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The Long Run Survival of Small States ^{*}

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JEL classification:H25, H73, O30, O43, F13, F15.

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1 Introduction

It is well-known that small states suffer from very limited capital and labour resources both in amount and in variety. Their small home market size prevents them from exploiting scale and scope economies. It is therefore not surprising that small states are highly open to international trade and capital flows (Alesina and Wacziarg, 1998). Because of their smallness these countries are highly depending on forces outside their control, which could threaten their economic viability (Briguglio, 1995). This explains the general view that the economic performance of these small states is associated with vulnerability¹ to external shocks. A good example is the case of Iceland whose GDP has been quite volatile due to shocks in the aluminium, fishing, or banking sector. Nevertheless, the strong growth performance of some small states suggests that it is possible to at least partially offset this vulnerability and increase their resilience by means of appropriate endogenous policies (Armstrong and Read, 2002). Armstrong, De Kervenoael and R. Read (1998) show that one country's economic smallness does not necessarily have a negative effect on its performance. This can be explained by the fact that small countries develop abilities and use instruments to overcome their natural handicaps. For example, some of the richest countries in the world are small states such as Luxembourg². This is an illustration of what Briguglio et al. (2009) call the "Singapore Paradox", which is the situation where a country highly vulnerable to exogenous shocks still manages to attain high economic performance.

Since domestic capital is relatively scarce in very small economies, it follows that attracting foreign investments is an important way to fill in this gap. As a matter of fact those economies tend to get more private capital from abroad as a ratio of total capital

¹Economic vulnerability indices mostly depend on a high level of openness and therefore are typically associated with smallness. The Commonwealth Secretariat and the UN have developed complex and rigorous vulnerability indices (Armstrong and Read, 2002).

²Note that high vulnerability and good economic performance are not contradictory aspects of very small economies. Indeed, Iceland has been an example for good economic performance but has shown great fragility in the context of the latest financial crisis.

formation (Streeten, 1993). Moreover, capital inflow may also be a critical contributor to the growth and development of small states (Read, 2008). Some empirical evidence shows a positive impact of FDI on economic growth and the possibility of spillover effects to local firms (Castellani and Zanfei, 2006).

In this paper, we extended the static version of Pieretti and Zanaj (2011) by assuming that the small country tries to attract foreign investments through low taxes and/or high level of public goods, which enhance firms' productivity into a dynamic framework. Public goods can cover a wide range of infrastructures, services and regulations provided by the local and/or the central government are attractive to firms if they enhance their productivity³. Accordingly, capital locates according to differentials in offered public good levels and tax differentials. Specifically, a country may not reduce its attractiveness by a unilateral increase in taxes if foreign investors are compensated by more infrastructure provision. Put differently, foreign investors are ready to pay higher taxes because infrastructure become more valuable to them (e.g. Haufler, 1998, Pieretti and Zanaj, 2011).

The literature has investigated the role of jurisdictions' size on their capacity to attract capital. Recent papers show that small economies can be attractive not only for tax reasons but also for their provision of public infrastructures (Justman et al., 2005, Zissimos and Wooders, 2008, Hindriks et al., 2008, Pieretti and Zanaj, 2011). This paper extends this literature by modelling the dynamics of a small economy's strategies to attract foreign investments. More precisely, we study a small state's inter-temporal

³In this context, we may consider transportation infrastructures, universities and public R&D investment, but also property rights enforcement, capital market regulations, labor and environmental regulations and the absence of red tape procedures. It follows that countries' ability to attract foreign investment may also be attractive for the quality of their institutions. In the Oxford Handbook of Entrepreneurship (2007), it is argued that the abundance of entrepreneurs in a country depends, among other factors, on the existence of regulations, property rights, accounting standards and disclosure requirements. Furthermore, in recent years there has been a surge of country and cross-country studies relating economic development to institutions, especially those affecting capital market development and functionality (La Porta et al. (1997) among others).

choice of optimal taxes which are used to afford public goods that enhance firms' productivity. Applying Pontryagin's maximum principle(see, for example, Boucekckine et al(2007)) we then characterize the potential steady states attainable by the small economy. The main findings of the paper can be summarized as follows. We show that there exists three types of steady states. One in which the size of the initially small country attracts sufficiently external capital to grow as big as the foreign economy. One in which the small economy is no more economically viable since it loses all its productive capital. Finally, an intermediate configuration in which the domestic economy survives while remaining small. In this scenario, there exists at least one intermediate steady state which exhibits saddle point stability. If the small economy does not undergo the optimal path which leads to one of these intermediate equilibria it may converge to the worst case and disappear. The survival of the small economies is thus an important public policy issue which implies an appropriate choice of initial conditions and a dynamic update of the tax policy.

The dynamic interactions among jurisdictions to attract mobile factors have already been analyzed using the framework of repeated games. The main issue studied by this literature is the tax coordination problem between symmetric regions (Cardarella et al., 2002, Catenaro and Vidal, 2006, Itaya et al., 2008). The purpose of this paper is however not to model a game between jurisdictions. We rather focus on the strategic choices of a very small open economy facing exogenously given choices of the rest of the world. The world is thus divided into two unequal sized regions where size refers to the magnitude of the population, which coincides with the number of capital-owners who are simultaneously entrepreneurs and workers. Specifically, our paper tries to offer an insight into the dynamic policy behavior of a very small country trying to guarantee the long run survival of its economy. We thus analyze, in an infinite time horizon, the dynamics of its size and its policy instruments for exogenous foreign levels of taxes and public goods. Like Alesina and Spolaore (1997) we do not regard the size of a country as being exogenously given. These authors assume a country's borders to be subject to the same analysis as any other human made institution ⁴. In

⁴It is interesting to recall that since 1990, 33 new countries have been formed and, many of them are

our paper however we consider that the economic country size can vary endogenously as a consequence of public policy but that its political (geographic) magnitude remains unchanged.

The paper is organized as follows. The next section presents the model and the optimal conditions via Pontryagin maximum principle. Section 3 provides the type and the analysis of steady states and its convergence. Finally section 4 concludes.

2 The model

The world is composed of two regions of unequal population size⁵. In the rest of the paper, we consider the smallest region as the small or the home country and the largest as the foreign country or the rest of the world indifferently. We assume that the members of both jurisdictions are at the same time entrepreneurs and workers and each of them owns one unit of productive capital. We thus assume that the endowment in human resources and physical capital grows in proportion to the human population. Furthermore, the size of a country is equivalent to the number of firms located in its territory.

At time $t = 0$, these jurisdictions are represented on an interval $[-S(0), S^*(0)]$.⁶ The size of the small country is $S(0)$ and extends from $-S(0)$ to 0 which corresponds to the border. The rest of the world has a size of $S^*(0)$ with $S(0) < S^*(0)$ and extends from 0 to $S^*(0)$. The firm-owners in both jurisdictions are evenly distributed on their respective sub-interval according to their disposition to invest outside their home location. As in Ogura (2006), we assume that the population of investors is heterogeneous in the degree of their attachment to home⁷.

very small like Kosova, Macedonia, Slovenia, East Timor, Marshall Islands, Namibia, Palau, etc. The dynamics of countries creation is thus a very current and lively process.

⁵Country size may be defined by its population, by its area, or by its national income (Streeten, 1993). In our paper, we focus on the *population* aspect rather than on the spatial size.

⁶The substrict " * " refers to the large (foreign) jurisdiction.

⁷Heterogeneity in home attachment was first considered in the fiscal competition literature by Man-

In our spatial setting we assume that the closer firms are located to the extremes the more they are attached to their current location. Conversely, the closer firms are to the border 0, the less they are attached to their territory and the easier they are able to relocate⁸ abroad. This means that a firm of type $\alpha \in [-S(0), 0]$ located in the home country incurs a disutility of relocating abroad which equal $k \cdot x$, where $x = d(\alpha, 0)$, i.e. the distance between 0 and α . The coefficient k represents the unit cost of moving capital abroad which can also be interpreted as the degree of international integration.

Now assume that each population member of both jurisdictions owns one unit of capital which she combines with her labor to set up a firm to produce $q + a_i$ ($a_i = a, a^*$) units of a final good, where q is the private component of (gross) productivity. The fraction a (a^*) of the produced good depends on the public input supplied by the home (foreign) jurisdiction⁹. Public infrastructure investment a_i may be an improvement of existing regulations that potentially increase the performance of the financial services industry. This makes the country more attractive to foreign financial firms and increases the attachment to home of domestic financial firms, *ceteris paribus*. The produced output is sold in a competitive (world) market at a given price normalized to one. Assuming that both countries have equal access to a common market this assumption implies that the small jurisdiction does not suffer from a reduced home market. We further suppose that the unit production cost is constant and equal to zero without loss of generality.

We now adopt a temporal perspective of the above setting. Each period $t \in [\Delta t, +\infty)$,

soorian and Myers (1993).

⁸For reasons of simplicity, we assume that relocation if any is only possible in the neighboring jurisdiction.

⁹The public input satisfies the local public good characteristic, which means that it is jointly used without rivalry by firms located in the same jurisdiction. It follows that the benefits and the costs of these good only accrue at the jurisdictional level. As in Zissimoss and Wooders (2008), we shall abstract from congestion costs . Taking account of congestion would complicate our framework without improving qualitatively the results. Moreover, if the public input represents immaterial goods as law and regulations (protecting intellectual property, specifying accurate dispute resolution rules,...), the absence of congestion is easily justified by the particular nature of these goods.

(for any $\Delta t > 0$) governments update their choice in terms of offered public goods and taxes¹⁰. We assume that the total number of entrepreneurs, $S(t) + S^*(t)$ will be constant over time t and normalized to one. Since firms may move, the relative size of both jurisdictions will change with t . In the following, we focus on the behavior of a small country. Therefore, we suppose that the home country's size $S(t)$ is small enough to consider the rest of the world's choices as exogenously given. Consequently, the state constraint writes as

$$0 \leq S(t) < \frac{1}{2} \text{ for } t \in [0, +\infty). \quad (1)$$

Providing firms with public infrastructures is costly. The public technology which serves to produce each period the public input is given by the function $f(S(t), T(t))$ where $T(t) \in [0, \widehat{T}]$ denotes the tax levied on one unit of capital at time t , and $\widehat{T} \in (0, \infty)$ is a constant. Supposing that the public good depreciates at a rate δ , we can write the following motion equations of public input

$$\dot{a}(t) = h[a(t)] - \delta a(t) + f[S(t), T(t)], \quad (2)$$

$$\dot{a}^*(t) = h[a^*(t)] - \delta a^*(t) + f[S^*(t), T^*(t)], \quad (3)$$

where $h[\dots]$ represents the flow of public inputs produced by the use of public inputs. For simplicity, we shall work with the following functions: $h[a(t)] = \xi a(t)$, $h[a^*(t)] = \xi a^*(t)$, $f[S(t), T(t)] = \zeta S(t)T(t)$ and $f[S^*(t), T^*(t)] = \zeta S^*(t)T^*(t)$ where ξ is a non-negative unit fee charged for the use of the public infrastructure and ζ represents a non-negative productivity factor. Furthermore, we assume that these coefficients verify $0 \leq \xi < \delta$. This condition rules out a breakdown in the production of public infrastructures.

Assume now that an entrepreneur of type $\alpha(t)$ initially located in the small country considers to stay at home or to invest her/his physical capital abroad. If she/he decides not to move, her/his profit is given by¹¹

$$\pi(t) = q(t) + a(t) - T(t) \quad (4)$$

¹⁰Notice that we assume there are no sunk cost on the investment or that our unit of time t is long enough to cancel the sunk cost of investment.

¹¹For sake of simplicity, we assume that q is such that the profit of each firm is positive for all equilibrium level of public goods and taxes.

If she invests abroad, her/his profit becomes

$$\pi^*(t) = q(t) + a^*(t) - T^*(t) - k \cdot x(t)$$

Furthermore, consider that this capital-owner is indifferent between investing abroad and staying at home. Then it follows that

$$q(t) + a(t) - T(t) = q(t) + a^*(t) - T^*(t) - k \cdot x(t).$$

After setting $b^*(t) = \frac{a^*(t) - T^*(t)}{k}$, we obtain

$$x(t, a, a^*, T, T^*) = b^*(t) - \frac{a(t) - T(t)}{k}. \quad (5)$$

In other words, the foreign country attracts capital ($x > 0$) from the small jurisdiction if the net gain of investing abroad, i.e. $a^*(t) - T^*(t)$, is higher than the net gain of staying at home, $a(t) - T(t)$, after taking into account the mobility cost kx .

The motion equation of the size variable $S(t)$ of the small economy is then given by

$$\dot{S}(t) = -\rho x = \rho \left(\frac{a(t) - T(t)}{k} - b^*(t) \right), \quad (6)$$

with the initial condition $\frac{1}{2} > S(0) > 0$ and positive constant ρ which represents some kind of density function that does not affect the analysis and the results.

Note that the relocation of a subset of firms at each period alters the ranking of firms' attachment to home. In the following, we adopt the following rule. For all $\tilde{\alpha}(t) \in [-S(t), S^*(t)]$, we define $\tilde{\alpha}(t) = \tilde{\alpha}(t - \Delta t) + x$, where $\tilde{\alpha}(t) = \begin{cases} \alpha(t) \in [-S(t), O(t)] \\ \alpha^*(t) \in [O(t), S^*(t)] \end{cases}$ and $O(t)$ stands for the origin at period t .

We thus assume that the preferences for the home location will change according to the relative attractiveness of the competing jurisdictions in the following way. For the firms that do not move, attachment to home will increase by x if the small economy is attractive to foreign investors ($-x < 0$) and it will decrease if the foreign location attracts capital from the small country ($x > 0$). For the capital owners who relocate

abroad, the attachment to the new location decreases with the attachment they had to the country they left.

In the rest of the paper we focus on the small jurisdiction. We analyze, in an infinite time horizon, the dynamics of its size $S(t)$ and its policy instruments $T(t)$ and $a(t)$ for exogenous foreign levels of taxes T^* and public goods a^* . We thus consider that the rest of the world does not react to the small country's decisions. We also analyze the convergence of the variables $S(t)$, $T(t)$ and $a(t)$ towards possible steady states.

We further assume that policy makers maximize the discounted linear-quadratic utility that depends on tax revenues, $S(t) \cdot T(t)$, net of the adjustment cost of public inputs $a^2(t)$, with unit adjustment cost $\frac{\beta}{2} > 0$. The objective-function of the small economy is given by

$$\begin{aligned} \max_{T(t)} W &= \int_0^\infty e^{-rt} \left[S(t)T(t) - \frac{\alpha}{2} (S(t)T(t))^2 - \frac{\beta}{2} a^2(t) \right] dt, \\ \text{s.t. } \dot{a}(t) &= h[a(t)] - \delta a(t) + f[S(t), T(t)], \\ \dot{S}(t) &= -\rho x = \rho \left(\frac{a(t) - T(t)}{k} - b^*(t) \right) \\ 0 &\leq S(t) < \frac{1}{2} \end{aligned} \quad (7)$$

where α represents a cost parameter of collecting taxes¹². We assume furthermore that the linear-quadratic utility is increasing and concave with respect to the total tax, that is, we assume $\hat{T} < \frac{1}{2\alpha_2}$.

We now characterize the inter-temporal optimal tax strategy chosen by the policy makers in the small country. Applying Pontryagin's maximum principle, we derive a canonical system of ordinary differential equations that has to be satisfied by the optimal trajectories. Since the Hamiltonian of the dynamic optimization problem is concave with respect to the state variables, the Maximum principle provides not only

¹²The time preference parameter r represents the degree of "impatience" of the home country's population.

necessary but also sufficient optimality conditions for interior solutions (see e.g. Theorem 4.2, Dockner et al. (2000), Chiang (2000), Hartl et al. (1995) or Sethi and Thompson (1981). Denote by μ and ν the costate variables corresponding respectively to the state variables $S(t)$ and $a(t)$.

Proposition 1 *For any state trajectory $(S(t), a(t))$ that corresponds to an optimal taxation strategy of the policy maker, there exist piecewise absolutely continuous costates $\mu(t), \nu(t)$ and two multipliers $\theta_1(t) \geq 0, \theta_2(t) \geq 0$, such that the optimal choice variable $T(t)$ satisfies*

$$T(t)S^2(t) = \frac{(1 + \zeta\nu)S - \frac{\rho}{k}(\mu - \theta_1 + \theta_2)}{\alpha}. \quad (8)$$

In addition to (6) and (2) the costate equations become

$$\dot{\mu} = r\mu - \zeta T\nu - (1 - \alpha TS)T, \quad (9)$$

$$\dot{\nu} = (r + \delta - \xi)\nu + \beta a - \frac{\rho}{k}(\mu - \theta_1 + \theta_2). \quad (10)$$

We further have

$$\dot{S} = -\rho \left(b^* - \frac{a - T}{k} \right) \leq 0, \text{ if } S(t) = \frac{1}{2}; \theta_1(t) \left(\frac{1}{2} - S(t) \right) = 0, \theta_1 \geq 0, \quad (11)$$

$$\dot{S} = -\rho \left(b^* - \frac{a - T}{k} \right) \geq 0, \text{ if } S(t) = 0; \theta_2(t)S = 0, \theta_2 \geq 0. \quad (12)$$

Finally, the transversality conditions $\lim_{t \rightarrow \infty} e^{-rt} \mu S = 0$ and $\lim_{t \rightarrow \infty} e^{-rt} \nu a = 0$ are satisfied.

In the following section, we characterize the potential steady states of the system and we analyze how the steady states can be attained.

3 Steady states and convergence analysis

Steady states are defined as rest points of the dynamic equations (2), (6), (9) and (10) with assumption that in the long run a^* and T^* are constants. Due to the state space

constraints, there are two types of possible steady states: *two constraint steady states* and *unconstraint steady state(s)*. The constraint steady states can be the upper bound, $\bar{S} = \frac{1}{2}$, of the small's country population or its lower bound $\underline{S} = 0$. In the following, we analyse three possibilities: the steady state in which the small country survives as such in the long run; the steady state in which the small economy collapses; and finally the steady state in which the small economy expands to the point that $S = \frac{1}{2}$.

3.1 Survival as small state

We first study the interior steady states \hat{S} ($0 < \hat{S} < \frac{1}{2}$). The existence of such steady states is crucial. Indeed, this would mean that there exists a policy mix consisting of taxes and public infrastructures able to guarantee the long term survival of a small country. In these interior steady states \hat{S} ($0 < \hat{S} < \frac{1}{2}$) where the boundary constraints are both not binding. Hence, $\hat{\theta}_1 = 0$ and $\hat{\theta}_2 = 0$ and the interior rest points of the dynamic system (6), (2), (9) and (10) are specified in the following proposition¹³.

Proposition 2 *For any given parameters $\rho, \delta, \xi(< \delta), \zeta, k$, and for any foreign policy choices, a^* and T^* made by the rest of the world, there is always one steady state¹⁴*

$$\hat{a} = \frac{\zeta(r + \delta - \xi)}{\alpha(r + \delta - \xi)(\delta - \xi) + \beta\zeta^2} (> 0), \quad (13)$$

$$\hat{S} = \frac{(\delta - \xi) \hat{a}}{\zeta \hat{T}}, \quad (14)$$

$$\hat{T} = \hat{a} - (a^* - T^*) \quad (15)$$

and the two costate variables are

$$\hat{\mu} = 0, \quad (16)$$

$$\hat{\nu} = -\frac{\beta}{(r + \delta - \xi)} \hat{a} (< 0). \quad (17)$$

¹³The proof is given in the appendix.

¹⁴In addition to the above interior steady state, other interior solutions may appear for special parameter and coefficient combinations. We present these cases in the appendix.

This steady state is a saddle point of the canonical system (2), (6), (9) and (10). Moreover, it is one dimensional locally asymptotically stable, if $r > \frac{\rho\hat{T}}{k\hat{S}}$.¹⁵ Otherwise, if $r < \frac{\rho\hat{T}}{k\hat{S}}$, it is two dimensional locally asymptotically stable.

The stability of the above interior steady state is the result of the presence of at least one negative eigenvalue in the 4-dimensional eigenvalue space; such a presence guarantees always the convergent path. Then, depending on the other parameters, there may appear another eigenvalue which is negative and hence this would lead to the convergent trajectory. Except for the above parameter conditions, the convergence or not of the system only depends on the initial conditions of the economy, that is, $a(0)$ and $S(0)$. There is one (or two) trajectory(ies) that converge(s) to the optimal steady state, while all the other trajectories lead to the two possible corner solutions: $\underline{S} = 0$ and $\overline{S} = \frac{1}{2}$.¹⁶

First note that according to (13) it is optimal for the small state to equate the net (of taxes) amount of provided public goods $\hat{a} - \hat{T}$ to that of the foreign economy $a^* - T^*$. This equality is necessary for migration to cease. It also appears that the amount of public infrastructure offered in the steady state does not depend on the rest of the world's decision variables. This is not the case for the equilibrium tax rate¹⁷ \hat{T} and consequently for the equilibrium size \hat{S} . We see that the steady state level of public infrastructure does not apparently depend on the foreign country's decision variables. What only matters are the technological parameters of producing public inputs and once the steady state level \hat{a} is set the tax amount will be determined by equation (13). Therefore, a small state may reach a steady state if it possesses the right technology to produce the public infrastructure, given its power to levy taxes.

¹⁵The values of \hat{T} and \hat{S} are given in (13), and the conditions are given to parameters, while \hat{T} and \hat{S} just for short notations.

¹⁶For more discussion on the saddle point stability see, for example, in Azariadis (2000), de la Croix and P. Michel (2002) or Galor (2007), and the references therein.

¹⁷According to (13) we could have chosen to express \hat{T} (or \hat{S}) as an independent solution of the foreign decision instruments. In this case \hat{a} and \hat{S} (or \hat{a} and \hat{T}) would depend on a^* and T^* .

We can now turn to some comparative statics. We see that the steady state provision of public goods increases with the time preference r since $\frac{\partial \hat{a}}{\partial r} > 0$. The reason is that the more the home country is impatient the more it will be reluctant to postpone to invest in public infrastructures. The impact of an increase of the productivity in providing public goods is however ambiguous. Indeed, according to (13) we have $\frac{\partial \hat{a}}{\partial \zeta} > 0$ if $\zeta < \bar{\zeta}$ ($\bar{\zeta} = \sqrt{\frac{\alpha(\delta-\xi)(r+\delta-\xi)}{\beta}}$) and $\frac{\partial \hat{a}}{\partial \zeta} < 0$ if $\zeta > \bar{\zeta}$. In other words, if productivity increases but remains at a low level ($\zeta < \bar{\zeta}$), the home country has an incentive to increase its attractiveness by augmenting public infrastructures. If the threshold $\bar{\zeta}$ is exceeded, then an increase in productivity induces too much public investment for a given level of taxes. Consequently, the home country increases its attractiveness to foreign investment by reducing its tax rate and thus by reducing the provision of public goods¹⁸.

If the policy variables set by the rest of the world change, the budget condition in the small open economy always holds. This follows from the fact that we always have $(\delta - \xi)\hat{a} = \zeta\hat{S}\hat{T}$ since $\dot{a} = 0$. This implies that the tax revenue $\hat{S}\hat{T} = \frac{(\delta-\xi)\hat{a}}{\zeta}$ is independent of the foreign decision variables. It follows that the costate variable corresponding to S is zero in the steady state, $\hat{\mu} = 0$. Furthermore, the negative value of $\hat{\nu}$ implies that increasing the public goods provision decreases the social welfare, due to the presence of adjustment costs in the objective function of the policy maker.

Next we analyze the impact of a change in \hat{a} originating, for example, from a shock affecting ζ or r on the steady state size of the home economy. It is straightforward to show that $\frac{\partial \hat{S}}{\partial \hat{a}}$ has the opposite sign¹⁹ of $a^* - T^*$. Since condition $\hat{T} = \hat{a} - (a^* - T^*)$ must hold in the steady state, both regions are equally *attractive* if the net amount of public goods offered by the small and large economies are either positive ($a^* > T^*$ and $\hat{a} > \hat{T}$). Conversely, both regions are equally *unattractive* if $T^* > a^*$ and $\hat{T} > \hat{a}$. The derivative $\frac{\partial \hat{S}}{\partial \hat{a}}$ is negative in the first case and positive in the second case. The impact of \hat{a} on \hat{S} can now be interpreted in the following way. If both regions are equally attractive,

¹⁸Since $\hat{T} = \hat{a} - (a^* - T^*)$, and thus $\frac{\partial \hat{a}}{\partial \zeta}$ and $\frac{\partial \hat{T}}{\partial \zeta}$ are equal.

¹⁹ $\frac{\partial \hat{S}}{\partial \hat{a}} = -\frac{1}{\zeta} \frac{(\delta-\xi)}{(\hat{a}+T^*-a^*)^2} (a^* - T^*)$

entrepreneurs have (for given moving costs) a preference for the country which levies lower taxes. If the small country increases the provision of public goods, it has also to increase its tax rate according to the above steady state condition. It follows that capital flows out of the small country and the size \hat{S} shrinks consequently. If both regions are equally unattractive, entrepreneurs have (for given moving costs) a preference for the region which offers comparatively more public goods. Hence, an increase in \hat{a} results in a capital inflow into the small economy and the size \hat{S} expands consequently.

3.2 Small state collapse

Let us first consider the case where the small economy could suffer from a possible economic collapse, i.e. $S(t) = 0$. In this case, the constraint on the state variable must be binding in order to exclude a negative population value and the steady state values of $\theta_1 = 0$ and θ_2 has to be positive. Hence, the condition (12) in Proposition 1, should hold. Furthermore, once $S(t)$ has attained the lower bound it can no more decrease and it should be either constant or increasing, that is, $\dot{S}(t) \geq 0$, as it is shown in Proposition 1. In this case, we get

$$\underline{S} = 0, \underline{\theta}_1 = 0, \underline{\theta}_2 = -\mu, \quad (18)$$

$$\underline{a} = 0, \underline{T} = T^* - a^*, \quad (19)$$

$$\underline{\nu} = 0, \underline{\mu} = \frac{1}{r}(T^* - a^*)(< 0). \quad (20)$$

Hence, the following result obtains.

Proposition 3 *For given parameters $\rho, \delta, \xi(< \delta), \zeta, k$, and for any policy variables set by the rest of the rest of the world a^* and T^* , if $T^* < a^*$ ²⁰, the small state may heading towards an economic collapse ($\underline{S} = 0$) which is specified by (18). In this case, the multiplier θ_2 is strictly positive.*

²⁰This is a necessary condition only, rather than sufficient.

It follows that a necessary condition for the small state's collapse is that the rest of the world is able to tax more than the value of the public infrastructure that it supplies. The big country can be able to tax more than the value of its public goods and still be able to attract capital for different reasons. For instance, simply because of the bigger size, the burden $T^* - a^*$ in the path toward the steady state can be smaller in the big economies than in the small country. Therefore, the slower exctration of the wealth of its citizens in the big country $T^* - a^* < \underline{T} - \underline{a}$ leads to flow out from the small to the rest of the world up to the point of collapse. Thus, at the steady state this gives $\underline{a} = 0$.

This result is crucial because it shows that if countries tax more than they supply in terms of public goods, big countries are able to render problematic the survival of small countries.

3.3 Convergence to equal size

Let us now consider the case in which the small country could converge to the upper bound $S(t) = \frac{1}{2}$. Under this configuration size asymmetry between countries disappears. Note that the attainment of such a limit requires us to abandon the small country assumption which implies passivity of the rest of the world with regard to the home country's policy choices. We however explore that case and show its possible economic relevance without modifying the small country assumption.

If $S(t) = \frac{1}{2}$, the constraint on the state variable becomes binding and the steady state value of θ_1 has to be positive and $\theta_2 = 0$. Hence, the condition (11), in Proposition 1, should hold. Whenever $S(t) = \frac{1}{2}$, population cannot increase any more, and therefore, the change of population should be constant or decreasing: $\dot{S}(t) \leq 0$. In the appendix, we show how the following steady state values are obtained

$$\bar{S} = \frac{1}{2}, \bar{a} = \frac{\zeta}{2(\xi - \delta) + \zeta}(a^* - T^*), \bar{T} = \bar{a} - (a^* - T^*). \quad (21)$$

Moreover, the costate variables become

$$\bar{\nu} = \frac{1}{r + \delta - \xi - \frac{\zeta}{2}} \left(\frac{1}{2} - \frac{\alpha \bar{T}}{4} - \beta \bar{a} \right), \quad (22)$$

$$\bar{\mu} = \frac{\bar{T}}{r} \left(1 - \frac{\alpha \bar{T}}{2} + \zeta \bar{\nu} \right), \quad (23)$$

$$\bar{\theta}_1 = \left(1 - \frac{2kr}{\rho \bar{T}} \right) \bar{\mu} > 0, \quad (24)$$

$$\bar{\theta}_2 = 0. \quad (25)$$

The above analysis lead to the following conclusion.

Proposition 4 *For given parameters $\rho, \delta, \xi (< \delta), \zeta, k$, and for any policy variables set by the rest of the world a^* and T^* , if (a) $\xi < \delta < \xi + \frac{\zeta}{2}$ and $a^* > T^*$, or (b) $\delta > \xi + \frac{\zeta}{2}$ and $a^* < T^*$, the economy may converge to its upper-limit size, given by (21). In this case, the multiplier θ_1 is given by (24) and is strictly positive with μ and ν given respectively by (23) and (22).*

According to the above condition, it appears that the small economy can converge to its limit-size $\bar{S} = \frac{1}{2}$ only if it offers the same net benefit to foreign investors as in the rest of the world ($\bar{a} - \bar{T} = a^* - T^*$). When $a^* - T^* > 0$ (case a), the home country may converge to the upper bound value $\bar{S} = \frac{1}{2}$ if its productivity in providing infrastructures, ζ , is high enough ($\zeta > 2(\delta - \xi) > 0$). It follows that the home country's size may move to $\frac{1}{2}$ by equating $\bar{a} - \bar{T}$ to $a^* - T^*$, without being hindered by the large economy.

If the net benefit to investors offered by the large economy is negative ($a^* - T^* < 0$)(case b)), the initially small country could end up in the situation in which it attracts all the world's capital. This does however not occur if $\delta > \xi + \frac{\zeta}{2}$ (see case (b) of Proposition 4)²¹. In other words, the home country's size may converge to $\bar{S} = \frac{1}{2}$, if

²¹If the depreciation rate and the user fee of public infrastructure are equal to zero ($\delta = \xi = 0$), the small country will never be able to attract all the world's capital.

$a^* - T^* < 0$ under the condition that the productivity factor ζ is bounded from above by $2(\delta - \xi) > 0$. It is however not realistic to assume that the large economy will remain passive and will not try to restore its attractiveness by reversing the sign of $a^* - T^*$.

4 Conclusion

Many authors recognize that small countries dramatically lack (quantitatively and qualitatively) fundamental productive resources. These deficiencies appear especially in the form of limited productive capital, entrepreneurs and human capital. For simplicity, we merged these three types of production factors in one entity by assuming that capital owners, firm owners and workers are the same individuals bearing different mobility preferences.

The particular situation we just featured contains a potential risk of collapse in the small economy. One way to escape this danger is to set up public policy strategies aimed at attracting foreign investments. These policies can be realized through different channels. The instruments which decision-makers are supposed to use in our paper, are tax instruments and the provision of public infrastructures enhancing private producers' productivity. We focused on the strategic choices available to a small economy given the policy choices of the rest of the world. In other words, we did not model a game in which the large economy would react to the small country's decisions. This assumption was justified by the fact that the home jurisdiction is supposed to be so small that it does induce any reaction from the large country. This assumption risks to become fragile if the initially small country is able to continually attract firms (and workers) from abroad. Such a possible occurrence lead us to be careful in the interpretation of the model's steady states. More precisely, we had to exclude cases which otherwise would have contradicted the small country assumption. In a future work, however, our framework should be able to model a non cooperative game between the small and the both jurisdictions. Accordingly, it would be of a great interest to show how the new modelling would change the likely occurrence of the small country's eco-

conomic collapse.

Using an inter-temporal framework we characterize in our model the optimal strategic taxation path chosen by the policy makers in the small country. Applying dynamic optimization techniques, we derive a set of steady states and their stability conditions. One of three types of steady states may emerge. A first one in which the size of the initially small country attracts sufficiently external capital to grow as big as the foreign economy. A second equilibrium in which the small economy is no more economically viable because it loses all its productive capital. Finally, there may occur an intermediate situation in which the domestic economy survives while remaining small. In this scenario, there exists at least one intermediate steady state which exhibits saddle point stability. If the small economy does not undergo the optimal path leading to one of these intermediate equilibria it may converge to the worst case. The survival of a small economy is thus an important public policy issue which implies an appropriate choice of initial conditions and a dynamic update of the tax policy.

Appendix

Proof of Proposition 1

Define the current value of the Hamiltonian corresponding to the underlying economy

$$\mathcal{H}(T, S, a, \mu, \nu) = \left[ST - \frac{\alpha}{2}(ST)^2 - \frac{\beta a^2}{2} \right] - \mu \rho \left(b^*(t) - \frac{a(t) - T(t)}{k} \right) + \nu [(\xi - \delta)a(t) + \zeta ST]$$

and the following Lagrangian which accounts for the state constraints²² of the model

$$\mathcal{L}(T, S, a, \mu, \nu, \theta_1, \theta_2) = \mathcal{H}(T, S, a, \mu, \nu) + \theta_1 \rho \left(b^*(t) - \frac{a(t) - T(t)}{k} \right) - \theta_2 \rho \left(b^*(t) - \frac{a(t) - T(t)}{k} \right).$$

It is easy to see that $\mathcal{H}(T, S, a, \mu, \nu)$ is concave with respect to the state variables S and a . Hence the first order conditions are necessary and sufficient for the existence

²²See, for example, Chiang, page 301-302.

of an optimum. Deriving the first order conditions from the Hamiltonian we obtain (8) with respect to T , while we get (9) and (10) with respect to both state variables. The multipliers of the state boundary constraints check (11) and (12). That finishes the proof. \square

Proof of Proposition 2

We first state the existence of possible interior steady state(s) in addition to that given in Proposition 2. Then we give the proof.

Proposition 5 *The following additional steady state(s) may appear.*

(II.1) *If $a^* - T^* = -\frac{kr(\delta-\xi)}{4\zeta\rho} (< 0)$, there is a further interior steady state specified by*

$$\widehat{T}_1 = \frac{kr(\delta - \xi)}{2\zeta\rho}, \quad \widehat{a}_1 = (a^* - T^*) + \widehat{T}_1, \quad \widehat{S}_1 = \frac{\rho\widehat{T}_1}{kr} \quad (26)$$

*and two costate variables*²³

$$\widehat{\nu}_1 = \frac{\alpha\widehat{S}_1^2\widehat{T}_1 + \beta\widehat{a}_1 - \widehat{S}_1}{\zeta\widehat{S}_1 - r - \delta + \xi}, \quad \widehat{\mu}_1 = \frac{k[(r + \delta - \xi)\widehat{\nu}_1] + \beta\widehat{a}_1}{\rho}. \quad (27)$$

(II.2) *If $a^* - T^* = 0$, the additional steady state is*

$$\widehat{T}_2 = \frac{kr(\delta - \xi)}{\zeta\rho} \quad (28)$$

and we obtain the remaining steady state variables by replacing the subscript 1 by 2 in (26) and (27).

(II.3) *If $a^* - T^* > 0$, the second steady state is specified by*

$$\widehat{T}_3 = \frac{kr}{2\zeta\rho} \left[(\delta - \xi) + \sqrt{(\delta - \xi)^2 + \frac{4\zeta\rho}{kr}(\delta - \xi)(a^* - T^*)} \right] \quad (29)$$

²³The condition $\zeta\widehat{S}_1 - r - \delta + \xi \neq 0$ must hold.

and the others are the same as in (26) and (27) by replacing the subscript 1 to 3.

(II.4) If $-\frac{kr(\delta-\xi)}{4\zeta\rho} < a^* - T^* < 0$, there are another two interior steady states where

$$\widehat{T}_{4,5} = \frac{kr}{2\zeta\rho} \left[(\delta - \xi) \pm \sqrt{(\delta - \xi)^2 + \frac{4\zeta\rho}{kr}(\delta - \xi)(a^* - T^*)} \right] \quad (30)$$

and the remaining the others are the same as in (26) and (27) by replacing the subscript 1 by 4 and 5.

Proof.

At the interior steady state, $\theta_1 = 0, \theta_2 = 0$ and we can rewrite the first order condition as follows

$$\left\{ \begin{array}{l} T = \frac{S - \frac{\rho\mu}{k} + \zeta S\nu}{\alpha S^2} \\ \dot{S} = -\rho(b^* - \frac{a-T}{k}), \\ \dot{a} = (\xi - \delta)a + \zeta ST, \\ \dot{\mu} = r\mu - \zeta T\nu - (1 - \alpha ST)T, \\ \dot{\nu} = (r + \delta - \xi)\nu + \beta a - \frac{\rho}{k}\mu. \end{array} \right. \quad (31)$$

We rewrite the first equation as follows

$$\frac{\rho\mu}{kS} = 1 + \zeta\nu - \alpha ST. \quad (32)$$

Substituting (32) into the 3rd equation and arranging leads to the

$$\dot{\mu} = r\mu - \frac{\rho T\mu}{kS}.$$

Hence, $\dot{\mu} = 0$ leads to two cases: $\widehat{\mu} = 0$ or $r = \frac{\rho\widehat{T}}{k\widehat{S}}$.

We consequently have two groups of steady states: general one $\widehat{\mu} = 0$ or/and special one where $r = \frac{\rho\widehat{T}}{k\widehat{S}}$.

(I) $\widehat{\mu} = 0$.

It is easy to check that the interior steady states are given by (13) and (16). To determine their stability, we consider the corresponding Jacobian

$$J_I = \begin{pmatrix} \frac{\rho}{k\alpha} \frac{1+\zeta\widehat{\nu}}{\widehat{S}^2} & \frac{\rho}{k} & \frac{\rho^2}{\alpha k^2 \widehat{S}^2} & -\frac{\rho\zeta}{\alpha k \widehat{S}} \\ 0 & \xi - \delta & -\frac{\zeta\rho}{\alpha k \widehat{S}} & \frac{\zeta^2}{\alpha} \\ 0 & 0 & -\frac{\rho}{k\alpha} \frac{1+\zeta\widehat{\nu}}{\widehat{S}^2} + r & 0 \\ 0 & \beta & -\frac{\rho}{k} & r + \delta - \xi \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\rho\widehat{T}}{k\widehat{S}} & \frac{\rho}{k} & \frac{\rho^2}{\alpha k^2 \widehat{S}^2} & -\frac{\rho\zeta}{\alpha k \widehat{S}} \\ 0 & \xi - \delta & -\frac{\zeta\rho}{\alpha k \widehat{S}} & \frac{\zeta^2}{\alpha} \\ 0 & 0 & -\frac{\rho\widehat{T}}{k\widehat{S}} + r & 0 \\ 0 & \beta & -\frac{\rho}{k} & r + \delta - \xi \end{pmatrix}.$$

It is easy to show that the eigenvalues of the Jacobian are given by

$$e_1 = \frac{\rho\widehat{T}}{k\widehat{S}} > 0, \quad e_2 = r - \frac{\rho\widehat{T}}{k\widehat{S}} > 0 \text{ (or } < 0),$$

$$e_{3,4} = \frac{r}{2} \pm \frac{1}{2} \sqrt{r^2 + 4 \left[\frac{\beta\zeta^2}{\alpha} + (r + \delta - \xi)(\delta - \xi) \right]}.$$

Hence, $e_3 > 0$ and $e_4 < 0$, which guarantees one dimensional convergence to the steady state. The other part of the convergence depends on e_2 is negative or not, that is, the relation of r with respect to the other parameters and exogenous variables.

(II) $r = \frac{\rho\widehat{T}}{k\widehat{S}}$.

In this case, we have $S = \frac{\rho T}{kr}$. $\dot{S} = 0$ leads to $a = (a^* - T^*) + T$ and $\dot{a} = 0$ gives $\zeta ST = (\delta - \xi)a$. Combining these conditions, we obtain

$$\frac{\zeta\rho}{kr} T^2 - (\delta - \xi)T - (\delta - \xi)(a^* - T^*) = 0,$$

which yields to two roots

$$T = \frac{kr}{2\zeta\rho} \left[(\delta - \xi) \pm \sqrt{(\delta - \xi)^2 + \frac{4\zeta\rho}{kr}(\delta - \xi)(a^* - T^*)} \right]$$

and if $\Lambda = (\delta - \xi)^2 + \frac{4\zeta\rho}{kr}(\delta - \xi)(a^* - T^*) > 0$, both roots are real. Furthermore, depending on Λ is larger or smaller than $(\delta - \xi)^2$, we have different conditions which leads to positive T s and which serve as the other steady states. \square

Proof of Proposition 4

In the upper-corner solution case, $S = \frac{1}{2}$ and, hence, $\theta_2 = 0$ and $\theta_1 > 0$. According to the complementary slackness conditions, we must have $\bar{a} - \bar{T} = a^* - T^*$, that is, $\bar{T} = \bar{a} - (a^* - T^*)$.

From $\dot{a} = 0$, we obtain

$$\bar{a} = \frac{\frac{\zeta}{2}(a^* - T^*)}{\xi + \frac{\zeta}{2} - \delta}. \quad (33)$$

The solution \bar{a} is positive if and only if $\delta > \xi + \frac{\zeta}{2}$ and $a^* < T^*$, or $(\xi <) \delta < \xi + \frac{\zeta}{2}$ and $a^* > T^*$.

The condition $\dot{\mu} = 0$ leads to

$$r\bar{\mu} = \frac{\bar{T}}{2} \frac{\rho}{k} (\bar{\mu} - \bar{\theta}_1) \quad (34)$$

or

$$\bar{\theta}_1 = \left(\frac{2kr}{\rho\bar{T}} - 1 \right) \bar{\mu}. \quad (35)$$

Similarly, $\dot{\nu} = 0$ leads to

$$(r + \delta - \xi)\bar{\nu} = \frac{\rho}{k}(\bar{\mu} - \bar{\theta}_1) - \beta\bar{a}. \quad (36)$$

On the other hand, (8) can be rewritten as

$$\alpha\bar{S}\bar{T} = 1 - \frac{\rho\bar{\mu} - \theta_1}{k\bar{S}} + \zeta\bar{\nu} \quad (37)$$

which gives

$$-\frac{\rho}{k}(\bar{\mu} - \bar{\theta}_1) = \frac{\alpha\bar{T}}{4} - \frac{1}{2} - \frac{\zeta\bar{\nu}}{2}.$$

Combining with (36), it follows

$$\bar{\nu} = \frac{1}{r + \delta - \xi - \frac{\zeta}{2}} \left[\frac{1}{2} - \beta\bar{a} - \frac{\alpha\bar{T}}{4} \right] \quad (38)$$

and

$$\bar{\mu} = \frac{\bar{T}}{r} \left(1 + \zeta\bar{\nu} - \frac{\alpha\bar{T}}{2} \right). \quad (39)$$

Hence, we obtain a complete solution for the steady state $\bar{S} = \frac{1}{2}$. The above solutions are meaningful if and only if $\bar{\theta}_1 > 0$. \square

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