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Bargaining with uncertain commitment: on the limits of disagreement.

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Abstract

The role of uncertain commitment on disagreement in bargaining is studied in symmetric information environments. Two players make simultaneous demands of a unit sized pie. Following incompatible demands they simultaneously choose whether to stick to their demand or accept the other's offer. Accepting the other's offer is costly with the cost being uncertain when demands are made but common knowledge before the second stage. Both parties sticking to their incompatible demands leads to disagreement. When the revoking costs have continuous densities and intervals for support, the model can be made tractable by using a global game information structure. Disagreement is shown to not exist in equilibrium if both players face the same but uncertain revoking cost. Even when the revoking costs are independently distributed disagreement cannot arise if the cost distributions first order stochastically dominate the uniform distribution. These results are found to be in sharp contrast with predictions from models where the cost has a discrete distribution with values of either zero or greater than one and models where the success of a commitment attempt is determined exogenously.

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Bargaining with Uncertain Commitment: On the Limits of Disagreement *

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1 Introduction

Does the ability to attempt commitment to aggressive demands lead to disagreement in bargaining between two rational agents, when the success of the commitment attempt is ex ante uncertain? A successful commitment attempt here refers to a bargainer being unable to back down from a stated demand. While Schelling(1960) forcefully answers in the affirmative through a series of informal but carefully formulated arguments, the question clearly needs to be further qualified for a definitive answer in a formal model. Importantly, the model needs to specify the requirements for a commitment attempt to be successful. Crawford(1982) frames the question in a formal game theoretic model, where two bargainers simultaneously choose whether to attempt commitment, and if so, what share of the surplus to demand. Following incompatible demands each agent decides simultaneously whether to back down and accept the other agents offer or stick to her original demand. Crucially, however, backing down is costly with each agent getting to know only her own backing down (revoking) cost, after the initial demands are made. The uncertainty shared by the two agents regarding the costs of backing down, when they make their initial demands, is captured by a pair of distribution functions that are commonly known. An agent, therefore, *achieves commitment* if in equilibrium following incompatible demands she chooses to stick to her demand. Disagreement results if both agents commit to incompatible demands and then do not back down. Crawford(1982) shows that while efficient (no disagreement) equilibria typically exist when the probability of the revoking cost being high is high, when this

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probability is low disagreement occurs. Indeed when this probability is low enough all equilibria feature a positive probability of disagreement.

This paper extends the analysis of Crawford(1982) to symmetric information environments. The basic framework adhered to through most of the paper is the following. Two agents bargain over a pie of size 1 by simultaneously announcing their demands in the first stage. If these demands are compatible (add up to less than 1) then each agent gets her own demand and half the remaining surplus, if any. Following incompatible demands each agent gets to know the revoking cost faced by both agents.[1] Subsequently a second stage game is played where each agent must decide whether to stick to her original demand or accept the other's offer. If player 1 sticks to her demand while player 2 concedes then player 1 gets her original demand. Player 2 gets the share offered by player 1 and also pays her revoking cost. If both players concede then both get their opponents offer, pay their respective revoking costs and split in half the excess of the surplus over the sum of their offers. Both players sticking to their incompatible demands results in disagreement with a resulting payoff of 0 to both. When making their demands both players only know the distributions for the revoking costs.

Given this general framework it is shown that assumptions regarding the distribution of revoking costs crucially determine the qualitative predictions of the model. In particular, as is done in Crawford(1982), the model can be made relatively simpler by assuming that the revoking cost for each player can either be 0 or some value greater than 1. Assuming the distributions are independent, it is shown that both players demanding the entire pie can in fact be supported as an equilibrium. Indeed, this is true irrespective of the probability of the cost being high. Further if the probability of the cost being 0 is greater than half then all equilibria must involve disagreement. It turns out that even the assumption of the distributions being independent can be dispensed with. Even if both players in the first stage know for sure that they will face exactly the same (but uncertain) revoking cost, disagreement persists. Both parties demanding the entire pie continues to be supported in equilibrium. One may consider these results to provide strong support to Schelling's view. However, it is shown that while gaining tractability by considering discrete distributions, a crucial yet subtle aspect of such bargaining environments is lost. In particular these models feature equilibria where the probability with which a player backs down in the second stage does not depend upon the amount demanded by either player. Notice that a player has no option but to stick to her demand when her revoking cost is high. Consequently if an agent faces a revoking cost of 0 against one who faces the high cost, the unique second stage equilibrium behavior would involve the former backing down. The existence of multiple equilibria in the second stage game when both agents face 0 costs makes supporting disagreement essentially a question of selecting an appropriate equilibrium. Crucially this equilibrium selection does not have to systematically depend upon the amounts demanded. This results in disagreement all too readily.

[1] Section 2.1 considers an asymmetric information environment where each agent only gets to know her own revoking cost.

The paper then studies the case where players *do not* believe that intermediate revoking costs are impossible. In particular, the density functions for the revoking costs are assumed to be strictly positive and continuous over an interval between and including 0 and some value greater than 1. While such density functions exacerbate the second stage multiplicity problem mentioned above, it also allows for the use of a global game information structure. It is assumed that before playing the second stage game each player gets to know the revoking costs faced by both players, but with a small amount of noise. The equilibrium properties of this model is studied for the limiting case when the amount of noise is made arbitrarily small. The use of such global game arguments resolves the second stage multiplicity, and reveals markedly different equilibrium behavior. In particular it is shown that when both players know that they will face the same (but uncertain) cost of backing down, if the amount of noise is small enough, disagreement can not be supported in equilibrium, irrespective of the density function. Efficient equilibria, on the other hand, always exists. Even if the revoking cost distributions are assumed to be independent, it is shown that for any distribution function that *first order stochastically dominates* the uniform distribution, not only can the efficient demand profile $(1/2, 1/2)$ be supported, disagreement can not be supported in equilibrium.

To understand the intuition behind these results it will help to spell out the counteracting forces involved in the model. Disagreement arises if both parties make high demands that consequently are incompatible, since there is always a state of the world where neither player can back down following such a demand profile. Player 1's incentive to make a higher demand is driven by the possibility that following incompatible demands she will face a high revoking cost (and therefore *achieve commitment*), while player 2 faces a low cost and is therefore better off conceding. The corresponding disincentive to making a high demand arises from the possibility that following incompatible demands both players will face high costs resulting in disagreement and the resultant loss of the entire surplus. These two features are present in both the discrete and continuous distribution models. The continuous distribution models along with the global games information structure, however, gives rise to a second disincentive to making higher demands. A higher demand makes it more difficult for one's opponent to concede thereby conferring a greater probability of success to the latter's commitment attempt. This in turn reduces the payoff an agent can hope to get by making the higher demand. It is the addition of this disincentive that results in the lack of disagreement in the continuous density models. The global game structure results in the risk dominant outcome of the second stage being played as a result of iterated elimination of dominated strategies whenever there would otherwise be multiple equilibria. This argument is especially acute for the case where both agents face the same revoking cost. Given an incompatible demand profile, in equilibrium, if one player makes a (sufficiently) higher demand than the other, then in the second stage either both players stick to their demands (when the cost is high enough) or the player with the higher demand backs down while the one with the lower demand gets her way. When the distributions are independent, the players essentially weigh the benefits of making a higher demand against the subsequent shrinking of the risk dominant region (of the state space) where she actually gets her demand.

The basic framework of the present analysis is almost identical to that of Crawford(1982). The only difference is the payoffs that result following incompatible demands if both player's choose to back down. In Crawford(1982) the payoff is given by an exogenously set compromise payoff, while in the present model each player gets what the other offered and gets half the remaining surplus. This assumption is also made in Kambe(1999), Abreu and Gul(2000) and Compte and Jehiel(2002). To show that this difference preserves the arguments leading to disagreement in the asymmetric information model in Crawford(1982), the asymmetric information disagreement results are replicated using the present model. Given that the analysis gets rid of an additional parameter (the compromise solution), the disagreement result of Crawford(1982) can in fact be seen in a simpler setting.

It should be noted that Crawford(1982) initially presents a model in which the revoking costs are continuously distributed and have intervals for support. Unfortunately, however, this setting turns out to be intractable. The analysis then focuses on discrete distributions where the costs can either be 0 or so high that concession is found to be too costly irrespective of the demands made. However, even in this simpler setting, it must be noted that given the asymmetric information environment making a higher demand does increase the probability of success of an opponent's commitment attempt. So this crucial feature, while preserved in asymmetric information environments when dealing with discrete distributions, is lost in symmetric information environments with discrete distributions. This issue itself necessitates the study of continuous densities in the present paper. The symmetric information environment then allows the use of global game arguments to make the model tractable. The results from when the revoking costs are independent are qualitatively similar to that of Crawford(1982) in that if the revoking cost is expected to be high (the ex ante probability of a successful commitment is high) then disagreement does not occur.

The results, however, are in sharp contrast with the findings in Ellingsen and Miettinen(2008)(henceforth EM) who also analyze symmetric information environments. EM show that presence of uncertain commitment always results in disagreement. This surprising finding is delivered by the ability of the agents in their model to use random commitment devices. Upon making use of such a device and following incompatible demands, the bargainer is either forced to stick to her demand, or forced to back down, with exogenously fixed probabilities. While it is easy to think of devices that may force someone to stick to their demand (a high realized value of the cost of backing down, for example), it is not clear why with some fixed probability a player would be forced to back down, irrespective of the other players action. To see the difference between such an outcome and the possibility of facing low costs of backing down, consider the case where both players demand the entire surplus. The use of such devices would result in a state of the world (with positive probability) that involves both parties backing down. This is not the same as costs being low for both parties, since in the latter case one agent backing down would give a strict incentive for the other to stick to her demand. Not surprisingly, the EM model predicts disagreement all too readily. Importantly it also involves both players making the highest demand possible. The key modeling difference here is that in the present paper *achieving commitment* is required to be the result of equilibrium behavior as in Crawford(1982).

Following the replication of the asymmetric information results from Crawford(1982), Section 2 extends the disagreement results to the symmetric setup where both players get to know each others cost before the second stage, and finally to the setup where both players face exactly the same cost of backing down, that they commonly learn before the second stage game. Throughout section 2, the distribution for the cost always has only two points in its support. Section 3 considers the case where both parties face the same cost that is distributed over an interval. It states the agreement result that arises when players get to know the cost with a sufficiently small amount of noise. Section 4 deals with the case when the revoking costs are independent. Section 5 concludes.

2 Replication and extension of disagreement results

For the rest of the section the following basic model applies. Each subsection will add a different set of assumptions to this framework. Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1, 2\}$, with j being the other player, $j \in \{1, 2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0, 1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with $d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k_1, 1 - z_1 + d - k_2$	$1 - z_2 - k_1, z_2$
Stick	$z_1, 1 - z_1 - k_2$	$0, 0$

Table 1: Payoffs following incompatible demands

2.1 Two point independent distributions and asymmetric information case

Add to the game defined above, the assumption that players in the first stage do not know the value of k_i. They only know that they are independent random variables with $Pr(k_i > 1) = q$ and $Pr(k_i = 0) = 1 - q$. Following incompatible demands and before playing the second stage game, players get to know their own but not their opponent's k_i.

Proposition 1. *For any value of $q \in (0,1)$ there exists an equilibrium with a positive probability of disagreement.*

Proof. Fix $q \in (0,1)$. Let $z = \frac{q+1}{2}$. Following an incompatible demand profile (z_1, z_2), in the second stage Bayesian game, player i must always play the strictly dominant action *Stick* when $k_i > 1$. Equilibrium behavior when $k_i = 0$ needs to be pinned down. In this regard notice that playing *Accept* when $k_i = 0$ for both i, would constitute a Bayesian Nash Equilibrium if the following two inequalities hold.

$$q(1 - z_2) + (1 - q)(1 - z_2 + d) \geq (1 - q)z_1 \tag{1}$$

$$q(1 - z_1) + (1 - q)(1 - z_1 + d) \geq (1 - q)z_2 \tag{2}$$

The left hand (right hand) side of the inequalities gives the expected payoff to the player with $k_i = 0$ from playing *Accept* (*Stick*) when her opponent's strategy involves playing *Accept* when the cost is zero and *Stick* when it is greater than 1. (1) and (2) hold with equality if $z_1 = z_2 = z = \frac{q+1}{2}$.[2] Clearly the demand profile (z, z) is incompatible.

Consider now the following strategies. Each player demands z. Following the demand profile (z, z) player i plays *Accept* when $k_i = 0$ and *Stick* when $k_i > 1$. Following a demand profile where $z_i = z$ but $z_j > z$, player i plays *Stick* irrespective of k_i while j plays *Accept* when $k_j = 0$ and *Stick* when $k_j > 1$. Following an *incompatible* demand profile where $z_i = z$ but $z_j < z$, both players play *Accept* when their cost is 0 and *Stick*, when it is high. The strategies also subscribe actions that constitute a BNE for any subgame not considered above. It will be shown that such a strategy profile constitutes a Perfect Bayesian Nash Equilibrium of the game.

Consider first, behavior in the second stage subgames. Only the behavior of the types facing $k_i = 0$ needs to be checked, since i must always play *Stick* when $k_i > 1$ as it is the strictly dominant action in that case. Following the profile (z, z) both players with 0 cost play *Accept*. It has been shown earlier that for this to be a BNE (1) and (2) must be satisfied. Given the derivation of z, this is in fact the case. For incompatible demand profiles where $z_i = z$ and $z_j > z$, the strategies suggest that the low type of player i should play *Stick* while player j with $k_j = 0$ should play *Accept*. Given j's strategy i's low type choice would be optimal if

$$q(1 - z_j) + (1 - q)(1 - z_j + d) < (1 - q)z \tag{3}$$

Given that this relation holds with equality when $z_j = z$ and that the left hand side is strictly decreasing in z_j, it must be that for $z_j > z$, (3) is indeed satisfied. Further given that player i plays *Stick* always, player j does strictly better by playing *Accept* when $k_j = 0$. Finally for incompatible demand profiles with $z_i = z$ and $z_j < z$, notice that the inequalities (1) and (2) continue to be

[2] Note that $d(z, z) = \frac{q}{2}$

satisfied. As a result the strategies involving low cost types playing *Accept* does induce a BNE in such subgames. As for the first stage decisions, consider player 1. The expected payoff to 1 from demanding z when 2 demands z is given by $q(1-q)z+(1-q)[q(1-z)+(1-q)(1-z+(2z-1)/2)]$. If 1 demands less than z, $(z_1 < z)$ her expected payoff is $q(1-q)z_1+(1-q)[q(1-z)+(1-q)(1-z+(z+z_1-1)/2)]$ which is clearly less than her payoff from not deviating. If 1 demands $z_1 > z$ then her expected payoff is merely $(1-q)(1-z)$, again strictly less than if she had not deviated. It remains to be shown that no player would want to deviate from the profile (z, z) to making the compatible demand $1-z$. Suppose this is a profitable deviation. Then it must be that,

$$\begin{aligned}
& q(1-q)z+(1-q)[q(1-z)+(1-q)(1-z+d)] < 1-z \\
\Rightarrow & q(1-q)z+(1-q)(1-z)+(1-q)^2 d < 1-z \\
\Rightarrow & q(1-q)z-q(1-z)+(1-q)^2\frac{q}{2} < 0 \\
\Rightarrow & z-zq-1+z+\frac{(1-q)^2}{2} < 0 \\
\Rightarrow & 2z-1-zq+\frac{(1-q)^2}{2} < 0 \\
\Rightarrow & q-\frac{q+1}{2}q+\frac{(1-q)^2}{2} < 0 \\
\Rightarrow & 2q-q^2-q+1-2q+q^2 < 0 \\
\Rightarrow & q > 1
\end{aligned} \tag{4}$$

(4) contradicts the initial assumption of $q \in (0, 1)$. As a result no player would want to deviate to making a compatible offer, from the incompatible profile (z, z). □

Proposition 2. *If* $0 < q < \frac{1}{2}$ *then any equilibrium must entail a positive probability of disagreement.*

Proof. Suppose not. Let the compatible demand profile supported in equilibrium be (z_1, z_2) where $z_1 + z_2 = 1$. WLOG let $z_1 \leq z_2$. Notice that substituting z_1 and z_2 into the inequalities (1) and (2) makes the inequalities strict. Further $d(z_1, z_2) = 0$. In particular, $q(1-z_2)+(1-q)(1-z_2) > (1-q)z_1$. Consequently if player 1 makes a higher demand, $z_1+\delta$, the inequality will still be satisfied for small enough values of δ. Indeed, to satisfy the inequality (1), δ should satisfy, $q(1-z_2)+(1-q)(1-z_2+(\delta/2)) \geq (1-q)(z_1+\delta)$, which in turn implies that,

$$\delta \leq \frac{2qz_1}{1-q} \tag{5}$$

To ensure that such a deviation maintains the second inequality it must be that, $q(1-z_1-\delta)+(1-q)(1-z_1-\delta+(\delta/2)) \geq (1-q)z_2$. This in turn, simplifies to,

$$\delta \leq \frac{2qz_2}{1+q} \tag{6}$$

So if δ satisfies both (5) and (6), then following such a deviation, the subgame involving the incompatible demand profile, $(z_1+\delta, z_2)$, would involve both players playing *Stick* when the cost is high and *Accept* when it is 0. To see that no other BNE exists in the second stage game, note that both low types playing *Stick* cannot occur in equilibrium. Further given that the inequalities (5) and (6) are satisfied, if one of the low types plays *Accept* then the low type of the other player must also play *Accept.* The expected payoff to player 1 from such a profile would therefore be, $q^2(0)+q(1-q)(z_1+\delta)+(1-q)[q(1-z_2)+(1-q)(1-z_2+(\delta/2))]$. For this deviation to be profitable it must be that,

$$\begin{aligned}
&[q(1-q)+(1-q)]z_1+q(1-q)\delta+(1-q)^2(\delta/2)>z_1\\
\Rightarrow &q(1-q)\delta+(1-q)^2(\delta/2)>z_1q^2\\
\Rightarrow &(1-q^2)\delta>2z_1q^2\\
\Rightarrow &\delta>\frac{2z_1q^2}{1-q^2} \qquad (7)
\end{aligned}$$

Let $z_1>0$. Then for such a deviation to exist, it simply needs to be shown that there exists $\delta>0$ that simultaneously satisfies (5), (6) and (7). Notice that $\frac{2z_1q^2}{1-q^2}<\frac{2qz_1}{1-q}\Leftrightarrow\frac{q}{1+q}<1$, and is satisfied for all $q>0$. Further $\frac{2z_1q^2}{1-q^2}<\frac{2qz_2}{1+q}\Leftrightarrow\frac{z_1q}{1-q}<z_2$. Given that $z_1\leq z_2$, this is satisfied for all $q<1/2$. Consequently, if $z_1>0$ and $0<q<1/2$, there always exists a profitable deviation for player 1.

For the case where $z_1=0$ and $z_2=1$. If 1 deviates by demanding $\delta>0$ that satisfies $\delta<\frac{2q}{1+q}$, the inequality (1) would be reversed and hold strictly. In other words following the demand profile $(\delta,1)$, if player 2 plays *Accept* when $k_2=0$ and *Stick* otherwise, then player 1 would play *Stick* always. Also, given that 1 plays *Stick* always, 2's optimal action when $k_2=0$ is indeed to play *Accept* since it gives a payoff of $1-\delta$ as opposed to the payoff of 0 if *Stick* is played. So these strategies constitute a BNE of the subgame following $(\delta,1)$. Both players playing *Stick* always is not a BNE of this subgame since the low type of player 2 would strictly prefer to play *Accept*, as just described. The low types of both players playing *Accept* cannot happen due to the strict reversal of the inequality (1). So the only other potential BNE of this subgame involves player 2 playing *Stick* always while the low type of player 1 plays *Accept.* This would require the low type of player 2 to choose *Stick*, requiring, $q(1-\delta)+(1-q)(1-\delta+(\delta/2))\leq(1-q)(1)$. But, this inequality is violated if $\delta<\frac{2q}{1+q}$. The only BNE following a deviation to δ, therefore involves player 1 always playing *Stick* with the low type for player 2 playing *Accept.* Since this deviation gives a strictly positive payoff to player 1 it is a profitable deviation.

So it has been shown that given any compatible demand profile (z_1,z_2) with $z_1\leq z_2$ as long as $0<q<1/2$, there always exists a profitable deviation for player 1. Clearly, a symmetric argument applies for $z_2\leq z_1$. Consequently with $0<q<1/2$ there cannot be any equilibrium involving compatible demands. $\square$

2.2 Two point independent distributions and symmetric information case

In this subsection, in addition to the basic model outlined earlier, it is assumed that while the costs of backing down are uncertain to both players at the demand stage, they become common knowledge following incompatible demand profiles. In particular, in the first stage it is common knowledge that player i faces cost k_i which takes a value greater than 1 with probability q while $Pr(k_i = 0) = 1 - q$. The two random variables are assumed to be independent. Following incompatible demands the true values of k_1 and k_2 are made common knowledge before the second stage game is played. The only difference from **Subsection 2.1** lies in the elimination of asymmetric information in the second stage game.

Proposition 3. *For* $0 < q < 1$ *the incompatible demand profile* $(1, 1)$ *can be supported in equilibrium, resulting in disagreement with probability* q^2*.*

Proof. Consider the following strategies. Both players demand 1 in the first stage. Following any incompatible demand profile (z_1, z_2), player i plays *Stick* when $k_i > 1$. If $k_i = 0$ and $k_j > 1$, then player i plays *Accept*. If $k_1 = k_2 = 0$, then player 1 plays *Stick* while player 2 plays *Accept*.

Table 1 makes it clear that the strategies outlined above induce a Nash Equilibrium in every subgame following incompatible demand profiles. Notice that these subgames are dominance solvable except for the case where $k_1 = k_2 = 0$. In the latter case both $(Accept, Stick)$ and $(Stick, Accept)$ are Nash Equilibria. The particular selection made in this case is entirely arbitrary, but sufficient to support the incompatible profile as an equilibrium outcome.

The expected payoff to player 1 from the strategies above is $q(1-q)(1)+(1-q)(1-q)(1)$. Deviating to any lower incompatible demand z_1 gives an expected payoff, $q(1-q)(z_1) + (1-q)(1-q)(z_1)$, while making a compatible demand gives a payoff of 0. So player 1 has no incentive to deviate. Player 2's expected payoff from the stated strategies is $q(1-q)(1)$. Deviating to a lower but still incompatible demand, z_2, gives her $q(1-q)z_2$. Finally deviating to a compatible demand gives her 0. As a result player 2 also has no incentive to deviate. □

Proposition 4. *For* $0 < q < 1/2$*, no efficient equilibrium exists.*

Proof. Suppose not. Let (z_1, z_2) be supported in equilibrium, where $z_1 + z_2 = 1$. Suppose player i deviates to demanding $\tilde{z}_i = 1$. Player i's expected payoff from such a deviation must be no less than $q^2(0) + q(1-q)(1) + (1-q)q(1-z_j) + (1-q)^2(1-z_j) = q(1-q) + (1-q)z_i$. For such a deviation to not be profitable it must be that $z_i \geq q(1-q) + (1-q)z_i$. This implies, $z_i \geq 1 - q$. Given that $q < 1/2$ and $z_1 + z_2 = 1$, it must be that for some $i \in \{1, 2\}$, $z_i < 1 - q$ holds. Such a player i would then do strictly better by deviating to a demand of 1. □

2.3 Two point identical distribution and symmetric information case

The final extension involves getting rid of the assumption that the random variables determining the cost of backing down are independent. In particular, it is still assumed that both players are uncertain about their cost of backing down while making their first stage demands. Further $Pr(k_i > 1) = q$ and $Pr(k_i = 0) = 1 - q$. In addition it is assumed that $Pr(k_1 = k_2) = 1$; both players know that they will face exactly the same cost following incompatible demands. Let the common cost be denoted k. This cost becomes common knowledge after an incompatible demand profile and before the second stage game is played.

Proposition 5. *For $0 < q < 1$ the incompatible demand profile $(1, 1)$ can be supported in equilibrium, resulting in disagreement with probability q^2.*

Proof. When $k > 1$, the unique Nash Equilibrium in the second stage game involves both players playing *Stick*. $k = 0$, on the other hand, results in two pure strategy NE, namely $(Accept, Stick)$ and $(Stick, Accept)$. Consider the following strategies. Both players demand 1. Following any incompatible demand profile (z_1, z_2), if $k = 0$, player 1 plays *Stick* while 2 plays *Accept*. Facing $k > 1$, both players play *Stick*. As mentioned earlier, the subgame strategies constitute Nash Equilibria. Player 1 gets an expected payoff of $1 - q$. By deviating to making any other demand z_1, the expected payoff would become strictly less, $(1 - q)z_1$. Player 2, on the other hand, would always get 0 irrespective of her first stage demand and therefore has no incentive to deviate. Consequently the strategies support the demand $(1, 1)$ in equilibrium. The subsequent probability of disagreement is therefore q^2. □

3 Identical costs of backing down and agreement

Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1, 2\}$, with j being the other player, $j \in \{1, 2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0, 1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with $d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k, 1 - z_1 + d - k$	$1 - z_2 - k, z_2$
Stick	$z_1, 1 - z_1 - k$	$0, 0$

Table 2: Payoffs following incompatible demands

In the first stage, when choosing their demands, players' prior regarding the cost of backing down k is given by a random variable K which takes values in

$[0, \bar{k}]$ where $\bar{k} > 1$. Having announced their demands, each player i gets a noisy signal, k_i^ϵ about k before playing the simultaneous move game. In particular, player i observes a realization of the random variable K_i^ϵ that is defined by

$$K_i^\epsilon = K + \epsilon E_i, \qquad i = 1, 2$$

where E_i is a random variable taking values in $\mathbb{R}$ and $\epsilon > 0$ serves as the scale parameter for the noise. A strategy for player i, comprises of a demand $z_i \in [0, 1]$ and a measurable function $s_i(z_1, z_2)$ for every incompatible demand profile, that gives the probability of playing *Accept* as a function of the the observed cost of backing down k_i^ϵ. So, $s_i(z_1, z_2) : [-\epsilon, \bar{k} + \epsilon] \to [0, 1]$. Γ^ϵ is used to denote this two stage game for a particular value of ϵ.

The following assumptions are made on the parameters of the model.

A1. *K admits a density h that is continuously differentiable on $(0, \bar{k})$, strictly positive, continuous and bounded on $[0, \bar{k}]$.*

A2. *The vector (E_1, E_2) is independent of K and admits a density φ.*

A3. *The support of each E_i is contained in the interval $[-1, 1]$ in $\mathbb{R}$ and φ is continuous on $[-1, 1] \times [-1, 1]$.*

I am interested in the perfect equilibrium prediction of Γ^ϵ for small values of ϵ. To this effect the following proposition holds.

Proposition 7. *Given $A1, A2, A3$, and for sufficiently small $\epsilon > 0$, if players use pure strategies for their first stage demands, there is never any disagreement in any perfect equilibrium of the game Γ^ϵ.*

It should be pointed out that the assumptions $A1, A2, A3$, are slightly weaker than the corresponding assumptions made for the one-dimensional case in Carlsson and van Damme(1993) (henceforth CvD). In particular the noise density function is allowed to be discontinuous at the boundary points of its support in the present study, while this is ruled out by the assumptions in CvD.[3]

The outline of the proof is as follows. *Lemma 1* establishes the crucial result that the conditional distribution of player 1's observation conditional on player 2's observation is symmetric to the conditional distribution of player 2's observation conditional on player 1's observation, in the sense that they add up arbitrarily close to 1. *Lemma 2* makes sure that even with the slightly weaker assumptions made in this paper, given measurable strategies, the probability with which player i chooses a particular action and the expected value of the true cost, conditional on player j making some observation, is continuous in player j's observation. The intuitive result that for very high value of observations both players will chose to play *Stick* is established first. It is then argued

[3]Indeed, the motivating example in CvD involves noise with a uniform density, and does not satisfy the assumptions of their paper. However the discontinuity at the boundary points merely requires a little more work as is done in *Lemma 2*, and does not endanger the equilibrium selection argument in CvD. I thank Hans Carlsson for clearing my doubt regarding this issue.

that following incompatible demands (z_1, z_2) if z_i is sufficiently larger than z_j, then there will always be observation values for which the unique dominance solvable outcome would involve i backing down while j plays *Stick*. *Lemmas 5 - 7* then show that following such an incompatible demand profile, either for all lower observations i will continue to back down with j playing *Stick*, or there will be two observation values really close to each other where the two players will switch their actions. *Lemma 8*, the critical part of the proof, then shows that if z_i is sufficiently larger than z_j, such switch points cannot exist and therefore player i will continue to back down with j playing *Stick*. This result is a consequence of the global games information structure used in the model for small enough $\epsilon > 0$. Player i backing down with j sticking to her demand turns out to be the risk dominant outcome whenever K takes values giving rise to multiple equilibria in the second stage following such incompatible profiles. Given that backing down for player i always pays her less than if she had simply made a compatible offer in the first stage, such incompatible profiles cannot constitute an equilibrium. The analysis also allows for a characterization of the expected payoffs such incompatible demands entail. I then consider the choice of first stage demands. It can be easily seen that demands that add up to less than 1 always allow for deviations. Next I establish a lower bound that the sum of the demands must satisfy for any incompatible profile from which neither player wants to deviate to a compatible profile. Finally it is shown that if an incompatible profile of demands involves z_1 and z_2 that do not differ much in value but sum up to greater than the bound mentioned above, then there is always a player i who could strictly improve her payoff by making a lower but still incompatible demand. This lower demand by i forces j, in equilibrium, to always back down in the second stage. These arguments together exhaust the possible set of incompatible demand profiles. Consequently it is shown that equilibria involving pure strategies in the first stage cannot involve incompatible demands, thereby eliminating the possibility of disagreement.

First I define a few terms for the game Γ^ϵ that allow the use of *Lemma 4.1* in Carlsson and van Damme(1993), henceforth (CvD). Let $F_i^\epsilon(k_j|k_i)$ and $f_i^\epsilon(k_j|k_i)$ be the distribution and density functions, respectively, of K_j^ϵ conditional on $K_i^\epsilon = k_i$. Let φ^ϵ be the joint density of $(\epsilon E_1, \epsilon E_2)$. Then,

$$f_i^\epsilon(k_j|k_i) = \frac{\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk}{\int\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_j dk} \tag{8}$$

The following lemma is the one dimensional version of *Lemma 4.1* in *CvD* that applies to the present model.

Lemma 1 (CvD). *Let $k_1, k_2 \in [-\epsilon, \bar{k} + \epsilon]$. Then their exists a constant $\kappa > 0$ such that for sufficiently small $\epsilon > 0$,*

$$|F_1^\epsilon(k_2|k_1) + F_2^\epsilon(k_1|k_2) - 1| \le \kappa\epsilon \tag{9}$$

Proof. Let $l = \max_{k\in[0,\bar{k}]} |h'(k)|$, where $h'(k)$ is the derivative of the function h at k for $k \in (0, \bar{k})$ with $h'(0)$ and $h'(\bar{k})$ defined as $\lim_{k\to 0} h'(k)$ and $\lim_{k\to\bar{k}} h'(k)$, respectively. Given **A1**, l is well defined with $l \geq 0$. Let $\nu = \min_{k\in[0,\bar{k}]} h(k)$. Given that h is continuous and strictly positive on $[0, \bar{k}]$, ν is well defined with $\nu > 0$. Let ϵ be such that $l\epsilon < \nu/2$. Then (8) leads to the following inequality for all $k_i, k_j \in [0, \bar{k}]$,

$$f_i^\epsilon(k_j|k_i) \leq \frac{(h(k_i) + l\epsilon)\int \varphi^\epsilon(k_1 - k, k_2 - k)dk}{(h(k_i) - l\epsilon)\int\int \varphi^\epsilon(k_1 - k, k_2 - k)dk_j dk} = \frac{(h(k_i) + l\epsilon)\psi^\epsilon(k_1 - k_2)}{h(k_i) - l\epsilon}$$

ψ^ϵ is the density function for $\epsilon E_1 - \epsilon E_2$ and is equal to the integral in the numerator of the second term for given values of k_1 and k_2 . Note that the double integral in the denominator of the second term above is equal to 1. Similarly, $\frac{(h(k_i)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(k_i)+l\epsilon} \leq f_i^\epsilon(k_j|k_i)$. For $k_i \in [-\epsilon, 0]$ the relevant inequality is $\frac{(h(0)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(0)+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)-l\epsilon}$. If $k_i \in [\bar{k}, \bar{k} + \epsilon]$ then the inequality is $\frac{(h(\bar{k})-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(\bar{k})+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})-l\epsilon}$. Therefore,

$$(1 - \frac{2l\epsilon}{h(k_i) + l\epsilon})\psi^\epsilon(k_1 - k_2) \leq f_i^\epsilon(k_j|k_i) \leq (1 + \frac{2l\epsilon}{h(k_i) - l\epsilon})\psi^\epsilon(k_1 - k_2)[4]$$

Further let $\kappa = \frac{8l}{\nu}$. Now,

$$\begin{aligned} 1 + \frac{2l\epsilon}{h(k_i) - l\epsilon} &\leq 1 + \frac{2l\epsilon}{\nu - l\epsilon} \\ &\leq 1 + \frac{2l\epsilon}{\nu/2} \end{aligned}$$

Also,

$$\begin{aligned} 1 - \frac{2l\epsilon}{h(k_i) + l\epsilon} &\geq 1 - \frac{2l\epsilon}{h(k_i) - l\epsilon} \\ &\geq 1 - \frac{2l\epsilon}{\nu - l\epsilon} \\ &\geq 1 - \frac{2l\epsilon}{\nu/2} \end{aligned}$$

Then,

$$\psi^\epsilon(k_1 - k_2)(1 - (\kappa\epsilon)/2) \leq f_i^\epsilon(k_j|k_i) \leq \psi^\epsilon(k_1 - k_2)(1 + (\kappa\epsilon)/2) \tag{10}$$

$$\Rightarrow \int_{y\leq k_2} \psi^\epsilon(k_1 - y)dy - (\kappa\epsilon)/2 \leq F_1^\epsilon(k_2|k_1) \leq \int_{y\leq k_2} \psi^\epsilon(k_1 - y)dy + (\kappa\epsilon)/2 \tag{11}$$

[4] For values of k_i in $[-\epsilon, 0]$ and $[\bar{k}, \bar{k} + \epsilon]$ replace $h(k_i)$ by $h(0)$ and $h(\bar{k})$, respectively.

(10) also implies,

$$\int_{z\leq k_1}\psi^\epsilon(z-k_2)dz-(\kappa\epsilon)/2\leq F_2^\epsilon(k_1|k_2)\leq\int_{z\leq k_1}\psi^\epsilon(z-k_2)dz+(\kappa\epsilon)/2$$
$$\Rightarrow\int_{z\geq k_1}\psi^\epsilon(z-k_2)dz+(\kappa\epsilon)/2\geq 1-F_2^\epsilon(k_1|k_2)\geq\int_{z\geq k_1}\psi^\epsilon(z-k_2)dz-(\kappa\epsilon)/2$$
$$\Rightarrow\int_{y\leq k_2}\psi^\epsilon(k_1-y)dy+(\kappa\epsilon)/2\geq 1-F_2^\epsilon(k_1|k_2)\geq\int_{y\leq k_2}\psi^\epsilon(k_1-y)dy-(\kappa\epsilon)/2 \tag{12}$$

Subtracting (12) from (11) gives the required inequality.

□

Next, it is shown that player i's expectation regarding the true value of k and the probability with which j plays *Accept*, conditional on observing k_i^ϵ are continuous functions of k_i^ϵ. Given j's second stage strategy s_j, let the probability with which i, conditional on observing k_i^ϵ, expects that j will play *Accept* be denoted by $Pr(A_j|k_i^\epsilon, s_j)$.[5] So,

$$Pr(A_j|k_i^\epsilon,s_j)=\int s_j(k_j)f_i^\epsilon(k_j|k_i^\epsilon)dk_j \tag{13}$$

Also, let i's expectation of k given her observation k_i^ϵ be denoted as $E^\epsilon(k|k_i^\epsilon)$.

Lemma 2. *For a given incompatible demand profile (z_1, z_2) and strategies s_i, s_j, $Pr(A_j|k_i^\epsilon, s_j)$ and $E^\epsilon(k|k_i^\epsilon)$ are continuous in player i's observation k_i^ϵ.*

Proof. The continuity of φ^ϵ is implied by the continuity of φ assumed in *A2*. Consider the numerator in the expression for $f_i^\epsilon(k_j|k_i)$ as expressed in (8). WLOG take a sequence k_1^n that converges to k_1, such that $k_1^n \in [-\epsilon, \bar{k}+\epsilon]$ for all n. Given the continuity of φ^ϵ it is immediate that holding k_2 fixed, $h(k)\varphi^\epsilon(k_1^n-k, k_2-k)\to h(k)\varphi^\epsilon(k_1-k,k_2-k)$, *almost everywhere* in $[0,\bar{k}]$. Further $h(k)\varphi^\epsilon(k_1^n-k,k_2-k)\leq h(k)\bar{\varphi}^\epsilon$ for all n and k, where $\bar{\varphi}^\epsilon$ is the maximum value taken by the function φ on $[-1,1]\times[-1,1]$. Consequently by the *Dominated Convergence Theorem*, $\int h(k)\varphi^\epsilon(k_1-k,k_2-k)dk=\lim_{n\to\infty}\int h(k)\varphi^\epsilon(k_1^n-k,k_2-k)dk$. In other words, $\int h(k)\varphi^\epsilon(k_1-k,k_2-k)dk$ is continuous in k_i. For the denominator in (8), consider first the marginal density. Fix k. Let $k_1\notin\{k-\epsilon,k+\epsilon\}$. Then for any sequence k_1^n that converges to k_1 it must be the case that $\varphi^\epsilon(k_1^n-k,k_2-k)\to\varphi^\epsilon(k_1^n-k,k_2-k)$ for all values of k_2, by **A3**. Again by the *Bounded Convergence Theorem*, the marginal $\int\varphi^\epsilon(k_1^n-k,k_2-k)dk_2$ for a given value of k is found to be continuous at all k_1 other than potentially two points, $k-\epsilon$ and $k+\epsilon$. Consequently for any sequence k_1^n that converges to k_1, it is true that $h(k)\int\varphi^\epsilon(k_1^n-k,k_2-k)dk_2\to h(k)\int\varphi^\epsilon(k_1-$

[5] The dependence of s_j on the demand profile (z_1, z_2) is suppressed for notational convenience, but it should be noted that the arguments are for a given pair of incompatible demands.

$k, k_2 - k)dk_2$ for all values of k other than possibly $k_1 - \epsilon$ and $k_1 + \epsilon$. Further, $h(k) \int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2 \leq h(k)\bar{\varphi}^\epsilon$ for all k, n. By the *Dominated Convergence Theorem*, it must be that $\int h(k) \int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2 dk$, the denominator in (8), is continuous in k_1. Given **A1** and the additive structure of the noise, the denominator is also strictly positive for all $k_1 \in (-\epsilon, \bar{k} + \epsilon)$. Therefore for all $k_1, k_2 \in [-\epsilon, \bar{k} + \epsilon]$, $f_i^\epsilon(k_j|k_i)$ is continuous in k_i. $f_i^\epsilon(k_j|k_i)$ is also continuous in k_j, since k_j does not affect the denominator of (8), while its influence on the numerator is symmetric to that of k_i. So let $\bar{f}_\epsilon$ be the maximum value taken by $f_i^\epsilon(k_j|k_i)$ for $k_1, k_2 \in [-\epsilon, \bar{k}+\epsilon]$. Then for any measurable function s_j, it must be that $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \to s_j(k_j)f_i^\epsilon(k_j|k_i)$ if $k_i^n \to k_i$ and $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \leq s_j k_j \bar{f}_\epsilon$, for all values of k_j. Therefore by the *Dominated Convergence Theorem*, $Pr(A_j|k_i^\epsilon, s_j) = \int s_j(k_j)f_i^\epsilon(k_j|k_i^\epsilon)dk_j$ is continuous in k_i^ϵ.

To show that $E^\epsilon(k|k_i^\epsilon)$ is continuous in k_i^ϵ consider first the conditional density of the true k given an observation k_i.

$$f_i^\epsilon(k|k_i) = \frac{\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_j}{\int \int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_j dk} \tag{14}$$

Continuity of the denominator of (14) in k_i has already been established before. The numerator for a given k is the product of the strictly positive $h(k)$ and the marginal density of k_i. It has been shown earlier that for a given k the marginal density of k_i is continuous at all k_i other than possibly when $k_i \in \{k - \epsilon, k + \epsilon\}$, the boundary points. As a result, for a given k, $f_i^\epsilon(k|k_i)$ is continuous for all k_i other than the two boundary points. Therefore for a sequence k_i^n that converges to k_i, $kf_i^\epsilon(k|k_i^n) \to kf_i^\epsilon(k|k_i)$ for all k other than possibly when $k \in \{k_i - \epsilon, k_i + \epsilon\}$. Further since the denominator in (14) is bounded below and the numerator bounded above, the *Dominated Convergence Theorem* delivers the continuity of $E^\epsilon(k|k_i^\epsilon) = \int kf_i^\epsilon(k|k_i^\epsilon)dk$ in k_i^ϵ. □

Equilibrium behavior in the second stage game following an incompatible demand profile is considered next. The payoffs specified in Table 2 make it evident that if the observed cost is high enough the player would strictly prefer to play *Stick*. The following lemma captures this immediate but useful implication of observing such high costs of backing down.

Lemma 3. *In equilibrium, following an incompatible demand profile (z_1, z_2), conditional on observing $k_i^\epsilon > 1 - z_j + \epsilon$,* Stick *is the strictly dominant action for player i.*

Proof. Given the payoffs in Table 2, it is clear that whenever j chooses *Accept*, i always does strictly better by choosing *Stick*. Upon observing $k_i^\epsilon > 1 - z_j + \epsilon$ player i knows that for all the possible values that k can take she would get a strictly negative payoff by playing *Accept* if j plays *Stick*. As a result i would still strictly prefer to play *Stick* since it guarantees a payoff of 0 as opposed to the negative expected payoff from playing *Accept*, when j plays *Stick*. Consequently, upon observing $k_i^\epsilon > 1 - z_j + \epsilon$, *Stick* is the strictly dominant action for player i. □

Lemma 3 shows that for high enough observation values (i.e. greater than $1-\min\{z_1, z_2\}+\epsilon$) the unique dominance solvable outcome in the second stage game is (*Stick, Stick*).

The next lemma shows that if the higher of the two incompatible demands is sufficiently larger than the lower demand, there will be an interval of observations that would always lead to a unique dominance solvable outcome in the second stage game where the player with the higher demand plays *Accept* while the other plays *Stick*.

Fix a small $\delta > 0$.

Lemma 4. *For an incompatible demand profile (z_1, z_2) such that $z_i-z_j > \delta+4\epsilon$, the unique dominance solvable outcome of the second stage game following both players making an observation in $(1 - z_i + 3\epsilon, 1 - z_j - \epsilon)$, involves i playing* Accept *and j playing* Stick.

Proof. From lemma 3 it is already known that j plays *Stick* for every observation $k_j^\epsilon > 1-z_i+\epsilon$. Player i making an observation $k_i^\epsilon \in (1-z_i+3\epsilon, 1-z_j-\epsilon)$ learns two things. Firstly, she knows that j must have observed $k_j^\epsilon > 1 - z_i + \epsilon$ and must therefore be playing the strictly dominant *Stick*. Secondly, she knows that the true state k must lie in the interval $(1 - z_i + 2\epsilon, 1 - z_j)$. Conditional on j playing *Stick* for any such value of k, playing *Accept* strictly dominates playing *Stick* for i. The dominance solvable outcome following such an observation, therefore, involves i playing *Accept* while j plays *Stick*. □

Given an equilibrium of Γ^ϵ and a pair of incompatible demands (z_1, z_2) where $z_i - z_j > \delta + 4\epsilon$, let $k_i^{\epsilon*}$ denote the highest observation value k_i^ϵ below $1-z_i+3\epsilon$ for which i chooses to play *Stick*. Similarly let $k_j^{\epsilon*}$ denote the highest observation value k_j^ϵ below $1 - z_i + 3\epsilon$ for which j chooses to play *Accept*. It is assumed that if i following some observation strictly greater than $-\epsilon$ is indifferent between her actions she chooses to play *Stick* while when j is indifferent he plays *Accept*. The next lemma shows that $k_i^{\epsilon*}$ and $k_j^{\epsilon*}$ are well defined. Let $B_i^\epsilon(z_1, z_2)$ denote the set of observations $k_i^\epsilon > -\epsilon$ such that $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ and i plays *Stick* for such observations (i.e. $s_i(k_i^\epsilon) = 0$). Similarly let $B_j^\epsilon(z_1, z_2)$ denote the set of observations $k_j^\epsilon > -\epsilon$ such that $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ and j plays *Accept* for such observations (i.e. $s_j(k_j^\epsilon) = 0$).

Lemma 5. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j > \delta + 4\epsilon$, either $B_i^\epsilon(z_1, z_2)$ is empty or $k_i^{\epsilon*} = \max\{x|x \in B_i^\epsilon(z_1, z_2)\}$ is well defined.*
Similarly, either $B_j^\epsilon(z_1, z_2)$ is empty or $k_j^{\epsilon*} = \max\{x|x \in B_j^\epsilon(z_1, z_2)\}$ is well defined.

Proof. Suppose the statement is false for player i, who makes the higher demand. This means that $B_i^\epsilon(z_1, z_2)$ is non empty but $y = \sup\{x|x \in B_i^\epsilon(z_1, z_2)\} \notin B_i^\epsilon(z_1, z_2)$. So there exists a sequence of observations k_i^n that converge to y, with i playing *Stick* for all n but she plays *Accept* upon observing y. i's expected payoff from playing *Accept* following an observation k_i is given by $1 - z_j - E^\epsilon(k|k_i) + dPr(A_j|k_i)$ while it is $z_iPr(A_j|k_i)$ from playing *Stick*. Given that i

plays *Stick* for all observations in the sequence k_i^n it must be that $z_i Pr(A_j|k_i^n) \geq 1 - z_j - E(k|k_i^n) + dPr(A_j|k_i^n)$. By *Lemma 2*, $E^\epsilon(k|k_i)$ and $Pr(A_j|k_i)$ are continuous in k_i for all measurable strategies, s_j. So if $k_i^n \to y$ it must be that $z_i Pr(A_j|y) \geq 1 - z_j - E(k|y) + dPr(A_j|y)$. Given the tie break rule mentioned earlier this implies that i would play *Stick* upon observing y. This contradicts the earlier claim and proves the lemma for i. A symmetric argument proves the lemma for player j. $\square$

Lemma 6. *If $B_i^\epsilon(z_1, z_2)$ or $B_j^\epsilon(z_1, z_2)$ is empty then they are both empty.*

Proof. Let $B_i^\epsilon(z_1, z_2)$ be empty. Then for all observations $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ player i chooses to play *Accept*. In that case whenever player j receives a signal $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ it is conditionally dominant for him to play *Stick*. This would imply that $B_j^\epsilon(z_1, z_2)$ is empty.

Now if $B_j^\epsilon(z_1, z_2)$ is empty then for all observations $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ player j chooses to play *Stick*. Player i following an observation $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ knows that the true value of k is such that $1 - z_j - k > 0$. Consequently conditional on j playing *Stick*, she is strictly better off playing *Accept*. As a result $B_i^\epsilon(z_1, z_2)$ is empty. $\square$

The next lemma establishes a relation between $k_i^{\epsilon *}$ and $k_j^{\epsilon *}$ when they are well defined.

Lemma 7. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j > \delta + 4\epsilon$ if the terms are well defined then, $k_i^{\epsilon *} < k_j^{\epsilon *} + 2\epsilon$.*

Proof. Let $k_j^{\epsilon *} + 2\epsilon \leq k_i^\epsilon \leq 1 - z_i + 3\epsilon$. Conditional on such an observation player i knows that for all the possible values of k, $1 - z_j - k > 0$ and hence she would strictly prefer to play *Accept* if j plays *Stick*. Further such an observation implies that j has observed $k_j^\epsilon > k_j^{\epsilon *}$ implying that j would certainly play *Stick*. Consequently i's conditionally dominant action is to play *Accept*. $\square$

The next lemma contains the crucial argument that drives the result, since it shows that for incompatible demands with the higher demand sufficiently larger than the smaller one, the player with the higher demand always concedes whenever the observed cost is in the range that generated multiplicity in the complete information game.

Lemma 8. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j \geq \max\{\delta + 4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, the sets $B_i^\epsilon(z_1, z_2)$ and $B_j^\epsilon(z_1, z_2)$ are empty.*

Proof. Suppose not. Then, by *Lemmas 5 and 6*, $k_i^{\epsilon *}, k_j^{\epsilon *} > -\epsilon$ are well defined. Let player i's payoff from playing *Accept* and *Stick* upon observing $k_i^{\epsilon *}$ be denoted as $u_i(A_i|k_i^{\epsilon *})$ and $u_i(S_i|k_i^{\epsilon *})$ respectively. Given the payoffs in Table 2, $u_i(A_i|k_i^{\epsilon *}) = 1 - z_j - E^\epsilon(k|k_i^{\epsilon *}) + dPr(A_j|k_i^{\epsilon *})$. Also $u_i(S_i|k_i^{\epsilon *}) = z_i Pr(A_j|k_i^{\epsilon *})$.

Given that i chooses *Stick* after such an observation, it must be that $u_i(S_i|k_i^{\epsilon*}) \geq u_i(A_i|k_i^{\epsilon*})$. This in turn implies the following inequality,

$$Pr(A_j|k_i^{\epsilon*}) \geq \frac{1 - z_j - E^{\epsilon}(k|k_i^{\epsilon*})}{z_i - d} \tag{15}$$

Similarly, player j choosing *Accept* upon observing $k_j^{\epsilon*}$ implies that $u_j(A_j|k_j^{\epsilon*}) \geq u_j(S_j|k_j^{\epsilon*})$. Writing out the payoffs, $1-z_i-E^{\epsilon}(k|k_j^{\epsilon*})+dPr(A_i|k_j^{\epsilon*}) \geq z_j Pr(A_i|k_j^{\epsilon*})$. This gives rise to the following inequality,

$$Pr(A_i|k_j^{\epsilon*}) \leq \frac{1 - z_i - E^{\epsilon}(k|k_j^{\epsilon*})}{z_j - d} \tag{16}$$

Now, player j plays *Stick* following any observation $k_j^{\epsilon} > k_j^{\epsilon*}$. Therefore, it must be that,

$$Pr(A_j|k_i^{\epsilon*}) \leq F_i^{\epsilon}(k_j^{\epsilon*}|k_i^{\epsilon*}) \tag{17}$$

On the other hand, player i plays *Accept* for observations $k_i^{\epsilon} > k_i^{\epsilon*}$ as long as $k_i^{\epsilon} < 1 - z_j - \epsilon$. For values of k_i^{ϵ} that are within 2ϵ of $k_j^{\epsilon*}$ it must be that $k_i^{\epsilon} < 1 - z_j - \epsilon$ since $k_j^{\epsilon*} \leq 1 - z_i + \epsilon$ by *Lemma 3* and $1 - z_i + \epsilon < 1 - z_j - \delta - 2\epsilon$ by assumption. As a result the following inequality holds.

$$Pr(A_i|k_j^{\epsilon*}) \geq 1 - F_j^{\epsilon}(k_i^{\epsilon*}|k_j^{\epsilon*}) \tag{18}$$

Subtracting (18) from (17) and using (9) from *Lemma 1* gives the inequality,

$$Pr(A_j|k_i^{\epsilon*}) - Pr(A_i|k_j^{\epsilon*}) \leq \kappa\epsilon \tag{19}$$

Finally combining (15), (16) and (19) gives,

$$\kappa\epsilon \geq \frac{1 - z_j - E^{\epsilon}(k|k_i^{\epsilon*})}{z_i - d} - \frac{1 - z_i - E^{\epsilon}(k|k_j^{\epsilon*})}{z_j - d} \tag{20}$$

$$\geq \frac{1 - z_j - k_i^{\epsilon*} - \epsilon}{z_i - d} - \frac{1 - z_i - k_j^{\epsilon*} + \epsilon}{z_j - d} \tag{21}$$

$$> \frac{1 - z_j - k_j^{\epsilon*} - 3\epsilon}{z_i - d} - \frac{1 - z_i - k_j^{\epsilon*} + \epsilon}{z_j - d} \tag{22}$$

(20) $\Rightarrow$ (21) by the fact that $E^{\epsilon}(k|k_i^{\epsilon*}) \leq k_i^{\epsilon*} + \epsilon$ and $E^{\epsilon}(k|k_j^{\epsilon*}) \geq k_j^{\epsilon*} - \epsilon$. While the inequality from *Lemma 7*, namely $k_i^{\epsilon*} < k_j^{\epsilon*} + 2\epsilon$, makes (21) $\Rightarrow$ (22).
(22) $\Rightarrow$

$$\begin{aligned}
&\kappa\epsilon(z_i - d)(z_j - d) > (z_j - z_i)(1 - k_j^{\epsilon*} + d) - (z_j^2 - z_i^2) - 3\epsilon z_j - \epsilon z_i + 4\epsilon d \\
\Rightarrow &\kappa\epsilon(1 - (z_i - z_j)^2) > (z_j - z_i)(1 - k_j^{\epsilon*} - (z_i + z_j) + d) + \epsilon(z_i - z_j) - 2\epsilon \\
\Rightarrow &\kappa\epsilon(1 - (z_i - z_j)^2) > (z_i - z_j)(k_j^{\epsilon*} + d + \epsilon) - 2\epsilon \\
\Rightarrow &k_j^{\epsilon*} + d + \epsilon < \frac{\kappa\epsilon(1 - (z_i - z_j)^2)}{z_i - z_j} + \frac{2\epsilon}{z_i - z_j} \\
\Rightarrow &k_j^{\epsilon*} < -\epsilon + \frac{\kappa\epsilon}{z_i - z_j} + \frac{2\epsilon}{z_i - z_j} - d - \kappa\epsilon(z_i + z_j) \\
\Rightarrow &k_j^{\epsilon*} < -\epsilon + \frac{(\kappa + 2)\epsilon}{z_i - z_j} - d && (23)
\end{aligned}$$

Given that $k_j^{\epsilon*}$ must be a value strictly greater than $-\epsilon$, (23) delivers a contradiction to the initial claim if,

$$z_i - z_j \geq \frac{(\kappa+2)\epsilon}{d} \tag{24}$$

The premise in the lemma satisfies (24) and therefore it must be that $B_j^{\epsilon}(z_1, z_2)$ is empty. *Lemma 6* then guarantees that $B_i^{\epsilon}(z_1, z_2)$ is empty too. □

Lemma 8 makes it immediate that following an incompatible demand profile (z_1, z_2), where $z_i - z_j \geq \max\{\delta + 4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, player j plays *Stick* irrespective of the observation k_j^{ϵ}. On the other hand player i plays *Stick* for $k_i^{\epsilon} > 1 - z_j + \epsilon$ while playing *Accept* for $k_i^{\epsilon} < 1 - z_j - \epsilon$. This allows for a characterization of the expected payoffs in the first stage, from making such incompatible demands. Let $y_i(z_1, z_2)$ and $y_j(z_1, z_2)$ denote i and j's expected payoff in equilibrium from making demands z_i and z_j. The following lemma is delivered simply by calculating payoffs given the characterization of equilibrium behavior in the second stage discussed in *Lemmas 3, 4 and 8*.

Lemma 9. *In any equilibrium of Γ^{ϵ} following a pair of incompatible demands (z_1, z_2) where $z_i - z_j \geq \max\{\delta + 4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, it must be that*

$$z_j F_i^{\epsilon}(1 - z_j - \epsilon) \leq y_j \leq z_j F_i^{\epsilon}(1 - z_j + \epsilon) \tag{25}$$

$$y_i \leq \int_0^{1-z_j} (1 - z_j - w) h(w) dw \tag{26}$$

The analysis can now turn to the choice of first stage demands. Let the set of demand profiles that can be supported by equilibrium strategies in Γ^{ϵ} be denoted by Eq^{ϵ}. Further let $\phi(d) = \max\{\delta + 4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$

Lemma 10. *If (z_1, z_2) satisfies either of the following conditions,*

a. $z_1 + z_2 < 1$

b. $z_1 + z_2 > 1$ *and* $|z_1 - z_2| \geq \phi(d)$

then, $(z_1, z_2) \notin Eq^{\epsilon}$.

Proof. (a) is immediate, since player i has an incentive to demand $1 - z_j$ and strictly increase her payoff by $1 - z_j - z_i > 0$. *Lemma 9* shows that following an incompatible demand profile such as (b), the player with the higher demand, say i, has an expected payoff $y_i \leq \int_0^{1-z_j} (1 - z_j - w) h(w) dw < 1 - z_j$ and could do strictly better by simply making the compatible demand $1 - z_j$. □

Let $\hat{k} = \int \min\{k, 1\} h(k) dk$. The following lemma shows that for an incompatible demand profile to be supported in equilibrium, the excess demand must be above a positive lower bound.

Lemma 11. *If $z_1 + z_2 > 1$ and $d < \hat{k}/2$ then $(z_1, z_2) \notin Eq^{\epsilon}$.*

Proof. Following an incompatible demand profile, the payoffs are determined by outcomes in the second stage game described in Table 2. Notice that following any possible realization, k, the maximum total payoff would be $\max\{1-k, 0\}$. As a result the expected payoffs from making incompatible demands must satisfy, $y_1 + y_2 \leq 1 - \hat{k}$. Now for the incompatible profile (z_1, z_2) to be supported as an equilibrium in Γ^ϵ, it must be that neither player gains by making a compatible demand instead. This means, $y_i \geq 1 - z_j$. Summing across the two players gives, $y_1 + y_2 \geq 2 - z_1 - z_2$, which in turn implies, $2 - z_1 - z_2 \leq 1 - \hat{k}$. Given that $d = (z_1 + z_2 - 1)/2$ it must be that $d \geq \hat{k}/2$. □

Recall that $\phi(d) = \max\{\delta + 4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$. Let $\phi^* = \phi(\hat{k}/8)$. The next lemma shows that incompatible demands that are close to each other but result in an excess demand that exceeds the bound from *Lemma 11* cannot be supported in equilibrium.

Lemma 12. *If $z_1 + z_2 > 1$, $d \geq \hat{k}/2$ and $|z_1 - z_2| < \phi(d)$ then $(z_1, z_2) \notin Eq^\epsilon$ for small enough δ and ϵ.*

Proof. Equilibrium behavior in the second stage game involves a total payoff of 0 if both parties play *Stick* or $1 - k$ if (*Accept*, *Stick*) or (*Stick*, *Accept*) is the outcome. Players using mixed strategies results in the total payoff lying in the interval $[0, \max\{0, 1-k\}]$. *Lemma 2* makes it clear that if $k > 1 - \min\{z_1, z_2\} + 2\epsilon$ then the players would always play (*Stick*, *Stick*). So it can be said for certain that following an incompatible demand profile, the total expected payoff in equilibrium must be no more than $(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon)$. This in turn implies that following incompatible demands there exists i with an expected payoff,

$$y_i \leq \frac{1}{2}(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon) \quad (27)$$

$d \geq \hat{k}/2$ implies $z_i + z_j - 1 \geq \hat{k}$. Also by the definition of ϕ, it must be that $\phi(d) \leq \phi^*$ since $d \geq \hat{k}/2$. So,

$$\begin{aligned} &|z_i - z_j| < \phi(d) \leq \phi^* \\ \Rightarrow &2\min\{z_1, z_2\} + \phi^* - 1 \geq \hat{k} \\ \Rightarrow &\min\{z_1, z_2\} \geq \frac{1}{2} + \frac{\hat{k}}{2} - \frac{\phi^*}{2} \end{aligned} \quad (28)$$

Let ϵ and δ be small enough such that $\phi^* < \frac{\hat{k}}{8}$.
Then,

$$(28) \Rightarrow \min\{z_1, z_2\} > \frac{1}{2} + \frac{7}{16}\hat{k} \quad (29)$$

Now consider what happens if player i, who receives the payoff mentioned in (27), deviates to making a *still incompatible* demand of $\tilde{z}_i = 1/2$. Note that $z_j - \tilde{z}_i > \frac{7}{16}\hat{k} > \phi^*$. Further $d(\tilde{z}_i, z_j) > \frac{7}{32}\hat{k}$ which implies that $\phi(d) \leq \phi^*$.

Therefore $z_j - \tilde{z}_i > \phi(d(\tilde{z}_i, z_j))$. As a result, the new demand profile satisfies the condition of *Lemma 9*, which implies that player i following such a deviation must expect a payoff $\tilde{y}_i$,

$$\tilde{y}_i \geq \frac{1}{2}F_i^{\epsilon}(\frac{1}{2} - \epsilon) \geq \frac{1}{2}H(\frac{1}{2} - 2\epsilon) \tag{30}$$

Player i's initial payoff inequality described in (27) along with (29) implies,

$$y_i < \frac{1}{2}H(\frac{1}{2} - \frac{7}{16}\hat{k} + 2\epsilon) \tag{31}$$

For small enough values of ϵ, it is clear that $y_i < \tilde{y}_i$. Given that such a profitable deviation exists, $(z_1, z_2) \notin Eq^{\epsilon}$. □

Proof of Proposition 1

Proof. Proposition 1 follows immediately from the observation that *Lemmas 10, 11 and 12* exhaust the entire set of incompatible demand profiles. □

4 Independent costs of backing down

Work in progress.

5 Conclusion

Work in progress.

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