-1}(\cdot)$ denotes the inverse function of $m^f(\cdot)$. It is easy to see that the job creation rate is increasing in π^e and decreasing in c. Equilibrium in the labor market is given by $\left\langle \theta, \underline{y} \right\rangle$ that solve the job destruction (2.8) and the job creation condition (2.12). These conditions resemble the equilibrium conditions of a fully dynamic version of the model (e.g. (Pissarides, 2000)) except that expected profits are not discounted and the job destruction condition does not incorporate possible future shocks to y. A first result is given by the following proposition.

Proposition 1. *An increase in firing costs leads to a reduction in both, the job creation rate (through lower labor market tightness) as well as the job destruction rate. The effect on the employment level is ambiguous.*

The proof is provided in appendix A. This result is well understood and intuitive. An increase in F pushes the cut-off value of production $\underline{y}$ and consequently the job destruction rate falls. At the same time firing costs reduce expected profits as stated in (2.11). Clearly, firms will create less vacancies and the workers' job finding rate falls. The effect on wages is summarized in the next proposition.

Proposition 2. *An increase in firing costs leads to an increase in the wage distribution. Whether the average wage falls or rises is solely determined by the nature of the underlying distribution of Y.*

[10]For an explicit relationship one could consider a typical Cobb-Douglas specification, e.g. $m^f = \phi\theta^{-\eta}$ and consequently $m = \theta m^f = \phi\theta^{1-\eta}$. Then labor market tightness is given as $\theta = \phi^{\frac{1}{\eta}} \left[\frac{\pi^e}{c}\right]^{\frac{1}{\eta}}$ and the job creation rate would be $m = \phi^{\frac{1}{\eta}} \left[\frac{\pi^e}{c}\right]^{\frac{1-\eta}{\eta}}$.

The proof is provided in appendix A. The first part is intuitive as firing costs prevent firms from destructing jobs with low profitability which pay low wages. Consequently, the wage distribution is widened on the left tail. The result that this does not automatically imply a decrease in the average wage stems from the fact that all wage rise by the constant term ω that workers can snatch in the bargain for every unit of F.

3 Increase in revenue risk

This section discusses how an increase in the riskiness of the value of production or revenue changes the equilibrium allocation. This increase could in principle have very different motivations, like what happens in an environment where firms are forced to engage in more risky projects. Another story, told by this paper, explains how international market integration can imply increased chances as well as increased threats to local firms, hence spreading out the distribution of potential revenues of firms after integration. I will be more explicit about this in section 4 where a simple 2 country model is used to show that international market integration implies a spread in the revenue distribution of the form that will be analyzed in a general and abstract manner in what follows now.

For simplicity and tractability I assume that the increase in revenue volatility takes place in the simple form of a mean preserving single crossing spread from Y to Y'.

Definition 1. *A mean preserving single crossing spread (MPSCS) from Y to Y' is given if the following two conditions hold*

a) Mean preservation (MP):

$$E(Y) = E(Y') = \int_{-\infty}^{\infty} y dG_Y(y) = \int_{-\infty}^{\infty} y dG_{Y'}(y) = \mu_Y$$

b) Single crossing spread (SCS):

$$\exists \hat{y} : G_Y(y) \geq G_{Y'}(y), \forall y \geq \hat{y} \quad \textit{and} \quad G_Y(y) \leq G_{Y'}(y), \forall y \leq \hat{y}$$

The notion of MP is self-explanatory while the SCS characteristic just implies that the cdf of the spread random variable Y' crosses the cdf of Y just once from above at the crossing point[11] $\hat{y}$ as illustrated in figure D.1. Assuming this one can analyze how job flows are affected but such a volatility increase.

Lemma 1. *The job destruction rate G is weakly increasing for any kind of SCS of Y if $\underline{y} \leq \hat{y}$.*

[11]Note that the single crossing point $\hat{y}$ coincides with the mean μ_Y if the distribution is symmetric.

Proof. This follows directly from the SCS definition. ∎

The intuition for this result is clear and also illustrated in figure D.1. While the cut-off $\underline{y}$ is unaffected, a bigger mass is now located below it, leading to a higher probability of separation. Let us address the effect on job creation now.

Lemma 2. *The job creation rate m is weakly increasing for any kind of MPSCS of Y.*

The proof is provided in appendix A. The basic intuition is again simple to grasp and best understood by looking at figure D.2. While the unconditional expectations of both distributions are the same, only the revenues above the cut-off $\underline{y}$ are actually realized. This means that the increase in the mass of low potential revenues does not hurt the firm because those jobs would not have been operated anyways while the firm profits from a higher probability of drawing a high revenue. This pushes up expected profits and consequently the job creation rate m. Hence, labor turnover is accelerated. Note the analogy to the effect of EP just with opposite direction. As in the case of EP the effect of a revenue spread on employment is undetermined while effects on welfare are clear cut as will be shown in section 5. The following corollary gives some supplement results.

Corollary 1. *Average output y^e, average wage w^e and the wage dispersion are weakly increasing for any kind of MPSCS of Y if $\underline{y} \leq \hat{y}$.*

The proof is provided in appendix A. If one wants to compute the exact changes in G and m, e.g. to be able to sign the change in employment, one has to be more specific about the nature of the MPSCS. The rest of this section is used to illustrate how comparative statics can be done for an example of a parameterized MPSCS. In order to model the volatility increase as a change in a specific parameter define the spread as

$$y' = y + \sigma(y - \mu_Y) \equiv f(y) \tag{3.1}$$

Every realization y is shifted outwards by a multiplicative constant σ. Clearly, the variance of the spread random variable is higher, $Var(Y') = (1+\sigma)^2 Var(Y)$, while the unconditional mean is preserved, $E(Y') = E(Y)$. Instead of working with $G_{Y'}(\cdot)$ one can rephrase the problem by change of variables[12] and use $G_Y(\cdot)$. The new separation rate G' and its reaction to the spread variable are given by

$$G_{Y'}(\mu - F) = G_Y\left(f^{-1}(\mu - F)\right) = G_Y\left(\frac{\mu - F + \sigma\mu_Y}{(1+\sigma)}\right) \equiv G' \tag{3.2}$$

$$\frac{dG'}{d\sigma} = g_Y\left(\frac{\mu - F + \sigma\mu_Y}{(1+\sigma)}\right)\frac{\mu_Y - (\mu - F)}{(1+\sigma)^2} \tag{3.3}$$

[12] See appendix C.1.

Hence, the separation rate increases in σ for all possible $F \geq 0$ as long as $\mu_Y > \mu$, which I will assume from now on. Next, one can derive an expression for $\frac{dm}{d\sigma}$ which has to be non-negative because the relevant term $\Lambda \equiv \int_{\underline{y}}^{\infty}(y-\underline{y})dG_Y(y)$ in the job creation condition reacts positively to a MPSCS as proven in lemma 2. Λ changes as follows

$$\Lambda' = \int_{\underline{y}}^{\infty}(y-\underline{y})dG_{Y'}(y) = \frac{1}{1+\sigma}\int_{\underline{y}}^{\infty}(y-\underline{y})g_Y\left(\frac{y+\sigma\mu_Y}{(1+\sigma)}\right)dy \tag{3.4}$$

Hence, $\frac{dm}{d\sigma} = \frac{1-\eta}{\eta}\frac{m}{\pi^e}(1-\omega)\frac{d\Lambda'}{d\sigma} > 0$. While the last paragraphs just gave an illustration of how comparative statics can be done for a given spread, the next section develops a microfoundation for the spread itself.

4 Market integration and revenue risk

The effect of an increase in revenue risk was discussed in the last section. This part of the paper presents a simple stylized extension to the labor market model described in section 2 that explicitly models a channel through which openness to trade affects the distribution of firms' revenues. So far the random variable Y, summarizing the distribution of potential values of production, was taken as given. More model structure will be put on the production side of the economy now. First, I will discuss this for the closed economy before allowing for free trade in intermediates with a symmetric second country. Motivation for increasing product market integration is obviously manifold. One could think of it as a result of increased standardization of intermediate goods or a removal of or reduction in prohibitive non-tariff trade barriers and so forth.

4.1 Closed economy

While the value of production was simply denoted y and followed a given distribution so far, I am more explicit about this now. The value of production y consists of a revenue component y^r and a non-labor cost component y^c, reflecting idiosyncratic productivity, such that $y = y^r - y^c$ represent before-wage profits and $y - w(y) = y^r - y^c - w(y^r - y^c)$ describes a firm's net earnings which in expectation have to cover labor market entry costs. The non-labor cost component y^c is drawn independently from a given distribution G_{Y^c}. The revenue component y^r captures demand for and price of a specific output.

Production in the economy occurs in a continuum of intermediate good sectors and a final good sector. Every worker-firm pair produces a different intermediate good or variety. Hence, due to the identity: 1 worker, 1 firm, 1 variety, one can index them all with $i \in [0,1]$. As every firm represents a whole domestic industry and revenues across domestic industries will not be correlated, the assumption in the reduced model from before that every firm receives an i.i.d. draw of y is not violated. Further, one can discuss the demand structure in one industry in isolation and I will therefore drop the index i if appropriate.

The production technology of the variety producers exhibits economies of scale of the following simple form. Upon paying the wage of a worker w and non-labor costs y^c a firm can produce any quantity q in the production set $[0, 2]$. Importantly, the marginal costs of producing a second output are lower than for producing the first unit. To keep things simple and tractable I normalized the marginal costs of producing a second unit to zero without changing the basic qualitative results. Recall that although w and y^c can be interpreted as fixed costs, they are reversible in case a worker is fired, in contrast to the search costs c which are sunk. The revenue is then given by $y^r = q^d \cdot p$, where p is the industry specific price of the variety and q^d is the according demand for that given price p. Hence, a firm will set the price that maximizes revenue y^r under the condition that $q^d \in [0, 2]$. Observe that this is independent of the realized non-labor cost component and the wage and therefore the same problem for every variety producer. Consequently, every variety producer will set the same price p_i. Before solving this problem one has to derive demand. The final good sector is characterized by a representative competitive firm that uses no labor but only the varieties as inputs to assemble them using a simple 1:1 technology. The final good is an all-purpose good that is used for consumption of the workers and for covering the firms costs y^c, c, and F and serves as numéraire. The production function is specifically given as

$$Q = \int_0^1 \min\,[1, q_i]\, di \tag{4.1}$$

and incorporates two distinct features worth discussing. First, similar to Grossman and Rossi-Hansberg (2008)'s idea of tasks there is no substitution between the varieties, i.e. the final good producer cannot make full use of the economies of scale technology of the variety producers by just sourcing from the cheapest supplier in the whole economy. Although an intermediate good firm would want to produce and sell more than one unit, the technology in the final good sector limits demand to 1. Second, in contrast to Grossman and Rossi-Hansberg (2008) not every single 'task' has to be done, or in this framework's interpretation: not every single 'variety' has to be produced, to get positive output of the final good. This assumption is obviously necessary as some of the intermediate firms will lay off workers and shut down production. Hence, the interpretation is that every variety adds to the amount or quality of the final good[13] in a linear way. The final good

[13] Instead of adding a final good sector one could alternatively define love of variety preferences directly.

producer is assumed to be competitive which implies that the final good's price for one unit, P, is pushed down to the costs needed to produce this unit. Hence, the price is given by $P = \int_0^1 p_i di$ which is normalized to 1. The accounting is simple and as all variety producers set the same price, the normalization implies $p_i = 1, \forall i$. In equilibrium $1 - e$ varieties are produced, assembled to $1-e$ units of final output which is sold at price 1 and used to cover all realized non-labor and labor costs y^c and w as well as vacancy posting costs c.

As demand q^d is limited to 1 this means that the revenue component is always $y^r = 1$. After subtracting the non-labor cost component, one gets $y = 1 - y^c$. Hence, the random variable Y is just a simple transformation of Y^c, with $E(Y) = 1 - E(Y^c)$, $Var(Y) = Var(Y^c)$ and the following distribution and density:

$$G_Y(y) = 1 - G_{Y^c}(1-y) \qquad \text{and} \qquad g_Y(y) = g_{Y^c}(1-y) \tag{4.2}$$

As before one can solve the labor market model just by inserting (4.2) in *Stage 4*.

4.2 Open economy

I will now allow international exposure to have an effect on the labor market. The effect is propagated through the product markets, namely through the potential revenues firms can make. Assume that there are a 'home' and 'foreign' country indexed by H and F. Both are symmetric in every aspect[14]. In contrast to typical new trade models the integration of both countries does not imply that the number of potential varieties doubles. The technology in the final good sector is such that a home variety i cannot be substituted for home variety j, but it is a perfect substitute for foreign variety i. This is very similar to the trade in tasks framework of Grossman and Rossi-Hansberg (2008) with the difference that instead of North-South trade where the South has a systematic price advantage, I tell a North-North trade story where the price advantage is stochastic. In one sector the domestic variety producer is the 'strong' firm receiving a higher market share than the foreign producer, called the 'weak' firm, while the pattern might be reversed in another sector. The notion of 'strong' firm is defined as having received a lower non-labor costs realization than the corresponding firm in the other country.

I focus on the stylized limiting case of perfect international market integration with zero

[14]A strong but very convenient assumption is that job matching is perfectly correlated in both countries. Hence, one does not have to consider the case that a variety producer is matched with a worker while the corresponding firm in the other country is not matched. One can therefore ignore that firms have to form expectations about how likely it is that this event will occur. Consequently, one can directly compare all potential revenues conditional on both firms in this industry being matched with a worker.

transportation or market entry costs such that both final good producers can freely choose whether to source variety i from the home or the foreign country. Hence, a variety producer might end up suppling both countries or none at all. This creates a potential competition situation that was not present in the closed economy. Recall that in autarky changing p_i in equilibrium did not affect q^d but had only a pure price effect through 1:1 changes in P. This is different now because a variety producer can in principle gain market shares by undercutting the corresponding variety producer from the other country as the two final good producers[15] will always source from the cheaper possibility. The case of pure Bertrand competition serves as a threat point and will never be realized as the two firms are allowed to bargain. In the process of bargaining both firms will split the surplus generated from not competing. As the outside option of the strong firm is to win the Bertrand competition game, which is higher than what the weak firm would get, namely zero revenues, the bargaining solution looks like the following: both firms continue to supply their home markets at the same price $p = 1$ that maximized revenues in the closed economy. In addition the weak firm pays a transfer in terms of revenue or market share to the strong firm for not entering. Alternatively and qualitatively equivalent the bargaining solution could look like: the strong firm gets the whole world market and pays a transfer in terms of revenue of market share to the weak firm. I will discuss the bargaining solution and its stability later in this section. The sequence of events in the redefined labor market game looks as follows.

Stages 1–3. As before.

Stage 4a. Idiosyncratic costs y_H^c and y_F^c (two i.i.d. draws from $G_{Y^c}(\cdot)$)[16] are revealed to both, the home and the foreign producer of a variety.

Stage 4b. Both firms enter a price announcement game, with the possibility of bargaining over market shares and revenues y_H^r and y_F^r. Given the resulting values of production $y_H = y_H^r - y_H^c$ and $y_F = y_F^r - y_F^c$ the most unprofitable workers are laid off.

Stage 5. As before.

I will now determine the revenues y_H^r and y_F^r as functions of the realized non-labor costs. Suppose that in some sector the random draws revealed $y_H^c < y_F^c$, i.e. the variety producer at home is more productive and referred to as the 'strong' firm. Let us define the inside

[15]As both final good producers face exactly the same problem one can alternatively think of one big assembling firm with the production function $Q = \int_0^1 \min[2, q_i]\, di$ generating world demand for intermediate goods.

[16]I assume that Y^c is bounded between 0 and $2-\mu$. The upper bound of $2-\mu$ guarantees that market entry is always a credible threat, no matter how high the costs are.

options for the home (HI) and the foreign variety producer (FI):

$$HI = \underbrace{2 - y_H^c - \tilde{\gamma}_H}_{y_H^I} - w(y_H^I) \quad \text{and} \quad FI = \underbrace{0 - y_F^c + \tilde{\gamma}_H}_{y_F^I} - w(y_F^I) \tag{4.3}$$

where $\tilde{\gamma}_H$ is the transfer from the 'weak' (foreign) to the 'strong' (domestic) firm. The outside option of both producers is Bertrand competition in which case the foreign firm, taking the price index P as given, will drive down the price to p^{zero} such that

$$\underbrace{2p^{zero} - y_F^c}_{y_F^{zero}} - w(y_F^{zero}) = 0 \tag{4.4}$$

From anticipating the result of the Nash-bargaining between the foreign firm and its worker one knows that $w(y_F^{zero}) = \mu$. Hence, the lowest price that the foreign firm will be willing to set is $p^{zero} = \frac{\mu + y_F^c}{2}$. The corresponding outside options HO and FO are therefore:

$$HO = \underbrace{2p^{zero} - y_H^c}_{y_H^O} - w(y_H^O) \quad \text{and} \quad FO = \underbrace{0 - y_F^c}_{y_F^O} - w(y_F^O) \tag{4.5}$$

Firms anticipate the result of the wage Nash-bargaining which implies that $y - w(y) = (1-\omega)\left[y - \mu\right] - \omega F$. Insert this expression in all inside and outside values and apply the typical Nash bargaining rule

$$HI - HO = FI - FO \tag{4.6}$$

where the bargaining power was chosen to be 1/2 because of symmetry. This will result in

$$HI = (1-\omega)\left[2 - y_H^c - \tilde{\gamma}_H - \mu\right] - \omega F \quad \text{and} \quad HO = (1-\omega)\left[y_F^c - y_H^c\right] - \omega F \tag{4.7}$$

$$FI = (1-\omega)\left[\tilde{\gamma}_H - y_F^c - \mu\right] - \omega F \quad \text{and} \quad FO = (1-\omega)\left[-y_F^c - \mu\right] - \omega F \tag{4.8}$$

and consequently the agreed side payment is

$$\tilde{\gamma}_H = \frac{2 - \mu - y_F^c}{2} \tag{4.9}$$

which lies between 0 and 1 given the assumptions. Because of symmetry there is the following transfer $\tilde{\gamma}_F = \frac{2-\mu-y_H^c}{2}$ from the foreign to the domestic variety producer if $y_F^c < y_H^c$. As mentioned before there is an alternative but equivalent scenario in which both firms supply their local markets and the 'weak' firm pays the strong firm the transfer γ of revenues or market share. In that case, the side payment is given as $\gamma_H = \frac{\mu + y_F^c}{2} \equiv \gamma(y_F^c)$ if the domestic firm is the 'strong' firm and as $\gamma(y_H^c)$ if the foreign firm has lower costs.

Intuitively the side payment a firm receives increases in the cost of the other firm. For the domestic firm this implies the following value of production y'_H after integration:

$$y'_H = \begin{cases} 1 + \gamma(y^c_F) - y^c_H & \text{if } y^c_H < y^c_F \\ 1 - \gamma(y^c_H) - y^c_H & \text{if } y^c_F < y^c_H \end{cases} \tag{4.10}$$

y'_F can again be trivially derived by symmetry. Before-wage profits as given in (4.10) have a nice interpretation. While in the closed economy before-wage earnings are always $1 - y^c$ they will be higher or lower now with probability $1/2$ each. A productive firm has a potential of stealing business from an unproductive foreign firm and raise its market share, while at the same time the opposite could happen. As firms are ex-ante unaware of whether they will be the 'strong' or the 'weak' firm this implies an increase in revenue risk for a variety producer. It is easy to see that both γ_H and γ_F are equal in expectation which implies that opening the economy implies potential losses or gains of equal size on average such that the unconditional mean of Y' will be the same as before.

Lemma 3. *The integration of both product markets implies that the mean of Y' is preserved while $Var(Y') > Var(Y)$.*

The proof is provided in appendix A. Lemma 3 hints to the main result that international integration as describe above indeed increases the spread of before-wage profits Y with all the discussed consequences in a way[17] that was taken as given in the labor market analysis so far. The following proposition summarizes the main implications.

Proposition 3. *The integration of both product markets leads to higher labor turnover and has ambiguous employment effects. Average and total net output, the average wage and the wage dispersion increase.*

Proof. This follows directly from combining lemma 3 with lemmata 1 and 2 and corollary 1. ∎

Increased job creation and destruction as well as the ambiguous effect on employment is a direct result of market integration leading to a spread in revenues and before-wage profits as discussed in the previous section. While gross output, given my assumptions, only varies with employment, average costs decrease as it is more likely that unproductive firms shut down production. Consequently, average net output increases. The next section will reveal that also total net output rises unambiguously. As wages contain an element proportional to firms' before-wage profits it is clear that the average wage goes

[17]Recall that one requires the slightly stronger assumption of SCS instead of an increase in variance. Market integration will imply also a SCS if the pdf of Y^c, $g_{Y^c}(\cdot)$, is not 'too convex'. A compact and comprehensive proof is still work in progress. Please contact the author directly.

up while the wage distribution becomes more dispersed.

The section closes with some comments on the chosen set-up. Let us first discuss the stability of the bargaining solution. If prices could be adjusted at any time the 'weak' firm has an incentive to receive the transfer γ first but then still to undercut the 'strong' firm afterwards. This means the bargaining solution would not be stable and the only Nash equilibrium would be Bertrand competition. The issue of non-stability is directly connected to simplistic set-up of the model as a static one period game. While the static framework is rather innocuous for the predictions of the labor market model which can be interpreted as a reasonable approximation of a more complex dynamic set-up, the outcome of the revenue determination is highly affected. In a more complex dynamic game, the same two variety producers would face each other every period in order to determine revenues after some shock to y_H^c and y_F^c. It is known from the literature on repeated games that a bargaining solution could be stable in such an environment. To mimic that result and guarantee that the bargaining solution is stable also in this simplistic static framework I introduce the assumption of a price announcement game with sequential moves of the following form. The final good producers call for tender and will decide once they received both price offers which are binding in case of production. Each firm has three strategies: announce a price immediately, wait for the other player and undercut his price by ϵ, or enter bargaining. If both wait both get a 0 market share, also if one waits and the other wants to enter bargaining. In absence of the bargaining option the only Nash equilibrium would be to announce 'Bertrand prices' simultaneously. Assume that if both agree to bargain they have to play the same game again afterwards. Further assume that the solution to the bargain can be more than just a transfer payment but could also include conditional rules such as the following. Consider for example the solution that the 'strong' firm pays γ to the 'weak' firm for announcing a price $1+\epsilon$, with $\epsilon > 0$. Afterwards it can announce a price of 1 and win the tender which gives the solution from above. Observe that given this option non of the both firms has an incentive to deviate.

I further like to add that the presence of the bargaining option with a stable solution is not as crucial for the main idea of the paper as it may appear at first glance but it is modeling wise very convenient. Imagine for the moment that bargaining was not possible or not stable. It is clear that the price would be pushed down such that the 'weak' firm makes non-positive profits and the 'strong' firm (supposed to be the domestic firm here) gets $(1-\omega)\left[y_F^c - y_H^c\right] - \omega F$ for given y_F^c and y_H^c. At a first glance profits seem to have decreased for both firms. But in equilibrium all prices in all sectors will be cut leading to lower price index P with which the expression above has to be corrected. It might consequently happen that profits for the 'strong' firm are increasing in real terms which would mimic the winner-loser pattern from before. The price index $P = \int_0^1 \frac{\mu+\max\left[y^c_{H,i}, y^c_{F,i}\right]}{2} di$ is a more complicated object which makes it hard to disentangle real from price effects. The bargaining solution circumvents this problem as there are no price effects and market integration just leads to a redistribution of revenues which makes it easier to compare with the closed economy benchmark.

5 Welfare analysis

This section presents the welfare analysis of the described model. First, I continue to assume that workers are risk neutral. This gives a trivial first best implementation with no government intervention in the case of 'balanced' search externalities. In a second step it is analyzed how EP implies welfare losses and how these losses are amplified in open economies with increased necessity for labor reallocation while opening the economy in principle boosts welfare in absolute terms. In the last part of this section I check the robustness of these results by introducing risk aversion and UI. The firing externality created by UI gives a motive for using EP as argued by Blanchard and Tirole (2008). It is shown that if endogenous job creation is taken into account the results derived in a risk neutral framework are not very likely to change qualitatively.

5.1 First best allocation

Recall that all workers are ex-ante identical w.r.t. abilities and are assumed to always have the same value of home production. They are indexed by $i \in [0, 1]$. The social planner is subject to the search frictions and the idiosyncratic shocks in the value of production. He has to choose a sequence of wages w_i and unemployment benefits b_i, labor market tightness θ and the cut-off $\underline{y}$ in order to maximize utilitarian welfare subject to a resource constraint. Given risk neutrality the social planner's problem reads:

$$\max_{\{w_i\},\{b_i\},\theta,\underline{y}} \int_0^1 \left[ew_i + (1-e)\mu_i\right] di \tag{5.1}$$

subject to equilibrium employment (2.1) and the following resource constraint

$$\int_0^1 \left[ew_i + (1-e)b_i\right] di = ey^e - c\theta \tag{5.2}$$

This implies

$$(1-\eta)\left[y^e - h\right](1-G) = \frac{c}{m^f} \tag{5.3}$$

$$\underline{y} = h \tag{5.4}$$

Let us compare these conditions with the decentralized equilibrium conditions

$$(1-\omega)\left[y^e-\mu+F\right](1-G)-F=\frac{c}{m^f} \tag{5.5}$$

$$\underline{y}=\mu-F \tag{5.6}$$

First note that because of risk neutrality the optimal UI is zero, i.e. $b=0$. Second, it is clear that one requires $F=0$ for implementation of the first best, given that the Hosios (1990)-condition ($\eta=\omega$) holds which I will assume. Hence, one can work with the welfare generated in a laissez-faire economy as the first best benchmark.

5.2 Second best allocation

Denote the welfare of a laissez-faire economy as $\Omega(0)$ while the welfare in an economy with positive firing costs, $F>0$ is denoted $\Omega(F)$. Defined in the equivalent (net) output terms representation[18] this is

$$\Omega(F)\equiv ey^e+(1-m)\mu+mG\left[\mu-F\right]-c\theta \tag{5.7}$$

subject to the equilibrium values of $\theta,\underline{y},m,G,e$.

Proposition 4. *Welfare is decreasing in firing costs for any non-negative level of firing costs, i.e.* $\frac{d\Omega(F)}{dF}<0$ *for all* $F\geq 0$.

The proof is provided in appendix A. This comes at no surprise as there is no inefficiency present that could justify $F>0$ as already argued in the derivation of the first best allocation. Proposition 4 extends this result by showing that welfare is monotonically decreasing. I will analyze the effect of F on welfare in an environment that is characterized by additional inefficiencies in section 5.3.

Proposition 5. *Welfare is weakly increasing for any MPSCS of* Y.

The proof is provided in appendix A. Note that proposition 5 even holds if a MPSCS leads to a fall in employment which is always overcompensated by the increase in average output.

Proposition 6. *The welfare loss due to firing costs is weakly increasing in job turnover if* $\eta=\omega\leq 0.5$.

The proof is provided in appendix A. The key message is that the welfare loss of EP is particularly severe for economies with high firing costs in combination with a high

[18] Both, utility and output maximization coincide in the case of risk neutrality as shown in appendix B.

matching elasticity i.e. if the number of matches is more responsive to an increase in vacancies (small η). Note that proposition 6 states a weak condition for the welfare loss to increase in job turnover. The same can be true for considerably higher levels of η and ω. Given the results of the previous section this implies that an integrated or open economy suffers relatively more in efficiency terms from firing restrictions. It still enjoys higher welfare than a closed or less open economy in absolute terms, while the distance to a possible first best rises. Firing restrictions can hence lead to a substantial failure in reaping the possible welfare gains that could result from economic integration by preventing necessary labor reallocation.

5.3 Risk aversion, unemployment insurance, and firing externalities

In this section I relax the assumption of risk neutrality and impose risk aversion. This implies that UI should be positive which creates a firing externality as described in Blanchard and Tirole (2008). Firms do not internalize the costs created by an UI system when they decide to lay off a worker. This externality can be counteracted by EP in form of a firing tax. Before the welfare analysis is presented it is discussed how equilibrium is affected by the introduction of risk aversion, implying $u'(x) > 0$ and $u''(x) < 0$. Note that the wage bargaining condition changes as follows

$$\frac{\omega}{u(w) - u(\mu)} u'(w) = \frac{1 - \omega}{y - w + F} \tag{5.8}$$

One can first-order Taylor approximate $u(x)$ around w and evaluate the function at $x = \mu$ which gives

$$u(w) - u(\mu) \approx u'(w)(w - \mu) \tag{5.9}$$

Using this handy approximation results in the same wage schedule as before

$$w(y) = (1 - \omega)\mu + \omega \left[y + F\right] \tag{5.10}$$

Hence, the equilibrium allocation is again determined by

$$(1 - \omega) \left[y^e - \mu + F\right] (1 - G) - F = \frac{c}{m^f} \tag{5.11}$$

$$\underline{y} = \mu - F \tag{5.12}$$

This conveniently implies that the employment and output level is independent of the degree of risk aversion. Consequently, propositions 4 to 6 are also true in the environment with risk aversion if the terms 'welfare', defined as the sum of workers' utilities, is replaced by 'output'. This distinction is important as output and welfare do not coincide anymore. Hence, in contrast to output, welfare will depend on the degree of risk aversion. The subsequent part of this section discusses the optimization of welfare.

5.3.1 First best allocation

In case of risk aversion the social planner's problem reads:

$$\max_{\{w_i\},\{b_i\},\theta,\underline{y}} \int_0^1 \left[eu(w_i) + (1-e)u(\mu_i)\right] di \tag{5.13}$$

subject to equilibrium employment (2.1) and the resource constraint (5.2) The FOC for w_i and b_i state that

$$u'(w_i) = \lambda = u'(\mu_i) \tag{5.14}$$

where λ is the Lagrange multiplier. (5.14) has two implications: first every employed worker receives the same wage, i.e. $w_i = w$, and every unemployed receives the same benefits $b_i = b$. Second, there is full insurance, i.e. $w = \mu = b + h$. Inserting the full insurance result in the FOC for θ and $\underline{y}$ results in

$$(1-\eta)\left[y^e - h\right](1-G) = \frac{c}{m^f} \tag{5.15}$$

$$\underline{y} = h \tag{5.16}$$

Let us consider a possible first best implementation now. Note by comparing (5.12) and (5.16) that optimal job destruction requires $b = F$ (as in Blanchard and Tirole (2008)). But observe how $b = F > 0$ will always lead to inefficiently low job creation which is not present in Blanchard and Tirole (2008). The only first best allocation would be given by $b = F = 0$ and $w = h$. This raises one problem: $w = h$ is incompatible with Nash bargained wages[19]. Hence, there does not exist a decentralized implementation of the first best.

5.3.2 Second best allocation

Let us look for a second best allocation, i.e. the market solution that maximizes welfare, now. To allow for more flexibility an additional financing instrument in form of simple non-distortionary[20] lump-sum taxes T is introduced. Note that in the Blanchard and Tirole (2008) framework without endogenous job creation this would not change the result that a firing tax should be used to internalize the firing externality which otherwise leads firms

[19]See Michau (2009) for a detailed discussion of this issue in a dynamic setting.

[20]This implies that the gross wage $w(y)$ is independent of T and is still given by 2.7.

to excessive job destruction. The problem changes as follows

$$\Omega(F) \equiv \max_{b} m \int_{\underline{y}}^{\infty} u(w(y) - T) dG(y) + (1-e) u(\mu - T) \tag{5.17}$$

subject to the equilibrium values of θ, $\underline{y}$, $w(y)$, m, G, and e and the budget constraint where I assume that F can be completely collected as a firing tax

$$T = b(1-e) - mGF \tag{5.18}$$

Let us define $\tilde{w} = w(y) - T$ and $\tilde{\mu} = \mu - T$. Use the envelope theorem to compute $\frac{d\Omega(F)}{dF}$. Insert for $\frac{de}{dF} = (1-G)\frac{dm}{dF} - m\frac{dG}{dF}$, expand and rewrite the expression as

$$\begin{aligned} \frac{d\Omega(F)}{dF} =& \frac{dm}{dF} \int_{\underline{y}}^{\infty} \left[u(\tilde{w}) - u(\tilde{\mu})\right] dG(y) + m\omega \int_{\underline{y}}^{\infty} u'(\tilde{w}) dG(y) \\ &+ \frac{dT}{dF} \left[m \int_{\underline{y}}^{\infty} \left[u'(\tilde{\mu}) - u'(\tilde{w})\right] dG(y) - u'(\tilde{\mu}) \right] \end{aligned} \tag{5.19}$$

All three integral expressions are positive. The first is so because $w(y) \geq \mu, \forall y \geq \underline{y}$. The third integral expression is positive because of concavity of $u(\cdot)$. As $\frac{dm}{dF} < 0$ and the change in T

$$\frac{dT}{dF} = -b\frac{de}{dF} - \frac{dm}{dF}GF - \frac{dG}{dF}mF - mG \tag{5.20}$$

cannot be signed, the effect on total welfare is in principle ambiguous, even for $F = 0$. Why is this result different from Blanchard and Tirole (2008) where one would have $\frac{d\Omega}{dF}|_{F=0} > 0$? The main difference is the endogenous job creation margin. It is still true that firms do not internalize the social costs of firing, but they also do not internalize the social benefit from hiring. The intuition is that if $\frac{de}{dF} < 0$ then marginally increasing F implies that the welfare loss of too little hiring outweighs the welfare gain of reduced firing. To investigate this thought let us check how $\frac{d\Omega(F)}{dF}$ is signed at $F = 0$. There are two cases:

1. if $d\Omega/dF > 0$ at $F = 0$ then $F^* > 0$

2. if $d\Omega/dF < 0$ at $F = 0$ then $F^* = 0$

I will address this issue with a small numerical example[21] to get a hint which of both cases seems to be more plausible. The functional forms and parameters were chosen in accordance with the literature and to replicate an unemployment rate of $1 - e = 0.1$. The

[21]The results of this calibrated static model should be interpreted with great care, as the model was mainly designed to derive qualitative results. For a more realistic assessment concerning the magnitude of the featured effects one should employ a full dynamic version of the model, which is left for future research.

choice of UI $b = 0.1$ was determined by welfare maximization according to (5.17) and represents a gross replacement rate of $\frac{b}{w^e} = 0.3$. The parameters and results are shown in table D.1 and indicate that for the chosen calibration the second case in which optimal EP is equal to zero is relevant. Hence, the result that EP leads to a welfare loss is likely to hold also in the case of risk aversion.

6 Concluding remarks

A parsimonious static version of the Diamond-Mortensen-Pissarides model with endogenous job creation and job destruction is combined with a North-North intermediate goods trade framework. The effects of openness to trade are propagated to the labor market through changes in revenues which are normally assumed to be constant in the canonical search and matching model. The final good production technology is such that there is no substitution between different domestic intermediate goods, while a foreign variety could replace a home variety in the same industry as an input. International market integration implies that within an industry, consisting now of a home and foreign intermediate good producer, market shares and revenues are redistributed from the less to the more productive firm. As firms are ex-ante unaware of their productivities, openness to intermediate goods trade widens the distribution of revenues and profits a firm can expect. The intuition is that a domestic firm has to form expectations not only about its own productivity as in the closed economy benchmark but also about the productivity of the competing rival firm abroad and the consequences for market shares. The increased risk in potential revenues leads to more job creation, more job destruction, higher output, welfare, and wage inequality while the effect on employment is ambiguous. It is shown that the effects of international market integration are qualitatively identical to a reduction in employment protection in form of costly firing restrictions, except for wage inequality which would decrease. Further, the positive welfare effects of opening to trade are decreasing in the level of firing costs which can therefore lead to a substantial failure in reaping the benefits that could result from economic integration, by preventing necessary labor reallocation.

Some further concluding comments are in order. First, a trade shock was analyzed by comparing the limiting cases of a closed versus a trade friction free open economy. Hence, the presented model is ignorant about in-between cases of gradual trade liberalization. Although not formalized, an according extension could look as follows. Suppose that trade costs entered the model in a way that reduces the surplus in the revenue bargaining game. Consequently, this would also decrease the side payment and the ex-ante revenue volatility. A reduction in trade costs would therefore have the same qualitative effects as the scenario of opening to trade without restrictions. Second, I used the term 'market integration' hinting at a fusion of two markets into one, which is not entirely the case. In contrast to many trade models the amount of varieties does not double, but stays constant inducing more competition within every variety sector. But there is no reason to rule out that in the long run variety producers can adapt their production techniques and that a persistent specialization pattern evolves. Hence, one should interpret the present model rather in a medium run perspective. Third, the model was designed in a very stylized way in order to analyze the effects of openness to trade and employment protection in a qualitative way. For quantitative exercises such as a cross-country welfare evaluation of existing firing restriction legislations given the observed openness to trade, one would have to employ a dynamic version of the model. This is left for future research.

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Appendix

A Proofs and derivations

Comparative statics w.r.t. F and **proof of proposition 1**:

Proof.

$$\frac{d\theta}{d\pi^e} > 0, \frac{d\theta}{d\phi} > 0, \frac{d\theta}{dc} < 0, \frac{d\theta}{d\eta} < 0 \tag{A.1}$$

$$\frac{d\underline{y}}{dF} = -1, \frac{dy^e}{d\underline{y}} = \frac{g_Y(\underline{y})(y^e - \underline{y})}{1 - G} > 0 \tag{A.2}$$

$$\frac{d\pi^e}{d\underline{y}} = -(1-\omega)(1-G) < 0 \tag{A.3}$$

$$\frac{d\pi^e}{dF} = -\left[1 - (1-\omega)(1-G)\right] < 0 \tag{A.4}$$

$$\frac{dm}{dF} = \frac{1-\eta}{\eta}\frac{m}{\pi^e} \cdot \frac{d\pi^e}{dF} = \frac{1-\eta}{\eta}\frac{m^2}{c\theta} \cdot \frac{d\pi^e}{dF} < 0 \tag{A.5}$$

$$\frac{dG}{dF} = -g_Y(\underline{y}) < 0 \tag{A.6}$$

$$\frac{de}{dF} = (1-G) \cdot \frac{dm}{dF} - m \cdot \frac{dG}{dF} \tag{A.7}$$

$$\frac{de}{dF} = -(1-G)\frac{1-\eta}{\eta}\frac{m^2}{c\theta}\left[1 - (1-\omega)(1-G)\right] + mg_Y(\underline{y}) \overset{?}{\gtreqless} 0 \tag{A.8}$$

■

Proof of proposition 2:

Proof. The first part follows directly from the drop in the cut-off value $\underline{y}$, according to equation (2.8). For the second part write compute the marginal change of the average wage $w^e = (1-\omega) + \omega(y^e + F)$.

$$\frac{dw^e}{dF} = \omega \left(1 - \underbrace{\frac{g(\underline{y})(y^e - \underline{y})}{1 - G(\underline{y})}}_{\chi} \right)$$

Whether the term χ is smaller or greater than 1 depends on the underlying distribution. For many distributions like uniform, normal, log-normal (with sufficiently small variance), beta (in the range used in this paper) etc. χ is bound between 0 and 1 for all $\underline{y}$ which consequently implies that F increases the average wage. ∎

Proof of lemma 2:

Proof. First, rewrite the MP condition. By the definition of MP one knows that $\int_{-\infty}^{\infty} y dG_{Y'}(y) - \int_{-\infty}^{\infty} y dG_Y(y) = 0$. Integrate by parts to get

$$\lim_{y\to\infty} \{y\,[G_{Y'}(y) - G_Y(y)]\} - \lim_{y\to-\infty} \{y\,[G_{Y'}(y) - G_Y(y)]\} - \int_{-\infty}^{\infty} [G_{Y'}(y) - G_Y(y)]\, dy = 0$$

as both limits tend to zero because of the definition of $G_Y(\cdot)$ and $G_{Y'}(\cdot)$ as cdfs it was established that

$$\int_{-\infty}^{\infty} [G_{Y'}(y) - G_Y(y)]\, dy = 0 \tag{A.9}$$

Split the integral such that

$$\int_{-\infty}^{\underline{y}} [G_{Y'}(y) - G_Y(y)]\, dy + \int_{\underline{y}}^{\infty} [G_{Y'}(y) - G_Y(y)]\, dy = 0, \quad \forall \underline{y} \in \mathbb{R} \tag{A.10}$$

I will now establish that the first term in (A.10) is non-negative while the second term is non-positive. Consider two cases, first $\underline{y} \geq \hat{y}$. Integrate the SCS condition from above to get

$$\int_{\underline{y}}^{\infty} G_Y(y)dy \geq \int_{\underline{y}}^{\infty} G_{Y'}(y)dy \tag{A.11}$$

Note that the same is true in the second case $\underline{y} \leq \hat{y}$: Integrate the SCS condition from below and use (A.10) to arrive at (A.11) which now consequently holds $\forall \underline{y} \in \mathbb{R}$. To show that job creation is increasing it suffices to prove that the term $\Lambda \equiv \left[y^e - \underline{y}\right](1 - G_Y(\underline{y})) = \int_{\underline{y}}^{\infty} (y - \underline{y}) dG_Y(y)$ is increasing as a result of a MPSCS. Hence, one has to show that $\Lambda' - \Lambda \equiv \Xi$ is non-negative, i.e.

$$\Xi = \left[\int_{\underline{y}}^{\infty} y dG_{Y'}(y) - \big(1 - G_{Y'}(\underline{y})\big)\, \underline{y} \right] - \left[\int_{\underline{y}}^{\infty} y dG_Y(y) - \big(1 - G_Y(\underline{y})\big)\, \underline{y} \right] \geq 0 \tag{A.12}$$

Integrate this expression by parts and take the limits to get

$$\Xi = -\underline{y}\left[G_{Y'}(\underline{y}) - G_Y(\underline{y})\right] - \int_{\underline{y}}^{\infty}\left[G_{Y'}(y) - G_Y(y)\right]dy - \left(1 - G_{Y'}(\underline{y})\right)\underline{y} + \left(1 - G_Y(\underline{y})\right)\underline{y} \tag{A.13}$$

Terms cancel, leaving $\Xi = -\int_{\underline{y}}^{\infty}\left[G_{Y'}(y) - G_Y(y)\right]dy$ which, as has been established before, has to be non-negative. ■

Proof of corollary 1:

Proof. Recall that in the proof of lemma 2 it was established that $\Lambda \equiv (1-G)(y^e - \underline{y})$ is increasing for any kind of MPSCS. Lemma 1 states that G is increasing for any kind of SCS if $\underline{y} \leq \hat{y}$. Both results can only be true at the same time if y^e is also increasing. Inserting in the wage equation (2.7) automatically reveals that the average wage w^e has to rise as well. The increase in the wage dispersion is a direct result of the MPSCS of Y. ■

Proof of lemma 3:

Proof. First note that $E(Y) = 1 - E(Y^c)$ and $Var(Y) = Var(Y^c)$. The same holds for Y' as the following lines show. Remember that the random variable Y' was defined as follows

$$y' = \begin{cases} 1 + \gamma(y_F^c) - y_H^c & \text{if } y_H^c < y_F^c \\ 1 - \gamma(y_H^c) - y_H^c & \text{if } y_F^c < y_H^c \end{cases}$$

To save notation define the cases $H : y_H^c < y_F^c$ and $F : y_F^c < y_H^c$. The expected value of Y' is given as

$$\begin{aligned} E(Y') &= P(H)E\left(1 + \gamma(Y_F^c) - Y_H^c|H\right) + \\ &\quad P(F)E\left(1 - \gamma(Y_H^c) - Y_H^c|F\right) \\ &= 1 - E(Y_H^c) + {}^1\!/\!_2 E\left(\gamma(Y_F^c)|H\right) - {}^1\!/\!_2 E\left(\gamma(Y_H^c)|F\right) \\ &= 1 - E(Y_H^c) + {}^1\!/\!_2 \gamma\left(E(Y_F^c|H) - E\left(Y_H^c|F\right)\right) \\ &= 1 - E(Y^c) \end{aligned}$$

where I used lemma 4 from appendix C.2, the linearity of $\gamma(\cdot)$ and the fact that Y_H^c and Y_F^c are equally distributed. Apply the same logic for computing the variance using $Var(Y') = E(Y'^2) - E(Y')^2$. It has to be shown that $Var(Y') > Var(Y)$. As $E(Y') = E(Y)$ it is sufficient to show that $E(Y'^2) > E(Y^2) = 1 - 2E(Y^c) + E(Y^c)^2$. $E(Y'^2)$ is given as

$$\begin{aligned}
E(Y'^2) &= P(H)E([1+\gamma(Y_F^c) - Y_H^c]^2\,|H) + \\
&\quad P(F)E([1-\gamma(Y_H^c) - Y_H^c]^2\,|F) \\
&= {}^1\!/\!_2 E\left(1 - 2Y_H^c + (Y_H^c)^2|H\right) + {}^1\!/\!_2 E\left(2\gamma(Y_F^c) - 2Y_H^c\gamma(Y_F^c) + \gamma(Y_F^c)^2|H\right) + \\
&\quad {}^1\!/\!_2 E\left(1 - 2Y_H^c + (Y_H^c)^2|F\right) + {}^1\!/\!_2 E\left(-2\gamma(Y_H^c) + 2Y_H^c\gamma(Y_H^c) + \gamma(Y_H^c)^2|F\right) \\
&= E(Y^2) - E(Y_H^c\gamma(Y_F^c)|H) + E(Y_H^c\gamma(Y_H^c)|F) + E(\gamma(Y_F^c)^2|H) \\
&= E(Y^2) - E(Y_H^c\gamma(Y_F^c)|H) + E(Y_F^c\gamma(Y_F^c)|H) + E(\gamma(Y_F^c)^2|H)
\end{aligned}$$

where I made use of the law of total expectation, lemma 4, and the equality of both cost distributions. The proof is complete if one can show that

$$E(Y_H^c\gamma(Y_F^c)|H) < E(Y_F^c\gamma(Y_F^c)|H) + E(\gamma(Y_F^c)^2|H)$$

which is true if

$$\begin{aligned}
& y_H^c\gamma(y_F^c) < y_F^c\gamma(y_F^c) + \gamma(y_F^c)^2, \qquad \forall y_H^c < y_F^c \\
\iff\ & y_H^c < y_F^c + \gamma(y_F^c), \qquad \forall y_H^c < y_F^c
\end{aligned}$$

The last statement was derived by using $\gamma(y_F^c) > 0$ and is clearly always true. ∎

Proof of proposition 4:

Proof. The derivative of 5.7 w.r.t. F reads

$$\begin{aligned}
\frac{d\Omega(F)}{dF} =& \frac{dm}{dF}\int_{\underline{y}}^{\infty} y\,dG_Y(y) + mg(\underline{y})(\mu - F) \\
& - \frac{dm}{dF}\mu + \frac{dm}{dF}G(\mu - F) - mG - mg(\underline{y})(\mu - F) - c\frac{d\theta}{dF}
\end{aligned} \tag{A.14}$$

Use $\frac{dm}{dF} = (1-\eta)m^f \cdot \frac{d\theta}{dF}$ and rewrite this equation as

$$\frac{d\Omega(F)}{dF} = \left(m^f\left[(1-\eta)(y^e - \mu + F)(1-G) - F\right] - c\right)\frac{d\theta}{dF} + m^f\eta F\frac{d\theta}{dF} - mG \tag{A.15}$$

Given that the Hosios-condition holds the first term is zero because of the free entry condition (5.5). Using $\frac{d\theta}{dF} = -\frac{m}{\eta c}\left[\omega + (1-\omega)G\right]$ leaves

$$\frac{d\Omega(F)}{dF} = -\frac{m^2}{\theta}\left[\omega + (1-\omega)G\right]\frac{F}{c} - mG < 0 \tag{A.16}$$

∎

Proof of proposition 5:

Proof. Note that the welfare function (5.7) can be rewritten as

$$\Omega(F) = e\omega\left[y^e - \mu + F\right] + \mu = \omega m\Lambda + \mu \tag{A.17}$$

as explained in appendix B. Given the assumptions, Λ and m are both non-decreasing as a result of a MPSCS as established in the proof of lemma 2. ∎

Proof of proposition 6:

Proof. It has to be established under which conditions (A.16) is decreasing in job turnover, i.e. m and G. The condition $\eta \leq 0.5$ guarantees that the term $\frac{m^2}{\theta}$ in (A.16) is increasing in θ. In that case $\frac{d\Omega(F)}{dF}$ is decreasing for any increase in m and/or G.

∎

B Utility and output maximization

Both concepts, utility and output maximization, coincide in case of risk neutral workers. Consider the second best welfare function for utility maximization

$$\Omega^{util} = m\int_{\underline{y}}^{\infty} w(y)dG(y) + (1-e)\mu \tag{B.1}$$

subject to the equilibrium conditions. Insert the wage schedule (2.5) and rearrange to get

$$\Omega^{util} = e(1-\omega)\mu + e\omega y^e + e\omega F + (1-e)\mu \tag{B.2}$$

Rewrite the free entry condition (5.5) as $e(1-\omega)\left[y^e - \mu + F\right] - mF - c\theta = 0$ and add it to (B.2) to get

$$\Omega^{util} = ey^e + (1-e)\mu - mGF - c\theta = \Omega^{output} \tag{B.3}$$

which is identical to the output maximization objective function (5.7).

C Math sheet

This section summarizes mathematical concepts that have been heavily used in this paper for easier reference.

C.1 Change of variables

Let X be a univariate continuous random variable with cdf $G_X(\cdot)$ and pdf $g_X(\cdot)$. Let f be a monotone transformation such that $Y = f(X)$. Then the cdf and pdf of Y are defined as follows

a) if $f(\cdot)$ is increasing:

$$G_Y(y) = G_X(f^{-1}(y)) \quad \text{and} \quad g_Y(y) = g_X(f^{-1}(y))\frac{1}{f'(f^{-1}(y))}$$

b) if $f(\cdot)$ is decreasing:

$$G_Y(y) = 1 - G_X(f^{-1}(y)) \quad \text{and} \quad g_Y(y) = -g_X(f^{-1}(y))\frac{1}{f'(f^{-1}(y))}$$

C.2 Sum and difference of random variables

Let X and Y be two independent univariate continuous random variables with the according cdfs and pdfs. Define $Z = X + Y$. Then $G_Z(\cdot)$ is the convolution

$$G_Z(x) = (G_X * G_Y)(y) = \int_{-\infty}^{\infty} G_X(x-y)dG_Y(y) = \int_{-\infty}^{\infty} G_Y(x-y)dG_X(y)$$

$$g_Z(x) = (g_X * g_Y)(y) = \int_{-\infty}^{\infty} g_X(x-y)g_Y(y)dy = \int_{-\infty}^{\infty} g_Y(x-y)g_X(y)dy$$

Now define $Z = X - Y$. Then $G_Z(\cdot)$ is the cross-convolution

$$G_Z(x) = (G_X \star G_Y)(y) = \int_{-\infty}^{\infty} \left[1 - G_Y(y-x)\right] dG_X(y) = \int_{-\infty}^{\infty} G_X(x+y)dG_Y(y)$$

$$g_Z(x) = (g_X \star g_Y)(y) = \int_{-\infty}^{\infty} g_Y(y-x)g_X(y)dy = \int_{-\infty}^{\infty} g_X(x+y)g_Y(y)dy$$

All values are assumed to be real-valued.

Lemma 4. *Let X and Y be two independent univariate continuous random variables that are equally distributed. Then the probability $P(x < y)$, where x and y are independent random draws, is equal to $P(y < x) = 1/2$.*

Proof. First note that $G_X(x) = G_Y(x), \forall x \in \mathbb{R}$. Define $Z = X - Y$. Clearly,

$$
\begin{aligned}
&P(x<y) = P(z<0) = G_Z(0) = \\
&\int_{-\infty}^{\infty} [1 - G_Y(y)]\, dG_X(y) = \int_{-\infty}^{\infty} G_X(y) dG_Y(y) = \\
&\int_{-\infty}^{\infty} [1 - G_Y(y)]\, dG_Y(y) = \int_{-\infty}^{\infty} G_Y(y) dG_Y(y) = \\
&1 - \int_{-\infty}^{\infty} G_Y(y) dG_Y(y) = \int_{-\infty}^{\infty} G_Y(y) dG_Y(y) \Leftrightarrow \\
&{}^1\!/_2 = \int_{-\infty}^{\infty} G_Y(y) dG_Y(y) = G_Z(0) = P(x<y)
\end{aligned}
$$

∎

D Tables and figures

Table D.1: Numerical example

Parameters	Results
$u(x) = \frac{x^{1-\psi}-1}{1-\psi}$	$\theta = 2.81$
$m = \phi\theta^{1-\eta}$	$\underline{y} = 0.15$
$Y \sim beta(\alpha, \beta)$	$e = 0.9$
$b = 0.1$	$m = 0.96$
$F = 0$	$m^f = 0.34$
$\psi = 3$	$G = 0.06$
$\eta = 0.5$	$\frac{w^e}{b} = 0.3$
$\omega = 0.5$	$T = 0.01$
$\alpha = 2$	$\frac{de}{dF} = -1.97$
$\beta = 2$	$\frac{dm}{dF} = -2.87$
$h = 0.05$	$\frac{dG}{dF} = -0.77$
$c = 0.06$	$\frac{dT}{dF} = 0.14$
$\phi = 0.57$	$\frac{d\Omega}{dF} = -28.33$

Figure D.1: MPSCS - cdfs

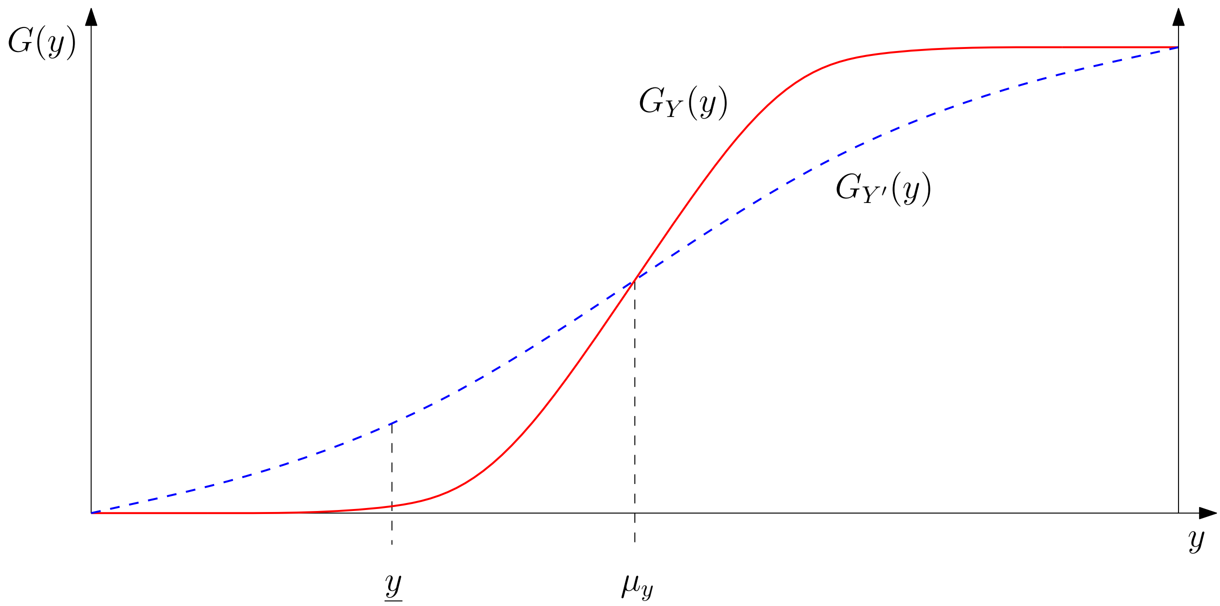

Figure D.2: MPSCS - pdfs