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Abstract

Paired Kidney Exchanges (PKE) programs increase the number of kidney transplants by arranging swaps of living donors among patients who have an incompatible donor. The number of transplants performed can be dramatically increased with the participation of compatible donor-patient pairs. We analyze matching rules that provide incentives to patients with compatible donors to enroll in PKE programs by giving them chances to get a kidney with higher expected graft survival. Our results are based on the observation that the characteristics of the donor which have a major impact on the expected graft survival in living donor kidney transplantation –namely, donor's age and health status– affect all compatible patients in the same way and therefore call for a natural restriction on the domain of patients' preferences. Hence, we define a restricted domain of patients' preferences such that patients prefer to get a younger (and healthier) kidney to an older one. We identify non manipulable and (constrained) efficient matching rules in this setting.

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1 Introduction

The literature of allocation of indivisible objects in economies without money has found a remarkable application in the design of Paired Kidney Exchange (PKE) programs. These programs try to overcome the incompatibility (between blood or tissue-types) of living donor-patient pairs by arranging swaps of donors among several pairs ([4, 5, 25]). Besides the difficulties of finding mutually compatible donor-patient pairs, the design of PKE programs needs to address many constraints that are absent in the standard allocation problems. PKE programs usually involve the cooperation and coordination of transplantation units of different medical centers. All the participants in a kidney exchange (donors and patients) need to be readied for surgery at the same time and multiple operating rooms and teams of surgeons have to work simultaneously.[1] At this point, real-life PKE programs have generally focused on maximizing the number of compatible simultaneous exchanges among two donor-patient pairs, although swaps among more than two pairs have been occasionally carried out. In order to solve situations where a donor-patient pair is involved in more than one viable exchange, PKE programs usually rely on (sequentially) giving priority to some patients in a similar fashion employed for the allocation of kidneys obtained from cadaveric donors.[2]

In spite of all the difficulties, PKE programs are a remarkable example of how to generate efficiency enhancement exchanges without any need of monetary compensations. It is common view however that PKE programs could attain even better outcomes if more donor-patient pairs participate in those programs. Clearly, the more donor-patient pairs available in the pool, the greater is the probability of identifying mutually compatible kidney exchanges and the better is the match between donors and recipients. An imme-

[1]There is also an increasing interest on the practice of non-simultaneous, extended altruistic-donor chains. In these cases, a donor chain starts from an altruistic donor who wish to donate a kidney to someone in need of a kidney transplant without having a related recipient ([2, 22, 33]).

[2]This is the case for the New England Paired Kidney Exchange NEPKE (Roth et al. [26, 27]). Other centralized kidney exchange programs implemented in different countries as Korea, Netherlands, and Spain follow similar protocols ([12, 19, 21]).

diate way to enlarge this pool is to provide incentives to compatible patient-donor pairs to enroll in PKE programs. Compatible pairs are usually excluded from PKE programs although they may be very helpful in finding additional mutually compatible swaps for incompatible pairs ([7, 27, 28].) To this end, it is necessary to give to the patient of a compatible pair the chance of a better match that increases the expected survival of the graft with respect to her compatible willing donor kidney. Medical research supports the idea different compatible kidneys may lead to substantially different outcomes in terms of expected survival of the graft. In fact, the age and health status of the donor have a major impact on the expected survival of the graft ([7, 8, 32]). For instance, recent studies by Øien et al. [16] highlight that age and health status also play a crucial role in case of living donations. Donor age greater than 65 is a risk factor for graft loss in all time periods after transplantation.[3]

In this paper, we suggest a protocol such that compatible pairs are willing to participate to PKE programs since patients have the chances to be matched with younger donors than their compatible related donors and therefore can receive a kidney with higher expected graft survival. The existence of such rules is far from being obvious. Previous works ([15, 26]) have shown that it is hard to abandon the dichotomous preference domain, where it is assumed that patients are indifferent between any pair of compatibles kidneys. In fact, if quality concerns regarding the compatible kidneys are taken into consideration, there are no rules that are efficient and induce patients to truthfully reveal their preferences in an unrestricted preference domain. This negative result depends on the presence of feasibility constraints in the maximum number of exchanges that can be simultaneously performed. However, since donors' age and health status affect all patients in the same direction, they call for a natural restriction on patients' preferences over kidneys. In this restricted preference domain we are able to design rules which satisfy strong normative requirements and take into account the different quality of the available kidneys.

[3] There is some controversy on the medical literature regarding the effects of other medical features as closeness of tissue types between patient and donors on the survival of the graft ([4, 9, 17, 18]).

We incorporate the implications of donors' features on patients' preferences in a PKE framework. We analyze a model of non-monetary exchange of indivisible goods with initial endowments where patients' preferences are naturally restricted. Patients preferences over donors' kidneys depend on the mutual compatibility and on the quality of the kidney. The compatibility between a donor and a patient depends on multiple issues that are not publicly observable and are private information for the patients. The quality of the donors' kidney depend on directly observable features as age and health status of the donor. The pool of donors' kidneys can be partitioned in equivalence classes of kidneys of the same quality. Thus, for each pair of kidneys belonging to different classes, every patient for which those kidneys are compatible, ranks those kidneys in the same way. We dub such domains as age-based preferences, because they reflect the idea that patients prefer younger compatible donors to older compatible donors.

PKE programs periodically use the medical details of donors and patients in order to find compatible exchanges in the pool of donor-patient pairs. Of course, this information needs to be elicited from the patients (or they doctors) and patients may have incentives to provide false information in order to improve their chances of getting a better outcome in the process. Hence, we model PKE protocols as rules that map patients' preferences over kidneys (or the medical features of patients and donors) to assignments of the donors' kidneys to the patients. We analyze rules defined on the domain of age-based preferences that satisfy the *individual rationality*,[4] *efficiency* restricted by the logistic constraints on the number of donor-patient pairs involved in the exchanges, and *strategy-proofness*.[5] We first show that those properties are incompatible for rules that admit simultaneous swaps involving more than two donor-patient pairs. Hence, without loss of generality, we focus on rules that only admit pairwise exchanges. We present a family of rules –*age-based priority rules*–that adapt the real-life protocols to our restricted domain of preferences.

[4]A rule satisfies *individual rationality* if patients never prefer the initial situation to the assignment prescribed by the rule.

[5]A rule satisfies *strategy-proofness* if patients never have incentives to provide a false report of their preferences.

According to age-based priority rules, patients select the assignments they prefer from the set of individually rational assignments, and patients' positions in the sequential choice procedure is determined by the quality of their donors. Specifically, if patients can strictly rank all compatible kidneys, then the age-based priority rule is the unique rule that satisfies *individual rationality*, $2-$*efficiency*, and *strategy-proofness.* If patients may be indifferent among kidneys, then age-based priority rules still fulfill those properties. We also analyze the relevant extension of the problem in which patients may have more than one potential donor. In this scenario, it seems desirable to investigate rules that never provide incentives for the patients to withhold some of their potential donors. We extend the definition of age-based priority rule and fine-tune these rules in order to accomplish to this goal.

Our paper contributes to two branches of the matching literature. On the one hand, the literature on kidney exchange, on the other, the literature on strategy-proof allocation of indivisible goods in economies without money when agents have private endowments.

The literature on kidney exchange started by Roth et al. [25], that shows the potential benefits of PKE in terms of increments in the number of kidneys transplantation. This paper assumes that patients' preferences are not restricted and they do not take into account feasibility constraints in the minimum number of operations required in the exchange. Alternatively Roth et al. [26, 27] present the theoretical model that supports the priority protocols applied in the NEPKE. These papers explicitly introduce the problem of feasibility constraints and preference domain restrictions in PKE. Basically, they analyze the incentives and equity properties of priority rules when kidneys exchanges are restricted to involve only two donor-patient pairs and patients are indifferent among compatible kidneys. In that framework, Roth et al. [29] show that efficiency gains could be attained (and almost exhausted) if kidney exchanges among three donor-patient pairs were admitted, under the assumption that compatibility issue only depends on blood–type compatibility. Hatfield [10] characterizes all the rules that satisfy and *constrained efficiency* and *strategy-proofness* for arbitrary feasibility restrictions. Ünver [33] explicitly incorporates a dynamic analysis of the problem, investigating the design of efficient exchange programs still assuming that patients consider all compatible kidneys to be homogeneous. Ausubel and Morrill [2][6] We depart from these papers in introducing a richer domain of preferences that admits strict preference over compatible kidneys, and that inevitably narrows the class of rules that satisfy our properties. Finally, Nicoló and Rodríguez-Álvarez [15] show that these properties are incompatible for unrestricted preferences under feasibility constraints and analyze patients' behavior if they are extremely risk averse. In this paper, we show that such incompatibility can be avoided if patients preferences are restricted to be age-based.

Regarding the literature on strategy-proof allocation of objects, there is a bast body of research that studies the agents' incentives in the so called housing markets proposed by Shapley and Scarf [30]. Shapley and Scarf [30] show that when agents are endowed with a unique object the set of (strict) core assignments (in the sense that no group of agents can improve by swapping objects among themselves) coincides with the competitive equilibrium correspondence. When preferences are strict, the core is single-valued ([24]) and the rule that selects the core allocation has many desirable properties. ([3, 14, 23]). In fact, it is the only rule that satisfies *individual rationality*, (unconstrained) *Pareto-efficiency*, and *strategy-proofness*([13, 31]). However, most of the desirable properties do not hold anymore when indifferences are admitted and/or there are feasibility constraints. The (strict) core itself may not exist ([30]), *Pareto efficiency* and *group strategy-proofness* turn out to be incompatible ([6]). The recent papers by Jaramillo and Manjunath [11] and Alcalde-Unzu and Molis [1] show that there are rules that satisfy *individual rationality*, *Pareto-efficiency*, and *strategy-proofness*, but they never satisfy *anonymity*. We contribute to this literature by analyzing a domain restriction where there exist rules, which satisfy most of the desirable properties, in presence of weak preferences and feasibility. Finally,

[6]Besides Ünver [33], Zenios [34] also considers PKE in a dynamic setting. Zenios focuses on the optimal assignment of donor-patient pairs to direct exchange programs or indirect exchange programs, where patients may exchange their incompatible donor to gain priority in the waiting list. In this paper the planner perfectly knows patients' preferences and no information has to be elicited.

Pápai [20] studies trades in general markets where individuals are endowed with multiple heterogeneous indivisible goods and a feasible allocation is a reallocation of the indivisible goods among the agents. Given the nature of the problem we study, we also consider the case when individuals may have multiple donors in their initial endowment but we assume that only one object is exchanged. Moreover, we depart from [20] by assuming that agents may have private information not only regarding their preferences but also the set of initial objects. Hence, we also focus on agents' incentives to truthfully report the information about both their preferences and their initial endowment.

Before we start the formal analysis, we outline the remainder of this paper. In Section 2, we present the model of kidney assignment problems and basic notation. In Section 3, we introduce the concept of age–based preferences. In Section 4, we analyze the framework where each patient may have only one willing donor. In Section 5, we extend the analysis to the multiple donor case. In Section 6, we present concluding remarks and further applications of the framework. In the Appendix, we collect all the proofs.

2 Basic Notation

Consider a finite society consisting of a set $N = \{1, \ldots, n\}$ of patients ($n \geq 3$) who need a kidney for transplantation. Each patient has a potential donor, and $\Omega = \{\omega_1, \ldots, \omega_n\}$ denotes the set of kidneys available for transplantation. For each patient i ω_i refers to the kidney of patient i's donor. We assume for the moment that all available kidneys are obtained through living donors and each patient has only one potential donor.[7]

Each patient i is equipped with a complete,reflexive, and transitive preference relation $\succsim_i$ on Ω. We denote by $\succ_i$ the associated strict preference relation and by $\sim_i$ the associated indifference relation. Let $\mathcal{P}$ denote the set of all preferences. We call $\succsim \in \mathcal{P}^N$ a preference profile. For each $T \subseteq N$ and each $\succsim \in \mathcal{P}^N$, $\succsim_T \in \mathcal{P}^T$ denotes the restriction of the profile $\succsim$ for the members of T. We usually assume that patients' preferences are further

[7]We dispense with the later assumption in Section 5.

restricted, so for each patient i her preferences belong to a subset $\mathcal{D}_i \subset \mathcal{P}$. We denote by $\mathcal{D} \equiv \times_{i \in N} \mathcal{D}_i \subseteq \mathcal{P}^N$ a domain of preferences over kidneys.

An ***assignment*** is a bijection from kidneys to patients. We denote an arbitrary a as an n-tuple of pairs $a = [(1, \omega), \ldots, (n, \omega')]$ such that for each $i, j \in N$, $i \neq j$ and each $\omega, \omega' \in \Omega$, if $(i, \omega), (j, \omega') \in a$, then $\omega \neq \omega'$. For each patient i and each assignment a, a_i denotes the kidney assigned to i by a. For each assignment a, if a patient is assigned her donor's kidney ($a_i = \omega_i$), we interpret that either she continues in dialysis or –if she is compatible with her donor– she receives her donor's kidney. Let $\mathcal{A}$ be the set of all assignments.

In every assignment, kidneys are allocated by forming exchange cycles of patient–donors couples. In each cycle, every patient receives a kidney from the donor of some patient in the cycle and simultaneously her donor's kidney is transplanted to another patient in the cycle.

For each assignment a, let π_a be the finest partition of the set of patients such that for each $p \in \pi_a$ and each $i \in p$, there are $j, j' \in p$, with $a_i = \omega_j$ and $a_{j'} = \omega_i$.[8]

Clearly, for each assignment a the partition π_a is unique and well-defined. We define the ***cardinality of*** a as the $\max_{p \in \pi_a} \#p$.

The cardinality of an assignment refers to the size of the largest cycle formed in the assignment. Of course, the concept of cardinality is crucial for our notion of feasibility.

For each $k \in \mathbb{N}$, $k \leq n$, we say that the assignment a is k–***feasible*** if a's cardinality is not larger than k. Let $\mathcal{A}^k$ be the set of all k–feasible assignments.

An interesting case of feasibility restrictions appears when only immediate exchanges between two couples are admitted. An assignment a is a ***pairwise-exchange*** assignment ($a \in \mathcal{A}^2$) if a satisfies that if for some $i, j \in N$ $(i, \omega_j) \in a$, then $(j, \omega_i) \in a$.

[8]Note that $j = j'$ and $i = j = j'$ and then $a_i = \omega_i$ are allowed.

In this paper, we are interested in rules that select a (kidney) assignment for each preference profile. Hence, a rule defined in the domain $\mathcal{D}$ is a mapping $\varphi : \mathcal{D} \to \mathcal{A}$.

Finally, we present formal definition of the standard desirable conditions for rules.

Individual Rationality. For each $i \in N$ and each $\succsim \in \mathcal{D}$, $\varphi_i(\succsim) \succsim_i \omega_i$.

$k-$***Efficiency.*** For each $\succsim \in \mathcal{D}$, $\varphi(\succsim) \in \mathcal{A}^k$ and there is no $a \in \mathcal{A}^k$ such that for each $i \in N$ $a_i \succsim_i \varphi_i(\succsim)$ and for some $j \in N$, $a_j \succ_j \varphi_j(\succsim)$.

Strategy-Proofness. For each $i \in N$, each $\succsim \in \mathcal{D}$, and each $\succsim'_i \in \mathcal{D}_i$,

$$\varphi_i(\succsim) \succsim_i \varphi_i(\succsim'_i, \succsim_{-i}).$$

3 Age-Based Preferences

In this section we present a new domain restriction that is directly inspired by the structure of paired kidney exchange. Before we describe the domain restriction, we introduce some useful notation.

For each patient i and each preference $\succsim_i \in \mathcal{P}$, we define i's ***set of desirable kidneys*** $D(\succsim_i) \equiv \{\omega \in \Omega \mid \omega \succ_i \omega_i\}$, and analogously i's ***set of undesirable kidneys*** $ND(\succsim_i) \equiv \{\omega \in \Omega \setminus \{\omega_i\} \mid \omega_i \succsim_i \omega\}$.

We interpret each patient's set of desirable kidneys as all those kidneys which lead to an improvement with respect to her donor's organ. Of course, if the patient is not compatible with her donor (and the transplantation is not viable), then the set of desirable kidneys includes all the compatible available kidneys. On the other hand, if the patient is compatible with her donor, then the set only includes the compatible organs that strictly improve upon her own donor's organ.

Kidney transplantations from living donors have excellent long-term outcome irrespective of matching according to HLA type ([4, 9]). However, donor's characteristics, like age or health status have a significant impact on graft survival. We incorporate this fact by assuming that patients divide the set of desirable kidney in subsets of kidneys of homogeneous quality (indifference classes). Therefore, patients are not always indifferent between pairs of compatible kidneys. Specifically, the quality of the match depends on characteristics of the donors which are observable by the transplant coordinator (TC) and *these characteristics affect all patients in the same direction.* In fact, donor's age turns out to represent the most relevant characteristic to determine the probability of long-term graft survival in case of living donations too.

Let denote by $\Pi = \{\Pi(1), \ldots, \Pi(l)\}$ be a partition of Ω. For each patient $i \in N$, the preference relation $\succsim_i \in \mathcal{P}$ is a Π***-based preference*** if for each $\omega, \omega' \in D(\succsim_i)$ and for each $\bar{\omega}, \bar{\omega}' \in ND(\succsim_i)$:

(i) $\omega \in \Pi(j)$ and $\omega' \in \Pi(k)$ and $j < k$ imply $\omega \succ_i \omega'$, and

(ii) $\omega, \omega' \in \Pi(j)$ implies $\omega \sim_i \omega'$.

(iii) $\omega_i \succ_i \bar{\omega}$ and $\bar{\omega} \sim_i \bar{\omega}'$.

Let $\mathcal{D}_i^\Pi$ denote the set of all Π-based preferences for patient i and let $\mathcal{D}^\Pi \equiv \times_{i \in N} \mathcal{D}_i^\Pi$.

According to Π-based preferences, Π divides (in decreasing order) the set of available kidneys in subsets of homogeneous quality kidneys. For expositional clarity, from now on we assume that Π partitions the set of available organs for transplantation according to the donors' age. We call the induced preference domain $\mathcal{D}^\Pi$ the Π-age-based preference domain. Thus, $\Pi(1)$ contains the highest quality kidneys, $\Pi(2)$ the second highest quality kidneys, and $\Pi(l)$ the lowest quality kidneys. In the simplest setting, the set of available kidneys is divided in two disjoint subsets, young donors and mature donors. This partition induces a natural restriction on patients' preferences. Whether being a young or a mature donor kidney does not determine the compatibility between the donor and the patient, a desirable kidney from a young donor is preferred to a desirable kidney from a mature donor. Moreover, patients are indifferent between any pair of desirable kidney from young (mature) donors.

We assume that the partition Π is public information. Patients report which is the set of desirable (i.e. compatible) kidneys. It is worth to highlight that since we focus on rules that satisfy *individual rationality*, all the relevant information that is private for patient i is contained in her sets of desirable and undesirable kidneys. For each $\succsim_i \in \mathcal{D}^{\Pi}$, if both $\omega, \omega' \in D(\succsim)$, the order in which i ranks ω with respect to ω' only depends on the elements of the partition Π they belong to.

Throughout the paper we rule out degenerate situations where the set of kidneys is partitioned in two sets and one of them contains a unique element.

Assumption. The partition of the set of kidneys Π is such that either $\#\Pi \geq 3$ or $\Pi = \{\Pi(1), \Pi(2)\}$ and $\#\Pi(1) \geq 2$ and $\#\Pi(2) \geq 2$.

Without any loss of generality and in order to simplify notation, henceforth, we assume that for each partition Π, for each $i, j \in N$, with $\omega_i \in \Pi(l)$ and $\omega_j \in \Pi(l')$, $i < j$ implies $\Pi(l) \leq \Pi(l')$.

4 Priority Rules and Age–Based Preferences

In this restricted age-based domain, we show that positive results may emerge and we are able to present rules that satisfy *individual rationality*, *k-efficiency*, and *strategy-proofness*. We can escape impossibility results because donor's characteristics have a common effect on patients' preferences. However, we start this section with an impossibility result that highlights the tension between *k-efficiency* and *strategy-proofness*. Even in the restricted age-based environment, we have to focus on rules that only admit pairwise exchanges.

Theorem 1. *For each partition Π and each $k \in \mathbb{N}$ such that $3 \leq k \leq n-1$, no rule $\varphi : \mathcal{D}^{\Pi} \rightarrow \mathcal{A}^k$ satisfies* individual rationality*,* k-efficiency*, and* strategy-proofness*.*

The literature on kidney exchange has focused on priority mechanisms which are commonly used in most transplant centers to allocate cadaver organs ([26, 27]). In this section, we analyze how priority mechanisms need to be tailored in the age-based preferences environment.

A ***priority ordering*** σ is a permutation of patients ($\sigma : N \to N$) such that the k-th patient in the permutation is the patient with the k-th priority. Let σ^* denote the natural priority ordering (for each $i \in N$, $\sigma^*(i) = i$). For each partition Π and each priority ordering σ, we say that σ ***respects*** Π if for each $i, j \in N$ with $\omega_i \in \Pi(l)$, $\omega_j \in \Pi(l')$, $l < l'$ implies $\sigma(i) < \sigma(j)$.

For each $\succsim \in \mathcal{P}^N$, and each $l \leq n$, let $\mathcal{I}(\succsim) \equiv \{a \in \mathcal{A}^2 \mid \text{for each } i \in N \ a_i \succsim_i \omega_i\}$ denote the ***set of all individually rational pairwise assignments***.

Priority Algorithm. Fix a permutation of the patients σ, and a preference profile $\succsim \in \mathcal{P}^N$:

- Let $\mathcal{M}_0^\sigma(\succsim) = \mathcal{I}(\succsim)$.
- For each $k \leq n$, let $\mathcal{M}_k^\sigma \subseteq \mathcal{M}_{k-1}^\sigma$ be such that:

$$\mathcal{M}_k^\sigma(\succsim) = \left\{a \in \mathcal{M}_{k-1}^\sigma \mid \text{for no } b \in \mathcal{M}_{k-1}^\sigma(\succsim),\ b_{\sigma^{-1}(k)} \succ_{\sigma^{-1}(k)} a_{\sigma^{-1}(k)}\right\}.$$

Note that $\mathcal{M}_n^\sigma$ is well defined, non-empty, and essentially single-valued.[9]

Let $\mathcal{D}$ be an arbitrary domain of preferences. A rule φ is a ***pairwise priority rule*** if there is a priority ordering σ such that for each $\succsim \in \mathcal{D}$, $\varphi(\succsim) \in \mathcal{M}_n^\sigma(\succsim)$. We denote by ψ^σ the pairwise priority rule with priority ordering σ. Analogously, let $\psi^* \equiv \psi^{\sigma^*}$. Finally, for every partition Π and priority ordering σ, the pairwise priority rule ψ^σ is an ***age-based priority rule*** if σ respects Π.

At this point, before we continue with the analysis of priority rules, we introduce definitions related to the concept of the core in PKE problems.

[9] A set is essentially single-valued if either it is single-valued, or if it contains more than one element, all the patients are indifferent between any two elements in the set. That is, for each patient i, each $\succsim \in \mathcal{P}^N$, and each $a, a' \in \mathcal{M}_n^\sigma(\succsim)$, $a_i \sim_i a'_i$.

For each pair of pairwise assignments $a, b \in \mathcal{A}^2$, each coalition $T \subseteq N$, and each $\succsim \in \mathcal{P}^N$, a ***weakly dominates*** b via T at $\succsim$ if:

(i) For each $i \in T$, there is $j \in T$ such that $a_i = \omega_j$,

(ii) for each $i \in T$, $a_i \succsim_i b_i$,

(iii) there is $j \in T$, such that $a_j \succ_j b_j$.

In this case we say that coalition T ***weakly blocks*** b under $\succsim$ via a. For each $\succsim \in \mathcal{P}^N$, an assignment $a \in \mathcal{A}^2$ is in the ***strict core*** of the pairwise exchange problem associated to $\succsim$ if a it is not weakly dominated by any assignment $b \in \mathcal{A}^2$. Similarly, a ***strongly dominates*** b via T at $\succsim$ if:

(i) For each $i \in T$, there is $j \in T$ such that $a_i = \omega_j$,

(ii) for each $i \in T$, $a_i \succ_i b_i$.

An assignment $a \in \mathcal{A}^2$ is in the ***weak core*** of the pairwise exchange problem associated to $\succsim$ if a it is not strongly dominated by any assignment $b \in \mathcal{A}^2$.[10]

We first consider situations where patients are never indifferent between two acceptable kidneys. Let Π^* denote the complete partition according to the natural order, that is, $\Pi^* \equiv \{\Pi(1), \ldots, \Pi(n)\} = \{\{1\}, \ldots, \{n\}\}$. When the set of available kidneys is finely partitioned and each indifference class consists of a single donor, the partition induces strict preferences over the set of available kidneys. Note that since preferences are strict, for each $\succsim \in \mathcal{D}^{\Pi^*}$, $\mathcal{M}_n^{\sigma^*}(\succsim)$ is always single-valued. In our next result, we state the close relation between age-based priority rules and the strict core in the domain $\mathcal{D}^{\Pi^*}$.

Proposition 1. *For each $\succsim \in \mathcal{D}^{\Pi^*}$, $\psi^*(\succsim)$ is the unique assignment in the strict core of the pairwise exchange problem associated to $\succsim$.*

[10]Note that the notions of strict and weak core coincide if preferences are strict.

With Proposition 1 at hand, we characterize the age-based priority rule as the unique rule that satisfies *individual rationality*, pairwise *k-efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$.

Theorem 2. *A rule* $\varphi : \mathcal{D}^{\Pi^*} \to \mathcal{A}^2$ *satisfies* individual rationality*,* 2-efficiency *and* strategy-proofness *if and only if* φ *is the age-based priority rule* ψ^**.*

The previous result does not extend to coarser partitions of the set of available kidneys. For arbitrary partitions Π, there are many profiles of age based preferences such that the strict core is empty. The arguments in the proof of Theorem 2 only apply to domains that always generate non-empty strict cores (See Sönmez [31]). This fact notwithstanding, with the arguments in the proof of Proposition 1, it is immediate to check that every age-based priority rule selects an assignment in the weak core. We devote the rest of this section to highlight the relevance of age-based priorities in the general framework by analyzing the implications of additional properties that incorporate the notion of group incentive compatibility.

Group Strategy-Proofness. For each $\succsim \in \mathcal{D}$, there is no $T \subseteq N$ and $\succsim'_T \in \mathcal{D}_T$ such that for each $i \in T$, $\varphi_i(\succsim'_T, \succsim_{N\setminus T}) \succ_i \varphi_i(\succsim)$.

Group strategy-proofness requires that truthful report of preferences be a dominant strategy for each patient, and coalitions of patients never have incentives to coordinate to jointly misreport their preferences. Note that our definition of *group strategy-proofness* only considers situations in which all the members of a deviating coalition strictly improve upon the initial report.[11] *Group strategy-proofness* is a relevant property for kidney assignment rules for the following reason. Patients belonging to the same transplant unit often know each others very well and have strong emotional relations among them, which makes quite reasonable to assume that they could and want to jointly misreport their

[11] A stronger definition of *group strategy-proofness* that admits that some members of the coalition to remain indifferent would be incompatible with other properties in our framework. See [6].

preferences if this could benefit all of them. Moreover often patients' preferences are reported by their doctors to TC. Doctors can easily help the exchange of information and the coordination among their patients.

If patients may be indifferent between desirable kidneys, then only *age-based priority rules* satisfy *strategy-proofness* among all *priority rules*. Furthermore, *age-based priority rules* do not provide incentives for any group of agents to misreport their preferences.

Theorem 3. *For each partition* Π *and each priority ordering* σ*, the priority rule* ψ^{σ} *satisfies* strategy-proofness *in* $\mathcal{D}^{\Pi}$ *if and only if* σ *respects* Π *and* ψ^{σ} *is an* age-based priority rule*. Moreover, if* σ *respects* Π*, then the age-based priority rule* ψ^{σ} *satisfies* group strategy-proofness *in* $\mathcal{D}^{\Pi}$*.*

Theorem 3 shows the relevance of *age-based priority rules* in the class of *priority rules* and their nice properties in terms of group incentives. We conclude this sections providing further evidence on the central position of *age-based priority rules* among the rules that satisfy *individual rationality*, 2-*efficiency*, and *strategy-proofness* in the domain of age-based preferences with indifferences. To perform this task, we consider the following property.

Non-Bossiness. For each $i \in N$, each $\succsim \in \mathcal{D}$, and each $\succsim_i' \in \mathcal{D}_i$, $\varphi_i(\succsim) = \varphi_i(\succsim_i', \succsim_{-i})$ implies $\varphi(\succsim) = \varphi(\succsim_i', \succsim_{-i})$

Non-Bossiness requires that if any patient i gets the same kidney under two preference profiles which differ only for patient i's preferences, then all patients get the same kidney under the two profiles. In fact, a rule that violates *non-bossiness* could be prone to (illegal) bribes among donor-patient pairs. If a patient i changes her report of preferences and she affects the outcome of patient j, then i may have incentives to accept any monetary compensation from j to reverse her report. This property seems particularly compelling in our setting where only pairwise exchange are admissible and monetary transactions are prohibited.

It is clear also that *age-based priority rules* satisfy *non-bossiness*. Moreover, *non-bossiness* in combination with our initial properties has interesting implications. First, this set of axioms precludes the maximization of the number of mutually compatible swaps.

Lemma 1. *If $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}^2$ satisfies* individual rationality*,* 2-efficiency*,* strategy-proofness*, and* non-bossiness *then for each $i, j \in N$ and $\succsim \in \mathcal{D}^{\Pi}$, $\omega_i \in D(\succsim_j)$ and $\omega_j \in D(\succsim_i)$ imply either $\varphi_i(\succsim) \succsim_i \omega_j$ or $\varphi_j(\succsim) \succsim_j \omega_i$ (or both).*

It is worthy to note that Lemma 1 above hods by replacing *strategy-proofness* and *non-bossiness* with *group strategy-proofness*.

Example 1. *Let $N = \{1, 2, 3, 4\}$ and $\Pi(1) = \{\omega_1, \omega_2\}$, $\Pi(2) = \{\omega_3, \omega_4\}$. Consider the preference profile $\succsim \in \mathcal{D}^{\Pi}$ such that*

$\succsim_1$	$\succsim_2$	$\succsim_3$	$\succsim_4$
ω_2	ω_1	ω_1	ω_2
$\omega_3 \sim_1 \omega_4$	$\omega_3 \sim_1 \omega_4$	ω_3	ω_4
ω_1	ω_2	$\dots$	$\dots$

Note that $[(1, \omega_3), (2, \omega_4), (3, \omega_1), (4, \omega_2)] \in \mathcal{I}(\succsim)$. However, for every rule φ that satisfies individual rationality*,* 2-efficiency*,* strategy-proofness*, and* non-bossiness*,*

$$\varphi(\succsim) = [(1, \omega_2), (2, \omega_1), (3, \omega_3), (4, \omega_4)].$$

Adding *non-bossiness* to the set of desirable properties of the matching rule has a second important consequence. In fact these four properties call for rules that always pick assignments in the weak core of the associated pairwise exchange problem.

Proposition 2. *If $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}^2$ satisfies* individual rationality*,* 2-efficiency*,* strategy-proofness*, and* non-bossiness *then for each $\succsim \in \mathcal{D}^{\Pi}$, $\varphi(\succsim)$ selects an assignment in the weak core of the of the pairwise exchange problem associated to $\succsim$.*

While a full characterization of the rules that satisfy *individual rationality*, 2-*efficiency*, *strategy-proofness*, and *non-bossiness*, seems to be out of reach, we can take a further important step in understanding the structure of such rules.

In order to simplify the exposition, we conclude this section analyzing the case in which there are only two types of donors, young and mature. Thus, $\Pi = \{\Pi(1), \Pi(2)\}$. In this simple scenario, we can provide a clear description of the implications of *individual rationality*, 2-*efficiency*, *strategy-proofness*, and *non-bossiness*. The results presented here can be immediately extended to the general framework with arbitrary types of kidneys. Since the extension calls for additional notations and the interpretation is not straightforward, we leave it for the interested reader and it is relegated to the Appendix.

For each $a \in \mathcal{A}^2$, define:

$$M_{1,1}(a) \equiv \{i \in N \,|a_i \neq \omega_i \text{ \& } \omega_i \in \Pi(1) \text{ and } a_i \in \Pi(1)\},$$
$$M_{1,2}(a) \equiv \left\{ i \in N \,\middle|\, a_i \neq \omega_i \text{ \& } \begin{array}{c} \text{either } \omega_i \in \Pi(1) \text{ and } a_i \in \Pi(2) \\ \text{or } \omega_i \in \Pi(2) \text{ and } a_i \in \Pi(1) \end{array} \right\},$$
$$M_{2,2}(a) \equiv \{i \in N \,|a_i \neq \omega_i \text{ \& } \omega_i \in \Pi(2) \text{ and } a_i \in \Pi(2)\}.$$

A rule φ is a ***sequential matching maximizing rule*** if for each $\succsim \in \mathcal{D}^{\Pi}$:

i) $\#M_{1,1}(\varphi(\succsim)) \geq \#M_{1,1}(a)$, for each $a \in \mathcal{I}(\succsim)$,

ii) $\#M_{1,2}(\varphi(\succsim)) \geq \#M_{1,2}(a')$, for each $a' \in \mathcal{I}(\succsim)$ such that $a'_i = \varphi_i(\succsim)$ for each $i \in M_{1,1}(\varphi(\succsim))$,

iii) $\#M_{2,2}(\varphi(\succsim)) \geq \#M_{2,2}(a'')$, for each $a'' \in \mathcal{I}(\succsim)$ such that $a''_i = \varphi_i(\succsim)$ for each $i \in [M_{1,1}(\varphi(\succsim)) \cup M_{1,2}(\varphi(\succsim))]$.

Note that age-based priority rules are sequential matching maximizing rules. A sequential matching maximizing rule does not select an assignment that maximizes the number of compatible kidneys exchanges. Instead, it selects an assignment such that maximizes the number of swaps that imply the exchanges involving only young donor pairs. Then, it sequentially applies the same logic to the remaining donor-patient pairs. Thus, given the exchanges of among young donor pairs, it maximizes the number of swaps between young donor pairs and mature donor pairs. Finally, given the exchanges arranged in the previous stages, a sequential matching maximizing rule maximizes the number of swaps between mature donor pairs.

Proposition 3. *If* $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}^2$ *satisfies* individual rationality*,* 2-efficiency*,* strategy-proofness*, and* non-bossiness*, then* φ *is a* sequential matching maximizing rule*.*

It is worth to note that Proposition 3 implies that there may be assignments in the weak core that are never selected by a rule that satisfies our set of axioms. Consider $N = \{1, 2, 3, 4, 5, 6\}$ and $\Pi = \{\Pi(1), \Pi(2)\} = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5, \omega_6\}\}$. Let $\succsim \in \mathcal{D}^{\Pi}$ be such that $D(\succsim_1) = \{\omega_2, \omega_3\}$, $D(\succsim_2) = \{\omega_1, \omega_4\}$, $D(\succsim_3) = \{\omega_1, \omega_5\}$, $D(\succsim_4) = \{\omega_2, \omega_6\}$, $D(\succsim_5) = \{\omega_3\}$, and $D(\succsim_6) = \{\omega_4\}$. Let $a = ((1, \omega_2), (2, \omega_1), (3, \omega_5), (4, \omega_6), (5\omega_3), (6, \omega_4))$. Clearly, a is in the weak core of the pairwise exchange problem. It involves three compatible swaps, but only one among patients with a donor in $\Pi(1)$. Hence, a cannot be selected because it is possible to carry out two exchanges involving donor-patients pairs in $\Pi(1)$.[12]

5 Multiple Donors

Sometimes patients in the waiting list may find more than one potential donors. If patients with multiple potential donors are keen to participate to PKE programs, algorithms have to take account of this aspect. Even if only one among the potential donors of a patient donates her kidney, the fact that a patient have many potential donors can greatly increase the chances to find mutually compatible pairs. Since it is reasonable to assume that the information about how many potential donors a patient have is private information, rules should provide incentives to patients to reveal this valuable information. In order to

[12]For instance, consider the assignment $b = ((1, \omega_3), (2, \omega_4), (3, \omega_1), (4, \omega_2), (5\omega_5), (6, \omega_6))$.

analyze this general case, we need to slightly modify the framework and to incorporate some additional notation.

Let $N = \{i, \ldots, n\}$ be a set of patients and $\Omega = \{\omega_1, \ldots, \omega_{n'}\}$ be a set of available kidneys from living donors, $n \leq n'$.[13] For each patient i let Ω_i denote the set of kidneys from i's donors. Clearly, $\cup_{i \in n} \Omega_i = \Omega$ and for each patient $j \neq i$, $\Omega_i \cap \Omega_j = \varnothing$.

In the multiple donor case, we analyze patients' incentives to manipulate by reporting different set of potential donors. Hence, the set of donors is as an argument of the kidney assingment rule. For each patient i let $\mathcal{K}_i$ be the set of non-empty subsets of Ω_i. Hence, $K_i \in \mathcal{K}_i$ is a set of donors reported by patient i to the TC. Abusing notation, let $\mathcal{K} \equiv \times_{i \in N} \mathcal{K}_i$ and we denote by $\mathbf{K} = (K_1, \ldots, K_n)$ a generic element of $\mathcal{K}$, let $\mathbf{\Omega} = (\Omega_1, \ldots, \Omega_n)$. Abusing notation, for each $\mathbf{K} \in \mathcal{K}$ and for each $S \subseteq N$, $\mathbf{K}_S$ denotes the restriction of $\mathbf{K}$ to the members of S.

The definition of assignment needs to be extended in order to accommodate the multiple donors case. An ***(generalized) assignment*** a is an n-tuple of pairs $a = [(1, \omega), \ldots, (n, \omega')]$ such that

(i) for each $i, j \in N$, $i \neq j$ and each $\omega, \omega' \in \Omega$, if $(i, \omega), (j, \omega') \in a$, then $\omega \neq \omega'$;

(ii) for each $i \in N$, if for some $j \in N$, $a_j \in \Omega_i$, then $a_k \notin \Omega_i$ for all $k \neq j$.

We need to introduce this second requirement to convey the idea that for each patient at most one donor donates her kidney.

For each $\mathbf{K} \in \mathcal{K}$, we say that an assignment a is $\mathbf{K}$-*feasible* if for each $i, j \in N$, $a_i \in \Omega_j$ implies $a_i \in \mathbf{K}_j$. Making slight abuse of notation, let $\mathcal{A}(\mathbf{K})$ be the set of all $\mathbf{K}$-*feasible* (generalized) assignments, and for each $k \leq n$ let $\mathcal{A}^k(\mathbf{K})$ be the set of all $\mathbf{K}$-*feasible* (generalized) assignments with cardinality smaller or equal to k. Let $S \subseteq N$. A ***reduced assignment*** a_S is an assignment among the members of S. For a partition of

[13] Note that kidneys' indexes do not longer refer to the patient who introduces the donor in the pool.

N, $\{S, S', \ldots, S''\}$ and a list of reduced assignments $a_S, a_{S'}, \ldots, a_{S''}$, we abuse notation and refer to the assignment formed by the reduced assignments as $[a_S, a_{S'}, \ldots, a_{S''}]$.

A ***generalized (kidney assignment) rule*** is a mapping $\Phi : \mathcal{D} \times \mathcal{K} \to \mathcal{A}$ such that for each $\succsim \in \mathcal{D}$ and each $\mathbf{K} \in \mathcal{K}$, $\Phi(\succsim, \mathbf{K}) \in \mathcal{A}(\mathbf{K})$.

The definition of *k-efficiency* directly applies to the multiple donor scenario. *Individual rationality* may be immediately extended just by applying its logic to all the potential donors of each patient.

Individual Rationality for generalized rules. For each $i \in N$, each $\succsim \in \mathcal{D}$, each $\mathbf{K} \in \mathcal{K}$, and each $\omega \in \mathbf{K}_i$, $\Phi_i(\succsim, \mathbf{K}) \succsim_i \omega$.

The extension of the notion of *strategy-proofness* is more delicate. In the general framework, it is necessary to take into account patients' incentives to manipulate the PKE outcome by withholding their potential donors.

Extended Group Strategy-Proofness (EGSP). There are no $T \subseteq N$, $\succsim \in \mathcal{D}$, $\succsim'_T \in \mathcal{D}_T$, and $\mathbf{K}'_T \in \times_{i \in T} \mathcal{K}_i$, such that for each $i \in T$, $\Phi_i(\succsim'_T, \succsim_{N \setminus T}, (\mathbf{K}'_T, \mathbf{\Omega}_{N \setminus T})) \succ_i \Phi_i(\succsim, \mathbf{\Omega})$.

In the multiple donors scenario, we assume that patients only care about the kidney they may receive and do not have preferences over whom be the willing donor involved in the kidney exchange. Hence, patients' preferences are still defined over all potential donors Ω. Here, the assumption on the irrelevance of the donor for patients' preferences allows to focus on the case in which patients could misreport the set of their potential donors in order to get a better kidney. In order to conclude with the description of patients' preferences, we say that a kidney is desirable for patient i if it improves upon all i's potential donors' kidneys. That is, for each patient i, i's ***set of desirable kidneys*** is the set $D(\succsim_i) \equiv \{\omega \in \Omega \mid \forall \omega' \in \Omega_i,\ \omega \succ_i \omega'\}$. With this definition of desirable kidneys, for each partition $\Pi = \{\Pi(1), \ldots, \Pi(l)\}$ of Ω, the notion of age-based preferences in the multiple donor scenario simply replicates the definition in the single-donor case.

The results in the previous section directly apply to the multiple donor framework. Hence, we focus on pairwise exchanges and *age–based priority rules*. However, because patients may have donors whose kidneys belong to different classes of the partition Π, there are alternative extensions of the notion of *age-based priority rules* to the multiple donor case. We devote this section to show the difficulties that arise in this setting and how to tailor age-based priority rules in order to preserve the nice properties they have in the single-donor scenario.

Example 2. *Let* $N = \{1, 2, 3\}$, $\Omega = \{\omega_1, \ldots, \omega_6\}$, $\Pi = \{\Pi(1), \Pi(2)\}$ *such that* $\Pi(1) = \{\omega_1, \omega_2\}$, $\Pi(2) = \{\omega_3, \omega_4, \omega_5, \omega_6\}$ *and* $\Omega_1 = \{\omega_1, \omega_4\}$, $\Omega_2 = \{\omega_2, \omega_5\}$ *and* $\Omega_3 = \{\omega_3, \omega_6\}$. *Let the generalized rule* Φ *be defined in such a way that , for each* $\mathbf{K} \in \mathcal{K}$, $\Phi(\cdot, \mathbf{K})$ *is a priority rule with priority ordering* σ^*. *Hence,* Φ *assigns priority to patients with a donor in* $\Pi(1)$. *Let* $\succsim \in \mathcal{D}^\Pi$ *be such that* $D(\succsim_1) = \{\omega_3\}$, $D(\succsim_2) = \{\omega_6\}$ *and* $D(\succsim_3) = \{\omega_2, \omega_4\}$. *Let* $\mathbf{K}, \mathbf{K}'$ *be such that* $\mathbf{K} = \mathbf{\Omega}$, $K'_j = \Omega_j$ *for* $j \in \{1, 2\}$ *and* $K'_3 = \{\omega_6\}$. *Note that*

$$\Phi(\succsim, \mathbf{K}) = [(1, \omega_3), (2, \omega_2), (3, \omega_4)],$$

but

$$\Phi(\succsim, \mathbf{K}') = [(1, \omega_1), (2, \omega_6), (3, \omega_2)].$$

Hence, $\Phi_3(\succsim, \mathbf{K}') \succ_3 \Phi_3(\succsim, \mathbf{K})$, *and* Φ *violates EGSP. Moreover, patient* 3 *could obtain the same outcome just by reporting* $\succsim'_3 \in \mathcal{D}^\Pi_3$ *such that* $D(\succsim'_3) = \{\omega_2\}$.

With multiple donors, it is necessary to define a multi-stage mechanism in order to maintain the non-manipulability of the age-based priority rule. For instance, in the case that potential kidneys can be divided in young and mature kidneys, Theorem 3 suggests that patients with young donors have priority over patients with mature donors. However, it is not immediate to assign the priorities when patients may simultaneously have young and mature donors. The problem steams from the fact that a patient may have donors belonging to different elements of the partition Π. In order to preserve *strategy-proofness*, it is necessary that better kidneys are offered first. Before we define the multiple-donor generalization of age-based priority rules, we introduce additional notation.

For each preference profile $\succsim \in \mathcal{P}$ and each profile of available kidneys $\mathbf{K} \in \mathcal{K}$, let $\mathcal{I}(\succsim, \mathbf{K})$ denote the set of all individually rational pairwise assignments when the set of available kidneys is given by $\mathbf{K}$. For each coalition of patients S, we denote by $\mathcal{I}(\succsim_S, \mathbf{K}_S \mid S)$ the set of individually rational reduced assignments for S under $\succsim$ and $\mathbf{K}$. Finally for each $T \subset N$ and each permutation of the patients σ, we say that the permutation $\bar{\sigma} : T \to \{1, \ldots, T\}$ is the reduction of σ to T if for each $i, j \in T$, $\sigma(i) < \sigma(j)$ implies $\bar{\sigma}(i) < \bar{\sigma}(j)$.

Let $\Pi = \{\Pi(1), \ldots, \Pi(l)\}$ be a partition of Ω. Fix $\succsim \in \mathcal{D}^{\Pi}$ and $\mathbf{K} \in \mathcal{K}$. Let $\Sigma = \{\sigma_1, \ldots, \sigma_l\}$ be a list of permutations of the patients. For each preference profile $\succsim$, the generalized priority rule generated by the list of permutations Σ proceeds according to the following algorithm:

Let $N(1) \equiv N$ and

$$\mathcal{M}_0^{\Sigma,1} \equiv \left\{ \begin{array}{l} a \in \mathcal{I}(\succsim, \mathbf{K} \mid N(1)) \text{ such that} \\ \forall\, i, j \in N(1), \text{ if } a_i \in \Omega_j \text{ and } \sigma_1(i) < \sigma_1(j), \\ \text{then } a_j \in \Pi(1) \end{array} \right\}.$$

For each $k = 1, \ldots, N(1)$,

$$\mathcal{M}_k^{\Sigma,1} \equiv \left\{ a \in \mathcal{M}_{k-1}^{\Sigma,1} \;\mid\; \text{for no } b \in \mathcal{M}_{k-1}^{\Sigma,1},\; b_{\sigma_1^{-1}(k)} \succ_{\sigma_1^{-1}(k)} a_{\sigma_1^{-1}(k)} \right\}.$$

Note that $\mathcal{M}_{N(1)}^{\Sigma,1}$ is not empty and essentially single-valued.

Intuitively, according to the priority mechanism, the patients with higher priority offer to swap their donors' kidneys in $\Pi(1)$ to the other patients (maybe receiving a kidney not in $\Pi(1)$.) By the definition of $\mathcal{M}_{N(1)}^{\Sigma,1}$ and the fact that it is essentially single-valued, for every $a \in \mathcal{M}_{N(1)}^{\Sigma,1}$, the set of patients who do not receive a kidney from other patients' donors is the same. In the following stages, the algorithm proceeds by iteratively applying the same logic to the unmatched patients who are allowed to sequentially offer kidneys in the remaining elements of the partition Π.

Once $N(1)$, $\mathcal{M}_0^{\Sigma,1}$, and $\mathcal{M}_{N(1)}^{\Sigma,1}$ are defined, for each $m = 2, \ldots, l$; let $N(m) = \{i \in N(m-1) \mid \; \forall\, a \in \mathcal{M}_{N(m-1)}^{\Sigma,(m-1)},\; a_i \in \Omega_i\}$, $M(m-1) = N(m-1) \setminus N(m)$, let $\bar{\sigma}_m$ be the restriction of σ_m to $N(m)$, and let

$$\mathcal{M}_0^{\Sigma,m} \equiv \left\{ \begin{array}{l} a_{N(M)} \in \mathcal{I}(\succsim_{N(m)}, \mathbf{K}_{N(m)} \mid N(m)) \text{ such that} \\ \forall\, i,j \in N(m), \text{ if } a_i \in \Omega_j \text{ and } \bar{\sigma}_m(i) < \bar{\sigma}_m(j), \\ \text{then } a_j \in \Pi(m) \end{array} \right\}.$$

For each $k = 1, \ldots, N(m)$,

$$\mathcal{M}_k^{\Sigma,m} \equiv \left\{ a_{N(m)} \in \mathcal{M}_{k-1}^{\Sigma,m} \;\mid\; \text{for no } b \in \mathcal{M}_{k-1}^{\Sigma,m},\ b_{\bar{\sigma}_m^{-1}(k)} \succ_{\bar{\sigma}_m^{-1}(k)} a_{\bar{\sigma}_m^{-1}(k)} \right\}.$$

A (generalized) rule $\Phi : \mathcal{D}^{\Pi} \times \mathcal{K} \to \mathcal{A}^2$ is a ***generalized priority rule*** if there is a list of permutations $\Sigma = \{\sigma_1, \ldots, \sigma_l\}$ such that for each $\succsim \in \mathcal{D}^{\Pi}$, for each $m = 1 \ldots, l$, for each $a^m \in \mathcal{M}_{N(m)}^{\Sigma,m}$, $\Phi(\succsim) = \left[\Phi(\succsim)_{M(1)}, \ldots, \Phi(\succsim)_{M(l)}\right]$ and $\Phi(\succsim)_{M(m)} = a_{M(m)}^m$. For a partition Π, $\Psi : \mathcal{D}^{\Pi} \times \mathcal{K} \to \mathcal{A}^2$ is a ***generalized age-based priority rule*** if Ψ is a *generalized priority rule* and for each $i,j \in N$, for each $m = 1, \ldots, l$ $\Omega_i \cap \Pi(m) \neq \varnothing$ and $\Omega_j \cap \Pi(m) = \varnothing$ implies $\sigma_m(i) < \sigma_m(j)$.

Theorem 4. *Consider an assignment problem with multiple donors and a partition of the set of available kidneys* Π. *A* generalized age-based priority rule $\Psi : \mathcal{D}^{\Pi} \times \mathcal{K} \to \mathcal{A}^2$ *satisfies* individual rationality, 2-efficiency, *and* EGSP.

Before concluding this section it is worthy to point out that our rules are immune to other form of misrepresentation of the information about the set of donors a part from withholding some potential donors. We focus on it only because, according to our opinion, is the only relevant aspect on a practical ground.

Remark 1. *Note that (generalized) age-based priority rules are also immune to manipulation by the introduction of dummy donors. That is, a patient does not improve by presenting a donor whose kidneys are not compatible with any other patient. If a patient incorporates a "dummy" donor whose kidney is not compatible with the remaining patients,* Ψ *may give more priority for assigning the "dummy" donor but by* individual rationality*, this is irrelevant for the final outcome of the algorithm. That is, consider patient i and let $\Omega_i = \bar{\Omega}_i \cup \{\omega\}$. Let $\succsim \in \mathcal{D}^{\Pi}$ be such that such that for each patient $j \neq i$, $\omega \notin D(\succsim_j)$. Then, $\Psi(\succsim, \mathbf{\Omega}) = \Psi(\succsim, (\mathbf{\Omega}_{N\setminus\{i\}}, \bar{\Omega}_i))$.*

6 Conclusion

In this paper we provide a theoretical framework to design PKE protocols that encourage compatible pairs to participate into PKE program. The relatively low number of kidney paired exchanges in the US performed with respect to the number of transplants from living donors makes clear that there are still significant barriers to utilization.

We believe that the participation of compatible pairs may represent the most important factor in expanding the number of kidney paired exchanges. As regards the US system, different proposals have been presented in order to increase the chance of an incompatible pair of finding a compatible match: the introduction of a nationwide PKE registry, the use of matching algorithms that include three-way matches and of non-directed (altruistic) donors. Some of these strategies can be carried out together, but others cannot. Specifically, our paper enlightens the difficulties in implementing incentive compatible algorithms that allow three-way matches in our age-based preference domain, at least if compatibility is private information of the patients and their doctors.

Therefore, it is compelling to predict the impact of allowing compatible pairs to participate to PKE programs, in order to understand which type of proposal may be more effective in increasing the number of transplants and their quality. A recent study by Gentry et al. [7] uses simulated data to prove that there could be large benefits for both incompatible pairs and compatible pairs if compatible pairs were willing to participate to KPE programs. The reason why expanding the pool through participation by compatible pairs could be a very successful strategy, is mostly driven by the blood group imbalance in the pool of incompatible pairs. Most of group O-donors can directly donate to their intended recipients, and so group-O recipients in the KPE pool must rely on a scarce number of compatible donors and rarely can find a match. Hence, participation of compatible pairs could dramatically reduce such imbalance and would nearly double the match rate for incompatible pairs (28.2% to 64.5% for a single-center program and 37.4% to 75.4% for a national program). Compatible pairs could also benefit from participation. Having defined the benefit of participation as the possibility of being matched with a donor at least ten years younger than the related donor (and not considering other potential benefits as an increase in HLA matching),[14] a single compatible pair would have a 34% chance of finding a better match in a single center program and 48% in a national registry, while if all compatible pairs were willing to participate to a PKE program still 11.7% (single-center) and 14.7% of these compatible pairs would find a more favorable match.[15] In conclusion, Gentry et al. [7] presents an important supporting argument for the introduction of an algorithm, as we suggest in this paper, that encourages compatible pairs participation in PKE programs.

We would like to end the paper by commenting on further possible applications of our framework. Although our model is directly inspired by the structure of PKE problems, it applies to other problems of centralized allocation of indivisible goods without money. A pertinent example is the holidays vacant houses swap networks. These networks usually arrange pairwise swaps due to administrative restrictions. Agents' preferences may be based on the location or size and quality (facilities) of the house. Normally, different agents may consider different locations as attractive or not (so some house exchanges are not compatible). However, every agent ranks the houses in each location taking into account only the facilities of each house. So while which location turns out be attractive for an agent is her private information, how agents rank houses in the same location it is

[14]Another potential benefit taken into consideration for female recipient is the reduction of high immunological risk donor/recipient combination (child-to-mother or husband-to-wife) due to exposure in uterus to paternal HLA antigens.

[15]Since compatible pairs compete for a fixed number of incompatible pairs with young donors, the potential benefit of a compatible pairs of enrolling in a PKE program is decreasing in the number of compatible pairs participating in the program.

easy to predict and such a ranking is the same for all agents interested in this location.

7 Appendix

Proof of Theorem 1. The arguments follow the proof of [15, Theorem 1].

Let $3 \leq k \leq n-1$. Assume, by way of contradiction, that there are a partition Π and a rule φ that satisfies *individual rationality*, *k-efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi}$. Without loss of generality, by our Assumption, let $\omega_1 \in \Pi(l)$, $\omega_2 \in \Pi(l')$, $\omega_k \in \Pi(\bar{l})$ and $\omega_{k+1} \in \Pi(\bar{l}')$ with $l < l'$ and $\bar{l} < \bar{l}'$.[16] Let Table 1 represent patients' preferences over compatible kidneys according to the preference profile $\succsim \in \mathcal{D}^{\Pi}$.

$\succsim_1$	$\succsim_2$	$\ldots$	$\succsim_{k-1}$	$\succsim_k$	$\succsim_{k+1}$
ω_2	ω_3	$\ldots$	ω_k	ω_{k+1}	ω_1
ω_1	ω_2	$\ldots$	ω_{k+1}	ω_k	ω_2
$\ldots$	$\ldots$	$\ldots$	ω_{k-1}	$\ldots$	ω_{k+1}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

Table 1: $\succsim$, Theorem 1.

Let $\succsim' \in \mathcal{D}^{\Pi}$ be such that for each $i \neq k-1$, $\succsim_i = \succsim'_i$, and $D(\succsim'_{k-1}) = \{\omega_k\}$. Under profile $\succsim'$, by *individual rationality*, either no object is assigned to any patient $1, \ldots, k+1$, or patient $k+1$ receives ω_2, patient 1 receives ω_1, and every other patient i receives ω_{i+1} (the kidney of her next to the right neighbor). By *k-efficiency*:

$$\varphi(\succsim') = \begin{bmatrix} (1, \omega_1), & \\ (i, \omega_{i+1}), & \forall i = 2, \ldots, k \\ (k+1, \omega_2) & \end{bmatrix}.$$

[16]For instance, we can assume that $\omega_1 \in \Pi(1)$ and $\{\omega_2, \omega_k\} \in \Pi(2)$, and $\omega - k + 1 \in \Pi(3)$. Alternatively, we can have $\{\omega_1, \omega_k\} \subseteq \Pi(1)$ and $\{\omega_2, \omega_{k+1}\} \subseteq \Pi(2)$, and apply a convenient relabeling of patients and donors in order to satisfy our notational assumption. (See page 7, definition of Π.)

By *strategy-proofness*, $\varphi_{k-1}(\succsim) \succsim_{k-1} \varphi_{k-1}(\succsim') = \omega_k$. Note that, according to $\succsim_{k-1}$, ω_k is patient $k-1$'s preferred kidney. Then, $\varphi_{k-1}(\succsim) = \omega_k$. By *k-efficiency* and *individual rationality*, $\varphi(\succsim) = \varphi(\succsim')$.

Let $\succsim'' \in \mathcal{D}^{\Pi}$ be such that for each $i \neq k+1$, $\succsim_j = \succsim''_j$ and $D(\succsim''_{k+1}) = \{\omega_1\}$. The same arguments we employed to determine $\varphi(\succsim')$ apply here to obtain:

$$\varphi(\succsim'') = \begin{bmatrix} (i, \omega_{i+1})(\text{modulo } k+1), & \forall i \notin \{k, k-1\} \\ (k-1, \omega_{k+1}), & \\ (k, \omega_k) & \end{bmatrix}.$$

Note that $\omega_1 = \varphi_{k+1}(\succsim'') = \varphi(\succsim''_{k+1}, \succsim_{-(k+1)}) \succ_{k+1} \varphi_{k+1}(\succsim) = \omega_2$, which contradicts *strategy-proofness.* □

Proof of Proposition 1. Let $\succsim \in \mathcal{D}^{\Pi^*}$. Because preferences over desirable kidneys and each patients' donor kidney are strict, it is clear that the priority mechanism selects a unique assignment. Let $b \in \mathcal{A}^2$, $b \neq \psi^*(\succsim)$. We prove that b is not in the core. Assume first that $b \notin \mathcal{I}(\succsim)$. There is $j \in N$ such that $\omega_j \succ_j b_j$. Let $a \in \mathcal{A}^2$ such that $a_j = \omega_j$. Clearly, a weakly dominates b via coalition $\{j\}$. Next, assume that $b \in \mathcal{I}(\succsim)$. Let i be the patient such that $\psi^*(\succsim)_i \neq b_i$ and for each $i' < i$, $\psi^*_{i'}(\succsim) = b_{i'}$. Note that $\psi^*(\succsim) \in \mathcal{M}_i^{\sigma^*}(\succsim)$, for each $i' < i$, $\psi^*_{i'}(\succsim) = b_{i'}$, and $b \in \mathcal{I}(\succsim)$ imply that $\psi^*_i(\succsim) \succsim_i b_i$. Because preferences over acceptable kidneys are strict, $\psi^*_i(\succsim) \succ_i b_i$. Let $j \in N$ be such that $\psi^*_i(\succsim) = \omega_j$ (and $\psi^*_j(\succsim) = \omega_i$). Since for each $i' < i$, $\psi^*_{i'}(\succsim) = b_{i'}$, $\omega_i = \psi^*(\succsim)_j \succ_j b_j$. Hence, $\psi^*(\succsim)$ weakly dominates b via $\{i, j\}$. Finally, we prove that there is no $b \in \mathcal{A}^2$ such that b weakly dominates $\psi^*(\succsim)$. Assume to the contrary there is $b \in \mathcal{A}^2$ and $T \subseteq N$ such that b weakly dominates $\psi^*(\succsim)$ via T. Define the set $T' \equiv \{i \in T, \text{ such that } b_i \succ_i \psi^*_i(\succsim)\}$. We first prove that $1 \notin T'$. By the definition of the priority algorithm, for each $b \in \mathcal{A}^2$, such that $b_1 \succ_1 \psi^*_1(\succsim)$, $b \notin \mathcal{I}(\succsim)$. Hence, $1 \notin T$. Analogously, for patient 2, for each $b \in \mathcal{A}^2$, such that $b_2 \succ_2 \psi^*_2(\succsim)$, either $b \notin \mathcal{I}^2(\succsim)$, or $b_2 = \psi^*_1(\succsim)$. Note that for each $i \in N$ and each $\succsim_i \in \mathcal{D}^{\Pi^*}$, the kidneys that i prefers to $\psi^*_i(\succsim)$ are those that come either from a donor whose patient finds ω_i as undesirable under $\succsim$, or from the donor of a patient j such that

$j < i$ $(\sigma(j)^* < \sigma^*(i))$. Since patient 1 does not improve patient 2's donor kidney, $2 \notin T'$. Repeating the argument iteratively for each $i = 2, \ldots, n$ we obtain that $T' = \{\varnothing\}$, which contradicts that b weakly dominates $\psi^*(\succsim)$ via T. □

Proof of Theorem 2. By definition ψ^* satisfies *individual rationality*, and 2-*efficiency*. Hence we check that ψ^* satisfies *strategy-proofness* in $\mathcal{D}^{\Pi^*}$. Let $\succsim \in \mathcal{D}^{\Pi^*}$ and let $\succsim'_1 \in \mathcal{D}_1^{\Pi^*}$ be such that $\psi_1^*(\succsim'_1, \succsim_{-1}) \neq \psi_1^*(\succsim)$. Without loss of generality, let $\psi_1^*(\succsim'_1, \succsim_{-1}) = \omega_j$ and $\psi_1^*(\succsim) = \omega_i$. If $i > j$, then, by the definition of ψ^*, $\omega_j \notin (D(\succsim_1) \cup \{\omega_1\})$. Since ψ^* satisfies *individual rationality*, $\psi_1^*(\succsim) \succ_1 \psi_1^*(\succsim'_1, \succsim_{-1})$. If $i < j$, since $\succsim_1 \in \mathcal{D}^{\Pi^*}$ and ψ^* satisfies *individual rationality*, $\psi_1^*(\succsim) \succ_1 \psi_1^*(\succsim'_1, \succsim_{-1})$. Hence, for each $\succsim'_1 \in \mathcal{D}^{\Pi^*}, \psi_1^*(\succsim) \succsim_1 \psi_1^*(\succsim'_1, \succsim_{-1})$. Let $i \in N$ be such that $\psi_1^*(\succsim) = \omega_i$ and $\psi_i^*(\succsim) = \omega_1$. Because ψ^* satisfies *individual rationality*, $\omega_1 \in D(\succsim_i)$ Since $\succsim_i \in \mathcal{D}_i^{\Pi^*}$, for each $\omega \in \Omega \setminus \{\omega_1\}$, $\psi_i^*(\succsim) \succ_i \omega$. Therefore, for each $\succsim'_i \in \mathcal{D}_i^{\Pi^*}$, $\psi_i^*(\succsim) \succsim_i \psi_i^*(\succsim'_i, \succsim_{-i})$. Consider now patient 2. By the definition of ψ^*, for each $i \in N$ such that $\omega_i \succ_2 \psi_2^*(\succsim)$, either $\omega_2 \notin D(\succsim_i)$ or $\omega_i = \psi_1^*(\succsim)$. Therefore, by the definition of ψ^*, for each $\succsim'_2 \in \mathcal{D}^{\Pi^*}$, $\psi_2^*(\succsim) \succsim_2 \psi_2^*(\succsim'_2, \succsim_{-2})$. Repeating the same argument as many times as necessary, we obtain that ψ^* satisfies *strategy-proofness* in $\mathcal{D}^{\Pi^*}$.

Next, we prove necessity side. Let φ be a rule that satisfies *strategy-proofness*, *individual rationality*, and 2-*efficiency*. Note that, for each $i \in N$, each $a \in \mathcal{A}^2$, and each $\succsim_i \in \mathcal{D}^{\Pi^*}$, $a_i \sim_i \omega_i$ if and only if $a_i = \omega_i$. Moreover, for each $i \in N$, each $a \in \mathcal{A}^2$, and each $\succsim_i \in \mathcal{D}^{\Pi^*}$ such that $a_i \succsim_i \omega_i i$ there is $\succsim'_i \in \mathcal{D}^{\Pi^*}$ such that

(i) for each $b \in \mathcal{A}^2 \setminus \{a\}$, $b_i \succsim_i a_i$ if and only if $b_i \succsim'_i a_i$,

(ii) for each $b \in \mathcal{A}^2 \setminus \{a\}$, $a_i \succsim_i b_i$ if and only if $a_i \succsim'_i b_i$,

(iii) for each $b \in \mathcal{A}^2 \setminus \{a\}$, $a_i \succ_i b_i$ if and only if $a_i \succ'_i b_i$, and $a_i \succsim'_i \omega_i \succsim'_i b_i$.

Hence, the domain $\mathcal{D}^{\Pi^*}$ satisfies Assumptions A–B on the domain of preferences proposed by Sönmez [31]. By [31, Theorem 1], if there is a rule φ that satisfies *individual rationality*, 2-*efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$, for each $\succsim \in \mathcal{D}^{\Pi^*}$, φ selects an assignment in the (strict) core of the pairwise exchange problem associated to $\succsim$. We have just seen that ψ^* satisfies *individual rationality*, 2-*efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$. By Proposition 1, for each $\succsim \in \mathcal{D}^{\Pi^*}$, $\psi^*(\succsim)$ is the unique assignment in the core of the pairwise kidney exchange problem associated to $\succsim$. Therefore, $\varphi = \psi^*$. $\square$

Proof of Theorem 3. Consider a partition Π and a priority ordering σ that does not respect Π. Let $i, j \in N$, $k, k' \in \mathbb{N}$ be such that $\omega_i \in \Pi(k)$, $\omega_j \in \Pi(k')$, $l \leq l'$, and $\sigma(i) > \sigma(j)$. Let $m \in N$ such that for some $k'' \geq k'$, $\omega_m \in \Pi(k'')$ and $\sigma(j) < \sigma(m)$ (the existence of m is guaranteed by our Assumption. Let $\succsim \in \mathcal{D}^{\Pi}$ be such that $D(\succsim_i) = \{\omega_m\}$, $D(\succsim_j) = \{\omega_m\}$, and $D(\succsim_m) = \{\omega_i, \omega_j\}$. Clearly, $\psi^\sigma_m(\succsim) = \omega_j$. Let $\succsim'_m \in \mathcal{D}^{\Pi}_m$ be such that $D(\succsim'_m) = \{\omega_i\}$. Then, $\psi^\sigma_m(\succsim'_m, \succsim_{-m}) = \omega_i$, and $\psi^\sigma_m(\succsim'_m, \succsim_{-m}) \succ_m \psi^\sigma_m(\succsim)$, which proves that ψ^σ violates *strategy-proofness*.

Next, we prove that if σ respects Π, then ψ^σ satisfies *group strategy-proofness*. In order to simplify notation, we consider the natural priority ordering σ^*. The arguments apply directly for arbitrary priority orderings that respect Π. Assume to the contrary that ψ^* violates *group strategy-proofness*. Then, there is $T \subseteq N$, $\succsim \in \mathcal{D}^{\Pi}$, $\succsim'_T \in \times_{i \in T} \mathcal{D}^{\Pi}_i$ such that for each $i \in T$, $\psi^*_i(\succsim'_T, \succsim_{N \setminus T}) \succ_i \psi^*_i(\succsim)$. Let patient $j \in T$ be such that for each $k \in T$, $j \leq k$. Since $j \in T$, $\succsim_j \neq \succsim'_j$ and $\psi^*_j(\succsim'_T, \succsim_{N \setminus T}) \succ_j \psi^*_j(\succsim)$, necessarily $\psi^* i_j(\succsim'_T, \succsim_{N \setminus T}) \neq \psi^*_j(\succsim)$. There are two possibilities:

(i) For each $i < j$, $\psi^*_i(\succsim'_T, \succsim_{N \setminus T}) = \psi^*_i(\succsim)$ and for some patient k, $\omega_k = \psi^*_j(\succsim'_T, \succsim_{N \setminus T})$. By the definition of ψ^*, $\omega_j \in D(\succsim'_k) \setminus D(\succsim_k)$ and $\omega_k \succ_k \omega_j$. Because $\succsim_k \neq \succsim'_k$, $k \in T$. However, by ψ^*'s *individual rationality*, $\psi^*_k(\succsim) \succsim_k \omega_k \succ_k \omega_j = \psi^*_k(\succsim'_T, \succsim_{N \setminus T})$, which contradicts $k \in T$.

(ii) There is $i < j$ such that $\psi^*_i(\succsim'_T, \succsim_{N \setminus T}) \neq \psi^*_i(\succsim)$. Let $i' < j$ be such that for each $i < i'$ $\psi^*_i(\succsim'_T, \succsim_{N \setminus T}) = \psi^*_i(\succsim)$. If $\psi^*_{i'}(\succsim'_T, \succsim_{N \setminus T}) \succ_{i'} \psi^*_{i'}(\succsim)$, by the definition of age-based priority rule, there is $k \in T$ such that $\omega_{i'} \notin D(\succsim_k)$ and $\omega_{i'} \in D(\succsim'_k)$ and $\psi^*_{i'}(\succsim'_T, \succsim_{N \setminus T}) = \omega_k$ However, by *individual rationality*, $\varphi_k(\succsim) \succsim_k \omega_k \succ_k \omega_{i'} = \psi^*_k(\succsim'_T, \succsim_{N \setminus T})$, which contradicts $k \in T$. Finally, if $\psi^*_{i'}(\succsim) \succsim_{i'} \psi^*_{i'}(\succsim'_T, \succsim_{N \setminus T})$, then there is $k' \in T$ such that $\psi^*_{i'}(\succsim) = \omega_{k'}$. By the definitions ψ^* and i', $\omega_{i'} \in D(\succsim_{k'}) \setminus D(\succsim'_{k'})$, and for each $\succsim_{k'} \in \mathcal{D}^{\Pi}$, $\omega_{i'} = \psi^*_{k'}(\succsim) \succsim_{k'} \psi^*_{k'}(\succsim'_T, \succsim_{N \setminus T})$, which contradicts $k \in T$.

Because both cases exhaust all the possibilities, this suffices to prove *group strategy-proofness.* $\square$

Proof of Lemma 1. Assume to the contrary that there are $i, j \in N$ and $\succsim \in \mathcal{D}^{\Pi}$ such that $\omega_i \in D(\succsim_j)$ and $\omega_j \in D(\succsim_i)$ but $\omega_i \succ_j \varphi_j(\succsim)$ and $\omega_j \succ_i \varphi_i(\succsim)$. Let $\succsim'_i \in \mathcal{D}^{\Pi}_i$ be such that $D(\succsim'_i) = \{\omega_j, \varphi_i(\succsim)\}$. By *individual rationality* and *strategy-proofness*, $\varphi_i(\succsim) = \varphi_i(\succsim'_i, \succsim_{-i})$. By *non-bossiness*, $\varphi(\succsim) = \varphi(\succsim'_i, \succsim_{-i})$. Let $\succsim'_j \in \mathcal{D}^{\Pi}_j$ be such that $D(\succsim'_j) = \{\omega_i, \varphi_j(\succsim)\}$. Repeating the previous reasoning, $\varphi(\succsim'_{\{i,j\}} \succsim_{-\{i,j\}}) = \varphi(\succsim)$. Let $\succsim''_i \in \mathcal{D}^{\Pi}_i$ be such that $D(\succsim''_i) = \{\omega_j\}$. By *individual rationality* and *strategy-proofness*, $\varphi_i(\succsim''_i, \succsim'_j, \succsim_{-\{i,j\}}) = \{\omega_i\}$. By *individual rationality*, $\varphi_j(\succsim''_i, \succsim'_j, \succsim_{-\{i,j\}}) \in \{\omega_j, \varphi_j(\succsim'_{\{i,j\}} \succsim_{-\{i,j\}})\}$. By 2-*efficiency*, $\varphi_j(\succsim''_i, \succsim'_j, \succsim_{-\{i,j\}}) = \varphi_j(\succsim'_{\{i,j\}} \succsim_{-\{i,j\}})$. Finally, let $\succsim''_j \in \mathcal{D}^{\Pi}_j$ be such that $D(\succsim''_j) = \{\omega_i\}$. By *individual rationality* and *strategy-proofness*, $\varphi_j(\succsim''_{\{i,j\}}, \succsim_{-\{i,j\}}) = \{\omega_i\}$ and $\varphi_i(\succsim''_{\{i,j\}}, \succsim_{-\{i,j\}}) = \{\omega_i\}$, which violates 2-*efficiency.* $\square$

At this point we change the order in which we present the proofs with respect to the main text. First, we present a prove a general version of Proposition 3 (Proposition 4). Then, the proof of Proposition 2 immediately follows from Proposition 4. Before, we introduce some notation, that generalizes the definition of *sequential matching maximizing rules* to arbitrary partitions Π of the set of donors.

For each $a \in \mathcal{A}^2$, each $j, k \in \mathbb{N}$ with $j \leq k$ and $k \leq l$, define:

$$M_{j,k}(a) \equiv \left\{ i \in N \;\middle|\; a_i \neq \omega_i \;\&\; \begin{array}{c} \text{either } \omega_i \in \Pi(j) \text{ and } a_i \in \Pi(k) \\ \text{or } \omega_i \in \Pi(k) \text{ and } a_i \in \Pi(j) \end{array} \right\}.$$

That is, $M_{j,k}(a)$ contains the patients with a donor in $\Pi(j)$ who receive a kidney in $\Pi(k)$ and the patients with a donor in $\Pi(k)$ who receive a kidney in $\Pi(j)$. For each $a \in \mathcal{A}^2$, let $P_{1,1}(a) \equiv \varnothing$ and define recursively for each $j, k \in N$, such that $\{j, k\} \neq \{1, 1\}$, $j \leq k$ and $k \leq l$:

$$P_{j,k}(a) \equiv \begin{cases} P_{j,k-1}(a) \cup M_{j,k-1}(a) & \text{if } j < k, \\ P_{j-1,l}(a) \cup M_{j-1,l}(a) & \text{if } j = k. \end{cases}.$$

Proposition 4. *If φ satisfies* individual rationality*,* 2-efficiency*,* strategy-proofness*, and* non-bossiness*, then for each $\succsim\in\mathcal{D}^{\Pi}$, each $j,k\in\mathbb{N}$ with $=j\leq k$ and $k\leq l$,*

$$\#M_{j,k}(\varphi(\succsim))\geq\#M_{j,k}(a),$$

for each $a\in\mathcal{I}(\succsim)$ such that for each $i\in P_{j,k}\left(\varphi(\succsim)\right)$, $a_i=\varphi_i(\succsim)$.

Proof of Proposition 4. We prove first the result for $\{i,j\}=\{1,1\}$. Then, the arguments can be replicated iteratively to prove the general result.

Assume to the contrary that φ satisfies *individual rationality*, 2-*efficiency*, *strategy-proofness*, and *non-bossiness* but there is $\succsim\in\mathcal{D}^{\Pi}$ and an assignment $a\in\mathcal{I}(\succsim)$ such that $\#M_{1,1}(a)>\#M_{1,1}(\varphi(\succsim))$. (Note that for each $a'\in\mathcal{A}^2$, $P_{1,1}(a')=\varnothing$.) Let $b=\varphi(\succsim)$. Without any loss of generality, there is a set $T\subset N$ and $m,n\in N\setminus T$ such that for each $i\in T\cup\{m,n\}$, $\omega_i\in\Pi(1)$, $\varphi_m(\succsim)\notin\Pi(1)$, $\varphi_n(\succsim)\notin\Pi(1)$ and:

(i) For each $h\in T$, there is $h'\in T\setminus\{j\}$ such that $\varphi_h(\succsim)=\omega_{h'}$.

(ii) For each $g\in T\cup\{m,n\}$, there is $g'\in(T\cup\{m,n\})\setminus\{g\}$ such that $a_g=\omega_{g'}$.

(See Figure 1.)

Assume first that $T=\varnothing$. In this case, $a_m=\omega_n$ and $a_n=\omega_m$. Since $a\in\mathcal{I}(\succsim)$, $\omega_m\in D(\succsim_n)$ and $\omega_n\in D(\succsim_m)$. Because $\varphi_m(\succsim)\notin\Pi(1)$ and $\varphi_n(\succsim)\notin\Pi(1)$, this contradicts Lemma 1.

Assume $T\neq\varnothing$. Let $\succsim'\in\mathcal{D}^{\Pi}$ be such that for each $i\notin T\cup\{m,n\}$, $D(\succsim'_i)=\varphi_i(\succsim)$ and for each $i'\in T\cup\{m,n\}$, $\succsim'_{i'}=\succsim_{i'}$. Let $i\notin T\cup\{m,n\}$. By *individual rationality,* $\varphi_i(\succsim'_i,\succsim_{-i})\in\{\omega_i,\varphi_i(\succsim)\}$. By *strategy-proofness,* $\varphi_i(\succsim'_i,\succsim_{-i})\succsim'_i\varphi_i(\succsim)$. Then, $\varphi_i(\succsim'_i,\succsim_{-i})=\varphi_i(\succsim)$ and by *non-bossiness,* $\varphi(\succsim'_i,\succsim_{-i})=\varphi(\succsim)$. Repeating the same argument exchanging the preference of a patient at a time, we obtain $\varphi(\succsim')=\varphi(\succsim)$. Let $\succsim''_m\in\mathcal{D}^{\Pi}_m$ be such that $D(\succsim''_m)=a_m\in\Pi(1)$. By *individual rationality,* $\varphi_m(\succsim''_m,\succsim_{-m})\in\{\omega_m,a_m\}$. By *strategy-proofness,* $\varphi_m(\succsim')\succsim'_m\varphi_m(\succsim''_m,\succsim_{-m})$. Hence, $\varphi_m(\succsim''_m,\succsim_{-m})=\omega_m$. We have to consider three cases.

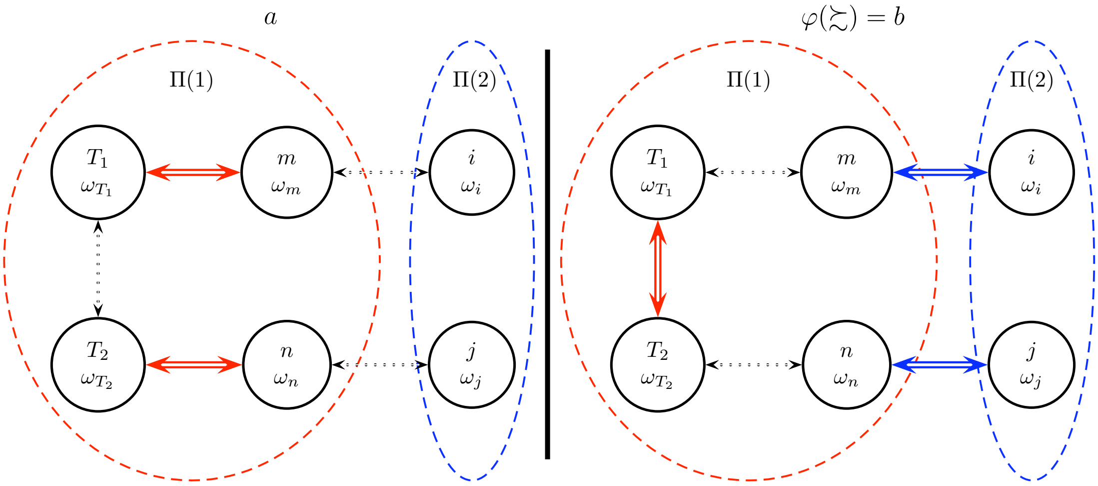

Figure 1: Proof of Proposition 4, $T = \{T_1, T_2\}$, $a \in \mathcal{I}(\succsim)$, $\varphi(\succsim) = b$.

(i) $\varphi_n(\succsim''_m, \succsim_n) = \omega_n$. Let $b \in \mathcal{A}^2$ be such that for each $k \in T \cup \{m, n\}$, $b_k = a_k$ and for each $i \notin T \cup \{m, n\}$, $b_i = \varphi_i(\succsim''_m, \succsim_n)$. Note the assignment b is such that all the patients in $T \cup \{m, n\}$ receive a kidney in $\Pi(1)$. Then, for each $i' \in N \setminus \{m\}$, $b_{i'} \succsim'_{i'} \varphi_{i'}(\succsim''_m, \succsim'_{-m})$, $b_m \succ''_m \varphi_m(\succsim''_m, \succsim_n)$, and $b_n \succ'_n \varphi_n(\succsim''_m, \succsim_n)$ which contradicts 2-*efficiency*.

(ii) $\varphi_n(\succsim''_m, \succsim'_{-m}) \in \Pi(1) \setminus \{\omega_n\}$. By *individual rationality* and the definition of the profile $(\succsim''_m, \succsim'_{-m})$, there are is $i \in T$, such that $\varphi_i(\succsim''_m, \succsim_{-m}) = \omega_i$. Consider the assignment $b \in \mathcal{A}^2$ defined in the previous paragraph. For each $i' \in N \setminus \{m\}$, $b_{i'} \succsim'_{i'} \varphi_{i'}(\succsim''_m, \succsim'_{-m})$, $b_m \succ''_m \varphi_m(\succsim''_m, \succsim_n)$, and $b_i \succ'_i \varphi_i(\succsim''_m, \succsim_n)$, which contradicts 2-*efficiency*.

(iii) $\varphi_n(\succsim''_m, \succsim_n) = \varphi_n(\succsim')$. Let $\succsim''_n \in \mathcal{D}^{\Pi}_n$ be such that $D(\succsim''_n) = \{a_n\}$. By an already familiar argument, *individual rationality* and *strategy-proofness*, imply that $\varphi_n(\succsim''_{\{m,n\}}, \succsim'_{-\{m,n\}}) = \omega_n$. By *individual rationality* and the definition of the profile $(\succsim''_{\{m,n\}}, \succsim'_{-\{m,n\}})$, there are at least two patients $i, i' \in T \cup \{m, n\}$, such that $\varphi_i(\succsim''_{\{m,n\}}, \succsim_{-\{m,n\}}) = \omega_i$ and $\varphi_{i'}(\succsim''_{\{m,n\}}, \succsim_{-\{m,n\}}) = \omega_{i'}$, which implies a contradiction with 2-*efficiency*.

Next, and just for the sake of completeness, we replicate the arguments to prove the result for $j = 1$ and $k = 2$. Assume to the contrary that φ satisfies *individual rationality*, 2-*efficiency*, *strategy-proofness*, and *non-bossiness* but there is $\succsim \in \mathcal{D}^{\Pi}$ and an assignment $a \in \mathcal{I}(\succsim)$ such that for each $i \in M_{1,1}(\varphi(\succsim)) = P_{1,1}(\varphi(\succsim))$, $\varphi_i(\succsim) = a_i$ but

$$\#M_{1,2}(a) > \#M_{1,2}(\varphi(\succsim)).$$

Without loss of generality there is a set $T \subset N$ and a pair of patients $m, n \in N \backslash T$ such that for each $i \in T \cup \{m, n\}$, $\omega_i \in (\Pi(1) \cup \Pi(2)$, $\omega_m \in \Pi(1)$, $\omega_n \in \Pi(2)$, $\varphi_m(\succsim) \notin (\Pi(1) \cup \Pi(2))$, $\varphi_n(\succsim) \notin \Pi(1)$, and:

(i) For each $h \in T$ with $\omega_h \in \Pi(1)$ there is $h' \in T$ with $\omega_{h'} \in \Pi(2)$ such that $\varphi_h(\succsim) = \omega_{h'}$.

(ii) For each $g \in T \cup \{m, n\}$ with $\omega_g \in \Pi(1)$ there is $g' \in T \cup \{m, n\}$ with $\omega_{g'} \in \Pi(2)$ such that $a_g = \omega_{g'}$.

Assume first that $T = \varnothing$. Then, repeating the argument in the proof for $\{i, j\} = \{1, 1\}$, we find a contradiction with Lemma 1.

Assume $T \neq \varnothing$. Let $\succsim' \in \mathcal{D}^{\Pi}$ be such that for each $i \notin T \cup \{m, n\}$, $D(\succsim'_i) = \varphi_i(\succsim)$. By repeated application of *individual rationality*, *strategy-proofness*, and *non-bossiness*, $\varphi(\succsim') = \varphi(\succsim)$. Let $\succsim''_m \in \mathcal{D}^{\Pi}_m$ be such that $D(\succsim''_m) = a_m \in \Pi(2)$. The arguments in the proof of Lemma 2 imply $\varphi_m(\succsim''_m, \succsim'_{-m}) = \omega_m$. We have to consider three cases that mimic the proof of Lemma 2.

(i) $\varphi_n(\succsim''_m, \succsim'_{-n}) = \omega_n$. Let $b \in \mathcal{A}^2$ be such that for each $i \in T \cup \{m, n\}$, $b_i = a_i$ and for each $i' \notin T \cup \{m, n\}$, $b_{i'} = \varphi_{i'}(\succsim''_m, \succsim_n)$. Note that, for each $i \in T \cup \{m, n\}$ with $\omega_i \in \Pi(1)$, $b_i \in \Pi(2)$ and for each $i' \in T \cup \{m, n\}$ with $\omega_{i'} \in \Pi(2)$, $b_{i'} \in \Pi(1)$. Hence, for each $i'' \in N \setminus \{m\}$, $b_{i''} \succsim'_{i''} \varphi_{i''}(\succsim''_m, \succsim'_{-m})$, $b_m \succ''_m \varphi_m(\succsim''_m, \succsim_n)$, and $b_n \succ'_n \varphi_n(\succsim''_m, \succsim_n)$ which contradicts 2-*efficiency*.

(ii) $\varphi_n(\succsim''_m, \succsim'_{-m}) \in \Pi(1)$. By *individual rationality* and the definition of the profile $(\succsim''_m, \succsim'_{-m})$, there is $i \in T$ with $\omega_i \in \Pi(2)$ such that $\varphi_i(\succsim''_m, \succsim_{-m}) = \omega_i$. The arguments of the previous paragraph immediately lead to a contradiction with 2-*efficiency*.

(iii) $\varphi_n(\succsim''_m, \succsim'_{-m}) = \varphi_n(\succsim')$. Combining the arguments in the previous paragraphs with the arguments in the proof of Lemma 2, we obtain a contradiction with 2-*efficiency*.

We can sequentially apply the same arguments to prove the result for $j = 1$ and $k = 3, \ldots, l$. Then, the arguments in the proof for $\{i, j\} = \{1, 1\}$ directly apply to the case $j = 2$ and $k = 2$ and iteratively to all the remaining steps. □

Proof of Proposition 3. Note that Proposition 3 is just a special case of Proposition 4 for partitions consisting of two elements $\Pi = \{\Pi(1), \Pi(2)\}$. □

Proof of Proposition 2. Assume to the contrary that φ satisfies *individual rationality*, 2-*efficiency*, *strategy-proofness*, and *non-bossiness*, and there are $a \in \mathcal{A}^2$ $T \subset N$ and $\succsim \in \mathcal{D}^\Pi$ such that a strictly dominates $\varphi(\succsim)$ via T. Note that for each $i \in M_{1,1}(\varphi(\succsim))$, because for each $\omega \in \Omega$, $\varphi_i(\succsim) \succsim_i \omega$, $i \notin T$. Analogously, for each $k = 2, \ldots, l$, if $j \in M_{1,k}(\varphi(\succsim))$ and $\omega_j \in \Pi(k)$, $j \notin T$.

Next, assume that there is $j \in M_{1,2}(\varphi(\succsim))$ with $\omega_j \in \Pi(1)$ and $j \in T$. Since $\varphi_j(\succsim) \in \Pi(2)$ and a blocks φ via T, there is $k \in T$ such that $a_j = \omega_k \in \Pi(1)$. Since $k \in T$, $a_k = \omega_j \succ_k \varphi_k(\succsim)$. Finally, let $b \in \mathcal{I}(\succsim)$ be such that for each $i \in M_{1,1}(\varphi(\succsim))$, $b_i = \varphi_i(\succsim)$, $b_j = a_j$, $b_k = a_k$, and for each $m \notin M_{1,1}(\varphi(\succsim)) \cup \{j, k\}$, $a_m = \omega_m$. Clearly, $b \in \mathcal{I}(\succsim)$ and $\#M_{1,1}(b) > \#M_{1,1}(\varphi(\succsim))$, which contradicts Lemma 2. Similarly, assume there is $j \in M_{1,3}(\varphi(\succsim))$ with $\omega_j \in \Pi(1)$ and $j \in T$. Since $\varphi_j(\succsim) \in \Pi(3)$ and a blocks φ via T, there is $k \in T$ such that $a_j = \omega_k \in \cup(\Pi(1) \cup \Pi(2))$. By the previous argument $\omega_k \in \Pi(2)$. Since $k \in T$, $a_k = \omega_j \succ_k \varphi_k(\succsim)$. Finally, let $b \in \mathcal{I}(\succsim)$ be such that for each $i \in (M_{1,1}(\varphi(\succsim)) \cup M_{1,2}(\varphi(\succsim))$, $b_i = \varphi_i(\succsim)$, $b_j = a_j$, $b_k = a_k$, and for each $m \notin (M_{1,1}(\varphi(\succsim)) \cup M_{1,2}(\varphi(\succsim))) \cup \{j, k\}$, $a_m = \omega_m$. Clearly, $b \in \mathcal{I}(\succsim)$, for each

$j \in M_{1,1}(\varphi(\succsim))$, $b_i = \varphi_i(\succsim) =$ but

$$\#\left\{ i \in N \;\middle|\; \begin{array}{l} \text{either } \omega_i \in \Pi(1),\ b_i \in \Pi(2), \\ \text{or} \omega_i \in \Pi(2),\ b_i \in \Pi(1) \end{array} \right\} > \# M_{1,2}(\varphi(\succsim)),$$

which contradicts Proposition 4. We can iteratively repeat the same argument as many times as necessary to show that there is no $i \in T$ with $\omega_i \in \Pi(1)$. The reasoning also apply for the remaining elements of the partition Π, to show that $T = \varnothing$. $\square$

Proof of Theorem 4. Let Ψ be a *generalized age-based priority rule* with permutations $\Sigma = \{\sigma_1, \ldots, \sigma_l\}$. *Individual rationality* for generalized rules directly follows from the definition of generalized age-based priority rule. For 2-*efficiency*, note that at each step of the algorithm a patient chooses her best preferred assignments in a set. Moreover, the kidneys that are *a priori* more desirable are the first kidneys to be assigned. Thus, we focus on EGSP. For each $\succsim \in \mathcal{D}^{\Pi}$, and each $\mathbf{K}' \in \mathcal{K}$ such that for each patient i, $K'_i \subset \Omega_i$, because $\mathcal{I}(\succsim, \mathbf{K}') \subseteq \mathcal{I}(\succsim, \Omega)$, by the iterative definition of Ψ, for every patient i, $\Psi_i(\succsim, \mathbf{\Omega}) \succsim_i \Psi(\succsim, \mathbf{K}')$. Then, for each $T \subseteq N$, each $\succsim \in \mathcal{D}^{\Pi}$, each $\mathbf{K}'_T \in \times_{i \in T} \mathcal{K}_i$, for each $i \in T$:

$$\Psi_i(\succsim, \mathbf{\Omega}) \succsim_i \varphi_i(\succsim, (\mathbf{\Omega}_{N \setminus T}, \mathbf{K}'_T)). \tag{1}$$

Repeating the arguments in the proof of Theorem 3, we obtain that there are no $T \subset N$, $\succsim \in \mathcal{D}^{\Pi}$, $\succsim'_T \in \mathcal{D}^{\Pi}_T$ and $K' \in \mathcal{K}$, such that for each $i \in T$:

$$\Psi_i(\succsim'_T, \succsim_{N \setminus T}, \mathbf{K}') \succ_i \Psi_i(\succsim, \mathbf{K}'). \tag{2}$$

Combining equations (1) and (2), and letting $\mathbf{K}' = (\mathbf{\Omega}_{N \setminus T}, \mathbf{K}'_T)$, we obtain that there are no $T \subset N$, $\succsim \in \mathcal{D}^{\Pi}$, $\succsim'_T \in \mathcal{D}^{\Pi}_T$, and $\mathbf{K}'_T \in \times_{i \in T} \mathcal{K}_i$ such that for each $i \in T$

$$\Psi_i(\succsim'_T, \succsim_{N \setminus T}, (\mathbf{\Omega}_{N \setminus T}, \mathbf{K}'_T)) \succ_i \Psi_i(\succsim, \mathbf{\Omega}).$$

$\square$

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