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The battle of the bourses

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The Battle of the Bourses *

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1 Introduction

The beginning of the bourses (or stock exchanges) goes back long ago in history. Any group of agents who agree to trade their assets form a bourse. Bourses evolved over time and big trading organizations appeared. By the XX century, the bourses were linked to their respective national countries. National bourses were understood as non-profit public organizations where outsiders were charged high commissions and trading fees in order to have access to their liquidity. But the evolution of bourses has never been so dramatic as in the last decade. In Europe, the Market in Financial Instruments Directive (Mifid), enacted in 2007 by the European Commission, facilitated competition across the region (the competitors are known as MTFs - Multilateral Trading Facilities -). The change in regulation and the new electronic trading technologies pushed the bourses into an unprecedented international integration movement, where the old-style stock exchanges progressively disappeared, being replaced by new entrepreneurial global trading organizations.

Being inspired by the recent wave of demutualizations, and knowing the importance of such institutions to the market, we provide a two stages general equilibrium model of formation of bourses and subsequent trading, that incorporates explicit important characteristics of this industry: possibly increasing gains from trade in larger bourse sizes, multiple memberships, and competition from "dark pools of liquidity"[1]. Our model passes the preliminary test of existence of a bourse structure equilibrium, and is able to give a good understanding on how efficiency is related to a process of demutualization. Also, under such a complete model, we address different issues that have been inspired by the following

Empirical evidence

Fact 1: In October 2006 the merge between the Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT) created the world' largest futures exchange. In May 2006 the New York Stock Exchange (NYSE) (itself a product of

[1]Wealthy market participants trading large blocks of shares have smaller costs if the trading occurs in a "dark pool of liquidity" than in standard "lit" exchanges. Those costs are reduced in dark pools since there is no need of a broker (intermediation cost) and also participants are protected against adverse share price movements since the trades are privately negotiated.

the union between the New York Stock Exchange and American Stock Exchange) acquired Euronext (in turn the result of the merge between Paris, Amsterdam and Brussels stock exchanges). In April 2007 Deutsche Börse acquired the International Securities Exchange (ISE).

By 2006 and 2007 the access to liquidity and new trading technologies seemed to be the final factors that induce traders to merge into a single large bourse.[2] In this paper, we provide numerical examples (Examples 1 and 2 below) that illustrate that when there are good complementarities among the traders, liquidity may lead to an equilibrium where all traders sort into a unique bourse. Complementarities are thus an important determinant of the liquidity in a bourse.[3]

Fact 2: In December 2008, NYSE Euronext and Deutsche Börse failed to merge.[4] Competition by small exchanges led to an increase in fragmentation in 2008 when the number of trading platforms increased. By 2009 MTFs accounted for about 20% of the trading in the largest shares across European main markets.

Fact 2 poses an empirical counter-argument to the claim that trading is a natural monopoly. It suggests that traditional big exchanges are ill-suited to certain types of institutions. Competition came from small exchanges such as offshore centres (such as Centres in Bermuda, Caribbean, Luxemburg and Dublin), trading consortia of banks and dark liquidity pools. But, *what are the forces competing against the tendency towards a unique bourse?* The new trading technology becomes a threat to the existing exchanges. Any consortia can now be created, at convenient low formation costs, to trade any assets with self-picked traders and assets, possibly gaining over the anonymity that large organizations impose over their members. Example 3 below portrays a scenario where a group of traders with good complementarities prefer a small bourse with low formation costs to a more expensive larger complex bourse where the additional traders lack good enough complementarities in the asset trading.

Fact 3: In January 2008 the US Department of Justice came into serious discussions concerning the potential merge between the Chicago Mercantile Exchange (CME) and the energy exchange Nymex.[5] Similar strategic unions were built up in

[2] See "The battle of the bourses", The Economist, May 25, 2006.

[3] A simple counterargument gives light to this idea: a bourse with a large number of traders, but all of them homogeneous in preferences and endowments, will have no trade of assets, and thus no liquidity.

[4] See Financial Times, "D Börse fails to agree merger", December 8, 2008.

[5] See "CME in $11bn move for Nymex", Financial Times, January 29, 2008.

Europe (Deutsche Börse/Kuwait Stock Exchange and NYSE Euronext/Qatar Investment Authority).

The concern of the US Department of Justice has to do with the possibility of anti-competitive behavior of big organizations. As we show in Example 5, fixing a bourse structure with bourses having monopolistic power in share trading may lead to price manipulation practices, which in turn decreases traders' welfare. This scenario of manipulation resembles the old-fashioned national bourses whose membership fees were high and far from competitive. This possibility poses the following question: *what are the gains of demutualization?*[6] As we remark after Example 5, policies that favor demutualization increase traders' welfare.

Fact 4: There are bourses whose asset structures cannot completely diversify traders' risks (e.g., CME and Nymex).

A complete asset structure, if possible, is technologically very costly to build up. But if it is possible, we can raise the following question: *are technology and trading complementarities determinant factors to the market incompleteness?* We answer in the affirmative using Example 4 below. The costs associated to a large bourse with a complex asset structure induce traders to prefer smaller bourses with an incomplete asset structure.

Relation with the literature

The question of how trade through organizations takes place has been an old issue that dates back to the end of the XIX century, when Lèon Walras (1900) focused his attention on the functioning of the Paris Böurse. Three branches of the economics literature are related: liquidity, incomplete markets and market microstructure.

Liquidity is an intricate concept that attracted the attention of researchers from long time ago. Seminal contributions are Keynes (1936, Ch. 13, 15), who attached to the concept of liquidity the notion of money[7], Hicks [1962]), who explored the riskiness dimension of an asset, and Pagano [1989], who analyzed the absorptive capacity of a market and the role of beliefs on the equilibrium.[8]

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[6] Demutualization is the process where traders freely move from their pre-assigned bourses (e.g., national bourses) to their most preferred ones, without any other restriction than the mere cost of paying the corresponding membership fee.

[7] Keynes [1936] : "the individual's liquidity preference is given by a schedule of the amounts of his resources, valued in terms of money (...), which he wish to retain in form of money in different sets of circumstances".

[8] Subsequent literature on liquidity and its relation with asset pricing is extensive. See

Our paper considers non-anonymous traders that are fully characterized by their preferences on the consumption of private goods and their budget constraints that account for their assets trading decisions. In that way, our fully non-anonymous analysis departs from Pagano (1989) and related works, who study liquidity in a framework where traders' decisions depend on the first and second order moments (mean and variance).

Under our specific two-sides (buy-sell) model with non-anonymous traders we can clearly assert that trading complementarities are an important component of the liquidity. Closer to this approach is the leading paper of Fostel and Geanakoplos (2008) that brings new insights on the liquidity inefficiencies generated by an incomplete market with heterogeneous traders' beliefs. In our paper, we show that traders' complementarities can be a decisive factor that leads small bourses (or dark liquidity pools) to be preferred in equilibrium to large bourses (see Example 3 below). This claim offers a complementary point of view to Pagano's proposition that large blocks of assets are traded in small markets.

Incomplete markets: Our paper also departs from Pagano [1989] in that we do not limit our analysis to only one stock (see Pagano's fn. 3). Instead, we consider different asset structures, possibly incomplete, with more than one asset. As shown below, the degree of incompleteness of the asset structure becomes an important determinant of the traders' sorting into bourses.

Previous works in the theory of general equilibrium with incomplete markets (GEI) portrayed an economy where all transactions take place in a single market.[9] As a consequence, these works did not explain how the well established institutional framework (bourse structure, monopoly power of the trading platform, set up costs of bourses, ...) affects traders' welfare. Neither the related field of security design properly models bourses as the place where traders issue and trade securities (see Allen-Gale (1991)).

Incomplete markets go hand by hand with default and collateral economies. Bourses can accommodate default by contracting the services of a clearinghouse.

Cochrane (2005) for a review of the recent literature. Other important works are Allen and Gale (1994), who studied how the sudden need for liquidity (say, from the arrival of news) exacerbates the impact of exogenous aggregate schocks, Huang and Wang (2009), who show how idiosyncratic shocks generate endogenous selling demand at the aggregate level, and Brunnermeier and Pedersen (2008) and Bottazzi, Luque and Pascoa (2010), who explore the funding liquidity dimension of an asset.

[9]See the reference book of Magill and Quinzii (1996).

Santos and Scheinkman (2001) focus their analysis on the clearing post-trading activity: given information problems, the clearinghouse requires a certain level of collateral in order to mitigate default. In this paper we do not attempt to model default, and thus the issue of the design of clearinghouses is left out in our model.

The endogeneity of the market incompleteness has been an old issue in the field of GEI. Up to our knowledge, endogenous market incompleteness has been explained by introducing some kind of frictions into the economies. In the field of optimal security design and financial intermediation, endogeneity arises through the issuing costs faced by intermediaries (see Bisin (1998) and Faias (2008))[10]. In the collateral economies, there is a subset of asset traded in equilibrium, which depend on the required level of collateral. Such assets selection may lead to a result of market incompleteness (see Geanakoplos and Zame (2007)). This contrasts with standard GEI no-default economies where the assets traded in equilibrium just depend on the Walrasian asset prices. In our paper, we give a different view to the issue of endogeneity of the market incompleteness. Here, the subset of assets chosen in equilibrium are those assets belonging to those bourses arising endogenously due to the sorting of the traders into bourses.

Our results on efficiency are related to Geanakoplos and Polemarchakis (1989). They show that inefficiency may arise (in the strong sense) if the asset structure is incomplete. This type of inefficiency is endogenous to our model. In our model it may happen that a bourse characterized with an incomplete asset structure results in equilibrium *even if* a bourse with a complete asset structure is available to the traders (Example 4). Technology, in the form of bourses formation cost, and trading complementarities play a crucial role in the determination of the degree of incompleteness of the asset structures.

Market microstructure: This paper is the first to provide the connection between the specific market micro-structure of trading in bourses (formed at some costs) and its consequences on the incompleteness, the volumes of assets traded and their prices (see Examples 1 and 4). Thus, this result adds to the market microstructure literature (e.g., Lo, Mamaysky and Wang (2004)) that relates the trading behavior to market-making activities - trading costs-.

By fully characterizing non-anonymous traders we also depart from Pagano [1989] in that we allow for every possible subset of traders to form a bourse, which

[10] In these papers intermediaries offer securities in a context of imperfect competition.

contrast with the two-bourses analysis of Pagano. To the best of our knowledge, our paper is the first to provide a general equilibrium theory on the existence and optimality of a bourse structure.

The study of the existence of an optimal bourse structure (our first stage) borrows from the results of Allouch and Wooders (2008). Our specific economy should let individuals belong to several bourses, so multiple memberships are allowed, but also we should permit trading platforms with unbounded size (e.g., a finite number of stock exchanges or even only one trading platform). Also, in our model of financial exchanges we aim to portrait an economy with a large number of traders who are characterized by a price taking behavior. These are the final justifications of the convenience of incorporating the Allouch and Wooders (2008) model into our economy.[11]

Our model of bourses reinterprets Allouch and Wooders (2008) to an specific scenario where the public project is one that provides the facility of asset trading. Paying for membership fees allows (non-anonymous) traders to trade in the bourses, possibly with traders evaluating differently the income associated with a given bourse by trading its assets. Also, one should observe that the optimal asset structure arises endogenously in our model once we let individuals choose their most preferred bourses (each specific group of traders (bourse) having its specific offer of assets). Thus public projects become endogenously determined.

We would like to emphasize that we consider a notion of a bourse substantially more general than an over-the-counter (OTC) market. A bourse is understood as a coalition of traders that engage in the public facility of asset trading. We model a bourse as a central market where the asset prices are common to all traders.[12] Instead, in an OTC market trading occurs bilaterally (usually between a trader and an intermediary) and an asset can have a different price depending on which is the couple of traders. In OTC markets, traders usually need to find an intermediary[13], which makes search models a good candidate to proxy

[11] Other papers in the club / local public goods literature (for instance, Cole and Prescott (1997) and Ellickson, Grodal, Scotchmer and Zame (2001) do not allow clubs to be unbounded in size. There are other papers (see for instance Konishi, Le Breton and Weber (1998) and Kovalenko and Wooders (2003)) that allow for an arbitrary number of jurisdictions (possibly only one) but do not consider a price taking equilibrium notion.

[12] An example of this type of bourses is the Tradingpoint Stock Exchange. It operates through the electronic system used on the Vancouver exchange. Direct access to investors is guaranteed, without the need of intermediaries for the trading.

[13] Financial intermediaries have been already studied in the literature and thus we do not

asset pricing in those particular markets (see Duffie et al. (2005)). In our model we do not assume intermediaries, and model traders going simultaneously to the bourse to trade at the equilibrium prices. Once the bourse is formed, asset trading occurs as in standard competitive general equilibrium models.

The rest of this paper is as follows. In Section 2 we provide several tractable numerical examples that illustrate the questions raised in this Introduction. In particular, how liquidity is related to trading complementarities, its consequence on the optimal bourses sizes, and how complementarities and technology, in the form of bourses formation costs, become a determinant factor to the degree of incompleteness of the asset structure. The model, in Sections 3 and 4, portraits an economy with the following characteristics: a large number of traders, possibly multiple membership in bourses, and bourses being possibly unbounded in size. Ever-increasing gains from trade in larger bourses is in fact a possibility, but it is not self-imposed into the model. Section 5 discusses how efficiency is related to policies of demutualization. There, we also address the issue of endogeneity of the incompleteness of the asset structure. Section 6 concludes.

2 Examples

Example 1 (*Bourse structure affects welfare through liquidity*): Our objective in the present example is to compare the welfare of two traders, namely 1 and 2, when they join together in a bourse with a *third* trader t, who can take two different identities, namely, $t = 3, 4$. Welfare will depend on the wealth and complementarities between the traders in a bourse. The bourse composed just by traders 1 and 2 is denoted by $S^1 = (1, 2)$. The bourses composed by the three traders are $S^2 = (1, 2, 3)$ or $S^3 = (1, 2, 4)$. There are two dates 1 and 2, and two states $\Sigma = \{2, 3\}$ at date 2. We consider a complete asset structure with two assets whose payoffs at date 2 are $(4, 1)$ for the first asset[14] and $(1, 5)$ for the second asset. The asset structure is denoted by $A(S) = \{(4, 1), (1, 5)\}$, for $S = S^1, S^2, S^3$. There is one good for consumption in each date.

At date 1 a trader chooses his preferred consumption of the good and engages

attempt to model these types of traders. For a dicussion of the mechanics of dealers see Duffie (2010). For a specific model of dealers' activity in repo markets see also Bottazzi, Luque and Pascoa (2010).

[14] The first asset pays 4 units of the good in state 2 and 1 unit in state 3.

in the asset trading with the traders of his bourse, given his good endowments. At date 2, a trader chooses his preferred consumption at each node, given endowments and asset payments. As in the model below, we denote trader i's short sale of asset j by $y_j^i < 0$ and the asset purchase by $y_j^i > 0$. Trader i's utility function is given by $u^i(x_1, x_2, x_3) = \alpha_1^i \ln x_1 + \alpha_2^i \ln x_2 + \alpha_3^i \ln x_3$. Traders 1 and 2' endowments and preference parameters are $(\omega_1^1, \omega_2^1, \omega_3^1) = (3, 2, 6)$, $(\alpha_1^1, \alpha_2^1, \alpha_3^1) = (1, 3/4, 0)$, $(\omega_1^2, \omega_2^2, \omega_3^2) = (2, 5, 2)$ and $(\alpha_1^2, \alpha_2^2, \alpha_3^2) = (1, 0, 4/5)$, respectively.

We do comparative statics by changing the utility and the endowments of the *third* trader. Trader 3 is rich *today* and prefers to consume *today*, i.e., $(\omega_1^3, \omega_2^3, \omega_3^3) = (6, 2, 2)$ and $(\alpha_1^3, \alpha_2^3, \alpha_3^3) = (1, 1/6, 1/6)$. Trader 4 is rich *today* and prefers to consume *tomorrow*, i.e., $(\omega_1^4, \omega_2^4, \omega_3^4) = (6, 2, 2)$ and $(\alpha_1^4, \alpha_2^4, \alpha_3^4) = (1/2, 4/5, 4/5)$. The next tables indicate the differences for different bourses in traders' indirect utilities,[15]

$\mathbf{S}^1 = (1, 2)$	$\mathbf{S}^2 = (1, 2, 3)$	$\mathbf{S}^3 = (1, 2, 4)$
$U^1(S^1) = 2.5$	$U^1(S^2) = 2.5$	$U^1(S^3) = 2.8$
$U^2(S^1) = 2.4$	$U^2(S^2) = 2.4$	$U^2(S^3) = 2.7$
n.a.	$U^3(S^2) = 2$	$U^4(S^3) = 2.8$

Table 1

trading volume,

	$\mathbf{S}^1 = (1, 2)$	$\mathbf{S}^2 = (1, 2, 3)$	$\mathbf{S}^3 = (1, 2, 4)$
Trader 1	$y_1^1(S^1) = 2.6$ $y_2^1(S^1) = -1.5$	$y_1^1(S^2) = 1.5$ $y_2^1(S^2) = -1.5$	$y_1^1(S^3) = 1$ $y_2^1(S^3) = -1.4$
Trader 2	$y_1^2(S^1) = -2.6$ $y_2^2(S^1) = 1.5$	$y_1^2(S^2) = -1.45$ $y_2^2(S^2) = 1.25$	$y_1^2(S^3) = -1.45$ $y_2^2(S^3) = 0.9$
Trader t	*n.a.*	$y_1^3(S^2) = -0.05$ $y_2^3(S^2) = 0.25$	$y_1^4(S^3) = 0.45$ $y_2^4(S^3) = 0.5$

Table 2

[15] The computations to the numerical examples of this paper can be found in the Supplementary Material (facilitaed upon request).

and asset prices

$q_1(S^1) = 1.4$	$q_1(S^2) = 1.7$	$q_1(S^3) = 3.8$
$q_2(S^1) = 1.4$	$q_2(S^2) = 1.8$	$q_2(S^3) = 4.1$

Table 3

Welfare: In Table 1 we can observe that the indirect utilities of traders 1 and 2 in bourse S^2 are the same as if they were alone in bourse S^1. This is because in bourse S^2 there are no good enough complementarities between trader 3 and traders 1 or 2.[16] Indeed, traders 1 and 2 have to trade "almost" between themselves as if they were the only traders in the bourse. In S^3 traders 1 and 2 can take advantage of the entrance of trader 4 in the bourse as the degree of trading complementarities is higher.[17] The introduction of trader 4 in the bourse makes traders 1 and 2' welfare to increase.

Assets trading volume: Let us first compare bourses (1,2) and (1,2,3). In bourse (1,2,3) trader 3 wants to buy asset 2, competing in this way with trader 2. This increases the price of asset 2. Trader 2 can purchase a smaller amount of asset 2 (just 1.25 units of asset 2). Thus, trader 2 needs less income for this purchase, and as a consequence, trader 2 short sells a smaller amount of asset 1. This in turn affects trader 1, who is now limited to purchase a smaller amount of asset 1 (only buys 1.5 of asset 1, instead of 2.6 in bourse (1,2)).

Now, in bourse (1,2,4), trader 4 has better complementarities with traders 1 and 2 than trader 3 had. Trader 4 wants to buy even more of asset 2 than trader 3. This increases the price of asset 2. Thus, trader 2 has to buy even less of asset 2, given what trader 1 wants to sell. But now, even when trader 2 can purchase a smaller amount, we have that he can do the same short sale in asset 1 as in bourse (1,2,3), as now trader 4 (with better complementarities) demands to buy a larger amount of asset 1. This increases the price of asset 1.

Asset pricing: We can see that asset prices are affected by the volumes of trades, which in turn depend on the degree of complementarities among the traders in a bourse. For instance, in bourse S^2 trader 3 does not affect much

[16] In S^2 trader 3 is rich today and prefers to consume today as well. Thus, trader 3 consumes his endowments today and, as we can see from his portfolio, trades very small volumes of the two assets. It follows that he is not interested in transferring his endowments to tomorrow through the bourse's asset trading facility.

[17] In S^3 trader 4 prefers to consume tomorrow but he is rich today. So, he is interested in trading in the bourse because his endowments of today can be transferred to tomorrow.

assets prices because the volumes of trades do not differ significantly from those in bourse S^1. However, in bourse S^3 the complementarities are stronger, the volume of assets transactions is higher, and, as a consequence, asset prices increase with respect to S^1.

Liquidity: Traders 1 and 2 experience a higher degree of liquidity in bourse (1,2,4). This means that, due to trader 4's willingness to trade in the bourse, traders 1 and 2 can transfer in an easier way their endowments to the state of nature they most prefer. The previous discussion brings us the important insight that the level of liquidity in the economy crucially depends on the bourse structure that is formed. Bourses S^1, S^2 and S^3 offer different trading opportunities, which imply different volumes of trade and asset prices.

Example 2: (*Large bourses are optimal if trading complementarities are good*): Let us consider an economy similar to Example 1 but we now have three dates: $0, 1$ and 2. At date 0 traders must pay for the membership fee to have access to the bourse at date 1. Membership fees cover the formation cost of the bourse. As before, we consider two states of nature at date 2, $\Sigma = \{2, 3\}$. Let the set of traders be $\mathbf{I} = \{1, 2, 4\}$, where trader 4 is the one introduced in Example 1. Traders 1, 2 and 4 have the same utility functions as in Example 1, except that here we also consider that traders consume at date 0. Trader i's utility functions is now given by $V^i(x_0, x_1, x_2, x_3) = \ln x_0 \left[\alpha_1^i \ln x_1 + \alpha_2^i \ln x_2 + \alpha_3^i \ln x_3\right]$. As before we consider just one good. The endowments at date 0 are $\omega_0^1 = 8$, $\omega_0^2 = 11$ and $\omega_0^4 = 7.25$. The possible bourses are $S^1 = (1, 2)$, $S^3 = (1, 2, 4)$, $S^6 = (1, 4)$ and $S^7 = (2, 4)$ (notation follows from Example 1).

We now compute the optimal membership fees required to enter in a bourse. The associated membership fee is denoted by $\pi^i(S)$. It can be shown (see Supplementary Material) that the non-anonymous[18] membership fee for a trader i in a two traders bourse $S = (i, k)$ is given by

$$\pi^i(S) = \frac{U^k(S)\omega_0^i - U^i(S)\omega_0^k}{U^i(S) + U^k(S)} + \frac{U^i(S)}{U^i(S) + U^k(S)} z(S)$$

whereas if it is a three traders bourse $S = (i, j, k)$ we would have

$$\pi^i(S) = \frac{\omega_0^i\left[U^j(S) + U^k(S)\right] - U^i(S)(\omega_0^j + \omega_0^k)}{U^i(S) + U^j(S) + U^k(S)} + \frac{U^i(S)}{U^i(S) + U^j(S) + U^k(S)} z(S)$$

[18]Membership fees are non-anonymous, which means that a bourse can distinguish among different traders.

These formulas give an efficient characterization of the membership pricing. Notice that in both cases the equilibrium membership fee equations consist of the sum of a pure transfer (first term on the right hand side) and a poll tax (second term). The poll tax is such that all traders share the bourse S's formation cost $z(S)$. The pure transfer prices reflect the trader's valuation of the trading opportunities in bourse S. Observe that membership fees cover the costs (i.e., $\sum_{i \in S} \pi^i = z(S)$). The membership fees in bourse S^1 are $\pi^1(S^1) = 1.44$ and $\pi^2(S^1) = 4.56$; by analogy, we can compute the traders' membership fees for the other bourses (see Supplementary Material).

Let us now see that liquidity, in the form of good complementarities, leads to the unique large bourse S^3. We consider the asset structures $A(S_k) = \{(4,1),(1,5)\}$ and formation costs $z(S_k) = 6$, for $S_k = S^1, S^3, S^6, S^7$. Our framework here is characterized by non-anonymity and market completeness. The associated indirect utilities are

	$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^6 = (1,4)$	$\mathbf{S}^7 = (2,4)$	$\mathbf{S}^3 = (1,2,4)$
Trader 1	$V^1(S^1) = 4.7$	$V^1(S^6) = 3.1$	$V^1(\omega^1) = 3.4$	$V^1(S^3) = 5.4$
Trader 2	$V^2(S^1) = 4.5$	$V^2(\omega^2) = 3$	$V^2(S^7) = 3.8$	$V^2(S^3) = 5.1$
Trader 4	$V^4(\omega^4) = 4$	$V^4(S^6) = 5.36$	$V^4(S^7) = 4.9$	$V^4(S^3) = 5.4$

Table 4

The resulting two-stages equilibrium bourse is $S^3 = (1,2,4)$, where traders 1, 2 and 4 are strictly better off than in bourses S^1, S^6 and S^7. All the above mentioned bourses share the same asset structure and therefore, we can conclude that traders prefer the higher degree of liquidity (measured in utility terms through the better complementarities and higher wealth) that the bourse S^3 provides.

Remark 1 (to Example 2): (*Anonymity versus non-anonymity: a welfare comparison*): Information problems may lead some bourses not to distinguish among traders' types. As we show here, this affects welfare. Let us reconsider the equilibrium bourse $S^3 = (1,2,4)$ with the same asset structure and the same formation cost, but now in a framework where bourses cannot distinguish among different traders (known as *anonymous pricing*). In the table below one can compare the welfare of traders 1, 2 and 4 in both frameworks, namely non-anonymous versus anonymous pricing.

Non-Anonymous	*Anonymous*
$V^1(S^3) = 5.4$	$V^1(S^3) = 5$
$V^2(S^3) = 5.1$	$V^2(S^3) = 5.9$
$V^4(S^3) = 5.4$	$V^4(S^3) = 4.6$

Table 5

The mere lack of information makes the richer trader (at date 0) to end up being better off in a context of anonymous pricing than in the non-anonymous one. The reason is that in an anonymous framework the bourse cannot extract all the surplus to the rich trader in order to give it to the poor.

Example 3: (*Size versus tailored efficiency*)[19]: In this example it is worth to consider larger replica bourses with N traders of each type. In particular, the bourse $S^1_N = (1,2)^N$ will denote a bourse composed by N traders of type 1 and N traders of type 2.[20] Our objective is to portray a scenario where the smallest possible bourse, $S^1 = (1,2)$, characterized by self-picked traders (with good complementarities) and assets, is preferred to a more expensive (complex) larger bourse $S^2_N = (1,2,3)^N$, with N large. As in Example 2, the asset structure of all bourses is $A(S) = \{(4,1),(1,5)\}$.

We consider the bourse $S^2_N = (1,2,3)^N$ (borrowed from Example 1)[21] with formation costs $z(S^1_N) = 6N$. At date 0 the endowment of a trader of type 3 is $\omega^3_0 = 6.5$. The formation cost of bourse S^1_N is $z(S^1_N) = 2N$. For $N = 1$, $z(S^1)$ differs from Example 2 to illustrate the idea that a smaller bourse is a less technologically difficult organization to establish and also that the tailored trades have associated a lower execution rate. The associated indirect utilities are[22]

	$\mathbf{S^1_N} = (1,2)^N$	$\mathbf{S^2_N} = (1,2,3)^N$
Trader 1	$V^1(S^1_N) = 5.4$	$V^1(S^2_N) = 4.9$
Trader 2	$V^2(S^1_N) = 5.1$	$V^2(S^2_N) = 4.6$
Trader 3	$V^3(\omega^3) = 3.8$	$V^3(S^2_N) = 3.4$

Table 6

[19] The properties of self-picked traders and assets motivate the expression *tailored-efficiency*.

[20] Observe that the equilibrium prices and allocations obtained in Example 1 still hold for any N replica bourse economy. See Supplementary Material.

[21] Recall that at S^2_N the complementarities between the traders are not good.

[22] In the same vein as in Example 2 we can compute the membership fees in bourses S^1_N and S^2_N in a non-anonymous framework.

From the table above, we see that all traders, including traders of type 3[23], have a higher utility in bourse S^1_N, possibly with $N = 1$. As a result, we can conclude that the small bourse (dark pool of liquidity) $S^1 = (1, 2)$ is preferred in equilibrium by traders of types 1 and 2 to the larger bourse $S^2_N = (1, 2, 3)^N$, for any N.

Remark 2: *What is the effect of the implementation of a Tobin tax on certain (but not all) bourses?* It can easily be shown that for two possible bourses (subject to legislation by regulators), with the same traders and asset structure, traders will participate in the bourse with lower execution rate for their trades. Higher execution rates can be accommodated in the form of a higher formation cost. This claim is intuitive given the results of Example 2 and Example 3 with $N = 1$. In both examples we consider the same bourse $S^1 = (1, 2)$. However, in Example 3 the formation costs are $z(S^1) = 2$, while in Example 2 we have $z(S^1) = 6$. The difference between the two costs can be interpreted as the Tobin tax. Now, looking to the respective tables we see that both traders 1 and 2 prefer the bourse without the tax.

We conclude that if traders are free to choose their preferred bourses, then a Tobin tax (see Tobin (1984)) on the financial transactions in some but not all bourses will not be effective, in the sense that those bourses with a Tobin tax imposed by their respective jurisdictions will not be created in equilibrium (Remark 2). This result adds to the current international debate of the imposition of an institutional Tobin tax on financial transactions (see Stiglitz (1989)).

Example 4: (*Market incompleteness as a consequence of bourse fixed formation costs and complementarities*): Let us consider again an economy with three dates: $0, 1$ and 2, and two states of nature at date 2, $\Sigma = \{2, 3\}$. Utility functions and endowments are the same as in Example 2 and 3. Let the possible bourses be now $S^1 = (1, 2)$, $S^2 = (1, 2, 3)$, $S^4 = (1, 3)$ and $S^5 = (2, 3)$. The respective asset structures are $A(S^1) = \{(4, 1)\}$ (incomplete asset structure) and $A(S^2) = A(S^4) = A(S^5) = \{(4, 1), (1, 5)\}$ (complete asset structure). The corresponding formation costs are $z(S^1) = 1$, $z(S^4) = z(S^5) = 10$ and $z(S^2) = 19$.[24]

[23] Traders of type 3 end up consuming their initial endowments as they do not prefer to enter in the bourse S^2_N.

[24] Recall that, as we explained in Example 3, we can interpret a larger number of traders in a bourse as a more technologically difficult bourse to create, and thus the associated costs are higher. Using the same reasoning we associate a higher cost to a bourse with a lower degree of market incompleteness.

The next table indicates the indirect utilities of the possible bourses.

$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^2 = (1,2,3)$	$\mathbf{S}^4 = (1,3)$	$\mathbf{S}^5 = (2,3)$
$V^1(S^1) = 4.4$	$V^1(S^2) = 2.2$	$V^1(S^4) = 1.4$	n.a.
$V^2(S^1) = 2.1$	$V^2(S^2) = 1.9$	n.a.	$V^2(S^5) = 1.9$
n.a.	$V^3(S^2) = 1.3$	$V^3(S^4) = 1.9$	$V^3(S^5) = 3.1$

Table 7

In Table 7 we see that traders 1 and 2 have a higher utility in bourse S^1 than in the larger bourse S^2. Also, traders 1 and 2 prefer to be in the two traders bourse S^1 than in another two traders bourse with trader 3. We conclude that traders 1 and 2 end up strictly preferring the incomplete asset structure associated to bourse S^1, even if a complete asset structure is available in another bourse.

3 Bourse economies

The economy last for three periods, $0, 1$ and 2. In period 0 traders form bourses; in period 1 they trade assets with the traders of their respective bourses; and in the last period these assets pay returns. The set of states of uncertainty in the last period is $\mathbf{\Sigma} \equiv \{1, ..., \Sigma\}$, with representative element ξ.

In each of the three periods all consumers trade commodities in a common market. The set of commodities is $\mathbf{L} \equiv \{1, ..., L\}$, with representative element ℓ. The number of state-time contingent goods is then $(2 + \Sigma) L$. Commodities are traded at prices $p = (p_0, p_1, p(1), ..., p(\Sigma)) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \mathbb{R}^{L \times \Sigma}_+$, where p_0, p_1 and $p(\xi)$ are the price vectors at dates 0, 1 and 2 (state ξ), respectively.

The set of traders in our economy is $\mathbf{I} \equiv \{1, ..., n\}$. We assume that a *trader* is an agent or institution that goes directly to a bourse to trade assets at given prices.

Let $\mathbf{\Theta}$ be a compact set of traders' *external characteristics*, endowed with a metric d. We denote by θ the representative element of the set $\mathbf{\Theta}$. An economy is a pair $(\mathbf{I}, \alpha)$, where $\alpha : \mathbf{I} \to \mathbf{\Theta}$ is an *attribute function*. Then, $\alpha(i) = \theta$ describes all trader i's relevant characteristics, such as endowments and preferences. We assume that traders' external characteristics are observable.

Trader i is endowed with a finite positive vector of private commodities $\omega^i = (\omega^i_0, \omega^i_1, \omega^i(\xi), \xi = 1, ..., \Sigma) \in \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \mathbb{R}^{L\Sigma}_+$. We assume that the total endowments of commodities is finite, that is, $\sum_{i\in I}\omega^i < \infty$. Let $x^i = (x_0^i, x_1^i, x^i(\xi), \xi = 1, ..., \Sigma) \in \mathbb{R}_+^L \times \mathbb{R}_+^L \times \mathbb{R}_+^{L\Sigma}$ be the trader i's consumption bundle and $x^I = (x^i \in \mathbb{R}_+^L \times \mathbb{R}_+^L \times \mathbb{R}_+^{L\Sigma} : i \in \mathbf{I})$.

Trader i enjoys consumption in the three periods. Let $u^i(x)$ with $x = (x_0, x_1, x(1), ..., x(\Sigma)) \in \mathbb{R}_+^{L(\Sigma+2)}$ denote trader i's utility function. In order to introduce a temporal distinction between period 0 (when traders choose bourses) and periods 1 and 2 (when traders already belong to the bourses) we assume that the utility function is separable as follows[25]

$$u^i(x_0, x_1, x(1), ..., x(\Sigma)) = u_0^i(x_0)u_1^i(x_1, x(1), ..., x(\Sigma)) \tag{1}$$

We assume that

A1: $u_1^i(x_1, x(1), ..., x(\Sigma))$ is continuous, increasing and strictly quasiconcave.

The set of assets available in the economy is $\mathbf{J} \equiv \{1, .., J\}$. We denote by $\mathcal{J}$ the set of all possible subsets of $\mathbf{J}$. Trading occurs in *bourses*. Bourses are thus public projects that allow traders to diversify the risks associated with their business activities. As such, a bourse becomes a source of liquidity to which traders have access to.

A *bourse* is a nonempty subset S of $\mathbf{I}$ that offers the specific activity of asset trading. Each trader in S is endowed with a membership license to trade assets in bourse S. The bourse has associated some assets for trading. For each coalition $S \subseteq \mathbf{I}$, we denote by $J(S)$ the different types of assets available for trade in bourse S. Thus $J(S)$ is an element of $\mathcal{J}$. This set of assets is exogenously given for bourse S, and can be thought as the assets that traders in bourse S agree to issue. We are thus describing primary markets.[26] We make the asset structure to depend on the specific sorting of the traders in financial exchanges. Then, there is a mapping $S \to A(S)$, where $A(S) = [a_j(\xi)]_{\Sigma\times J(S)}$ is the payoff matrix

[25] This specific functional form is considered just for presentation purposes. We emphasize that other types of functional representations of u_0^i and u_1^i are also admissible. However, we point out that the specific form $u_0^i + u_1^i$ is not interesting, as the trading opportunities that a bourse facilitates to a trader are not reflected in the trader's decision of what bourse to choose.

[26] Modeling secondary markets, where trade is done with already issued securities that traders have as endowment because of a previous purchase or security borrowing, would easily follow by considering traders' positive net supply of securities. In this case, traders' identities would matter as each trader has associated a given amount of securities supply. See Bottazzi, Luque and Páscoa (2010) for such a model with asset endowments and security borrowing through repo.

describing the financial structure for coalition S, where $a_j(\xi) \in \mathbb{R}^L_+$ (assets are real). We denote by $q(J(S)) \in \mathbb{R}^{J(S)}$ the vector of assets prices associated to $J(S)$. Observe that it may be possible that a same asset has different price and volume of trade in two different bourses (see Example 1), as the traders in the two bourses can have different endowments, degree of risk aversion and desire to smooth consumption on private goods.

We represent a bourse by a pair $(S, A(S))$. A *bourse structure* is given by $F(\mathbf{I}) = \{(S_1, A(S_1)), ..., (S_k, A(S_k)), ..., (S_K, A(S_K))\}$. We denote by $\mathbf{F}(\mathbf{I})$ the set of all possible bourse structures. Traders can belong to several bourses, and therefore, a bourse structure is not a partition in the sense that it may happen that $S_k \cap S_{k'} \neq \emptyset$. As in Allouch and Wooders (2008) we require the number of bourse memberships of every trader to be bounded.

Given an economy $(\mathbf{I}, \alpha)$, and a bourse structure $F(\mathbf{I})$, let us denote by $F[i; \mathbf{I}] = \{S_k \in F(\mathbf{I}) : i \in S_k\}$ the set of all bourses in $F(\mathbf{I})$ that contain trader i. Trader i can only trade assets with those traders in $F[i; \mathbf{I}]$. We thus can relate the trading volume of an asset $j \in J(S)$ to the number of traders in S and their wealth. This implies that a trader not only chooses a bourse because of the assets available for trade, but also because of the wealth associated to its traders. See Examples 1 and 2 for instance.

We denote by y^i_j the trader i's position on asset $j \in J(S)$, with $S \in F[i; \mathbf{I}]$. As usual, $y^i_j > 0$ denotes a purchase of asset j and $y^i_j < 0$ denotes a short sell of asset j. Let us denote $y^I = (y^i \in \mathbb{R}^{J(S_k)}_+ : S_k \in F(i; \mathbf{I}), i \in \mathbf{I})$.

There are costs associated to a bourse S, such as those of providing a trading technology or the costs incurred in the financial engineering of creating the assets. These costs can be seen as fixed costs to bourse S, and we denote them by $z(S) \in \mathbb{R}^L_{++}$ (in terms of inputs of the private goods). In fact, we may think of $z(S)$ as the cost of adopting the technology of asset trading for bourse S. In this way, we can characterize the asset market characteristics of bourse S by the pair $(A, z)(S)$.

In order to participate in a bourse S, each trader must pay a fixed cost, called *membership fee,* $\pi^i(S) \in \mathbb{R}_+$, which is dependent on his own type $\alpha(i)$. A participation price system is a set $\Pi = \{\pi^i(S) \in \mathbb{R} : S \subset I \text{ and } i \in S\}$.

For the sake of simplicity, we assume that the bourse charges no transaction fee to its traders. Trader i's budget constraint at the initial period, when he chooses a set of bourse memberships, is:

$$p_0(x_0^i - \omega_0^i) + \sum_{S_k \in F(i,I)} \pi^i(S_k) \leq 0 \tag{2}$$

The profits of bourse S are given by

$$\sum_{i \in S} \pi^i(S) - p_0 z(S) \tag{3}$$

The budget constraint in period 1, once trader i has chosen the bourses he wants to belong to, is

$$p_1(x_1^i - \omega_1^i) + \sum_{S_k \in F(i,I)} \sum_{j \in J(S_k)} q_j y_j^i \leq 0 \tag{4}$$

We assume that short sales are bounded. Trader i's budget constraints in period 2 are

$$p(\xi)(x^i(\xi) - \omega^i(\xi)) \leq p(\xi) \sum_{S_k \in F(i,I)} \sum_{j \in J(S_k)} a_j^i(\xi) y_j^i, \ \forall \xi \in \Sigma \tag{5}$$

Observe that in period 2 traders are paid the full promise (default is not allowed).

4 Equilibrium

We propose a two stages economy where the first stage corresponds to the formation of a bourse structure and the second stage corresponds to the activity of asset trading, given a bourse structure.

4.1 Second stage: trading in a fixed bourse structure

Definition 1 (*Equilibrium for the second stage*)**:** Given the bourse structure $F(\mathbf{I})$, an equilibrium for the second stage consists of a system $(x_1^I, x^I(1), ..., x^I(\Sigma), y^I, p, q)(F(\mathbf{I}))$, such that,

(B.i) given trader's bourses memberships $F(i; \mathbf{I})$, the trader chooses optimally his position of commodities and assets, that is, $(x_1^i, x^i(1), ..., x^i(\Sigma), y^i)(F(\mathbf{I})) \in \arg\max u_1^i(x_1, x(1), ..., x(\Sigma))$, subject to constraints (4) and (5).

(B.ii) commodity markets clear at periods 1 and 2, i.e. $\sum_{i\in I}(x_1^i - \omega_1^i) = 0$ and $\sum_{i\in I}(x^i(\xi) - \omega^i(\xi) - \sum_{S_k\in F(i,I)} \sum_{j\in J(S_k)} a^j(\xi)y_j^i = 0, \forall \xi \in \Sigma$.

(B.iii) the asset market clears *for each bourse*, i.e. $\sum_{i\in S_k} y_j^i = 0$, $\forall j \in J(S_k)$, $\forall S_k \in F(\mathbf{I})$.

We denote the set of equilibria of the second stage, given a bourse structure $F(\mathbf{I})$, by $E(F(\mathbf{I}))$. The above equilibrium concept is parameterized by the bourse structure and the assets offered for trade by each bourse. Examples 1-4 illustrate how such parametrizations are important determinants of traders' welfare.

4.2 First stage: demutualization

First, observe that, given the bourse structure $F(\mathbf{I})$, our specification of the utility function (1) allows us to write trader i's utility as follows

$$V^i(x_0, F(\mathbf{I})) \equiv u_0^i(x_0)U_1^i(x(F(\mathbf{I}))) \tag{6}$$

where

$$U^i(x(F(\mathbf{I}))) \equiv u_1^i((x_1, x(1), ..., x(\Sigma))(F(\mathbf{I}))) \tag{7}$$

denotes the trader i's indirect utility that results from maximizing his utility $u_1^i(\cdot)$ subject to his budget constraints (4) and (5). Observe that trader's bourse memberships enter indirectly into the utility u^i through the access to income and the risk sharing that he gains from trading the securities offered in these bourses. It is also important to observe that $V^i(x_0, F(\mathbf{I}))$ may represent ever-increasing gains from trade in larger bourse sizes, depending on how $u_0^i(x_0)$ and $u_1^i(x(F(\mathbf{I})))$ interrelate in the functional form dictated by $u^i(x_0, x(F(\mathbf{I}))$.[27] We assume that

A2: V^i satisfies Allouch and Wooders (2008) assumptions (a)-(h); namely, monotonicity, continuity, quasi-concavity, desirability of endowment, private goods are valuable, and continuity with respect to attributes.[28] We assume that $F[i; \mathbf{I}] \to x^i(F[i; \mathbf{I}])$ is a continuous correspondence on trader i's attribute.[29]

[27] As can be seen in Appendix, ever-increasing gains from trade in larger bourses is not a problem in assuring existence of a two stages equilibrium as we can guarantee existence of the first stage equilibrium using Allouch and Wooders (2008) result.

[28] See also these assumptions explicitly stated in the Supplementary Material.

[29] The latter assumption in A2 says that if we change the attribute of trader i slightly (say change endowments and preferences just a "bit"), then $F[i; \mathbf{I}]$ changes slightly and trader i's consumption at dates 1 and 2 will change smoothly when $F[i; \mathbf{I}]$ changes. This assumption corresponds to the "continuity with respect to attributes" of Allouch and Wooders (2008).

We now come to the definition of the equilibrium of an economy where the bourse structure is endogenously formed by the traders.

Definition 2 (*Equilibrium for the first stage*): An equilibrium for the first stage is an ordered triple $((x_0^I, F(\mathbf{I})), p_0, \Pi)$ that consists on an allocation of commodities x_0^I, with $x_0^I \equiv (x_0^i \in \mathbb{R}_+^{(2+\Sigma)L} : i \in \mathbf{I})$, a bourse structure $F(\mathbf{I})$, a commodity price vector p_0 and a participation price system Π, such that

(A.i) $\sum_{i \in \mathbf{I}} (x_0^i - \omega_0^i) + \sum_{S_k \in F(\mathbf{I})} z(S_k) \leq 0$

(A.ii) For each S_k, profits are non positive, i.e., $\displaystyle\sum_{i \in S} \pi^i(S) - p_0 z(S) \leq 0$.

(A.iii) Utility maximization given bourse formation costs and budget constraints: if $V^i(y_0^i, F[i, S]) > V^i(x_0^i, F[i, \mathbf{I}])$, then $p_0(y_0^i - \omega_0^i) + \displaystyle\sum_{S_k \in F(i,S)} \pi^i(S_k) > 0$.

The stated equilibrium concept above follows Allouch and Wooders (2008). An extra equilibrium condition would be to require that most consumers cannot be very far outside their budget constraints. As they remark, this condition can be derived from conditions of the model and other parts of the definition of equilibrium. Therefore, we omit such condition in Definition 2.

We could have also explicitly considered communication costs paid at period 0 (as in Allouch and Wooders (2008)), if some traders want to move to different bourses (coalitions). For example, the communication costs of a bourse of size $|S|$ can be $\varepsilon|S|\bar{1} \in \mathbb{R}_{++}^L$ with $\varepsilon > 0$ and $\bar{1} = (1, ..., 1) \in \mathbb{R}^L$. Then, condition (A.iii) should be modified (replace the right hand side element of the inequality in the thesis part for $\varepsilon|S|\bar{1}$). This possibility of explicitly introducing communication costs would allow us to accommodate specific searching frictions used in other models, such as Duffie et al. (2005).

It may also happen that, depending on the composition of the set of traders $\mathbf{I}$, some traders cannot be accommodated in their preferred bourses. Allouch and Wooders (2008) show that if the economy is large, then these set of traders constitute only a small proportion of the total population. For that, Allouch and Wooders accommodate the equilibrium notion to take account of these reminders. We prefer to avoid further notation and keep our notation simple and refer to the original paper for such refinement.

4.3 Subgame perfect equilibrium

Finally, we introduce a two-stages equilibrium concept for the process of formation and competition of bourses. The first stage corresponds to period 0, whereas the second stage is for periods 1 and 2.[30]

Definition 3: We say that the vector $(((x_0^I, F(\mathbf{I}), p_0, \Pi), (x_1^I, x^I(1), ..., x^I(\Sigma), y^I, p, q)(F(\mathbf{I})))$ constitutes a subgame perfect equilibrium*,* if

1) $(x_0^I, F(\mathbf{I}), p_0, \Pi)$ is an equilibrium for the first stage, and

2) $(x_1^I, x^I(1), ..., x^I(\Sigma), y^I, p, q)(F(\mathbf{I}))$ is an equilibrium for the second stage.

Observe that at date 2 there can be more than one equilibrium for a given bourse structure $F(\mathbf{I})$. It is well known that different beliefs among traders on the equilibrium realizations may lead to a problem of non-existence. To avoid this possibility we impose the "rational expectations hypothesis" (see Duta and Morris (1997)); that is, traders agree on the realization of prices at each state (consensus) and simultaneously believe that there is a single possible price in each state (degenerate beliefs). No information problems are considered here. Thus, traders' beliefs of the realization of prices in each state are self-fulfilling.

Finally, we must take care of the possibility that traders can make ever-increasing gains from larger and larger bourses.[31] In this scenario, equilibrium is assured to exist if the utility function $V^i\left(x_0, F(\mathbf{I})\right)$ satisfies the following "Desirability of wealth" condition, considered by Allouch and Wooders [2008].

A3: There is a bundle of goods $x_0^* \in \mathbb{R}_+^L$ and an integer η such that, for any economy $(\mathbf{I}, \alpha)$ and any consumer $i \in \mathbf{I}$, there is a coalition $S \subset \mathbf{I}$ with $|S| \leq \eta$ and a bourse structure $F(S)$ satisfying the condition that, for any bourse structure $F(\mathbf{I})$, $V^i(x_0^i + x_0^*, F(i; S)) \geq V^i(x_0^i, F(i; \mathbf{I}))$ for any $x_0^i \in \mathbb{R}_+^L$.

As Allouch and Wooders [2008] point out, this assumption permits ever-increasing gains from larger and larger "bourses" while, at the same time, allows for small "bourses". Assumption 3 is weak in the sense that x_0^* may not be feasible for a trader or even for the bourse. Moreover, Assumption 3 portraits

[30] Observe that for the first stage, utility $U^i\left(x^i(F(\mathbf{I}))\right)$ is given, whereas for the second stage, it is utility $u_0^i(x_0^i)$ the one that is given.

[31] We emphasize that ever-incresing gains from larger bourses is not imposed as an assumption. We are just allowing for this possibility to occur in the economy.

an economy with dark pools of liquidity, as it considers the possibility that sufficiently wealthy traders can substitute their bourse memberships with arbitrarily many members for a relatively small "dark liquidity pool" and achieve a preferred outcome.

Theorem 1: *Let assume A1, A2 and A3. Then, there exists a subgame perfect equilibrium for the two-stages economy with possibly ever-increasing gains from larger bourses.*

Observe that our notion of equilibrium is in fact a price-taking equilibrium. As Allouch and Wooders (2008) demonstrate, when the number of agents is large, agents behave as price takers. This price-taking behavior also translates into our first stage equilibrium notion. In the second stage, a finite but large number of traders also may resemble a price-taking economy. Asset prices are determined competitively by assuming that prices precede traders' maximization problems. This type of competitive model with a finite but large number of traders is common in the GEI literature (see Geanakoplos and Zame [2007]).

Departing from a price taking framework in the second stage would need to consider a strategic behavior by the traders in the economy. This is the case of Pagano [1989], who proposes a Nash bargaining solution where traders can influence the asset price by trading the asset. Although an strategic approach to price determination seems possible in some circumstances (say a group of hedge funds manipulating a thin market), in many other circumstances traders are price takers. Other types of equilibrium determination of asset prices and their consequences on traders' sorting into bourses seem interesting. In particular, the learning process portrayed in Biais, Hillon and Spatt (1999) seems an interesting approach.

5 Discussion on efficiency

Inefficiency and manipulation power of the bourses: We argue that if the bourse structure is fixed and the managers of the bourses have manipulation power to increase the charges to its traders, then traders' welfare may diminish, compared with a situation of free sorting of traders into bourses. This situation may occur in a context with oligopoly in the bourses industry or back to the historical impediment to the national traders of trading outside their national markets. The next example illustrates this point.

Example 5 (*Inefficiency and manipulation*): Let us consider the three traders bourse $S^3 = (1, 2, 4)$ (with the same utilities and the same endowments as in Example 2). This bourse has a formation cost of $z(S^3) = 6$. Let us think of this bourse as a national bourse with monopoly power. The bourse chooses a common membership fee $\hat{\pi}(S)$ in order to maximize a quadratic concave profit function[32]

$$\Gamma \equiv |S^3|\hat{\pi}(S^3) - \frac{\hat{\pi}(S^3)^2}{4} - z(S^3)$$

From the maximization problem it follows that the manipulated membership fee is $\hat{\pi}(S^3) = 2|S^3|$ which is compatible with positive profits. The indirect utilities associated to this national bourse are $\hat{V}^1(S^3) = 1.9$, $\hat{V}^2(S^3) = 4.3$ and $\hat{V}^4(S^3) = 0.6$. The indirect utilities in a competitive framework, where traders are free to choose their bourse memberships and the membership fees are efficient, are $V^1(S^3) = 5.4$, $V^2(S^3) = 5.1$ and $V^4(S^3) = 5.4$.[33] Now comparing the indirect utilities in the two cases we can assert that all traders are worse off in the framework with a monopolistic national bourse. This leads to the conclusion that, if bourses have manipulation power to increase the charges to their traders, inefficiency could arise. ■

Recovering efficiency through demutualization: Non-optimal bourse structures may arise because traders are not free to choose their preferred bourses. For example, at the time when bourses where national monopolies, traders were constraint to trade in the bourses of their respective countries. Not only the national bourses could have manipulated the membership fees as we have shown above (Example 5), but also it could be that the mere composition of traders in those national bourses were far from optimal. Here we argue that a process of *demutualization* (traders are free to move from their pre-assigned bourses (e.g., national bourses) to their most preferred ones, without any other restriction than the mere cost of paying the corresponding membership fee, has a profound effect on traders' welfare, the volume of assets traded and the assets prices. In Example 1 we can clearly see that the free sorting process has implications in the volume of assets traded and their prices. Examples 2 and 3 point into the direction that

[32]The term $-\frac{\hat{\pi}(S^3)^2}{4}$ of Γ is considered in order to avoid the bourse choosing an infinite membership fee.

[33]Here the indirect utilities are analogous to those defined in Example 2 for the three traders bourse S^3 in the non-anonymous setting.

when traders are free to choose, the bourses formed result to be optimal in size and members. In particular, we can claim that *larger bourses are not always efficient*, since the right selection of traders and assets in a bourse may be such that this bourse may be Pareto superior to another bourse larger in size (see Example 3).

The process of demutualization corresponds to the first stage of our game. We claim the following.

Claim 1: *The demutualization process leads to an optimal bourse structure.*

The efficiency of the equilibrium of the first stage is guaranteed by Allouch and Wooders (2008) (through the core). As a corollary we can assert that there is a membership price for each type of trader such that the sum of the membership prices covers the cost $z(S)$, and such that these membership prices internalizes the externalities that some members may put on others.

However, Claim 1 (efficient bourse structure in the first stage) does not guarantee the equilibrium of the second stage to be efficient.

Inefficiency and incomplete markets: The issue of efficiency in the second stage is related to previous results in the literature of GEI. Arrow (1951) showed that, in an economy with complete markets, any competitive equilibrium is Pareto optimal. Geanakoplos and Polemarchakis (1986) demonstrated that if market incompleteness is considered instead and assets pay off in the numeraire commodity, then competitive equilibrium allocations are typically Pareto suboptimal in a strong sense (the market does not make efficient use of the existing assets). These results naturally extent to our second stage economy when assets pay in numeraire[34] (where the bourse structure is fixed).

The sorting of traders into bourses gives light to new insights concerning efficiency. By analogy with Geanakoplos and Polemarchakis (1986), efficiency in the second stage can only be attained if the bourse structure chosen in the first stage is characterized by complete markets in every bourse. In other words, inefficiency occurs in an economy where the *asset structure* chosen by the traders is not from adequate to diversify traders' risks. This may occur in equilibrium as we illustrated in Example 4. There, even if a complete market structure is

[34]Observe that it is easy to rewrite our model with assets paying in only one of the commodites. In this case there are no further complications for the equilibrium existence proof of stage 2.

available to the traders in one of the bourses, the result might be another bourse characterized with market incompleteness. In that example, the reason is due to the higher fixed formation costs associated to a bourse with complete markets (higher degree of complexity). This in turn affects traders' utilities when facing their respective membership fees. Thus, inefficiency in the second stage arises even if a bourse with a complete asset structure was available to the traders. This argument suggests that

Claim 2: *The inefficiency of the equilibrium in the second stage is endogenous to the model, in the sense that it depends on the sorting of traders into bourses (stage one), given bourses formation costs.*

As a consequence of Claims 1 and 2,

Claim 3: *If the first stage equilibrium is characterized by a bourse structure where each bourse has a complete asset structure, then the subgame perfect price taking equilibrium is efficient. Otherwise, it is Pareto suboptimal in the sense of Geanakoplos and Polemarchakis (1986).*

6 Conclusions

This paper provides a general equilibrium model for the process of formation of bourses and subsequent trading. The model proposed is well behaved (in the sense that equilibrium exists). In general, inefficiencies may arise in these markets: 1) if the equilibrium bourse structure is characterized by some bourses with incomplete asset structures; this may happen for certain bourse formation costs and trading complementarities, which induce traders to sort into bourses with asset structures which are endogenously incomplete (see Example 4); 2) if the bourse structure is not optimal; this may occur in the case of national bourses; 3) if bourses have the manipulative power to increase the membership fees to theirs traders (Example 5). The first type of inefficiency pointed out here is endogenous to our model: the degree of incompleteness of the asset structure is determined by the free sorting of traders into bourses, given the bourses formation costs. However, we find that demutualization policies may alleviate the latter two types of inefficiencies. As we show in Examples 1, 2 and 3, the process of demutualization has important consequences on the volume of assets traded, their prices and the efficiency of the markets.

Ever-increasing gains from trade in larger bourses may lead to an equilibrium with a unique global trading platform. We illustrate this possibility with Example 2: large bourses are optimal if traders complementarities are good enough. But, as we point out in Example 3, it may occur that larger bourses are not always optimal. Self picked assets and traders associated to dark liquidity pools may be preferred to larger bourses. This provides support to the empirical evidence provided in Fact 2.

7 Appendix

The proof of Theorem 1 relies on previous results of existence of equilibrium in the literatures of GEI and local public goods. Here we outline the steps of the proof. First, we show that an equilibrium for the second stage exists, given a bourse structure. Secondly, we show that there exists a measurable selector of the equilibria of the second stage, which is the one in which we evaluate the indirect utility function used in the first stage. Thirdly, we show that an equilibrium for the first stage exists, given the parametrized outcomes in the second stage.

Proposition 1: *Let assume A1 and bounded short sales. Then, there exists an equilibrium for the second stage, where the bourse structure is fixed.*

The proof of the Proposition 1 departs from standard GEI in that trading is only carried on through bourses and, therefore, traders have only access to those assets offered in the bourses which they belong to. Asset prices become dependent on the structure of bourses (i.e., q_j, with $j \in J(S)$, is determined endogenously in bourse S) affecting in turn the allocation of assets in different bourses and the traders' utilities (through their budget constraints). Examples 1-4 illustrate these points. Apart from these subtleties, the proof of Proposition 1 is standard and is left for the Supplementary Material.

Now observe that the problem of maximizing $u_1^i(\cdot)$ subject to the budget constraints (4) and (5) may result in more than one solution. This would imply that there would be one indirect utility $U^i\left(x^i(F(\mathbf{I}))\right)$ for each solution $x^i(F(\mathbf{I}))$, and thus more than one function $V^i(x_0^i, F\left(\mathbf{I}\right))$. Existence of a two-stages equilibrium would require to choose a measurable selector $x^i(F(\mathbf{I}))$ from the maximization problem (B.i). The next proposition asserts that this requirement is possible.

Lemma 1: *The maximization problem in (B.i) admits a measurable selector* $x^i(F(I))$.

Proof of Lemma 1: The proof follows by the Kuratowski-Ryll-Nardzewski measurable selection theorem (a weak measurable correspondence with non-empty closed values into a separable metrizable space admits a measurable selection). In fact, we have that $\mathbf{F}(\mathbf{I})$ is a finite set and therefore the equilibrium correspondence $E(\cdot)$ defined in $\mathbf{F}(\mathbf{I})$ is trivially a weak measurable correspondence (see Aliprantis and Border (2006, p. 600)). The correspondence takes values in the positive coordinate subset of a finite dimensional space and therefore it follows immediately that is a separable metrizable space.

The correspondence $E(\cdot)$ takes closed values, i.e., if $(x_1^{I,s}, x^{I,s}(1), ..., x^{I,s}(\Sigma), y^{I,s}, p^s, q^s)$ is a sequence in $E(F(\mathbf{I}))$ that converges to $(x_1^I, x^I(1), ..., x^I(\Sigma), y^I, p, q)$, then $(x_1^I, x^I(1), ..., x^I(\Sigma), y^I, p, q)$ also belongs to $E(F(\mathbf{I}))$. Given an equilibrium sequence, if we consider the budget constraints of each trader and pass to the limit we obtain that in the limit the budget constraint of each trader is satisfied. The same reasoning allows us to prove that the market clearing also holds in the limit.

Finally, it remains to show that in the limit each trader is maximizing his utility. Suppose not, so for a trader i there exists another bundle $(\tilde{x}^i, \tilde{y}^i)$ which is budget feasible and such that $u_1^i(\tilde{x}^i) > u_1^i(x^i)$. Now, let $(\hat{x}^i, \hat{y}^i) = (\lambda x^{i,s} + (1 - \lambda)\tilde{x}^i, \lambda y^{i,s} + (1 - \lambda)\tilde{y}^i)$ with $\lambda \in [0, 1]$. Observe that $(\hat{x}^i, \hat{y}^i)$ is budget feasible for s large enough and for λ close to one. Moreover, by continuity we have that $u_1^i(\tilde{x}^i) > u_1^i(x^{i,s})$, for s large enough. Then, the strict quasiconcavity implies that $u_1^i(\hat{x}^i) = u_1^i(\lambda x^{i,s} + (1 - \lambda)\tilde{x}^i) > u_1^i(x^{i,s})$. This is a contradiction because $(x_1^{I,s}, x^{I,s}(1), ..., x^{I,s}(\Sigma), y^{I,s}, p^s, q^s)$ was an equilibrium for the given bourse structure $F(\mathbf{I})$.

An immediate consequence of Lemma 1 is that $V^i(x_0, F(\mathbf{I}))$ is well defined.

Proposition 2: *Let assume that A2 and A3 hold. Then a price taking equilibrium exists for the first stage of the economy with possibly ever-increasing gains from trade in larger bourses.*

The proof of Proposition 2 follows from Allouch and Wooders [2008, Theorem 2] and is thus omitted. Theorem 1 follows by Lemma 1 and Propositions 1 and 2.

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