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1 Introduction

Consider the problem of two countries engaged in preventing a common enemy from acquiring advanced weapons. Each country independently expends resources devoted to prevention in its sphere of influence. The enemy will try to acquire weapons from the least guarded location, but, if better-armed, the enemy may strike at either country. Thus, success of the joint effort depends on how secure the least protected source of weapons is. Also, each country possesses private information about the severity of the threat, obtained from its intelligence agencies, along with other information about the benefits of alternative uses of its resources. What determines each country's success in such a joint endeavor? Would an information-sharing agreement facilitate success? And would it be in each country's individual interest to truthfully reveal information, or would exaggerations prevail? How are information-sharing and prevention activities affected by the asymmetries between countries, e.g., better information, greater vulnerability to an enemys blow, the enemys declared intent to strike at one country, and the like?

In this paper, we analyze a situation that shares important characteristics with the above example:

- Distinct parties must take coordinated action to thwart a common enemy;
- Success of the common effort is determined, in large measure, by the party that expends the least effort (in other words, this is an example of weakest-link technology, e.g. Hirshleifer, 1983);
- All parties possess private information that may affect their willingness to take action;
- Information-sharing is voluntary and non-verifiable (e.g., because of concerns about the security of sources); and
- Contracts specifying side-payments are not enforceable in a court of law.

First, we characterize equilibrium of the symmetric, one-shot game without communication. Although multiple equilibria exist, we are able to identify the "best" one from the players' perspective: i.e., the equilibrium that maximizes the interim expected payoff of any type. Our characterization highlights a property of the distribution of private information about the marginal cost of effort that we label "regularity." We show that regularity is a necessary and sufficient condition for the best equilibrium to be strictly decreasing in the marginal cost of effort. However, even under regularity, all equilibria of the game—including the best—are inefficient, in any of the definitions in Holmström and Myerson (1983) that are appropriate for environments with private information.

We then perform comparative statics on the distribution of marginal costs for the symmetric case and focusing on the best equilibrium. Reasonably, we show that first-order stochastic dominance shifts that lead to higher costs result in players being worse-off. The results for second-order stochastic dominance (even with single-crossing cdfs) are less clear cut, especially without regularity, and we show examples in which the less-risky distribution may have a better (i.e. yielding higher interim expected payoffs) or a worse best equilibrium. We extend our comparative static analysis to regular, two-player situations where contributors are asymmetric. In particular, we show that if one player's cost distribution improves (worsens) in the sense of first-order stochastic dominance, then both player's contributions and utility increase (decrease).

Finally, we analyze communication. We introduce a pre-play exchange of messages, and we assume that post-message play continues with the appropriate best equilibrium. We consider two communication protocols. In the first communication is limited, a situtation meant to capture the fact that not all information may be transmissible—for instance, there may be overriding security reasons. In particular, we focus information transmission that only conveys whether the situation is favorable. We model this setup as having marginal costs that may be drawn from one of two distributions ordered by first-order stochastic dominance, and agents can send a message that identifies their cost distribution. In this case, we show that it is impossible for agents to be sincere in equilibrium. We then consider the case of unlimited communication, in which agents can reveal the exact realization of their cost parameter. In this setup, full-revelation of information is an equilibrium outcome, and agents are as well-off as in the best equilibrium of the game with full-information.

Two strands of literature are related to our work. First, our framework uses the weakest-link aggregator function, and thus belongs to the literature stemming from the seminal paper of Hirshleifer (1983). This extensive literature has analyzed many variations of the weakest-link public good game (e.g., Vicary 1990, Sandler and Vicary 2001, and Vicary 2002 consider different timings of contributions and the possibility of monetary or in-kind transfers), and has applied the analysis to various situations, from levees (Hirshleifer 1983) to security perimeters in alliances (Murdoch 1995), to cooperation in dredging successive stretches of a navigation channel (Harrison and Hirshleifer, 1989). However, we are not aware of the existence of a formal analysis of the effects of private-information within this framework, let alone of a treatment of communication.

The second strand of related literature concerns the voluntary provision of public goods with private information without an uninformed mediator who has full-commitment ability (see, e.g., Alboth *et al.* 2001, Barbieri and Malueg 2008a and 2008b, Laussel and Palfrey 2003, Martimort and Moreira 2010, and Menezes *et al.* 2001). This strand of the literature has analyzed different variations of the two-player subscription and contribution games (using the terminology of Admati and Perry, 1991), in which the success of the collective action depends on the sum of individual efforts and contributions may or may not be refunded. Also, within this strand of the literature, Agastya *et al.* (2007) provide an analysis of communication in a two-person contribution game. In contrast, we analyze the weakest-link technology, where success depends only on the lowest effort.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 characterizes equilibrium and analyzes its efficiency properties. Comparative statics for the symmetric and asymmetric case are in Section 4, while Section 5 analyzes communication. Section 6 concludes. Technical proofs are in the Appendix.

2 The Model

We study the problem of $n\,(\geq 2)$ players who simultaneously contribute to the funding of a public good. Each player's benefit from amount G of the public good is given by $v(G)$, where $v' > 0$ and $v'' < 0$ for all $G > 0$. Players differ in their costs of contributing to the public good. We let c_i denote the marginal cost to player i. We model this provision game as one of private information, where each player's cost is independently drawn from the cumulative distribution F having support $[\underline{c}, \bar{c}]$. For now we assume $\bar{c} \leq v'(0)$, so all types $c < \bar{c}$ are potentially willing to contribute. We can relax this later as desired. A player knows his own marginal cost of provision but no one else's. If player i contributes g_i and the realized value of the public good is G, then player i's net payoff is $v(G) - c_i g_i$.

In this paper we study the "Weakest-Link" "aggregator function": if the vector of contributions is $(g_1, \ldots, g_n)$, then the realized level of the public good is $G_{\mathrm{WL}} \equiv \min\{g_1, \ldots, g_n\}$.

3 Equilibria

We next derive Bayesian equilibria. Taking as given the strategies of the others, a player chooses his own contribution to maximize $-cg + \mathrm{E}[\,v(G)\,]$. We denote the equilibrium strategy function of player i as $g_i : [\underline{c}, \bar{c}] \to \Re_+$. Usual incentive compatibility arguments imply that every g_i will be weakly decreasing in cost. Moreover, a Bayesian equilibrium $(g_j)_j$ must satisfy $g_1(\underline{c}) = \cdots = g_n(\underline{c})$, for otherwise the player with the largest contribution at $\underline{c}$ could reduce his contribution (when his cost parameter is $\underline{c}$) to $\min_k g_k(\underline{c})$ and thereby raise his expected payoff (as the realized level of the public good would not change). Because equilibrium strategies are monotonic, they are almost everywhere differentiable, which yields the following necessary condition for equilibrium.

Lemma 1. *If $g_i(c_i)$ is an element of the best response of type c_i of player i to $\{g_j\}_{j\neq i}$, then the following* *two inequalities must hold for* $g_i(c_i) > 0$*:*

$$-c_i + v'(g_i(c_i)) \prod_{j \neq i} \lim_{\gamma \uparrow g_i(c_i)} \Pr(g_j \geq \gamma) \geq 0, \tag{1}$$

and

$$-c_i + v'(g_i(c_i)) \prod_{j \neq i} \lim_{\gamma \downarrow g_i(c_i)} \Pr(g_j \geq \gamma) \leq 0; \tag{2}$$

for $g_i(c_i) = 0$ *only* (2) *applies.*

The proof of Lemma 1 essentially is a check that at $g_i(c_i) > 0$ the left-hand derivative of player i's payoff is nonnegative and the right-hand derivative is nonpositive. In the next two propositions we apply Lemma 1 to considerably simplify the search for equilibria. The first proposition shows equilibrium strategies are continuous; the second shows all equilibria are symmetric.

Proposition 1. *All equilibrium strategies are continuous.*

We have already seen that $g_1(\underline{c}) = \ldots = g_n(\underline{c})$ in equilibrium, but this does not exclude the possibility that players' strategies are not all identical. The next proposition does this.

Proposition 2. *All equilibria are symmetric.*

We now characterize all equilibria, which, by Propositions 1 and 2, are continuous and symmetric. With the weakest-link technology, the level of the public good is determined by the player with the highest cost. It is therefore helpful to define $c_1^{[k]}$ as the maximum realization of k independent draws from the distribution F. Let H and h denote the cumulative distribution and density function of $c_1^{[n-1]}$. The symmetric equilibrium strategy g is characterized in the following proposition.

Proposition 3 (Characterization)**.** *Consider a continuous and non-increasing function* $g : [\underline{c}, \bar{c}] \to \Re_+$*. The function* g *is a symmetric equilibrium strategy if and only if, for almost all* $c \in [\underline{c}, \bar{c}]$*, all of the following conditions are satisfied if* $g(c) > 0$*:*

$$-\bar{c} + v'(g(\bar{c})) \geq 0, \tag{3}$$

$$-c + v'(g(c))H(\inf\{s : g(s) = g(c)\}) \leq 0, \quad \forall c \text{ such that } g \text{ is not strictly decreasing at } c, \tag{4}$$

$$-c + v'(g(c))H(\sup\{s : g(s) = g(c)\}) \geq 0, \quad \forall c \text{ such that } g \text{ is not strictly decreasing at } c; \tag{5}$$

$$\text{and } v'(g(c)) = \frac{c}{H(c)}, \quad \forall c \text{ such that } g \text{ is strictly decreasing at } c. \tag{6}$$

If $g(c) = 0$*, then only* (4) *applies.*

It is trivial to verify that $g(c) \equiv 0$ is a symmetric equilibrium strategy. It is more interesting to identify equilibria where g is, at least in part, strictly decreasing. Using (6) and the fact that v' is strictly decreasing, we see that g can indeed be strictly decreasing only if $c/H(c)$ is strictly increasing. We will say that the provision game with weakest link technology is *regular* if $c/H(c)$ is strictly increasing over $[\underline{c}, \bar{c}]$. Because $H(c) = (F(c))^{n-1}$ and $h(c) = (n-1)(F(c))^{n-2} f(c)$, we see that

$$0 < \frac{d(c/H(c))}{dc} = \frac{(F(c))^{n-2}\left[F(c) - (n-1)f(c)c\right]}{(F(c))^{2(n-1)}}$$

holds if and only if

$$\frac{f(c)}{F(c)} < \frac{1}{(n-1)c}. \tag{7}$$

We can use the above inequality to derive a necessary condition that F must satisfy for regularity. The inequality in (7) yields

$$-\log(F(c)) = \log(F(\bar{c})) - \log(F(c)) = \int_c^{\bar{c}} \frac{d\log(F(t))}{dt}\, dt < \left(\frac{1}{n-1}\right)\int_c^{\bar{c}} \frac{1}{t}\, dt = \left(\frac{1}{n-1}\right)\log\left(\frac{\bar{c}}{c}\right),$$

which in turn implies $F(c) > (c/\bar{c})^{\frac{1}{n-1}}$ for any $c \in (\underline{c}, \bar{c})$. From the previous inequality, we also conclude that if F is atomless and $\underline{c} > 0$, then the distribution is *not regular*.

Nonetheless it is possible to describe with relative ease all equilibria of the weakest-link private provision game with the help of Figure 1. The red curve depicts the function $c/H(c)$. The black line depicts the function $v'(g(c))$, for a symmetric equilibrium strategy $g(c)$. It is convenient to begin the description of equilibrium strategies at $\bar{c}$, noting that, by (3), $v'(g(\bar{c}))$ must lie on or above $\bar{c}/H(\bar{c}) = \bar{c}$ (recall the assumption that $v'(0) \geq \bar{c}$). Starting at $c = \bar{c}$ and moving backwards towards $\underline{c}$, there are only two ways for $v'(g(c))$ to move: it either follows $c/H(c)$ or it stays flat. However, the first option is open only if $v'(g(c)) = c/H(c)$ *and* if $c/H(c)$ is increasing. Therefore, in the regular case, if for some $\tilde{c} \in [\underline{c}, \bar{c}]$ we have $v'(g(\tilde{c})) > \tilde{c}/H(\tilde{c})$, then $v'(g(c))$ is constant on $c \in [\underline{c}, \tilde{c}]$. This is the case depicted in Figure 1. Similarly, if $c/H(c)$ is strictly decreasing (a situation guaranteed by F convex), then all equilibrium strategies are flat. Even when $c/H(c)$ displays many increasing and decreasing segments, we can easily identify the equilibrium with the largest interim expected contributions using the following procedure:

1. Start at $v'(g(\bar{c})) = \bar{c}/H(\bar{c}) = \bar{c}$ (this is possible because of the assumption that $v'(0) \geq \bar{c}$), and, moving in the direction of $\underline{c}$, begin by follow $c/H(c)$. Set the counter $z = 1$.

2. Proceed with $v'(g(c))$ that mirrors $c/H(c)$ until the latter function reaches a local minimum at, say, $\tilde{c}_z$.

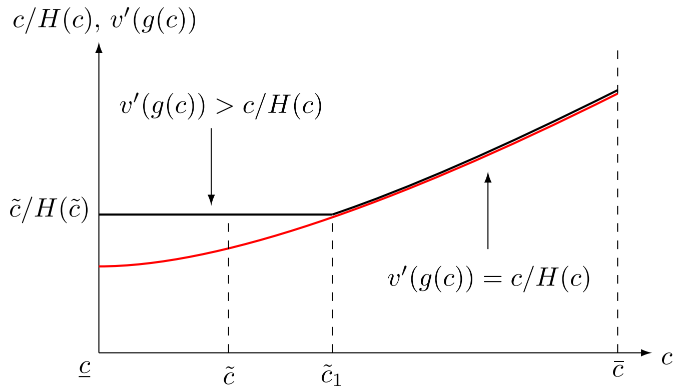

Figure 1: Multiple equilibria are possible even in the regular case.

3. Let $v'(g(c))$ remain flat at $v'(g(\tilde{c})) = \tilde{c}/H(\tilde{c})$ until $c = \underline{c}$ unless a further intersection with $c/H(c)$ occurs at, say, $c_z^* < \tilde{c}_z$,

4. Increase the counter z by 1 and repeat step 2

As depicted in Figure 2, the above procedure identifies the lowest $v'(g(c))$ in equilibrium and hence the largest possible $g(c)$.

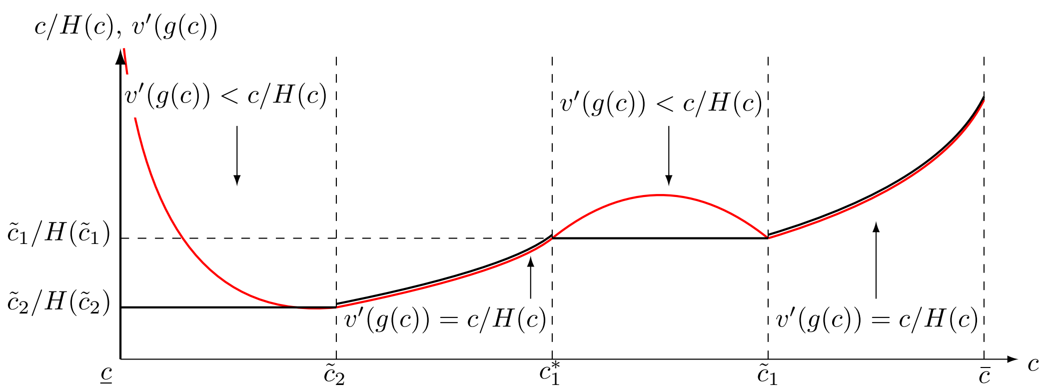

Figure 2: Need a caption here.

The above procedure maps out as the candidate $v'(g(c))$ for the best equilibrium the largest weakly increasing function that is bounded above by $c/H(c)$. Now it is easy to account for the situation where $v'(0) < \bar{c}$. The best equilibrium is found where $v'(g(c))$ is equal to the largest weakly increasing function bounded above by $\max\{c/H(c), v'(0)\}$.

The equilibrium identified by the previous procedure is especially important because it is the one that maximizes the interim expected payoff of any type. To see this, the next lemma restates the familiar incentive-compatibility result to fit our framework where the asymmetric information is about c, the marginal cost of contributions. We first note that, given a the symmetric strategy g, if player 1 has marginal cost c but acts as if his cost is c', his payoff will be

$$\begin{aligned} U(c' \mid c) &= -cg(c') + \int_{\underline{c}}^{c'} v(g(c'))\, h(\tilde{c})\ d\tilde{c} + \int_{c'}^{\bar{c}} v(g(\tilde{c}))\, h(\tilde{c})\ d\tilde{c} \\ &= -cg(c') + H(c')\, v(g(c')) + \int_{c'}^{\bar{c}} v(g(\tilde{c}))\, h(\tilde{c})\ d\tilde{c}. \end{aligned} \tag{8}$$

We now have the following.

Lemma 2 (Utility in equilibrium)**.** *Let the utility in any equilibrium be* $U^*(c) \equiv U(c' = c \mid c)$*, where* $U(c' \mid c)$ *is defined in* (8)*, for some equilibrium strategy* $g(c)$*. We then have*

$$U^*(c) = U^*(\bar{c}) + \int_c^{\bar{c}} g(s)\ ds.$$

By Lemma 2, interim utility is maximized by choosing the largest possible $g(c)$ and the largest $U^*(\bar{c})$. As discussed, the earlier procedure identifies the largest possible $g(c)$ in equilibrium. Moreover, in any symmetric equilibrium, type $\bar{c}$ is sure the quantity of public good provided is $g(\bar{c})$ for a payoff of $v(g(\bar{c})) - \bar{c}g(\bar{c})$. Therefore, and as prescribed by the previous procedure, $U^*(\bar{c})$ is maximized when $v'(g(\bar{c})) = \bar{c}$ (implicitly we have again used $v'(0) \geq \bar{c}$).

As our leading example of a value function we use $v(G) = G - \frac{1}{2}G^2$. In this case we see that, **assuming regularity**, the equilibrium contribution strategy is $g(c) = 1 - c/H(c)$. The associated equilibrium provision of the public good is therefore $\mathrm{E}[\, G_{\mathrm{WL}}\,] = \mathrm{E}\Big[\, g\left(c_1^{[n]}\right)\Big]$. The cdf of the maximum cost, $c_1^{[n]}$, is given by $M(t) = (F(t))^n$, with density $m(t) = n(F(t))^{n-1} f(t)$. Consequently,

$$\begin{aligned} \mathrm{E}[\, G_{\mathrm{WL}}\,] &= \int_0^1 g(t)\, m(t)\, dt && (9) \\ &= 1 - \int_0^1 \left(\frac{t}{(F(t))^{n-1}} \right) n(F(t))^{n-1} f(t)\, dt && (10) \\ &= 1 - n \int_0^1 t f(t)\, dt && (11) \\ &= 1 - n\mathrm{E}[\, c\,]\,. && (12) \end{aligned}$$

Example 1 (Power function costs)**.**

Suppose $F(c) = c^t$ on $[0, 1]$, where $t > 0$. Here $\mathrm{E}[\, c\,] = t/(t+1)$. Then $c/H(c) = c^{1-(n-1)t}$ is increasing if and only if $t < 1/(n-1)$, yielding the regular case. Suppose $v(G) = G - \frac{1}{2}G^2$. In the regular case, the equilibrium strategy is $g(c) = 1 - c^{1-(n-1)t}$, which is positive for all $c < 1$. Also, if $t < 1/(n-1)$, then the above analysis implies

$$\mathrm{E}[\,G_{\mathrm{WL}}\,] = 1 - n\left(\frac{t}{t+1}\right),$$

which approaches 0 as $t \uparrow 1/(n-1)$. □

We now investigate the efficiency properties of equilibrium. We show that no equilibrium is efficient, for any of the definitions of efficiency that are appropriate for environments with private information, as described by Holmström and Myerson (1983). Indeed, the main result of this section shows an alternative provision mechanism with an equilibrium allocation that *ex post* dominates any equilibrium allocation characterized in Proposition 3.

The alternative mechanism is very simple. First, each agent makes a report c_i' to the designer. Then, the public good produced G is such that $v'(G) = \max\{c_1', \ldots, c_n'\}$ and all players contribute G. For any equilibrium of the weakest-link game characterized in Proposition 3, let $u_i^{eq}(c_1, \ldots, c_n)$ denote the *ex post* payoff to player i; let $u_i^{alt}(c_1, \ldots, c_n)$ denote the *ex post* payoff to player i in the alternative mechanism described above, after truthful reporting by all players. We then obtain the following (the proof is in the Appendix).

Proposition 4 (Ex post incentive Pareto-dominance)**.** *The alternative mechanism is incentive compatible and individually rational. Moreover, $u_i^{alt}(c_1, \ldots, c_n) \geq u_i^{eq}(c_1, \ldots, c_n)$ for all $(c_1, \ldots, c_n)$, with strict inequality with positive probability.*

This Pareto improvement brought about by the alternative mechanism can be understood as follows. For each realized cost, c, a player has a most desirable level of the public good, namely, that for which $v'(G) = c$. The payoff $v(G) - cG$ is concave in G. The alternative mechanism has all players contribute to the level that funds the most-preferred level for the highest-cost player. All players are willing to contribute at least this much because all of their most-preferred levels of the public good are at least this large. In equilibrium, however, each player with $c < \bar{c}$ contributes less than necessary to fund his most-preferred level, so the realized level of the public good is less than the preferred level for the highest-cost player. Thus, for for almost all realizations of costs, the alternative mechanism moves the outcome closer to each player's most desired result, so that all players' payoffs increase.

4 Comparative statics

Here we explore how changes in the distribution of players' costs affects equilibrium strategies. One naturally expects that a shift to lower costs, in the sense of first-order stochastic dominance, would lead to contribution strategies that are greater, as for each realization of cost one believes the other players' costs are likely to be low and therefore they are more disposed toward greater contributions. Indeed, if F_2 first-order stochastically dominates F_1, then for any equilibrium strategy of the game with distribution of costs given by F_1, there is in the game with distribution F_2 an equilibrium strategy that is everywhere at least as large as when the distribution is F_1. This follows from the simple fact that for all c, $c/F_1(c) \le c/F_2(c)$, so the greatest weakly increasing function bounded above by $c/F_1(c)$ is everywhere no larger than the greatest weakly increasing function bounded above by $c/F_2(c)$.

Proposition 5 (First-order stochastic dominance)**.** *Suppose F_1 and F_2 are two distributions of cost, and suppose that $F_2(c) \le F_1(c)$ for all c, with strict inequality for some c. For contributors, the best equilibrium under F_1 is better than the best equilibrium under F_2.*

Proposition 6 (Single-crossing cdfs)**.** *Suppose there are n contributors. Suppose F_1 and F_2 are regular distributions and there exists $c_0 \in (\underline{c}, \bar{c})$ such that $F_2(c) < F_1(c)$ for $c \in (\underline{c}, c_0)$ and $F_2(c) > F_1(c)$ for $c \in (c_0, \bar{c})$. Let g_i denote the best equilibrium strategy under distribution F_i, $i = 1, 2$. Then $g_1(c) > g_2(c)$ for $c \in (\underline{c}, c_0)$ and $g_1(c) < g_2(c)$ for $c \in (c_0, \bar{c})$.*

The previous proposition includes the case where F_1 and F_2 are ordered by second-order stochastic dominance, with F_2 being less risky than F_1 and the cdfs crossing just once on $(\underline{c}, \bar{c})$. This general relationship does not extend to the case where distributions are not regular. In this case, the less-risky distribution may have a better or a worse best equilibrium, as illustrated by the next two examples. These examples rely on the beta distribution. Let $B(c \,|\, a, b)$ denote the cdf of a beta distribution on $[0, 1]$ with parameters a and b, so the mean of the distribution is $a/(a + b)$.

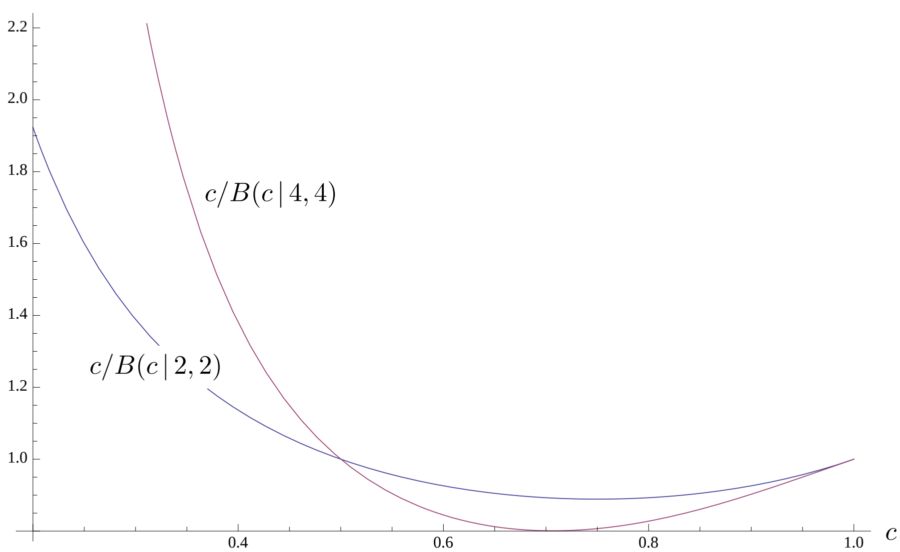

Figure 3: Graphs of $c/B(c \,|\, a, b)$ in Example 2

Example 2. *Consider the two beta distributions $B(c \,|\, 2, 2)$ and $B(c \,|\, 4, 4)$.*

These distributions are symmetric about 1/2, with $B(c\,|\,4,4)$ being less risky than $B(c\,|\,2,2)$. Over $[0,1/2)$, $c/B(c\,|\,a,b)$ exceeds 1 for both distribution. The following figure shows $c/B(c\,|\,2,2) > c/B(c\,|\,4,4)$ for all $c \in (1/2,1)$. Therefore the best equilibrium under the less risky distribution $B(\cdot\,|\,4,4)$ is better than the best equilibrium under distribution $B(\cdot\,|\,2,2)$. $\square$

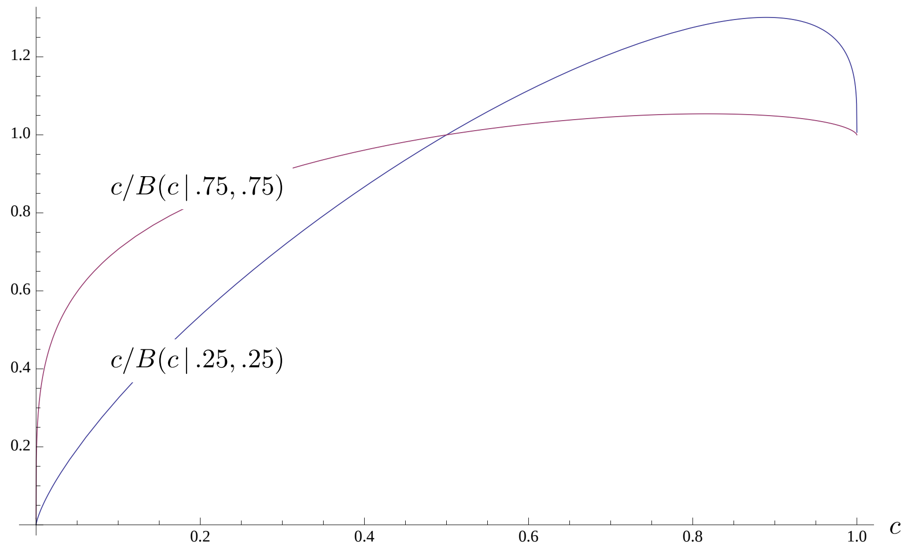

Figure 4: Graphs of $c/B(c\,|\,a,b)$ in Example 3

Example 3. *Consider the two beta distributions* $B(c\,|\,1/4,1/4)$ *and* $B(c\,|\,3/4,3/4)$.

These distributions are symmetric about 1/2, with $B(c\,|\,3/4,3/4)$ being less risky than $B(c\,|\,1/4,1/4)$. Over $(1/2,1)$, $c/B(c\,|\,a_i,b_i)$ exceeds 1, $i=1,2$; and, over $(0,1/2)$, $c/B(c\,|\,a_1,b_1)$ and $c/B(c\,|\,a_2,b_2)$ are strictly increasing with $c/B(c\,|\,.25,.25) < c/B(c\,|\,.75,.75)$. Therefore the best equilibrium under the less risky distribution $B(\cdot\,|\,.75,.75)$ is worse than the best equilibrium under distribution $B(\cdot\,|\,.25,.25)$. $\square$

4.1 Extension: Asymmetric Players

We now consider a two-player, asymmetric game. While we maintain the assumption that the support of marginal costs is $[\underline{c},\bar{c}]$ for each player, we now allow for distinct cumulative distribution of costs F_1 and F_2. Also, for simplicity we only focus on equilibrium strategies that are strictly decreasing. Therefore, fixing 2's strategy $g_2(c_2)$, type c_1's utility of contributing γ is

$$-c_1\,\gamma + v(\gamma)\ \Pr(g_2 \geq \gamma) + \int_{\{c_2:\ g_2(c_2)<\gamma\}} v(g_2(c_2))\ dF_2(c_2).$$

Denoting the inverse contribution function $\phi_2 \equiv g_2^{-1}$, and differentiating with respect to γ, we obtain the first-order condition

$$v'(\gamma)F_2(\phi_2(\gamma)) - c_1 = 0.$$

Replicating the same steps for Player 1 and noticing that, in equilibrium, $c_1 = \phi_1(g_1(c_1))$, we obtain this system of functional equations that characterizes equilibrium, for any equilibrium contribution level γ:

$$\begin{cases} v'(\gamma)F_2(\phi_2(\gamma)) - \phi_1(\gamma) = 0 \\ v'(\gamma)F_1(\phi_1(\gamma)) - \phi_2(\gamma) = 0. \end{cases} \tag{13}$$

We illustrate equilibrium in the next example, which reprises our earlier Example 1

Example 4 (Asymmetric Power function costs.)**.**

Suppose $F_1(c) = c^a$ and $F_2(c) = c^b$ on $[0,1]$, where $a < b$ and $ab < 1$. Also, suppose $v(G) = G - \frac{1}{2}G^2$. Condition (13) yields $(\phi_2(\gamma))^{1-ab} = (1-\gamma)^{a+1}$ and $(\phi_1(\gamma))^{1-ab} = (1-\gamma)^{b+1}$, thus, the equilibrium strategies are $g_1(c) = 1 - c^{\frac{1-ab}{1+b}}$, and $g_2(c) = 1 - c^{\frac{1-ab}{1+a}}$. As it is readily checked, we have $g_2(c) > g_1(c)$. Note that this occurs while F_2 first-order stochastically dominates F_1—which in our context means that costs for Player 2 are "larger." This is a reflection of the complementarity of efforts in the weakest-link framework: being matched with a worse partner makes a player contribute less. This interpretation is reinforced by the comparison to the symmetric case in Example 1, for which we take $F_1(c) = F_2(c) = c^a$ and we obtain an equilibrium contributions $g_0(c) = 1 - c^{1-a}$. Note that we obtain $g_2(c) < g_0(c)$. Indeed,

$$\begin{aligned} g_2(c) < g_0(c) &\iff 1 - c^{\frac{1-ab}{1+a}} < 1 - c^{1-a} \\ &\iff c^{1-a} < c^{\frac{1-ab}{1+a}} \\ &\iff \ln\left(c^{1-a}\right) < \ln\left(c^{\frac{1-ab}{1+a}}\right) \\ &\iff \frac{1-ab}{1+a} < 1-a && \text{(since } c<1\text{)} \\ &\iff 1-ab < 1-a^2 \\ &\iff a < b, \end{aligned}$$

and $a < b$ is assumed. Thus, the overall ranking of contribution functions is $g_0(c) > g_2(c) > g_1(c)$, which shows that, in response to a first-order-stochastic-dominance increase in the costs of Player 2, both players end up reducing their contributions. □

The patterns highlighted in Example 4 are general, as the next Proposition demonstrates.

Proposition 7 (Asymmetric first-order stochastic dominance shifts). *Consider the game where players' distributions are given by the pair (F_1, F_2) and denote the equilibrium contribution functions as g_1 and g_2. Now consider the game where players' distributions are given by the pair $(F_1, \hat{F}_2)$ and denote the equilibrium contribution functions as $\hat{g}_1$ and $\hat{g}_2$. Let $\hat{F}_2$ first-order stochastically dominate F_2. Then*

$$\hat{g}_1(c_1) < g_1(c_1) \text{ and } \hat{g}_2(c_2) < g_2(c_2).$$

Proof. We prove the proposition by showing that the types that contributes the same amount γ in the two games is smaller under $(F_1, \hat{F}_2)$. In other words, recalling our definition of inverse, we show $\hat{\phi}_1(\gamma) < \phi_1(\gamma)$ and $\hat{\phi}_2(\gamma) < \phi_2(\gamma)$. Since strategies are strictly decreasing, continuous, and end at the same level in both games—the level that maximizes (for g) $-\bar{c}g + v(g)$—this implies the conclusion of the proposition. We proceed graphically, first by fixing γ, and then by using the system of equations in (13) to define $\phi_1(\gamma)$ and $\phi_2(\gamma)$, as in Figure 4.1.

The first-order stochastic shift is represented by the curve in red. Clearly, each component of the solution of the system (13) becomes smaller. $\square$

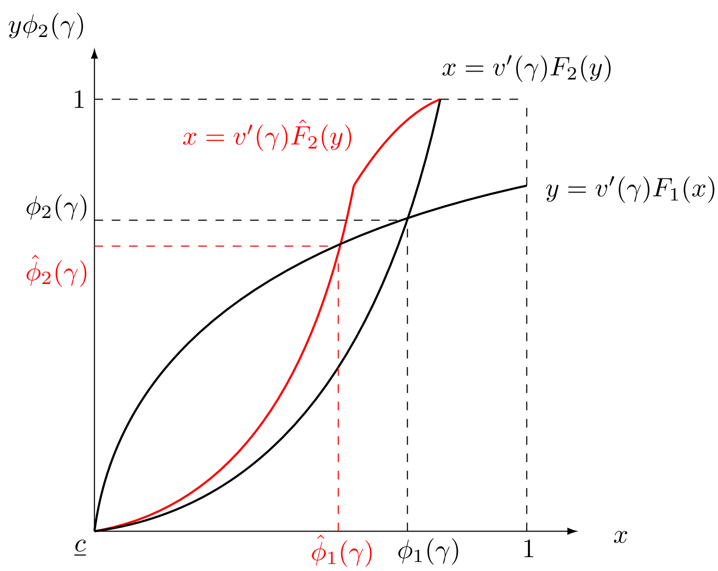

Figure 5: Graphs for the proof of Proposition 7.

5 Communication

We now turn to the issue of communication. We augment our basic game with a round of pre-play communication. Thus, the timing of the game is as follows:

1. Agents receive their private information;
2. Agents exchange non-binding messages;
3. Taking into account their private information, messages sent, and updating upon the messages received, agents make their contribution decisions; and then
4. Payoffs are received.

In this framework, the analysis in the previous sections corresponds to subgames starting at point 3. In particular, while we have shown that multiple equilibria exist, we focus on the best equilibrium, for simplicity and to avoid creating spurious multiplicity.

Similarly to Agastya *et al.* (2007), we consider two communication protocols: limited and unlimited. We cast limited communication in a framework meant to capture that, at times, it is impossible for agents to reveal all of their information in detail—for instance, the security of their sources may be compromised by such openness. We model this situation as one in which marginal costs are drawn from a favorable or an unfavorable distribution, and agents may communicate which distribution, but not the exact realization of the cost. We demonstrate that truthful revelation of information is not an equilibrium in the next example.

Example 5.

Consider two players and two possible distributions of the cost, F and $\hat{F}$, with F first-order stochasticallly dominated by $\hat{F}$. Each player first learns from which distribution its costs are drawn from. Then players may exchange a message identifying whether their cost distribution is "favorable" (F), or "unfavorable" ($\hat{F}$). Finally, agents receive their private information, observe the history of messages, and make their contribution decision. It turns out that truthful revelation of information is not possible. Indeed, it is an immediate consequence of Proposition 7 that Player 1 benefits from convincing 2 that its cost distribution is F rather than $\hat{F}$, since 2 will contribute more. This benefit accrues to 1 for any realization of its cost parameter. Thus, Player 2 cannot expect 1 to be truthful. □

We now turn to the case of unlimited communication, assuming that players' messages can be as specific to include their exact realization of the cost. In this case, it turns out that the results are surprisingly positive, as the next proposition demonstrates.

Proposition 8 (Full-revelation of information)**.** *There exists an equilibrium of the game with unlimited communication that implements the alternative mechanism allocation in Proposition 4, in which, corresponding to cost realizations* $(c_1, \ldots, c_n)$*, the level of public good produced is* G *is such that* $v'(G) = \max\{c_1, \ldots, c_n\}$ *and all players contribute* G*.*

Proof. The proof proceeds by backwards induction. First note that, assuming truthful, full revelation of information, not only is a contribution of G such that $v'(G) = \max\{c_1, \ldots, c_n\}$ an equilibrium of the full-information, weakest-link provision game; it is the best equilibrium, since we do not allow for transfers. This is the critical step of the argument. The rest of the argument follows as for Proposition 4. In particular, considering incentives at the interim-stage, note that this allocation is the same prescribed in the alternative mechanism allocation in Proposition 4. Thus, as demonstrated in the Proof of Proposition 4, and in particular in equation (19), truthful revelation of information is an equilibrium. □

In other words, Proposition 8 states that cheap-talk resolves all inefficiencies deriving from asymmetric information, as long as unlimited information revelation is possible.

Appendix

Proof of Lemma 1. We can rewrite type c_i's utility of contributing γ as

$$-c_i\,\gamma + v(\gamma)\prod_{j\neq i}\Pr(g_j \geq \gamma) + \sum_{j\neq i}\int_{\{c_j:\ g_j(c_j)<\gamma\}}\left[v(g_j(c_j))\cdot\prod_{s\neq i,j}\Pr(g_j(c_j) < g_s)\right] f(c_j)\ dc_j. \tag{14}$$

For a sufficiently small $\varepsilon > 0$ and for any j, g_j is either strictly decreasing when taking values in $(g_i(c_i) - \varepsilon, g_i(c_i))$, or there is no type c_j such that $s_j(c_j) \in (g_i(c_i) - \varepsilon, g_i(c_i))$. In the latter case, both the term $\Pr(g_j \geq \gamma)$ and the set $\{c_j :\ g_j(c_j) < \gamma\}$ are (locally) independent of γ, so they behave as constants in the first-order condition for optimality of $\gamma = g_i(c_i)$. In the former case, the inverse of g_j is locally well-defined. Denoting with J the set of players for whom this is the case, we can then rewrite (14) as

$$\begin{aligned}-c_i\,\gamma + v(\gamma)\prod_{j\neq i}\Pr(g_j \geq \gamma) &+ \sum_{j\neq i,\, j\notin J}\int_{\{c_j:\ g_j(c_j)<\gamma\}}\left[v(g_j(c_j))\cdot\prod_{s\neq i,j}\Pr(g_j(c_j) < g_s)\right] f(c_j)\ dc_j \\ &+ \sum_{j\neq i,\, j\in J}\int_{g_j^{-1}(\gamma)}^{\bar{c}_j}\left[v(g_j(c_j))\cdot\prod_{s\neq i,j}\Pr(g_j(c_j) < g_s)\right] f(c_j)\ dc_j,\end{aligned}$$

and the derivative with respect to γ of the above is

$$\begin{aligned}-c_i + v'(\gamma)\prod_{j\neq i}\Pr(g_j \geq \gamma) &+ \sum_{j\neq i,\, j\in J} v(\gamma) f(g_j^{-1}(\gamma))\frac{dg_j^{-1}(\gamma)}{d\gamma}\prod_{s\neq i,j}\Pr(g_s \geq \gamma) \\ &- \sum_{j\neq i,\, j\in J}\frac{dg_j^{-1}(\gamma)}{d\gamma}\left[v(g_j(g_j^{-1}(\gamma)))\cdot\prod_{s\neq i,j}\Pr\big(g_j(g_j^{-1}(\gamma)) < g_s\big)\right] f((g_j^{-1}(\gamma)).\end{aligned}$$

Thus, the necessary first-order condition from below simplifies as

$$-c_i + v'(\gamma)\prod_{j\neq i}\Pr(g_j \geq \gamma) \geq 0, \quad \forall\gamma \in (g_i(c_i) - \varepsilon, g_i(c_i)),$$

from which we derive (1). Clearly, if $g_i(c_i) = 0$, then the previously displayed condition is not applicable. The inequality in (2) can be derived along similar lines. □

Proof of Proposition 1. We prove this by showing that the best-reponse of type c_i to the equilibrium strategies $\{g_j\}_{j\neq i}$ contains only one value. By contradiction, suppose $\tilde{g} > \hat{g}$ are both optimal choices against $\{g_j\}_{j\neq i}$. Applying Lemma 1, and in particular (1) to $\tilde{g}$ and (2) to $\hat{g}$, we obtain

$$-c_i + v'(\tilde{g})\ \prod_{j\neq i}\lim_{\gamma\uparrow\tilde{g}}\Pr(g_j \geq \gamma) \geq -c_i + v'(\hat{g})\ \prod_{j\neq i}\lim_{\gamma\downarrow\hat{g}}\Pr(g_j \geq \gamma)$$

which is impossible unless $\prod_{j\neq i}\lim_{\gamma\downarrow\hat{g}}\Pr(g_j\geq\gamma)=\prod_{j\neq i}\lim_{\gamma\uparrow\tilde{g}}\Pr(g_j\geq\gamma)=0$, because $v'(\tilde{g})<v'(\hat{g})$ by strict concavity and because

$$\prod_{j\neq i}\lim_{\gamma\uparrow\tilde{g}}\Pr(g_j\geq\gamma)\leq\prod_{j\neq i}\lim_{\gamma\downarrow\hat{g}}\Pr(g_j\geq\gamma)\,,$$

since $\tilde{g}>\hat{g}$. If $\prod_{j\neq i}\lim_{\gamma\downarrow\hat{g}}\Pr(g_j\geq\gamma)=\prod_{j\neq i}\lim_{\gamma\uparrow\tilde{g}}\Pr(g_j\geq\gamma)=0$, it is immediate to verify that $\tilde{g}$ is not optimal: a small decrease in contribution leaves the expected value of the public good unchanged. □

Proof of Proposition 2. Without loss of generality, assume that the strategies of players 1 and 2 are not the same. There are two cases to consider. In the first, at the moment $\tilde{c}\geq\underline{c}$ in which g_1 and g_2 become different, both strategies are strictly decreasing. This implies that there exists an interval of contribution levels such as $\tilde{g}$ in Figure 5 in which the same contribution is made by two different types.

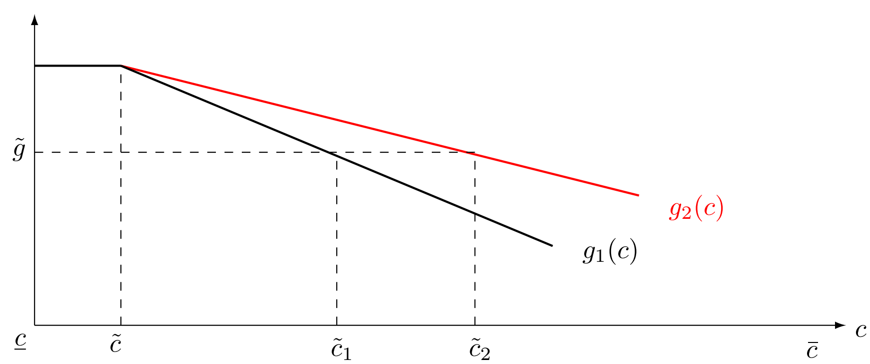

Figure 6: Both strategies are decreasing

Because the number of players is finite, we can choose $\tilde{g}$ so that no strategy of any player has a flat spot at $\tilde{g}$. Note how Lemma 1 leads to

$$v'(\tilde{g})F(\tilde{c}_2)\prod_{s>2}\Pr(g_s\geq\tilde{g})=\tilde{c}_1. \tag{15}$$

Indeed, the limits in (1) and (2) are identical as soon as there is no g_j that has a flat spot at $\tilde{g}$. Similarly, we obtain

$$v'(\tilde{g})F(\tilde{c}_1)\prod_{s>2}\Pr(g_s\geq\tilde{g})=\tilde{c}_2. \tag{16}$$

Equations (15) and (16) imply $F(\tilde{c}_1)\,\tilde{c}_1=F(\tilde{c}_2)\,\tilde{c}_2$, which is impossible because $F(c)\,c$ is strictly increasing and $\tilde{c}_1\neq\tilde{c}_2$. The previous argument can be extended to any segment of equilibrium strategies to conclude that there cannot exist a range of contributions such that equilibrium strategies are strictly decreasing and different. Therefore, in our quest for asymmetric equilibria, we can restrict attention to situations such that, at the moment in which the first divergence between strategies occurs, at least one strategy remains flat, as depicted in Figure 7 below, where g_1 and g_2 are flat at the beginning, but, for costs larger than $\tilde{c}_1$, g_2

becomes strictly decreasing at least up to $\tilde{c}_2$ while g_1 remains flat at least up to $\tilde{c}_2$.

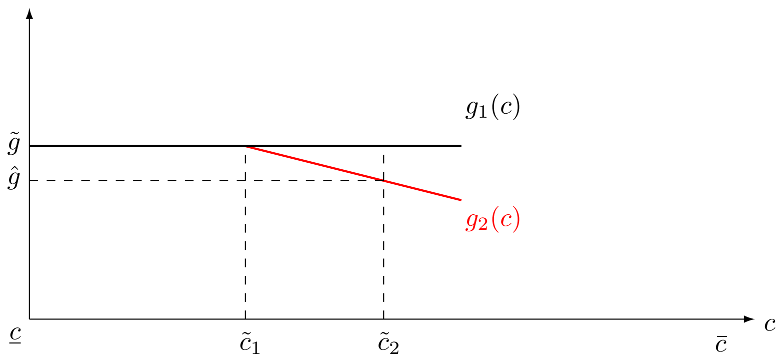

Figure 7: One strategy is locally constant and the other is decreasing.

Note that, by the previous discussion, there cannot be any contributions of player 1 between $\hat{g}$ and $\tilde{g}$, so by Proposition 1, this implies that player 1's strategy must be constant all the way up to $\bar{c}_1$. Now, applying equation (2) in Lemma 1 for type $\tilde{c}_2$ of player 2, we obtain

$$-\tilde{c}_2 + v'(\hat{g}) \prod_{j \neq 2} \lim_{\gamma \downarrow \hat{g}} \Pr(g_j \geq \gamma) \leq 0. \tag{17}$$

As for type $\tilde{c}_2$ of player 1, applying equation (1) in Lemma 1, we obtain the necessary condition

$$-\tilde{c}_2 + v'(\tilde{g}) \prod_{j \neq 1} \lim_{\gamma \uparrow \tilde{g}} \Pr(g_j \geq \gamma) \geq 0. \tag{18}$$

The conditions (17) and (18) are incompatible. To see this, observe that equations (17) and (18) imply

$$v'(\hat{g}) \prod_{j \neq 2} \lim_{\gamma \downarrow \hat{g}} \Pr(g_j \geq \gamma) \leq v'(\tilde{g}) \prod_{j \neq 1} \lim_{\gamma \uparrow \tilde{g}} \Pr(g_j \geq \gamma)\,,$$

which, because $v'(\hat{g}) > v'(\tilde{g})$, in turn implies

$$\prod_{j \neq 1} \lim_{\gamma \uparrow \tilde{g}} \Pr(g_j \geq \gamma) > \prod_{j \neq 2} \lim_{\gamma \downarrow \hat{g}} \Pr(g_j \geq \gamma)\,.$$

And this last inequality is contradicted by

$$\prod_{j \neq 1} \lim_{\gamma \uparrow \tilde{g}} \Pr(g_j \geq \gamma) \leq \prod_{j \neq 2} \lim_{\gamma \uparrow \tilde{g}} \Pr(g_j \geq \gamma) \leq \prod_{j \neq 2} \lim_{\gamma \downarrow \hat{g}} \Pr(g_j \geq \gamma)\,,$$

where the first inequality follows because g_1 is uniformly larger than g_2, and the second inequality follows because eventually all elements of the subsequence converging to $\tilde{g}$ are larger than all elements of the subsequence converging to $\hat{g}$, since $\tilde{g} > \hat{g}$. Therefore we can exclude the case depicted in Figure 7 as well, thus concluding the proof, because any other possible configuration would involve jumps, which are ruled out by Proposition 1. $\square$

Proof of Proposition 3. Necessity. Follows from Lemma 1. In particular, when g is strictly decreasing at c, the limits in (1) and (2) are the same, hence one obtains (6). *Sufficiency.* The derivative of $U(c' \,|\, c)$ (see (8)) with respect to c' is, almost always,

$$\frac{\partial U}{\partial c'} = g'(c')\left[-c + H(c')v'(g(c'))\right].$$

This, along with condition (6), implies that any deviation from $g(c)$ to a different contribution γ such that $\gamma = g(c')$ for some $c' \in [\underline{c}, \bar{c}]$ is not profitable. Indeed, consider for instance $c' > c$. We obtain

$$U(c' \,|\, c) = U(c' = c \,|\, c) + \int_c^{c'} g'(\tilde{c})\left[-c + H(\tilde{c})v'(g(\tilde{c}))\right] d\tilde{c}.$$

By condition (6), the integrand in the above expression is either zero (when $g'(\tilde{c}) = 0$), or it is strictly negative for any $\tilde{c} > c$:

$$g'(\tilde{c})\left[-c + H(\tilde{c})v'(g(\tilde{c}))\right] = g'(\tilde{c})\left[-c + \tilde{c}\right] < 0.$$

Therefore, since g is continuous, it follows that $U(c' \,|\, c) < U(c \,|\, c)$. The case $c' < c$ is similar. To conclude the proof, we now rule out deviations to contributions γ outside the range of g. Because of the weakest-link technology, it is obvious that if $\gamma > g(\underline{c})$, then a deviation to γ is dominated by one to $g(\underline{c})$. Consider now $\gamma < g(\bar{c})$. The utility of such deviation is $-c\gamma + v(\gamma)$, with derivative

$$-c + v'(\gamma) > -c + v'(\bar{c}) \geq 0,$$

where the last inequality follows from (3). Therefore, a deviation to $\gamma < g(\bar{c})$ is dominated by one to $g(\bar{c})$, thus concluding the proof of sufficiency. $\square$

Proof of Lemma 2. Using (8), we obtain

$$\begin{aligned}
dU^*(c)/dc &= -g(c) - cg'(c) + h(c)v(g(c)) + H(c)v'(g(c))g'(c) - v(g(c))h(c) \\
&= -g(c) - g'(c)(c - H(c)v'(g(c))) \\
&= -g(c),
\end{aligned}$$

where the last equality follows from Proposition 3, since almost everywhere either $g'(c) = 0$ or $g'(c) < 0$, in which case $v'(g(c))H(c) = c$. The lemma follows by integration of $dU^*(c)/dc = -g(c)$. $\square$

Proof of Proposition 4. For compactness, let $\phi \equiv (v')^{-1}$; so equilibrium contributions are $\phi(c/H(c))$ when strictly decreasing and the contributions in the alternative mechanism are $\phi(max\{c_1, \ldots, c_n\})$. We begin by showing incentive compatibility of the alternative mechanism. If all other agents are truthful, the interim expected utility of an agent of type c that announces c' is

$$U^{alt}(c'|c) = H(c')\,(v(\phi(c')) - c\phi(c')) + \int_{c'}^{\bar{c}} (v(\phi(\tilde{c})) - c\phi(\tilde{c}))\,h(\tilde{c})\;d\tilde{c},$$

so that

$$\frac{\partial U^{alt}(c'|c)}{\partial c'} = H(c')\,\phi'(c')\,(v'(\phi(c')) - c)\begin{cases} > 0 & \forall c' \in [\underline{c},\, c) \\ = 0 & \text{if } c' = c \\ < 0 & \forall c' \in (c,\, \bar{c}]\,, \end{cases} \tag{19}$$

where the signs follow because $\phi' < 0$ by strict concavity of v. Thus, truthful revelation is optimal and incentive compatibility is proven.

We now turn to *ex post* Pareto-dominance. We begin by establishing the following upper bound on equilibrium contributions:

$$\phi(c) > g(c)\;\forall c \in [\underline{c}, \bar{c}), \tag{20}$$

where g is any symmetric equilibrium strategy. To verify the condition in (20), it is convenient to work with the equivalent relation $c < v'(g(c))$. Note first that equation (6) in Proposition 3 yields the following implication:

$$g'(c) > 0 \Rightarrow v'(g(c)) = c/H(c) > c, \tag{21}$$

which, along with (3) and the continuity of g established in Proposition 1, implies that there exists $\varepsilon > 0$ such that

$$c < v'(g(c))\;\forall c \in (\bar{c} - \varepsilon, \bar{c}). \tag{22}$$

Next, by contradiction suppose $c^* \geq v'(g(c^*))$ for some c^*. Then, by continuity of g and by (22), there must exist $c^{**} > c^*$ such that $c^{**} \geq v'(g(c^{**}))$ and $g'(c^{**}) > 0$. The last two inequalities contradict (21), therefore condition (20) is verified and the upper bound on equilibrium contributions is established.

We conclude the proof of *ex post* Pareto-dominance focusing on player 1 for concreteness. Fix any realization of costs $(c_1, \ldots, c_n)$ and first suppose $c_1 \leq \tilde{c} < \bar{c}$. Then $u_1^{eq}(c_1, \ldots, c_n) = v(g(\tilde{c})) - c_1 g(c_1)$, so, because $g(\tilde{c}) \leq g(c_1)$, we obtain

$$u_1^{eq}(c_1, \ldots, c_n) \leq v(g(\tilde{c})) - c_1 g(\tilde{c}). \tag{23}$$

By $c_1 \leq \tilde{c} < \bar{c}$, we have

$$g(\tilde{c}) < \phi(\tilde{c}) \leq \phi(c_1), \tag{24}$$

where the first inequality follows from (20) and the second from v strictly concave. Note now that $v(\gamma) - c_1\gamma$ is strictly increasing in γ for $\gamma < \phi(c_1)$. Therefore, (23) and (24) yield

$$u_1^{eq}(c_1, \ldots, c_n) < v(\phi(\tilde{c})) - c_1\phi(\tilde{c}) = u_1^{alt}(c_1, \ldots, c_n).$$

Next suppose $\bar{c} > c_1 \geq \tilde{c}$. With a similar reasoning, we obtain

$$\begin{aligned} u_1^{eq}(c_1, \ldots, c_n) &= v(g(c_1)) - c_1 g(c_1) \\ &< v(\phi(c_1)) - c_1\phi(c_1) \\ &= u_1^{alt}(c_1, \ldots, c_n), \end{aligned}$$

thus concluding the proof that the alternative mechanism is *ex post* Pareto-dominant.

Finally, individual rationality of the alternative mechanism follows from individual rationality of the weakest-link equilibrium and from the *ex post* Pareto-dominance result above. □

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