Submission Number: PET11-11-00122

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Keywords: Curriculum, school choice, academic performance
JEL code: I21, J24, J78

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1 Introduction

Different students are best served under different curricula. The hierarchical nature of the learning process dictates that students have to internalize some basic concepts before moving to advanced topics. As a result, students with solid background knowledge can progress relatively quickly, while students on shaky footing need to slow down to truly learn a subject. When a student is matched to a curriculum at just the right level, his learning efficiency is maximized.

Unfortunately, it is not feasible to personalize the curriculum for each individual student. When many students go to the same school and share the same classes, the curriculum becomes a public good. Thus a mismatch can take place. An overmatch happens when the curriculum is too challenging, i.e., when the student knows less than what the curriculum assumes, so he struggles to keep up. An undermatch happens when the curriculum is not challenging enough, i.e., where the student knows more than what the curriculum assumes, so he gets bored by the slow pace. In either case, the student suffers a loss in his learning efficiency due to the mismatch.

The importance of curriculum in shaping a student's learning experience is our main focus in this paper. It has been hinted at in the empirical literature. For example, in terms of returns to schooling, there have been recent findings that returns to schooling, especially college education, are much lower for the marginal students than the average students. These differential benefits from schooling suggest differential learning efficiencies across students facing the same curriculum. In terms of affirmative action in college admission, there have been heated debates on whether affirmative action hurts the minority students that it intends to benefit. The "overmatch hypothesis" asks whether it is to their benefits to preferentially admit minority students into highly selective colleges and hence subject them to an overly challenging curriculum. Surprisingly, the role of curriculum has received little attention in the theoretical literature. This paper intends to fills in the gap and offers a theoretical model on curriculum designs.

A curriculum is characterized a two-parameter education technology. It imposes a minimum threshold on student qualification - the background knowledge he has to master, and a corresponding progress rate - how quickly we move from topic to topic. A challenging curriculum is one with high qualification threshold and fast progress rate, while an accommodating curriculum is one with low qualification threshold and slow progress rate. Students with high qualifications will maximize their learning efficiencies under a challenging curriculum, while students with low qualifications will maximize their learning efficiencies under an accommodating curriculum. The interpretation is analogous to absolute versus comparative advantage. For any given curriculum, students with high qualifications learn more than students with low qualifications, so they enjoy absolute advantage. On the other hand, comparing two different curricula, students with high qualifications enjoy comparative advantage in a challenging curriculum, while students with low qualifications enjoy comparative advantage in an accommodating curriculum. Thus, if students are academic driven, there is an optimal match between their qualifications and the suitable curriculum.

Of course, curriculum is not the only determining factor in students' academic outcomes. Nor is it the only concern in their school choice decisions. Along the academic dimension, besides curriculum, school quality measured as average expenditure per student also matters. The distinction between the curriculum effect and school quality effect is subtle yet important. Under identical curriculum, students tend to benefit more if they have smaller classes, better motivated teachers, more lab equipment, etc. This is the school quality effect. Under identical school quality, students with high qualifications benefit more if the class pushes the envelope and tackles challenging questions, while students with low qualifications benefit more if the class focuses on the foundation and offers many repetitions and examples. This is the curriculum effect. We show that when schools face no capacity constraints, adding the school quality effect changes students' decisions and outcomes quantitatively but not qualitatively. On the other hand, for empirical studies, if the curriculum effect is not properly accounted for, the estimated school quality effect may be significantly upward biased.

Besides the academic value of schooling, students may also care about other dimensions of their school experiences. When these factors are taken into account, their optimal school choices may be different from the optimal match between the school curriculum and their qualifications. In particular, if the students care for dimensions that are arguably negatively correlated with the academic program, for example, lower stress level or famed sport teams, they may optimally choose an undermatch. On the other hand, if the students care for some dimensions that are positively correlated with the academic program, for example, eliteness or networking opportunities, they may optimally choose an overmatch. In both cases, if we maintain the assumption of individual rationality, the loss in learning efficiency from a mismatch has to be more than offset by the gain in utility from other dimensions. In particular, we consider college affirmative action policies that give minority students preferential admission. We characterize how the observed school choices for similarly comparable minority students can be driven by unobservable heterogeneity. Again if the curriculum effect is not properly accounted for, empirical studies may lead to biased estimates for the mismatch effect.

The rest of the paper is structured as follows. Section 2 contains the literature review. Section 3 introduces the benchmark curriculum model and characterizes students' school choices. Section 4 extends the benchmark curriculum model to address specific policy issues like school quality, school financing, and potential mismatch under affirmative action. Section 5 provides some further discussions and draws the conclusion.

2 Literature review

This paper can be linked to several strands of the existing literature. First and foremost, starting from with Mincer (1974), there has been a vast literature of empirical labor economics on the identification and estimation of returns to schooling.[1] While earlier studies have focused on the average returns to schooling, recent studies have devoted increasing attentions to the heterogeneous returns to schooling across students. Heckman, Schmierer and Urzua (2010) find that not only returns to schooling vary across students, but more importantly, students take their idiosyncratic return into account when making schooling decisions. Carneiro, Heckman and Vytlacil (forthcoming) estimate the marginal returns to college education, and find that the marginal students induced to attend college under marginal policy changes face very low returns. One important source for the heterogeneous returns to schooling is undoubtedly the difference in students' abilities, which captures the "absolute advantage" aspect we discussed above. This paper points out another equally important and complementary source for the heterogeneous returns to schooling, namely the difference in school curricula and the resulted match between curriculum and students, which captures the "comparative advantage" aspect. Similar to ability, curriculum cannot be easily observed or measured, even though some

[1] For example, see Welch (1973), Griliches (1977, 1979), Garen (1984), Angrist and Kruger (1991), Card and Kruger (1992a, 1992b), Lam and Schoeni (1993), Ashenfelter and Kruger (1994), Altonji and Dunn (1996a, 1996b), and Ashenfilter and Zimmerman (1997), among many others. Card (1999) provides an excellent survey of the literature.

proxies exist and arguably capture certain aspects of the variable of interest. Thus, if the curriculum is important in determining students' academic outcomes, it poses another "omitted variable" problem, opens another dimension of self selection, and further complicates the econometric analysis.

Another strand of the related empirical literature examines the effect of affirmative action. Sander (2004) proposes a hypothesis of mismatch. He argues that when admitting minority students into highly selective universities and hence subjecting them to an overly challenging curriculum, affirmative action policies have negative impact on the academic outcomes of its intended beneficiaries. Heated debates ensue, for example, see Ayres and Brooks (2005), Chambers *et al* (2005), Ho (2005), Sanders (2005a, 2005b), and Barnes (2007). While that literature fails to reach a consensus, it does bring the importance of how curriculum affects students' academic achievements into focal attention. Where the observational data may fail to separate the curriculum effect from other factors such as the peer effect, the self selection effect, and the confounding effect from any other unobserved factors, a theoretical model can shed some light on the individual mechanism through which these factors operate. This paper intends to offer such a theoretical model.

On the theory side, a closely related literature in the economics of education is about school choice. The attention so far, however, has been on peer effect, school finance, and other observable measures such as teacher quality, class size, and education expenditures, etc. Epple and Romano (1998, 2002), Epple, Newlon and Romano (2002) and Epple, Figlio and Romano (2004) analyze school choice between private and public schools, and show that a hierarchy of school qualities can arise endogenously when private schools use the tuition as an exclusion mechanism. In their setup, all schools adopt the same curriculum (the same education technology), and difference in school quality is purely driven by the composition of the student population and hence the average student ability.[2] Along a similar line, Fernandez and Gali (1999) analyze the efficient match between student quality and school quality, which are complements in the production technology. Again, despite the different levels of inputs faced by different students, the overall production technology remains the same for all. They compare the allocative efficiency through a market versus that through a tournament, taking potential borrowing constraints into account. This paper adds to the literature by allowing schools to adopt different curricula

[2] For empirical analysis of such "cream skimming" effect of school choice, see Hoxby (2002, 2003) and Hsieha and Urquiolab (2006) for example.

(different production technologies). The optimal school choice will hinge on the optimal match between a curriculum and the student quality, besides all the other factors that have been identified in the literature.

Last but not the least, there is a growing literature on vertically differentiated education technologies. Driskill and Horowitz (2002) and Su (2004, 2006) model education as a sequence of hierarchical stages, where the human capital output from a lower stage acts as an input in the education technology of a higher stage. In that framework, the school choice problem concerns whether to go for higher education or not, and what is the macroeconomics impact on efficiency and inequality. In contrast, this paper focuses on horizontally differentiated education technologies. So the school choice problem is to find the optimal match between a curriculum and the student quality among many competing curricula.

3 Benchmark model of curriculum

Consider an economy with a continuum of heterogeneous students and a finite number T of schools. Students are indexed by $i \in [0, 1]$ and differ in their quality q_i, which is distributed with the density function $\phi(.)$ on $[\underline{q}, \overline{q}]$. The density is assumed positive and finite everywhere, and total measure of students is normalized to be 1. Schools differ in the curricula they offer, where each curriculum is characterized by the two parameters (A_t, c_t).

3.1 Curriculum as education production technology

When a student with qualification q_i enters school t with the curriculum (A_t, c_t), his human capital output from attending school t is given by the following education technology:

$$h_{it} = \begin{cases} 0 & \text{if } q_i \leq c_t \\ A_t(q_i - c_t)F(\mathbf{X}_t, \mathbf{Y}_i, \mathbf{Z}_{it}) & \text{if } q_i > c_t \end{cases} \tag{1}$$

The main focus is on the student's learning efficiency under the given curriculum $A_t(q - c_t)$. The parameter c_t measures the minimum qualification requirement under this curriculum, below which a student cannot benefit from this type of schooling. A higher c_t implies a more challenging program and can also be interpreted as imposing a higher academic standard.[3] On the other hand the

[3]Note that unlike Costrell (1994, 1997) and Betts (1998), where academic standard is a policy chosen threshold, in this paper, academic standard is intrinsic to a given education

parameter A_t measures the progress rate of the curriculum. A higher A_t implies faster progress during the schooling process.

Using a familiar example of teaching intermediate microeconomics, if c_t is high, i.e., when students are proficient in calculus, the course may set up the Lagrangian problem, take the first order condition, solve for the optimal choices, and probably even proceed to comparative statics, i.e., A_t is high. On the other hand, if c_t is low, i.e., when students barely know calculus, the course may start with how to take partial derivatives, how to solve algebraic problems, offer many examples and repetitions, and probably not advance beyond formula based approach, i.e., A_t is low.

As can be easily seen, a higher A_t must accompany a higher c_t to guarantee that this education technology is not dominated in production efficiency. Without loss of generality, it is assumed that $\{A_t, c_t\}_{t=1}^{T}$ is ranked in a strictly ascending order, so that school 1 implements the curriculum with the lowest academic standard and the slowest progress rate, while school T implements the curriculum with the highest academic standard and the fastest progress rate.

It is worth pointing out that even though the curricula can be ranked by their academic standards and progress rates, they cannot be ranked when it comes to the production efficiency. Students with different qualifications benefit most from different curricula, so there exists an optimal match. It is equally harmful for student learning when the curriculum is too challenging or not challenging at all, while at the right level his learning efficiency $A_t(q-c_t)$ can be maximized. On one hand, high qualification students have absolute advantage in any given curriculum than low qualification students. On the other hand, high qualification students have comparative advantage in a more challenging curriculum, while low qualification students have comparative advantage in a less challenging curriculum. Each curriculum is the best choice for a stratum of the students with certain qualifications.

On the other hand, all other inputs into the education technology can be captured in a general function $F(\mathbf{X}_t, \mathbf{Y}_i, \mathbf{Z}_{it})$. Here $\mathbf{X}_t$ is a vector of school-specific factors such as school size, average teacher quality, average student quality, average expenditure per student, etc. The vector $\mathbf{Y}_i$ captures student-specific factors such as parental education level, home environment, sibling effect, etc. The vector $\mathbf{Z}_{it}$ captures the student choices that may vary if he goes to different

technology.

school, such as student learning effort, tuition payment, etc.

Assumption 1. In the benchmark case, $\mathbf{X}_t = \mathbf{X}$, $\mathbf{Y}_i = \mathbf{Y}_i$, and $\mathbf{Z}_{it} = \mathbf{Z}$.

Assumption 1 allows us to focus on the role of curriculum-student match in the benchmark case, while abstracting from variations in the factors of $\mathbf{X}_t$, $\mathbf{Y}_i$, and $\mathbf{Z}_{it}$. Extensions and applications of the benchmark model will be analyzed in Section 4.

3.2 Optimal student-curriculum match

Undoubtedly, the curriculum effect is present at every stage of the education process: primary, secondary, and tertiary. However, in the analysis, we interpret the model in the framework of higher education (tertiary). This by no means implies that the curriculum effect is not important for basic education (primary and secondary). The fact that students may repeat or skip a grade in the K-12 process clearly demonstrates the significance of the curriculum effect. More specifically, students repeat a grade if their qualifications are deemed inadequate to meet the minimum threshold for the next grade. So instead of proceeding to the next level, whose curriculum is too challenging for them, they are better served repeating the grade where the curriculum is more accommodating. On the other hand, students skip a grade if their qualifications are deemed far surpassing the minimum threshold for the next grade. So instead of proceeding to the next level, whose curriculum is not challenging enough for them, they are better served going directly to a higher level where the curriculum is more challenging. This is precisely the comparative advantage that students with different qualifications enjoy in different curricula.

There are three main reasons why we interpret the current curriculum model in the framework of higher education. First, in higher education, students have more latitude in choosing their colleges across wide geographic boundaries. This is in contrast to basic education, where students' school choices are more constrained by the designated school district, which is in turn determined by their family's residential choices. Second, in higher education, students are hardly "tracked" within a college, so we can more easily equate their school choices are their curriculum choices. This is in contrast to basic education, there schools typically offer "gifted" classes and regular classes, which arguably represent two different curricula within the same school. Third, in the model, we allow the possibility that some students at the very low end of the qualification spectrum to stay out of schools if they fail to meet the minimum threshold of the least challenging curriculum. This is again in contrast to basic education, where compulsory schooling laws apply and students cannot stay out of schools. We adopt the higher education interpretation of ease of exposition. The same main results apply to basic education with minimal changes.

Abstracting from the component $F(\mathbf{X}_t, \mathbf{Y}_i, \mathbf{Z}_{it})$, a student chooses among the competing curricula as follows. Consider two adjacent curricula t and $t+1$ where his qualification q_i meets both the minimum requirements. Then his learning efficiency is equal to $A_t(q_i - c_t)$ under curriculum t, and $A_{t+1}(q_i - c_{t+1})$ under curriculum $t+1$. This leads to the following cutoff points:

$$\widehat{q}_{(t+1)t} = \frac{A_{t+1}c_{t+1} - A_t c_t}{A_{t+1} - A_t} = c_{t+1} + \frac{c_{t+1} - c_t}{\frac{A_{t+1}}{A_t} - 1} \tag{2}$$

It is easy to see that for for a student with qualification $q_i < \widehat{q}_{(t+1)t}$, curriculum t suits him better and leads to higher learning efficiency; while for a student with qualification $q_i > \widehat{q}_{(t+1)t}$, curriculum $t+1$ suits him better and leads to higher learning efficiency. The student would be indifferent between the two curricula if $q = \widehat{q}_{(t+1)t}$. Without loss of generality, we assume that in case of a tie, the student always chooses school t instead of school $t+1$.

Assumption 2. $\widehat{q}_{(t+2)(t+1)} > \widehat{q}_{(t+1)t}$ for $t = 1, 2, ..., T-2$.

Assumption 2 ensures that all curricula are at the technological frontier. Otherwise curriculum $t+1$ would be strictly dominated by curriculum t for $q_i < \widehat{q}_{(t+1)t}$, strictly dominated by curriculum $t+2$ for $q_i > \widehat{q}_{(t+2)(t+1)}$, and equivalent to both curricula t and $t+2$ for $q_i = \widehat{q}_{(t+1)t} = \widehat{q}_{(t+2)(t+1)}$. If that were the case, curriculum $t+1$ should never be adopted in the economy for efficiency concerns.

Assumption 3. $\underline{q} < c_2$ and $\overline{q} > \widehat{q}_{T(T-1)}$.

Assumption 3 ensures that all curricula are economically relevance, in that each of which has some students who strictly prefer it. The condition $\underline{q} < c_2$ implies that $\underline{q} < \widehat{q}_{21}$, where some students with low qualifications strictly prefer curriculum 1 curriculum 2, while the condition $\overline{q} > \widehat{q}_{T(T-1)}$ implies that some students with high qualifications strictly prefer curriculum T to curriculum $T-1$, so that both curricula at the extreme ends are not dominated in production efficiency.

Under Assumptions 1-3, we can characterize a student's preference over the curricula and hence the optimal student-curriculum match.

Proposition 1 *A student's preference over all curricula is single peaked, and there is perfect stratification of student qualification across schools. More specifically, for a student with qualification* q_i*, the optimal curriculum* $s_i^* = T$ *if* $q_i > \widehat{q}_{T(T-1)}$*,* $s_i^* = t+1$ *if* $\widehat{q}_{(t+1)t} \leq q_i \leq \widehat{q}_{(t+2)(t+1)}$ *for* $t = 1, 2, ..., T-2$*,* $s_i^* = 1$ *if* $c_1 < q_i \leq \widehat{q}_{21}$*, and* $s_i^* = 0$ *if* $q_i \leq c_1$*.*

Proof. *It is easy to check that Assumption 2 is equivalent to* $\widehat{q}_{(t+s)t} > \widehat{q}_{(t+1)t}$ *for any* $s > 1$*, so a student's preference over all curricula is single peaked. The perfect stratification follows directly from the definition of* $\widehat{q}_{(t+1)t}$*. Also note that since we do not assume* $\underline{q} \geq c_1$*, it is possible that some students with the lowest qualifications cannot benefit from schooling.* ■

This Proposition establishes the curriculum effect on students' school choice decisions. Given that different curriculum best serves students with different levels of qualifications, it is not surprising that there is perfect stratification of student qualifications across schools. This perfect stratification arises purely from the technological perspective. Students self select into the curriculum where they have comparative advantage, where their learning efficiency is maximized.

The benchmark case abstract from many important factors that arguably also contribute to students' decisions, such as school quality, school financing, affirmative action policy in admission, etc. In the next section, we extend the benchmark case to include these factors and compare the outcomes.

4 Extensions and applications

In this section, we get rid of Assumption 1 of the benchmark curriculum model, allow variations in the vectors of $\mathbf{X}_t$, $\mathbf{Y}_i$, and $\mathbf{Z}_{it}$, and analyze the effect of specific education policies.

4.1 School quality

In this extension, we introduce school quality as measured by the average education expenditure per student. We analyze how the enrollment size N_t changes with total education expenditure G_t at each school. More specifically, our focal interest is how school quality affects students' school choices and academic outcomes, so we abstract from the issue of school financing. For this reason, we assume that education expenditure G_t is purely publicly financed, a lump-sum tax τ is used to balanced the government budget, and students pay no tuitions. Consequently, students do not consider the school financing problem when making optimal school choices.

In this example, the benchmark equation (1) takes the following form:

$$h_{it} = \begin{cases} 0 & \text{if } q_i \leq c_t \\ A_t(q_i - c_t)F(G_t, N_t) & \text{if } q_i > c_t \end{cases} \tag{3}$$

Assumption 4 $F_1 > 0, F_2 < 0, F(0,.) = 0, F(.,0) = +\infty$

Assumption 4 implies that for a given level of enrollment, public education expenditure is an essential and productive input into the education technology. At the same time, for a given level of public education expenditure, high enrollment has a congestion effect, while low enrollment leads to concentration of the education expenditure and hence extremely high school quality for a few students. Overall, the term $F(G_t, N_t)$ can be interpreted as school quality, which is independent of the school curriculum.

This is the place where a sharp distinction can be seen between school quality and the curriculum. School quality, measured as a scalar, can be easily ranked across schools. On a per student basis, the high is the public education expenditure, the higher is the school quality such as better qualified teachers, smaller class sizes, better infrastructures, etc. A better school quality always contributes positively to a student's learning efficiency, everything else the same. On the other hand, it is not the case that a more challenging school curriculum always enhances a student's learning efficiency. Quite the contrary, each curriculum best serve a particular segment of students, depending on their qualifications.

Naturally, if two schools share identical curriculum but have different school quality, we expect different academic outcomes for students attending these two schools, even if they have identical qualifications. This effect of school quality on academic outcomes has been extensively studied in the empirical literature. However, if two schools have identical school quality but adopt different curricula, they can also lead to different academic outcomes for students attending these two schools, even if they have identical qualifications. This effect of curriculum on academic outcomes has been largely overlooked in the empirical literature. This is partly due to the difficulty of devising a numerical measure to capture the curriculum effect, and partly due to the difficulty of simultaneously estimating many horizontally differentiated education technologies. However, as will become clear later, omitting the curriculum effect from empirical studies may have confounding impact on the school quality effect.

Definition Taking $\{A_t, c_t, G_t\}_{t=1}^T$ as given, an enrollment equilibrium consists of students' optimal school choices $\{s_i^*\}_{i=0}^1$ and the school enrollment $\{N_t\}_{t=1}^T$ such that:

(1) Given $\{N_t\}_{t=1}^T$, $\{s_i^*\}_{i=0}^1$ maximizes each students' human capital output according to (3);

(2) Given $\{s_i^*\}_{i=0}^1$, $N_t = \displaystyle\int I(s_i^* = t)\phi(i)di$;

(3) The lump-sum tax τ balances government's' budget: $\tau = \sum_{t=1}^T G_t$.

Similarly to the benchmark case, taking $\{A_t, c_t, G_t, N_t\}_{t=1}^T$ as given, a student's optimal school choice can be determined as follows. For two adjacent curricula t and $t+1$ where his qualification q_i meets both of the minimum thresholds, his learning efficiency is equal to $A_t(q_i - c_t)F(G_t, N_t)$ under curriculum t, and $A_{t+1}(q_i - c_{t+1})F(G_t, N_t)$ under curriculum $t+1$. The optimal school choice is when his learning efficiency is maximized. This leads to the following cutoff points:

$$\widetilde{q}_{(t+1)t} = c_{t+1} + \frac{c_{t+1} - c_t}{\frac{A_{t+1}F(G_{t+1}, N_{t+1})}{A_t F(G_t, N_t)} - 1} \tag{4}$$

Proposition 2 *Suppose an enrollment equilibrium exists. Then in the equilibrium $\{\widetilde{q}_{(t+1)t}\}_{t=1}^{T-1}$ is strictly increasing with $\underline{q} < \widetilde{q}_{21}$ and $\overline{q} > \widetilde{q}_{T(T-1)}$. Furthermore, there is perfect stratification of student qualification across schools.*

Proof. *See Appendix.* ■

This proposition characterizes the properties of an enrollment equilibrium if it exists. These properties in turn facilitate the proof of the existence and uniqueness of the enrollment equilibrium in Proposition 3. It says that an equilibrium has to be such that after both the public education expenditure and the enrollment size are taken into account, none of the schools is dominated by others. Rational students adjust their school choices in response to different levels of public education expenditure at different schools, and the congestion effect from enrollment expansion counterbalances the advantage of having more education expenditure allocated to a particular school. Just like the benchmark curriculum model where all curricula are at the technological frontier, adding school quality into the problem does not change the pattern. All schools still remain at the technological frontier, and each appeals to a segment of the students with comparative advantage in that particular curriculum. Without uncertainty or other dimensions of student heterogeneity, the perfect stratification of student qualification across schools also resembles that in the benchmark case. Again the optimal student-school match is positively assortative, and there is no economic rent to be gained from an overmatch.

Proposition 3 *There exists a unique enrollment equilibrium with* $N_t > 0 \ \forall t$.
Proof. *See Appendix.* ■

This proposition establishes the existence and uniqueness of the enrollment equilibrium. It relies critically on the congestion effect of enrollment size, when students optimally adjust their school choices in response to the different levels of education resources at different schools. This in turns uniquely determines the school quality, as defined by $F(G_t, N_t)$. For example, if $F(G_t, N_t)$ takes the particular form $\frac{G_t}{N_t^{\alpha}}$, then $\alpha = 1$ captures the case when education resources are purely private goods and only the average education expenditure per student matters in the education technology, while $0 < \alpha < 1$ captures the case that some components of the education resources are public goods and there is economies of scale to some extent. As long as the education resources are not purely public good, the congestion effect guarantees the existence and the uniqueness of the enrollment equilibrium.

One immediate implication of Proposition 3 applies to empirical studies. There has been a large empirical literature on how school quality affects students' academic outcomes. Here we see that if the curriculum effect is not explicitly accounted for, it may bias the school quality effect. More specifically, if there is a positive correlation in the data between the selectiveness of schools, which can approximate the minimum threshold in its curriculum, and the average expenditure per student, then the estimated school quality effect would be upward biased. If there is a negative correlation in the data, then the estimated school quality effect would be downward biased. The bias is particularly severe if there is less variation in school quality than that in school curriculum. It remains an empirical question to determine the relative importance of the curriculum effect and the school quality effect.

Another implication of Proposition 3 arises when education policies target specific segments of the student population by providing extra funding to the corresponding school(s). For example, the "community college initiative" increases funding to community colleges, which arguably adopt less challenging curricula and serve students on the lower end of the qualification distribution. Proposition 3 shows that the effect is not fully contained within the targeted schools, but instead spill over to all students attending other schools. In particular, all students attending community colleges benefit directly from the improved quality; some marginal students switch from other schools, which lessens the congestion effect and improves quality at their original schools. This spillover effect moves successively upward, till a new enrollment equilibrium is reach and marginal students become indifferent between two neighboring schools. In the new equilibrium, all students that attend schools benefit. The students that fail to benefit, if any exists, are those with $q_i \leq c_1$. These students are excludes from schooling and cannot benefit from any public education expenditure, unless there are changes in the curriculum that lowers the minimum threshold.

4.2 Affirmative action

To analyze the impact of affirmative action on students' school choices and academic outcomes, we need to introduce frictions and additional heterogeneity into the benchmark model. Without frictions, all students attend their optimal schools, so affirmative action cannot have any real impact. Without additional heterogeneity, students' preferences across schools can again be perfectly stratified, so affirmative action cannot lead to any ostensible "mismatches" in the data. Naturally, affirmative action may give preferential treatment to any subgroups of the population that seems to be under-represented in the schools, according to race, gender, and veteran status, etc. For ease of exposition, we refer to the subgroup receiving preferential treatment as the minority group, and all the rest as the majority group.

Starting from the benchmark case, where students' most preferred schools can be characterized by the cutoff points $\{\widehat{q}_{(t+1)t}\}_{t=1}^{T-1}$ as defined in (2). Instead of open enrollment, in this example schools face capacity constraints, so not all students who prefer a particular school can be admitted. More specifically, let the exogenously given school capacity $\{\widehat{N}_t\}_{t=1}^{T}$ be such that:

$$\sum\nolimits_{t=m}^{T} \widehat{N}_t < \int_{\widehat{q}_{m(m-1)}<q_i<\overline{q}} \phi(i)di \text{ for } m \in \{2, 3, ..., T\} \tag{5}$$

This series of inequalities imply binding capacity constraints. Suppose schools are meritocratic, so they admit students with high qualifications before those with low qualifications. Starting at the top, not all students who prefer to enroll at school T can be admitted, so some at the low end of the spectrum $[\widehat{q}_{T(T-1)}, \overline{q}]$ are forced to enroll in school $T-1$, their second best choice. This corresponds to the inequality (5) for $m = T$. At school $T-1$, the capacity is not big enough to accommodate all the students who are forced out of school T, now at the high end of the spectrum for school $T-1$, and all the students who prefer to enroll at school $T-1$. As a result, some at the low end of the spectrum $[\widehat{q}_{(T-1)(T-2)}, \widehat{q}_{T(T-1)}]$ are forced to enroll in school $T-2$, their second best choice. This corresponds to the inequality (5) for $m = T-1$, and so forth. The systemic exclusion of students from their most preferred schools into the second best choices allows affirmative action to have a real impact. Also note that we do not impost the inequality (5) for $m = 1$, so the bottom school may or may not face a capacity constraint. The selectiveness of a given school is partly determined by its curriculum and partly determined by the cumulative capacity of schools at and above its own level.

Without affirmative action, the group label for each student does not matter, so they are measured against the same cutoff points $\{\widetilde{\widetilde{q}}_{(t+1)t}\}_{t=1}^{T-1}$ implied by the school capacity $\{\widehat{N}_t\}_{t=1}^{T}$:

$$\sum\nolimits_{t=m}^{T} \widehat{N}_t = \int_{\widetilde{\widetilde{q}}_{m(m-1)} < q_i < \overline{q}} \phi(i) di \text{ for } m \in \{2, 3, ..., T\} \tag{6}$$

The fact that there are binding capacity constraints implies an upward shift of all the cutoff points compared to the benchmark case, so we have $\widetilde{\widetilde{q}}_{(t+1)t} > \widehat{q}_{(t+1)t}$ for all $t \in \{1, 2, ..., T-1\}$. Only students with qualifications above the cutoff point $\widetilde{\widetilde{q}}_{(t+1)t}$ can enroll in school $T+1$.

On the students side, we introduce two more dimensions of heterogeneity. First, students choose their learning effort when enrolled in a given school, and their disutility from learning effort is captured by the preference parameter θ_ι. Second, besides academic outcomes, students also care about the nonacademic experiences and opportunities offered by each school, and their utility from the nonacademic outcomes is captured by the additive utility term u_{it}. Overall, a student chooses the optimal school to maximizes his utility as follows:

$$\max A_t(q_i - c_t)F(e_{it}) + \theta_i \ln(1 - e_{it}) + u_{it} \text{ if } q_i > c_t \tag{7}$$

Assumption 5 $F' > 0, F'_{e \to 0+} = +\infty, F'' < 0$

This assumption implies that learning effort is a productive input in the education technology. For any student-curriculum match where $q_i \leq c_t$, the optimal learning effort is trivially 0. On the other hand, for any student-curriculum match where $q_i > c_t$, the Inada condition on learning effort ensures that the optimal learning effort is an interior solution in the interval of $(0,1)$. Furthermore, the first order condition on e_{it} is given by

$$A_t(q_i - c_t)F'(e_{it}) = \frac{\theta_i}{1 - e_{it}} \tag{8}$$

Since the second order condition is negative, the optimal learning effort e_{it}^* is implicitly determined as the solution for (8).

For any given level of learning effort, the learning efficiency $A_t(q_i - c_t)$ is maximized by the same set of cutoff points $\{\widehat{q}_{(t+1)t}\}_{t=1}^{T-1}$ as in (2). So the optimal student-curriculum match is again characterized by Proposition 1, i.e., dependent on heterogeneity in student qualifications q_i but independent of student preferences for leisure θ_i. It is also easy to see that due to the complementarity, a student's optimal learning effort e_{it}^* is the highest under the optimal student-curriculum match, and decreases when there is either an overmatch or an undermatch. Thus a mismatch is detrimental to students' academic outcomes for two reasons. First, the mismatched curriculum reduces learning efficiency directly. Second, the lower learning efficiency reduces learning effort indirectly. The endogenous learning effort magnifies the curriculum effect.

Plugging the optimal learning effort e_{it}^* back in (7), we know that the academic value of school t is

$$V_t(q_i, \theta_i) = A_t(q_i - c_t)F(e_{it}^*) + \theta_i \ln(1 - e_{it}^*) \tag{9}$$

It is obvious from the Envelope Theorem that for students with qualifications $q_i > c_t$, we have $\frac{dV_t(q_i,\theta_i)}{dq_i} > 0$, so students with higher qualifications receive higher net benefit from any curriculum, i.e., the absolute advantage. At the same time we have $\frac{dV_t(q_i,\theta_i)}{d\theta_i} < 0$, so the academic value of any school decreases when students have strong preference for leisure. The total value -both academic and nonacademic- of school t is given by $V_t(q_i, \theta_i) + u_{it}$, and a student's optimal school choice is the one that leads to the highest value of $V_t(q_i, \theta_i) + u_{it}$.

Without affirmative action, students only risk an undermatch due to the capacity constraint. With affirmative action, minority students receiving preferential treatment may be offered opportunity of an overmatch. More specifically, if school $t+1$ admits minority students with qualifications $\widehat{q}_{(t+1)t} \leq q_i < \widetilde{\widetilde{q}}_{(t+1)t}$, there is no overmatch. These students have maximal learning efficiency if attending school $t+1$ and are only denied admission initially due to the capacity constraint. On the other hand, if school $t+1$ admits minority students with qualifications $q_i < \widehat{q}_{(t+1)t}$, there is an overmatch. These students are better served under less challenging curricula, and they are admitted for diversity concerns. However, if we maintain the assumption of individual rationality, students would only optimally choose an overmatch if they obtain higher total value from that school. Namely, the loss in the academic value from an overmatch has to be more than compensated by the gain in the nonacademic value to induce a student into an overmatch.

Whether a minority student prefers an overmatch or not depends not only on his qualification q_i, but also on his preference for leisure θ_i and the non-academic value he attaches to the different schools $\{u_{it}\}_{t=1}^{T}$. It is obvious that is u_{it} is decreasing in t, an overmatch is never optimal. To focus on the non-trivial case, we assume that u_{it} is increasing in t. One interpretation of the increasing u_{it} may be imperfect information in the labor market. For example, as typical in the statistical discrimination literature, if a student's human capital level is observed with some noise component, employers can use the average human capital level from the school as a prior and update the belief with the student's own signal. Given that students with high qualifications self select into schools with more challenging curricula, it is the case that the average human capital level is increasing with t. In this paper, we do not explicitly model the labor market to focus on the schooling decision. Instead we simply assume that the non-academic values for u_{it} are exogenously given and increasing in t for each individual i.

Proposition 4 *Let $u_{it} = u_t$ for all i. If a student with (q_i, θ_i) prefers an overmatch, then a student with (q_j, θ_j) where $q_j = q_i$ but $\theta_j > \theta_i$ would also prefer the same overmatch; if a student with (q_i, θ_i) prefers the original school, then a student with (q_j, θ_j) where $q_j = q_i$ but $\theta_j < \theta_i$ would also prefer the original school.*

Proof. *This follows directly from the fact that $\frac{dV_t(q_i,\theta_i)}{d\theta_i} < 0$.* ■

This Proposition identifies one source heterogeneity for self selection. Suppose schools observe student qualification q_i but not their preference for leisure θ_i, so they give preferential admission to minority students based on q_i but not θ_i. But who actually accept the preferential admission depends on θ_i. The loss in the academic value from an overmatch is less severe for a student with stronger preference for leisure, so he is more likely to prefer an overmatch thanks to the main in the non-academic value, which is independent of his learning efficiency or learning effort. In the extreme case, a student may prefer an overmatch where his learning efficiency is 0, as long as he gains substantially in the non-academic value.

For empirical studies, this self selection problem would widen the gap in the observed academic outcomes between similarly comparable students, some of which take the preferential admission and enroll in schools with more challenging curricula, while others reject the preferential admission and instead enroll in schools with less challenging curricula. If students' preference for leisure is not explicitly accounted for, the estimated mismatch effect would be upward biased.

Proposition 5 *Let $\theta_i = \theta$ for all i. If a student with $(q_i, u_{it}, u_{i(t+1)})$ prefers an overmatch, then a student with $(q_j, u_{jt}, u_{j(t+1)})$ where $q_j = q_i$ but $u_{j(t+1)} - u_{jt} > u_{i(t+1)} - u_{it}$ would also prefer the same overmatch; if a student with $(q_i, u_{it}, u_{i(t+1)})$ prefers the original school, then a student with $(q_j, u_{jt}, u_{j(t+1)})$ where $q_j = q_i$ but $u_{j(t+1)} - u_{jt} < u_{i(t+1)} - u_{it}$ would also prefer the original school.*

Proof. *This follows directly from the fact that $\frac{dV_t(q_i,\theta_i)}{dq_i} > 0$.* ■

This Proposition identifies a second source heterogeneity for self selection. Suppose schools observe student qualification q_i but not the non-academic values they attach to each school u_{it}. Under affirmative action, they give preferential admission to minority students based on q_i but not u_{it}. Again who actually accept the preferential admission becomes a self selection problem. The only channel u_{it} affect students' academic outcomes is through their school choices, which is perfectly observable. In this case, comparing the achievement gap between similarly comparable students with different school choices is a valid exercise. It remains an empirical question to determine how important the non-academic values students attach to different schools drive their school choice decisions.

5 Discussions and conclusion

This paper proposes a general theoretical model to study the effect of school curriculum. We should the impact of competing curricula on students' school choice decisions and their academic outcomes, even treating everything else constant. We then extend the benchmark curriculum model to analyze two specific policy issues: school quality measure as average expenditure per student; preferential admission and hence potential mismatch under affirmative action. In both cases, we show that the curriculum effect serves as an anchoring point for students' school choices. Instead of masking the importance of the curriculum effect, adding other factors in the education technology accentuates its significance. Examining the impact of various education policies through the lens of horizontally differentiated curricula yields interesting and new insights. In this section, we briefly discuss some related issues that our model can shed light on.

5.1 School financing and peer effects

As summarized in Section 2, there is a literature on school choices between private and public schools, where peer effects play a significant role in determining the school quality. The general idea is that if individuals have different willingness to pay for academic outcomes, the ones with high willingness to pay but low qualifications may cross subsidize the ones with low willingness to pay but high qualifications. This is how private schools operate: they charge high tuitions to screen out individuals with low willingness to pay, while offering scholarships to attract individuals with high qualifications. Thus, a hierarchy of school quality may arise endogenously, where better school quality represents higher peer effects.

That literature has so far assumed the same curriculum across schools, i.e., there is only one education production technology. If we introduce competing curricula into the problem, it is easy to see that different curricula serve as strong anchoring points for different students. Given different students enjoy comparative advantage in different curricula, it becomes more costly to put them together for the desired peer effects. If the school adopts an accommodating curriculum, the students with high qualifications would suffer more loss in their learning efficiency, on top of the negative externality from their peers with low qualifications. As a result, they need higher financial compensations to induce their school choices. On the other hand, if the school adopts a challenging curriculum, the students with low qualifications would suffer some loss in their learning efficiency, despite the positive externality they enjoy from their peers with high qualifications. As a result, their willingness to pay for the peer effects is reduced. In short, adding the curriculum effect restricts the range where such cross-subsidization can be mutually beneficial, and to some extent mitigates the "cream skimming" problem typically associated with private schools.

5.2 Asymmetric information

Another interesting interpretation of the curriculum effect applies to the asymmetric information problem in school choices. A recent paper by Arcidiacono *et al* (2010) addresses the potential mismatch hypothesis under affirmative action through asymmetric information. More specifically, they argue that if schools possess private information on the students' capability, schools may induce some minority students into an overmatch that is actually detrimental to their academic outcomes. The authors then test their conditions of asymmetric information using administrative data from Duke university. They find that Duke indeed possesses private information that is statistically significant in predicting students' post-enrollment academic outcomes.

An interesting question arises as to what is the source of Duke's private information. After all, the university obtains students' information from their own applications, such as gender, race, family background, high school grades, SAT or ACT test scores, etc. It seems difficult to imagine that the university knows something more about the student that the student himself. However, in light of the curriculum effect, it is easy to understand the source of asymmetric information. Even though tudent qualification is public information, the qualification-curriculum match quality can be hard to determine. For the students, their prior learning experience sheds little light on the nature of the curriculum for the next stage of education. For the schools, they repeatedly take in entering students with different qualifications and observe their academic outcomes through graduation. Thus, schools have much better data to infer the qualification-curriculum match quality than students.

One immediate follow-up question is whether it is efficient for schools to withhold information and "trick" underqualified students into an overmatch, i.e., pursuing diversity for the sake of diversity. One may expect that this asymmetric information problem is particularly severe for students with inferior family background, who have less direct or indirect experience to infer the match quality, and at the same time, risk bigger loss from an overmatch. It may be helpful, then, for schools to offer interpretations of the match quality to aid their school choice decisions.

References

Altonji, Joseph G. and Dunn, Thomas A. (1996a), "Using Siblings to Estimate the Effect of School Quality on Wages." Review of Economics and Statistics, 78, pp. 665-671.

——— and ——— (1996b), "The Effects of Family Characteristics on the Return to Schooling." Review of Economics and Statistics, 78, pp. 692-704.

Angrist, Joshua D. and Alan B. Kruger (1991). "Does Compulsory School Attendance Affect Schooling and Earnings." Quarterly Journal of Economics, 106, pp. 979-1014.

Arcidiacono, Peter, Aucejo, Esteban M., Fang, Hanming, and Kenneth I. Spenner (2010). "Does Affirmative Action Lead to Mismatch? A New Test and Evidence." Economic Research Initiatives at Duke (ERID) Working Paper No. 27.

Ashenfelter, Orley and Alan B. Kruger (1994). "Estimates of the Economic Return to Schooling for a New Sample of Twins." American Economic Review, 84, pp. 1157-1173.

——— and David Zimmerman (1997). "Estimates of the Return to Schooling from Sibling Data: Fathers, Sons and Brothers," Review of Economics and Statistics, 79, pp. 1-9.

Ayres, Ian, and Richard Brooks (2005), "Does Affirmative Action Reduce the Number of Black Lawyers?" Stanford Law Review, 57(6), pp. 1807-1854.

Barnes, Katherine Y. (2007) "Is Affirmative Action Responsible for the Achievement Gap Between Black and White Law Students?" Northwestern University Law Review, 101(4), pp. 1759-1808.

Betts, Julian R. "The Impact of Educational Standards on the Level and Distribution of Earnings." American Economic Review, March 1998, 88(1), pp. 266-275.

Card, David (1999), "The Causal Effect of Education on Earnings," Handbook of Labor Economics, in: O. Ashenfelter & D. Card (ed.), Handbook of Labor Economics, edition 1, volume 3, chapter 30, pages 1801-1863 Elsevier.

Card, David and Alan B. Kruger (1992a). "Does School Quality Matter: Returns to Education and the Characteristics of Public Schools in the United States." Journal of Political Economy, 100, pp. 1-40.

——— and —— (1992b). "School Quality and Black-White Relative Earnings: A Direct Assessment." Quarterly Journal of Economics, 107, pp. 151-200.

Carneiro, Pedro, Heckman, James J., and Edward J. Vytlacil (forthcoming), "Estimating Marginal Returns to Education." American Economic Review, forthcoming.

Chambers, David L., Clydesdale, Timothy T., Kidder, William C., and Richard O. Lempert (2005) "The Real Impact of Eliminating Affirmative Action in American Law Schools: An Empirical Critique of Richard Sander's Study." Stanford Law Review, 57(6), pp. 1855-1898.

Costrell, Robert M. "A Simple Model of Educational Standards." American Economic Review, September 1994, 84(4), pp. 956-971.

———. "Can Centralized Educational Standards Raise Welfare?" Journal of Public Economics, September 1997, 65(3), pp. 271-293.

Driskill, Robert A. and Horowitz, Andrew. "Investment in Hierarchical Human Capital", Review of Development Economics, Feb. 2002, 6(1), pp. 48-58.

Epple, Dennis, Figlio, David and Romano, Richard. "Competition between Private and Public Schools: Testing Stratification and Price Predictions." Journal of Public Economics, July 2004, 88(7-8), pp. 1215-1245.

———, Newlon, Elizabeth and Romano, Richard. "Ability Tracking, School Competition, and the Distribution of Educational Benefits." Journal of Public Economics, January 2002, 83(1), pp. 1-48.

——— and Romano, Richard E. "Competition between Private and Public Schools, Vouchers, and Peer-Group Effects." American Economic Review, March 1998, 88(1), pp. 33-62.

——— and ———. "Educational Vouchers and Cream Skimming." International Economic Review, November 2008, 49(4), pp. 1395-1435.

Fernandez, Raquel and Gali, Jordi. "To Each According to...? Markets, Tournaments, and the Matching Problem with Borrowing Constraints." Review of Economic Studies, October 1999, 66(4), pp. 799-824.

Garen, John (1984), "The Returns to Schooling: A Selectivity Bias Approach with a Continuous Choice Variable." Econometrica, Sept. 1984, 52(2), pp. 1199-1218.

Griliches, Zvi (1977), "Estimating the Returns to Schooling: Some Econometric Problems." Econometrica, Jan. 1977, 45(1), pp. 1-22.

——— (1979), "Sibling Models and Data in Economics: Beginning of a Survey." Journal of Political Economy, 87: pp. S37-S65.

Heckman, James J., Schmierer, Daniel, and Sergio Urzua (2010), "Testing the Correlated Random Coefficient Model." Journal of Econometrics, 158, pp. 177-203.

Ho, Daniel E. (2005) "Why Affirmative Action Does Not Cause Black Students to Fail the Bar." Yale Law Journal, 114(8), pp. 1997-2004.

Hoxby, Caroline M. "How school choice affects the achievement of public school students." Choice with Equity, Hoover Institution Press, 2002.

———. "School choice and school productivity." The economics of school choice, National Bureau of Economic Research Conference Report, 2003.

Hsieha, Chang-Tai and Urquiolab, Miguel. "The effects of generalized school choice on achievement and stratification: Evidence from Chile's voucher program." Journal of Public Economics, September 2006, 90(8-9), pp. 1477-1503.

Lam, David and Schoeni, Robert F (1993), "Effects of Family Background on Earnings and Returns to Schooling: Evidence from Brazil." Journal of Political Economy, Aug. 1993, 101(4), pp. 710-740.

Mincer, Jacob (1974), *Schooling, Experience and Earnings,* New York: NBER Press.

Sander, Richard H. (2004) "A Systemic Analysis of Affirmative Action in American Law Schools." Stanford Law Review, 57(2), pp. 367-483.

———. (2005a) "Mismeasuring the Mismatch: A Response to Ho." Yale Law Journal, 114(8), pp. 2005-2010.

——— (2005b) "Reply: A Reply to Critics." Stanford Law Review, 57(6), pp. 1963-2016.

Su, Xuejuan. "The Allocation of Public Funds in a Hierarchical Educational System." Journal of Economic Dynamics and Control, December 2004, 28(12), pp. 2485-2510.

———. "Endogenous Determination of Public Budget Allocation across Education Stages." Journal of Development Economics, 2006, 81(2), pp. 438-456.

Welch, Finis (1973), "Black-White Differences in Returns to Schooling." American Economic Review, Dec. 1973, 63(5), pp. 893-907.

Appendix

Proposition 2.

Proof. Suppose $\{\widetilde{q}_{(t+1)t}\}_{t=1}^{T-1}$ *were not strictly increasing, i.e.,* there exists a t such that $\widetilde{q}_{(t+2)(t+1)} \leq \widetilde{q}_{(t+1)t}$. *If this were the case, school* $t+1$ would be strictly dominated by school t for $q_i < \widetilde{q}_{(t+1)t}$, *strictly dominated by school* $t+2$ for $q_i > \widetilde{q}_{(t+2)(t+1)}$, and identical to both schools t and $t+2$ for $q_i = \widetilde{q}_{(t+1)t} = \widetilde{q}_{(t+2)(t+1)}$. The optimal school choices among students imply that $N_{t+1} = 0$. However, under Assumption 4, $F(.,0) = +\infty$, so $\widetilde{q}_{(t+1)t} = c_{t+1}$ and $\widetilde{q}_{(t+2)(t+1)} \geq c_{t+2}$, which is a contradiction.

Similarly, $N_1 = 0$ if $\underline{q} \geq \widetilde{q}_{21}$, *under Assumption 4, this implies* $\widetilde{q}_{21} \geq c_2$, *which leads to a contradiction to Assumption 3;* $N_T = 0$ *if* $\overline{q} \leq \widetilde{q}_{T(T-1)}$, *under Assumption 4, this implies* $\widetilde{q}_{T(T-1)} \leq c_T < \overline{q}_{T(T-1)}$, *again a contradiction to Assumption 3.*

The perfect stratification of student qualification across schools follows immediately. ■

Proposition 3.

Proof. We construct the unique enrollment equilibrium recursively. Let $0 \leq \mu \leq 1$ be the total measure of students that attend schools. It is trivial that $N_t^*(\mu) = 0$ for all $t \in \{1, 2, ..., T\}$ if $\mu = 0$. The non-trivial case is when $0 < \mu \leq 1$. Remember that we do not assume $\underline{q} < c_1$, so it is possible for some students to stay out of schooling. In that case, $\mu = 1 - \int_{\underline{q} < q_i < c_1} \phi(i)di < 1$.

When $T = 2$, denote $N_1 = \mu - N_2$. The optimal choice between school 1 and school 2 is determined by $\widetilde{q}_{21} = \frac{A_2 c_2 F(G_2, N_2) - A_1 c_1 F(G_1, N_1)}{A_2 F(G_2, N_2) - A_1 F(G_1, N_1)} = c_2 + \frac{c_2 - c_1}{\frac{A_2 F(G_2, N_2)}{A_1 F(G_1, \mu - N_2)} - 1}$, where $s^* = 1$ if $q_i < \widetilde{q}_{21}$, $s^* = 2$ if $q_i > \widetilde{q}_{21}$, and $s^* \in \{1, 2\}$ if $q_i = \widetilde{q}_{21}$. An enrollment equilibrium is thus a fixed point of the mapping $\Psi(N_2) = \int_{\widetilde{q}_{21} < q_i < \overline{q}} \phi(i)di$. Under Assumption 4, for given μ, we know $\frac{\partial \widetilde{q}_{21}}{\partial N_2} > 0$. So $\Psi(.)$ is a strictly decreasing mapping from $[0, \mu]$ into itself, where $\Psi(0) > 0$ and $\Psi(\mu) < \mu$. Thus a unique fixed point $N_2^*(\mu)$ exists, and consequently $N_1^*(\mu) = \mu - N_2^*(\mu)$. In this unique enrollment equilibrium for $T = 2$, $0 < N_t^*(\mu) < \mu$ and $\frac{dN_t^*(\mu)}{d\mu} > 0$ for $t \in \{1, 2\}$.

Suppose a unique enrollment equilibrium exists for $T = m \geq 2$. Namely for given $0 < \mu \leq 1$, we have $0 < N_t^*(\mu) < \mu$ and $\frac{dN_t^*(\mu)}{d\mu} > 0$ for $t \in \{1, 2, ..., m\}$. Now consider the case when $T = m + 1$. For given N_{m+1}, denote $\widetilde{\mu} = \mu - N_{m+1}$. Among the m schools, for given $\widetilde{\mu}$, a unique enrollment equilibrium exists $\{N_t^*(\widetilde{\mu})\}_{t=1}^m$, and $\frac{dN_t^*(\widetilde{\mu})}{d\widetilde{\mu}} > 0$ for $t \in \{1, 2, ..., m\}$. Now the optimal choice between school m and school $m+1$ is determined by $\widetilde{q}_{(m+1)m} = \frac{A_{m+1}c_{m+1}F(G_{m+1},N_{m+1})-A_m c_m F(G_m,N_m)}{A_{m+1}F(G_{m+1},N_{m+1})-A_m F(G_m,N_m)} = c_{m+1} + \frac{c_{m+1}-c_m}{\frac{A_{m+1}F(G_{m+1},N_{m+1})}{A_m F(G_m,N_m^*(\mu-N_{m+1}))}-1}$, where $s^* = m$ if $q_i < \widetilde{q}_{(m+1)m}$, $s^* = m+1$ if $q_i > \widetilde{q}_{(m+1)m}$, and $s^* \in \{m, m+1\}$ if $q_i = \widetilde{q}_{(m+1)m}$. An enrollment equilibrium is thus a fixed point of the mapping $\Psi(N_{m+1}) = \int_{\widetilde{q}_{(m+1)m} < q_i < \overline{q}} \phi(i)di$. Under Assumption 4, we have $\frac{\partial \widetilde{q}_{(m+1)m}}{\partial N_{m+1}} > 0$ for given μ, so $\Psi(.)$ is a strictly decreasing mapping from $[0, \mu]$ into itself, where $\Psi(0) > 0$ and $\Psi(\mu) < \mu$. Thus a unique fixed point $N_{m+1}^*(\mu)$ exists, and consequently, $\widetilde{\mu}^* = \mu - N_{m+1}^*(\mu)$, so $N_t^* = N_t^*(\widetilde{\mu}^*)$ for $t \in \{1, 2, ..., m\}$. In this unique enrollment equilibrium for $T = m+1$, $0 < N_t^*(\mu) < \mu$ and $\frac{dN_t^*(\mu)}{d\mu} > 0$ for $t \in \{1, 2, ..., m+1\}$.

Overall, a unique enrollment equilibrium $\{N_t^*(\mu)\}_{t=1}^T$ for any T, and $\frac{dN_t^*(\mu)}{d\mu} > 0$ for $t \in \{1, 2, ..., T\}$. ■