# Submission Number: PET11-11-00207 

# Endogenous Debt Constraint, Education Choice and Income Distribution 

Yang Tang<br>Washington University in St.Louis


#### Abstract

This paper is interested to study how credit market imperfection will affect agent's education choice. Essentially, we want to investigate the following question: will agent choose not to attend college or other level of education simply because they are less abeled or because they are borrowing constrained at the credit market? Moreover, we calibrate a life-cyle model into U.S economy and perform some quantitative analysis in order to find out how people's education choice respond to different credit market condition. Results suggest that improving credit market condition quantitatively could enhance the average education attainment level and decrease the the overall income inequality in the economy. Moreover the counter-factual exercise on taxation policy suggests that taxing unskilled agent and subsidy skilled agent might be potentially beneficial in term of lowering income inequality and improve the average education attainment level.


# Endogenous Debt Constraint, Education Choice and Income Distribution 

Yang Tang<br>Washington University in St.Louis

March 9, 2011


#### Abstract

This paper is interested to study how credit market imperfection will affect agent's education choice. Essentially, we want to investigate the following question: will agent choose not to attend college or other level of education simply because they are less abeled or because they are borrowing constrained at the credit market? Moreover, we calibrate a life-cyle model into U.S economy and perform some quantitative analysis in order to find out how people's education choice respond to different credit market condition. Results suggest that improving credit market condition quantitatively could enhance the average education attainment level and decrease the the overall income inequality in the economy.


## 1 Introduction

Different people make different education choice: some people choose to attend college; some people pursue furthermore for a Master or Doctoral degree while some others may not even finish high school. Different education choice may lead to a sharp life-time income difference across all agents. The empirical evidence from U.S data also suggest that an increasing number of people choose to receive more education and the income inequality from different types of education attainment tends to become sharper and sharper.


Number of people older than 25years by level of education (thousands)


It is generally believed that innate ability is the critical element that generates these heterogeneity among agents, other potential channels may also include initial wealth, human capital accumulation technology etc. This paper mainly focus on the channel of borrowing constraint. To be more precisely, we present a model to study how credit imperfection leads to different education choice. The essential question we attempt to ask is everything else being the same, how people's education choice vary according to the magnitude of borrowing constraint. Since people receive more income from higher education, hence studying of this question will also enable us to understand the extent of income inequality arising from credit market imperfection.

Credit market imperfections are pervasive in the case of education loans due, in part to the fact that human capital does not act as collateral for loans, there is a moral hazard issue in lending to finance education. Credit market imperfection in the paper is captured by the level of endogenous credit limit, which defines how much agent can borrow against their future income in youth in order to receiving education. Endogenous debt constraint is firstly proposed by Kehoe and Levine (1993). Endogenous credit limit is crucial to our analysis, and matter significantly in explaining the macroeconomic consequences of credit constraints for education.

The mechanism throughout the paper works explicitly as follows: agents' wage income is assumed to be monotonically increasing in term of both ability and education, and higher education is supposed to be more expensive. Therefore less abeled agent may be constrained from borrowing due to the credit market imperfection, and they have to go for lower education, and this may potentially amplify the income inequality among agents. The mechanism is embedded in a three-period OLG framework where agents are heterogenous in term of innate ability, and no initial wealth is assumed for all agents. Agents make the education choice in the youth, which will be affected by agent's innate ability and credit market condition. In the middle age, agents repay the loan and receive the wage income based upon their education and ability level. Agent continues to work through the old age. No unemployment and layoff is considered in the paper.

This paper contributes to the literature that connect income inequality with the credit market imperfection: Priya (2000) describes the credit market imperfection by assuming an exogenous constant probability of default. Galor (1993) assumes that a higher yet fixed interest rate from borrowing than from self-financing. To the best of our knowledge, Wang(2003) is the first paper that describes endogenous debt constraint in a context that studies the issue of income inequality, and in their work the default probability depends on the education dis-utility in youth. The main contribution of this paper can be recognized in the following respects: firstly, we model a continuum education choice instead of a binary decision such as to receive education or not. We believe that a contin-
uum of education will give us more insight on which education attainment level will be more affected if credit market condition gets improved, and also it will give a more precise measure on income inequality once more available education choices are included. Secondly, in our paper, the endogenous debt constraint depends on agent's innate ability, and thus also hinges on agent's intending education choice.

We extend the theoretical model into a life-cycle model and calibrate it to U.S economy. Quantitative analysis suggest that when credit market is perfect, the average education attainment level will be greatly increased, moreover income inequality across agents will also be reduced. We also discuss some potential credit policy such as extending or shortening the punishment periods in the event of default, we could quantify the contribution of a better credit market market to agent's education choice and income inequality.

The remaining of the paper is organized as follows: section 2 describes the general environment, we discuss the prefect credit market equilibrium in section 3 and section 4 defines the imperfect credit market equilibrium, some numerical examples are offered in section 5. Section 6 calibrates the model into U.S economy and Section 7 perform some counter factual exercise. Finally Section 8 concludes.

## 2 Model setup

### 2.1 Environment

The economy is populated by a unit mass of population. Each agent can live three periods: youth, middle age and old age. Agents are heterogenous in term of innate ability. Once agent is born, his ability is a random draw from a distribution function: $F$, and $F$ is defined on the interval $[\underline{a}, \bar{a}]$.

Suppose in the economy there are a continuum types of education available indexed by $i$, the tuition for education $i$ at period $t$ is denoted by $k_{t}(i)$, where $k_{t}($.$) is assumed to$ be continuous and twice differentiable. We also impose the assumption that $k_{t}^{\prime}(j)>0$ and $k_{t}^{\prime \prime}(j)>0$, hence higher education is more costly.

Young agent has no initial wealth, hence in order to receiving education $j$, they have to borrow $k_{t}(j) / R_{t}$ in the youth, and pay back $k_{t}(j)$ in the middle age.For agents with ability $a$, his middle age income from receiving education is given as $e(i, a)$, where $e(i, a)$ is assumed to be continuous and twice differentiable with respect to both $i$ and $a$. Moreover, we assume higher type of education deliver more income for every agent, that is $\partial e(i, a) / \partial i>0$, and also we assume $\partial e(i, a) / \partial i>0$, which means higher ability agents
will always earn more income from receiving any kind of education. In the old age, agent's income is assumed to be a fraction of middle age income, the fraction is denoted by $\lambda$.

Agent who is born at $t$ has life-time utility as follows:

$$
u\left(c_{t}^{t}, c_{t+1}^{t}, c_{t+2}^{t}\right)=u\left(c_{t+1}^{t}\right)+\beta u\left(c_{t+2}^{t}\right)
$$

## 3 Perfect Credit Market

In this section we assume credit market is perfect in a sense that agents are committed to repay their loan made in youth.

Throughout the whole paper, I am going to make the following assumptions:

## Assumption 1

$$
e_{11}(i, a)<0, e_{12}(i, a)>0
$$

## Assumption 2

$$
e_{1}(i, a)<k^{\prime}(i) \text { for each a }
$$

These two assumptions above are indeed very general, a CES utility function with elasticity of substitution less than one will satisfy assumption 1. Assumption 2 requires that the marginal cost from receiving education will be more than the increase of middle age income from receiving a marginal more unit of education.

Proposition 1: Given assumption 1, in the perfect credit market scenario, agents with higher ability will choose a higher level of education.

## [Insert Figure 1]

The formal proof of proposition 1 is offered in appendix. However it is very straightforward to show it in figure 1. All we need for higher ability agents choosing more education is the convexity of tuition function and the concavity of $e(i, a)$ in $i$, and both of them are guaranteed from assumption 1 . Now we will move on to define the perfect credit market equilibrium as follows:

Definition 1: The perfect credit market equilibrium consists of an interest rate $R$, educational choices $i(a)$ and consumption allocations $\left\{c_{1}(a), c_{2}(a)\right.$ such that


1. Given $R$, agent $a$ chooses education type $i(a)$ and $\left\{c_{1}(a), c_{2}(a)\right\}$ to maximize their life-time utility:

$$
\begin{array}{r}
\max \left\{U_{i}\right\} \\
\text { where } \quad U_{i}=\frac{c_{1}^{1-\sigma}}{1-\sigma}+\beta \frac{c_{2}^{1-\sigma}}{1-\sigma} \\
c_{1}+c_{2} / R=e(i, a)\left(1+\frac{\lambda}{R}\right)-k(i)
\end{array}
$$

2. Market clearing conditions:

$$
\begin{array}{r}
\int_{0}^{\bar{a}} s\left[R, y_{1}(a, R), y_{2}(a, R)\right] d F(a)= \\
\int_{0}^{\bar{a}} \frac{k\left(i^{*}(a, R)\right)}{R} d F(a)
\end{array}
$$

where

$$
\begin{aligned}
& y_{1}(a, R)=e\left[i^{*}(a, R), a\right]-k\left(i^{*}(a, R)\right) \\
& y_{2}(a, R)=\lambda e\left[i^{*}(a, R), a\right]
\end{aligned}
$$

Given the definition of perfect credit market equilibrium, we need to investigate the existence and uniqueness of such an equilibrium, before that we derive the following two lemmas. Lemma 1 essentially states that agent's optimal education choice will be a decreasing function of interest rate R . This result is intuitive in a sense that when interest rate is low, agents needs to repay more from the education loan made in youth, and this could potentially discourage agent from receiving more education. Moreover, the standard results that saving function is increasing in R also hold within current framework,
which can be proved through Lemma 2.

Lemma 1: Given Assumptions 1-2, it can be shown

$$
\frac{\partial i^{*}(a, R)}{\partial R}<0
$$

Lemma 2: saving function satisfies the following properties:

- $s(R, e(i, a)-k(i), \lambda(i) e(i, a))$ is increasing in $R$.

Since saving function is increasing in R and at the demand side, the borrowing is decreasing in R. In combination Lemma 1 with Lemma 2, we thus can derive the existence and uniqueness of equilibrium in the following proposition.

Proposition 2: Given assumptions 1-3, there exists a unique perfect credit market equilibrium with $R>0$.

## 4 Imperfect Credit market

In this section, we will move on to the imperfect credit market scenario. When the credit market is imperfect, agents are not committed to repay their loan in the youth. Also due to lack of commitment, agents are not allowed to borrow in the middle age. To simplify analysis, here we assume, if agents default, he will be caught with probability 1. If agents default, basically he will be restrained from both lending and borrowing in the middle age. Note that all the theoretical results can still go through by simply extending the model by assuming that agents will be caught with certain probability less than 1 in the event of default.

In the following, we will investigate the conditions under which agent will default. Essentially, agent will default if at least one of the two conditions satisfy: agent is a borrower in the middle age or his default payoff is greater than the solvency payoff, the later condition can be expressed as follows:

$$
u(e(i, a))+\beta u(\lambda(i) e(i, a))>(1+\beta) u\left(c_{1}(a)\right)+\beta u\left(c_{1}(a)(\beta R)^{\frac{1}{\sigma}}\right)
$$

where $c_{1}(a)=\frac{e(i, a)\left(1+\frac{\lambda}{R}\right)-k(i)}{1+\frac{\left(\frac{\beta R}{}\right)^{\frac{1}{\sigma}}}{R}}$
Proposition 3: If agent of ability $a$ does not default at education $i$, then agents with higher ability agents will not default either.

The proof of proposition 3 can be found from the appendix.

The equilibrium outcome from perfect credit market will not be supported in the imperfect credit market scenario, for instance, if any agent is a borrower in the middle age in the perfect credit market equilibrium, which means the following condition holds:

$$
\frac{e(i, a)-k(i)}{e(i, a)}<\frac{\lambda}{(\beta R)^{\frac{1}{\sigma}}}
$$

then this equilibrium will no longer be supported when credit market is imperfect. In the following, I illustrate a numerical example to show that it is actually the case that some agent will default from their perfect credit market equilibrium outcome when credit market is imperfect.

Now we move on to the determination of credit limit in the imperfect credit market scenario.

Lemma 3: when $(\beta R)^{\frac{1}{\sigma}}<\lambda$, then it is optimal for agents to borrow in the middle age, hence they are constrained and their credit limit is zero.

In the following, I denote $b b(i, a, R)$ to be the solution to the following equation:

$$
u(e(i, a))+\beta u(\lambda e(i, a))=u\left(c_{1}(a)\right)+\beta u\left((\beta R)^{\frac{1}{\sigma}} c_{1}(a)\right)
$$

where $_{c}(a)=\frac{e(i, a)\left(1+\frac{\lambda}{R}\right)-b b(i, a, R)}{1+\frac{(\beta R)^{\frac{1}{\sigma}}}{R}}$
Lemma 4: When $(\beta R)^{\frac{1}{\sigma}} \geq \lambda$, if the credit limit is set to be $b b(i, a, R)$, then agents will always lend in the middle age.

Therefore the credit limit is set as follows: Denote $b(i, a, R)$ to be the credit limit level for agent $a$ if she wants to achieve education $i$, then

$$
b(i, a, R)= \begin{cases}b b(i, a, R) & \text { if } \quad(\beta R)^{\frac{1}{\sigma}} \geq \lambda \\ 0 & \text { otherwise }\end{cases}
$$

Lemma 5: $b(i, a, R)$ satisfies the following properties:

- $b(i, a, R)$ is non-decreasing in $a$ for each $i$.
- $b(i, a, R)$ is non-decreasing in $i$ for each $a$.
- For any given $i, b(i, a, R) / e(i, a)$ is constant across all $a$

Denote $i^{o}(a)$ to be the solution of the following equation:

$$
R b(i, a, R)=k(i)
$$



Lemma 6: If agent $a$ is constrained by education $i^{o}(a)$, then he will also be borrowing constrained for all $i>i^{o}(a)$.
[Insertpicture]


Denote

$$
i(a, R)=\min \left\{i^{*}(a, R), i^{o}(a, R)\right\}
$$

Proposition 6: Given assumption 1-3, higher ability agents still choose more education in the imperfect credit market.

Definition 2:The Imperfect credit market equilibrium consists of an interest rate $R$, educational choices $e(i, a)$ and consumption allocations $\left\{c_{1}(i, a), c_{2}(i, a)\right\}$ such that:
(1) Given $R$, agent $a$ chooses education type $j$ and $\left\{c_{1}, c_{2}\right\}$ to maximize their life-time utility:

$$
\begin{array}{r}
\max \left\{U_{j}\right\} \\
\text { s.t } \quad R b(i, a, R) \geq k(i) \\
\text { where } \quad U_{j}=\max _{c_{1}, c_{2}} \ln \left(c_{1}\right)+\beta \ln \left(c_{2}\right) \\
c_{1}+c_{2} / R=e(i, a)\left(1+\frac{\lambda(i)}{R}\right)-k(i)
\end{array}
$$

(2) Market clearing conditions:

$$
\begin{array}{r}
\int_{0}^{\bar{a}} s\left[R, y_{1}(a, R), y_{2}(a, R)\right] d F(a)= \\
\int_{0}^{\bar{a}} \frac{k\left(i^{*}(a, R)\right)}{R} d F(a)
\end{array}
$$

where

$$
\begin{aligned}
& y_{1}(a, R)=e\left[i^{*}(a, R), a\right]-k\left(i^{*}(a, R)\right) \\
& y_{2}(a, R)=\lambda\left(i^{*}(a, R)\right) e\left[i^{*}(a, R), a\right]
\end{aligned}
$$

Proposition 7: Given all assumptions, there exists at least two equilibrium: autarky is always an equilibrium, the other equilibrium has $R \gg \underline{\lambda} / \beta$.

## Numerical Illustration

Before we proceed to the calibration part, we will offer a simple numerical example to illustrate how a perfect credit equilibrium may not be supported when the credit market is imperfect in the sense that when agent can not commit to repay the loan made in youth.

First I assume ability is uniformly distributed on interval [0, 1], the earning function has the form:

$$
e(i, a)=(1+i)^{0.5} a^{0.5}
$$

Moreover, the parameter values are assigned as follows:

$$
\begin{array}{r}
\beta=0.6 \\
\lambda=0.5 \\
\sigma=1 \\
k(1)=0.3
\end{array}
$$

If credit market is perfect, I can solve

$$
\begin{aligned}
a & =0.38 \\
R & =2.84
\end{aligned}
$$

Given this setting, if credit market is not perfect, I can solve

$$
\begin{aligned}
a & =0.58 \\
R & =4.92
\end{aligned}
$$



As shown in the graph above, when credit market is imperfect, there are some agents, who otherwise can become skilled if credit market condition is perfect, can not receive education.

## Quantitative Analysis

## A life cycle model

We now explore the quantitative implication of the theoretical model. Each agent is assumed to live $T$ periods, and we use $p$ to denote the age agent enters the labor market. Moreover, there is no consumption before entering labor market. Therefore agent's lifetime utility is represented as follows:

$$
\sum_{t=0}^{T-p} \beta^{t} u\left(c_{t}\right)
$$

The instantaneous utility function will take the CRRA form:

$$
u\left(c_{t}\right)=\frac{c_{t}^{1-\sigma}}{1-\sigma}
$$

In the quantitative analysis, we will drop the assumption on continuum education choice, and suppose there are several education choice available: college and advanced programme. Advanced programme refers to agents with schooling more than 19 years. Therefore agent can choose not to enter college, enter college or enter advanced programme.Every agent will retire at age $R$. Agent receives no income after retiring. The earning function is assumed to be linear in ability level and constant over time, and agents with more education will work at more productive jobs. To be precise, the income at each period will take form:

$$
\begin{array}{r}
w_{t}=\gamma_{0} \quad i=0 \\
w_{t}=\gamma_{i} a, \quad i \in\{1,2\}
\end{array}
$$

If agent chooses not to enter college, he will enter the labor market at age $p$ and no tuition cost will be incurred. The life-time budget constraint for not-college school agent is given as:

$$
\sum_{t=0}^{T-p} \frac{c_{t}}{(1+r)^{t}}=\sum_{t=0}^{R-p} \frac{\gamma_{0}}{(1+r)^{t}}
$$

If agent chooses to enter college, he will borrow tuition and repay them after entering labor market at age $p$. We assume the type of education loan requires to repay them at a constant amount $k_{1}$ for $d$ years after entering labor market. The life-time budget constraint for college agent is thus given as:

$$
\sum_{t=0}^{T-p} \frac{c_{t}}{(1+r)^{t}}=\sum_{t=0}^{R-p} \frac{\gamma_{1} a}{(1+r)^{t}}-k_{1}
$$

Similarly the life-time budget constraint for advanced programme agent is given as follows, the only difference is agent needs to repay the loan at a constant amount $k_{2}$ for $d$ years, where $k_{2}>k_{1}$.

$$
\sum_{t=0}^{T-p} \frac{c_{t}}{(1+r)^{t}}=\sum_{t=0}^{R-p} \frac{\gamma_{2} a}{(1+r)^{t}}-k_{2}
$$

The optimal consumption path for all agents satisfies:

$$
\frac{c_{t+1}}{c_{t}}=g=(1+r)^{\frac{1}{\sigma}}
$$

### 4.1 Perfect credit market scenario

Therefore when credit market is perfect in a sense that agent is committed to repay the loan, agent will choose to attend college if and only if he will get a higher life-time
utility from entering college:

$$
\frac{c_{1}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}>\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}
$$

where

$$
\begin{array}{r}
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{0} \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}} \\
c_{1} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{1} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{1}
\end{array}
$$

Similarly, agent will choose to attend advanced programme iff the following holds:

$$
\frac{c_{2}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}>\frac{c_{1}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}
$$

where

$$
\begin{aligned}
& c_{1} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{1} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{1} \\
& c_{2} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{2} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{2}
\end{aligned}
$$

Proposition: Agents with higher ability will choose more education.

### 4.2 Imperfect credit market scenario

The life-time utility for non-college agent is given as:

$$
v_{0}(a)=\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{0} \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}
$$

When agent can not commit to repay the loan, we assume if agent default on the loan, agent will be punished as autarkic for $\pi$ periods. Therefore agent will reenter the credit market at age $p+\pi$. Therefore the solvency payoff for college agent is given as:

$$
v_{1}(a)=\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{1} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{1}
$$

The default payoff for college agent is given as:

$$
V_{1}^{d}(a)=\frac{1-\beta^{\pi}}{1-\beta} \frac{\left(\gamma_{1} a\right)^{1-\sigma}}{1-\sigma}+\beta^{-\pi}\left\{\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p-\pi}}{1-\beta g^{1-\sigma}}\right\}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p-\pi}}{1-\frac{g}{1+r}}=\gamma_{1} a \frac{1-\frac{1}{(1+r)^{R-p-\pi}}}{1-\frac{1}{1+r}}
$$

Similarly, the solvency payoff for advanced program agent is given as:

$$
v_{2}(a)=\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p}}{1-\beta g^{1-\sigma}}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{2} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{2}
$$

The default payoff for advanced programme agent is given as:

$$
V_{2}^{d}(a)=\frac{1-\beta^{\pi}}{1-\beta} \frac{\left(\gamma_{2} a\right)^{1-\sigma}}{1-\sigma}+\beta^{-\pi}\left\{\frac{c_{0}^{1-\sigma}}{1-\sigma} \frac{1-\left[\beta g^{1-\sigma}\right]^{T-p-\pi}}{1-\beta g^{1-\sigma}}\right\}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p-\pi}}{1-\frac{g}{1+r}}=\gamma_{2} a \frac{1-\frac{1}{(1+r)^{R-p-\pi}}}{1-\frac{1}{1+r}}
$$

Denote $a_{1}^{o}$ and $a_{2}^{o}$ to be the solution of the following equations:

$$
\begin{aligned}
& V_{1}^{d}\left(a_{1}^{o}\right)=v_{1}\left(a_{1}^{o}\right) \\
& V_{2}^{d}\left(a_{2}^{o}\right)=v_{2}\left(a_{2}^{o}\right)
\end{aligned}
$$

Denote $a_{1}^{p}$ and $a_{2}^{p}$ to be the cutoff ability level for attending college and advanced programme respectively when agents can commit to repay the loan. Define $a_{1}^{*}$ and $a_{2}^{*}$ as follows:

$$
\begin{aligned}
& a_{1}^{*}=\max \left\{a_{1}^{p}, a_{1}^{o}\right\} \\
& a_{2}^{*}=\max \left\{a_{2}^{p}, a_{2}^{o}\right\}
\end{aligned}
$$

Therefore $a_{1}^{*}$ and $a_{2}^{*}$ will be the cutoff ability level for attending college and advanced programme respectively when agent can not commit to repay the loan.

Proposition: $a_{2}^{*}>a_{1}^{*}$ if and only if the following conditions hold:

$$
a_{1}^{o}<a_{2}^{o}
$$

Finally, the market clearing condition can be rewritten as:

$$
\begin{aligned}
\int_{0}^{a_{1}^{*}} C^{0}(a) d F(a)+\int_{a_{1}^{*}}^{a_{2}^{*}} C^{1}(a) d F(a)+ & \int_{a_{2}^{*}}^{1} C^{2}(a) d F(a)=\left[\int_{0}^{a_{1}^{*}} \gamma_{0} a d F(a)+\right. \\
& \left.\int_{a_{1}^{*}}^{a_{2}^{*}} \gamma_{1} a d F(a)+\int_{a_{2}^{*}}^{1} \gamma_{2} a d F(a)\right](R-P)
\end{aligned}
$$

where $C^{i}(a)$ denotes life-time consumption level of agent $a$ who chooses education $i$. For non-college agents:

$$
C^{0}(a)=c_{0} \frac{1-g^{T-p}}{1-g)}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{0} \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}
$$

For college agents:

$$
C^{1}(a)=c_{0} \frac{1-g^{T-p}}{1-g}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{1} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{1}
$$

For advanced programme agents:

$$
C^{2}(a)=c_{0} \frac{1-g^{T-p}}{1-g}
$$

where $c_{0}$ satisfies:

$$
c_{0} \frac{1-\left(\frac{g}{1+r}\right)^{T-p}}{1-\frac{g}{1+r}}=\gamma_{2} a \frac{1-\frac{1}{(1+r)^{R-p}}}{1-\frac{1}{1+r}}-k_{2}
$$

### 4.3 Parameters

We now discuss the parameter values used for the quantitative analysis. We assume ability is distributed on the interval $[0,1]$ with distribution function given as:

$$
F(a)=a^{\alpha}
$$

In the baseline model, $\alpha$ will be assigned to be 1 , which implies ability obeys uniform distribution. $\alpha$ will be subjected to sensitive analysis in later chapter. We set $\sigma=2$, which implies an IES of 0.5 . $\beta$ is chosen to be 0.9 .

Given all the baseline parameters, the three endogenous variables are $k_{1}, k_{2}$ and $r$. In the following I assume all the conditions that guarantee high ability agent will choose more education are satisfied here. Given all the endogenous variables solved below, I then all those conditions are indeed satisfied.

The income level among agents is plot below.

Table 1: Baseline Model Parameters

|  | Table 1: Baseline Model Parameters |  |  |
| :--- | :---: | :---: | :---: |
| Parameter | Value | To match |  |
| Chosen parameters |  |  |  |
| $\sigma$ | 2 |  |  |
| $\beta$ | 0.9 | U.S Legal Environment |  |
| $\pi$ | 10 |  |  |
| $\alpha$ | 1 | U.S Demographics |  |
| $\gamma_{1}$ | 1 |  |  |
| $p$ | 25 |  |  |
| $R$ | 65 |  |  |
| $T$ | 80 |  |  |
| Calibrated parameters |  |  |  |
| $a_{1}$ | 0.6 | fraction of non-college agents is 0.6 |  |
| $a_{2}$ | 0.8 | faction of advanced programme agents is 0.2 |  |
| $\gamma_{0}$ | 0.175 | the median no-college agents annual income |  |
|  | to median college agents annual income is 4 |  |  |
| $\gamma_{2}$ | 2.33 | the median advanced programme agents annual income |  |
|  |  | to median college agents annual income is 3 |  |

Table 2: Endogenous Variable

| Parameter | Value |
| :--- | :---: |
| $k_{1}$ | 1.42 |
| $k_{2}$ | 4.42 |
| $r$ | 0.066 |

### 4.4 Perfect credit market results

In this section, we look at the perfect credit market results by keeping the same tuition level solved from last section and all the baseline parameters will remain the same as well. We are interested to see, if credit market is perfect, what the individual education choice will be. The results are given below:

Table 3: Perfect Credit market results

| Parameter | Value |
| :--- | :---: |
| $a_{1}$ | 0.36 |
| $a_{2}$ | 0.36 |
| $r$ | 0.11 |

The results suggest that when credit market is perfect, more agent will choose type 2 education, fewer agent will not receive education and no one will go for type 1 education. Overall the average education attainment level increases. Furthermore, we compute the gini coefficient for agent's present value of life-time income within both scenarios where credit market is perfect or not. The results are listed as follows:

| Table 4: Gini Coefficient Comparison |  |  |
| :---: | :---: | :---: |
| Perfect credit |  |  |
| Imperfect credit |  |  |
| Gini | 0.36 | 0.45 |

It can be found out that when credit market is perfect, the gini coefficient decrease about $20 \%$ as opposed to the situation where agents can not commit to repay the loan. Therefore a potential policy suggestion that can be drawn from this exercise is to reduce the extent of credit market imperfection will be helpful in term of decreasing income inequality.

### 4.5 Counterfactual Exercise

In this section, we will do some counterfactual exercise such as extending the punishment periods for default; designing some taxation scheme; we want to study the policy implication which can help reduce the income inequality and enhance the average education level among agents.

The first exercise we perform is to manipulate the value $\pi$, which is essentially the length of punishment periods in the event of default. The results seem to be consistent with the intuition: a higher value of $\pi$, which implies a more severe punishment will discourage agent from defaulting, and thus this could potentially increase the credit limit
level for each agent, hence more agent will not be borrowing constrained and more agent will get to make their optimal education choice.

Table 5: Effects of $\pi$

| . |  |  |  |
| :--- | :---: | :---: | :---: |
|  | $a_{1}$ | $a_{2}$ | $r$ |
| Baseline | 0.6 | 0.8 | 0.066 |
| $\pi=8$ | 0.62 | 0.82 | 0.058 |
| $\pi=12$ | 0.59 | 0.79 | 0.0716 |

The second exercise we intend to implement some taxation scheme among agents. The goal is to check what could be the potentially taxation policy that might help reduce income inequality and enhance the average education attainment level in the economy. Explicitly, we suppose tax rate on agents with at least certain college degree ( $a>a_{2}$ ) are same, and on agents who have never been to college are same ( $a \leq a_{2}$ ). Denote $\tau_{1}$ to be the tax rate on agents with at least certain college degree ( $a>a_{2}$ ), and $\tau_{0}$ is the tax rate on agents who have never been to college.

| Variable | Imperfect |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $a_{1}$ | $a_{2}$ | R | gini | $\tau_{0}$ |
| $\tau_{1}=0$ | 0.477 | 0.783 | 2.456 | 0.36 |  |
| $\tau_{1}=0.1$ | 0.590 | 0.904 | 2.381 | 0.323 | -0.125 |
| $\tau_{1}=0.2$ | 0.705 | 0.969 | 2.3483 | 0.273 | -0.1114 |
| $\tau_{1}=-0.1$ | 0.537 | 0.867 | 2.398 | 0.34 | 0.268 |

## Conclusion

[To be added]

## Appendix

## Proof to Proposition 1:

Denote $i^{*}(a, R)$ to be the solution of the following problem:

$$
\max _{i} \quad e(i, a)\left(1+\frac{\lambda}{R}\right)-k(i)
$$

Then $i^{*}(a, R)$ satisfies the following equation:

$$
e_{1}(i, a)\left(1+\frac{\lambda}{R}\right)-k^{\prime}(i)=0
$$

Take derivative with respect to $a$ gives:

$$
\frac{\partial i^{*}(a, R)}{\partial a}=\frac{e_{12}(i, a)\left(1+\frac{\lambda}{R}\right)}{k^{\prime \prime}(i)-e_{11}\left(1+\frac{\lambda_{1}}{R}\right)}
$$

Therefore, given assumption 1, we can show: $\partial i^{*}(a, R) / \partial a>0$.

## Proof to Lemma 1:

Take the derivative of $i^{*}(a, R)$ with respect to $R$ gives:

$$
\frac{\partial i^{*}(a, R)}{\partial R}=\frac{e_{1}(i, a) \frac{\lambda(i)}{R^{2}}}{e_{11}(i, a)-k^{\prime \prime}(i)}
$$

In combination with assumption 1 , we can show: $\partial i^{*}(a, R) / \partial R<0$.

## Proof to Lemma 2:

Given CRRA utility form, the saving function $s(R, e(i, a)-k(i), \lambda e(i, a))$ has the following form:

$$
s(R, e(i, a)-k(i), \lambda(i) e(i, a))=[e(i, a)-k(i)] \frac{(\beta R)^{\frac{1}{\sigma}}}{R+(\beta R)^{\frac{1}{\sigma}}}-\frac{\lambda e(i, a)}{R+(\beta R)^{\frac{1}{\sigma}}}
$$

Given lemma 1 and assumption 2, it suffices to show that saving function is increasing in $R$.

## Proof to Proposition 2:

From lemma 2, we know the saving function is increasing in $R$, moreover, it can be shown that:

$$
\partial\left(k\left(i^{*}(a, R)\right) / R\right) / \partial R=\frac{R k^{\prime}(i) \frac{i^{*}(a, R)}{\partial R}-k}{R^{2}}
$$

From lemma 1, it can be shown that:

$$
\frac{\partial i^{*}(a, R)}{\partial R}<0
$$

Therefore the lending function is decreasing in R , and thus the equilibrium is unique.

## Proof to Proposition 3:

The incentive of not defaulting is to smooth consumption between middle and old age, therefore intuitively, the less the ratio of old age income to middle age income, the more likely agent will choose not to default. The ratio of old age income to middle age income is given as:

$$
\frac{\lambda e(i, a)}{e(i, a)-k(i)}
$$

The ratio is decreasing in term of $a$ for each $i$, hence it suffices to show If agent of ability $a$ does not default at education $i$, then agents with higher ability agents will not default either.

## Proof to Lemma 3:

The saving function can be expressed as follows:

$$
[e(i, a)-k(i)] \frac{(\beta R)^{\frac{1}{\sigma}}}{R+(\beta R)^{\frac{1}{\sigma}}}-\frac{\lambda e(i, a)}{R+(\beta R)^{\frac{1}{\sigma}}}
$$

It can be shown that when $(\beta R)^{\frac{1}{\sigma}}<\lambda$, then $s(R, e(i, a)-k(i), \lambda e(i, a))$ is negative, and thus agent is a borrower in the middle age.

## Proof to Lemma 4:

For log case ( $\sigma=1$ ), solving $b b(i, a, R)$ from

$$
\ln (e(i, a))+\beta \ln (\lambda(i) e(i, a))=(1+\beta) \ln \left(c_{1}(a)\right)+\beta \ln (\beta R)
$$

where $c_{1}(a)=\frac{1}{1+\beta}\left(e(i, a)\left(1+\frac{\lambda(i)}{R}\right)-b b(i, a, R)\right)$ gives:

$$
\left(\frac{\lambda(i)}{\beta R}\right)^{\frac{\beta}{1+\beta}}(1+\beta)=1+\frac{\lambda(i)}{R}-\frac{b b(i, a, R)}{e(i, a)}
$$

Since $\beta R>\lambda(i)$, thus $\left(\frac{\lambda(i)}{\beta R}\right)^{\frac{\beta}{1+\beta}}>\frac{\lambda(i)}{\beta R}$, which can also be written as:

$$
\left(\frac{\lambda(i)}{\beta R}\right)^{\frac{\beta}{1+\beta}}(1+\beta)-\frac{\lambda(i)}{R}>\frac{\lambda(i)}{\beta R}
$$

Therefore this proves:

$$
1-\frac{b b(i, a, R)}{e(i, a)}>\frac{\lambda(i)}{\beta R}
$$

Hence agents will always save in the middle age.
When $\sigma>1$, solve $b b(i, a, R)$ from

$$
c_{1}^{1-\sigma}\left[1+R^{\frac{1-\sigma}{\sigma}} \beta^{\frac{1}{\sigma}}\right]=e(i, a)^{1-\sigma}\left[1+\beta \lambda^{1-\sigma}\right]
$$

where $c_{1}(a)=\frac{e(i, a)\left(1+\frac{\lambda}{R}\right)-b b(i, a, R)}{1+\frac{(\beta R) \frac{1}{\sigma}}{R}}$ gives:

$$
b b(i, a, R)=e(i, a)\left(1+\frac{\lambda}{R}\right)-e(i, a)\left(1+\beta \lambda^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\left(1+R^{\frac{1-\sigma}{\sigma}} \beta^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

It can be shown that:

$$
1-\frac{b b(i, a, R)}{e(i, a)}>\frac{\lambda}{(\beta R)^{\frac{1}{\sigma}}}
$$

Above is equivalent to showing:

$$
\left(1+\beta \lambda^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\left(1+R^{\frac{1-\sigma}{\sigma}} \beta^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}>\frac{\lambda}{R}+\frac{\lambda}{(\beta R)^{\frac{1}{\sigma}}}
$$

which is also equivalent to showing:

$$
\left(1+\beta \lambda^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\left(1+R^{\frac{1-\sigma}{\sigma}} \beta^{\frac{1}{\sigma}}\right)^{\frac{1}{\sigma-1}}(\beta R)^{\frac{1}{\sigma}}>\lambda
$$

Above inequality holds when $(\beta R)^{\frac{1}{\sigma}}>\lambda$. This completes the proof.

## Proof to lemma 5:

Following lemma 4, it is straightforward to see that $b b(i, a, R)$ is linear in $e(i, a)$.

## Proof to Proposition 6:

Under imperfect credit market, agent's optimization problem becomes:

$$
\begin{array}{r}
\max \left\{U_{j}\right\} \\
\text { s.t } \quad R b(i, a, R) \geq k(i) \\
\text { where } \quad U_{j}=\max _{c_{1}, c_{2}} u\left(c_{1}\right)+\beta u\left(c_{2}\right) \\
c_{1}+c_{2} / R=e(i, a)\left(1+\frac{\lambda(i)}{R}\right)-k(i)
\end{array}
$$

Follow similar proof as proposition 1, and it can establish the proof.
Denote $i^{o}(a)$ to be the solution of the following problem:

$$
e(i, a) * \text { cons }=k(i)
$$

where

$$
\text { cons }=\left(1+\frac{\lambda}{R}\right)-\left(1+\beta \lambda^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\left(1+R^{\frac{1-\sigma}{\sigma}} \beta^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

since $b b(i, a)=$ cons $* e(i, a)$, and $1-\frac{b b(i, a, R)}{e(i, a)}>\frac{\lambda}{(\beta R)^{\frac{1}{\sigma}}}$, therefore it can be shown that:

$$
\text { cons }<1
$$

Therefore given assumption 2, it can be shown that:

$$
\frac{d i^{o}(a)}{d a}=\frac{e_{2}(i, a)}{\frac{k^{\prime}(i)}{\text { cons }}-e_{1}(i, a)}>0
$$

## Proof to Proposition 7:

When interest rate is too low, every agent has incentive to default, and thus in equilibrium the credit limit is set to be zero for each agent, and thus lending is zero, which means

Autarky is the only equilibrium. When $R>\underline{\lambda} / \beta$, follow similar proof as proposition 2 , the other equilibrium can be guaranteed.
It can be shown that:

$$
\frac{\partial i^{o}(a, R)}{\partial R}\left(e_{1}(i, a)-\frac{k^{\prime}(i)}{R}\right)=\frac{-k(i)}{R^{2}}-e(i, a) \frac{\partial c o n s}{\partial R}
$$

Moreover, it can be shown that:

$$
\frac{\partial c o n s}{\partial R}>0
$$

## References

[1] Aiyagari,R.,J.Greenwood, and A.Seshadri "Efficient Investment in Children," Journal of Economic Theory, 102 (2), 290-321.
[2] Andofalfatto,D. and M.Gervais, (2006)."Human Capital Investment and Debt Constraint," Review of Economic Dynamics, 9 (1), 52-67.
[3] Camerson,S.,and C.Taber (2004),"Estimation of Educational Borrowing Constraints Using Return to Schooling", Journal of Political Economy, 109, 455-499.
[4] Chatterjee,S.,D.Corbae,M.Nakajima,J.-V.,Rios-Rull(2007),"A Quantitative Theory of Unsecured Consumer Credit with Risk of Default", Econometrica, 75(6), 1529-89.
[5] Kehoe,T., and D.Levine(1993), "Debt-Constrained Asset Markets", Review of Economic Studies, 60(4), 865-888.
[6] Lochner,L. and A.Monge-Naranjo (2008), "The Nature of Credit Constraints and Human Capital," NBER Working Paper 13912.

