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Party bosses vs. Party Primaries: quality of legislature under different selectorates

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Party-bosses vs. party-primaries: quality of legislature under different *selectorate*s (DRAFT)

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Abstract

Using a theoretical model, we study the equilibrium quality of the legislature under two selectorates: party-principal and party-primary. In the model, two parties compete in three districts; each party has three candidates who differ in their quality. Each voter prefers higher quality, but the median voters in each district differ in their most-preferred policies: two are home districts of each party while the third district is a battleground district characterized by weaker policy preference. In the special case when neither party has an advantage (in the quality of its candidates or the popularity of its policies), we find that the quality of the legislature is never lower (and, generally, strictly higher) under party-primaries. In the general case with no restrictions on the relative strength of either party, still, if an equilibrium legislature of optimal quality exists under party-principals, then one will exist under party-primaries, but the reverse is not true.

> The aim of every political Constitution, is or ought to be, first obtain for rulers who possess most wisdom to discern, and most virtue to pursue, the common good of the society; and in the next place to take the most effectual precautions for keeping them virtuous whilst they continue to hold their public trust. (Madison, 1788)

1 Introduction

Like many other "goods", there is a scarcity of high-quality (able and honest) politicians. Elections are part of the mechanism through which this resource is allocated; voters desiring high-quality candidates can vote for them. But, voters have preferences over issues other than candidate quality (such as the policy of the candidate); so, a high quality candidate from a given party may end up losing the election if he runs in a district where the policy of the party is highly unpopular. Then, how much of its scarce resource (the high-quality politicians) a society is able to use also depends on where these candidates are fielded. The political body that fields

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the candidates is called the selectorate (blending the words selector and electorate, Paterson (1967) coined the word). Different selectorates have different objectives; thus, it is no surprise that the allocation of candidates may differ under them. In this paper, we study how the quality of elected legislatures differs under two types of selectorates: a party-principal and a party-primary. We find that under a large set of parameters, party-primaries have more desirable features in terms of the resulting legislature quality.

In the model we study, two political parties compete over three single-member districts in a legislative election. Each party has a pool of three candidates who differ only in their quality (or, valence (Stokes, 1963)), i.e., non-policy characteristics that are desirable by all voters such as honesty and competence. Each party has a fixed policy. The (median voters of the) districts differ in their policy preferences. Two are the home districts of each party (partisan districts where the median voter strictly prefers the policy of one party). The remaining district is a battleground district, in which partisanship, if there is any, is weaker than in the home districts.

The candidates are fielded either by an exclusive and centralized selector(ate), which we refer to as the party-principal, or by an inclusive and decentralized selectorate which we refer to as the party-primary.[1] Under both selectorates the candidates propose (and, when elected support) the party policy.[2] Under either selectorate, the candidate allocation problem can be modeled as a game. In the first case, the players are party-principals; each principal's objective is to maximize the number of expected seats his party wins. In the second case, the players are the candidates (the party-primary always chooses the candidate with the highest quality); the candidates are selfish with the objective of each to maximize the probability that he wins a seat. Given two outcomes in which this probability is the same, he chooses the one where his party wins more seats.

We first study a symmetric election in which the battleground district is indifferent between the policies of the parties and neither party has an advantage in terms of its resources (the quality of candidates or the strength of support for its policies in its home district). In this case, we find that when each party's home district is safe (the second highest-quality candidate from the party can defend the district against the highest-quality candidate from the other party), each party-principal nominates his best candidate in the battleground district. Intuitively, nominating a highest-quality candidate in a safe district is a waste of resources when there is another district that only this candidate has any chance of winning. When both highest-quality candidates are nominated in the same district, only one of these highest-quality candidates win the election, resulting in suboptimal legislature quality. Exactly how

[1]Rahat, (2007 p. 160) notes that until 2000, the leader of Mexico's long-ruling Institutional Revolutionary Party handpicked all of its Senate candidates, yet, nowadays, a party-leader determining alone the party's candidates is less common among established democracies exceptions are several populist parties on Europe's extreme right, including parties in Denmark, Italy, and Norway. Our analysis applies when one replaces the party-principal with a centralized party committee that has the same objective (to maximize the expected number of seats won).

[2]Discussing the adoption of party-primaries in US, Ansolobehere, Hirano and Snyder (2007, p. 22) note that the overall assessment in the literature for US is that there is less party discipline when candidates are fielded through the party-primaries. Yet, they also note that "[t]he ratio of conjecture to hard evidence in this literature is quite high, however, and the evidence that exists is decidedly mixed." In Canada where the dues-paying local party members are the selectorate, the party discipline has been considerably high (Malloy, 2006).

suboptimal depends, in part, on the strength of partisanship in the home districts. When these are sufficiently partisan, there exists a pure strategy Nash Equilibrium (PSNE) in which each party-principal nominates his lowest-quality candidate in the home turf of the party, leading to a legislature with a majority of lowest-quality politicians. When the home district of each party is not safe, but still defendable by the lowest-quality candidate from the party against the second highest-quality candidate from the other party, there exists a PSNE in which each party nominates its best candidate in the other party's home district (each of them win these districts). In that PSNE, the second highest-quality candidates compete in the battleground district; each win with probability one half. The resulting equilibrium legislature is of optimal quality. Finally, when the home districts are not safe nor can be defended by a lowest-quality candidate against a second highest-quality candidate from the other party, there exists no PSNE. All mixed strategy Nash equilibria under party-principals in the symmetric case result in a stochastic legislature quality that is suboptimal with non-zero probability.

In the symmetric case, under party-primaries the equilibrium legislature quality is always optimal. The (selfish) highest-quality candidate from each party runs in the primary of a district that he will certainly win in the legislative election, such as the home district of his party. As a result, each of the highest-quality candidates and one of the candidates with the next-highest quality win a seat in the legislature. Thus, while the home districts gain under party-primaries (in terms of representative quality), the battleground districts may lose.

We then examine the most general case, allowing (almost) any configuration of candidate quality between the two parties and only restricting the districts to the degree that at least one district favors each party's policies. It is impossible to calculate exactly what happens for each possible distribution of candidate valences and district preferences: depending on the relative size of these two types of parameters, under party-principals there are more than two-hundred thousand different games. Another feature of the general case, which makes our predictions less precise, is the existence of multiple equilibria with different legislature quality under the same selectorate and in the same game (election).

Still, in this most general framework, we prove that if under party-principals optimal legislature quality is an equilibrium outcome, then under party-primaries it will be as well (Theorem 1). We also find that there are cases in which under neither selectorate optimal legislature quality is an equilibrium outcome. Yet, such cases describe uncommon scenarios, such as a setup in which the lowest-quality candidate from one party is better than the highest-quality candidate from the other party, while the home-district of the latter party has such strong policy preferences that even the best candidate from the former party has no chance of winning the seat.

We rule out such unlikely cases (and, with them about the ten percent of the possible games under party principals), and consider elections in which the best candidate from each party is better than the worst candidate from the other party.[3] Then, we find that for an optimal quality legislature to be an equilibrium outcome under party-principals, a necessary condition is that when each party's home district is safe, the highest quality candidate from one party must be able to win the battleground district against any candidate from the other party. Intuitively, when

[3] In this case, the optimal quality legislature includes at least one candidate from each party.

the home districts are safe, the party-principals will not nominate their best candidates in the battleground district only when one of these candidates has no chance of winning the district. Although in our model there is no uncertainty about the policy preferences (or, locations) of voters when the candidates are fielded, in Section 4.4 we extend the model to show that whenever home districts are safe, any uncertainty about the winner of the race between the highest-quality candidates in the battleground district result in a legislature with suboptimal quality under party-principals.

Under party-primaries, we find that if the home districts are safe and the highest-quality candidate from each party is not worse than the lowest-quality candidate from the other party, both highest-quality candidates win a seat in the legislature. Even these assumptions are not sufficient to guarantee that the third member of the optimal quality legislature (a second highest-quality candidate from one of the parties) will win a seat.

Finally, in Section 4 we examine our model under alternative assumptions and show that our results are quite robust to such changes. The alternatives we consider includes (i) parties with identical policies, (ii) candidate pools where two or more of the candidates from the same party has the same quality, (iii) alternative voter preferences, (iv) uncertainty about the voter preferences in the battleground district, and (v) parties having several additional low-quality candidates in their candidate pool.

Observers of politics and political scientists alike have noted that party-leaders (or, strong politicians within a party) quite often reward their low-quality loyalists by nominating them in "safe" seats. Although this may be the case in several countries, we find that party-principals whose only purpose is winning as many seats as possible may nominate lower-quality candidates in safe districts, leading to a legislature with suboptimal quality. Thus, low legislature quality may simply result from the party-principal's solution to his resource (candidate) allocation problem.

One explanation for the observed low-quality legislators in several countries is that high-quality individuals do not participate in politics in these countries. Caselli and Morelli (2004) study individual's incentives to enter into politics when some of politics' rewards depend on the overall quality of all politicians. They show that low-quality incumbents reduce entry by higher-quality individuals, since when most politicians are dishonest and incompetent, being a politician is not at all desirable. In their analysis, Caselli and Morelli (2004) assume that once a high-quality individual becomes a candidate, he will always be elected to the legislature. We find, however, that under a party-principal even the highest-quality politicians do not necessarily win an office. Thus, the low equilibrium legislature quality predicted in Caselli and Morelli (2004) is likely to be an upper bound when candidates are fielded by party-principals.

Political scientists have long recognized the importance of the selectorate.[4] Sartori (1976, p. 74) defined candidate selection as the defining characteristics of a political party: "A party is any political group that presents at elections, and is capable of placing through elections, candidates for public office." Yet, the workings of selectorate is not always open; Duverger (1954, p. 354) noted that candidate selection is a "private act which takes place within the party," while Ranney (1981,

[4]The following three quotes are from Hazan and rahat (2010).

p. 75) defines candidate selection as "the predominantly extralegal process.." With the exception of US (and, to some degree Germany, New Zealand, and Finland), legal regulations on candidate selection are almost non-existent. Given all these, it is no surprise that there has been very few formal analysis of the workings of a selectorate. Recently, several authors studied a question that is complementary to the question we study here: how candidate selection through party-primaries could increase voters welfare by revealing information about the quality of the candidates. Serra (2010) provides such an analysis and a detailed review of this literature.

2 The Model

We study a legislative election held in three single-member districts, $d \in \{l, b, r\}$. Two political parties $P \in \{L, R\}$ compete by fielding a candidate in each district. The policy of each party $\Psi_P \in \mathbb{R}$ is fixed such that[5]

$$\Psi_L < \Psi_R. \tag{1}$$

Each party has three candidates who differ in non-policy characteristics that are desirable to all voters, e.g., wisdom, ability, and honesty. Below, we refer to these characteristics as the quality, or, the valence (Stokes, 1963) of the candidate. Let $v_j^P \in \mathbb{R}$ be a measure of the quality of candidate j from party P. To identify the candidates with different valence from party P, we refer to them as Candidates 1, 2, and 3, where

$$v_1^P > v_2^P > v_3^P. \tag{2}$$

The winner of the election in a given district d is the candidate receiving a majority of votes in d. There is a continuum of voters in each district. The voting is sincere: when deciding which candidate to vote for in her district, a voter takes into account both the policy of the party and the quality of the candidate. By abusing the notation, let i denote the voter whose most-preferred policy is $i \in \mathbb{R}$. The preferences of i are represented by the utility function

$$U_i(\Psi_P, v_j^P) = -L(\|i - \Psi_P\|) + v_j^p, \tag{3}$$

where $L(\cdot)$ is strictly-increasing and continuous. We also assume that $L(\cdot)$ is convex; thus, in each district a majority votes for the candidate whom the median voter prefers (Groseclose, 2007, Lemma 1).

Let m_d denote the (ideal policy of the) median voter in district d. We use

$$\lambda_d = -L(\|m_d - \Psi_L\|) + L(\|m_d - \Psi_R\|)$$

to measure the policy preference for party L in district d. If $\lambda_d > (<)0$, the median voter in district d strictly prefers the policy of party L (R). Alternatively, $\lambda_d = 0$ implies that the median voter is indifferent between these policies, i.e., $m_d = (\Psi_R + \Psi_L)/2$.

The districts l and r are partisan districts for (or, the home district of) parties

[5] In section 4, we discuss the equilibria when $\Psi_L = \Psi_R$, as well as equilibria under several alternative assumptions.

(resp.) L and R while district b is a battleground district: we have[6]

$$\lambda_l > 0,\ \lambda_r < 0,\ \text{and}\ \lambda_b \in (\lambda_r, \lambda_l) \tag{4}$$

That is, the median voter in battleground district b may or may not prefer one party based on its policy (we consider both cases below). We assume only that if she does, her policy preferences are weaker compared to those of the median voters in the home districts l and r. Since many of our results depend on the relative strength of partisanship in each of the home districts, to help facilitate the discussion, we introduce the following definition.

Definition 1 *The home district of P is safe (super-safe) if Candidate 2 (Candidate 3) from P can defend the seat against Candidate 1 from P' in the legislative election.*

Clearly, any district that is super-safe is also safe. By no means does this definition cover all of the meaningful cases of partisanship and valence differences. For example, even when a district is neither safe nor super-safe, the lowest-quality candidate from P may be able defend it against Candidate 2 from P'. Since the districts differ in their policy preferences, there is no guarantee that the highest-quality candidate(s) will win a seat. Such a candidate may end up losing the election, if, for instance, he is nominated in the opposing party's super-safe home district.

Let $\mathbf{V} = [v^L_j, v^L_{j'}, v^L_{j''}, v^R_k, v^R_{k'}, v^R_{k''}]$ denote an allocation of candidates, where candidates j, j', and j'' (candidates k, k', and k'') from L (from R) are nominated in districts (resp.) l, b, and r. Given any allocation (as well as each candidate's valence and each district's strength of partisanship), one can determine the outcome, i.e., which candidates win (or tie for) a seat in the legislature. To evaluate the equilibrium quality of elected legislature, the first-best or benchmark outcome in our analysis is the optimal-quality leislature; intuitively, it is a legislature that includes no candidate of strictly lower quality than another candidate outside of the legislature. More formally, optimal-quality legislature can be thought of as a subset of the set of all candidates, with the associated probability that each is elected. Note first that only the candidates with the three highest valence levels will be elected with non-zero probability (at most four candidates as no candidates within a party have the same valence). When there are only three candidates in this subset, the optimal quality legislature is simply those three candidates (with the highest quality) each winning a seat with probability one. When two candidates both have the third-highest valence, any outcome in which the two highest-valence candidates win a seat with certainty and the sum of the probabilities that third-highest valence candidates win a seat is equal to one is an optimal-quality legislature. For example, when $v^L_1 > v^L_2 > v^R_1 = v^L_3$, any allocation of candidates in which Candidates 1 and 2 from L win a seat for sure and the probability that Candidate 1 from R wins a seat and the probability that Candidate 3 from L wins a seat sum to one.

The quality of the elected legislature depends on the districts in which the candidates are fielded. The candidates are fielded by the *selectorate*. A blend of words "selector" and "electorate", the term coined by Paterson (1967) refers to the "body that selects the party's candidates for public office" (Hazan and Rahat, 2010, p.

[6]In terms of the primitives of the model, this simply means that $\|m_l - \Psi_L\| < \|m_b - \Psi_L\| < \|m_r - \Psi_L\|$, $\|m_r - \Psi_R\| < \|m_b - \Psi_R\| < \|m_l - \Psi_R\|$.

33). We compare the equilibrium legislature quality under a fully-centralized selectorate (CS), such as a party-principal (or, a centralized party-committee), to the equilibrium quality under a fully-decentralized selectorate (DS), such as a local party-primary (or, a vote among the local party-members). It is worth noting that these two selectorate are diametrically opposed both in terms of centralization and exclusiveness. We investigate under which (if, any) one of these selectorates is an optimal-quality legislature a PSNE outcome.

Before we specify the objectives and model the decision-making problem of each selectorate, let us make clear that we examine the equilibrium quality of the legislature and not the equilibrium social welfare under different selectorates. The latter depends on not only the quality of elected candidates, but also their policy.[7] Since, unlike the quality of a candidate, policy is an issue over which the voters disagree, one needs to make interpersonal comparisons, and, thus needs to know both (i) the measure and the distributions of voters in each district, and (ii) the exact shape of the voter utility function. Additionally, for welfare calculations, the cost of each candidate selection method needs to be taken into account. For these reasons, we instead focus only on the quality dimension and consider a general setup. If one imposes the reasonable assumptions that when there are multiple equilibria under DS, the probability that the players will be able to coordinate on each equilibria is the same, then in the special case of symmetric elections (studied in Section 3.1), our result allows one to make welfare comparisons. Then, under either selectorate each party's policy will win with equal probability, and, the voter welfare is solely determined by the quality of legislature.

2.1 Description of the game under each selectorate

Under CS: The objective of each player (party-principal) is to maximize the (expected) number of seats his party wins in the legislative election.[8] The preferences of each party-principal satisfy the expected utility hypothesis, and each principal is risk-neutral.[9] A strategy for the principal of P is an ordered sequence of valences $v_j^P v_{j'}^P v_{j''}^P$, where playing $v_j^P v_{j'}^P v_{j''}^P$ simply means nominating candidates with valences v_j^P, $v_{j'}^P$, and $v_{j''}^P$ in districts (resp.) l, b, and r. A strategy profile for this game is denoted by $V_{CS} = (v_j^L v_{j'}^L v_{j''}^L, v_k^R v_{k'}^R v_{k''}^R)$.

It is worth noting that we assume neither that candidates differ in their loyalty to the party-principal nor that the party-principals prefer a higher-quality legislature. These considerations, we believe, are, at most, secondary criteria while fielding the candidates: a party-principal will not give up a seat to the other party just because otherwise a non-loyal or low-quality candidate from his own party would win that seat.

Under DS. The candidates are the players, each deciding in which district's primary to run. When two or more candidates from party P decide to run in the same

[7]The quality of elected legislatures affects voter welfare: the corrupt legislatures will steal public funds and the ones with less ability or wisdom will produce badly written laws with damaging loopholes or with extensive uncertainty (Londregan, 2000, p.29).

[8]Throughout the paper we use the terms centralized selectorate (CS) and party-principal interchangeably.

[9]Thus, for instance, a party-principal is indifferent between outcome A in which his party ties in all districts and outcome B in which his party ties only in one district, wins one district and loses the other one.

primary, the local members of P vote to decide which candidate will run in the (general) legislative election.[10] Since in our model, all the candidates from a given party will offer the same policy in the election, the candidate with highest quality wins the primary. Thus each candidate has three strategies, $\{l, b, r\}$. A strategy profile for this game is denoted by $V_{DS} = (d, d', d'', k, k', k'')$ indicating that Candidates 1, 2, and 3 from $L(R)$ run in the primaries of the districts (resp.) d, d', and $d''(k, k'$, and $k'')$. For example, a strategy profile such as (l, b, r, l, r, b) gives rise to an allocation $\mathbf{V} = [v_1^L, v_2^L, v_3^L, v_1^R, v_3^R, v_2^R]$.

We assume that the candidates are selfish: each candidate cares first and foremost about his own success in the legislative election. Let $(\circ_j, o_j, O)$ denote candidate j's results from a given strategy profile under DS, where $\circ_j \in \{win_j, loss_j\}$ indicates how j has performed in the primary (by (2), there are no ties in the party-primaries), $o_j \in \{win_j, tie_j, loss_j\}$ indicates how j has performed in the legislative election (if he loses the primary, then he automatically loses the election as well) and O indicates his party's expected seats. We assume that each j has lexicographic preferences over (the outcomes of) strategy profiles where priority is given to his performance, i.e., he prefers $(\circ_j, o_j, O)$ to $(\circ'_j, o'_j, O')$ if and only if (i) $\circ_j \succ \circ'_j$, or, (ii) $\circ_j \sim \circ'_j$, but $o_j \succ o'_j$, or, (iii) both $\circ_j \sim \circ'_j$ and $o_j \sim o'_j$ with $O_j > O'_j$. In other words, he ranks two outcomes in terms of his party's overall performance in the legislature only when in both cases his performance is the same in the election.

3 Equilibrium legislature quality under two selectorates

The model as described above imposes almost no restriction on the set of parameters, giving rise to a large number of possible games. In some of these games, the question we study is not interesting. For instance, when the quality difference between the (candidate pools of the) parties is so large that all the candidates from one party will win the election under *any* selectorate, we gain no insight into which type of selectorate typically produces higher-quality equilibrium legislatures. Therefore, in this section we first study a case in which neither party has an advantage (the symmetric case) to establish intuition about the differences in equilibrium legislature quality under different selectorates, before returning to the general case.

3.1 A special case: neither party has an advantage

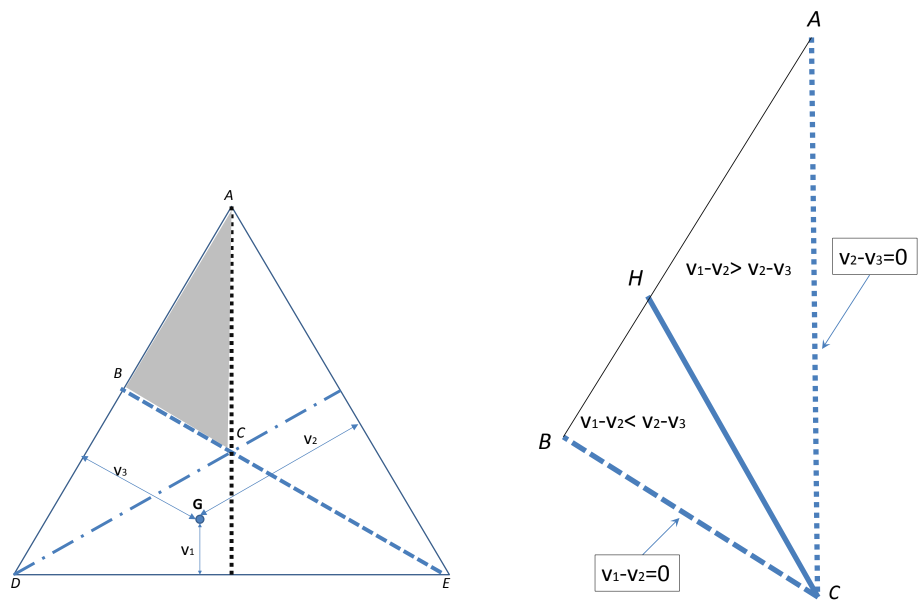
In this (symmetric) case, neither party has an advantage either in terms of the quality of its candidates, $v_j^L = v_j^R$ for all $j \in \{1, 2, 3\}$, or the voter support for its policy, $\lambda_l = -\lambda_r > 0 = \lambda_b$.[11]

3.1.1 Equilibrium quality under CS

When the party-principals field the candidates, each principals's problem is a resource allocation problem. The appropriate use of his resources (candidates), de-

[10] As long as they do not vote strategically, allowing independents and members of the other party to vote in the primaries, i.e., considering semi-open and open primaries, would have no effect on our results.

[11] The candidates from each party still differ in their qualities, (2) holds, and the parties still propose different policies, (1) holds. We remove these assumptions in Section 4.



(a) (b)

Figure 1: For a given total valence $(v_1 + v_2 + v_3)$, the triangle ADE depicts the set of all valence distributions while the shaded triangle ABC, detailed further in panel (b), depicts the set of all valence distributions that satisfy (2). In any valence distribution above $[HC)$, the highest-valence candidate is the outlier in the party $(v_1 - v_2 > v_2 - v_3)$. In any valence distribution below $[HC)$, the lowest-quality candidate is the outlier $(v_1 - v_2 < v_2 - v_3)$. Last, in any valence distribution lying on $[HC)$, the quality difference between candidates is equal $(v_1 - v_2 = v_2 - v_3)$.

pends on the relative strength of home district partisanship (in the symmetric case, the median district is not at all partisan) and the valence differences between the candidates. Since we do not impose any restrictions on these, even in this special case, there are several subcases with potentially different equilibrium outcome. To clarify this point, Figure 1 depicts all possible valence distributions and the resulting valence differences between the candidates. The set of all possible valence distributions when a party has a total valence of $v_1 + v_2 + v_3$ can be depicted by the equilateral triangle with sides of length $\frac{2\sqrt{3}}{3}(v_1 + v_2 + v_3)$ in Figure 1.a. Any point in this triangle can be considered a valence distribution where the distances from the point to the base, the left side, and the right side of the triangle are equal to (resp.) v_1, v_2, and v_3. The ordering in (2), $v_1 > v_2 > v_3$, helps us to eliminate several permutations of the same valence distribution. The set of all valence vectors satisfying (2) correspond to the triangle ABC (except for the sides AC and BC), and are further described in Figure 1.b.[12]

The valence differences matter differently in the battleground district and the home districts. When the battleground district is indifferent between the policies of

[12]On $[BC)$ we have $v_1 = v_2 > v_3$, on $[AC)$ we have $v_1 > v_2 = v_3$, and at point C we have $v_1 = v_2 = v_3$. We discuss the equilibria of these non-generic cases in Section 4.2; Figure 4 presents all the equilibria for these cases.

the parties, the winner in b is determined solely by the valence difference between the candidates from L and R (if there is no valence difference, then each wins with equal probability). Winning the home districts, however, depends on each party's policy (or, partisanship) advantage in its own home district.

Let λ denote the policy advantage of a party in its home-district when the game is symmetric:

$$\lambda = \lambda_l = -\lambda_r.$$

To capture the other party's home-district a party(-principal) must nominate a candidate whose valence advantage over the other party's candidate in that district is larger than λ. Since the valence distribution is the same within both parties, there are three valence differences to consider:[13]

$$\begin{aligned}\Delta_1 &= v_1 - v_2,\\ \Delta_2 &= v_2 - v_3,\\ \Delta_3 &= v_1 - v_3.\end{aligned}$$

Depending on the relative strength of λ, and the valence differences between the candidates, the payoffs from a given strategy (and, thus, possibly the PSNE) will change. Before we present all the PSNE in all these subcases, let us note some similarities in these equilibria.

Lemma 1 *When neither party has an advantage in the election, under CS*
(i) in any PSNE, the expected outcome is a tie,
(ii) PSNE exists if and only if $\lambda \geq \min\{\Delta_1, \Delta_2\}$,
(iii) there is no PSNE in which a principal nominates his lowest-quality candidate in the battleground district or his highest-quality candidate in his own party's home district.

Intuitively, each party-principal can guarantee a tie by playing the mirror image of the strategy played by the other principal, thus, each must tie in any PSNE. The condition $\lambda \geq \min\{\Delta_1, \Delta_2\}$ simply requires that the partisan (home) districts are genuinely different from the battleground district. That is, when the strength of policy preference in a partisan district is less than the smallest quality difference between the candidates, the election between two candidates with different valences would have the same result in any district.[14] Although strictly speaking this is not a Blotto game (the principals are maximizing the number of seats, not simply trying to get a majority in the legislature), the intuition is similar to the failure of the existence of PSNE in a Blotto game with three identical targets. The last part of Lemma 1 reflects the fact that in this game the candidates are resources: nominating the lowest-quality candidate in the battleground (or, nominating the highest-quality candidate in the party's home) district is a waste as long as home districts are different from the battleground district.

[13] Note that only under valence distributions that satisfy (2) are none of these valence differences equal to zero.

[14] If j from L wins against j' from R in district b with $v_j \neq v_{j'}$, then j wins against j' in any district as well. When $\lambda < \min\{\Delta_1, \Delta_2\}$, the only role of the partisanship advantage is to break the ties, i.e., it only matters when $v_j = v_{j'}$.

The PSNE under CS are presented in Figure 2.[15] There are two types of equilibria. When the home districts are safe ($\lambda > \Delta_1$), in all equilibria both of the highest-valence candidates (Candidate 1's) run in the same (battleground) district. Since only one of them can win, the legislature quality is always suboptimal. Under lower partisanship ($\lambda < \Delta_1$), in equilibrium each Candidate 1 runs in the opposing party's home district, resulting in optimal legislature quality.[16] Thus,

Proposition 1 *When neither party has an advantage in the election, under CS, the equilibrium legislature quality is always suboptimal unless,* $\Delta_2 \leq \lambda \leq \Delta_1$.

Under CS, how suboptimal the equilibrium legislature quality will be depends on the strength of policy preferences in partisan districts: the majority of the legislature may consist of candidates with the next-highest quality (Candidate 2's), candidates with the lowest quality (Candidate 3's), or a mixture of these two type of candidates. When the home districts are super-safe ($\lambda > \Delta_3$), the party-principals may nominate their lowest-quality candidates there, leading to a legislature with a majority of lowest-quality candidates. Such poor equilibrium legislature quality arises under super-safe home districts because even if a principal nominates his *worst* candidate in the party's home district, the party will still carry that district: nominating the candidate with the highest valence in the home district of *either* party is a waste of resources, while nominating any other candidate (including a Candidate 2) there is not.[17] In this case, if the party-principal has secondary preferences on quality or other characteristics of the candidates, then these will determine the equilibrium outcome. For instance, if the party-principal is policy motivated (and, if higher valence candidates can propose higher valence policy proposals that are more likely to be implemented, as Londregan (2000, p. 32) argues), then he may try to maximize the quality of the party's caucus in the legislature. A much more commonly suggested motivation for party-principals is that they simply award their low quality loyalists with safe seats.

In fact, that low-quality but loyal (to the centralized selector) politicians are often nominated to safe seats by party-principals has already been used to explain observed low-quality among politicians; see, for instance, Best and Cotta (2000). Our analysis provides an alternative (and, in certain cases, complementary) explanation, for low-quality legislatures, i.e., that low legislature quality may simply be an artifact of party-principals maximizing the usefulness of their resources. Our analysis also shows that to solidify his support in the legislature, a party-principal may

[15] The MATLAB code employed to calculate the PSNE under CS is available upon request.

[16] When $\lambda = \Delta_1$ both types of equilibria exist if $\Delta_2 \leq \Delta_1$ (otherwise only an equilibrium with suboptimal legislature exists), as well as a hybrid where one Candidate 1 runs in the battleground and the other in the opposing party's home district. In the hybrid case, both Candidate 3's run against each other, thus legislature quality is suboptimal.

[17] Note that when the strength of partisanship is exactly equal to the valence difference between Candidates 1 and 3 (Δ_3), the intuition (and, thus, the equilibria) discussed above remains the same. Now, nominating the highest-valence candidate in the other party's home turf could secure a tie in that district when the other party nominates its worst candidate there. However, this is not a net gain, as by moving Candidate 1 from b the party is now losing the battleground district in which was previously tied. More importantly, the opposing party's best response (playing Candidate 2 in their home district, Candidate 1 in the battleground district, and Candidate 3 in the remaining district), results in the opposing party winning two seats. Thus, both party-principals still nominate their best candidates in the battleground district in any PSNE.

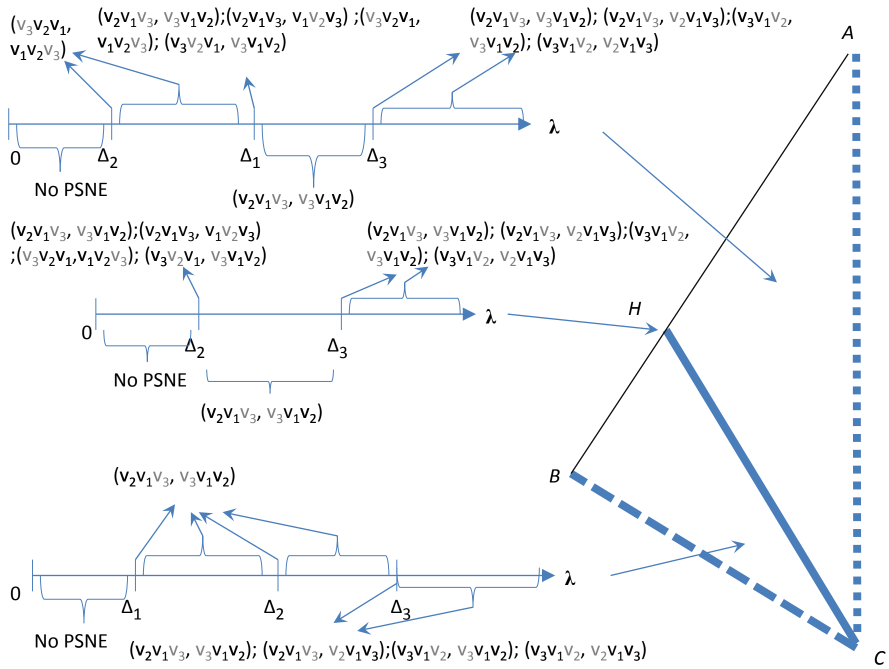


Figure 2: The PSNE under CS when neither party has an advantage. (In each strategy profile in the figure, the candidates with the darkest (lighest) fonts win (lose) the election in the district they run; the ones with regular font color tie.)

nominate his (loyal, but, possibly dishonest or incompetent) supporters in super-safe seats. Yet, the main source of low-quality legislatures is the centralized candidate nomination system: that lowest-quality candidates are nominated in super-safe seats is an implication of the party-principal's resource allocation calculus, and, it may happen even when the principal cares not at all about the loyalty of the party's members in the legislature and only tries to maximize the expected number of seats his party controls.

It is worth noting that in the symmetric case such suboptimal quality legislature will exist even if the battleground district is not indifferent between the two parties as long as the neither one of the highest-quality candidates can secure the battleground district with certainty. That is, suppose when both Candidate 1's run in b, Candidate 1 from R will win the election with a probability of 9/10. Still, the PSNE of the game remains intact as long as the home districts are safe. Intuitively, when the home districts are safe, and a Candidate 1 has a chance, however small, to win b, nominating him in any other district is a waste (see Section 4.4, for more on this.)

Another point to note is that when neither party has an advantage and the candidates are fielded centrally, the home district of a party will never be represented by a high quality candidate from that party. Further, the quality of elected legislature is optimal if and only if both home-districts are won by a highest-valence candidate from the other party. From Figure 2, one can see that it goes even further: with the exception of non-generic case in which the strength of policy preference in a given district is exactly equal to the quality difference between the highest quality and the next highest quality candidate, when neither party has an advantage and the candidates are fielded centrally, the legislature will include both of the highest-quality candidates if and only if the home district of each party is won by a highest-quality candidate from the other party.

3.1.2 Equilibrium quality under DS

Recall that under decentralized selectorate each candidate decides in which primary to run. As each candidate is selfish, caring foremost about his own success, the higher-quality candidates who, in essence, can choose to run in (and win) any primary they choose, will run in primaries in which they know they will also win in the general election. Lower-quality candidates may not have that possibility (of winning the general election) but will be sure to pick a district in which they can win at least the party-primary. Thus, under party-primaries there will be exactly one candidate running in each party's primary in each district.[18]

This observation leads to Lemma 2, which discusses several features of PSNE under DS.

Lemma 2 *When neither party has an advantage in the election, under DS*

(i) an expected tie and one party winning a majority of seats are both PSNE outcomes,

(ii) a PSNE always exists,

(iii) there is no PSNE in which candidate j from P wins a seat unless every candidate j' from P with $v^P_{j'} > v^P_j$ wins a seat,

(iv) there is no PSNE in which both of the highest- (or, the lowest-) quality candidates from each party run in the same district.

Proof. Part (i), simply follows from Figure 3 (the calculations for Figure 3 are available from the authors).

To see (ii), consider the strategy profile in which each candidate with valence v_1 (v_2) runs in the primary of his party's home district (of the battleground district), while each candidate with valence v_3 runs in the opposing party's home district. None of the candidates have incentive to deviate. Thus, this is a PSNE independent of the valence distribution and the strength of the partisanship.

To see (iii), assume otherwise, i.e., that there exists an equilibrium in which candidates from each party have chosen the primaries in which to run, and, as a result, a candidate from P with valence v_j runs in the primary of d winning both the primary and the legislative election, while another candidate from P with valence $v_{j'} > v_j$ runs in the primary of d' and does not win the legislative election there. This cannot be an equilibrium as by deviating from d' to d, candidate j' will win the seat representing d.

To see (iv) simply note that if Candidate 1 from each party runs in the primary of the same district, then each will win the primary, but, in the legislative election,

[18]This result is due to two factors: there are only three candidates in each party, and the quality of the candidates are known by the selectorate. In Section 4.5, we consider the case in which each party has many lowest-quality candidates. Then, there will be party-primaries with several contestants, but, as we discuss there, this has no effect on our results. The effects of uncertainty about quality of the candidates, however, is more complicated and it has been left for future research.

one will lose (if the election is held in one party's home district) or both will tie (if in the battleground district). Yet, either Candidate 1 can guarantee a seat by simply running in the primary of his own party's home district. Thus, the highest-quality candidates from L and R never face each other in the legislative election; they run in separate districts, and each wins a seat. The remaining seat in the legislature is not won by a candidate with the lowest quality (as would be the case if the lowest-quality candidates face each other in the legislative election): by (iii), one party's Candidate 2 wins that seat. ■

Note that even when neither party has an advantage, under DS, one party may win more seats than the other (as we show in Figure 3, there are also equilibria in which the two parties win the same number of expected seats).[19] Intuitively, under DS, a party can not coordinate its candidates; and, thus, it may end up losing the battleground district to the other party. The existence of such an equilibrium also shows the strength of the party-principals' incentives to coordinate (influence) the districts in which the candidates run.[20]

Last two parts of Lemma 2 implies that

Proposition 2 *Under DS, the equilibrium quality of the legislature is always optimal.*

In contrast to equilibria under CS, under DS,, at least one highest-quality candidate represents a home district. Further, when the home districts are weakly safe ($\lambda \geq \Delta_1$) this is a candidate from the party supported by the district; see Figure 3.[21] Additionally, when home districts are (strictly) safe under DS, the battleground district may not be represented by a highest-quality candidate (as is the case under CS). Thus, while the home districts gain under DS (in terms of representative quality), the battleground districts may lose. The legislature never includes a lowest-quality candidate, the third member is a candidate with the second highest quality.

To summarize, we find (in the fully-symmetric case) that party primaries always lead to an optimal-quality legislature while party-principals fielding candidates rarely does so. To explain this contrast, it helps to note the difference between the objectives of the candidates and party-principals. Although neither group of players

[19]For instance, the best candidate from L (R) running in the primary of l (b), while Candidate 2 from R runs in r (with the other three candidates playing any strategy such that all candidates from the same party run in separate primaries) constitutes a PSNE. In this PSNE Candidate 1 from L wins only district l, while Candidates 1 and 2 from R win both districts b and r.

[20]In a two-party setup with a dominant party, Snyder and Ting (2010) studies whether the dominant party will decide that (both of the parties) must select candidates by primary. Although this is not the question we study here, existence of such equilibria also shows why even when primaries produce a higher-quality legislature, the parties may be reluctant to unilaterally adopt them when neither party is dominant: if one party-principal could manipulate his party's primaries while the other one does not, he could guarantee an election victory even when his party has no advantage. In such a case the legislature has an optimal quality, but one principal meddling in the affairs of his own party is not an equilibrium; when he can, the other principal, too, will do the same, leading to a suboptimal legislature except in the situation mentioned in Proposition 1, i.e., when the quality of legislature is optimal under CS.

[21]When the policy preferences at home districts are weak enough ($\Delta_1 > \lambda$), it is also possible to have, as we found under CS, the home district of each party being represented by the best candidate from the other party (while Candidate 2 from each party runs in b). Note that under DS we do not need $\lambda \geq \Delta_2$ as is required under CS for this equilibrium to exist.

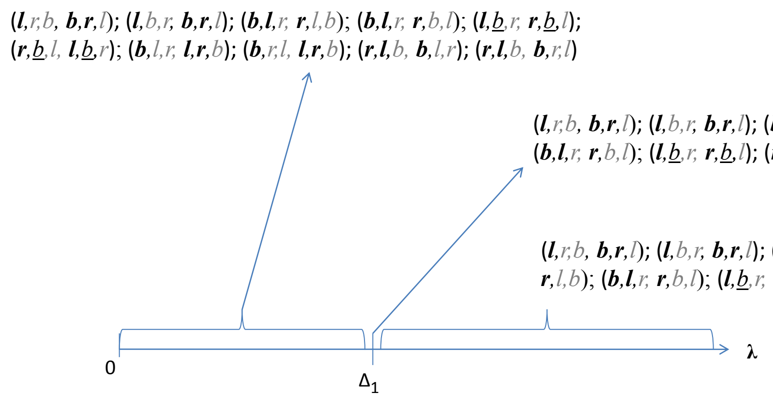


Figure 3: The PSNE under DS when neither party has an advantage in the elections. (In each strategy profile in the figure, the candidates whose strategy is in the darkest (lighest) fonts, win (lose) the election in the district they run; the ones with regular font color tie.)

care about the quality of elected legislature (in our model everybody is selfish), the incentives and tools at the disposal of these players are different.

The party-principals maximize the expected number of total seats their parties win. Given that the number of seats is fixed, a party-principal's objective can also be thought of as minimizing the expected number of seats the other party wins. So, even if a high-quality candidate does not win a seat with certainty, and, instead, reduces the probability that (a high-quality candidate from) the other party wins that seat, the allocation may still be optimal from the party-principal's point of view.

Alternatively, the candidates want, primarily, to get elected. Therefore, under DS, each candidate tries to find a district (there may be more than one) from which he will be elected with highest probability. Given that no party has an overall advantage in terms of its candidate quality or support for its policy, each party's highest-quality candidate can always find a district from which he will win. By similar reasoning (also by Lemma 2, part (iii)), the remaining member of the legislature will always be a Candidate 2 from one of the parties. Since both have equal valence, regardless of which Candidate 2 wins, the legislature quality will always be optimal.

3.2 General case

Both of the arguments at the end of the last section are heavily based on the assumption that although the parties, the districts, and the candidates all differ from each other, they differ in a symmetric way. Under CS, suboptimal quality legislatures exists when each principal fields his best candidate in the battleground district. In addition to the existence of safe districts, the symmetry of the situation makes nominating higher-quality candidates elsewhere a waste. As the candidates are of the same quality and the district is indifferent between the policies of the parties, the candidates tie. Had any one of these assumptions been violated one of the highest-quality candidates would win the election for sure, and then, there may exists another equilibrium in which both highest quality candidates win a seat in the legislature. Similarly, if the battleground district strictly prefers the policy of, say, L, then under DS we can have a suboptimal quality legislature in which Candidate 2 from L (with $v_2^L < v_2^R$) wins this seat.

The following example from the close neighborhoods of the symmetric case, illustrates both points. Consider for a small $\epsilon < v_1^P - v_2^P$, candidate valences given by $v_j^L = v_j^R + (3-j)\epsilon$ for $j \in \{1, 2\}$) with $v_3^L = v_3^R$ (the top two candidates from L have a slight valence advantage), and policy preferences in which the battleground district slightly prefers Ψ_R such that b would elect Candidate 2 from R when both Candidate 2's compete but Candidate 1 from L when both Candidate 1's compete (say, $\lambda = -\frac{3}{2}\epsilon$) and home districts are safe. It is straightforward to show that under CS the strategy profile $(v_2^L v_1^L v_3^L, v_3^R v_2^R v_1^R)$ is a PSNE. In this equilibrium the top two candidates from L and the top candidate from R each win a seat with probability one: the equilibrium legislature is of optimal quality. Yet, in the same setup, under DS, the strategy profile $(l, b, r,\ b, r, l)$, is a PSNE, and the elected legislature in this equilibrium (the top two candidates from R and the top candidate from L each win a seat with probability one) is of suboptimal quality. That is, when one considers asymmetric cases it is possible to have an equilibrium under CS that produces a higher-quality legislature than an equilibrium under DS.

We should note that the above example is not an example of dominance by CS: in this election, under DS, too, there exists a PSNE, resulting in an optimal-quality legislature, $(b, l, r,\ r, b, l)$. Similarly, our point is not that when one removes the symmetry everything will be different. Rather, our point is that in the general case studied below, we quite often have several equilibria with different equilibrium legislature quality under the same selectorate and in the same election. Thus, the results one can obtain are inevitably less precise.

Our most general result is that in the general case, CS never dominates DS in the sense that,

Theorem 1 *If for a given election an optimal-quality legislature is an equilibrium outcome under CS, then for the same election an optimal-quality legislature is an equilibrium outcome under DS.*

Theorem 1 is evaluates the performance of the two selectorates in relative terms. It does not say when (and, if) one selectorate may produce an optimal quality legislature. This is because, when one imposes no restriction on the set of parameters, there are elections in which the equilibrium legislature quality is suboptimal under both selectorates.

Proposition 3 *There exist elections in which the equilibrium quality of the legislature is always suboptimal under either selectorate.*

Proof. Consider an election in which we have $v_3^L > v_1^R$ with $v_1^L - v_3^R < -\lambda_r$. Note that none of the candidates from R should be in the optimal quality legislature. Under CS, the principal of party R will always win r (by nominating any candidate there). So, even though the optimal legislature does not include any politician from R, in any PSNE at least one candidate from R will be in the legislature. Similarly, under DS, Candidate 1 from R will be in the legislature, as he can always run in the primary of r, winning both the primary and the legislative elections in that district. ■

This impossibility result applies to an unusual election. One party (L) completely dominates the other in terms of its candidate quality (the worst candidate from L is better than the best candidate from R); the optimal-quality legislature should include only candidates from L. Additionally, R's home district is super-safe. We speculate that there exists no other candidate selection mechanism that would implement the optimal-quality legislature in that election. In general, if one party dominates the other in candidate quality or the home-district partisanship is very strong, the optimal-quality legislature may be hard to achieve.

When we investigate the optimality of equilibria under CS and DS in the general case, we find that we often can say significantly more about the equilibrium quality when we consider cases in which neither party completely dominates the other in candidate quality.

Definition 2 *Neither party dominates the other in candidate quality if for each $P \neq P' \in \{L, R\}$, we have $v_1^P > v_3^{P'}$.*

Note that when neither party dominates the other in candidate quality, the optimal quality legislature includes the highest-quality candidate from each party; the identity of the third member of the legislature is not determined. All we can say is that in addition to the highest-quality candidate from each party, the optimal-quality legislature must include a Candidate 2. This third member will be the Candidate 2 from R (L) if $v_2^L < (>) v_2^R$, and either of these Candidate 2's if $v_2^L = v_2^R$.

3.2.1 Equilibrium quality under CS

Removing symmetry comes with a cost in calculating the PSNE under CS. Even though our code can be used to calculate PSNE under CS in any given game (payoff matrix), the number of possible games (or, payoff matrices) when one removes the symmetry explodes to at least two hundred thousand, making it impossible to calculate or present all the equilibria for all the games.[22] A related difficulty is that the precise conditions under which a PSNE exists under CS are not easy to determine.

When studying the equilibria of such a large number of games, we look at the features of elections that had an effect on the quality of equilibrium legislature in the symmetric games. In the symmetric case, the most important feature of elections was the existence of safe districts. In the general case, we find that

Proposition 4 *If the home districts are at least safe, then under CS, a PSNE with suboptimal-quality legislature always exists.*

Proof. Assume that the home districts are super-safe (safe), then it is straightforward to show that a strategy profile in which each party-principal nominates his lowest (second-lowest) quality candidate in his party's home district while nominating his highest-quality candidate in b (and, nominating the third candidate in the other party's home district is a PSNE). The equilibrium quality is suboptimal as one of the Candidate 1's lose the election while a Candidate 2 or Candidate 3 from the same party wins. ■

With safe home districts, this type of equilibrium (both Candidate 1's nominated in b) was the only type of equilibrium under CS in the symmetric case.[23] As our

[22]In the symmetric case studied above, depending on the partisanship advantage in the home-districts and the valence differences between candidates, there were nineteen different games under CS, c.f. Figure 2. When one looks at slightly asymmetric games, a slight asymmetry in only one of the candidate valences or in the policy preferences of only one district, the number of possible games increases to (resp.) fifty-one and sixty-three. When there are symmetry restrictions on neither valence vectors (increasing the number of possible valence difference parameters to as high as fifteen from only three in the symmetric case) nor on the strength of party preferences (increasing the number of possible district specific policy preferences to three from only one in the symmetric case), there are at least one hundred and forty thousands possible games under CS. This is a lower bound: when no candidate from either party has valence equal to any other candidate's valence, then there are twenty possible valence orderings, each with fifteen valence differences. The relative size of the fifteen valence differences can be permuted at least one hundred twenty meaningful ways. Meaning that for a fixed partisanship, we have no less than one thousand six hundred eighty games. Varying the partisanship would increase the number of games to at least 208,800.

[23]One small difference is that in the general case the highest quality candidates from each party can differ in their valences. And, if the median of the battleground district prefers the policy of this party sufficiently, then it is possible that in the equilibrium the lower-valence Candidate 1 is elected while the higher-valence Candidate 1 cannot win a set, i.e., Candidate 1 from L could be elected even when $v_1^R > v_1^L$, if $\lambda_b > v_1^R - v_1^L$.

example in the beginning of this section illustrates, in the general case there may exist additional equilibrium legislatures with different (and, sometimes optimal) quality even with safe home districts. These additional equilibria arise because in the general case the battleground district is not necessarily indifferent between the two parties. Then nominating his highest-quality candidate in the party's home district is not always a waste of resources for a principal. He might do so when his highest-quality candidate cannot win against the other party's best candidate in b; either because of the valence difference between these Candidate 1's, or because the policy preferences of the median in b. Then, in equilibrium both Candidate 1's are elected. The resulting equilibrium legislature would be of optimal-quality under certain conditions.[24] As a result, Proposition 4 does not state that strong policy preferences necessarily rule out equilibria with optimal-quality equilibria (rather that equilibria with suboptimal legislature quality will exist).

When, in addition, neither party can capture the battleground district for sure, as in the symmetric case, then nominating the highest-quality candidates elsewhere becomes a waste of resources, and an optimal quality legislature becomes impossible.

Proposition 5 *When each of the home districts are safe (or super-safe), under CS the equilibrium legislature is always of suboptimal quality if neither of the highest quality candidate will win the battleground district with certainty ($\lambda_b + v_1^L = v_1^R$).*

Proof. Assume that (i) $\lambda_b + v_1^L = v_1^R$, (ii) $\lambda_l + v_2^L > v_1^R$, and (iii) $-\lambda_r + v_2^R > v_1^L$. Then, it is straightforward to show that $(v_2^L v_1^L v_3^L, v_3^R v_1^R v_2^R)$ is a PSNE. By (ii) and (iii), in this PSNE each party wins its home district, and, by (i), ties in b. To see that in all PSNE Candidate 1's must be nominated in b, note that, if P does not nominate its Candidate 1 in b, then P' can always guarantee two seats in the legislature by nominating its Candidate 1 in b and its Candidate 2 in its own home district. Thus, in all PSNE, each Candidate 1 is not in the legislature with probability 1/2. By (2), at least one of these Candidate 1's must always be in the optimal quality legislature: the legislature quality is suboptimal in any PSNE. ■

Yet, the suboptimal quality under CS is not an implication of a contestable battleground district alone. That is, Candidate 1's not tying in b is not a sufficient condition for the existence of an optimal quality legislature either. To see this, let us examine the case when neither party completely dominates the other in terms of its candidate quality ($v_1^R > v_3^L$ and $v_1^L > v_3^R$). By limiting the valence distribution in such a way, we eliminate less than ten percent of the possible games under CS, but are able to say much more about the equilibrium legislature quality.

Proposition 6 *Consider a setup in which each of the home districts are at least safe and neither party dominates the other in candidate quality.*

(i) When the second-highest quality candidates in each party differ from each other in quality ($v_2^L \neq v_2^R$), the equilibrium quality of legislature is always suboptimal if the battleground district sufficiently prefers the policy of the party with the lower-quality Candidate 2 ($\lambda_b + v_1^L < (>) v_1^R$ when $v_2^L > (<) v_2^R$).

[24] For instance, if $\lambda_b > v_1^R - v_1^L$, in one PSNE the principal of R nominates his highest-quality candidate in his home-district, and, as long as l is safe, the principal of L nominates his Candidate 2 in l. Thus, if $v_1^R > v_3^L$ and $v_2^L > v_2^R$, the resulting legislature (which includes Candidate 1 and 2 from L and Candidate 1 from R) is an optimal-quality legislature.

(ii) When the Candidate 2's have identical quality $v_2^L = v_2^R$, there always exists a PSNE with optimal legislature quality as long as the Candidate 1's from each party do not tie in b ($\lambda_b + v_1^L \neq v_1^R$).

Proof. To see (i), assume, without loss of generality, that $v_2^L > v_2^R$ and $\lambda_b + v_1^L < v_1^R$. No matter where the principal of L fields his candidates, the principal of R can win two seats by nominating his Candidate 1 in b and his Candidate 2 in r (home districts are safe). Since the optimal-quality legislature includes only one candidate from R, and in equilibrium R wins two seats, the equilibrium legislature is always of suboptimal quality. (To see the existence of a PSNE, note that when his home district is super-safe (safe), the principal of L nominating any candidate in l is a PSNE.)

To see (ii), assume, without loss of generality, that $\lambda_b + v_1^L < v_1^R$. Then, R nominating Candidates 1 and 2 in districts b and r and L nominating Candidate 1 in l is a PSNE, regardless of where the party-principals nominate the rest of their candidates. Note that in this PSNE, an optimal quality legislature (Candidates 1 and 2 from R with Candidate 1 from L) is elected. ■

So, in the general case with safe home districts, for optimal-quality legislature to be a possibility under CS, the battleground district must have policy preferences that do not too strongly favor a party with only one of the three highest-valence candidates. Otherwise, this party can win its home district as well as the battleground district, resulting in a suboptimal-quality legislature. Note that this condition, would typically rule out the Candidate 1's tying in b as well.

The second part of the Proposition 6 notes that Candidate 1's not tying in b is a sufficient condition for optimal quality legislature to be an equilibrium outcome when there is some degree of symmetry in the game. Having Candidate 2's with equal quality in each party does facilitate existence of a PSNE with optimal quality legislature. Unlike the case in which both Candidate 1's have the same quality, when there are two Candidate 2's with the same quality, either one winning a seat would be enough for the optimal quality legislature.

Finally, we consider the case in which the districts are not safe (thus, also not super-safe). In the symmetric case, we find that under CS only one equilibrium strategy profile leads to an optimal-quality legislature. Next, we investigate when this type of PSNE exists under CS in the general case.

Proposition 7 *Assume that (i) neither home district is safe, but that each home district could be defended by the home party's Candidate 3 against Candidate 2 from the other party, and (ii) neither party dominates the other in candidate quality. If the higher-valence Candidate 2 wins the battleground district when he competes with lower-valence Candidate 2, then there always exists a PSNE with optimal quality of legislature.*

Proof. It is straightforward to show that under the conditions above, each party nominating its highest valence candidate at the other party's home district, and its second-highest valence candidate at b, is always a PSNE. ■

As noted above, In the symmetric case, too, an optimal-quality legislature was also an equilibrium outcome under these conditions, i.e., when the home districts can be captured by the highest-quality candidate from the other party unless the own party nominates its best candidate there.[25] Unlike in the symmetric case, in the general case neither Candidate 2 has the same valence nor is the battleground district indifferent between the policies of the parties. As in Proposition 6, in the general case an equilibrium optimal-quality legislature still exists as long as the (median of the) battleground district does not to prefer too strongly the policy of the party that has only one candidate in the optimal-quality legislature.

3.2.2 Equilibrium quality under DS

Let us first note that the strategy profile we presented to prove Lemma 2.(ii), –each Candidate 1 (Candidate 3) standing in the primary of his own party's (the other party's) home district while each Candidate 2 stands in the primary of b,– is a PSNE under DS in the general case as well. Thus,

Lemma 3 *Under DS a PSNE always exists.*

In our analysis of the symmetric case, we find that whether the home districts are safe or not has important implications on the equilibrium quality under CS, but not under DS. Because even when these districts are not safe for Candidates 2 and 3, a Candidate 1, always a member of the optimal quality legislature in the symmetric case, can win both the primary and the legislative election in his party's home district. In the general case, both Candidate 1's are members of the optimal quality legislature only when neither party dominates the other in terms of candidate quality. When this is true, in the general case, too, both Candidate 1's will be elected in a PSNE..

Proposition 8 *Assume that neither party dominates the other in candidate quality. Under DS, there always exists a PSNE in which the highest-valence candidate from each party (and, thus two of the three members of the optimal quality legislature) is elected. Further, if under DS there also exists a PSNE in which this is not the case, then under CS there exists no PSNE in which the highest-valence candidate from each party is elected.*

Note that under DS, too, multiple equilibria is a possibility, and, even when neither party dominates the other in terms of its candidate quality, there may exist PSNE in which the Candidate 1 with lower valence is not elected. This would happen when the policy of that Candidate 1's party is not strongly preferred by the medians in any of the districts and the quality of the candidates from the other party is sufficiently high, e.g. under DS there exists a PSNE in which Candidates 1, 2, and 3 from R run in the primaries of districts (resp) l, b, and r, each of them winning the legislative election in these districts as well when $(v_1^L, v_2^L, v_3^L, v_1^R, v_2^R, v_3^R) = (2, 1, 3/4, 4/5, 1/2, 0)$ with $(\lambda_l, \lambda_b, \lambda_r) = (1/2, -1/20, -1/10)$.

Adding sufficiently partisan home districts gives us uniqueness.[26]

[25] The other condition on valence in Proposition 6 is required for the existence of PSNE.

[26] We actually do not need the home districts to be safe for this result to hold. A weaker condition, that each home district can be defended by own Candidate 1 against the Candidate 1 from the other party, is sufficient.

Remark 1 *When neither party dominates the other in candidate quality and the home districts are safe, under DS, the highest-valence candidate from each party (and, thus two of the three members of the optimal quality legislature) are always elected.*

The second part of Proposition 8 notes that if under DS an equilibrium in which only one of the Candidate 1's is elected to the legislature exists, then under CS there exists no equilibrium in which both Candidate 1's are elected. Intuitively, if such an equilibrium exists under DS (in this equilibrium all the members of the legislature must belong to the same party; see Lemma 2, part (iii)), under CS the principal of party R will field his candidates in these districts: if the best candidate from L cannot win a seat against this combination, then none of the candidates from L will win a seat. Thus, the possibility of a Candidate 1 not being elected under DS does not make CS a better candidate selection mechanism.

Under the assumption that neither party completely dominates the other in terms of candidate quality, the identity of the third member of the optimal-quality legislature is not determined. All we can say is that in addition to the highest-quality candidates from both parties, the optimal quality legislature must include a Candidate 2. This third member will be the Candidate 2 from R (L) if $v_2^L < (>) v_2^R$, and either of these Candidate 2's if $v_2^L = v_2^R$. Under DS, there exists a PSNE in which this candidate, too, will always be elected as long as the median voter of the battleground district does not overly favor the policy of the party to which the lower-quality Candidate 2 belongs.

Proposition 9 *When neither party dominates the other in candidate quality,*

(i) if the optimal-quality legislature includes Candidate 2 from L (R), then, $\lambda_b + v_1^L > v_2^R$ ($\lambda_b + v_2^L < v_1^R$) *is a sufficient condition for an optimal-quality equilibrium legislature to exist under DS.*

(ii) if either one of the Candidate 2's can be in the optimal-quality legislature ($v_2^L = v_2^R$)*, then the optimal quality legislature is always an equilibrium outcome under DS.*

In other words, for optimal legislature quality to be an equilibrium outcome under DS, none of the parties dominating the other one in terms of candidate valences is often[27] a necessary but not sufficient condition. With the additional condition that the battleground district does not overly favor the policy of the party with the lower-quality Candidate 2, (i) in Proposition 9, we have a set of conditions that are sufficient for the existence of an equilibrium giving rise to the optimal quality legislature under DS.

To see that these conditions, too, are sufficient but not necessary, first suppose $v_2^L < v_2^R$. Consider a strategy profile such as (b, l, r, r, b, l) under DS. Candidate 2 from L may win L's home district, while Candidate 1 from L wins the battleground district, and Candidate 1 from R wins r. In scenarios in which l is safe and $\lambda_b + v_1^L >$

[27] If the home district advantage of the dominated party is larger than the valence difference between the highest-quality candidates from each party, no PSNE with optimal legislature quality exists. For smaller home district advantages, some PSNE with optimal legislature quality will exist. If the home district advantage of the disadvantaged party is less than the valence difference between the lowest-quality candidate of the dominating party and the highest-quality candidate of the dominated party, then all PSNE will have optimal legislature quality.

v_2^R, this is a PSNE and equilibrium legislature quality is suboptimal. Part (ii) notes that if the third member of the optimal legislature can be any Candidate 2, then the policy preferences of the median voter in b become irrelevant. Intuitively, we cannot have an equilibrium in which the lower-quality Candidate 2 runs in b and wins the election against the higher-quality Candidate 2 if both have the same quality.

4 Robustness check and discussion

In this section we examine the robustness of our model to many low-quality candidates, parties proposing the same policy, convex voter policy, multiplicative valence, and non-generic valence distributions. Then, we briefly discuss legislature quality in mixed strategy Nash Equilibria under CS, and several other modeling assumptions such as no district-specific valence and deterministic elections.

4.1 Parties with policy differences

Among the existing studies of policy competition between two parties in a multi-district setup, Callander (2005) and Eyster and Kittsteiner (2007) predict policy divergence while Hinich and Ordeshook (1974) predict policy convergence (at the median of the district medians). In our analysis, we assume that the policies of the parties are not identical, $\Psi_L \neq \Psi_R$. When we analyze the case in which both parties adopt the same policy ($\Psi_L = \Psi_R$), we find that (i) under DS, the quality of legislature is always optimal, (ii) under CS, a PSNE does not exist under a large set of parameters. More specifically, under CS there exists no PSNE unless the valence ordering is such that none of the six candidates have the same valence and when all candidates are ordered according to valence, the ordering is one of the following three: $v_1^P > v_2^P > v_3^P > v_1^{P'} > v_2^{P'} > v_3^{P'}$, $v_1^P > v_2^P > v_1^{P'} > v_2^{P'} > v_3^{P'} > v_3^P$, and $v_1^P > v_1^{P'} > v_2^{P'} > v_3^{P'} > v_2^P > v_3^P$.

Proposition 10 *When parties propose identical policies ($\Psi_L = \Psi_R$),*

(i) under DS, a PSNE always exists, and in the unique equilibrium outcome the legislature is always of optimal quality;

(ii) under CS, a PSNE exists if and only if all candidates differ in their valence and for at least one P there exists no $v_j^{P'} \in [v_3^P, v_1^P]$, where $j \in \{1,2,3\}$ and $P' \neq P$.

Intuitively, when there is no policy difference between the parties, the policy preferences of voters have no effect on the outcome; voters in all districts vote purely based on the quality of the candidates. Under DS, the candidates with higher-valences can (and, will) win the election in any district in which they run; a PSNE exists. Under CS, however, each of the seat-maximizing party principals has an incentive to nominate his high-valence candidates in a district where they not only win the election, but also prevents the other party's high-valence candidates from winning the election. When the valence ordering is such that neither party can be viewed as a block (in other words an ordering such that $..v_1^P > v_2^P > v_3^P..$ does not occur for either party $P \in \{L, R\}$), then both party-principals will have a best response such that the two parties collectively win more than three seats. Since clearly it is impossible for both party-principals to simultaneously play their best responses, no PSNE exists.

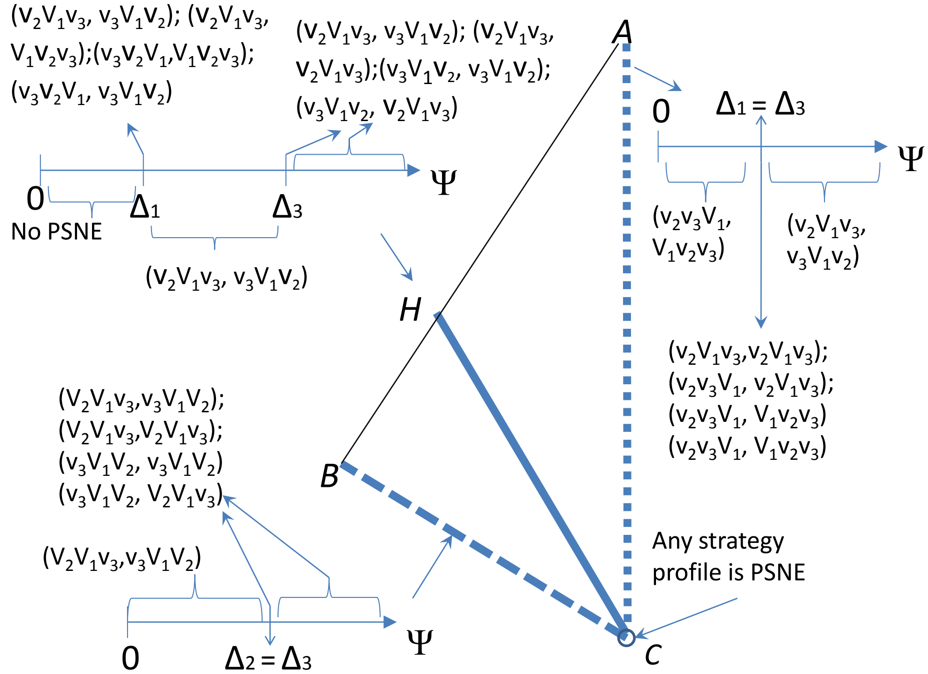


Figure 4: The PSNE under non-generic valence distributions with a centralized selector (neither party has an electoral advantage).

4.2 Candidates with valence differences

We assume that within a party, all candidates differ in their valence, ruling out valence distributions in which two (or, more) of the candidates from the same party have the same valence. This restriction is imposed mostly for the convenience it provides when calculating the PSNE under CS. Ruling out these possibilities limits the number of possible cases.

Consider the fully-symmetric case. In addition to the three cases presented in Figure 2, allowing candidates within a party to have identical valence introduces three further cases (two candidates with valence v_1^P, two candidates with valence v_3^P, and all three candidates with valence v^P) under CS. For each of these three cases, we also have subcases based on the relative importance of policy preferences and valence differences. We have not checked all of these possibilities in the general case, yet we speculate that there will not be any qualitative changes. Encouraging this speculation is the PSNE under CS when neither party has an advantage and when two (or, all) of the candidates from the same party have the same valence; see Figure 4.

As one can note by comparing Figures 2 and 4, when two of the candidates have the same valence, the equilibrium quality of legislature does not qualitatively differ from what we found under (2) in Section 3.1. That is, still when the home-districts are sufficiently partisan, equilibrium quality of legislature under CS will be suboptimal.[28]

[28]One case perhaps worth noting is that this inefficiency occurs even when each party has two candidates with the highest valence ($v_1^P = v_2^P$ for both $P \in \{L, R\}$). When $\lambda > v_1^P - v_3^P$, in this

Candidates from the same party would differ in their valences, for instance, when the candidate valences in each party are realizations of continuous i.i.d. random variables.[29] The opposite valence-generating process, which results in all candidate valences from a given party being perfectly correlated, would give rise to a setup in which each party has a given quality. In that scenario, both selectorates perform equally and not necessarily optimally.

Proposition 11 *If each candidate from party P has valence v^P, then, (i) a PSNE always exists and, effectively, it is unique, (ii) the equilibrium legislature quality is identical under CS and DS, and (iii) when there is no quality difference between the parties ($v^L = v^R$), the equilibrium legislature quality is always optimal, while otherwise ($v^L > (<) v^R$), the equilibrium legislature quality is optimal iff $\left\| v^L - v^R \right\| > \lambda_r(\lambda_l)$.*

4.3 Voters with convex and additive preferences

We assume that voter preferences are represented by the additive utility function in (3) while the utility from policy is concave (the loss function is convex). This is by far the most commonly employed utility function in the literature (where the disutility from policy difference is generally assumed to be either quadratic or linear in that difference). We employ these preferences in order to compute the electoral winners knowing only the median voter's most preferred policy. As Groseclose (2007, Lemma 1) shows, additivity with convexity are sufficient conditions for the majority to always vote for the same alternative as the median voter.[30] Groseclose (2001, 34) shows that his result is quite general and it holds under several alternative utility functions. Under any of these functional forms, our results remain unchanged as well.

For instance, Groseclose (2001, 34) notes that additivity can be replaced with multiplicativity (this is what Groseclose (2001) refers as the *competency* form). When we replace (3) with the following functional form

$$U_i(\Psi_P, v_j^P) = \frac{-L(\|i - \Psi_P\|)}{v_j^p},$$

all our analysis and results still apply; all we need to change is the way we measure policy preferences in a district and the valence differences between candidates (and, to assume that for all P and j now we have $v_j^P > 0$). More specifically, since the payoff matrices and the equilibria are determined by the relative magnitude of these two advantages only, when we define (i) P's policy advantage in a given district by $\widehat{\lambda}_d = \frac{L(\|m_d - \Psi_L\|)}{L(\|m_d - \Psi_R\|)}$, and (ii) the valence advantage between two given candidates as the ratio of their valences, $\frac{v_j}{v_{j'}}$, the games become isomorphic.

case, under CS there exists an equilibrium legislature in which the majority of the members are low quality candidates (with valence v_3^P), even though four of the six competing candidates of highest quality.

[29]Then the probability that two of the candidates from the same party having the same valence is zero. Yet, if one considers such a framework, the case in which neither party has an advantage in elections (studied in Section 3.1) is also a non-generic case.

[30]Our notation is different form that in Groseclose (2007): we use $-L(.)$ to measure utility from policy while he uses $u(.)$. As a result, he finds that for the majority to prefer the same alternative that the median voter prefers, a sufficient condition is that $u(z)$ is concave, i.e., $L(.)$ is convex.

Similarly, one can remove convexity, but this is much costlier. When $L(\cdot)$ is strictly concave, one can still obtain a setup in which the majority votes for the same alternative as the median voter, but several case-specific restrictions on the size of the support of the voter ideal points, the valence difference between the candidates, and the curvature of $L(\cdot)$ are required.[31]

4.4 No uncertainty

We assume that the voters know the quality of candidates and that the politicians (both the party-principals and the candidates) know the policy preferences of the median voters. How our results change when there is uncertainty about all of these parameters is beyond the scope of this paper. Yet, let us note that there are cases in which our results remain the same.

For example, suppose that the exact location of the median voter in b is uncertain, specifically that m_b is a random variable such that each party will have a partisanship advantage in the b with non-zero probability. Note that to determine the precise manner in which uncertainty in m_b impacts λ_b (the strength of partisanship in the battleground district) we would need to specify each party's policies and the form of the loss function.[32]

However, without specifying the loss function we are still able to make strong predictions with regards to equilibrium legislature quality as long as λ_b favors each party with non-zero probability. We also continue to assume that $\lambda_b < \lambda$ with certainty (the battleground district remains the battleground district in all realizations of m_b).

Proposition 12 *If λ_b favors both parties with non-zero probability, then equilibrium legislature quality under each type of selectorate is the same as under the deterministic, fully-symmetric case.*

To be more specific, under CS, if home districts are safe, both party-principals nominate Candidate 1 in the battleground district and the equilibrium legislature is of suboptimal quality. Only if $\Delta_2 < \lambda < \Delta_1$ will the equilibrium legislature be of optimal quality under CS (in such a case each party nominating Candidate 1 in their opponent's home district and Candidate 2 in the battleground district is always a PSNE).

Note that under DS, equilibrium quality of legislature will always be optimal. Intuitively, each party's highest-quality candidate could always run in his own party's home district and win. In such a case, both Candidate 2's will run in the battleground (to do otherwise would involve losing either the primary or general election with certainty). In fact, if the degree of uncertainty is small, all equilibria under DS

[31] Formally, for a given $L(\cdot)$, as the support of the voter ideal points becomes longer (and, for a given support of the voter ideal points, the curvature of $L(\cdot)$ decreases), it becomes more likely that the utility difference $-L(\|i - \Psi_P\|) + L(\|i - \Psi_{P'}\|)$ will change sign three times over the support of the voter ideal points.

[32] If Ψ_L and Ψ_R are -1 and 1 respectively, then some commonly-used loss functions make the transformation of uniform uncertainty in the position of the median voter into uncertainty in λ_b quite simple. For example, if a quadratic loss function is used, then $m_b \sim U[-x, x]$ results in $\lambda_b \sim U[-4x, 4x]$. Similarly, if a linear loss function is used, then $m_b \sim U[-x, x]$ results in $\lambda_b \sim U[-2x, 2x]$.

remain the same as in the fully-symmetric, deterministic case.[33] Thus, still, under this specific type of uncertainty, Theorem 1 holds, and under DS, an equilibrium exists with legislature quality at least as high as the highest-quality equilibrium legislature under CS. In fact, in this case we can further say that, as in the fully-symmetric case, DS (weakly) dominates CS.

4.5 A shortage of ***low-quality*** candidates in each party

Our model investigates the choices by the selectorate when there is a scarcity of high-quality candidates in each party. Yet, one implication of the assumption that there are only three candidates in each party's candidate pool is that there is a scarcity of low-quality candidates as well. Our results would change only minimally (and no qualitative predictions change) when each party P has more than one candidate with the lowest valence, v_3^P. More specifically, the equilibrium legislatures under DS are identical if there are many Candidate 3's, while under CS, no equilibrium is eliminated, but in many cases additional equilibria exist. However, having additional Candidate 3's under CS will not give rise to equilibria with higher-quality legislatures than in the case where each party has only one lowest-quality candidate.

Lemma 4 *Under DS, when there are many lowest-quality candidates, in any PSNE, all lowest-quality candidates from the same party run in the same primary.*

Proof. Candidate preferences dictate that they care first and foremost about their own success in the primary. Therefore, each candidate will run in a primary that he will lose with certainty only if no primary exists where he may tie. Thus, for a given party, the highest-quality candidate and the second-highest quality candidate will never run in the same primary. All lowest-quality candidates will run in the primary of the remaining district as all have a non-zero probability of advancing to the general election if and only if they run in the primary without either Candidate 1 or Candidate 2. ■

Proposition 13 *When each P has many lowest-quality candidates,*

(i) under DS, the set of equilibrium legislatures is identical to the set when each party has only one lowest-quality candidate.

(ii) under CS, no equilibrium legislature of higher quality than the highest-quality legislature when each party has only one lowest-quality candidate exists.

Note that the results under DS are dependent on the assumption that candidates care first and foremost about their success in the primary. If we change candidate preferences such that candidates care first about whether they win the general election and then about the party's success (and not at all about whether they win in the primary), then Proposition 8 changes slightly. While it is still the case that any equilibrium under one Candidate 3 per party is an equilibrium when there are many Candidate 3's per party, the converse is no longer true.[34]

[33] Specifically if under all realizations of m_b do both $\lambda_b + v_1^L > v_2^R$ and $\lambda_b + v_2^L < v_1^R$ hold.

[34] For example, the strategy profile (l, b, r, r, b, l) is no longer a PSNE in the fully-symmetric case if there is an additional lowest-quality candidate running in the primary of l. In such a case, the highest-quality candidate from L could instead run in district b, winning that seat for L with certainty. The remaining lowest-quality candidate in district l would win the general election in

4.6 Mixed strategy equilibrium

We focus on the pure strategy Nash Equilibrium (PSNE) even though under CS the games we study will typically have mixed strategy Nash Equilibrium (MNE). The equilibrium outcome in an MNE is typically a random variable: some candidates who do not belong to the optimal-quality legislature will win a seat with non-zero probability. Yet, especially under CS, there are so many possible games that an MNE with a deterministic set of winners may exist. We ignore this possibility as, under CS, the number of all possible games is in hundred thousands and the analysis of all MNE in all of these games is not feasible.

Still, using Gambit^{TM} we solved for all the MNE of all the games in the symmetric case, i.e., including games for which no PSNE exists, and games for which PSNE exists, whether the PSNE legislature quality is optimal or suboptimal. Non-degenerate MNE exist only when $\lambda < \Delta_3$. In any non-degenerate MNE, a party-principal always puts equal weight on the strategies that he mixes and mixes at most three strategies. We find that in none of the MNE is the equilibrium quality of the legislature always optimal.

4.7 District-independent valence

We assume, under both DS and CS, that a candidate has the same valence in each district. However, under many scenarios, this may not be the case. Consider the case of carpetbaggers[35] in the United States. While some carpetbaggers are ultimately successful in their candidacy, the term is often used to denigrate opponents, presumably hoping to lower their perceived valence.[36]

Other reasons for a candidate having district-dependent valence seem also quite plausible. For example, characteristics such as race or ethnicity may increase a candidate's electability in one district while decreasing it in another. Even characteristics as seemingly innocent as marital status may have different valence repercussions in different districts. Alternatively, it seems reasonable that a candidate's valence may not differ greatly in three, generally homogenous, districts in high geographic proximity. In any case, it should be duly noted that this assumption, while helpful for tractability (the number of cases analyzed is large enough without considering state-dependent valence) is not innocuous. We hope that despite this limitation, our analysis both helps explain a meaningful difference between the two types of selectorates we analyze and encourages future research in this topic.

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that district. This would improve L's outcome in the election (2 expected seats as opposed to 1.5) while the highest-quality candidate still wins his seat. Thus this would be a profitable deviation for Candidate 1 from L.

[35] The term carpetbagger refers to politicians who move to a particular district in order to run for office representing that district (sometimes the term parachute politician is used in the same context).

[36] Prominent examples of successful carpetbagger campaigns include the US Senate campaigns of Hillary Clinton (in 2000 from New York), Robert Kennedy (in 1964 from New York), and Elizabeth Dole (in 2002 from North Carolina). Unsuccessful carpetbaggers are myriad but notably Alan Keyes ran a late campaign against Barack Obama for a US Senate seat from Illinois in 2004 after the original Republican candidate, Jack Ryan, withdrew following a scandal.

5 Discussion and Conclusion

In this paper we study the quality of legislature under two selectorates: the party-principal and the party-primary, where the former is representative of any kind of centralized selector whose objective is to maximize the expected number of seats his party wins in the election. Under many plausible scenarios we find that the legislature quality is higher under party-primaries.

6 Appendix A: Proofs

Proof of Lemma 1. For (i), note that the expected maxminimized payoff for each player is a tie: under symmetry, for any given strategy of, say, L, the party-principal in R can always choose to play the mirror image of this strategy, i.e., playing (v_2, v_3, v_1) against (v_1, v_3, v_2), tying the election. Since each party can guarantee a tie, if a PSNE exists, then they must tie.

For (ii), note that when $\lambda < \min\{\Delta_1, \Delta_2\}$, the following strategy guarantees an election victory (winning two districts) to a party-principal: nominate Candidate 3 in the district where the other party nominates its Candidate 1, nominate your Candidate 1 where the other party nominates its Candidate 2, and nominate your Candidate 2 where the other party nominates its Candidate 3. (Since by nominating a candidate with the next highest quality, a party can win any district, this argument does not depend on the districts in which the other party fields its candidates.) Since each party cannot have an election victory in two (out of three) districts at the same time, no PSNE exists when $\lambda < \min\{\Delta_1, \Delta_2\}$. Finally, the for the existence under $\lambda \geq \min\{\Delta_1, \Delta_2\}$ see Figure 2, and the code that generates it (provided as an online supplement).

For (iii), note that by (i), we know that in equilibrium the parties will tie. Thus, if we can show that when a party-principal nominates Candidate 3 in b or Candidate 1 in his own home-turf, the principal of the other party can guarantee winning two districts, then we prove our claim. Assume that the principal of, say, L, nominates Candidate 3 in b. Then, R can guarantee an election victory in districts b and r by nominating its Candidate 2 in b and Candidate 1 in r. Similarly, assume that the principal of L nominates Candidate 1 in l. Then, R can guarantee an election victory in districts b and r by nominating its Candidate 1 in b and Candidate 2 in r. ■

Proof of Theorem 1. Assume that for a given set of parameters (valence vectors, policies of parties, and policy preferences) under CS there exists a PSNE with the optimal quality legislature as the equilibrium outcome. Let $\mathbf{V}_{CS} = (v^L_j v^L_{j'} v^L_{j''}, v^L_k v^L_{\widetilde{k}} v^L_{\widetilde{\widetilde{k}}})$ denote the strategy profile giving rise to this PSNE. Note that from $\mathbf{V}_{CS}$ we can always construct a strategy profile for the game under DS, a $\mathbf{V}_{DS}$, in the following way: if under $\mathbf{V}_{CS}$ a candidate from P runs in d, then under $\mathbf{V}_{DS}$ the same candidate alone will run in the primary of d. Now, we claim that under DS, the strategy profile $\mathbf{V}_{DS}$ is a PSNE giving rise to an optimal quality legislature.

Let us proceed by contradiction. Assume that the constructed strategy profile $\mathbf{V}_{DS}$ is not a PSNE. This implies that at least one of the candidates can change districts and either (i) increase the number of expected seats his party wins or (ii) improve his own outcome (tie or win if he was originally losing or win if he was originally in a tie).

But, regarding (i), if the candidate can switch seats and increase the number of expected seats his party wins, then the party-principal could have done the same under CS, implying that $\mathbf{V}_{CS}$ is not a PSNE. $\Rightarrow\Leftarrow$

Regarding (ii), a candidate could improve his own outcome in two ways. One way is by switching to a district his party would either tie or lose and (resp.) win or tie (or win) that district. However, by doing so he would increase the number of expected seats his party wins, implying that $\mathbf{V}_{CS}$ is not a PSNE. $\Rightarrow\Leftarrow$

The other way a candidate could improve his own outcome is by switching to a district his party would win or tie, win the primary in that district, and then maintain the same outcome as before (either win or tie the district). The candidate winning the primary implies that he has higher valence than one of his party members who is a member of the legislature with non-zero probability (recall that we assume that no two members of the same party have the same valence). This implies that $\mathbf{V}_{CS}$ does not result in optimal legislature quality. $\Rightarrow\Leftarrow$

Therefore, $\mathbf{V}_{DS}$ must be a PSNE. And since the same candidates are running in the same districts as in $\mathbf{V}_{CS}$, the resulting legislatures are identical and $\mathbf{V}_{DS}$ results in optimal legislature quality as well. ■

Proof of Proposition 8. The strategy profile (l, b, r, r, b, l) always results in both party's highest quality candidates being members of the legislature if $v_1^P > v_3^{P'}$ for all $P, P' \in \{L, R\}$. We proceed by showing that this strategy profile is always a PSNE (No candidate has incentive to deviate).

Consider the highest-quality candidates from both parties. Since both are winning their districts, only if deviating improves their party's outcome will they have incentive to do so. Note that if they deviate however, the party will lose the district in which they currently run with certainty. Thus neither can improve his party's outcome and has no incentive to deviate.

Consider the middle-quality candidates from both parties. By deviating, these candidates can either move to a district where the top candidate from their own party is running (which they will not do since they would lose in the primary) or to the opponent's home turf to run against the top candidate from the other party. Note that the deviating candidate could only win or tie in the opponent's home turf against Candidate 1 if he is currently winning in b against Candidate 2 (he would be running against a better candidate in a less favorable district). But in such a case, the candidate would not improve his own outcome nor would the party be better off (the reasoning is the same as in the case for the Candidate 1's. Thus no middle-quality candidate has incentive to deviate.

Consider the lowest-quality candidates. Neither will deviate because that would mean running in a primary which they would lose with certainty (each currently wins a primary with certainty).

Since none of the candidates have incentive to deviate (l, b, r, r, b, l) will always be a PSNE if neither party dominates the other and thus there will always be a PSNE in which both party's highest quality candidates are in the legislature.

For the last claim, consider the PSNE under DS in which the highest-valence candidate from, say, R cannot win a seat in the legislature. Then, by Proposition 2, none of the candidates from R can win a seat in this PSNE. But, then, a seat-maximizing party-principal can always nominate his candidates in that same order, guaranteeing three seats in the legislature. Thus, there exists no PSNE under CS in which L wins less than three seats. ■

Proof of Proposition 9. *Part (i)*: Assume, without loss of generality that $v_2^L > v_2^R$. It is straightforward to show that under DS, the strategy profile (b, l, r, r, b, l) is a PSNE if and only if $\lambda_b + v_1^L > v_2^R$.

Part (ii): When $v_2^L = v_2^R$, it is straightforward to show that under DS the strategy profile (b, l, r, r, b, l) is always PSNE. ■

Proof of Proposition 10. Part (i): To see the existence of PSNE under DS, note that any strategy profile in which the candidates with the first three highest valences are running in the primaries of three different districts is a PSNE. To see that the equilibrium legislature is unique under DS, assume that candidate j wins a seat, while candidate j' with $v_{j'} > v_j$ does not. This cannot be an equilibrium as j' could simply deviate to the primary of the district in which j runs, winning both the primary and the legislative election there. (Independent of which party he is from, j' must win the party-primary there since j was winning the seat even though both parties propose the same policy.)

Part (ii): Since the set of all possible valence orderings is quite large, we show that a PSNE exists only under these three types of valence orderings by first providing a classification of all possible orderings. Let us order *all* the candidates according to their valences (from highest to lowest). When all candidates differ in their valences, the candidate with the highest valence will be the first candidate in our ordering and the candidate with the lowest valence will be the sixth candidate. Yet, it is possible that two candidates from different parties have the same valence,– by (2), candidates from the same party cannot have the same valence. When this is the case, we assign the lower rank to the candidate with the lower ranking in his party. That is, when, for instance, $v_1^P > v_2^P > v_3^P = v_1^{P'}$, we say that Candidate 3 from P is the fourth candidate. It is possible that the candidates with the same valence also have the same ranking in their parties, in this case one can use either permutation (when, for instance, $v_1^P = v_1^{P'} > v_2^P$, one may say Candidate 1 from P or P' is the first (second) candidate). Since our argument does not depend on the name of the parties, either permutation is accounted for in our argument below.

Now, consider the fourth candidate in the order described above. We have three possibilities: this is either a Candidate 1, or a Candidate 2, or a Candidate 3, from party $P \in \{L, R\}$. To show that the condition in Proposition 5 is a *necessary* condition for the existence of PSNE, below we show that in each possibility (case), there is no PSNE when either two candidates have the same valence or the valence ordering is different from the ones provided in Proposition 5. Our method is proof by contradiction: we simply note that when either one of these two conditions is violated, by placing his candidates in districts where certain candidates of the other party are nominated, for any strategy of his rival, the principal of each party P can guarantee g_p seats, where $g_L + g_R > 3$. (Since the total number of seats each principal can guarantee by reshuffling his candidates exceed the total number of available seats, no PSNE exists.)

Case (i): the fourth candidate is Candidate 1 from P. In this case, our condition is satisfied by default. There is neither an equality nor a hole in the ranking (given our rule, Candidate 3 from P' would be ranked as the fourth candidate had we have $v_3^{P'} = v_1^P$); we have $v_1^{P'} > v_2^{P'} > v_3^{P'} > v_1^P > v_2^P > v_3^P$.

Case (ii): the fourth candidate is Candidate 2 from P. In this case no PSNE exists if any candidate from P' has valence that falls in the interval $[v_1^P, v_2^P]$, i.e., if there is a hole in the ranking to the left of the fourth candidate. To see why, let us study each of the possible subcases in which there are $K \in \{1,2,3\}$ candidates from P' whose valences lie in this interval (K must be less than three; otherwise, Candidate 2 from P cannot be the fourth candidate).

If $K = 3$, then we must have $v_1^{P'} = v_1^P$ with $v_3^{P'} = v_2^P$. Then, the principal of P' can guarantee two seats (by nominating his Candidate 1(3) in the district in which Candidate 2(3) from P is nominated), while the principal of P can guarantee an expected one and half seats (by nominating his Candidate 1(2) in the district in which Candidate 2(3) from P' is nominated).

If $K = 2$, then we must have either $v_2^P > v_3^{P'}$ or $v_2^P = v_3^{P'}$. In the first case we must have $v_1^{P'} \geq v_1^P > v_2^{P'} \geq v_2^P$, while in the second case we must have $v_2^{P'} = v_1^P > v_3^{P'} = v_2^P$. In the former case, given any strategy by his opponent, the principal of P' (P) can guarantee two seats by nominating his Candidate j in the district in which Candidate $j+1$ from the other party is nominated, where $j \in \{1,2\}$. In the latter case, the principal of P' can guarantee three seats (by nominating his Candidate j in the district in which Candidate j from the other party is nominated, where $j \in \{1,2,3\}$), while the principal of P can guarantee one seat by nominating his Candidate 1 in the district in which P' nominates its Candidate 3.

If $K = 1$, we must have $v_1^{P'} > v_1^P$ with $v_2^{P'} \in [v_2^P, v_1^P]$. If $v_2^{P'} < v_1^P$, then the principal of P' can guarantee two seats by nominating his Candidate 1(2) in the district in which Candidate 1(3) from P is nominated, while the principal of P can guarantee at least an expected 1.5 by nominating his Candidate 1(2) in the district in which Candidate 2(3) from P' is nominated. Thus, no PSNE exists when $K = 1$ with $v_2^{P'} < v_1^P$. If $v_2^{P'} = v_1^P$, then the principal of P' can guarantee two seats when $v_3^{P'} < v_3^P$ (2.5 seats when $v_3^{P'} = v_3^P$, and three seats if $v_3^{P'} > v_3^P$) while the principal of P' can guarantee an expected 1.5 seats when $v_3^{P'} < v_3^P$ (1.5 seats when $v_3^{P'} = v_3^P$, and one seat when $v_3^{P'} > v_3^P$) by placing his Candidate 1 where Candidate 2 from P' is nominated and nominating his Candidate 2 where the other party nominates its Candidate 3.

So far, we established that when the fourth candidate is Candidate 2 from P, a necessary condition for PSNE to exist is that no candidate from P' has valence that falls in the interval $[v_1^P, v_2^P]$: we must have $v_1^{P'} > v_2^{P'} > v_1^P > v_2^P$. Given this, again, for the case in which the fourth candidate is Candidate 2 from P, next we prove that no PSNE exists when $v_3^{Pprime} \in [v_2^P, v_3^P]$: assume that $v_3^{Pprime} \in [v_2^P, v_3^P]$, then the principal of P' can guarantee at least an expected two and a half seats by nominating his candidate $j \in \{1,2,3\}$ where the principal of P nominates his candidate j. Yet, the principal of P can guarantee at least a seat (by nominating his Candidate 1 in the district in which Candidate 3 from P' is nominated). Contradiction. Thus, when the fourth candidate is Candidate 2 from P, a necessary condition for PSNE to exist is that $v_1^{P'} > v_2^{P'} > v_1^P > v_2^P > v_3^P > v_3^{P'}$.

Case (iii): when the fourth candidate is Candidate 3 from P, a necessary condition for the existence of PSNE is $v_1^{P'} > v_1^P$. When $v_1^{P'} \leq v_1^P$, the principal of P can guarantee two and a half expected seats (three seats, if $v_1^{P''} < v_1^P$) by nominating Candidate 1 in the same district as Candidate 1 from P' and nominating his other two candidates in the districts in which P' nominates its Candidates 2 and 3 while the principal of P' can guarantee one seat (half a seat if $v_1^{P'} = v_3^P$) by nominating his Candidate 1 in the district in which Candidate 3 from P runs in the election.

Thus, we show that the conditions in Proposition 5 above are necessary. It is straightforward to show that when these conditions hold, any strategy profile in which the party principals nominate the three highest-valence candidates in different districts is a PSNE. ■

Proof of Proposition 11. Parts (i), (ii), and (iii) follow from a simple observation: when $v_j^P = v^P$ for all $j \in \{1, 2, 3\}$, effectively, each party has a single strategy (and, there is a single strategy profile under CS): nominate the candidate with valence v^P in any given district. Similarly under DS, in which primary a candidate runs does not matter as long as he is the only candidate from his party running in that primary.

For the equilibrium quality, note that when $v^L = v^R$, any three of the six candidates would make an optimal quality legislature. When $v^L > (<) v^R$, the optimal quality legislature consists of only members of L (R). This will happen if and only if a candidate from L (R) wins the other party's home district. ■

Proof of Proposition 12. Let π_P be the probability that party $P \in \{L, R\}$ is favored in district b ($\pi_L = Pr(\lambda_b < 0$, $\pi_R = 1 - \pi_L)$. By playing Candidate 2 in its home turf, Candidate 1 in b, and Candidate 3 in its opponent's home turf P wins its home turf with certainty and b with probability π_P ($1 + \pi_P$ expected seats).

First, we prove this strategy is a PSNE by showing that no profitable deviation exists for P.

Assume P deviates from this strategy. P can do so in two ways; by (i) nominating Candidate 1 in a district other than b or (ii) switching the districts in which Candidate 2 and 3 are nominated.

Suppose P nominates Candidate 1 in a district other than the b. P will lose b with probability $\pi_P - Pr(\lambda_b \in (0, \Delta_k))$ where $k \in \{1, 3\}$ (depending on whether P replaces Candidate 1 in b with Candidate 2 or 3). P will also lose the opponent's home turf with certainty ($\lambda > \Delta_1$). Therefore, P wins $1 + \pi_P - Pr(\lambda_b \in (0, \Delta_k))$ seats. Since $1 + \pi_P - Pr(\lambda_b \in (0, \Delta_k)) < 1 + \pi_P$, P will not deviate in this way.

Suppose P switches the districts in which it nominates Candidate 2 and 3 (Candidate 2 is now in its opponent's home turf while Candidate 3 is in P's home turf). P will, at best (when $\lambda > \Delta_2$), win its home district, always lose the opponent's home turf, and still win with probability π_P in b. As a result, P wins, at most, $1 + \pi_P$ expected seats by deviating and is indifferent between the two strategies.

Since no profitable deviation exists for P, nominating Candidate 1 in b, Candidate 2 in P's home turf, and Candidate 3 in the opponent's home turf is a PSNE.

We now show this is a unique PSNE. We show above that if P proposes any strategy in which Candidate 1 is not nominated in b, P will win $1 + \pi_P - Pr(\lambda_b \in (0, \Delta_k))$ expected seats if the opponent plays Candidate 1 in b and Candidate 3 in P's home turf. And since $1 + \pi_P - Pr(\lambda_b \in (0, \Delta_k)) < 1 + \pi_P$, P will not play such a strategy.

The only remaining candidate strategy for P is to nominate Candidate 3 in P's home turf, Candidate 1 in b, and Candidate 2 in the opponent's home turf. By doing so P will win, at most, 1 expected seat if the opponent nominates its Candidate 1 in P's home turf, its Candidate 2 in its home turf, and its Candidate 3 in b. ($L(R)$ will win $Pr(\lambda_b > (<)\Delta_3)$ expected seats. Therefore, P will not propose such a strategy and nominating Candidate 1 in b, Candidate 2 in P's home district, and Candidate 3 in the opponent's home district is the unique PSNE. ■

Proof of Proposition 13. (i) First assume that C^{PSNE} is an equilibrium legislature when each party has only one lowest-quality candidate. Denote the districts that Candidate j of Party P runs in the primary as d_j^P. As noted in the proof of Lemma 4, Candidates 1, 2, and 3 from each party will locate in different districts for the primary. Now consider the case when each party has many lowest-quality candidates. Assume the candidates locate during the primary such that Candidate j of Party P locates in the same district d_j^P as when there is only one Candidate 3 of each party. This is an equilibria as well that gives rise to the same legislature C^{PSNE}.

Now assume that C^{PSNE} is an equilibrium legislature when each party has many lowest-quality candidates. By Lemma 4, note that all lowest-quality candidates from each party will run in the same primary (and only those of lowest quality will run in that primary). Denote the districts that Candidate j of Party P runs in the primary as d_j^P. Consider the case when there is only one Candidate 3 from each party. Assume candidates locate during the primary such that each Candidate j of Party P locates in the same district d_j^P as when there are many lowest-quality candidates in each party. This will be an equilibria as well with the same legislature C^{PSNE}.

(ii) Consider two cases, first those equilibria with only one lowest quality candidate nominated per party and second those equilibria with many lowest quality candidates. We show that in each case, any equilibria under many Candidate 3's has a corresponding equilibrium which has (weakly) higher legislature quality under one Candidate 3 per party.

Let CS^{many} be a PSNE when both parties have more than one lowest-quality candidate. Note that if in CS^{many} both parties nominate only one Candidate 3 each, then CS^{many} will be an equilibrium strategy when both parties have only one Candidate 3 as well (since neither party can increase the number of expected seats by introducing one of the "unused" lowest-quality candidates, the conditions for equilibrium are the same under both scenarios). Any such equilibria under many Candidate 3's will thus result in an equilibrium legislature quality no higher than equilibria under only one Candidate 3 per party.

Now, let CS^{many} be a PSNE when both parties have more than one lowest-quality candidate such that one or both parties nominate more than one Candidate 3. In such a scenario, note that a party that does so, leaves higher quality candidates unused. Thus, for such CS^{many} there is a corresponding equilibrium, $CS^{many}*$, that is identical to CS^{many} except that each party replaces all but one of the lowest-quality candidates with a higher-quality one (note that this strategy weakly dominates all strategies with more than one Candidate 3). As shown above, $CS^{many}*$ is also an equilibrium when both parties have only one Candidate 3. Further note that the legislature in $CS^{many}*$ cannot be of lower quality. Thus for any PSNE with more than one Candidate 3 deployed by either party, then there is a corresponding equilibria of no lower quality under the scenario with only one Candidate 3 per party.

Therefore, for any equilibrium when parties have many Candidate 3's, under DS, there exists a corresponding equilibrium of no lower legislature quality when parties have only one Candidate 3. ■

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