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Carlos E Da costa
Fundacao Getulio Vargas

Thiago Pereira
Fundacao Getulio Vargas

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Carlos E. da Costa
Fundação Getulio Vargas
carlos.eugenio@fgv.br

Thiago Pereira
Fundação Getulio Vargas
tpereira@fgvmail.br

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We investigate efficiency properties of an income tax schedule derived under the minimum equal sacrifice criterion in a Mirrlees (1971) setting. The shape of equal sacrifice tax schedules does not depend on the underlying distribution of productivities which is in contrast with schedules resulting from the maximization of a social welfare functional. It is not hard, then, to find examples of economies for which equal sacrifice tax schedules are inefficient. This result is, however of little practical relevance since we do not 'choose' the underlying distribution of skills. Our focus is on the more relevant question of whether inefficiencies are likely to arise for preferences and distribution of productivities compatible with empirical distributions of income. We show that the methodology in Werning (2007) can be used to address this question. We find that, although under reasonable parameters for preferences and the distribution of productivities an equal sacrifice schedule may be inefficient, for our best specifications, inefficiency only arises for extremely high levels of income.

1 Introduction

Following the seminal work of Mirrlees (1971) almost all discussions about the design of income tax schedules is based on the optimization of a social welfare function under implementability constraints. Yet, for a long time, it was vertical equity and the idea of 'equal sacrifice' that was dominant.[1]

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[1]In the words of Adam Smith "whatever sacrifices the government requires should be made to bear as nearly as possible with the same pressure upon all".

Formal discussions of tax schedules based on the equal sacrifice principle are found at least as early as Samuelson (1947). Despite having all but disappeared from the academic debate following the development of the optimal taxation literature, the idea of equal sacrifice has found renewed interest in the late 1980's—Young (1987, 1988, 1990); Richter (1983).

In a very intriguing work, Young (1990) has shown that most actual tax systems can be approximated by an equal sacrifice schedule. That is, for a given observed distribution of before and after tax incomes, one may find a common (and empirically sound) utility function that equalizes the utility loss of all individuals, and such that this loss is minimal to finance the government revenue requirements.[2] This work thus provides some indirect evidence that the equal principle may have found its way into the political debate and ultimately influenced the design of actual tax systems in different moments in time and different places.

A shortcoming of Young (1990)'s work and, for that matter, of all the early literature on equal sacrifice is that it (implicitly) takes taxable income to be independent of the tax schedule. This is unfortunate because incentive effects and the question of efficency could not be examined, despite suggestions, by Young (1990) himself, that efficiency concerns may underlie the poor fit of equal sacrifice schedules to actual ones at the high end of the distribution of income.[3]

To the best of our knowledge, the first work to explicitly take into account labor supply incentives in the equal sacrifice literature was Berliant and Gouveia (1993). In a Mirrlees (1971) setting, individuals with identical preferences differ with respect to their labor market productivities, which is private information. Allocations are derived under the restriction that they respect incentive compatibility: a tractable way of incorporating endogenous non-linear budget sets. Although Berliant and Gouveia (1993) raise the issue of efficiency, they do not address it formally.[4]

In this paper, we use a Mirrlees (1971) setting to investigate efficiency of an equal sac-

[2]More specifically, he tests and is not able to reject the hypothesis that almost all tax schedules that prevailed in the United States Federal Tax during the period 1957-1987 is based on the equal sacrifice principle. The same is true for Germany, Italy, Japan, and, to a lesser degree, the United Kingdom.

[3]According to Young (1990), p.264 "For high incomes, therefore, the departure from equal sacrifice may be due to efficiency considerations while for low income it is probably due to revenue requirements."

[4]Berliant and Gouveia (1993) start by declaring that "One of the aspects of the model we still need to clarify are its welfare properties" and continue by suggesting that inefficiency should result since "The condition of a zero marginal tax rate at the top ability level, emphasized in Sadka (1976) and Seade (1977), is not generally satisfied."

rifice tax system along the lines of Berliant and Gouveia (1993). The economy is inhabited by a continuum of individuals which differ with respect to their labor market productivity, w. A tax system induces labor supply choices with an associated equilibrium utility profile $v_1(.)$, where $v_1(w)$ is the utility attained by an individual with productivity w. The question we aim at answering is whether there is a Bergson-Samuelson social welfare function $W(v)$, increasing in v, such that the tax system generated by the equal sacrifice principle is the one which maximizes $W(v)$.We shall then say that $W(v)$ rationalizes the tax schedule. This is, of course, equivalently to asking whether there is an alternative tax schedule which generates no less revenue and induces a utility profile $v^*(.)$ such that $v^*(w) \geq v_1(w)\ \forall w$ with strict inequality for a subset of positive measure of types.

Throughout the paper we adopt a separable specification for preferences, which is particularly convenient for the discussion of equal sacrifice tax schedules. Given the functional form used here, one can show that taxable income is invariant to the level of sacrifice. Using this invariance property of labor income we derive the progressive or regressive nature of tax schedules as a direct function of the curvature of the utility function. We then follow the characterization of Pareto efficient tax schedules derived by Werning (2007) to investigate the efficiency of equal sacrifice schedules.

Because the shape of efficient tax schedules depends on the underlying distribution os skills while the shape of equal sacrifice schedules does not, it is not hard to build model economies for which the equal sacrifice schedules induce constrained inefficient allocations.[5]

Although interesting from a theoretical perspective, this latter result is of limited value from a policy perspective since real world distribution of types is not a choice variable. We then concentrate our analysis on real world economies. We first show that, if preferences are of the ln type and if productivities follow a Pareto distribution with a fast enough decay,[6] then there is a level of government above which equal sacrifice leads to Pareto dominated allocations. Using a typical parametrization for the US economy, the level of expenditures at which the tax schedule becomes inefficient is, however, above 50% of GDP.

[5]A trivial example is the case with ln preferences and a distribution of productivity with bounded support. Most recent works in applied optimal taxation, however, depart from the assumption of a bounded support—e.g., Saez (2001)—by viewing this as a rather unrealistic description of the data. As it turns, even with unbounded support, counterexamples of inefficient schedules are always possible if one is allowed to choose the distribution of productivities.

[6]To be precise, a coefficient greater than 2.

Next we try different parametrization for preferences, that bring us closer to the findings by Young (1990) regarding the shape of the tax schedule. That is to rationalize the progressivity of the US income tax schedule using the equal sacrifice principle a coefficient of relative risk aversion greater than one is needed.[7] We derive regions of inefficiency for marginal tax rates for different parametrizations of preferences and levels of Government expenditures and ask whether they may justify the poor fit for high incomes suggested by Young (1990). For the best specifications for preferences associated with levels of risk aversion aligned with those estimated by Young (1990) we find that inefficiency only arises at the high end of the distribution of income.

The rest of the paper is organized as follows. Section 2 describes the economy. Implementable allocations are described in Section 3. In Section 4 we derive the shape of equal sacrifice schedules for different parameters of risk aversion. The main results of this paper are found in Sections 5 and 6. Section 7 concludes. The appendix gathers the derivation of some of the main results.

2 The Environment

The economy is inhabited by a continuum of measure one of individuals with identical preferences defined over consumption, c, and effort, l.

Preferences are represented by

$$U(c,l) = u(c) - h(l), \tag{1}$$

where u and h are smooth functions such that $u', -u'', h', h'' > 0$.

Individuals differ from one another along a single dimension: labor market productivity, $w \in W \subset R_+$, where W is a closed convex set. w is, therefore, the parameter that captures heterogeneity across individuals. We assume that w is distributed according to $F(w)$ with associated density $f(w) > 0$ for all $w \in W$.

An individual with productivity w that makes effort l produces an output $y = lw$. Output is measured in units of the consumption good. Technology is, therefore, very simple, one efficient unit of effort lw is converted one for one into one unit of consumption.

[7] Young (1990) finds that a coefficient between 1.5 and 1.7 provides the best fit. Our focus on a coefficient of relative risk aversion greater than one is, then, due to its empirical content. Still, for sake of completeness we explore regressive tax schedules induced by equal sacrifice when the coefficient of relative risk aversion is less than one.

We assume that the economy is so that an individual is paid his output. We shall then refer to y as output and taxable income, interchangeably.

Following Mirrlees (1971), we assume that w is private information. That is, neither w nor l are observed separately, only the product $lw = y$ is observed by all.

In the economy there is also a government that must finance a given level of expenditures B which we take as exogenous to the problem. The government budget constraint is,

$$B \leq \int T(y)dG(y),$$

where, $T(y)$ is the tax schedule, a function that maps an individual's output y into his tax obligations $T(y)$, and $G(y)$ is the distribution of income induced by the tax system.

Our environment is, therefore, exactly that of Mirrlees (1971) with the specialization for the case of separable preferences. Contrary to Mirrlees (1971), we do not consider a social welfare functional that will be maximized by the planner to the obtain the tax schedule. Instead, we apply the principle of equal sacrifice: to determine the tax schedule, $T(.)$, that will be used to finance B, the government imposes the same utility loss on all individuals. At this point it is worth noting that U, as defined in (1), may be viewed as a 'social norm' that will ultimately reflect how society will perceive what sacrifice is being made by each individual.

The first step in deriving the equal sacrifice schedule is to find a reference point in the sense of an initial utility level from which sacrifice will be defined.

We shall take as a reference point a world in which $B = 0$, and $T(y) = 0 \ \forall y$,

$$v_0(w) = \max_y \{u(y) - h(y/w)\}.$$

We shall refer to this reference point as the 'no-sacrifice world'.

Since our goal is to investigate efficiency of the tax schedule it is crucial that we take into account the behavioral responses to changes in budget sets. As it turns it is very straightforward to attack the problem using a mechanism design approach. That is, we shall follow Berliant and Gouveia (1993) in focusing directly on equal sacrifice allocations while requiring them to be incentive compatible. This is what we do in the next section.

3 Incentive-compatible equal-sacrifice systems.

An allocation is a mapping $(c, y) : W \longmapsto R^2_+$ that associates to each type, w a pair $(c(w), y(w))$.

Let $\Gamma(w)$ denote the set of choices (budget sets) available for an agent of productivity w. By working in the space of pairs (c, y) instead of (c, l), we may take $\Gamma(w) = \Gamma$ for all w. An allocation is incentive compatible if and only if $(c(w), y(w)) \in \arg\max_{(c,y)\in\Gamma}\{u(c) - h(y/w)\}$ for all w.

Let us, then, focus on the no-sacrifice allocation. The budget set for a type w individual in this case is $\Gamma^0 \equiv \{(c, y); c \leq y\}$, which, as we have pointed out, does not depend on w. Let $v_0(w) \equiv \max_{(c,y)\in\Gamma^0}\{u(c) - h(y/w)\}$. Under the assumptions adopted so far, $v_0(w)$ is differentiable and $v_0'(w) = h'(y_0(w))y_0(w)/w^2$, where $y_0(.)$ maps productivity to output when the budget set is Γ^0.

Let us now describe the direct mechanism associated with the equal sacrifice taxation problem. The planner asks each individual his or her type, w, and uses the (possibly false) report $\hat{w}$ to assign a bundle $(c(\hat{w}), y(\hat{w}))$. By the revelation principle we can focus on a truthful mechanism for which the planner chooses an allocation $(c, y) = (c(w), y(w))_{w\in W}$ such that

$$w \in \arg\max_{\hat{w}\in w}\{u(c(\hat{w})) - h(y(\hat{w})/w)\}.$$

This global condition of incentive compatibility is satisfied if and only if the envelope condition,

$$v'(w) = h'\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^2}, \tag{2}$$

and the monotonicity condition,

$$y'(w) \geq 0, \tag{3}$$

are satisfied.

Under the assumptions that $h(.)$ is strictly increasing and strictly convex we may use the implicit function theorem to derive a relationship

$$\frac{y(w)}{w} = \varphi(v'(w)w), \tag{4}$$

where φ is a strictly increasing function.

Our focus in this paper is on the iso-elastic specification for preferences,

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

for $\rho > 0, \rho \neq 1$, $u(c) = \ln c$ for $\rho = 1$ and

$$h(l) = l^{\gamma}/\gamma$$

for $\gamma > 1$.[8]

Nowhere in this discussion have we used the level of utility, only its variation. This is a very interesting consequence of separability: under incentive compatibility, the change in utility pins down the level of output produced by individuals. Such property will prove useful for characterizing the allocations in the equal-sacrifice problem.

Let $T(y)$ denote the tax function, then, if $v_1(w) = \max_y \{u(y - T(y)) - h(y/w)\}$, and define the sacrifice, $s(w)$, through

$$s(w) \equiv v_0(w) - v_1(w).$$

The equal sacrifice principle amounts to requiring $s(w)$ to be constant.

Therefore, equal sacrifice yields $v_0'(w) = v_1'(w)$. An immediate consequence of (4) is, therefore, that $y_1(w) = y_0(w)$ for all w. Individuals must produce the exact same output they were producing at the reference state![9] In this case, it is immediate to see that under our iso-elastic specification, $y(w) = y_0(w) = w^{\frac{\gamma}{\gamma+\rho-1}}$.

Once we realize that everyone is making the same effort as they were before the introduction of taxes, then it must be the case that all the sacrifice is due to reduced consumption:

$$\begin{aligned} s &= u(c_0(w)) - u(c_1(w)) \\ &= u(y_0(w)) - u(y_0(w) - T(y_0(w))) \end{aligned} \tag{5}$$

[8]For the iso-elastic case, we may define $\theta \equiv w^{-\gamma}$ and note that (2) may also be written $v'(\theta) = h(y(\theta))$. Hence, given a path for $v(\theta)$ we uniquely define $y(\theta)$. Because there is a one to one relationship between θ and w, from the path for $v(w)$ we recover a unique path for $y(w)$.

[9]This result was first derived by Berliant and Gouveia (1993) — see their Proposition 4.

Let $\xi(.) = u^{-1}$, then

$$\xi(u(y_0(w)) - s) = y_0(w) - T(y_0(w)). \tag{6}$$

4 The Shape of Equal Sacrifice Tax Schedules

The term progressivity is sometimes to denote a tax schedule for which *average* taxes weakly increase with income and sometimes to denota a schedule for which *marginal* taxes weakly increase with income. Progressivity in the first sense has been shown to imply that after tax income is more equaly distributed than before tax income, which makes it a very appealing notion.[10] Marginal tax rate progressivity, however, seems to get much attention as the discussions following the zero top marginal tax rates found in the optimal taxation literature makes clear.

Marginal and Average Tax Rates Assume that the tax function, $T(.)$, is twice continuously differentiable. Differentiating (5) and rearranging terms yields

$$1 - \frac{u'(y_0(w))}{u'(y_0(w) - T(y_0(w)))} = T'(y_0(w)). \tag{7}$$

That is, the marginal tax rate faced by any individual is (one minus) the ratio of his or her marginal utility of income before and after the introduction of taxes.

Abusing notation somewhat, let $u_0(w) = u(c_0(w))$. Because utility differences are the same for all w, we can also write the expression above as

$$1 - \frac{\xi'(u_0(w) - s)}{\xi'(u_0(w))} = \tau(w),$$

where $\xi'(u)$ is the marginal cost in consumption terms of delivering utility u and $\tau(w) = T'(y(w))$. Note that ξ is an increasing convex function of u which means that $0 < \tau < 1$ for all $s > 0$.

If one assumes that average tax rates are increasing, then it must be the case that marginal tax rates are everywhere greater than marginal tax rates. This is still compatible with declining marginal tax rates, though. As we shall show next, (7), further restricts

[10] To compare two distribution of incomes the Lorenz criterion is used.

the behavior of $T(.)$ in such a way as to guarantee that this cannot be the case for iso-elastic preferences. Indeed, differentiating (7) we get

$$\begin{aligned}\frac{T''(y_0(w))}{1-T'(y_0(w))} &= \left\{\frac{u''(y_0(w))}{u'(y_0(w))} - \frac{u''(y_0(w)-T(y_0(w)))}{u'(y_0(w)-T(y_0(w)))}\left[1-T'(y_0(w))\right]\right\} \\ &= -\frac{1}{y_0(w)}\left\{r(c_0(w)) - r(c_1(w))\frac{1-T'(y_0(w))}{1-\varsigma(y_0(w))}\right\}, \end{aligned} \tag{8}$$

where $\varsigma(y) = T(y)/y$ is the average tax rate and $r(c)$ is the coefficient of relative risk aversion at consumption level c.

If $u(c) = c^{1-\rho}(1-\rho)^{-1}$, for $\rho \neq 1$, $u(c) = \ln c$ for $\rho = 1$, then, $r(c) = \rho$ for all c and the expression above reduces to

$$-\left.\frac{d\ln(1-T'(y))}{d\ln y}\right|_{y=y_0(w)} = \rho\left\{1 - \frac{1-T'(y_0(w))}{1-\varsigma(y_0(w))}\right\}.$$

For our purposes it will also be important to write the expression above as follows

$$\rho\left\{\frac{\tau(w)-\varsigma(y_0(w))}{1-\varsigma(y_0(w))}\right\}\frac{d\ln y_0(w)}{d\ln w} = \frac{d\ln\tau(w)}{d\ln w}\frac{\tau(w)}{1-\tau(w)}. \tag{9}$$

If one defines progressivity in terms of increasing *average* taxes—i.e., $\varsigma(.)$ increasing in y—then, it is immediate to verify that a smooth tax schedule is progressive if and only if $T' \geq \varsigma$ almost everywhere. But, in this case, $T'' \geq 0$. That is, marginal tax rates will increase (resp. decline) with y: the tax schedule is progressive (resp. regressive) in the sense of increasing (resp. decreasing) *marginal* tax rates. Hence, there is no ambiguity in the case of iso-elastic preferences, progressivity in one sense implies progressivity in the other.

4.1 Risk Aversion and the Shape of the Tax Schedule

In what is probably the first attempt to relate equal sacrifice and progressivity, Samuelson (1947) has shown that an equal sacrifice schedule is progressive if and only if the coefficient of relative risk aversion of the chosen utility function is greater than one. If we follow Samuelson (1947) by disregarding incentives and using a utility function that only depends on income, the point is quite simple to make.

Let U be the standing utility representation, assumed to be strictly increasing and concave. Differentiating $s = U(y) - U(y - T(y))$ and rearranging terms we get

$$\frac{U'(y)y}{U'(y - T(y))[y - T(y)]} = \frac{1 - T'(y)}{1 - \varsigma(y)}.$$

Since $T'(y) \geq \varsigma(y)$ is necessary and sufficient for the tax schedule to be progressive in the sense of increasing average taxes, the result is immediate.

If preferences depend not only on consumption but also on leisure and incentives are considered this need not hold, in general. However, for the special case of separable preferences, the result still obtains.

First, recall that, $\tau(w) = 1 - h'(y(w)/w)/u'(c(w))w$. Because y does not vary with s, how marginal tax rates vary with sacrifice only depends on how the marginal utility of consumption varies with s,

$$\frac{d\tau}{ds} = \frac{h'(y_0(w)/w)}{u'(c(w))w}\frac{u''(c(w))}{u'(c(w))}\frac{dc(w)}{ds},$$

or

$$\frac{d\tau}{ds}\frac{s}{1-\tau} = r(c(w))\frac{dc(w)}{ds}\frac{s}{c(w)}.$$

Now, equal sacrifice for all s is equivalent to $u'(c_0(w))dc(w) = u'(c_0(\hat{w}))dc(\hat{w})$ for all w and $\hat{w}$. With CRRA preferences,

$$\frac{dc(w)/c(w)}{dc(\hat{w})/c(\hat{w})} = \left(\frac{c(\hat{w})}{c(w)}\right)^{1-\rho}, \forall w, \hat{w}.$$

Hence, if $\rho < 1$ (resp. $\rho \geq 1$) equal sacrifice requires a greater variation of consumption in percentage terms from those who are initially consuming less (resp. more). As argued before, a regressive (resp. progressive) tax system obtains.

Figure 1 displays marginal and average tax rates derived from the use of the equal sacrifice principle for different values of risk aversion. The level of sacrifice is chosen in such a way as to generate total tax revenues of 20 percent of GDP. This first figure makes explicit the fact that progressivity in both senses characterizes tax schedules when $\rho \geq 1$ and regressivity characterize them otherwise. Figure 2 superimposes average and marginal taxes for the two cases. The case $\rho = 1$, i.e., $u(c) = \ln(c)$ yields linear tax rates. We shall further characterize the optimal schedule for the ln case to illustrate how the equal sacrifice principle allows for simple characterization of tax schedules.

Incentive compatibility is equivalent to $v(w) = v(\underline{w}) + \int_{\underline{w}}^{w} h'(y(\tilde{w})/\tilde{w})y(\tilde{w})/\tilde{w}^2 d\tilde{w}$ and monotonicity of $y(.)$. Recalling the definition $v(w) = u(c(w)) - h(y(w)/w)$, we have that consumption for a type w at any incentive compatible allocation is given by

$$c(w) = u^{-1}\left(v(\underline{w}) + \int_{\underline{w}}^{w} h'\left(\frac{y(\tilde{w})}{\tilde{w}}\right)\frac{y(\tilde{w})}{\tilde{w}^2}d\tilde{w} + h\left(\frac{y(w)}{w}\right)\right). \tag{10}$$

If $u(.) = \ln(.)$, (10) may be written

$$c(w) = A(v(\underline{w}))B(w),$$

where

$$A(v(\underline{w})) = e^{v(\underline{w})},$$

and

$$B(w) = \exp\left\{\int_{\underline{w}}^{w} h'\left(\frac{y(\tilde{w})}{\tilde{w}}\right)\frac{y(\tilde{w})}{\tilde{w}^2}d\tilde{w} + h\left(\frac{y(w)}{w}\right)\right\}.$$

Hence,

$$T'(y(w)) = 1 - h'\left(\frac{y(w)}{w}\right)A(v(\underline{w}))\frac{B(w)}{w}.$$

If our reference point is $T(y) = 0$ for all y,

$$h'\left(\frac{y_0(w)}{w}\right)\frac{B_0(w)}{w} = \frac{1}{A(v_0(\underline{w}))},$$

where $B_0(w)$ is defined as $B(w)$ with $y_0(w)$ substituting for $y(w)$.

Using the fact that for any equal sacrifice schedule $y(w) = y_0(w)$ for all w, we have that $B_0(w) = B(w)$ and

$$T'(y(\theta)) = 1 - \frac{A(v(\underline{w}))}{A(v_0(\underline{w}))},$$

which is independent of y.

5 Sacrifice and Efficiency

The next question we shall address, the focus of our work, is whether tax schedules obtained by applying the equal sacrifice principle are Pareto efficient. One possibility for addressing this issue is to answer the following question. Given a tax schedule derived from the use of the equal sacrifice principle is it always the case that we may find a Paretian social welfare function that rationalizes it as the optimal choice?

If we take this question in a very strong sense, the answer is on the negative: we may easily build an environment for which the equal sacrifice principle leads to an inefficient tax schedule. This result is an immediate consequence of Proposition 2 in Werning (2007), which states that "For any tax schedule $T(y)$ and its resulting allocation, there is a set of skill distributions $F(\theta)$ and net endowments $-B$ for which the outcome is Pareto Efficient and another set of skill distributions $F(\theta)$ and net endowments, $-B$ for which it is Pareto inefficient."[11] That inefficiency could arise for *some* environment was to be expected. For a compact W, for example, we know that the marginal tax rate at the top is non-positive for any Pareto efficient schedule while it is positive for any progressive ($\rho \geq 1$) schedule and positive sacrifice.

The result above is useful to highlight the fact that, because equal sacrifice schedules are independent of the distribution of types, there will always be a distribution of types that makes the induced schedule inefficient for a positive level of sacrifice. This is not, however, the question we are interested in from a policy perspective. What we really want to know is whether an equal sacrifice schedule is inefficient given the actual distribution of productivities for the society we study.

What we shall do, then, is try to establish whether for the distributions of productivity that characterize real world economies there is a level of sacrifice which makes the tax schedule thus derived inefficient. If this is the case, the next question is what is the associated level of expenditures as a fraction of GDP above which the tax schedule is inefficient?

To try and answer these questions, we start by replicating—see appendix—the findings in Werning (2007) to our setting.[12] We show that a smooth tax schedule is efficient if and only if $\tau(w)$ is such that

$$\tau(w) \leq \frac{\gamma}{\gamma + \Phi(w)}, \ \forall w, \tag{11}$$

where

$$\Phi(w) = (\gamma - 1)\,\frac{d\ln y}{d\ln w} - (\gamma + 1) - \frac{d\ln \tau}{d\ln w} - \frac{d\ln f}{d\ln w}. \tag{12}$$

[11] θ is as defined in footnote 8. See Werning (2007), p. 6.

[12] An alternative approach is to use the procedure advanced by Bourguignon and Spadaro (2008). They assume that the observed tax schedule solves an optimization problem and use optimal tax formulae to recover the derivative of the social welfare function for each observation of y. A social welfare function is Paretian if and only if the derivative is non-negative.

It is important to notice that $d\ln y/d\ln w$ is *not* the elasticity of taxable income with respect to w. Instead, it is the cross-sectional derivative of taxable income with respect to w, i.e. the percentage change in taxable income when we consider individuals whose productivities vary by one percent. The two values will differ under a non-linear schedule since on the cross-section since the virtual income, as defined by Hausman (1985), will also differ across individuals. This makes the application of (11) quite simple in our case, since $y(w)$ coincides $y_0(w)$ which has a simple form under the iso-elastic specification we use: $d\ln y/d\ln w = \gamma/(\gamma+\rho-1)$.

As for $d\ln\tau/d\ln w$, expression (9) allows us to see how it depends on the difference between marginal and average tax rates, as well as the cross-sectional variation of taxable income, $d\ln y/d\ln w$. However due to the term in curly brackets in the left had side of (9) it does not lend itself to trivial back of the envelope calculations, except for the case $\rho = 1$. We turn to it next.

To illustrate how inefficiency easily obtains in this setting we shall focus on the case $u(.) = \ln(.)$, which, as we have pointed out yields a very simple equal sacrifice schedule: a linear one. Notice also that, as is the case for all equal sacrifice schedules, conditional on s, the tax schedule is independent of the distribution of types $F(w)$. This feature will allow us to construct an example of inefficient tax.

Before, however, it is worth making explicit $v(w)$:

$$\begin{aligned} v(w) &= \ln y_0(w)\,[1-\tau] - h(y(w)/w) \\ &= \ln\left(\frac{A(v(\underline{w}))}{A(v_0(\underline{w}))}\right) + \underbrace{\ln y_0(w) - h(y_0(w)/w)}_{v_0(w)}. \end{aligned}$$

I will assume an unbounded distribution of productivities, which induces an unbounded distribution of income. The focus of the discussion that follows will be on the distribution of income for this is what we observe in the data.

Assume that the tax system induces a distribution of income that has a Pareto distribution with support $\left[\underline{y},\infty\right)$, $\underline{y} > 0$, and associated density $g(y) = \kappa y^{-\alpha}$, $\alpha > 1$, where $\kappa = (\alpha-1)\underline{y}^{\alpha-1}$.[13] This is a commonly used specification for the distribution of income, at least for part of the distribution—e.g., Saez (2001); Diamond (1998).

[13]The distribution is $G(y) = 1 - \left(\frac{y}{\underline{y}}\right)^{\alpha-1}$.

To apply the result above, note that

$$\frac{d\ln f}{d\ln w} = \frac{d\ln g(y)}{d\ln y} = -\alpha,$$

since with $u(.) = \ln$, $h(l) = l^{\gamma}/\gamma$ and linear taxes $y = w$.[14]

Proposition 1 *Let*

$$U(c,l) = \ln c - \frac{l^{\gamma}}{\gamma},$$

and assume

$$F(w) = 1 - \left(\frac{\underline{w}}{w}\right)^{(\alpha-1)},$$

with $\alpha > 2$, $w \in [\underline{w}, \infty)$. *Then, there is a level of government per capita expenditure,* $\overline{B} < \int yf(y)dy$, *such that the use of the equal sacrifice principle leads to an inefficient tax schedule for all* $B \geq \overline{B}$.

Proof. When preferences are such that $u(c) = \ln c$, then $d\ln y/d\ln w = 1$, and $d\ln\tau/d\ln w = 0$, for an equal sacrifice schedule. Therefore, $\Phi(w) = -2 - d\ln f/d\ln w$. Using, our choice for $f(w)$, (11) becomes

$$\tau \leq \frac{\gamma}{\alpha+\gamma-2}, \tag{13}$$

thus replicating expression to $\overline{\tau}$ in Werning (2007). Note that (13) imposes an upper bound on the marginal tax rate when $\alpha > 2$. Next, one just has to notice that if a government must raise a given value B where $B < \int ydF(y)$, then $B = \int T(y)dF(y) = \tau\int ydF(y)$, which leads to the violation of (13) for any B such that $B/\int ydF(y)$ is greater than the right hand side of (13). ■

A couple of things about this proposition are worth discussing. First, we have imposed the restriction $B < \int ydF(y)$, so that the result is non-trivial. The endogeneity of $F(y)$ tends to make this type of restrictions hard to interpret. Not in this case, however, since it can be imposed prior to the imposition of taxes due to the fact that $y_1(w) = y_0(w)$ $\forall w$. Second, Proposition 1 allows for simple back of the envelope calculations that illustrates actual applications of our procedure using equation (13), hence, its underlying assumptions.

[14]For $\rho \neq 1$ we use, instead, $F(w) = 1 - (\underline{w}/w)^{\varphi-1}$ for $\varphi = [\alpha(\gamma+\rho-1) - (\rho-1)]/\gamma$. This simple calculation is possible for an equal sacrifice schedule when preferences are separable, since $y_1(w) = y_0(w)$.

Saez (2001) considers the following values for α: 1.5, 2 and 2.5 for the US economy. As we have seen, for the first two values, the condition does not have a bite. Let $\epsilon = 1/(\gamma - 1)$ be the Frisch elasticity of labor supply. In the case $\alpha = 2.5$, for extreme values of ϵ the the maximum value for τ is close to 70%, and it is 75% for $\epsilon = 2$, for example. If instead of using the values found by Saez (2001) we use $\alpha = 3$, which is used in Werning (2007), then, depending on the value of the Frisch elasticity of demand, ϵ the right hand side will vary from 1, when $\epsilon = 0$ to 0.5 when $\epsilon \to \infty$. Most studies consider values for ϵ not greater than 4, in which case, the highest fraction of GDP that can be efficiently financed using an equal sacrifice principle is 55%. Because most transfers should be excluded from this calculation, this fraction is still very far from what one observes in the US.

It is also important to note that, because our model is static the parameter γ can be used to provide us with some flexibility to vary the society's perception of ability to pay, by varying the parameter of relative risk aversion ρ, while holding the elasticity of labor supply constant.

6 Sacrifice and Efficiency in Practice

The discussions in the previous sections were based on a specification for the utility of consumption of the form $u(.) = \ln(.)$. This is an important benchmark since preferences representable by this functional form induce inelastic labor supply, which does seem to adhere reasonably well to the data, for prime age males at least.

However, equal sacrifice schedules induced by ln preferences are linear and linear tax schedules need not provide the best approximation of actual tax schedules for real world economies.[15] In fact, taking taxable income as independent of the tax schedule, Young (1990) has shown that the US tax schedules that prevailed for most of the period from 1957 to 1987 can be rationalized by the equal sacrifice principle if one accepts a value for ρ in the range $[1.5, 1.7]$. As we have seen, under this parametrization for preferences the equal sacrifice principle induces a progressive tax schedule, which seems to provide a better description of the US tax system than a linear one for the period considered by Young (1990).

Another drawback of using the ln specification is that we cannot disentangle the society's perception of individuals' ability to pay, as captured by the curvature of the utility

[15] One should however bear in mind that a linear specification is often used as an approximation of the actual system for many purposes–e.g., Saez (2001).

function, and labor supply elasticity. The point is that the parameter ρ affects not only labor supply but also how society 'measures' utility losses. The problem with this specific choice of ρ that induces a (however plausible) labor supply elasticity equal to zero, is that we end up commiting ourselves to a specific view of the society's perception of the relative sacrifice induced by any given relative reduction in consumption. A view which is, for example, in contrast with what is found by Young (1990), as we have just mentioned.

If we choose, instead, $\rho \neq 1$, we retain some degree of freedom to explore different social perceptions of ability to pay while still holding labor supply elasticity fixed at an empirically relevant level. Ideally we would like to vary ρ while holding the elasticity constant at *any* empirically sound level. Unfortunately, we do not have full flexibility for disentangling the two since ρ pins down the sign of the elasticity independently of the value of γ. Still this added flexibility will be of great value for us.

When $\rho \neq 1$ we cannot directly apply (13) since the equal sacrifice tax function is not linear. We must instead rely directly on (11). What makes this procedure potentially hard to implement is that we do not observe the 'reference' distribution of income. Hence, we must back it up somehow from the data. We do this in two steps. We first recover the distribution of w, $F(w)$, using the actual distribution of income $G(y)$. Then we calculate the optimal choices for each individual when $T(y) = 0$ for all y.

The second step is trivial. As for the first step the issue here is how to take into account the effect of the tax system on labor supply.

An assumption that greatly simplifies the first step is the one adopted by Saez (2001), namley that the tax system may be reasonably approximated by a linear one, $T(y) = \tau y$. Note that, by choosing this approximation we are either departing from the assumption that the current system is an equal sacrifice one, or we are restricting ourselves to the ln specification. Either view is in contrast with what Young (1990) has argued to be the best description of the data for the 1957-1987 period. We shall do so under the implicit assumption that the most recent tax system, the one that induces the distribution of income from which our distribution of types is recovered, is not based on the equal sacrifice principle.[16]

Main results for the US economy To check whether equal sacrifice tax schedules are inefficient for the US economy we first choose a level of expenditures as a percentage of

[16]Or, for the case in which we use $u(.) = \ln(.)$ that the society has geared toward a vision of sacrifice that is more closely described by $\rho = 1$.

GDP that approximates the actual one. Note however that the logic of an equal sacrifice schedule means that we should exclude from our calculations all transfers. We have chosen to set expenditures at 30% of GDP.

The next step is to recover $F(w)$ from $G(y)$. With, $\rho \neq 1$ and $h(l) = l^{\gamma}/\gamma$, we get $w_{\tau}(y) = y^{\frac{\rho+\gamma-1}{\gamma}}(1-\tau)^{\frac{1-\rho}{\gamma}}$. This function associates to each output, y, the productivity, w, of an individual who is supplying it, given the approximated tax system. We can invert this function to recover $F(w)$ and then, $y_0(w)$ and $G(y_0)$.[17]

An important fact about the calculation of equal sacrifice schedules is related to the use of F. While having a closed form distribution of productivities is important for the derivation of optimal taxes, from a computational viewpoint this is not so for the derivation of equal sacrifice schedules. Nor is it for our procedure for testing for efficiency.

Note that, conditional on s, the equal sacrifice schedule is independent of F. Moreover, because $y(w)$ is independent of s under a separable specification, tax revenue is a monotonic function of s: i.e., there are no Laffer effects from equal sacrifice schedules. The consequence is that for a given F we raise s until revenues match the empirically relevant value. Similarly a functional form can be easily dispensed with in the calculation of $\Phi(w)$ by our using the numerical value of $d \ln f / d \ln w$.

Yet, to make our results comparable to the ones found in the optimal taxation literature we follow Saez (2001) and Werning (2007) in considering that earnings, y, follow a Pareto distribution, with $\alpha = 3$. According to Saez (2001), a Pareto distribution fits the US empirical earnings very well from the mode value to the top of distribution. In our economy, the poorest individual earns at least \$34,000 dollars per year.[18]

Figure 3 displays our results for the case $\rho < 1$ and $\gamma = 1.5$. The dashed line is the upper bound and it is given by equation (11). The solid line is the marginal tax rate according to equal sacrifice to finance expenditures at 30% of GDP. The condition to be a Pareto efficient tax schedule is that the $\bar{\tau}$ does not cross the tax schedule to anybody.

Given the level of expenditures as a fraction of GDP we have used, the tax schedule obtained by applying the equal sacrifice principle is Pareto efficient. Figure 3 shows that the tax schedule would be inefficient for someone facing a marginal tax rate equal or

[17] For our purposes, of course, one needs not recover $F(w)$ to derive the equal sacrifice distribution of income under our specification for preferences and a linear tax system. Indeed, it is clear that $y_0 = y_{\tau}(1-\tau)^{\frac{\rho-1}{\rho+\gamma-1}}$, where y_{τ} is the level of income induced by the tax system $T(y) = \tau y$. In possession of $y(w)$ we need only use (6) to derive $T(y)$. We use $\tau = 0.30$.

[18] We collect the data of labor income from Panel Study of Income Dynamic (PSID) in 2007.

above 33%.

The trouble with As we said in previews section, when $\rho < 1$ the tax schedule is characterized as regressivity. Even though, the case when $\rho < 1$ is theoretically interesting due to the regressivity of tax schedule, in reality, it is not often to find a tax schedule with this characteristic.

Although we use different parametrizations for preferences that allows us to discuss how they lead to inefficiencies at different ranges of the distribution of income, most of our discussions will focus on the case for which $\rho > 1$. As we said, because the tax schedules examined are all progressive, the underlying value for ρ is necessarily greater than one. We shall try to evaluate the efficiency of these schedules here. Indeed, it is also suggested in the paper that the poor fit for high incomes may be due to incentive concerns. Our approach allows for addressing this claim.

Figure 4 shows the earnings versus marginal tax rate and upper bound tax rate. We vary the risk aversion and keep γ uncharged. We consider the same range to risk aversion parameter than Young (1990) to US economy. Again, the dashed line is upper bound tax rate and and solid line is the marginal tax rate.

In all situations, the tax schedules derived from the use of the equal sacrifice principle is Pareto inefficient. As the tax schedule is progressive, the inefficiency occurs on the top of distribution. It is easy to observe that when we increase ρ the inefficient earning value is dislocated gradually from the top to the center of distribution.

When we increase risk aversion, there are two effects in the marginal tax rate and in the $\bar{\tau}$. The first effect is the increase of marginal tax rate on the top. The second effect is to raise the $\bar{\tau}$ curve. In the case $\rho = 1.5$, $\bar{\tau}$ is around 95% on the bottom and 84% on the top of distribution. While $\rho = 1.7$, $\bar{\tau}$ is around 113% on the bottom and 91% on the top. We can see the impact of increasing risk aversion into marginal tax rate and $\bar{\tau}$ throughout equations (7) and (11).

According our results, we establish that for the distributions of productivity that characterize real world economies there is a level of sacrifice which makes the tax schedule thus derived inefficient. Using parameters of US economy, to finance the expenditures at 30% of GDP, the tax schedules derived from the equal sacrifice principle is Pareto inefficient. However, the tax schedule may be Pareto efficient whether we consider a lower level of expenditures

7 Conclusion

In a series of papers in the late 1980's Young (1987, 1988, 1990) has forcefully argued that the income tax schedule for the US for the period period from 1957 to 1987 could be rationalized by direct applications of the equal sacrifice principle. The body of work that followed allows one to pin down the restrictions imposed on observed tax schedules by the equal sacrifice system—Mitra and Ok (1996); Ok (1995)—and to understand the consequences of taking incentives into account explicitly—Berliant and Gouveia (1993).

We use a separable iso-elastic specification for preferences to derive a tax system built using the principle of equal-sacrifice. The use of a separable specification for preferences greatly facilitates the characterization of the shape of equal sacrifice schedules and allows for an explicit evaluation of efficiency of such schedules since it lends itself to efficiency tests based in the methodology developed by Werning (2007).[19]

When utility of consumption is logarithmic and the cross-sectional distribution of productivities is Pareto with realistic parameters, there is always a level of per capita government spending above which an equal sacrifice tax schedule is inefficient. Back of the envelope calculations indicate that these threshold values are much higher than the average expenditures for the United States for normal times.

When the parameter of risk aversion is greater than one a progressive income tax schedule results from the equal sacrifice principle. For most parametrizations we have used, either all equal sacrifice schedules are efficient or they only become inefficient for very high levels of income.

References

Berliant, Marcus and Miguel Gouveia, "Equal Sacrifice and Incentive Compatible Income Taxation," *Journal of Public Economics*, 1993, *51* (2), 219–240.

Bourguignon, François and Amadeo Spadaro, "Tax-Benefit Revealed Social Preferences," Working Paper EM9/08, EUROMOD October 2008.

[19]Werning (2007) offers an interesting interpretation of these efficiency tests—see, p. 9. He shows that bounds of efficiency may be found by comparing the income distribution induced by the equal sacrifice system and that induced by an optimal Rawlsian tax schedule. That is, one calculates the optimal Rawlsian tax schedule for the relevant level of government expenditures, and check whether the equal sacrifice income distribution is first order stochastically dominated by the one induced by the Rawlsian tax schedule or its multiples.

Diamond, Peter A., "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rat," *American Economic Review*, 1998, *88* (1), 83–95.

Hausman, Jerry A., "Taxes and Labor Supply," in Alan Auerbach and Martin Feldstein, eds., *Handbook of Public Economics*, Vol. I, Elsevier, 1985, chapter 4, pp. 213–263.

Mirrlees, James A., "An Exploration in the Theory of Optimal Income Taxation," *Review of Economic Studies*, 1971, *38*, 175–208.

Mitra, Tapan and Efe A. Ok, "Personal Income Taxation and the Principle of Equal Sacrifice Revisited," *International Economic Review*, 1996, *37* (4), 925–948.

Ok, Efe A., "On the Principle of Equal Sacrifice in Income Taxation," *Journal of Public Economics*, 1995, *58*, 453–467.

Richter, Wolfram F., "From ability to pay to concepts of equal sacrifice," *Journal of Public Economics*, 1983, *20* (2), 211–229.

Sadka, Efraim, "On income distribution incentive effects and optimal income taxation," *Review of Economic Studies*, 1976, *43*, 261–268.

Saez, Emmanuel, "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 2001, *68*, 205–229.

Samuelson, Paul, *Foundations of Economic Analysis*, Harvard University Press, Cambridge Massachusetts, 1947.

Seade, Jesus, "On the shape of optimal tax schedules," *Journal of Public Economics*, 1977, *7*, 203–236.

Werning, Iván, "Pareto Efficient Income Taxation," 2007. MIT working paper.

Young, Peyton, "Progressive Taxation and the Equal Sacrifice Principle," *Journal of Public Economics*, 1987, *32*, 203–214.

_ , "Distributive Justice in Taxation," *Journal of Economic Theory*, 1988, *44*, 321–335.

_ , "Progressive Taxation and Sacrifice," *American Economic Review*, 1990, *80*, 253–266.

A Appendix

In this appendix we follow Werning (2007) in deriving necessary conditions for an allocation to be efficient in our setting. Because our goal is to check whether the equals sacrifice principle leads to inefficiencies, necessary conditions are enough. Yet, it is possible to show—see Werning (2007)—that these conditions are also sufficient.

The problem the government faces is that of

$$\max_{y(.),v(.)} \int [y(w) - e(v(w), y(w), w)] f(w) dw$$

s.t.,

$$v'(w) = \frac{y(w)^\gamma}{w^{\gamma+1}}, \tag{14}$$

$$y(w) \text{ increasing,} \tag{15}$$

and

$$v(w) \geq \bar{v}(w) \, \forall w. \tag{16}$$

Disregarding the monotonicity constraint (14), we may write the Lagrangian

$$\int \left\{ [y(w) - e(v(w), y(w), w)] f(w) + \mu(w) \left[v'(w) - \frac{y(w)^\gamma}{w^{\gamma+1}} \right] + \lambda(w) [v(w) - \bar{v}(w)] \right\} dw,$$

where

$$v(w) = u(e(v(w), y(w), w)) - \frac{y(w)^\gamma}{\gamma w^\gamma}$$

Integrating by parts

$$\int \left\{ [y(w) - e(v(w), y(w), w)] f(w) - \mu'(w)v(w) - \mu(w)\frac{y(w)^\gamma}{w^{\gamma+1}} + \lambda(w) [v(w) - \bar{v}(w)] \right\} dw \\ + \mu(\bar{w})v(\bar{w}) - \mu(\underline{w})v(\underline{w})$$

First order conditions are

$$\left(1 - e_y(v(w), y(w), w)\right) f(w) = \mu(w)\gamma \frac{y(w)^{\gamma-1}}{w^{\gamma+1}} \tag{17}$$

and

$$-e_v(v(w), y(w), w) f(w) = \mu'(w) - \lambda(w) \tag{18}$$

which implies,

$$-e_v(v(w), y(w), w) f(w) \leq \mu'(w) \tag{19}$$

Using the fact that $1 - e_y = \tau > 0$, (17) can be written in logs

$$\ln \tau + \ln f = \ln \mu + \ln \gamma + (\gamma - 1) \ln y - (\gamma + 1) \ln w$$

which implies

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} = \frac{d \ln \mu}{d \ln w} + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1)$$

Next note that

$$\frac{d \ln \mu}{d \ln w} = \frac{\mu'}{\mu} w \geq -\frac{e_v f}{\mu} w = -\frac{\gamma y^{\gamma - 1} e_v f w}{\left(1 - e_y\right) f w^{\gamma + 1}} = -\frac{\gamma}{\tau} e_v \frac{y^{\gamma - 1}}{w^{\gamma}} = -\frac{\gamma}{\tau}(1 - \tau)$$

Hence,

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} \geq -\frac{\gamma}{\tau}(1 - \tau) + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1).$$

That is,

$$\tau(w) \leq \frac{\gamma}{\gamma + \Phi(w)},$$

where

$$\Phi(w) = (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1) - \frac{d \ln \tau}{d \ln w} - \frac{d \ln f}{d \ln w}.$$

Figure 1: Marginal Tax Rate and Marginal Tax Rate

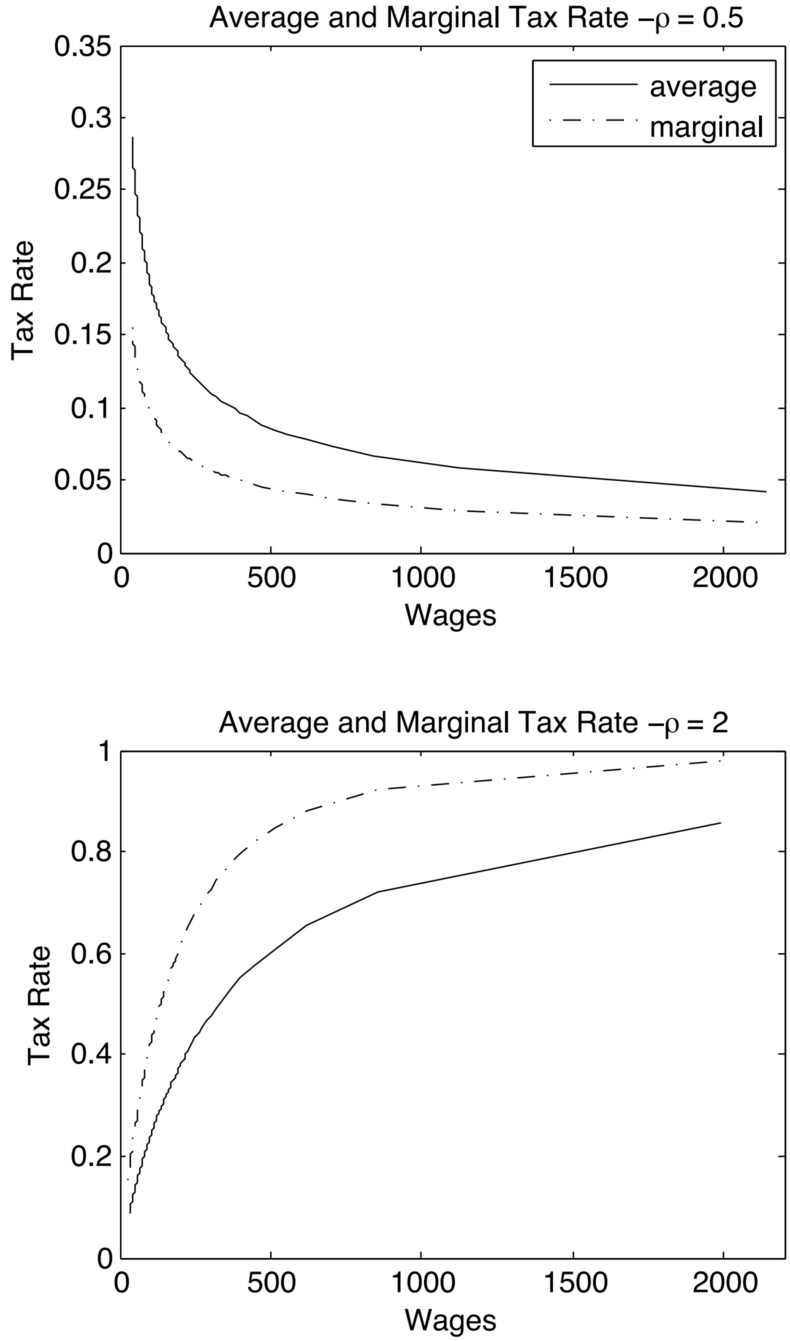

Figure 2: Marginal Tax Rate and Marginal Tax Rate

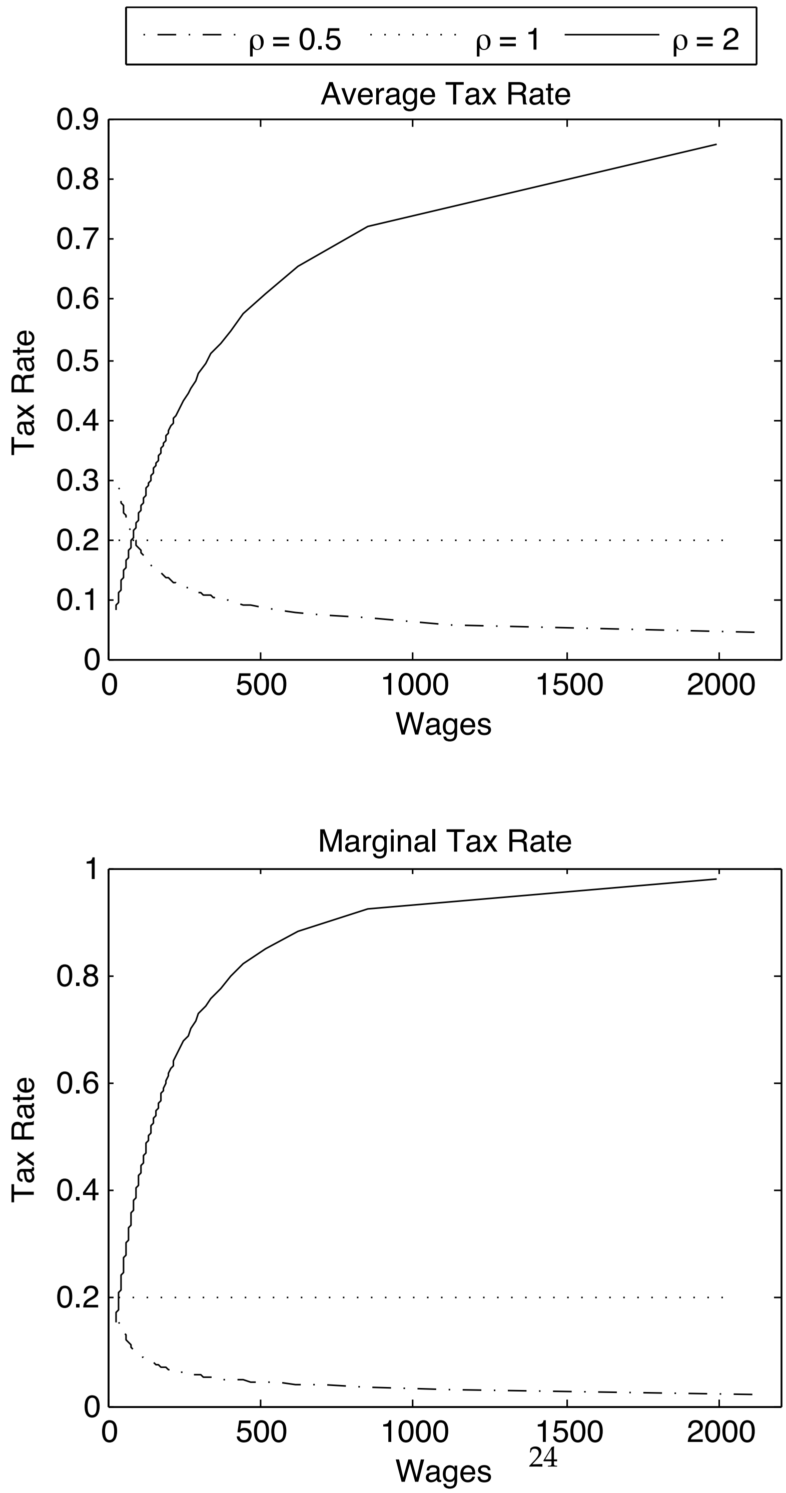

Figure 3: Marginal Taxes Rate and $\bar{\tau}$ - $\rho < 1$

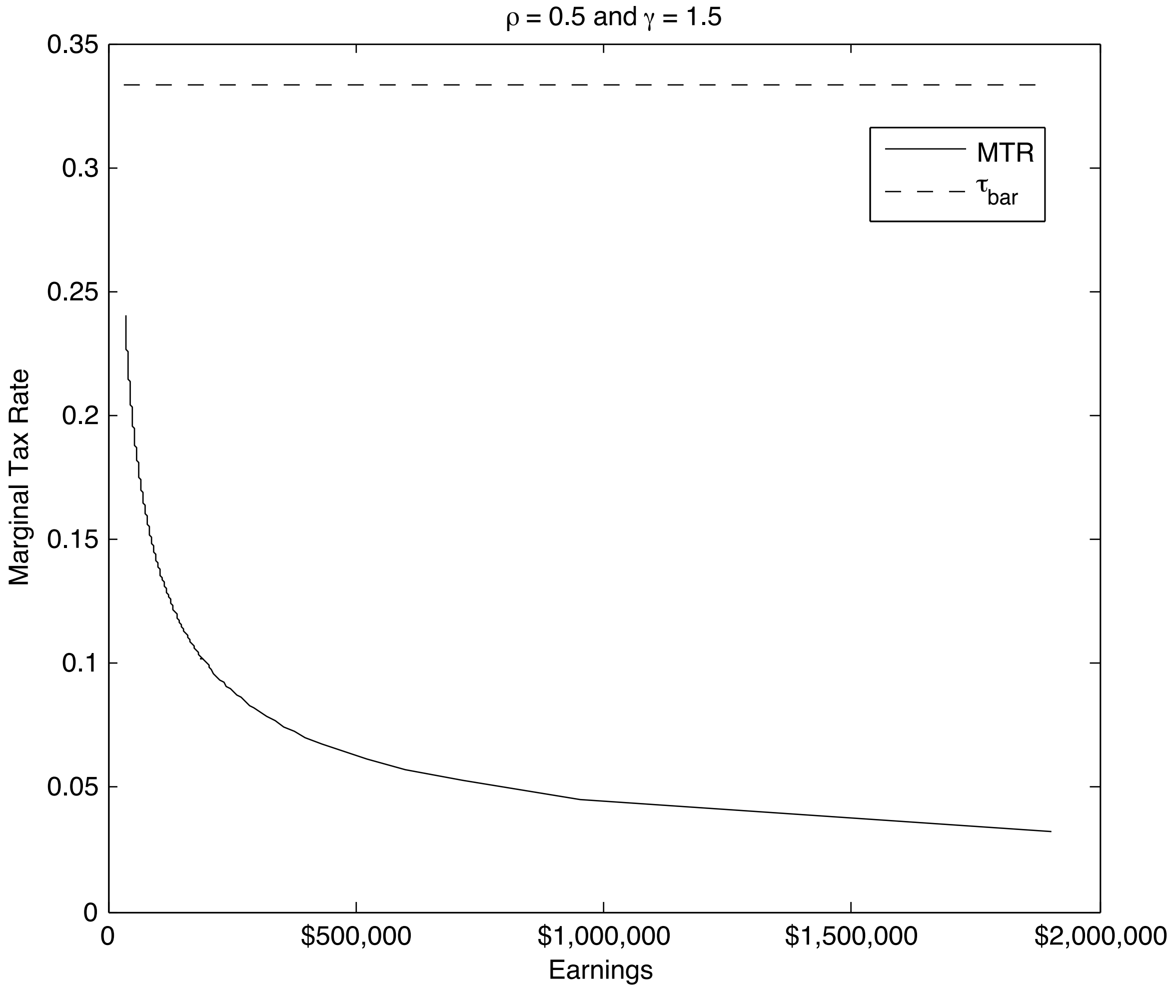

Figure 4: Marginal Taxes Rate and $\bar{\tau}$ - $\rho > 1$