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Dynamic tax competition and coordination under asymmetric productivity of public capital

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In this paper, we expand the static tax competition models in symmetric small regions, which were indicated by Zodrow and Mieszkowski (1986) and Wilson (1986), to a dynamic tax competition model in large regions, taking consideration of the regional asymmetry of productivity of public capital and the existence of capital accumulation. We then analyzed the consequence from tax competition and the impact to economic welfare by tax coordination. It is assumed that public capital contributed as a public input is formed on the basis of the capital tax of local governments. Supposing that it is under a situation with regional asymmetry in the productivity effects of public capital, it becomes clear from theoretical analysis that the regional welfares shall be improved under long-term steady state by raising the capital tax rate from the Nash equilibrium. Simulation analysis shows that there are more than one potential coordinated solutions. Moreover, the simulation taking consideration of transition process shows that there are potential coordinated solutions whose welfare becomes worse than the Nash equilibrium depending on the social time preference rate. It means that a transformation of tax coordination such as dropout or change of potential coordinated solutions occurs under an analysis with transition process.

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Dynamic Tax Competition and Coordination under Asymmetric Productivity of Public Capital

－A Welfare analysis using an overlapping generations model among two large regions－

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Abstract

In this paper, we expand the static tax competition models in symmetric small regions, which were indicated by Zodrow and Mieszkowski (1986) and Wilson (1986), to a dynamic tax competition model in large regions, taking consideration of the regional asymmetry of productivity of public capital and the existence of capital accumulation. We then analyzed the consequence from tax competition and the impact to economic welfare by tax coordination.

It is assumed that public capital contributed as a public input is formed on the basis of the capital tax of local governments. Supposing that it is under a situation with regional asymmetry in the productivity effects of public capital, it becomes clear from theoretical analysis that the regional welfares shall be improved under long-term steady state by raising the capital tax rate from the Nash equilibrium. Simulation analysis shows that there are more than one potential coordinated solutions. Moreover, the simulation taking consideration of transition process shows that there are potential coordinated solutions whose welfare becomes worse than the Nash equilibrium depending on the social time preference rate. It means that a transformation of tax coordination such as dropout or change of potential coordinated solutions occurs under an analysis with transition process.

As aforementioned, it was confirmed that there were candidates of coordinated solutions in the direction of the tax rate increase. It was also confirmed that shifts of such candidates depending on the social time preference rate of the local government were commonly acknowledged in both allocation methods of public capital rent, one where all the contributions to the production were imputed to the labor income and the other where those were imputed to the capital income. But, the welfare improvement level by the tax rate increase was higher in the case where the contributions were imputed to the labor income.

1. Introduction.

In tax competition theory started by Zodrow and Mieszkowski (1986) and Wilson (1986), the regional migration of private capital as an input and the stability of capital supply in homogeneous small regions and the entire economy were assumed on the premise that tax competition had led to undertax and underprovision of public goods. Since then, many qualitative and quantitative analyses have been attempted to clarify if a consequence from tax competition would change by modifying the premise of models, including Noisit and Oakland (1995), Matsumoto (1998), and Kellermann (2006) which all introduced public capital as a public input, Bucovetsky (1991) and Wilson (1991) which assumed regional heterogeneity by the population size, and Batina (2009) which adopted a dynamic framework with changing capital supply.

Tanaka and Hidaka (2010) used an overlapping generations model among two large regions with consideration of capital accumulation process while assuming different situations with various productivity effects of public capital in the public input between the regions, and made simulation analysis of the consequence of a dynamic tax competition. It made clear that there were cases when the optimal capital tax rate became zero and when it became positive by the taxation method imposed on older generations and younger generations on the assumption that different taxation rights had been allocated between the central government and local governments, and that there were more than one potential tax coordinated solutions that improved the economic welfare in both regions better than the Nash equilibrium.

Zodrow and Mieszkowski (1986) and Wilson (2010) showed the conclusion that tax competition (or tax coordination) would bring deterioration (or improvement) of welfare. Tanaka and Hidaka (2010) also confirmed the same conclusion with a dynamic tax competition model, however, the analysis was only made with the comparison of two steady states. It is thus possible that a different conclusion may be led with regards to the impact to welfare when policy variables change, when transition process to long-term steady state is taken into consideration.

It was assumed that public capital rent incorporated as a public input was imputed to the capital income in Tanaka and Hidaka (2010). Therefore, thorough consideration was not made about what impact the allocation method of rent would give to tax competition or tax coordination. By doing simulation analysis of transition process with multiple allocation methods of rent, it would be possible to clarify whether potential tax coordinated solutions would still exist even with transition process, whether the values of coordinated solutions would vary, or whether how to treat public capital rent would give any impact on the condition of dynamic tax coordination.

Here in this paper, based on the aforementioned awareness, we do qualitative analysis and simulation analysis using an overlapping generations model among two regions assuming that there is asymmetry in the productivity effects of public capital between the regions. We also consider what impact the competition and coordination over the capital tax between local governments would give to the regional economic welfare. We particularly focus on clarifying if multiple potential tax coordinated solutions which are shown to be actualized in long-term steady state would still become candidates for coordinated solutions in an analysis with transition process, and also if any difference would appear in the condition of tax coordination by the allocation method of public capital rent, either imputed to the labor income or to the capital one.

In Chapter 2, we review the related literature of theoretical analyses over tax competition, and lay out our position in this paper. In Chapter 3, we describe households of each region, the optimal action of firms, the market equilibrium and the object function and budget constraint of local governments with regards to the overlapping generations model among two regions used here, and also do qualitative analysis about the impact to welfare by capital tax coordination under steady state. Then in Chapter 4, we do simulation analysis about policy conclusion of capital tax coordination in long-term steady state and transition process on the premise of asymmetry of productivity effects between the regions. Finally, in Chapter 5, we summarize the conclusion in this paper, and discuss a few remaining issues for further analysis.

2. Related literature.

Many researches on fiscal competition theory have been done since late 1980s. The theory has evolved to a framework which clarifies the consequence of competition over various policy variables among local governments, including tax competition, expenditure competition, and redistribution competition. It has now been recognized as one of the major research area in public economics. Zodrow and Mieszkowski (1986) and Wilson (1986) marked the beginning on the basis of capital tax competition. Their studies were focused on tax-related regional migration such as capital or labor, and on what impacts competitive and uncooperative policy decisions by local governments might give on the regional public goods provision.

Zodrow and Mieszkowski (1986) and Wilson (1986) theoretically clarified that the capital tax competition among local governments under capital flow, in the economy where many homogeneous small regions exist, could induce undertax and underprovision of public goods, and consequently the reduction of resident welfare because local governments would tend to reduce the tax rate to avoid the private capital outflow from their regions.

Many researches have modified or expanded the capital tax competition model of Zodrow and Mieszkowski (1986) and Wilson (1986). Their main interests have been to clarify theoretical consequences of the capital tax competition in the following three cases: (1) when the regions are heterogeneous; (2) the capital amount of the entire economy fluctuates; and (3) when public capital (public goods) contributes to the improvement of local productivity as a public input.

Bucovetsky (1991), Wilson (1991) and Peralta and Ypersele have focused on the aforementioned case (1), and made theoretical analyses on the capital tax competition between two heterogeneous regions with different population size and different initial storage of private capital. According to their studies, coordinated actions between local governments over the tax rate and other issues have been different from the conclusion in homogeneous regional models which has shown Pareto improvement in both regions. Since there was a region that did not show Pareto improvement by coordinated action, the region with expected welfare deterioration would possibly remain at the Nash asymmetric equilibrium.

Batina (2009) and Shinozaki, Kato and Kunizaki (2010) have discussed the aforementioned case (2). These studies have relaxed assumption that the volume of capital supply in economy is constant, and have theoretically reviewed the consequence from dynamic tax competition and tax coordination between homogenous regions on the basis of the overlapping generations model considering the intertemporal choice of consumption and savings at different time points. Batina (2009) has used a horizontal capital tax competition model between local governments, and Shinozaki, Kato and Kunizaki (2010) has used a vertical capital tax competition model between the central and regional governments, to examine how the tax rate change would cause fluctuation of private capital and what impact it would give to economic welfare.

Noisit and Oakland (1995), Matsumoto (1998), Kellermann (2006) and (2007) have discussed the aforementioned case (3). Among these, Kellermann (2006) and (2007) have used the same overlapping generations model as Batina (2009) and Shinozaki, Kato and Kunizaki (2010), however, they have incorporated public capital as a public input and have made a theoretical analysis of dynamic capital tax competition within homogenous regions. They have suggested that there would be a possibility for capital tax competition within symmetric regions to induce inefficiency in resource allocation, even with the assumption that capital accumulation should exist. Their studies were based on an assumption of small regions where the fluctuation of capital tax rate would not give any influence on the rate of return on capital. Thus, they have not analyzed the consequence of capital tax competition in the cases when the interest rate is endogenously determined.

The research on capital tax competition, which was started by Zodrow and Mieszkowski (1986) and Wilson (1986) has extended from an analysis of symmetric equilibrium in homogeneous regions to that of asymmetric equilibrium in heterogeneous regions, and furthermore from a static framework without the choice of consumption and savings to a dynamic framework with such issue. Nevertheless, there are few researches using a dynamic capital tax competition model, which explicitly treated public capital as a public input. Tanaka and Hidaka (2010) is one of the very few studies that have dealt with a dynamic capital tax competition among large regions where the interest rate is endogenously determined. In addition, analyses using static models have been done only at long-term steady state, and transition process has not been taken into consideration.

Here in this paper, we focus on dynamic capital tax competition and tax coordination including the following three issues which have been individually analyzed in preceding studies: (1) heterogeneity between the regions; (2) existence of capital accumulation; and (3) productivity effect of public capital. More specifically, we use the overlapping generations model of Diamond (1965) and construct a dynamic capital tax competition model in asymmetric large regions incorporating public capital as a public input. We, on the basis of qualitative and simulation analysis, clarify how much impact capital tax competition and coordination would give to economic welfare.

3. Theoretical model.

In this paper, we give an analysis on tax competition based on the case where two regions procure public investment funds through capital tax. It is assumed that public capital provided for by public investment is used as a public input for each region. Representative firms in each region produce goods with public inputs of labor, private capital and public capital based on their own production techniques. Population, utility function of representative households and goods produced in both regions are identical. The two regions are differentiated only by the production technique.

We use the overlapping generations model of Diamond (1965) for a dynamic process of public as well as private capital accumulation, and expand the model to two regions by putting in public capital from governments as a public input. Each region has younger generations born in time t and older generations born in time t-1. When their populations are respectively L_t^i and L_{t-1}^i, and the population growth rate is n, the equation $L_t^i = (1+n)L_{t-1}^i$ is formed. The population in each region is equal and there is no regional migration. In what follows, we describe behaviors of firms and households and the market equilibrium, and then summarize the behaviors of local governments under tax competition and coordination.

3-1. Behaviors of firms.

Firms in region i (i = 1,2) produce goods (Y_t^i) using the linear homogeneous production function $F_t^i\left(L_t^i, K_t^i, G_t^i\right)$ by means of labor (L_t^i), private capital (K_t^i), and public capital (G_t^i) as inputs. Firms solve the following profit maximization problem with public capital (G_t^i) and production technique:

$$\underset{L_t^i, K_t^i}{Max} \quad \Pi_t^i = Y_t^i - w_t^i L_t^i - r_t^i K_t^i \tag{1}$$

$$s.t. \qquad Y_t^i = F^i\left(L_t^i, K_t^i, G_t^i\right) \tag{2}$$

w_t^i means the wage rate and r_t^i means the rate of return on capital. Due to the first order condition of profit maximization, $w_t^i = \partial F^i / \partial L_t^i$ and $r_t^i = \partial F^i / \partial K_t^i$ are derived. In accordance with the assumption of linear homogeneity of production function, $Y_t^i = \left(\partial F^i / \partial L_t^i\right) L_t^i + \left(\partial F^i / \partial K_t^i\right) K_t^i + \left(\partial F^i / \partial G_t^i\right) G_t^i$ are established and the profit shall be represented as $\Pi_t^i = \left(\partial F^i / \partial G_t^i\right) G_t^i$.

It is assumed, in this paper, that the profit here is distributed to labor L at the rate of ε and to capital K at the rate of $1-\varepsilon$ [1]. When the profit rate is ρ, $\rho_t^i = r_t^i + (1-\varepsilon)\Pi_t^i / K_t^i$ shall be formed. The income distribution shall be shown as $Y_t^i = w_t^i L_t^i + \rho_t^i K_t^i$. With the assumption that the labor supply is fixed similarly to the Diamond model, the production volume, private capital and public capital shall be described as 1 unit of labor. With the assumption of $f = F/L$, $g = G/L$, $y = Y/L$ and $k = K/L$, the rate of return on capital and income distribution shall be:

[1] It complies with Feehan and Batina (2007). It discussed that public capital rent was given as an exogenous distribution parameter, and that the impact such value would give to the labor force or private capital at the competition equilibrium. It also made an analysis on the relevence between the distribution method of rent and the optimal tax policy.

$$\rho_t^i = \frac{\partial f^i\left(k_t^i, g_t^i\right)}{\partial k_t^i} + (1-\varepsilon)\frac{\partial f^i\left(k_t^i, g_t^i\right)}{\partial g_t^i}\frac{g_t^i}{k_t^i} \tag{3}$$

$$y_t^i = f^i\left(k_t^i, g_t^i\right) = w_t^i + \rho_t^i k_t^i \tag{4}$$

Capital demand k_t^i can be represented with functions of g_t^i and ρ_t^i from equation (3) as follows:

$$k_t^i = k^i\left(\rho_t^i, g_t^i\right) \tag{5}$$

By assigning equation (5) to k_t^i of equation (4), wage rate w_t^i can be shown with functions of g_t^i and ρ_t^i as follows:

$$\begin{aligned} w_t^i &= f^i\left(k_t^i, g_t^i\right) - \rho_t^i k^i\left(\rho_t^i, g_t^i\right) \\ &= w^i\left(\rho_t^i, g_t^i\right) \end{aligned} \tag{6}$$

For the purpose of this paper, it is assumed that labor has no regional migration whereas capital freely moves between the regions. Capital tax shall be imposed on the rate of return on capital ρ_t^i with the tax rate τ_t^i in each region. When households in both regions decide where to invest by watching the rate of return on capital $\left(1-\tau_t^i\right)\rho_t^i$ after tax, the rate of return on capital after tax shall become equal as the result of arbitrage. It means that when the rate of return on capital is θ_t, the following equation shall be formed:

$$\theta_t = \left(1 - \tau_t^i\right)\rho_t^i \tag{7}$$

Using equation (7), the capital demand and wage rate shall be:

$$k_t^i = k^i\left(\frac{\theta_t}{1-\tau_t^i}, g_t^i\right) \tag{5'}$$

$$w_t^i = w^i\left(\frac{\theta_t}{1-\tau_t^i}, g_t^i\right) \quad (6)'$$

and shall be represented as the variable g_t^i of time t, and the function of θ_t and τ_t^i.

3-2. Behaviors of households.

It is assumed here that households in each region earn wage income w_t^i during earlier life, and that they make consumption c_t^{yi} during earlier life (time t) and make consumption c_{t+1}^{oi} during older life (time t+1). The budget constraint equations for each time are $s_t^i = w_t^i - c_t^{yi}$ and $c_{t+1}^{oi} = (1+\theta_{t+1})s_t^i$ when savings are s_t^i.

Assuming that households are facing the following utility maximization problem under diachronic budget constraints,

$$\underset{c_t^{yi}, c_{t+1}^{oi}}{Max} \quad u_t^i = u^i\left(c_t^{yi}, c_{t+1}^{oi}\right) \quad (8)$$

$$s.t. \quad w_t^i = c_t^{yi} + \frac{c_{t+1}^{oi}}{1+\theta_{t+1}} \quad (9)$$

The following consumption function and savings function for earlier life are derived by solving this utility maximization problem:

$$c_t^{yi} = c^{yi}\left(w_t^i, \theta_{t+1}\right) \quad (10)$$

$$\begin{aligned} s_t^i &= w_t^i - c^{yi}\left(w_t^i, \theta_{t+1}\right) \\ &= s^i\left(w_t^i, \theta_{t+1}\right) \end{aligned} \quad (11)$$

3-3. Market equilibrium.

Produced goods and private capital are transferable within the two regions. The equilibrium of capital market can be reached when the capital demand in both regions become equivalent to their capital supply, or $\sum_{i=1}^{2} L_t^i s_t^i = \sum_{i=1}^{2} K_{t+1}^i$.

By assigning equations (5)' and (11) to the capital demand and supply, the balance equation of capital market during time t+1 can be derived as follows:

$$\sum_{i=1}^{2} s^i\left(w_t^i, \theta_{t+1}\right) = \sum_{i=1}^{2} (1+n) k^i\left(\frac{\theta_{t+1}}{1-\tau_{t+1}^i}, g_{t+1}^i\right) \tag{12}$$

w_t^i means the function of variables $\left(\theta_t, \tau_t^i, g_t^i\right)$ of time t according to equation (6)'. g_{t+1}^i is determined by variables $\left(\tau_t^i, g_t^i, k_t^i, \rho_t^i\right)$ of time t according to equation (13). When policy variables $\left(\tau_t^i, \tau_{t+1}^i, g_t^i, g_{t+1}^i\right)$ are given, the equilibrium equation of capital market in equation (12) shall be shown as a dynamic system of $\left(\theta_t, \theta_{t+1}\right)$[2].

3-4. Behaviors of local governments.

Local governments here impose capital tax on firms in each region, divert it to the funds for public investment $IG_t^i = \tau^i{}_t \rho_t^i K_t^i$. Public capital increases by public investment as $G_{t+1}^i = G_t^i + IG_t^i$. The budget constraint equation of local governments, with public capital per worker, can be described as follows:

$$(1+n) g_{t+1}^i = g_t^i + \tau_t^i \rho_t^i k_t^i \tag{13}$$

Tax competition between local governments can be formulated as a maximization problem for the following social welfare function with constraint of equations (12) and (13):

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[2] When variables $\left(\tau_t^i, y_t^i, g_t^i, k_t^i, c_t^{yi}, c_t^{oi}, w_t^i, \theta_t\right)$ of time t are given and policy variable τ_{t+1}^i changes during time t+1, $y_{t+1}^i, k_{t+1}^i, g_{t+1}^i, c_{t+1}^{yi}, c_{t+1}^{oi}, w_{t+1}^i, \theta_{t+1}$ shall be determined to satisfy the equilibrium of goods and capital market.

$$\underset{\tau_t^i}{Max} \qquad SW_t^i = \sum_{t=0}^{T} \frac{u_t^i}{\left(1+\phi^i\right)^t} \tag{14}$$

ϕ^i here represents the social time preference rate. The bigger ϕ^i gets, the more shortsighted it becomes. τ_t^i, τ_t^j actualized at the equilibrium is the Nash equilibrium solution which local governments obtain through capital tax competition with mutual tax rate.

On the other hand, tax coordination between local governments can be formulated as a maximization problem for the following object function with constraint of equations (12) and (13), with the assumption that negotiations between the regions over coordination should be proceeded based on the Nash negotiation game:

$$\underset{\tau_t^i, \tau_t^j}{Max} \qquad \left(SW_t^i\right)^{\upsilon} \left(SW_t^j\right)^{1-\upsilon} \tag{15}$$

υ here represents the relative size of negotiating power of region 1. Coordinated solutions $\left(\tau^i, \tau^j\right)$ mean those for the aforementioned objective function, and shall be chosen from candidates for coordinated solutions which satisfy Pareto efficiency, in accordance with negotiating power υ.

3-5. Influence of tax coordination on welfare under steady state.

Does economic welfare in both regions increase under the Nash equilibrium solution when both regions take a coordinated action to raise tax rate τ^i from the Nash equilibrium solution actualized under long-term steady state as a result of capital tax competition? In what follows, we discuss the influence of tax coordination on welfare under long-term steady state on the basis of comparative statics.

When the utility function of households under steady state is described as the indirect utility function $v^i = v^i(w^i, \theta) = v^i(w(\rho^i, g^i), \theta)$, the following equation can be formed based on the Nash equilibrium solution $\tau_n = (\tau_n^i, \tau_n^j)$ under steady state:

$$\frac{dv^i}{d\tau_n^i} = v_w^i w_\rho^i \left(\frac{\partial \rho^i}{\partial \tau_n^i} + \frac{\partial \rho^i}{\partial \theta} \frac{\partial \theta}{\partial \tau_n^i} \right) + v_w^i w_g^i \frac{\partial g^i}{\partial \tau_n^i} + v_\theta^i \frac{\partial \theta}{\partial \tau_n^i} = 0 \tag{16}$$

When the coordinated solution is $\tau_c = (\tau_c^i, \tau_c^j)$ with the assumption that both regions raise their capital tax rate at the same time from the rate under the Nash equilibrium solution, the equation shall be:

$$\begin{aligned} dv^i = v_w^i w_\rho^i & \left\{ \left(\frac{\partial \rho^i}{\partial \tau_c^i} + \frac{\partial \rho^i}{\partial \theta} \frac{\partial \theta}{\partial \tau_c^i} \right) d\tau_c^i + \frac{\partial \rho^i}{\partial \theta} \frac{\partial \theta}{\partial \tau_c^j} d\tau_c^j \right\} \\ & + v_w^i w_g^i \left(\frac{\partial g^i}{\partial \tau_c^i} d\tau_c^i + \frac{\partial g^i}{\partial \tau_c^j} d\tau_c^j \right) + v_\theta^i \left(\frac{\partial \theta}{\partial \tau_c^i} d\tau_c^i + \frac{\partial \theta}{\partial \tau^j} d\tau_c^j \right) \end{aligned} \tag{17}$$

By using equation (16), equation (17) can be rewritten as follows, provided that $\frac{\partial \rho^i}{\partial \theta} = \frac{1}{1-\tau_c^i}$:

$$\frac{dv^i}{d\tau_c^j} = \left(v_w^i w_\rho^i \frac{1}{1-\tau_c^i} + v_\theta^i \right) \frac{\partial \theta}{\partial \tau_c^j} + v_w^i w_g^i \frac{\partial g^i}{\partial \tau_c^j} \tag{18}$$

Economic welfare in both regions will improve by tax coordination, when equation (18) is positive or $\frac{dv^i}{d\tau_c^j} > 0$.

As explained in Appendix B, $\frac{\partial g^i}{\partial \tau_c^j} > 0, \frac{\partial \theta}{\partial \tau_c^j} < 0$ shall be represented under long-term steady state. With $v_w^i > 0, v_\theta^i > 0, w_\rho^i < 0$, and based on the assumption of $w_g^i > 0$, the 3rd term of the right hand side will become positive. The sufficient condition for equation (18) to be positive will be:

$$v_w^i w_\rho^i \frac{1}{1-\tau_c^i} - v_\theta^i < 0 \tag{19}$$

From equations (3) and (4), $w^i_\rho = -k^i - k^i_\rho (1-\varepsilon) f^i_g \frac{g^i}{k^i}$. $v^i_\theta = v^i_w c^{oi} (1+\theta)^{-2} = v^i_w s^i (1+\theta)^{-1}$ can be formed according to Roy's identity. Equation (19) can be rewritten by using them as follows, provided that z^i is the capital outflow from region i which is described as $z^i = s^i - (1+n)k^i$:

$$v^i_w w^i_\rho \frac{1}{1-\tau^i_c} - v^i_\theta = \frac{v^i_w}{1+\theta}\left[\left(n-\rho^i\right)k^i - \frac{\tau^i_c}{1-\tau^i_c}k^i + z^i - (1-\varepsilon)\frac{1+\theta}{1-\tau^i_c}k^i_\rho f^i_g \frac{g^i}{k^i}\right] < 0 \qquad (20)$$

The 1st term of the right hand side of equation (20) shows the golden rule. It suggests that welfare improvement in region i should be expected by tax coordination raising the capital tax rate as long as economy satisfies dynamic efficiency ($n < \rho^i$). Also, it can be interpreted that welfare improves when the value is negative, or capital inflow, whereas welfare deteriorate when the value is positive, or capital outflow, by tax coordination, because the 3rd term of the right hand side represents the term which represents the influence the capital outflow of region i would give on welfare.

Moreover, the 4th term shall be positive as a whole due to $k^i_\rho < 0$. It can be interpreted that the effect to improve welfare by tax coordination increases as ε becomes bigger within the range under 1, since the absolute value of the 4th term becomes smaller.

4. Simulation analysis.

In this chapter, we give a simulation analysis on the consequence of dynamic tax competition using the overlapping generations model among two regions which was made in Chapter 3, based on the assumption that there is asymmetry in labor and productivity of public capital between the regions. A tax competition theory indicates that competition over the tax rate among local governments which have been authorized with the taxation right should cause tax externality under decentralized economy on the premise of tax-based regional migration, and that it leads to undertax and underprovision of public goods.

Now, we make two cases of simulation to clarify the impact of dynamic tax competition and tax coordination, with asymmetry of productivity effect, on economic welfare of both regions, in accordance with the consequence of the above tax competition theory[3].

First in 4-1, we evaluate multiple potential coordinated solutions[4] which can be realized by establishing the Nash equilibrium solution of capital tax competition under long-term steady state and tax coordination. We move on to discuss the transition of economic welfare within the two regions during transition process, and then we point out that shifting from the Nash equilibrium solution to a potential tax coordinated solution can deteriorate economic welfare in both regions at the initial point.

In 4-2, we focus on the transition process of capital tax competition, and attempt to clarify whether the multiple potential tax coordinated solutions evaluated in 4-1 can still become candidates for coordinated solution by a simulation analysis per time horizon. When regional governments respectively take a shortsighted case ($\phi = 0.075, 0.100$), all potential tax coordinated solutions under long-term steady state at both $\varepsilon = 0$ and $\varepsilon = 1$ drop out of the candidates. When they do not take a shortsighted case ($\phi = 0.000, 0.025, 0.050$), such coordinated solutions under long-term steady state at both $\varepsilon = 0$ and $\varepsilon = 1$ shrink or disappear, or another new coordinated solutions sometimes appear. We also discuss these cases hereinafter.

Upon the simulation analysis of transition process, we specify production function and utility function as shown below. As for production function, it is assumed to be a Cobb-Douglas function and $F^i(L_t^i, K_t^i, G_t^i) = A^i (L_t^i)^{\beta^{L^i}} (K_t^i)^{\beta^{K^i}} (G_t^i)^{1-\beta^{L^i}-\beta^{K^i}}$. A^i here means a scale parameter. As for utility function, it is assumed that the elasticity of alternatives at different time points should be constant, and it is specified as

[3] Empirical analyses, including Kawasaki (2007), intended to evaluate the productivity of each region have indicated that there is a larger gap among regions in Japan in labor productivity and public capital productivity than in private capital productivity. Productivity gap can usually be resolved through regional migration of input, however, the regional migration of labor is considered to be less active than that of private capital in Japan. It maybe interpreted that these empirical analyses show the reality that there has been a gap in labor productivity among regions. With this in mind, we discuss the cases with different productivity in labor and public capital.

[4] In this paper, the combination of capital tax rate is called "potential tax coordinated solution," which possibly realizes tax coordination through negotiation between local governments.

$u^i(c_t^{yi}, c_{t+1}^{oi}) = \frac{(c_t^{yi})^{1-1/\mu^i} - 1}{1 - 1/\mu^i} + \frac{1}{1+\delta^i}\frac{(c_t^{oi})^{1-1/\mu^i} - 1}{1 - 1/\mu^i}$. It is assumed that social time preference rate ϕ for calculating discounted utility in both regions should be a value in the range 0.000～0.010 based on the above function form and $n^1 = n^2 = 1, \delta^1 = \delta^2 = 1, \mu^1 = \mu^2 = 1, A^1 = A^2 = 100$ to simplify.

It will be discussed in the simulation below that what impact would be given to welfare of both regions through the shift of economic variables by the change of capital tax rate. Transition process will also be analyzed. The analysis will be made with the assumption that the period before any policy change is the initial steady state and it continues to time 0. Policy change is made after time 1, and gives a change to discounted utility of each time. The new steady state achieved by such policy change is called long-term steady state. It will be followed by a welfare comparison with simulation analysis considering transition process.

4-1. Capital tax Competition and coordination under steady state.

First, we discuss the consequence of capital tax competition under long-term steady state. The simulation below shows that labor and public capital productivities are asymmetric in the two regions. It is assumed that the elasticity value of public capital against RGP in region 1 is relatively high, whereas that of labor is relatively low. Parameter of production function in region 1 is $\beta^{L1} = 0.6, \beta^{K1} = 0.3, \beta^{G1} = 0.1$ whereas it is $\beta^{L2} = 0.65, \beta^{K2} = 0.3, \beta^{G2} = 0.05$ in region 2. As for Japan, region 1 would be an urban area, and region 2 would be a rural area[5].

Figures 4-1-1 and 4-1-2 show the Nash equilibrium solution within asymmetric regions with different productivity and the Pareto improving zone, where welfare in both regions improves from the Nash equilibrium solution, respectively in the cases of $\varepsilon = 0$ and $\varepsilon = 1$. In both cases, undertax and underprovision of public capital occur after capital tax competition, and the Nash equilibrium solution which deteriorates

[5] It has been indicated in Honma and Tanaka (2004) using an empirical analysis that the elasticity value of public capital against RGP in urban areas is 0.22, which is higher than that in rural areas (0.06).

economic welfare in both regions appears. Accordingly, it shows a simulation result which is consistent with the consequence of tax competition theory suggesting inefficiency of a decentralized policy decision.

According to Figures 4-1-1 and 4-1-2, multiple combinations of capital tax rate are found to improve economic welfare in both regions better than Nash equilibrium solution, and there are seventeen (17) in the case $\varepsilon = 0$ and twenty six (26) in the case $\varepsilon = 1$. Appendix D discusses the influence of the difference of ε over the welfare improving effect by tax coordination. It suggests that the welfare improving effect of $\varepsilon = 1$ exceeds that of $\varepsilon = 0$, when comparing the two cases[6]. After the simulation, the Pareto improving zone of $\varepsilon = 1$ becomes broader than that of $\varepsilon = 0$. It is consistent with the result of qualitative analysis.

Figure 4-1-1. Nash equilibrium solution and Pareto improving zone under long-term steady state (ε=0).

			τ2									
			0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
u1	τ1	0.3	8.208	8.211	8.215	8.219	8.223	8.227	8.231	8.235	8.239	8.243
		0.32	8.208	Nash;8.212	8.216	8.220	8.224	8.229	8.233	8.237	8.241	8.245
		0.34	8.208	8.212	CT1;8.216	CT2;8.220	8.224	8.228	8.233	8.237	8.241	8.245
		0.36	8.205	8.209	CT3;8.214	CT4;8.218	CT5;8.222	8.227	8.231	8.235	8.240	8.244
		0.38	8.202	8.206	8.210	CT7;8.215	CT8;8.219	CT9;8.224	8.228	8.233	8.237	8.242
		0.4	8.197	8.201	8.206	8.210	CT11;8.215	CT12;8.219	CT13;8.224	8.229	8.233	8.238
		0.42	8.190	8.195	8.200	8.204	8.209	CT15;8.214	CT16;8.219	CT17;8.223	8.228	8.233
		0.44	8.183	8.187	8.192	8.197	8.202	8.207	8.212	CT20;8.217	CT21;8.222	8.227
		0.46	8.173	8.178	8.183	8.188	8.193	8.198	8.204	8.209	CT24;8.214	8.219
		0.48	8.163	8.168	8.173	8.178	8.183	8.189	8.194	8.199	8.205	8.210
			τ2									
			0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
u2	τ1	0.3	7.805	7.806	7.804	7.801	7.796	7.790	7.783	7.775	7.766	7.756
		0.32	7.811	Nash;7.811	7.810	7.806	7.802	7.796	7.789	7.781	7.772	7.762
		0.34	7.816	7.817	CT1;7.815	CT2;7.812	7.808	7.802	7.795	7.787	7.778	7.768
		0.36	7.822	7.822	CT3;7.821	CT4;7.818	CT5;7.814	7.808	7.802	7.794	7.785	7.775
		0.38	7.827	7.828	7.827	CT7;7.824	CT8;7.820	CT9;7.815	7.808	7.800	7.792	7.782
		0.4	7.833	7.834	7.833	7.830	CT11;7.826	CT12;7.821	CT13;7.814	7.807	7.798	7.789
		0.42	7.839	7.840	7.839	7.836	7.833	CT15;7.827	CT16;7.821	CT17;7.814	7.805	7.796
		0.44	7.845	7.846	7.845	7.843	7.839	7.834	7.828	CT20;7.821	CT21;7.812	7.803
		0.46	7.851	7.852	7.851	7.849	7.846	7.841	7.835	7.828	CT24;7.820	7.810
		0.48	7.857	7.858	7.858	7.856	7.852	7.847	7.842	7.835	7.827	7.818
			τ2									
			0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
u1+u2	τ1	0.3	16.013	16.017	16.019	16.020	16.019	16.018	16.015	16.010	16.005	15.999
		0.32	16.019	Nash;16.023	16.026	16.027	16.026	16.025	16.022	16.018	16.013	16.007
		0.34	16.024	16.028	CT1;16.031	CT2;16.032	16.032	16.031	16.028	16.024	16.019	16.014
		0.36	16.027	16.032	CT3;16.035	CT4;16.036	CT5;16.036	16.035	16.033	16.029	16.025	16.019
		0.38	16.029	16.034	16.037	CT7;16.039	CT8;16.039	CT9;16.038	16.036	16.033	16.029	16.024
		0.4	16.030	16.035	16.038	16.040	CT11;16.041	CT12;16.040	CT13;16.039	16.036	16.032	16.027
		0.42	16.029	16.035	16.038	16.041	16.041	CT15;16.041	CT16;16.040	CT17;16.037	16.034	16.029
		0.44	16.028	16.033	16.037	16.040	16.041	16.041	16.040	CT20;16.037	CT21;16.034	16.030
		0.46	16.024	16.030	16.035	16.037	16.039	16.039	16.038	16.036	CT24;16.034	16.030
		0.48	16.020	16.026	16.031	16.034	16.035	16.036	16.036	16.034	16.032	16.028

6 As shown in 3-5 and Appendix D, it is generally suggested that when $\varepsilon(0 \leq \varepsilon \leq 1)$ is larger, welfare improving effect of tax coordination by raising the tax rate becomes greater as well.

Figure 4-1-2. Nash equilibrium solution and Pareto improving zone under long-term steady state (ε=1).

u1	τ1	τ2: 0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
	0.3	8.289	8.294	8.299	8.304	8.309	8.314	8.319	8.325	8.330	8.335
	0.32	8.290	Nash; 8.295	8.300	8.305	8.310	8.315	8.321	8.326	8.332	8.337
	0.34	8.289	8.294	CT1; 8.299	CT2; 8.304	8.310	8.315	8.321	8.326	8.332	8.337
	0.36	8.286	8.291	CT3; 8.297	CT4; 8.302	CT5; 8.308	CT6; 8.313	8.319	8.325	8.330	8.336
	0.38	8.282	8.288	8.293	CT7; 8.299	CT8; 8.304	CT9; 8.310	CT10; 8.316	8.322	8.328	8.334
	0.4	8.277	8.283	8.288	8.294	CT11; 8.300	CT12; 8.306	CT13; 8.311	CT14; 8.317	8.324	8.330
	0.42	8.270	8.276	8.282	8.288	8.293	CT15; 8.300	CT16; 8.306	CT17; 8.312	CT18; 8.318	8.325
	0.44	8.262	8.268	8.274	8.280	8.286	8.292	CT19; 8.299	CT20; 8.305	CT21; 8.311	CT22; 8.318
	0.46	8.252	8.258	8.264	8.271	8.277	8.283	8.290	CT23; 8.297	CT24; 8.303	CT25; 8.310
	0.48	8.241	8.247	8.254	8.260	8.267	8.273	8.280	8.287	8.294	CT26; 8.301

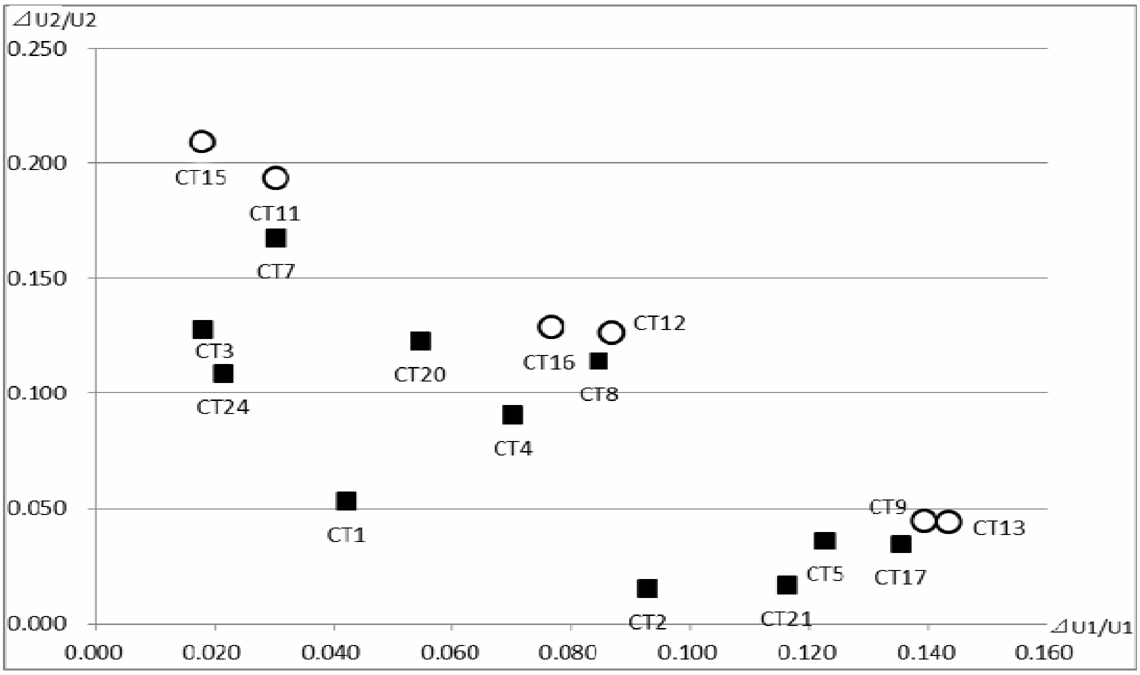
u2	τ1	τ2: 0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
	0.3	7.910	7.911	7.911	7.908	7.905	7.900	7.894	7.887	7.879	7.870
	0.32	7.915	Nash; 7.917	7.916	7.914	7.911	7.906	7.900	7.893	7.885	7.876
	0.34	7.921	7.922	CT1; 7.922	CT2; 7.920	7.916	7.912	7.906	7.899	7.892	7.883
	0.36	7.926	7.928	CT3; 7.927	CT4; 7.926	CT5; 7.922	CT6; 7.918	7.912	7.906	7.898	7.889
	0.38	7.932	7.933	7.933	CT7; 7.931	CT8; 7.928	CT9; 7.924	CT10; 7.919	7.912	7.905	7.896
	0.4	7.937	7.939	7.939	7.937	CT11; 7.934	CT12; 7.930	CT13; 7.925	CT14; 7.919	7.911	7.903
	0.42	7.943	7.945	7.945	7.943	7.941	CT15; 7.937	CT16; 7.932	CT17; 7.925	CT18; 7.918	7.910
	0.44	7.949	7.951	7.951	7.949	7.947	7.943	CT19; 7.938	CT20; 7.932	CT21; 7.925	CT22; 7.917
	0.46	7.954	7.956	7.957	7.956	7.953	7.949	7.945	CT23; 7.939	CT24; 7.932	CT25; 7.924
	0.48	7.960	7.962	7.963	7.962	7.959	7.956	7.951	7.946	7.939	CT26; 7.932

u1+u2	τ1	τ2: 0.16	0.18	0.2	0.22	0.24	0.26	0.28	0.3	0.32	0.34
	0.3	16.199	16.205	16.210	16.212	16.214	16.214	16.214	16.212	16.209	16.205
	0.32	16.205	Nash; 16.211	16.216	16.219	16.221	16.221	16.221	16.219	16.217	16.213
	0.34	16.210	16.216	CT1; 16.221	CT2; 16.224	16.226	16.227	16.227	16.226	16.223	16.220
	0.36	16.213	16.219	CT3; 16.224	CT4; 16.228	CT5; 16.230	CT6; 16.231	16.231	16.230	16.228	16.226
	0.38	16.214	16.221	16.226	CT7; 16.230	CT8; 16.233	CT9; 16.234	CT10; 16.235	16.234	16.232	16.230
	0.4	16.214	16.222	16.227	16.231	CT11; 16.234	CT12; 16.236	CT13; 16.237	CT14; 16.236	16.235	16.233
	0.42	16.213	16.221	16.227	16.231	16.234	CT15; 16.236	CT16; 16.237	CT17; 16.237	CT18; 16.236	16.235
	0.44	16.211	16.218	16.225	16.229	16.233	16.235	CT19; 16.237	CT20; 16.237	CT21; 16.237	CT22; 16.235
	0.46	16.207	16.215	16.221	16.226	16.230	16.233	16.235	CT23; 16.236	CT24; 16.235	CT25; 16.234
	0.48	16.201	16.210	16.216	16.222	16.226	16.229	16.231	16.233	16.233	CT26; 16.232

Note 1: The values within the figures show those of economic welfare.
Note 2: Double solid lines show Nash equilibrium solution, colored areas show the zones where welfare in both regions improve higher than Nash equilibrium solution (Pareto improving zone), and bold lines represent potential tax coordinated solution which can be actualized by tax coordination.

In the Pareto improving zone, there is a possibility for mutual welfare to improve by a coordinated action of local governments. It is thus suggested that the local governments have an incentive to drop out from the Nash equilibrium solution and promote tax coordination. A question here is which combination would become potential tax coordinated solution under long-term steady state, among seventeen (17) at $\varepsilon=0$ and twenty six (26) at $\varepsilon=1$ of Pareto improving capital tax rate combinations. The answer is shown at Figures 4-1-3 and 4-1-4.

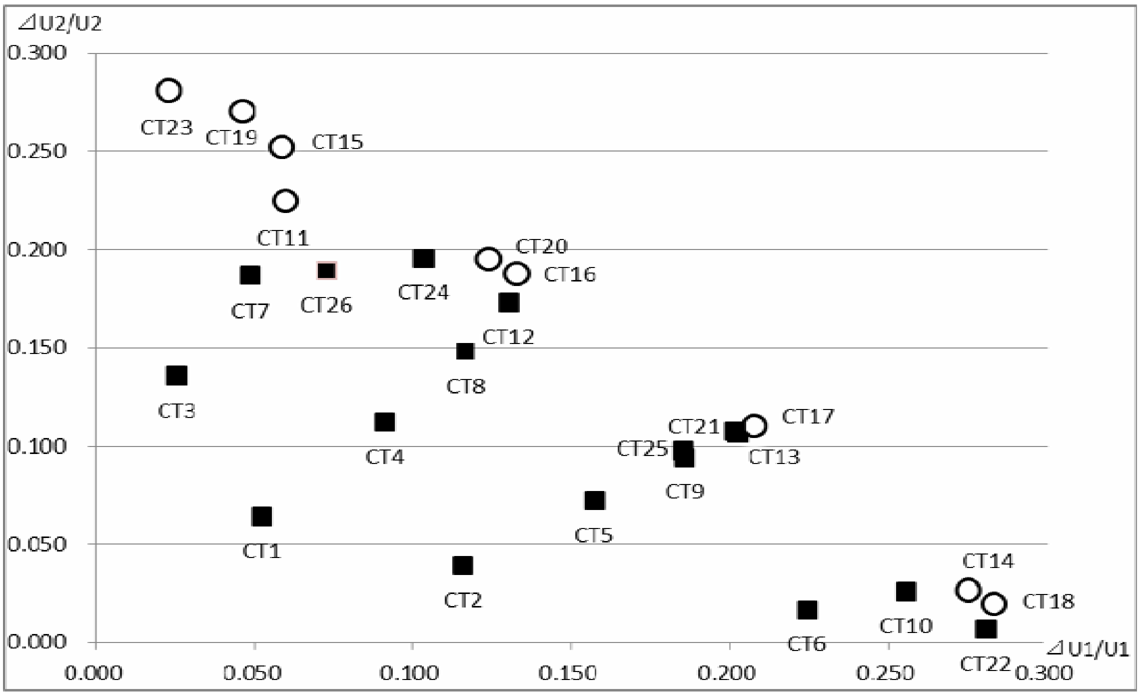
Figure 4-1-3. Potential tax coordinated solution under long-term steady state (ε=0).



Note 1: Items within the figure correspond to the tax rates shown in Figure 4-1-1.
Note 2: ○ represents potential tax coordinated solution.

The vertical and horizontal axes in both figures respectively indicate the percentage of the welfare change ($\Delta u^1/u^1, \Delta u^2/u^2$) in both regions by shifting from the Nash equilibrium solution to the Pareto improving zone. It shows that welfare improves further towards the upper right. Among seventeen (17) or twenty six (26) combinations of Pareto improving tax rates, six (6) combinations at the case $\varepsilon = 0$, namely CT9, CT11, CT12, CT13, CT15, and CT16, and nine (9) at the case $\varepsilon = 1$, namely CT11, CT14, CT15, CT16, CT17, CT18, CT19, CT20, and CT23, are found farthest from the point of origin.

Figure 4-1-4. Potential tax coordinated solution under long-term steady state (ε=1).



Note 1: Items within the figure correspond to the tax rates shown in Figure 4-1-2.
Note 2: ○ represents potential tax coordinated solution.

These six (6) or nine (9) combinations show better Pareto improvement than Nash equilibrium solution. It means that negotiations with any combination of these have a high possibility to achieve tax coordination, and these are the combinations which have relatively high welfare improving effect.

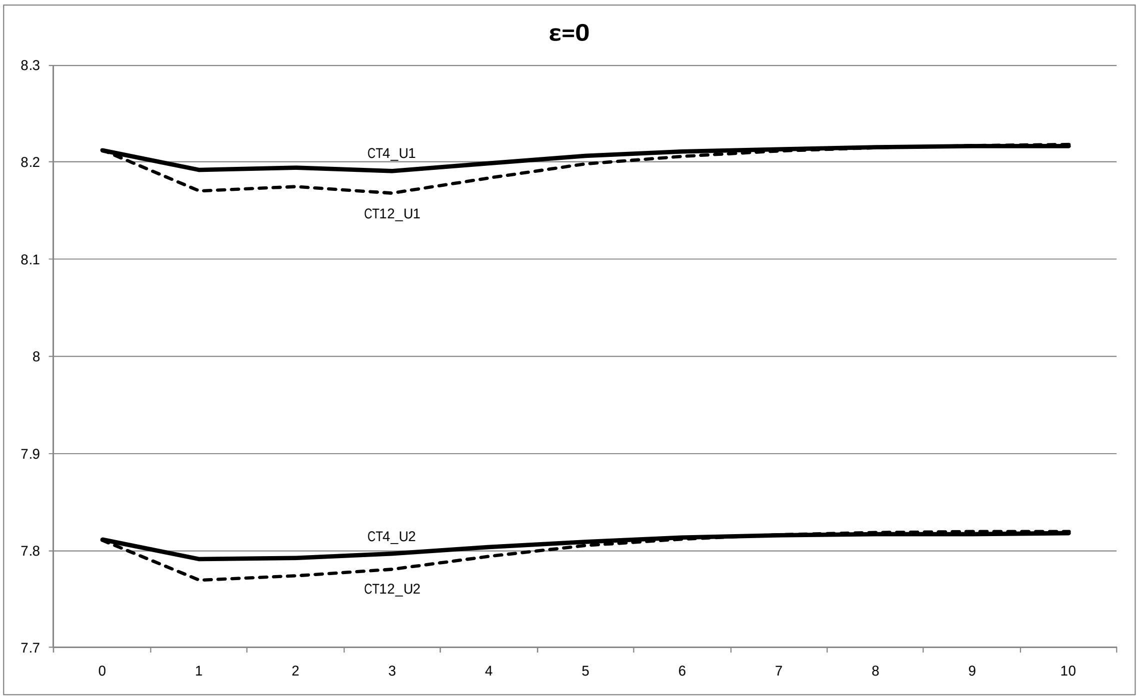
Furthermore, there is no relative merit in resource allocation among six (2) or nine (9) combinations on the nearly same line. It can lead to suggest that these six (6) at the case $\varepsilon = 0$ and nine (9) at the case $\varepsilon = 1$ could become potential tax coordinated solutions[7]. It depends on negotiation power υ, however, as to which combination would become the coordinated solution ultimately.

Can they also become candidates for coordinated solutions in transition process? We here review the transition of economic welfare in the two regions and analogize the tax coordination during transition process as a preliminary consideration, before discussing details in 4-2 using a simulation analysis targeted for transition process.

[7] It can be assumed that the utility of regions 1 and 2 should be found nearly on the same utility possibility frontier. Thus, it is not possible to determine which is superior among six (6) or nine (9) potential coordinated solutions from the standpoint of the efficiency of resource allocation.

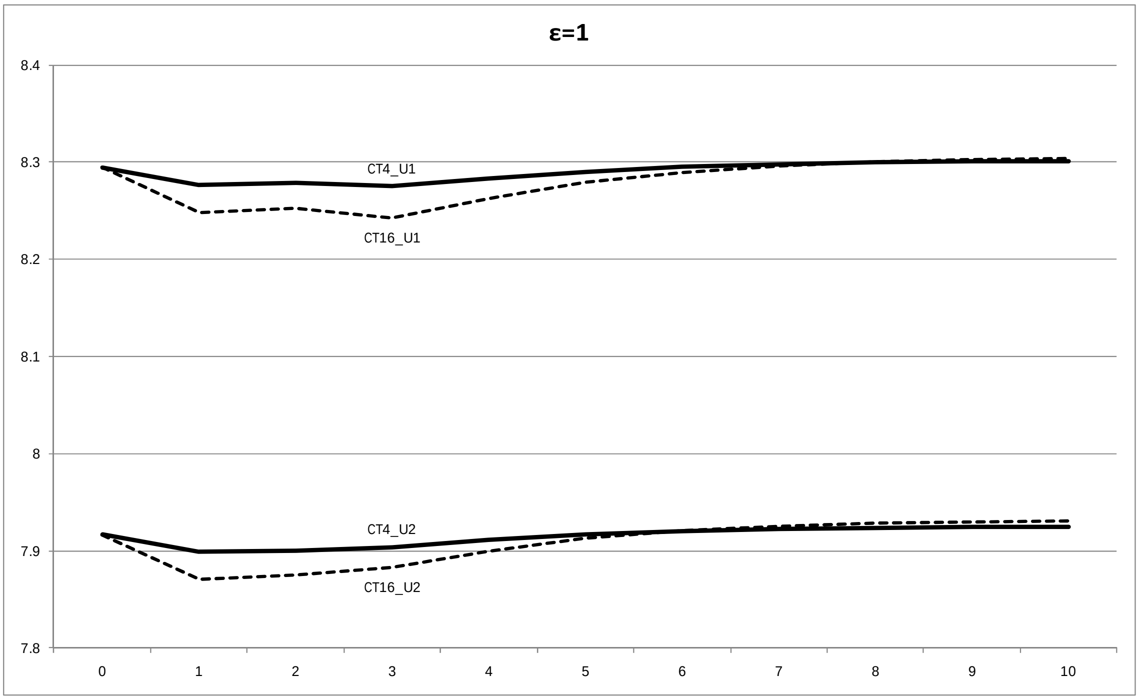
Figures 4-1-5 and 4-1-6 show the transition of economic welfare in regions 1 and 2 during transition process (T=10). According to the figures, it seems clear that the shift from Nash equilibrium solution to potential coordinated solution can deteriorate[8] the welfare of both regions at the initial point. The reason why welfare deteriorates at the initial point can be explained by that private capital temporarily decreases and productivity declines, then income and consumption of households decrease because the shift to the Pareto improving zone makes the tax rate go higher than the Nash equilibrium solution.

Figure 4-1-5. Transition of economic welfare by the region during transition process ($\varepsilon = 0, T = 10$)



[8] It has been confirmed that the utility at the initial point decreases in other potential coordinated solutions, although typical cases are shown here as CT4 and CT12 for the case $\varepsilon = 0$ and CT4 and CT 16 for the case $\varepsilon = 1$.

Figure 4-1-6. Transition of economic welfare by the region during transition process ($\varepsilon = 1, T = 10$)



The decline of welfare is milder at the initial point of CT4 than CT12 or CT16. The reason can be explained that relative range of capital tax rate increase from Nash equilibrium solution is narrower at CT4, and the initial drop of capital accumulation is smaller. Considering the decline of welfare at the initial point along with tax coordination, there maybe no potential tax coordinated solution which is Pareto improving better than Nash equilibrium solution during transition process, or there maybe cases which realize different potential tax coordinated solutions from long-term steady state, depending on the time horizon.

4-2. Capital tax Competition and coordination under transition process.

We here discuss the consequence of dynamic tax competition in transition process, based on the same simulation as 4-1. Asymmetry is assumed for productivity of labor and public capital in the two regions. It is also assumed that local governments build public capital from the capital tax. Parameters are the same as those used in 4-1. Local governments compete against each other over the capital tax rate, and face different social time preference rate ϕ from long-term steady state. The time horizon appears when considering discounted utility during transition process.

Five different values, $\phi = 0.000, 0.025, 0.050, 0.075, 0.100$, are assumed for the social time preference rate. A simulation analysis will be done here as to how the welfare of regions 1 and 2 changes by shifting from the Nash equilibrium solution to the seventeen (17) or twenty six (26) combinations which become Pareto improving under long-term steady state. The measurement of discounted utility shall be up to time 300 ($T = 300$).

Figure 4-2-1 represents the change of discounted utility ($\Delta u^1/u^1, \Delta u^2/u^2$, shown in %) for each region by shifting the Nash equilibrium solution to a combination of the capital tax rate. The values in the figure is shown positive when the combination improves welfare better than the Nash equilibrium solution, whereas shown positive when it deteriorates, as the consequence of the tax rate competition.

Focusing on long-term steady state, six (6) potential coordinated solutions and seventeen (17) values in the Pareto improving zone at $\varepsilon = 0$ are positive, whereas nine (9) solutions and twenty-six (26) values are positive at $\varepsilon = 0$. As indicated in 4-1, it means that shift from the Nash equilibrium solution to potential tax coordinated solution effects Pareto improvement under long-term steady state.

The value of discounted utility corresponding to the Nash equilibrium solution and the combination of other capital tax rate varies, when the social time preference rate is different. First, take a look at the cases of $\phi = 0.075$ and 0.010 which have rather large social time preference rate.

According to Figure 4-2-1, all seventeen (17) combinations which are in the Pareto improving zone under long-term steady state in the case $\varepsilon = 0$ become negative at least at one of $\Delta u^1/u^1, \Delta u^2/u^2$. On the other hand, twenty four (24) , excluding CT1 and CT4, of twenty six (26) combinations at $\phi = 0.075$ and all twenty six (26) at $\phi = 0.010$ show negative.

Figure 4-2-1. Welfare change from Nash equilibrium solution by the social time preference rate

			長期定常				φ=0				φ=0.025				φ=0.05				φ=0.075				φ=0.1			
			ε=0		ε=1		ε=0		ε=1		ε=0		ε=1		ε=0		ε=1		ε=0		ε=1		ε=0		ε=1	
	τ1	τ2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2	⊿u1/u1	⊿u2/u2
CT1	0.34	0.20	0.042	0.053	0.052	0.064	0.039	0.051	0.050	0.061	0.023	0.035	0.033	0.046	0.008	0.020	0.018	0.031	-0.005	0.008	0.005	0.019	-0.015	-0.002	-0.005	0.008
CT2	0.34	0.22	0.093	0.015	0.116	0.039	0.089	0.012	0.112	0.035	0.067	-0.009	0.089	0.014	0.046	-0.028	0.067	-0.006	0.028	-0.044	0.048	-0.022	0.013	-0.056	0.032	-0.036
CT3	0.36	0.20	0.018	0.127	0.026	0.136	0.014	0.123	0.021	0.132	-0.011	0.099	-0.002	0.109	-0.034	0.076	-0.025	0.086	-0.053	0.055	-0.042	0.066	-0.068	0.038	-0.057	0.050
CT4	0.36	0.22	0.070	0.091	0.091	0.112	0.065	0.086	0.086	0.107	0.034	0.057	0.055	0.078	0.005	0.028	0.025	0.050	-0.019	0.005	0.001	0.027	-0.039	-0.015	-0.019	0.007
CT5	0.36	0.24	0.123	0.036	0.157	0.072	0.116	0.030	0.151	0.066	0.080	-0.002	0.113	0.033	0.044	-0.033	0.076	0.001	0.015	-0.058	0.046	-0.025	-0.010	-0.079	0.020	-0.047
CT6	0.36	0.26	-	-	0.224	0.016	-	-	0.217	0.010	-	-	0.172	-0.026	-	-	0.128	-0.061	-	-	0.091	-0.088	-	-	0.060	-0.111
CT7	0.38	0.22	0.030	0.167	0.049	0.187	0.024	0.161	0.042	0.180	-0.014	0.123	0.005	0.143	-0.050	0.086	-0.030	0.107	-0.079	0.054	-0.058	0.076	-0.102	0.028	-0.081	0.050
CT8	0.38	0.24	0.085	0.114	0.117	0.148	0.077	0.107	0.109	0.141	0.033	0.066	0.065	0.100	-0.009	0.026	0.023	0.059	-0.044	-0.008	-0.012	0.026	-0.072	-0.036	-0.040	-0.003
CT9	0.38	0.26	0.139	0.045	0.186	0.094	0.131	0.038	0.177	0.086	0.080	-0.006	0.125	0.042	0.032	-0.047	0.076	-0.001	-0.008	-0.081	0.034	-0.036	-0.041	-0.110	0.000	-0.065
CT10	0.38	0.28	-	-	0.255	0.026	-	-	0.245	0.018	-	-	0.187	-0.028	-	-	0.129	-0.072	-	-	0.082	-0.108	-	-	0.042	-0.138
CT11	0.40	0.24	0.030	0.194	0.059	0.225	0.021	0.186	0.051	0.216	-0.029	0.135	0.002	0.167	-0.076	0.086	-0.045	0.118	-0.114	0.044	-0.082	0.077	-0.145	0.009	-0.112	0.043
CT12	0.40	0.26	0.087	0.127	0.130	0.173	0.077	0.118	0.121	0.164	0.020	0.065	0.064	0.111	-0.033	0.014	0.010	0.060	-0.077	-0.029	-0.034	0.017	-0.113	-0.064	-0.070	-0.019
CT13	0.40	0.28	0.143	0.044	0.202	0.107	0.132	0.035	0.191	0.097	0.070	-0.018	0.127	0.042	0.010	-0.070	0.066	-0.010	-0.040	-0.112	0.015	-0.054	-0.081	-0.147	-0.028	-0.090
CT14	0.40	0.30	-	-	0.275	0.027	-	-	0.263	0.017	-	-	0.191	-0.038	-	-	0.122	-0.091	-	-	0.064	-0.134	-	-	0.016	-0.170
CT15	0.42	0.26	0.018	0.210	0.058	0.252	0.007	0.199	0.048	0.242	-0.054	0.137	-0.012	0.181	-0.112	0.077	-0.069	0.121	-0.158	0.025	-0.115	0.071	-0.196	-0.018	-0.152	0.028
CT16	0.42	0.28	0.076	0.129	0.133	0.188	0.065	0.118	0.121	0.177	-0.003	0.055	0.053	0.114	-0.067	-0.006	-0.012	0.053	-0.120	-0.057	-0.064	0.002	-0.163	-0.099	-0.108	-0.041
CT17	0.42	0.30	0.136	0.034	0.207	0.111	0.123	0.024	0.194	0.100	0.049	-0.039	0.119	0.036	-0.022	-0.099	0.046	-0.026	-0.081	-0.149	-0.014	-0.077	-0.129	-0.190	-0.063	-0.120
CT18	0.42	0.32	-	-	0.283	0.020	-	-	0.269	0.009	-	-	0.185	-0.055	-	-	0.105	-0.116	-	-	0.038	-0.166	-	-	-0.018	-0.207
CT19	0.44	0.28	-	-	0.046	0.271	-	-	0.034	0.259	-	-	-0.036	0.187	-	-	-0.102	0.117	-	-	-0.156	0.058	-	-	-0.199	0.008
CT20	0.44	0.30	0.055	0.123	0.123	0.195	0.041	0.110	0.110	0.183	-0.037	0.038	0.032	0.110	-0.111	-0.032	-0.042	0.040	-0.171	-0.091	-0.103	-0.019	-0.220	-0.140	-0.153	-0.069
CT21	0.44	0.32	0.116	0.017	0.202	0.107	0.102	0.004	0.187	0.095	0.017	-0.067	0.101	0.022	-0.064	-0.135	0.018	-0.048	-0.130	-0.192	-0.050	-0.106	-0.185	-0.238	-0.106	-0.154
CT22	0.44	0.34	-	-	0.281	0.007	-	-	0.265	-0.006	-	-	0.170	-0.078	-	-	0.080	-0.146	-	-	0.004	-0.202	-	-	-0.058	-0.249
CT23	0.46	0.30	-	-	0.023	0.281	-	-	0.009	0.267	-	-	-0.070	0.186	-	-	-0.145	0.106	-	-	-0.205	0.039	-	-	-0.254	-0.017
CT24	0.46	0.32	0.021	0.109	0.104	0.195	0.006	0.095	0.088	0.181	-0.081	0.013	0.001	0.100	-0.163	-0.065	-0.082	0.020	-0.231	-0.131	-0.150	-0.046	-0.285	-0.186	-0.205	-0.101
CT25	0.46	0.34	-	-	0.185	0.097	-	-	0.169	0.084	-	-	0.073	0.003	-	-	-0.019	-0.075	-	-	-0.094	-0.140	-	-	-0.156	-0.194
CT26	0.48	0.34	-	-	0.073	0.189	-	-	0.056	0.174	-	-	-0.040	0.084	-	-	-0.131	-0.004	-	-	-0.205	-0.077	-	-	-0.265	-0.138

Note 1. The unit $\Delta u^1/u^1, \Delta u^2/u^2$ of is %.

Note 2. Colored areas show potential tax coordinated solution corresponding to each social time preference rate.

It indicates that a shift from the Nash equilibrium solution to seventeen (17) or twenty six (26) combinations does not achieve Pareto improvement when the social time preference rate is large and local governments are shortsighted ($\phi = 0.075, 0.100$).

There was a room for local governments to agree on tax coordination in multiple potential tax coordinated solutions under long-term steady state. When the social time preference rate is large, however, no potential tax coordinated solution exists which are more Pareto improving than the Nash equilibrium solution, and possibility of tax coordination, which was expected to achieve under steady state, disappears.

Under these circumstances, negotiations between local governments would not reach to an agreement for any coordinated actions, and they would deadlock at the Nash equilibrium solution.

Then, will there be any changes in potential tax coordinated solutions when the social time preference rate is comparatively small? Figures 4-2-2 and 4-2-3 illustrate the changes of economic welfare when shifted from the Nash equilibrium solution to the capital tax rate combinations.

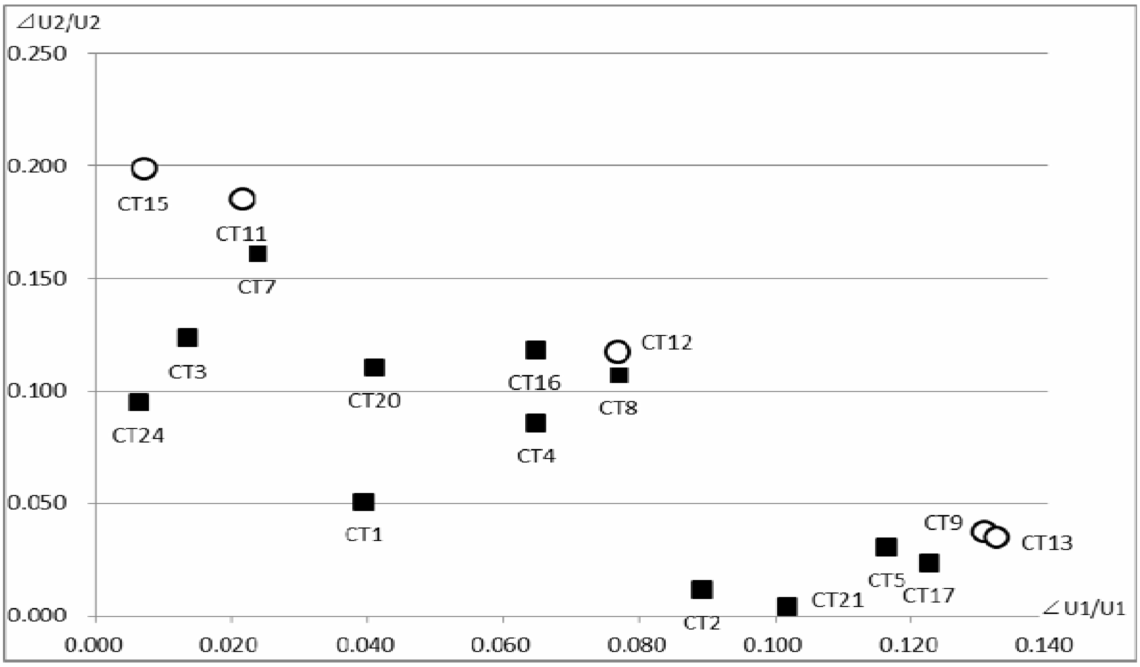
The horizontal and vertical axes respectively indicate the percentage of welfare change ($\Delta u^1/u^1, \Delta u^2/u^2$) when shifted from the Nash equilibrium solution. The positive value shows welfare improvement whereas the negative shows welfare deterioration. It means that the tax rate combinations in the first quadrant are the Pareto improving zone where welfare would improve in both regions. It can be interpreted that the welfare improving effect is bigger towards the upper right.

Five (5) combinations (CT9, CT11, CT12, CT13 and CT15) shown with ○ in Figure 4-2-2 and ten (10) combinations (CT10, CT11, CT14, CT15, CT16, CT17, CT18, CT19, CT20 and CT23) shown with ○ in Figure 4-2-3 are found on the nearly same line in the farthest zone from the point of origin. It can be said that they are the combinations which have relatively high welfare improving effect from the Nash equilibrium solution among the combinations.

The combination of either five (5) or ten (10) tax rates is assumed to have no relative merits with regards to resource allocation. Thus, it is highly possible that local governments would come to an agreement on either combination when negotiating over tax coordination. Specifically, it can be considered that the potential coordinated solutions under $\phi = 0.000$ will be CT9, CT11, CT12, CT13 and CT 15 at $\varepsilon = 0$, and CT10, CT11, CT14, CT15, CT16, CT17, CT18, CT19, CT20 and CT 23 at $\varepsilon = 1$.

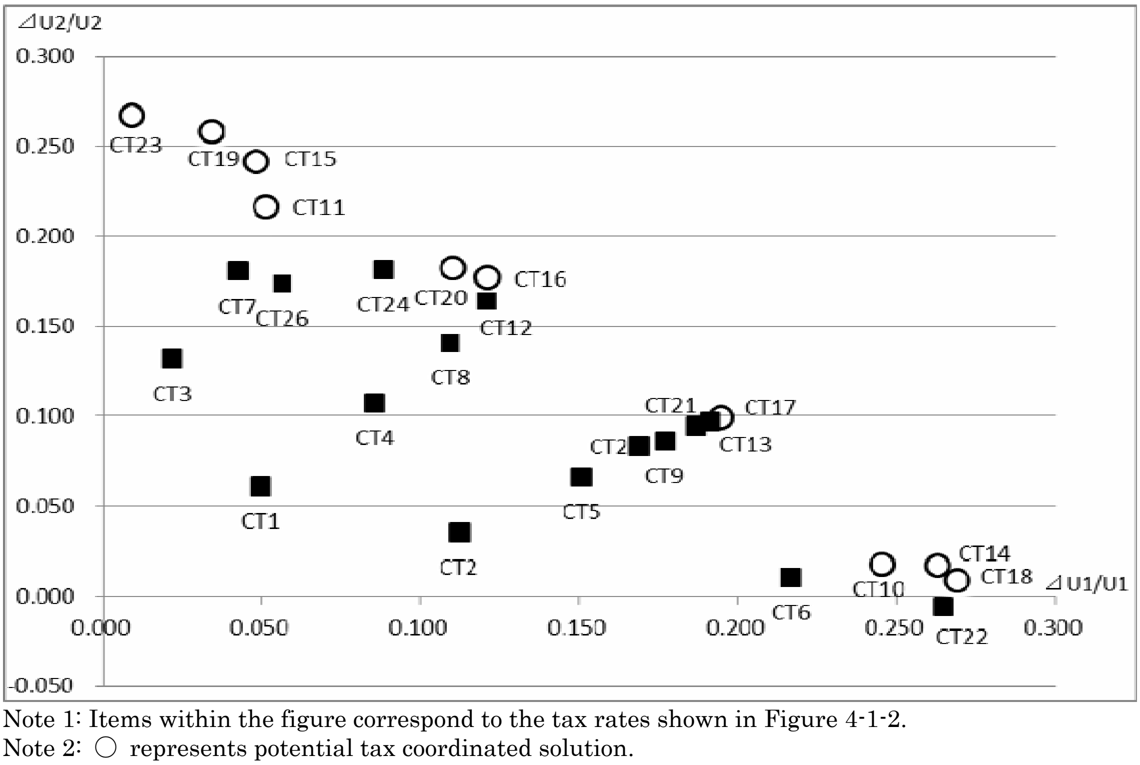
Meanwhile, in the cases $\phi = 0.025$ and $\phi = 0.050$ which have rather small social preference rate, the potential tax coordinated solution for $\varepsilon = 0$ will be CT4 and CT8 in the case $\phi = 0.025$ and CT7, CT8, CT11, CT12, CT13 and CT16 in the case $\phi = 0.050$, according to Figures 4-2-4, 4-2-5, 4-2-6 and 4-2-7.

Figure 4-2-2. Potential tax coordinated solution during transition process. $(\varepsilon = 0, \phi = 0.000)$



Note 1: Items within the figure correspond to the tax rates shown in Figure 4-1-1.
Note 2: ○ represents potential tax coordinated solution.

Figure 4-2-3. Potential tax coordinated solution during transition process. $(\varepsilon = 1, \phi = 0.000)$



Note 1: Items within the figure correspond to the tax rates shown in Figure 4-1-2.
Note 2: ○ represents potential tax coordinated solution.

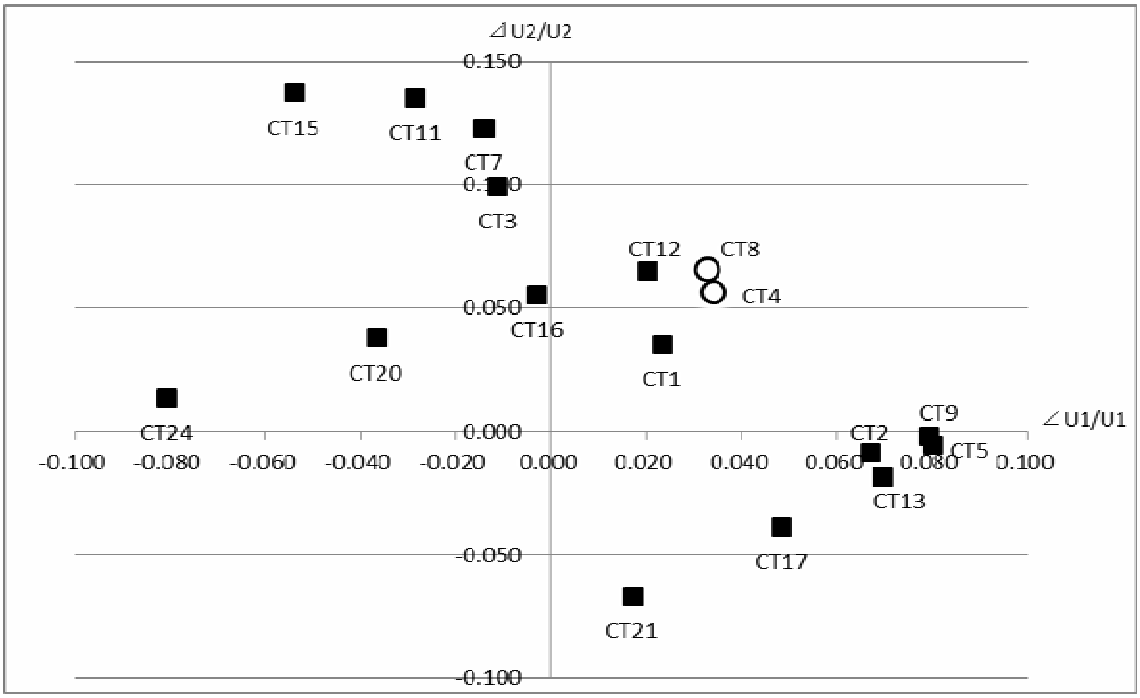
In the cases $\phi = 0.025, 0.050$, the number of potential tax coordinated solutions decreases as the social time preference rate changes. There are six (6) in the case $\varepsilon = 0$ and nine (9) in the case $\varepsilon = 1$ under long-term steady state, however, it decreases to two (2) at $\varepsilon = 0$ and six (6) at $\varepsilon = 1$ for $\phi = 0.025$, and two (2) at $\varepsilon = 0$ and four (4) at $\varepsilon = 1$ for $\phi = 0.050$, and the candidates for coordinated solutions are replacing. It is confirmed that the potential tax coordinated solutions which were found under long-term steady state have all dropped out from the candidates in the case $\phi = 0.050$, and another combinations have become candidates in both cases $\varepsilon = 0$ and $\varepsilon = 1$.

We now summarize the result of our simulation. The analysis under transition process showed a totally different condition of tax coordination from long-term steady state, although it had no change in terms of the consequence of tax competition and actualization of the Nash equilibrium solution. It was indicated that in the cases ($\phi = 0.075, 0.010$) where the social time preference rate was comparatively high, the potential tax coordinated solutions found under steady state could not be candidates, and the possibility of tax coordination disappeared except CT1 and CT4 at $\varepsilon = 1, \phi = 0.075$. At $\phi = 0.000$, they nearly overlapped on those under long-term steady state. In contrast, in the cases $\phi = 0.025$ and 0.050, there was a possibility of tax coordination, but the potential coordinated solutions under long-term steady state gradually dropped out from the candidates, and different ones started to appear.

The potential tax coordinated solutions under long-term steady state, especially the combinations with relatively high tax rates, gradually dropped out from the Pareto improving zone or the first quadrant as ϕ becomes larger under transition process.

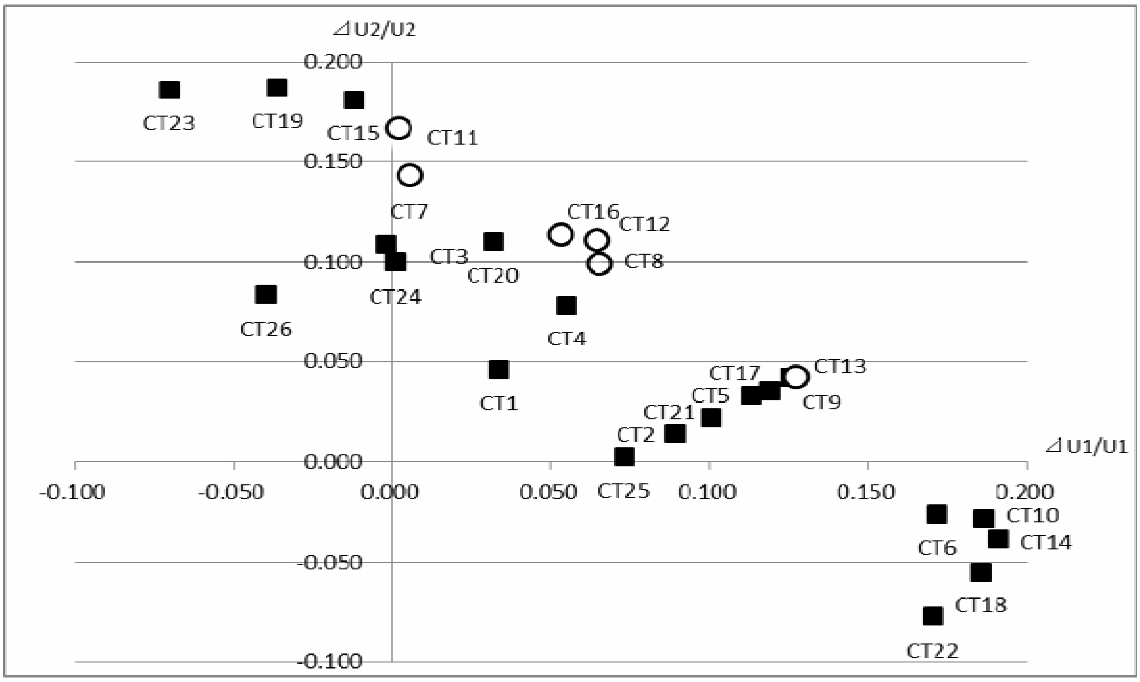
The reason can be explained as follows: The initial welfare deterioration due to the tax rate change was not obvious under long-term steady state. It left room for tax coordination at a high capital tax rate which could lead to the accumulation of public capital with increased capital tax income, and could lead to the utility increase brought by improved productivity.

Figure 4-2-4. Potential tax coordinated solutions under transition process ($\varepsilon = 0, \phi = 0.025$)



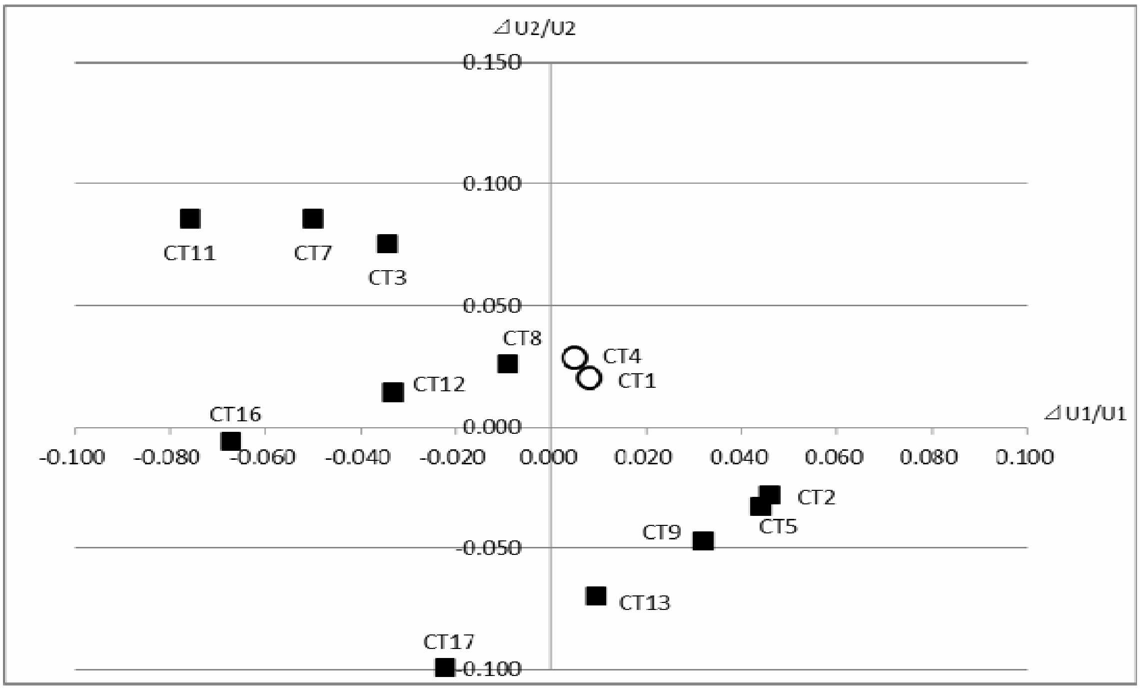
Note 1: Items within the figure correspond to the tax rate combination shown in Figure 4-1-1.
Note 2: ○ represents potential tax coordinated solution.

Figure 4-2-5. Potential tax coordinated solutions under transition process ($\varepsilon = 1, \phi = 0.025$)



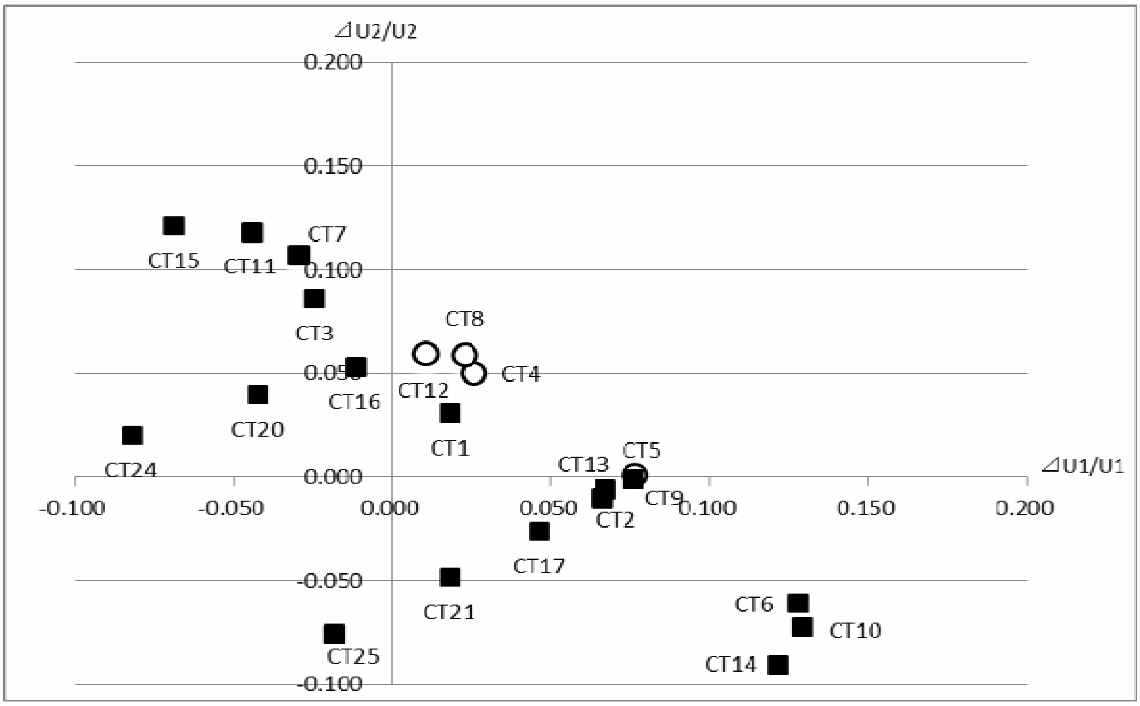
Note 1: Items within the figure correspond to the tax rate combination shown in Figure 4-1-2.
Note 2: ○ represents potential tax coordinated solution.

Figure 4-2-6. Potential tax coordinated solutions under transition process ($\varepsilon = 0, \phi = 0.050$)



Note 1: Items within the figure correspond to the tax rate combination shown in Figure 4-1-1.
Note 2: ○ represents potential tax coordinated solution.

Figure 4-2-7. Potential tax coordinated solutions under transition process ($\varepsilon = 1, \phi = 0.050$)



Note 1: Items within the figure correspond to the tax rate combination shown in Figure 4-1-1.
Note 2: ○ represents potential tax coordinated solution.

On the contrary, the initial welfare deterioration would be considered under transition process and the combination with high capital tax rates could not be the candidates. It can be considered that it is possible to actualize tax coordination with a combination of lower capital tax rates as the social time preference rate becomes larger.

When local governments are shortsighted, capital accumulation will be accelerated and it will become difficult to actualize tax coordination with the tax rate which makes economy more efficient in the long run. Larger social time preference rate can cause more serious evaluation on the initial welfare deterioration. Thus, potential tax coordinated solutions improving welfare in both regions better than the Nash equilibrium solution disappear and tax coordination cannot be achieved. The potential solutions found under steady state reduce or disappear under transition process, and another candidates begin to appear. It can be considered that a "transformation of tax coordination" has occurred.

5. Conclusion.

In this paper, we attempted a welfare analysis using an expansion of theoretical model and simulation as to the consequence of a dynamic tax competition within asymmetric regions and as to the improvement of economic welfare brought by the shift from the capital tax competition solution, or the Nash equilibrium solution, to the coordinated solution.

This paper features that there is a difference in productivity among regions in terms of public capital used as a public input for each region. It modelizes a dynamic framework that public capital is going to be accumulated by public investment. Further, it is qualitatively and quantitatively analyzed that the ratio which the public capital rent attributes to the wage and capital income is exogenously given by distribution parameter ε, and what kind of changes ε would give to the result of tax coordination.

The conclusion is summarized in what follows. First, it was indicated in the qualitative analysis made in Chapter 3 that welfare improvement could be realized by raising the tax rate in the case where a dynamic efficiency was being established ($n < \rho^i$), when both regions did tax coordination to achieve welfare improvement from the Nash equilibrium of tax competition. It suggested that tax coordination could be actualized by raising the tax rate in both regions since tax competition led to tax rate deduction in under the condition of dynamic efficiency. Such result from the qualitative analysis was supported by the simulation made in Chapter 4 which indicated that the Pareto improving zone would exist when the tax rate was raised from the Nash equilibrium solution.

A welfare evaluation was made in Chapter 4 for long-term steady state and transition process. There were seventeen (17) combinations of capital tax rate at $\varepsilon = 0$ and twenty six (26) at $\varepsilon = 1$, which actualized Pareto improving from the Nash equilibrium solution. As a result, it was clarified that changes would occur in potential coordinated solutions depending on the social time preference rate set by local governments. More specifically, it appeared clear that there was a possibility to hinder tax coordination, considering transition process, when local governments weighed the initial welfare deterioration heavily, even in the case where the shift to a coordinated solution became Pareto improving at the welfare level under steady state.

As described above, candidates for coordinated solutions would exist in the direction of tax hike, and such candidates would change according to the social time preference rate of local governments. These were common regardless of distribution parameter ε. On the other hand, the influence the value of ε would give to tax coordination appeared in the size of welfare improving effect through the wage rate change as explained in Appendix D. It was consistent with the simulation result, which indicated that the welfare improvement level was higher at $\varepsilon = 1$ than $\varepsilon = 0$ in any combination of the tax rates and there were more Pareto improving combinations in the first quadrant at $\varepsilon = 1$ at any discount rate ϕ.

It was indicated in the theoretical analysis in this paper that welfare improvement under steady state existed with regards to the tax hike from the Nash equilibrium. As to transition process, however, welfare evaluation was made based on the result of simulation analysis. As shown in Figures 4-1-5 and 4-1-6, the utility of transition process did not change monotonically and showed asymmetric movement in the two regions. Thus, it was not possible to trace such complicated transition. A simulation analysis was chosen in this paper instead of theoretical analysis to clarify the complicatedness in that the asymmetry of public capital was happening due to the asymmetric nature of the targeted regions, and the elevation of tax rate, which was policy variables, caused a change on the demand of private capital through the accumulation of public capital which eventually influenced on the rate of return of capital or the wage rate.

Now, we would like to note two pending issues. Firstly, a deeper analysis should be made as to the relation between the welfare improving effect of tax coordination and economic environment. In this paper, we adopted an economic environment with different distribution volume of public capital rent, however, we did not discuss further as to the relation of a dynamic efficiency and tax coordination. The analysis result shown in this paper may possibly be dependent on the parameters of specified utility function and production function in the simulation. We cited the dynamic efficiency as a sufficient condition for welfare improvement for theoretical model, and followed this premise at our simulation. The relation between tax coordination and economic environment should be more clarified by doing an analysis on what consequence would be led by tax competition and tax coordination under a dynamic inefficiency.

The second remaining issue is to expand options for policy instruments in order to further study the possibility of policy coordination among regions including the tax rate. Although it was indicated here that tax coordination might not be achieved considering transition process, only capital tax rate was used as a policy instrument. It can be considered that there is room to improve the welfare of both regions by incorporating other policy instruments. A strong candidate is an inter-generational redistribution which can complement the initial welfare deterioration under transition process causing a “transformation of coordination.” It is generally known that such inter-generational redistribution can give an influence on capital accumulation through the change of savings. Therefore, it may be plausible to analyze the welfare effect of policy coordination including inter-generational redistribution in order to further discuss the relation between dynamic efficiency and policy effect.

Appendix A: Stable Condition

The model used in this paper is a dynamic system of three variables (θ_t, g_t^i, g_t^j). When policy variable τ^i is constant, the condition which equilibrium is locally stable shall be set as follows. The three equations can be obtained by totally differentiating equations (12) and (13) and organizing by using the relation of $d\theta_t = (1-\tau^i)d\rho_t^i$ and $d\rho_t^i = \frac{d\theta_t}{1-\tau^i} = \frac{d\theta_t}{\theta_t/\rho_t^i} = (\rho_t^i/\theta_t)d\theta_t$ [9].

$$
\begin{aligned}
&(s_w^i w_\rho^i \rho^i/\theta + s_w^j w_\rho^j \rho^j/\theta)d\theta_t + s_w^i w_g^i dg_t^i + s_w^j w_g^j dg_t^j \\
&= -(s_\theta^i + s_\theta^i)d\theta_{t+1} + (1+n)(k_\rho^i \rho^i/\theta + k_\rho^j \rho^j/\theta)d\theta_{t+1} + (1+n)k_g^i dg_{t+1}^i + (1+n)k_g^j dg_{t+1}^j \\
&\tau^i(k^i + k_\rho^i \rho^i)(\rho^i/\theta)d\theta_t + (1+\tau_t^i \rho_t^i k_g^i)dg_t^i = (1+n)dg_{t+1}^i \\
&\tau^j(k^j + k_\rho^j \rho^j)(\rho^j/\theta)d\theta_t + (1+\tau_t^j \rho_t^j k_g^j)dg_t^j = (1+n)dg_{t+1}^j
\end{aligned}
$$

The matrix form of the above equations shall be as follows,

$$
\begin{bmatrix} m_a & (1+n)k_g^i & (1+n)k_g^j \\ 0 & (1+n) & 0 \\ 0 & 0 & (1+n) \end{bmatrix}
\begin{bmatrix} d\theta_{t+1} \\ dg_{t+1}^i \\ dg_{t+1}^j \end{bmatrix}
=
\begin{bmatrix} m_z & s_w^i w_g^i & s_w^j w_g^j \\ m_d^i & m_y^i & 0 \\ m_d^j & 0 & m_y^j \end{bmatrix}
\begin{bmatrix} d\theta_t \\ dg_t^i \\ dg_t^j \end{bmatrix}
$$

however, provided that

$m_a = (1+n)(k_\rho^i \rho^i + k_\rho^j \rho^j)/\theta - (s_\theta^i + s_\theta^j)$, $m_z^i = (s_w^1 w_\rho^1 \rho^1 + s_w^j w_\rho^j \rho^j)/\theta$, $m_y^i = 1 + \tau^i \rho^i k_g^i$, $m_y^j = 1 + \tau^j \rho^j k_g^j$,
$m_d^1 = \tau^i(k^1 + k_\rho^i \rho^i)\rho^i/\theta$, $m_d^j = \tau^j(k^j + k_\rho^j \rho^j)\rho^j/\theta$.

Assuming $A = \begin{bmatrix} m_a & (1+n)k_g^i & (1+n)k_g^j \\ 0 & (1+n) & 0 \\ 0 & 0 & (1+n) \end{bmatrix}$, $B = \begin{bmatrix} m_z & s_w^i w_g^i & s_w^j w_g^j \\ m_d^i & m_y^i & 0 \\ m_d^j & 0 & m_y^j \end{bmatrix}$, the stable

[9] The relation of ρ^i and θ is based on the assumption that τ^i is constant.

condition is that the determinant $|M| = |A^{-1}B| = |A^{-1}||B|$ of the coefficient matrix $M = A^{-1}B$ should be negative. With the assumption of $s_\theta^1 > 0,\ s_\theta^2 > 0$, it is $|A^{-1}| = |A|^{-1} = \{(1+n)^2(m_a)\}^{-1} < 0$, and $|B| > 0$ should be the stable condition. In other words, $|B| = m_z m_y^i m_y^j - m_y^i m_d^j s_w^j w_g^j - m_y^j m_d^i s_w^i w_g^i > 0$ is formed.

Because $m_z^i = (s_w^i w_\rho^i \rho^i + s_w^j w_\rho^j \rho^j)/\theta < 0,$ $m_y^i = 1 + \tau^i \rho^i k_g^i > 0,$ and $m_y^j = 1 + \tau^j \rho^j k_g^j > 0$, $m_d^i < 0, m_d^j < 0$ needs to be fulfilled in order for $|B| > 0$ to be formed. Therefore, it can be interpreted that the condition for the dynamic system in this paper to be stable should be that the interest elasticity $\sigma = \dfrac{\partial k^i}{k^i} \Big/ \dfrac{\partial \rho^i}{\rho^i}$ of the private capital demand is less than -1.

Appendix B: the influence on θ and g^i changes under steady state when the capital tax rate τ^j.

It is assumed that both regions in accord raise capital tax rate τ^i from the Nash equilibrium solution. By totally differentiating equations (12) and (13) and by organizing with the relation of $d\rho^i = \dfrac{d\theta}{1-\tau^i} + \dfrac{\rho^i d\tau^i}{1-\tau^i} = \dfrac{d\theta}{\theta/\rho^i} + \dfrac{\rho^i d\tau^i}{1-\tau^i}$, the following three equations can be obtained:

$$s_w^i w_\rho^i \left(\frac{d\theta}{\theta/\rho^i} + \frac{\rho^i d\tau^i}{1-\tau^i} \right) + s_w^j w_\rho^j \left(\frac{d\theta}{\theta/\rho^j} + \frac{\rho^j d\tau^j}{1-\tau^j} \right) + s_\theta^i d\theta + s_\theta^j d\theta + s_w^i w_g^i dg^i + s_w^j w_g^j dg^j$$
$$= (1+n)\left(k_\rho^i \left(\frac{d\theta}{\theta/\rho^i} + \frac{\rho^i d\tau^i}{1-\tau^i} \right) + k_\rho^j \left(\frac{d\theta}{\theta/\rho^j} + \frac{\rho^j d\tau^j}{1-\tau^j} \right) \right) + (1+n)(k_g^i dg^i + k_g^j dg^j)$$

$$(1+n)dg^i = \rho^i k^i d\tau^i + (\tau^i k^i + \tau^i k_\rho^i \rho^i)\left(\frac{d\theta}{\theta/\rho^i} + \frac{\rho^i d\tau^i}{1-\tau^i} \right) + (1 + \tau^i k_g^i \rho^i) dg^i$$

$$(1+n)dg^j = \rho^j k^j d\tau^j + (\tau^j k^j + \tau^j k_\rho^j \rho^j)\left(\frac{d\theta}{\theta/\rho^j} + \frac{\rho^j d\tau^j}{1-\tau^j} \right) + (1 + \tau^j k_g^j \rho^j) dg^j$$

The matrix form of the above equations shall be as follows,

$$\begin{bmatrix} m_g & m_e^i & m_e^j \\ -m_d^i & m_c^i & 0 \\ -m_d^j & 0 & m_c^j \end{bmatrix}\begin{bmatrix} d\theta \\ dg^i \\ dg^j \end{bmatrix} = \begin{bmatrix} m_f^i \dfrac{d\tau^i}{1-\tau^i} + m_f^j \dfrac{d\tau^j}{1-\tau^j} \\ m_d^i \dfrac{\theta}{\rho^i}\dfrac{d\tau^i}{1-\tau^i} \\ m_d^j \dfrac{\theta}{\rho^j}\dfrac{d\tau^j}{1-\tau^j} \end{bmatrix}$$

however, provided that

$m_a = (1+n)(k_\rho^i \rho^i + k_\rho^j \rho^j)/\theta - (s_\theta^i + s_\theta^j)$, $m_c^i = (n - \tau^i \rho^i k_g^i)$, $m_c^j = (n - \tau^j \rho^j k_g^j)$, $m_d^i = \tau^i (k^i + k_\rho^i \rho^i)\rho^i/\theta$,

$m_d^j = \tau^j (k^j + k_\rho^j \rho^j)\rho^j/\theta$, $m_e^i = (1+n)k_g^i - s_w^i w_g^i$, $m_e^j = (1+n)k_g^j - s_w^j w_g^j$, $m_f^i = \{s_w^i w_\rho^i - (1+n)k_\rho^i\}\rho^i$,

$m_f^j = \{s_w^j w_\rho^j - (1+n)k_\rho^j\}\rho^j$, $m_g = m_a - (s_w^i w_\rho^i \rho^i + s_w^j w_\rho^j \rho^\rho)/\theta = -m_f^i/\theta - m_f^j/\theta - (s_\theta^i + s_\theta^j)$.

When $M_L = \begin{bmatrix} m_g & m_e^i & m_e^j \\ -m_d^i & m_c^i & 0 \\ -m_d^j & 0 & m_c^j \end{bmatrix}$, $M_L^{-1} = \dfrac{1}{|M_L|}\begin{bmatrix} m_c^i m_c^j & -m_e^i m_c^j & -m_e^j m_c^i \\ m_d^i m_c^j & m_g m_c^j + m_d^j m_e^j & -m_e^j m_d^i \\ m_d^j m_c^i & -m_e^i m_d^j & m_g m_c^i + m_d^i m_e^i \end{bmatrix}$.

Thus, the above matrix can be rewritten as follows:

$$\begin{bmatrix} d\theta \\ dg_i \\ dg_j \end{bmatrix} = \frac{1}{|M_L|}\begin{bmatrix} m_c^i m_c^j & -m_e^i m_c^j & -m_e^j m_c^i \\ m_d^i m_c^j & m_g m_c^j + m_d^j m_e^j & -m_e^j m_d^i \\ m_d^j m_c^i & -m_e^i m_d^j & m_g m_c^i + m_d^i m_e^i \end{bmatrix}\begin{bmatrix} m_f^i \dfrac{d\tau^i}{1-\tau^i} + m_f^j \dfrac{d\tau^j}{1-\tau^j} \\ m_d^i \dfrac{\theta}{\rho^i}\dfrac{d\tau^i}{1-\tau^i} \\ m_d^j \dfrac{\theta}{\rho^j}\dfrac{d\tau^j}{1-\tau^j} \end{bmatrix}$$

$$= \frac{1}{|M_L|}\begin{bmatrix} m_c^j\left(m_c^i m_f^i - m_e^i m_d^i \dfrac{\theta}{\tau^i}\right)\dfrac{d\tau^i}{1-\tau^i} + m_c^i\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\tau^j}\right)\dfrac{d\tau^j}{1-\tau^j} \\ m_d^i\left\{m_c^j m_f^i + (m_g m_c^j + m_d^j m_e^j)\dfrac{\theta}{\rho^i}\right\}\dfrac{d\tau^i}{1-\tau^i} + m_d^i\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\rho^j}\right)\dfrac{d\tau^j}{1-\tau^j} \\ m_d^j\left(m_c^i m_f^i - m_e^i m_d^i \dfrac{\theta}{\rho^i}\right)\dfrac{d\tau^i}{1-\tau^i} + m_d^j\left\{m_c^i m_f^j + (m_b m_c^i + m_d^i m_e^i)\dfrac{\theta}{\rho^j}\right\}\dfrac{d\tau^j}{1-\tau^j} \end{bmatrix}$$

Here, $|M_L| = m_b m_c^i m_c^j + m_d^j m_e^j m_c^i + m_d^i m_e^i m_c^j$, so it can be derived that

$$d\theta = \frac{m_c^j\left(m_c^i m_f^i - m_e^i m_d^i \dfrac{\theta}{\rho^i}\right)\dfrac{d\tau^i}{1-\tau^i} + m_c^i\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\rho^j}\right)\dfrac{d\tau^j}{1-\tau^j}}{m_g m_c^i m_c^j + m_d^j m_e^j m_c^i + m_d^i m_e^i m_c^j}.$$

According to the condition of stability, $m_d^1 < 0, m_d^2 < 0$. Assumed

$m_c^1 > 0, m_c^2 > 0,\ m_e^1 < 0, m_e^2 < 0,\ m_f^1 < 0, m_f^2 < 0$ and also $\left|s_\theta^1\right|, \left|s_\theta^2\right|$ is small enough, the numerator should be negative and the denominator should be positive because $m_g^1 > 0, m_g^2 > 0,$. The influence $\left(\dfrac{d\theta}{d\tau^j}\right)$ to θ given by tax coordination shall be as follows:

$$\frac{d\theta}{d\tau^j} = \frac{\dfrac{m_c^i}{1-\tau^j}\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\rho^j}\right)}{m_g m_c^i m_c^j + m_d^j m_e^j m_c^i + m_d^i m_e^i m_c^j} < 0$$

In contrast, dg^i can be described as:

$$dg^i = \frac{m_d^i\left\{m_c^j m_f^i + \left(m_g m_c^j + m_d^j m_e^j\right)\dfrac{\theta}{\tau^i}\right\}\dfrac{d\tau^i}{1-\tau^i} + m_d^j\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\tau^j}\right)\dfrac{d\tau^j}{1-\tau^j}}{m_g m_c^1 m_c^2 + m_d^2 m_e^2 m_c^1 + m_d^1 m_e^1 m_c^2}$$

Accordingly, the following equation shall be made as to $\dfrac{dg^i}{d\tau^j}$:

$$\frac{dg^i}{d\tau^j} = \frac{\dfrac{m_d^i}{1-\tau^j}\left(m_c^j m_f^j - m_e^j m_d^j \dfrac{\theta}{\rho^j}\right)}{m_g m_c^i m_c^j + m_d^j m_e^j m_c^i + m_d^i m_e^i m_c^j} = \frac{m_d^i}{m_c^i}\frac{d\theta}{d\tau^j} > 0$$

Appendix C: Welfare improving effect by tax coordination.

Assumed that tax coordination should be made to raise the capital tax rate from the Nash equilibrium solution. It was shown in equation (18) that what impact a coordinated action of local governments would give to mutual economic welfare. When it is positive, it can be said that welfare improvement is expected to occur by tax coordination. As shown in Appendix B, $\frac{\partial g^i}{\partial \tau_c^j} > 0, \frac{\partial \theta}{\partial \tau_c^j} < 0$ under long-term steady state. Based on the assumption that $v_w^i > 0, v_\theta^i > 0, w_\rho^i < 0$, the 3rd term of the right hand side of equation (18) should be positive when $w_g^i > 0$. Accordingly, the sufficient condition for equation (18) to be positive can be given by equation (19). As shown in equations (3) and (4), $w_\rho^i = (f_k^i - \rho^i)k_\rho^i - k^i = -(1-\varepsilon)k_\rho^i f_g^i \frac{g^i}{k^i} - k^i$. According to Roy's identity, $v_\theta^i = v_w^i c^{oi}(1+\theta)^{-2} = s^i(1+\theta)^{-1}$ can be formed. Equation (19) can be rewritten as:

$$
\begin{aligned}
v_w^i w_\rho^i \frac{1}{1-\tau_c^i} - v_\theta^i &= v_w^i \left[\left\{ -k^i - (1-\varepsilon)k_\rho^i f_g^i \frac{g^i}{k^i} \right\} \frac{1}{1-\tau_c^i} + \frac{1}{1+\theta} \left\{ z^i + (1+n)k^i \right\} \right] \\
&= v_w^i \left\{ \left(\frac{1+n}{1+\theta} - \frac{1}{1-\tau_c^i} \right) k^i + \frac{1}{1+\theta} z^i - \frac{1}{1-\tau_c^i} (1-\varepsilon) k_\rho^i f_g^i \frac{g^i}{k^i} \right\} \\
&= \frac{v_w^i}{1+\theta} \left[\left(n - \rho^i \right) k^i - \frac{\tau_c^i}{1-\tau_c^i} k^i + z^i - (1-\varepsilon) \frac{1+\theta}{1-\tau_c^i} k_\rho^i f_g^i \frac{g^i}{k^i} \right] < 0
\end{aligned}
$$

The sufficient condition to meet the aforementioned will be that the 1st and the 3rd terms of the right hand side are negative. The 1st term of the right hand side shows the condition of a dynamic efficiency ($n < \rho^i$). The sign of the 3rd term can be determined by whether capital inflow or outflow. Consequently, it is the sufficient condition for the welfare of both regions to be improved by tax coordination raising the tax rate that economy satisfies a dynamic efficiency and it is a capital outflowing region.

Appendix D: Influence of ε on welfare improving effect by tax coordination.

Now, we discuss here what influence ε would give to an economic welfare improving effect, when both regions take a coordinated action to raise the tax rate under long-term steady state. We pick up the cases of $\varepsilon = 0$ and $\varepsilon = 1$, and we attempt to see the difference in welfare improving effect by focusing on the size of $\left| w_\rho^i \right|$ and $\left| w_g^i \right|$.

According to equations (3) and (4), w_ρ^i and w_g^i should be given by the following equation:

$$w_\rho^i = (f_k - \rho^i)k_\rho^i - k^i = -(1-\varepsilon)k_\rho^i f_g \frac{g^i}{k^i} - k^i$$

$$= -k_\rho^i f_g \frac{g^i}{k^i} - k^i \qquad (\varepsilon = 0)$$

$$= -k^i \qquad (\varepsilon = 1)$$

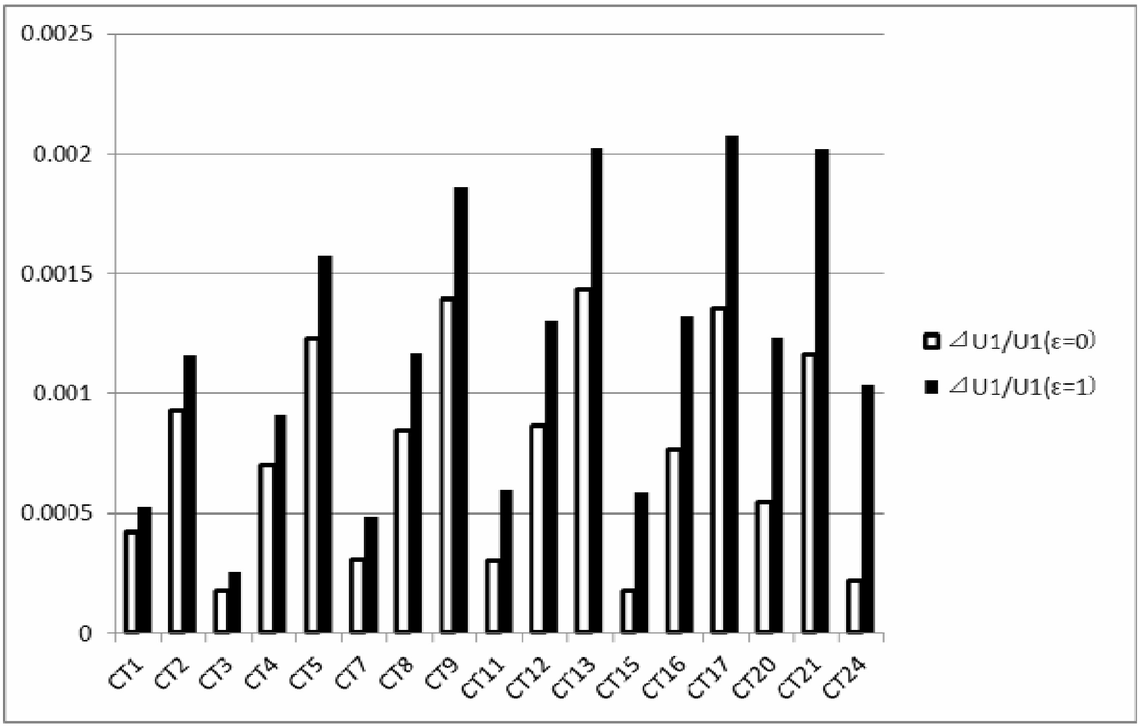
$$w_g^i = f_g + f_k k_g^i - \rho^i k_g^i = -(1-\varepsilon)k_g^i f_g \frac{g^i}{k^i} + f_g$$

$$= -k_g^i f_g \frac{g^i}{k^i} + f_g \qquad (\varepsilon = 0)$$

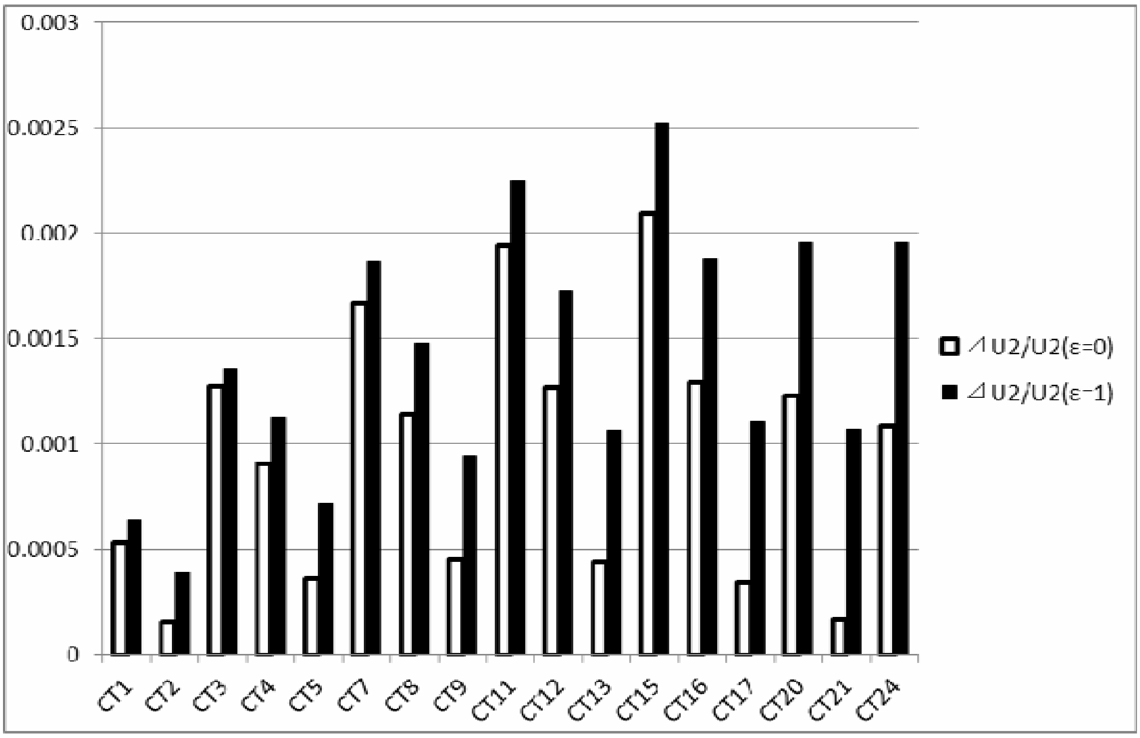
$$= f_g \qquad (\varepsilon = 1)$$

When $\left|w_\rho^i\right|$ and $\left|w_g^i\right|$ at $\varepsilon = 0$ and $\varepsilon = 1$ are respectively $\left|w_\rho^i\right|^{\varepsilon=0}$, $\left|w_\rho^i\right|^{\varepsilon=1}$, $\left|w_g^i\right|^{\varepsilon=0}$ and $\left|w_g^i\right|^{\varepsilon=1}$, it will be $\left|w_\rho^i\right|^{\varepsilon=0} < \left|w_\rho^i\right|^{\varepsilon=1}$ and $\left|w_g^i\right|^{\varepsilon=0} < \left|w_g^i\right|^{\varepsilon=1}$. Therefore, $\frac{\partial g^i}{\partial \tau_c^j} > 0, \frac{\partial \theta}{\partial \tau_c^j} < 0$. Based on $v_w > 0, v_\theta > 0, w_\rho^i < 0, w_g^i > 0$, the welfare improving effect of $\varepsilon = 1$ is shown to be higher than that of $\varepsilon = 0$, according to equation (18). It is generally considered that the welfare improving effect of tax coordination by raising the tax rate, under long-term steady state, can be higher when ε is larger within the range $0 \leq \varepsilon \leq 1$.

Supplemental Figure 1. Influence on economic welfare under long-term steady state by ε (U1).



Supplemental Figure 2. Influence on economic welfare under long-term steady state by ε (U2).



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