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Dynamic Technological Innovation with Dual Quality Ladders

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We develop a dynamic game theoretic model of innovation when the productive output of one firm can be licensed for use in the research activities of another competing firm. The model builds upon the structure of the familiar quality ladder framework instead assuming that each firms' technology progresses on its own quality ladder with outputs that compete in a market with potentially differentiated products. We analyze both the subgame perfect equilibria of a two-period game and the markov perfect equilibria of the infinitely repeated game.

Keywords: Sequential Innovation, Research Licensing, Product Differentiation.

JEL Codes: C7, D21, D43, L13, L24, O3.

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1 Introduction

This paper presents a model of sequential innovation in the presence of differentiated products. This has been explored previously by Grossman and Helpman (1991) who examine innovation in the context of economic growth. In their model innovation for each differentiated product follows a quality ladder. That is, innovation brings about quality improvements of existing products. We focus on use license pricing when the output of one firm can be used (if a license is purchased) in the R&D process of a rival. Access to a rivals output increases the probability of a firms R&D efforts producing an innovation.

Moschini and Yerokhin (2008) analyze a dynamic model of sequential innovation to study the impact of different property rights assignments on R&D, "research exemption" versus "experimental use". They allow for the possibility that one firms output can be used as a research input for a competing firm. However, their model assumes that all innovations move along one quality ladder with the innovating firm capturing the entire market share. This is inconsistent with many industries and in particular for the agricultural biotechnology industry they describe. Multiple firms engage in R&D to create new seed strains which they can then bring to market. However, different strains are targeted for farmland which fits a specific set of characteristics. Firms produce strains which target drought resistance and others target pesticide resistance among others. Given that a firm targets a specific characteristic, R&D often takes the form of making improvements to their proprietary strain. The firm maintains its target yet tries to improve the quality (yield) of its seed for its' target farm characteristics. Each firms seed then competes against each other as farm buy the seed which maximize their profits given the varieties available and the yields they would produce for their own farm characteristics. Moschini and Yerokhin (2008) also do not explore many of the important aspects of the strategic environment that stem from licensing.

The model of Moschini and Yerokhin (2008) originates from the quality ladder literature which is concerned with what size of innovation (how large a jump on the quality ladder) should there be before a firm patents its' innovation. This literature is summarized nicely in Scotchmer (2004). While the questions of when a firm should patent (so as to maximize profits) and how much breadth should the regulating body require in order to award a new patent (when does an innovation infringe on a previous patent versus deserve a new patent) are very important topics, they are not the focus of this research. (*add citations*) Instead we suppress issues of patentability and breadth to instead focus on the competing interests of a firm that can increase its' own probability of innovation if it purchases a use license from its' rival while if it sells a use license for its own product it gains revenue from the sale yet faces the prospects of an increased probability of its rival advancing along its quality ladder.

The model we present borrows the dynamic aspects from Moschini and Yerokhin (2008) yet models each firms R&D process as following its own quality ladder with product differentiation in the output market and sales and purchases of research licensing. Section 2 gives an overview of the model. Section 3 analyzes a model with two time periods while section 4 presents analysis of the model in an infinitely repeated game. We demonstrate the applicability of the model through a number of examples provided in section 5. We then conclude with section 6.

2 Model Overview

Two firms produce differentiated products which they sell either directly to consumers or to other producers as intermediate goods. In the first time period the game begins with firms, indexed by i, producing and selling their current product y_t^i. Each firm earns a profit which depends on the current product quality of both firms, $\pi^i(y_t^i, y_t^j)$. Each firm may produce an innovation which improves the quality of its output in the following time period. We model innovation for each firm following the well known quality ladder model (see Scotchmer (2004)). In the initial time period firm i begins with a product quality somewhere on the quality ladder as depicted in figure 1. In the figure the subscript does not represent time but rather represents the number assigned to the each rung along the quality ladder.[1] Rung's on the quality ladder are spaced $\Delta > 0$ apart. If firm i has a successful innovation then in the next time period firm i's product will move one step along the quality ladder.

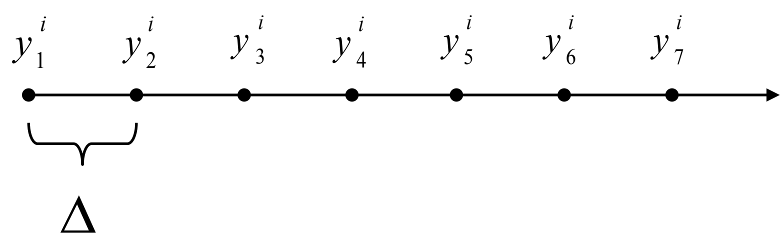

Figure 1: Quality Ladder

Firms interact strategically both in the product market and in the purchase and sale of research licensing for each firms product. Possession of a research license for a rival firms product in any time period increases the probability of an innovation for that firm (moving up one step on the quality ladder).

In each time period the firms play a two stage game. In stage one each firm simultaneously sets a price it will charge the other (ρ^i is the price i sets to charge j) and in the second stage each firm will simultaneously decide to either purchase the research license to their rivals product or not. Firm i get profits from their sales in the output market based on the current product qualities of both firms, $\pi^i(y^i_{t+1}, y^j_{t+1})$, and get any revenue, ρ^i from license sales and pay any costs, ρ^j from license purchases. Finally, based on purchase decisions made, nature will select whether each firm innovates (moves to the next step on its' quality ladder) or not.

[1]In the fully dynamic setting the ladder could have infinitely many rungs.

We will be limiting out attention to markovian strategies. We further assert that for a wide class of such markovian games current period payoffs will not depend on the exact step each firm is at on the quality ladder; payoffs depend only on the difference in quality across the two firms. That is having a high quality product doesn't influence demand, rather it is the differential quality between rival products that determines demand. Let k_t be the number of rungs that firm i is ahead of firm j in time t. Therefore, $\Delta k_t = y_t^i - y_t^j$ and if $k_t = 0$ the firms have products of identical quality, if $k_t \geq 1$ firm i has higher quality and if $k_t \leq 1$ firm j has higher quality. Therefore we can write payoffs which only depend on the state variable k_t as $\pi^i(k_t)$. We suppress the time subscript throughout the remainder of the paper as the markovian assumption rules them unnecessary.

3 Model: Two Time Periods

We begin by considering the simplest case of two time periods and no stochastics in the R&D process. If i purchases a license from j then i advances along her quality ladder by one rung with certainty and if i does not purchase a license then i stays at the same rung with certainty. The second time period is the terminal node with no pricing or purchase decisions made. Payoffs are simply awarded based on the decisions that were made in time period 1.

Let δ be the common discount factor. Then, the discounted payoffs to each player if neither purchases a license is

$$\pi^i(k) + \delta\pi^i(k).$$

The discounted payoffs to each player if both purchase a license is

$$\pi^i(k) + \delta\pi^i(k) + \rho^i - \rho^j.$$

In both these cases, the technology gap remains unchanged as time progresses from the initial to the terminal stage however when both purchase license's the payoffs are effected by the revenue and cost from selling and purchasing a license.

If player i purchases a license but player j does not, then in the terminal stage the state variable k will increase by one. The discounted payoff to player i is

$$\pi^i(k) - \rho^j + \delta\pi^i(k+1)$$

and the discounted payoff received by player j is

$$\pi^j(k) + \rho^j + \delta\pi^j(k+1).$$

If player j purchases a license but i does not, then in the terminal stage the state variable k will decrease by one. The discounted payoff to the i is

$$\pi^i(k) + \rho^i + \delta\pi^i(k-1)$$

and the discounted payoff received by player j is

$$\pi^j(k) - \rho^i + \delta\pi^j(k-1).$$

Let $q^i \in [0,1]$ be the probability that player i purchases a license. We now proceed to solve for the best response functions for the license purchase decision as it depends on the state variable and license prices: k, ρ^i, and ρ^j.

3.0.1 ρ^i, and ρ^j exogenous

To keep matters clear it is timely to introduce a couple of items of notation. To limit repetition of equations and result we make use of an indicator function $I^i \equiv$ defined below.

$$I^v \equiv I(v) = \begin{cases} 1 & if \quad v = i \\ -1 & if \quad v = j \end{cases} \tag{1}$$

Define the discounted loss in profit due to decreasing the quality gap as

$$L^i(k) = \delta \left(\pi^i(k) - \pi^i(k - I^i) \right) \tag{2}$$

and the discounted gain in profit due to increasing the quality gap as

$$G^i(k) = \delta \left(\pi^i(k + I^i) - \pi^i(k) \right). \tag{3}$$

Explicitly we have the following payoff function:

$$\Pi^i \left(\rho^i, \rho^j, q^i(\rho^j), q^j(\rho^i) \right) =$$

$$q^j(\rho^i) \left[q^i(\rho^j)(\pi^i(k) + \delta\pi^i(k) + \rho^i - \rho^j) + (1 - q^i(\rho^j))(\pi^i(k) + \delta(\pi^i(k - I^i)) + \rho^i) \right] + \tag{4}$$

$$(1 - q^j(\rho^i)) \left[q^i(\rho^j)(\pi^i(k) + \delta\pi^i(k + I^i) - \rho^j) + (1 - q^i(\rho^j))(\pi^i(k) + \delta\pi^i(k)) \right]$$

which can be easily simplified using the above definitions to:

$$\Pi^i \left(\rho^i, \rho^j, q^i(\rho^j), q^j(\rho^i) \right) =$$

$$(1 + \delta)\pi^i(k) - q^i(\rho^j)\rho^j + q^j(\rho^i)\rho^i + q^j(\rho^i)(q^i(\rho^j) - 1)L^i(k) + (1 - q^j(\rho^i))q^i(\rho^j))G^i(k) \tag{5}$$

In words equation (5) simply states that firm i will receive current profits plus discounted next stage profits and when firm j purchases firm i will receive ρ^i and avoid the loss, $L^i(k)$, by paying ρ^j if firm i also purchases. If firm j does not purchase, then firm i will receive the gain, $G^i(k)$ by purchasing and forgo this amount if not purchasing.

Now, we can write the best response correspondence for the purchase decision. We make use of the following definition.

$$E^B[i,k,q^j] = q^j(\rho^i)L^i(k) + (1 - q^j(\rho^i))G^i(k) \tag{6}$$

The best response function for firm i is given by:

$$q^i(q^j,\rho^j) = \begin{cases} q^i = 1 & if \quad E^B[i,k,q^j] \geq \rho^j \\ \\ q^i = 0 & if \quad E^B[i,k,q^j] \leq \rho^j \\ \\ q^i \in [0,1] & if \quad E^B[i,k,q^j] = \rho^j \end{cases} \tag{7}$$

Given this best response function we are guaranteed the existence a Nash equilibrium to the second stage buy game for any prices $\rho^i \geq 0$ and $\rho^j \geq 0$.

3.0.2 ρ^i, and ρ^j endogenous

We now let all prices in the model be fully endogenous. Purchase decisions are made after prices have been set. Therefore, with endogenous prices the game played in each time period is 2 stage: in the first stage prices for licenses are simultaneously set, in the second stage players make simultaneous purchase decisions. Given this setup, prices are set given knowledge that purchase decisions will be made to maximize profits. Purchase strategies are therefore identical to those specified in the previous section.

A strategy for player i,s^i, is a pair $s^i = (q^i(\rho^j), \rho^i)$ where $q^i(\rho^j)$ is the probability that i purchases j's license and ρ^i is the price i charges j for i's license. We also need to define beliefs. Because we focus our solution to subgame perfect equilibrium we require that player i believe that player j will always make purchase decisions according to her best response function $q^j(\rho^i)$. However, in order to write well defined payoff functions each player must also have beliefs about the probability with which the other player will mix over buy and don't when the price makes them indifferent. Let $\mu^i(q^j)$ denote i's beliefs over j's buy decision.[2] The beliefs for player i are specified in the following equation where f^i is a probability density function over the unit interval.

$$\mu^i(q^j) = \begin{cases} \mu^i = 1 & if \quad E^B[j, k, q^i(\rho^j)] > \rho^i \\ \\ \mu^i = 0 & if \quad E^B[j, k, q^i(\rho^j)] < \rho^i \\ \\ \mu^i = f^i[0, 1] & if \quad E^B[j, k, q^i(\rho^j)] = \rho^i \end{cases} \tag{8}$$

The expected payoff to firm i from license prices and beliefs can now be written as in the following equation.

$$\Pi^i\left(\rho^i, \rho^j, \mu^i(q^j), \mu^j(q^i)\right) = (1+\delta)\pi^i(k)$$

$$\int_0^1 \int_0^1 \left[q^j\mu^i(q^j)\rho^i - q^i\mu^j(q^i)\rho^j + q^j\mu^i(q^j)(q^i\mu^j(q^i) - 1)L^i(k) + (1 - q^j\mu^i(q^j))q^i\mu^j(q^i)G^i(k)\right] dq^j dq^i \tag{9}$$

Let the mixed strategy that i expects j to play q^{je} be the mean of f^i. So $q^{je} = \int_0^1 q^j f^i dq^j$. With an additional definition it is possible to write the pricing best response function.

$$E^P[j, k, \rho^i] = \mu^i(q^j)G^j(k) + (1 - \mu^i(q^j))L^j(k) \tag{10}$$

$$\rho^i(\rho^j) = \begin{cases} E^B[j, k, \mu^j(q^i)] & if \quad E^B[j, k, \mu^j(q^i)] \geq E^P[i, k, \rho^j] \\ \\ E^B[j, k, \mu^j(q^i)] + \epsilon & if \quad E^B[j, k, \mu^j(q^i)] < E^P[i, k, \rho^j] \end{cases} \tag{11}$$

[2]In an abuse of notation I suppress the dependence of q^j on ρ^i.

Definition 3.1. *A subgame perfect equilibrium to the two time period pricing game is a strategy profile s and beliefs μ such that the following conditions are met.*

1. *μ^i takes the form as given in equation (8).*
2. *Each player plays a best response pricing strategy as given in equation (11) given beliefs μ^i and μ^j*

We now look for a subgame perfect equilibrium to the game. In order to characterize equilibria we make some simplifying assumptions about the underlying functions $\pi^i(k)$ given below.

1. Symmetry:

$$\pi^1(k) = \pi^2(-k) \qquad \forall k$$

2. Weakly increasing at a weakly increasing rate

$$\frac{\partial \pi^1(k)}{\partial k} \geq 0; \qquad \frac{\partial^2 \pi^1(k)}{\partial k^2} \geq 0$$

Lemma 1. *Assumptions (1) - (2) imply that $q^i(\rho^j)$ is weakly decreasing as ρ^j increases which further implies that $\rho^i(\rho^j)$ is weakly increasing in ρ^j.*

Proposition 1 establishes the conditions under which a subgame perfect equilibrium exists in which both players play a pricing strategy where $(q^i(\rho^j), q^j(\rho^i)) = (1, 1)$ is a best response. That is, it is a possible equilibrium outcome that both players i and j buy a research use license. However, these equilibria involve pricing strategies that induce a mixed strategy over the buy decision. Therefore, beliefs play a major role in the existence proof whenever $|k| \geq 1$; there are admissible beliefs for which this equilibrium does not exist.

Proposition 1. *Given assumptions (1) - (2) there exists a subgame perfect equilibrium where $\rho^i = E^B[j, k, \mu^j(q^i)]$ and $\rho^j = E^B[i, k, \mu^i(q^j)]$.*

1. $k \geq 1 \iff$
$$L^1(k) \leq G^2(k)$$

2. $k \leq -1 \iff$
$$L^2(k) leq G^1(k)$$

3. $k = 0$

More content to come.

4 Model: Infinite Number of Time Periods

More content to come.

5 Examples

We now derive the profit functions $\pi(k)$ for a variety of underling competitive environments and show that they satisfy the assumptions of the game in general.

5.1 Example 1: Agricultural Biotechnology

We borrow heavily from Malla and Gray (2005). Two firms indexed by $i = 1, 2$ compete in a market in which they produce a differentiated intermediate good (wheat seed). Their products are differentiated across two dimensions: quality (yield) and characteristic. Their output is purchased by firms (from now on called farms) in a competitive market as an input to production. Farms choose to purchase the seed that maximizes profits. Farms are indexed by j and are uniformly distributed along the unit interval, $\Psi_j \sim [0, 1]$, where Ψ_j is the ideal characteristic desired by farm j.

Yield of firm 1's variety at time t for a farm located at 0 is denoted as $y_t^1 + \tau$. The yield of the firm 1's variety for a farm depends on the ideal characteristic of the farm. The yield of firm 1's variety for a farm as it depends on farm location on the characteristic dimension is given by $y_t^1 + \tau(1 - \Psi)$. Yield of firm 1's variety for a firm decreases linearly at rate τ as farm location varies. Likewise, yield of firm 2's variety as it depends on farm location is given by $y_t^2 + \tau\Psi$.

From the yield equations we write the farm profits from purchasing each firm's variety. Let P_t be the price of wheat in time period t (whenever convenient we assume $P_t = 1$ throughout t because P_t is determined outside of the model) and W_t^i be the price of seed purchased from firm 1 or 2, respectively. Profits if a farm at location Ψ purchases seed from firm 1 are

$$y_t^1 + \tau(1 - \Psi) - w_t^1 \tag{12}$$

and if purchased from firm two are

$$y_t^2 + \tau\Psi - w_t^2. \tag{13}$$

We define the technology gap at time t as $\Delta k_t = (y_t^1 - y_t^2)$. Δ is the distance between consecutive rungs on each firms quality ladder and k_t is the number of rungs that the leader is ahead of the follower. From these two equations we can solve for the demand for firm 1's seed at time t, Ψ_t^{1*}, as follows

$$\Psi_t^{1*} = \frac{\Delta k_t + \tau + (w_t^2 - w_t^1)}{2\tau} \tag{14}$$

and demand for firm 2's seed at time t is $\Psi_t^{2*} = 1 - \Psi_t^{1*}$ which reduces to

$$\Psi_t^{2*} = \frac{-\Delta k_t + \tau + (w_t^1 - w_t^2)}{2\tau} \tag{15}$$

Firm 2 will choose the price of its seed so as to maximize its own profits. Let T be a parameter that represents the marginal cost of marketing and reproducing the seed. Then the profit maximizing decision for firm 1 at time period t is

$$\max_{w_t^1} \left(w_t^1 - T\right) \Psi_t^{1*}.$$

From the first order conditions we get the best response function for contemporaneous seed pricing as

$$w_t^1(w_t^2) = \frac{\tau + w_t^2 + T}{2} + \frac{\Delta k_t}{2}.$$

Similarly for firm F we get

$$\max_{w_t^2} \left(w_t^2 - T\right) \Psi_t^{2*}$$

and

$$w_t^2(w_t^1) = \frac{\tau + w_t^1 + T}{2} - \frac{\Delta k_t}{2}.$$

Nash Equilibrium contemporaneous seed prices are then

$$w_t^1(\Delta k_t) = \tau + T + \frac{\Delta k_t}{3} \tag{16}$$

and

$$w_t^2(\Delta k_t) = \tau + T - \frac{\Delta k_t}{3} \tag{17}$$

Equations 16 and 17 determine the price charged for each firms seed as firm yields (quality) change over time as the result of innovation in a repeated game. These can be used to write time period demand and profits from seed sales for each firm as below

$$\Psi_t^1(\Delta k_t) = \frac{1}{2} + \frac{\Delta k_t}{6\tau}$$

$$\Psi_t^2(\Delta k_t) = \frac{1}{2} - \frac{\Delta k_t}{6\tau}$$

$$\pi^1(k) = \frac{\tau}{2} + \frac{(\Delta k)^2}{18\tau} + \frac{\Delta k}{3}$$

$$\pi^2(k) = \frac{\tau}{2} + \frac{(\Delta k)^2}{18\tau} - \frac{\Delta k}{3}.$$

It is now straightforward to see that these $\pi(k)$ functions satisfy the assumptions lined out in assumptions (1) - (2).

5.1.1 Cellular Market

The model of the previous section can also be expressed in terms of utility maximizing consumers purchasing one of two differentiated products. This specification resembles competition in the cellular phone market where competing firms all offer slightly differentiated network features and also compete based on the speed and reliability (quality) of their network.

5.2 Software

More content to come

6 Conclusion

More content to come

A Proofs

Proof of equation (11) . 1. When j prices so that $\rho^j = E^B[i,k,\mu^i(q^j)]$ then the payoff to i can be rewritten as follows

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) =$$

$$(1+\delta)\pi^i(k) + \int_0^1 \left[q^j\mu^i(q^j)\rho^i - q^{ie}\rho^j + q^j\mu^i(q^j)(q^{ie}-1)L^i(k) + (1-q^j\mu^i(q^j))q^{ie}G^i(k)\right]dq^j \tag{18}$$

Suppose that i sets $\rho^i < E^B[j,k,\mu^j(q^i)]$, then $\mu^j = f^j$ and the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) + \rho^i - q^{ie}\rho^j + (q^{ie}-1)L^i(k)$$

and is increasing in ρ^i.

Suppose that i sets $\rho^i > E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) - q^{ie}\rho^j + q^{ie}G^i(k).$$

Suppose that i sets $\rho^i = E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) + q^{je}\rho^i - q^{ie}\rho^j + q^{je}(q^{ie}-1)L^i(k) + (1-q^{je})q^{ie}G^i(k).$$

Therefore it is the best response for i to set $\rho^i = E^B[j,k,\mu^j(q^i)]$ if $E^B[j,k,\mu^j(q^i)] \geq q^{ie}G^i(k) + (1-q^{ie})L^i(k)$ if not then set $\rho^i = E^B[j,k,\mu^j(q^i)] + \epsilon$ with $\epsilon > 0$.

2. If j prices so that $\rho^j < E^B[i,k,\mu^i(q^j)]$ then $\mu^j = 1$ and the payoff to i is

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) =$$

$$(1+\delta)\pi^i(k) + \int_0^1 \left[q^j\mu^i(q^j)\rho^i - \rho^j + (1-q^j\mu^i(q^j))G^i(k)\right]dq^j \tag{19}$$

Suppose that i sets $\rho^i < E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) + \rho^i - \rho^j$$

and is increasing in ρ^i.

Suppose that i sets $\rho^i > E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) - \rho^j + G^i(k)$$

Suppose that i sets $\rho^i = E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) + q^{je}\rho^i - \rho^j + (1-q^{je})G^i(k)$$

Therefore it is the best response for i to set $\rho^i = E^B[j,k,\mu^j(q^i)]$ if $E^B[j,k,\mu^j(q^i)] \geq G^i(k)$ if not then set $\rho^i = E^B[j,k,\mu^j(q^i)] + \epsilon$ with $\epsilon > 0$.

3. If j prices so that $\rho^j > E^B[i,k,\mu^i(q^j)]$ then $\mu^j = 0$ and the payoff to i is

$$\begin{gathered}\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = \\ (1+\delta)\pi^i(k) + \textstyle\int_0^1 \left[q^j\mu^i(q^j)\rho^i - q^j\mu^i(q^j)L^i(k)\right]dq^j\end{gathered} \tag{20}$$

Suppose that i sets $\rho^i < E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k) + \rho^i - L^i(k)$$

and is increasing in ρ^i.

Suppose that i sets $\rho^i > E^B[j,k,\mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i,\rho^j,\mu^i(q^j),\mu^j(q^i)\right) = (1+\delta)\pi^i(k)$$

Suppose that i sets $\rho^i = E^B[j, k, \mu^j(q^i)]$, then the payoff to i is given by

$$\Pi^i\left(\rho^i, \rho^j, \mu^i(q^j), \mu^j(q^i)\right) = (1+\delta)\pi^i(k) + q^{je}\rho^i - q^{je}L^i(k)$$

Therefore it is the best response for i to set $\rho^i = E^B[j, k, \mu^j(q^i)]$ if $E^B[j, k, \mu^j(q^i)] \geq L^i(k)$ if not then set $\rho^i = E^B[j, k, \mu^j(q^i)] + \epsilon$ with $\epsilon > 0$.

$\square$

Proof of Proposition 1. From the best response pricing function, equation (11) we see that firm i and firm j will only purchase if

$$E^B[j, k, q^i] \geq E^P[i, k, \rho^j]$$

is satisfied for both i and j. This amounts to the following equation for both i and j:

$$q^i(\rho^j)L^j(k) + (1 - q^i(\rho^j))G^j(k) \geq q^i(\rho^j)G^i(k) + (1 - q^i(\rho^j))L^i(k).$$

These equations will only be true if the following holds for the purchase decision

$$q^i(L^j - G^i) + (1 - q^i)(G^j - L^i)$$

$$q^1 \leq \frac{G^2 - L^1}{(G^2 - L^1) + (G^1 - L^2)}$$

$$q^2 \leq \frac{G^1 - L^2}{(G^2 - L^1) + (G^1 - L^2)}$$

Thus, $G^1 - L^2 \geq G^2 - L^1$.

$\square$

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