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Prices for superstars can flatten out

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Abstract

We reconsider Rosen's economics of superstars model establishing that the relationship between price and quality could only be convexified, not concavified. We show that this result is false and explain why. The concavity or convexity of the relationship is related to the way multiple types of consumers (unequal connoisseurship) and suppliers (unequal talent) match. With convexity because of a stronger fixed cost independent of quality, non-connoisseurs turn towards high priced, highly gifted superstars and symmetrically, connoisseurs, who prefer low increase in prices of quality, turn towards the low quality (and the bottom of talent). When the relationship is concave, the matching works reversely. Connoisseurs go towards high talents whose price of quality flattens out and non-connoisseurs stop at lower prices, lower talents because the marginal pricing of quality is strong. The global shape of the price to quality relationship (concave or convex) is determined by market clearing conditions and more crucially by the distribution of agents on both sides. If non-connoisseurs are relatively more numerous, that may increase the star phenomenon and convexity of price may occur. If conditions for convexity are satisfied, non-connoisseurs pile up towards the top of talent and convexity is re-unforced then. On the contrary, prices of quality may be concave when less numerous connoisseurs go to the top leading prices to flatten out then.

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Prices for superstars can flatten out *†

Abstract

We reconsider Rosen's economics of superstars model establishing, in the case of single type consumers, a constant marginal price of quality and a convex relationship between earnings and talent. The author conjectured that, in the case of multiple types consumers, the bias toward a superstars' high revenue could only be stronger because the relationship between price and quality could only be convexified, not concavified. We show that this conjecture is false and explain why. The concavity or convexity of the relationship is related to the way multiple types of consumers and suppliers match. Consumers with a poor knowledge have heavier fixed costs (search costs : they need more time to know the criterias for assessing quality, for instance) independently of the quality they have to pay for on the market. Non-connoisseurs then, are less reluctant when prices increase strongly and they prefer location in the catalog where marginal appreciation of quality is high. With convexity, they turn towards superstars. Symetrically, convexity encourages connoisseurs, who prefer low increase in prices of quality, to turn towards the low quality (and the bottom of talent). When the relationship is concave, for the same reason, the matching works reversely : connoisseurs go towards high talents whose price of quality flatten out. Non connoisseurs stop at lower price, lower talents because the marginal pricing of quality is strong. The global shape of the price to quality relationship (concave or convex) is determined by market clearing conditions and more crucially by the distribution of agents on both sides. If non-connoisseurs are relatively more numerous, that may increase the star phenomenon and convexification of price may occur. If conditions for convexification are satisfied, non-connoisseurs pile up towards the top of talent and convexification is re-inforced then. On the contrary, prices may concavified when less numerous connoisseurs go to the top and prices flatten out then.
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Up to now, the literature on superstars has principally been concerned with which essential mechanism enables the most strongly gifted few to gain enormous rewards, combined with the ability to attract large audiences and fix a high price for talent differentials[1] . On prices, Rosen's seminal work conjectured that assuming multiple types in consumers' characteristics was likely to convexify the relationship between price and quality and to make the superstar phenomenon stand out in comparison to the single type case. We question this result and show that it is precisely the contrary that is likely to happen. Thus there is no oxymoron in the case of very highly talented artists (or sportsmen, designers, scientists) applying at equilibrium a flat price differential to their services compared to those of less gifted sellers. Indeed as a starting point, as stressed by Rosen, when all buyers are the same, a differential in price must make all sellers indifferent to consumers, including both the extreme top and the extreme bottom of the distribution of talent. Price then becomes the only factor of cost differentiation among sellers, and a non-degenerate distribution of talent requires that highly differentiated prices discourage the massification of all consumers towards the most-talented sellers. When buyers are not the same, if they differ in connoisseurship for instance, the need for price to perform this function is not necessarily the same, depending on how distributions of talent matches with distribution of connoisseurship[2] differentiates.

Evidence of such flat prices for high talents on a variety of markets can be found in many fields where connoisseurship is an important differentiating factor among amateurs or collectors : non-competitive sports, baroque chamber music, etching, engraving. Picasso, Dali and Miro, for instance, were known to provide the engraving market with a very large number of specimens to which they did not grant great technical care and sold them consistently to connoisseurs at rather low prices, considering their reputation... All precursors' markets have the same overall structure : connoisseurs are too few and thus precursors cannot ask them for too high a price. At the beginning of Art Brut, for instance, Dubuffet, although already considered a great master, was mainly in demand

[1]After Rosen, many authors sought new requirements for superstars to emerge, including either supply conditions that have a disproportionate effect on the best workers, as in Borghans and *alii* [4] and in Boldrin and *alii* [3], or untypical models of choice or preference that do not even require workers to be unequally gifted, as in Chung and *alii* [5] and in Adler [1], [2].

[2]Other characteristics could be considered in the same way as income, tastes, ...etc. To illustrate income differentiation, think that everyone knows Guernica and many other works of Picasso, although only one tiny fringe of the population can afford to buy a work of Picasso or *really* visit the Prado Museum in Madrid. Similarly, high talented participants can be restrictively demanded by a fringe of connoisseurs in a variety of markets, even if their reputation is great with everyone.

from a small fringe of well-informed amateurs, and his work was not yet commanding very high prices. Other examples of such markets where connoisseurs go to the top can be found when the fringe of high talented artists is very narrow (like in the case of precursors).

This paper shows that switching from the single type case to the multiple types case may also *concavify* the relation between price and quality instead of only convexifying it, as Rosen asserted. Section two and three recall the main features and results of Rosen's model on demand side; section four gives the characterization of an equilibrium similar to Rosen's and offers one example with a concave relationship when consumers are multiple types.

1 Economics for superstars

1.1 Demand side in Rosen modelling

We first recall the main features of modelling by Rosen of the economics for superstars. He assumes that one consumer is equipped with a "home production" function that allows him to reach the level y of a composite good by combining the quantiy n and the quality z that is offered on the market. Assuming that, the model implicitely constrains consumer to select only one level of quality. He assumes furthermore a constant elasticity of substitution in the production of y, such that $y = nz$. Utility thus follows as :

$$u(x, y) = u(x, nz), \tag{1}$$

where x denotes the numerary good. Consumers overcome two types of costs. Each one of the n units incurs a relative market price $p(z)$ which is assumed to depend on the quality z, plus a fixed[3] cost in time. Before buying one unit of the good, each consumer needs an incompressible time $t \in [\underline{t}, \bar{t}]$ depending on his or her connoisseurship level[4]. This fixed cost enters the constraint in time : $nt + t_w \leq T$, where T is the total time available, and t_w the time devoted to work. The monetary resource constraint is : $x + p(z)n \leq wt_w$, where w is the wage[5]. The n units result in a forgone earning wtn and both constraints

[3] "fixed" means "not depending on quality z".

[4] Highest knowledge levels need less time per unit n embodied in y. Connoisseurs are closer from $\underline{t}$ and non-connoisseurs closer from $\bar{t}$.

[5] As told by Rosen, the model is not perfectly general and take poorly into account the preference for variety. A more general model would assume a utility function $u\big(x, \int_z n(i)idi\big)$. The monetary constraint would be $px + \int_z p(i)n(i)di \leq wt_w$ and consumers would have to control $n(i)$ among all i. This is not

result in a global constraint :

$$x + (p(z) + s)n \leq wT, \tag{2}$$

the total time T valued at its monetary value is allocated to the numerary good x and to the global spendings on art whose every unit incurs a monetary cost $p(z)$ and a forgone earning cost in time $s = wt$. Consumers can be indexed according to this cost $s \in [w\underline{t}, w\overline{t}]$ whose the opposite $-s$ can be interpreted as an index of the knowledge they possess on y.

1.2 Prices catalog

It is not perfectly clear how the modelling adopted by Rosen tackles the relationship between the market prices and the quality z which is a rather abstract and not mesurable notion; in addition, it is not obvious wether this is an endogeneous or exogeneous variable in his model. We propose to disentangle this by assuming that nor consumers neither suppliers observe z. Instead we make the more realistic assumption that both supply and demand sides meet on the market in focusing on prices that are not the same for all authors or supplyers. Names of authors are signalled within catalogs[6] and can been ranked according to their prices. Artists with the same prices are necessarily considered to have the same level of intrinsic talent $q \in [\underline{q}, \overline{q}]$. For instance, each page q of the ranked catalogue gather all authors with same price, "very small" increment of prices can be assumed from page to page if the number of pages is "very big", and thus prices can be written as following a function of pages, or equivalently talent, $\varphi(q)$. Considering those prices, all consumer or producers make beliefs on the quality z expected for all q, such that expected quality can also be assumed to follow a function of page or talent $z^e = \psi(q)$. Quality has to be measurable along one unidimensional criteria. This could be the probability that an event, an exhibition of a film or a show, will be considered

possible here, maybe because of a special form of indivisibility which enforces consumers to choose only one level of quality, except in the indifference case. This indivisibility is equivalent to a constraint that enforce the quantity consumed to be the same $n_i = n$, for all i. We can understand this indivisibility as induced in short terms context. In case somebody needs a lawyer for a divorce, for instance, one does not pick up more than one quality on the market and just look to the quantity purchased of the unique supplier choosen. In a more general long term context, he or she could buy the complete set of quality with the best quantity n_i of each level i.

[6] This assumption is consistent with the empirical evidence on dealers' practice in rarely displaying prices alone. W.D. Grampp [7] stresses that in the case of paintings : "Attribution matters : Imagine how a dealer would fare if he alone in the market and none of his competitors did not provide information about the painting he offered for sale: no name, no title, no provenance... nothing but the price".

positively, after attending it, for instance[7]. Equipped with those beliefs, depending on the fixed cost in time s incured, he or she demands $n(\varphi(q), \psi(q), s)$ picked up at the page or talent q, that fits optimality conditions in the catalog $\varphi(q)$. On supply side, sellers q are assumed to be price-takers[8], and to choose to offer $m(\varphi(q))$ in achieving the maximum of profit affordable with his or her level $\varphi(q)$ in the catalog. In addition, the extrinsic quality fullfilled by an author of level q, after he or she have sold $m(\varphi(q)$, is given by a technical function $h(q, m(\varphi(q))$[9].
Assuming perfect foresights by :

$$\psi(q) = h(q, m(\varphi(q))), \tag{3}$$

the catalog $\varphi(q)$ is the set of prices that solves the clearing market conditions that make zero excess demands for all q, consistently with (3). *"The marriage of buyers s to sellers q, including the assignment of audiences to performers, of students to textbooks, patients to doctors, and so forth"*[10] is simply resulting from the choice of his or her best page or talent q by every consumer s in equilibrium.

2 Rosen's results and conjectures

Rosen's paper results splits in two folds. In a first case, it is assumed a single type consumer whose fixed cost is set at a given level $\tilde{s} = w\tilde{t}$, and in a second case, it is assumed instead that consumers are of multiple types that belong to a continuum $s \in [w\underline{t}, w\overline{t}]$.

Single type consumers In case consumer is one type, it is clear that the only possible outcome of the market with multiple types producers q is a catalog $\varphi(q)$ that leads to indifference among all of them. This results trivially in $\varphi'(q)/\psi'(q) = (\varphi(q) + s)/\psi(q)$ equal to a constant υ and the quality price relationship in the catalogue $\varphi(q)$ is linear[11] :

$$\varphi(q) = \upsilon\psi(q) - \tilde{s}. \tag{4}$$

[7]In other areas, this could be the probability of success for some purpose, conduct a successful operation to an end for a surgeon to win a trial for a lawyer, and so on.

[8]Unlike Borghans and Groot [4], we depart from Rosen's assumptions and assumed perfect competition. In the case of the arts, Filer [6] asserts that *"there is no evidence that markets for most art forms are particularly concentrated"* and obviously the "superstars concentration" effect is alleviated in the multiple types case.

[9]Following Rosen's assumptions, $\partial h(q, m)/\partial q > 0$ and $\partial h(q, m)/\partial m < 0$.

[10]Rosen [8] p.846.

[11]This is the equation $p(z) = \upsilon z - \tilde{s}$ in the paper from Rosen p.848.

Multiple types consumers Rosen was wrong in considering that only convex relationship was possible between quality and price in the multiple types consumers case[12]. We can explain that by entering into details of his modelization and by changing it slightly. After having substituted x from (2) in (1) we can easily switch from Rosen's modelling to ours in replacing $p(z)$ by $\varphi(q)$ and z by $\psi(q)$. Utility is then a function of n and q only : $v(n,q) = u(wT - (\varphi(q)+s)n, n\psi(q))$. First order conditions $v'_n = \partial v(n,q)/\partial n = 0$ and $v'_q = \partial v(n,q)/\partial q = 0$ imply :

$$\frac{u'_y}{u'_x} = \frac{\varphi(q)+s}{\psi(q)} = \frac{\varphi'(q)}{\psi'(q)}. \tag{5}$$

The equalization of marginal price of quality to average total price on the left of (5) can be misleading. It *does not* come from the direct comparison of both value, as could do a profit maximizer, but rather from the cross property of the best n^* and of the best q^*. The former is obtained by the usual condition in equalizing $MRS_{x/y}$ to the marginal *quantitative* cost, *i.e.* marginal utility of y must be equal to the average unit cost for a given quality, evaluated in terms of marginal value of x, $u'_y = [(\varphi(q)+s)/\psi(q)].u'_x$ (see fig. 1b). The later equalizes $MRS_{x/y}$ to marginal price of quality $\varphi'(q)/\psi'(q)$ that is the marginal *qualitative* cost for a given quantity, $u'_y = [\varphi'(q)/\psi'(q)].u'_x$ (see thicklines on fig. 1a). A key point is that consumer is taking into account *the rise and not only the level* of the cost of quality.

[12] Rosen [8] section IV p.854 and [9] p.461.

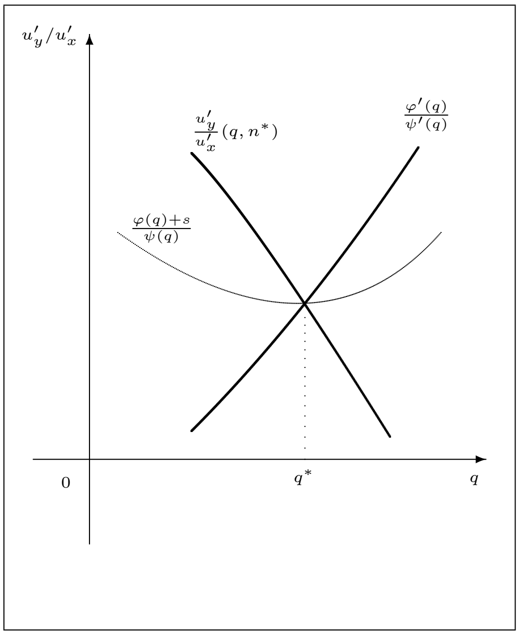

Fig. 1a

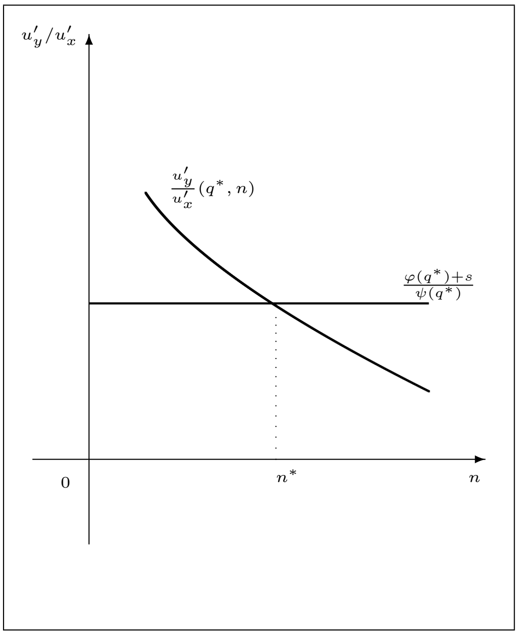

Fig. 1b

Therefore the marginal price may be decreasing, as in q_2^* in fig.2.

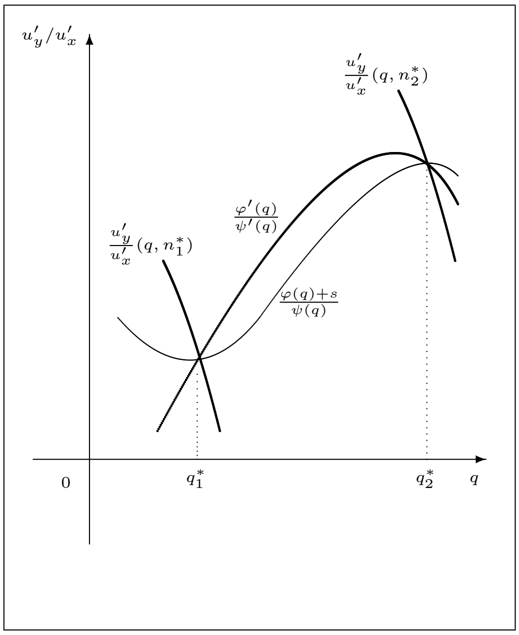

Fig. 2a

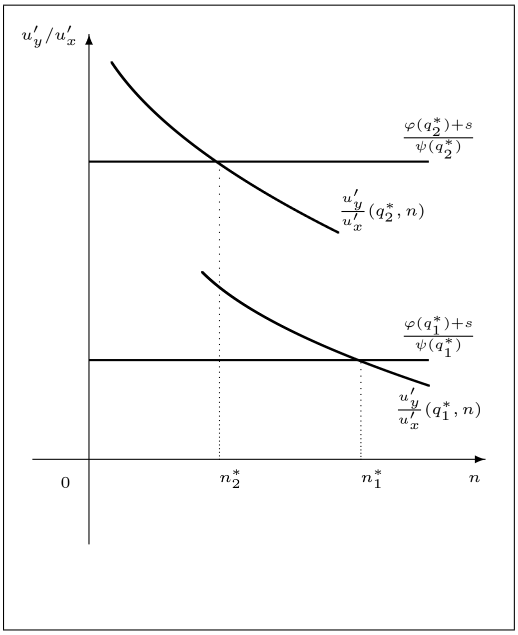

Fig. 2b

One have to pay a close attention to second order conditions. The slope of $u'_y(q,n^*)/u'_x(q,n^*)$ has to be negatively steaper than the one of $\varphi'(q)/\psi'(q)$. That is obviously satisfied when $\varphi'(q)/\psi'(q) > 0$, in q_1^* in Fig.2a, for instance, but that has to be checked more carefully when $\varphi'(q)/\psi'(q) < 0$ as in q_2^* in Fig. 2a [13].

The choice of q^* by one s follows naturally from the equalization (5) which can be told another way: s buys n^* and selects a page q^* when his or her total price elasticity of quality [14] is equal to one $\xi(q^*) = 1$. Symetrically, artists located at page q in the catalog are demanded by $s = (\varepsilon(q) - 1)\varphi(q)$, where $\varepsilon(q)$ is the elasticity quality of price[15] at the location q. For instance in Fig.3, s_1 chooses C_1, indifferently any point of (A, B) and C_1', when quality price relationship follows respectively the thicklines on the left, middle and right in the catalogue $\varphi(q)$.

[13]In case of a Cobb-Douglas utility, for instance, $u(x, nz) = (1-\gamma)\log x + \gamma \log nz$, the second order condition requires :

$$\frac{\psi''(q)\varphi'(q) - \varphi''(q)\psi'(q)}{(\psi'(q))^2} < \frac{1}{(1-\gamma)}\frac{\varphi'(q)}{\psi(q)},$$

wich is obviously satisfied when $\psi''(q)\varphi'(q) - \varphi''(q)\psi'(q) < 0$, in case of a convex relationship betwen price and quality, as in q_1^* in the figure 2a. This can be satisfied as well in the concave case as in q_2^* in figure 2a.

[14]$\xi(q) \equiv [(d\psi(q)/dq)/(d\varphi(q)/dq)]\,(\varphi(q) + s)/\psi(q) = [\psi'(q)/\varphi'(q)]\,(\varphi(q) + s)/\psi(q)$.

[15]$\varepsilon(q) \equiv [d\varphi(q)/dq)/d\psi(q)/dq]\,\psi(q)/\varphi(q) = [\varphi'(q)/\psi'(q)]\,\psi(q)/\varphi(q)$. It implies of course that $\xi\varepsilon = (\varphi(q) + s)/\varphi(q)$.

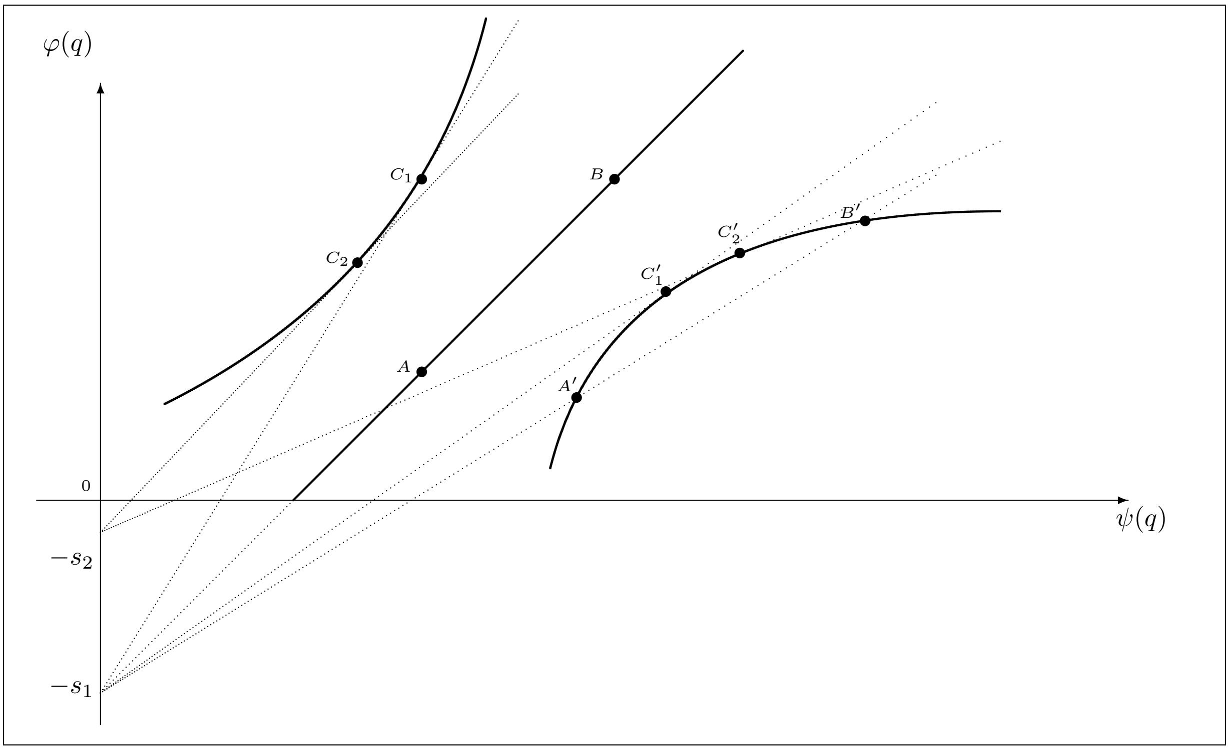

Figure 3 : Convex, linear and concave price to quality relationship in catalogs $\varphi(q)$.

If there is only one type of consumers s_1, it is obvious that (4) has to be satisfied then and that only a linear (A, B) shape can occur such that (5) is true for more than one page q and all consumers may buy at all pages indifferently. Otherwise the relationship $\varphi(q)$ to $\psi(q)$ cannot be linear, when there is more than one type of consumer. Connoisseurship indexed by $-s$ then increases incentive to take into account marginal appreciation of quality. As a consequence of that, connoisseurs with higher index value $-s_2$ match with artists located in pages where rise in price is lower that is : with lower quality C_2 when relationship is convex on the left and with higher quality C'_2 when relationship is concave on the right in Fig.3. The reverse is true, non connoisseurs with lower value $-s_1$ go to location where the relationship has a higher slope towards the top of quality C_1 or towards the bottom C'_1. Whether we are in a situation or the other depends on the market clearing conditions, and more crucially on the distributions of agents on both sides.

Rosen was mistaken by thinking that because linear relationship (A, B) were leading to indifference between all pages, then all straight lines starting from $-s_i$, like $(-s_1, C_1)$, $(-s_2, C'_2)$ or $(-s_1, A', B')$ for instance, should be describing indifference, whatever be the shape of the quality price relationship[16]. This is obviously wrong. Consider for instance A' and B' when the thickline on the right describes the quality price relationship in $\varphi(q)$: it is clear that indifference is impossible between them since (5) is not satisfied neither at A' nor at B'. If we consider now these two same points, knowing that the relationship is given by thickline *on the left*, the question is meaningless, since we know nothing about the value of $\varphi'(q)/\psi'(q)$ in A' nor in B'. The key point is that indifference can never be described only between two *points* of the map $(\varphi(q), \psi(q))$, but also by considering *directions*, due to the consideration of the slope $\varphi'(q)/\psi'(q)$ in (5). Except in case of linear relationship, thus we need more than a two-dimensional space to describe indifference between price and quality. Therefore, nothing prevents C_1' or C_2' to be maximising utility for s_1 and s_2 in the thickline on the right, provided that the usual second-order conditions are met at those points.

3 New results

3.1 Equilibrium

Assume price-takers competitive sellers distributed on $q \in]\underline{q}, \overline{q}]$ with density $\theta(q)$ that all have the same cost $C(m)$. Assume that buyers, equipped with (1) under constraint (2), may (or may not) differ according to $s \in [\underline{s}, \overline{s}]$ with density $\alpha(s)$. An equilibrium is a set $\left\{\varphi(q), n(\psi(q), \varphi(q), s), x(\psi(q), \varphi(q), s), m(\varphi(q)), \psi(q)\right\}_{q\in[\underline{q},\overline{q}], s\in[\underline{s},\overline{s}]}$ satisfying : 1) the supply condition leading price-takers sellers to adjust quantities $m(\varphi(q))$ until they equalize price to marginal cost $\varphi(q) = dC(m)/dm$, for all q; 2) demand conditions leading to consumptions $n(\psi(q), \varphi(q), s)$ and $x(\psi(q), \varphi(q), s)$ for all s; 3) The marriage of q and s by equalizing $s = \varsigma(q)$; 4) the perfect foresight condition : $\psi(q) = h(q, m(\varphi(q)))$, for all q; 5) market clearing conditions that can be distinguished from each other according to two cases. In the single type case, all buyers have the same $s = \tilde{s}$, the market clears when $\int_{\underline{q}}^{\overline{q}} \beta(v) n(\psi(v), \varphi(v), \tilde{s}) dv = \int_{\underline{q}}^{\overline{q}} \theta(v) m(\varphi(v)) dv$, where $\beta(q)$ is the density of population of buyers that can be supplied by sellers q for a given $\varphi(q)$. In the multiple type case, the market clears when $\int_{\underline{s}}^{\varsigma(q)} \alpha(u) n(\psi(\varsigma^{-1}(u)), \varphi(\varsigma^{-1}(u)), u) du = \int_{\underline{q}}^{q} \theta(v) m(\varphi(v)) dv$ for all $q \in [\underline{q}, \overline{q}]$ if $\varsigma(\underline{q}) < \overline{s}$ and $q \in [\varsigma^{-1}(\overline{s}), \overline{q}]$, otherwise in the decreasing case $\varsigma'(q) < 0$, and

[16] Were it true, C_1' would thus be the *minimum*, not maximum, of utility s_1 would reach in the catalog, because any straight line on the left would describe another indifference *locus* with higher quality for same prices. Within concave relationship, consumers, regardingless of their connoissorship $-s$, would ask only for the highest level of quality available, which is not consistent with multiple levels of talent.

symetrically in the increasing case $\varsigma'(q) > 0$.

3.2 One simple example : uniform distributions of agents on both sides

Equipped with $u(x, nz) = (1-\gamma)\log x + \gamma \log n + \gamma \log z$, every buyer consumes $n(\varphi(q), s) = \gamma wT/(\varphi(q)+s)$ and $x = (1-\gamma)wT$. On supply side, $C(m) \equiv (c_1/2)m^2 - c_2 m$ leads sellers q to produce $m(\varphi(q)) = (\varphi(q) + c_2)/c_1$. Define extrinsic quality as $h(q, m) \equiv (q+1)e^{-m}$, the assignment function thus simplifies to $\varsigma(q) = 1/\Big\{1/[(q+1)\varphi'(q)] - 1/c_1\Big\} - \varphi(q)$. A numerical example is given below for $c_1/100 = c_2 = wT = 10\gamma = 1$.

In the single type consumer case, all consumers distribute indifferently among sellers within a distribution of monotonously increasing density $\beta(q)$. The constant marginal price of quality v in (4) depends positively on the relative number of buyers to sellers $\int_{\underline{q}}^{\overline{q}} \beta(q)dq / \int_{\underline{q}}^{\overline{q}} \theta(q)dq$. In the multiple types case, the catalog $\varphi(q)$ and expected quality $\psi(q)$ verify in equilibrium $\theta(q).\{(\varphi(q)+c_2)/c_1\} = \alpha(\varsigma(q)).\{\gamma wT/(\varphi(q)+s)\}$. Thus, using (3)and (5), we have :

$$\theta(q).(\varphi(q) + c_2)\varphi'(q)\psi(q) = \alpha(\varsigma(q)).c_1\gamma wT\psi'(q), \tag{6}$$

$$\psi(q) = (q+1)\exp[-(\varphi(q) + c_2)/c_1]. \tag{7}$$

Assume $\theta(q)/\alpha(\varsigma(q)) = 1$, for instance with an uniform distributions on $[0, 1]$ for both populations of sellers according to q and buyers (when they differ) according to s, for all q. Then (6) and (7) reduce to :

$$(\varphi(q) + c_2)\varphi'(q)\psi(q) = c_1\gamma wT\psi'(q), \tag{8}$$

$$\psi(q) = (q+1)\exp[-(\varphi(q) + c_2)/c_1]. \tag{9}$$

The solution for (8) and (9) is :

$$\varphi(q) = -(\gamma wTc_1 + c_2) + \sqrt{(\gamma wTc_1 + c_2)^2 + 2\gamma wTc_1 \log(q+1) + K},$$

and the corresponding $\psi(q)$, for a constant K.

Within the numerical case, $K = 53.28$ ensures that all sellers $q \in [0, 1]$ are matched with buyers $s \in [0, 0.97]$ by $\varsigma(q)$. Income effect exceeds substitution effect, which is concretized in $d\varsigma(q)/dq < 0$: time-saver better-informed buyers (lowest s) select the most talented sellers with highest q, at the same time reaching higher utility levels. In all locations q, they consume $n = 0,1/(q+1)\varphi'(q)) - 0,1$ and $x = 0,9$. Buyers with the highest $s \in]0.97, 1]$ do not get anything. In contrast to Rosen's results, marginal price of quality $\varphi'(q)/\psi'(q)$ decreases, leading to a concave $p(z)$, market sizes and receipts[17] are concave according to q.

3.3 More results by changing distributions of agents on both sides

We solve by numerical integration (6) and (7). Of course, the general shape and the level of the quality/price relationship in the catalog depend on the density $\theta(q)$ and $\alpha(s)$ of the distributions of agents on both sides. We show that, in the simpler case of uniform distribution on both sides, for instance, prices raise with the number of consumers and fall with the number of producers. The uniform case is interesting since it produces an analytical solution. But it remains unrealistic and restrictive. For this purpose, we turn to more a flexible formulation. We consistently consider affine cases and also an exponential density on supply side, which reflects better the scarcity of high talents. We however rely on numerical integration for the computation of the solution since the formulation does not yield analytical solution.

In all those cases, density of agents is decreasing with talent on the producer side, $\theta'(q) < 0$, and decreasing with connoisseurship $-s$ on the consumer side, $\alpha'(s) > 0$. We split the presentation in two folds :

1. linear density on both sides,

2. and linear density on consumer side and exponential density on the producer side.

3.3.1 Linear density on both sides

In a first case, density of talent follows $\theta(q) = d - c.q$, and density of consumers follows $\alpha(s) = a.s + b$ where $a \geq 0$, $b \geq 0$, $c \geq 0$ and $d \geq 0$ are parameters. A higher value of a increases the number of non-connoisseurs with high s and thus the relative scarcity of connoisseurs among all consumers $\int_0^{\overline{s}} \alpha(v)dv$. A higher value of d increases the total number of artists uniformly on all fringes of artists, from very low-gifted talent (at the

[17] Partly due to price-takers assumption.

very "first" pages of the catalog) to the highest talent located at the end (it increases also the number of pages) and c is the slope of decreasing number of higher talent in the catalog. Of course, a higher value of d increases global number producers $\int_q \theta(u)du$ without changing the relative scarcity of higher talent. And a higher value of c decreases the global number and increases the scarcity of higher talent. We set parameters a, b, c and d according to table 1, where $\mathcal{U}_1$ refers to the uniform case on both sides and $\mathcal{A}_i$, $i = 1, 2, 3$ and 4 to four different affine cases. Others parameters are set as follows : $c_1 = c_2 = 1$, $T = 10$, $w = 2$, $\gamma = 1/2$.

cases	a_i	b_i	c_i	d_i
$\mathcal{U}_1$	0	3	0	4
$\mathcal{A}_1$	1	0	1	10
$\mathcal{A}_2$	5	0	1	10
$\mathcal{A}_3$	1	0	1	5
$\mathcal{A}_4$	1	0	3	10

Recall that in case of a concave relationship, low s match with high q and reciprocally. Then we find expected results on both sides. Looking at impacts induced on the price of quality induced by changes on producer side only, we find that, with higher d, prices fall all over the catalog uniformly (see fig. 4) and that, with higher c, prices fall more loudly for low talent (see fig. 5). Looking at impacts induced by changes on consumer side only, we find also that, with higher a, prices raise for low talent and that they fall for high talent (see fig. 6).

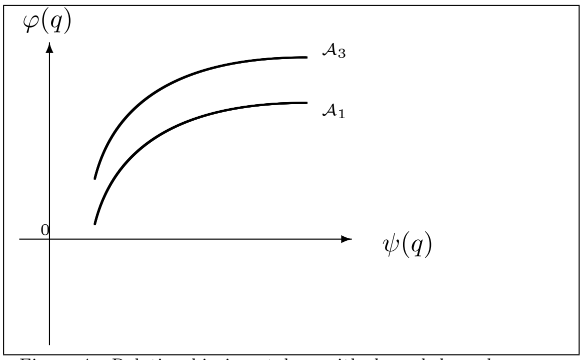

Figure 4 : Relationship in catalogs with d_1 and $d_3 < d_1$.

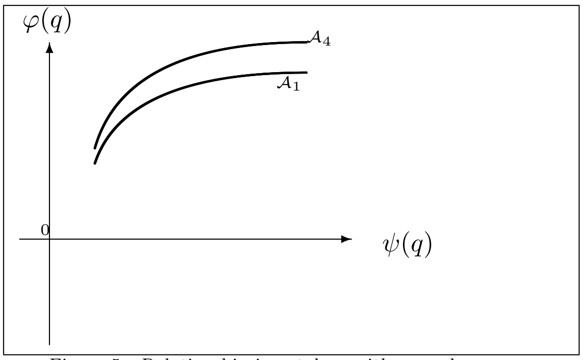

Figure 5 : Relationship in catalogs with c_1 and $c_4 > c_1$.

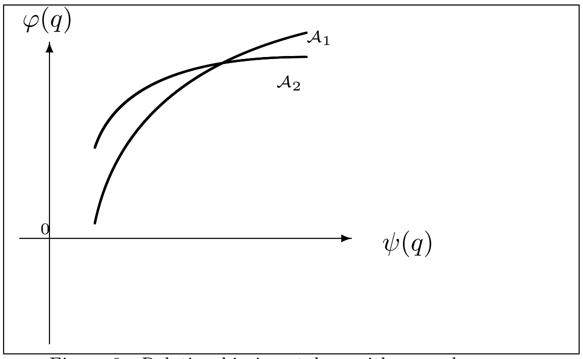

Figure 6 : Relationship in catalogs with a_1 and $a_2 > a_1$.

3.3.2 Linear density on consumer side and exponential density on the producer side

The most interesting case is the case when number of producers decreases with a constant rate along pages in the catalog $\theta'(q)/\theta(q) = -c$. Thus talent is distributed following a exponential density $\theta(q) = de^{-cq}$. With such a density, the fringe of high artist can be very tiny. Obviously, higher values of c decrease both the number of artists and the scarcity of higher talent. We keep a linear distribution of consumers $\alpha(s) = a.s + b$, as in the previous section.

cases	a_i	b_i	c_i	d_i
$\mathcal{E}_1$	1	0	1/100	1
$\mathcal{E}_2$	1	0	1/2	1
$\mathcal{E}_3$	1	0	1/2	3

We find that higher value of c, of course have an positive impact on the price but surprisingly, it changes a convex relationship in a concave relationship, so that scarcity of high talent *concavify* the price/quality relationship, instead of convexifying it. We can understand that very easily. The matching between both distributions of agent leads simply to change the way connoisseurs and non-connoisseurs turn to high or low talents. As c increases, high talents turn to match with connoisseurs wich are less numerous and the raise in price is higher for lower talents that meet the great mass of non-connoisseurs.

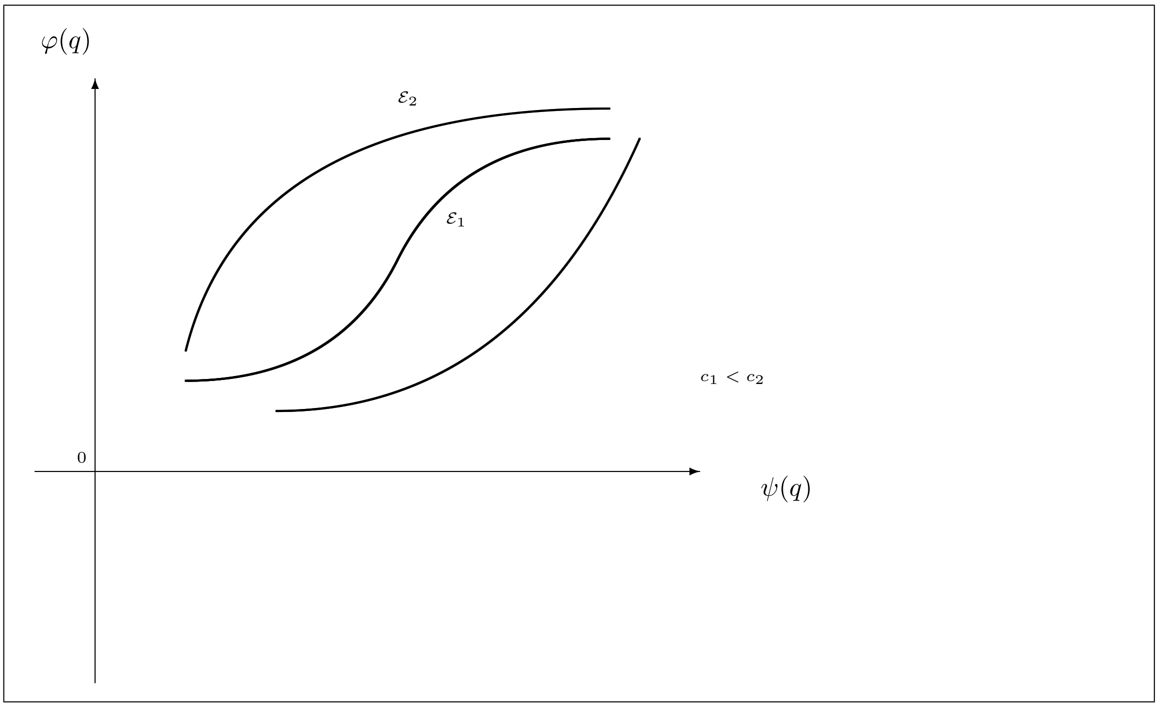

Figure 7 : Convex to concave relationship in catalogs by increasing c.

4 Conclusion

We show here that unequally talented sellers' prices may be flat at the top because of a concave relationship between price and quality when buyers are multiple types. The ability of superstars to accumulate the most frequent attributes of stardom, including large prices, rewards and markets, thus seems to be contingent on many factors about which much remains to be discovered. One of these factors is undoubtedly the demographic aspects of the equilibrium considered. First, we tackle uniform distributions on both sides (supply and demand) for obvious tractability reasons. But a great attention has also been paid in this paper to the part played by many other forms and more realistic distributions, to the way they match and thus decisively affect the main outcomes of the market. We conclude that higher scarcity in talent can lead to the *convexification* of the relation price/quality. For instance, on the market of precursors, high talented are very scarce, they meet connoissors that are not very numerous as well, and this matching may lead to a convexification of the relationship between price and quality.

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