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Key Words: voting, mass media, persuasion games, media bias, median voter theorem

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1 Introduction

This paper studies a role of mass media in electoral competitions from the view of strategic communication. In the reality, mass media report several information regarding elections, and their reports influence both candidates and voters. For example, mass media frequently report who is a candidate, what is his/her proposed policy, and what extent the policy is endorsed by voters. Most of voters use these outlets of mass media as an information source to decide which candidate is better, and candidates might change their policies if they are little endorsed by voters. Then, mass media could have incentives to strategically select contents and ways of presentation of news. This paper considers a question of how mass media affects elections if they strategically report relevant information.

In this paper, we analyze a simple Downsian voting model including one mass medium. There exist two candidates, one medium, and one voter. Compared with the standard models, the voter cannot directly observe the proposed policies by the candidates, but the medium can observe the policies. Hence, the medium sends a news regarding the proposed policies to the voter before voting occurs. In other words, we consider the following two stage game. At the first stage, the two candidates simultaneously propose policies, and only the medium can observe them. At the second stage, the medium sends a news regarding policies, and the voter chooses one of the candidates after observing the news. We assume that the information about proposed policies is hard information; it is verifiable by the third party or the voter after the election. Hence, strategic communication of the second stage is represented by a persuasion game. That is, the media can withhold unfavorable information, but cannot misreport it. Under this environment, we mainly pursue the following two questions. First, how much the relevant information does the medium disclose? Second, how does the existence of the medium distort electoral outcomes compared with the no media case.

The results are as follows. We show that the medium can conceal a part of unfavorable information in any equilibrium when the media bias is not small. Because of the media manipulation of information, appealing to the voter becomes difficult for the candidates. Then, we can observe a variety of policy distribution in equilibrium. In contrast with the standard Downsian models, policy divergence and policy randomization can be supported in equilibrium. In addition, the candidates could have incentives to control the medium's behaviors. In other words, the candidates choose policies in order to control the medium's behavior, not to appeal to the voter. Therefore, the median voter theorem could fail. As the result, equilibrium outcomes are distorted compared with the no media case. There are two channels of the distortion: *indirect distortion*, a distortion in the candidates' behaviors, and *direct distortion*, a distortion in the voter' behaviors.

The motivation of this research is explicitly describing strategic communication aspects of mass media in elections. Most of the standard Downsian voting models represent interactions between the candidates and the voters as direct ones in the sense that the players are the candidates and the voters, and the voters can directly observe the proposed policies. While, of course, this representation is quite useful to capture political dynamics, it significantly omits the roles of mass media. In real elections, the interactions are indirect in the sense that there exists mass media between the candidates and the voters. Mass media gather information regarding proposed policies, and send news as a message about the information. Because most of voters use the news as an information source of voting, instead of directly observing the information, we can say that mass media has notable influence over political outcomes.

In addition, it is well accepted that media outlets of reporting news is slanted due to media bias. One interpretation of the phenomena is that media bias reflects the preferences of journalists or owners of the media. If the preferences of the journalists or the owners are divergent from those of the majority of voters, then mass media have incentives to strategically slant the news in order to lead political outcomes to their preferred direction. This paper explicitly represents such strategic behaviors of mass media by a model of strategic communication, and considers how the strategic behaviors of mass media affect political outcomes.

This paper is organized as follows. In the following subsection, we briefly review related literature. Section 2 defines a formal model. In Section 3, we analyze a benchmark model without mass media, and consider a model involving mass media in Section 4. We discuss the robustness of the results in Section 5, and conclude this paper in Section 6.

1.1 Related literature

This paper is based on several branches of economics. We adopt the Downsian voting model, initiated by Downs (1957), as the basic framework of analysis. In the Downsian voting models, the median voter theorem holds. As Roemer (2001) explains, this is a persistent property in the Downsian models; the median voter theorem is robust to the voters' small mistakes or the candidates' uncertainty about the voters' types. There are a few papers reporting a failure of the median voter theorem. Palfrey (1986) derives the policy divergence between two candidates in the situation where the third candidate could enter the competition. Kartik and McAfee (2007) introduce a "character" of candidate, which affects the voter's payoff, and show that the median voter theorem fails. Different from the above papers, we consider a Downsian model including one mass medium, and derive the failure of the median voter theorem.

The strategic communication aspects of our model is based on persuasion games. These are sender-receiver games with verifiable private information first formalized by Milgrom (1981), and there is huge literature, for example, Milgrom and Roberts (1986), Shin (1994), Seidmann and Winter (1997), and Giovannoni and Seidmann (2007). Compared with cheap talk models, like Crawford and Sobel (1982), the sender's private information is verifiable, so the information cannot be misreported, but the sender can withhold unfavorable information. Our analysis is based on Miura (2010), which analyzes the general persuasion game used in this paper.[1]

There is growing literature in economics of mass media. One of the main research agenda is making clear reasons of slanted news reports. Mullainathan and Shleifer (2005) consider a model focusing on the demand side. In their environment, consumers want to read newspapers that are consistent with own beliefs. Then, in order to satisfy this demand, mass media write slanted news. In Gentzkow and Shapiro (2006), the media outlets are slanted to the consumers' prior beliefs in order to build good reputation about the quality of the contents of the news. On the other hand, Barron (2006) argues that slanted news comes from the supply side. That is, biased news reflects the preferences of editors or owners of the mass media. Due to career objectives of journalists, they tend to write articles consistent with preferences of editors or owners. In Gentzkow and Shapiro (2006), media competitions reduce the media biases, but in Mullainathan and Shleifer (2005) and Baron (2006), the competition may force mass media to write more slanted news.

Another research agenda of economics of media is studying roles of mass media in political contexts. Prat and Strömberg (2010) is a survey of this field. Chan and Suen (2008) consider a model with political and media competitions, and characterize the optimal editorial positions of profit-maximizing mass media. On the other hand, Chan and Suen (2009) consider the similar model except that the mass media's preferences are policy-motivated. They conclude that even if mass media have biased preferences, containing subjective opinions into the reports could be better than just reporting the objective facts. Strömberg (2004) constructs another model of media competitions with political competitions. Profit-

[1]The roles of the medium in this paper is similar to those in Chen (2007). She studies a trade-off between information transmission and information acquisition in the context of media.

maximizing mass media deliver news to groups of individuals that are valuable to advertising. Because news from mass media is a main information channel to voters, candidates propose policies that are attractable to the groups. Anderson and McLaren (2010) analyze a model of media competitions in which the media who have profit and policy-motivated preferences can withhold unfavorable information.

Our model is different from above models in several points. First, the interaction between the candidates and the voter is different. In this paper, that is represented by a persuasion game in which the voter can distinguish whether the medium withholds the information about the proposed policies. In Strömberg (2004), the interaction is not strategic communication; the media do not mislead the voters, and the voters naively believe the information. Chen and Suen (2008, 2009) represent the interaction as a cheap talk game, where the senders' private information is fundamentals of economy, not the proposed policies. Anderson and McLaren (2010) adopt a persuasion game model to describe the interaction, but the structure of the game is different from ours; in their model, the voter cannot distinguish whether the media withhold the information, like Shin (1994). Second, we omit the media competition aspects in order to focus on the roles of the medium as the mediated sender.

2 The Model

There exist four players: candidate 1, 2, the medium and the (median) voter.[2] They play the following two stage game. At the first stage, call *policy setting stage*, each candidate simultaneously proposes a policy, and only the medium can observe the proposed policies. At the second stage, call *information disclosure stage*, the medium sends a message about the proposed policies to the voter. After observing the message, the voter casts the ballot

[2]We treat the candidates and the voter as male and the media as female throughout this paper.

for one of the candidates. Then, the winning candidate implements the proposed policy.

Let $X \equiv \{0, a, 2a\}$ be the set of available policies of the candidates with $a > 0$. Let $x_i \in X$ be the policy proposed by candidate $i \in \{1, 2\}$, and $x \equiv (x_1, x_2) \in X^2 \subset \mathbb{R}^2_+$ describes a pair of the proposed policies by the candidates. We assume that the information regarding x is *hard information*; that is, this is verifiable information. In addition, we assume that the medium can correctly observe x, but the voter cannot. Hence, the information about x is the medium's private information at the information disclosure stage.[3]

Let $M(x) \equiv \{x, \phi\}$ be the message space of the medium when she observes policy pair x at the policy setting stage. The element x represents the medium's disclosure behavior. That is, the medium tells the voter what she observes. On the other hand, the element ϕ represents the medium's withholding. That is, the medium completely conceals what she observes and tells nothing to the voter.[4] It is worthwhile to note that the medium cannot misreport the information about x. The medium cannot say that the observed policy pair is x' when she observes $x \neq x'$ because the information is verifiable.[5] Let $M \equiv \cup_{x \in X^2} M(x)$ be the universal message space, and $m \in M$ be the generic notation of the medium's message. Let $Y \equiv \{y_1, y_2\}$ be the voter's action space, where y_i represents that the voter casts the ballot for candidate $i \in \{1, 2\}$, and use $y \in Y$ to describe the generic notation of the voter's action.

We assume that there are two types of the candidates: *non-policy type* and *policy type*. The non-policy type candidate is the standard office-motivated candidate. In other words,

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[3]These assumptions seem to be reasonable because, in the real world, this information is in the manifesto of each candidate, and anyone can check it if prefers. However, voters learn the information through reports of mass media. Voters seldom check the information by themselves.

[4]We can easily extend the model allowing the medium to partially disclose the information regarding x. However, we obtain similar results. So, to simplify the analysis, we restrict the medium's message space as above. We will revisit this point in Section 5.4.

[5]In other words, we implicitly assume that the medium bears huge costs for misreporting, for example, her bad reputation is widely known to people, due to the verifiability of the information.

the objective of the non-policy type candidate is only to win the election. He does not care about his proposed policy. On the other hand, the policy type candidate cares both taking office and his own proposed policy. That is, the policy type candidate would like to win the election by proposing the preferred policy. Let $\Theta \equiv \{\theta_N, \theta_P\}$ be the candidates' type space, and θ_N (resp. θ_P) represent the non-policy type (resp. policy type). We assume that candidate i's type $\theta_i \in \Theta$ is candidate i's private information, and θ_1 and θ_2 are independently determined. Let $p > 0$ be the probability that each candidate is the non-policy type, and assume that it is common knowledge.

The players' preferences are defined as follows. We assume that the medium and the voter have single-peaked preferences. Define the voter's von Neumann-Morgenstern utility function $v : X^2 \times Y \to \mathbb{R}$ by

$$v(x, y) \equiv \begin{cases} -|x_1| & \text{if } y = y_1 \\ -|x_2| & \text{if } y = y_2. \end{cases} \tag{1}$$

Similarly, define the medium's von Neumann-Morgenstern utility function $w : X^2 \times Y \times \mathbb{R} \to \mathbb{R}$ by

$$w(x, y, b) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1 \\ -|x_2 - b| & \text{if } y = y_2. \end{cases} \tag{2}$$

The voter's ideal policy is 0, but that of the medium is $b > 0$. Hence, the parameter b represents the difference between the voter's and the medium's preferences. We call the parameter *media bias* throughout this paper. We assume that the level of the media bias is common knowledge.

Define candidate i's von Neumann-Morgenstern utility function $u_i : \Theta \times X \times Y \to \mathbb{R}$ by

$$u_1(\theta_1, x_1, y) \equiv \begin{cases} 1 & \text{if } \theta_1 = \theta_N \text{ and } y = y_1 \\ 1 & \text{if } \theta_1 = \theta_P \text{ and } x_1 = a \text{ and } y = y_1 \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

$$u_2(\theta_2, x_2, y) \equiv \begin{cases} 1 & \text{if } \theta_2 = \theta_N \text{ and } y = y_2 \\ 1 & \text{if } \theta_2 = \theta_P \text{ and } x_2 = 2a \text{ and } y = y_2 \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

That is, if the candidate is the non-policy type, then he obtains positive payoff if and only if he wins the election. On the other hand, if candidate 1 is the policy type, then he obtains positive payoff if and only if he wins the election with proposing policy $x_1 = a$. Similarly, the policy type of candidate 2 obtains positive payoff if and only if he wins with proposing policy $x_2 = 2a$.

It is worthwhile to remark that this setup, in which the policy type candidates' ideal policies lie on the same direction from the voter's ideal policy represents a situation such that the candidates compete in levels of public spending or tax rates. For example, the voter has a preference such that the less the consumption tax rate is, the better off he is. On the other hand, the candidates of the policy type strongly would like to increase the tax rate in order to finance the public debt, but they disagree with the rates. Furthermore, in this setup, the medium also agrees with positive tax rates because, for example, she expects reduction of corporation tax as a compensation of the increased consumption tax. We will discuss another setup where the voter's ideal policy lies in the middle of those of the policy type candidates in Section 5.2.

The timing of the game is formalized as follows. At the policy setting stage, first, nature chooses candidate i's type $\theta_i \in \Theta$ according to the prior distribution p, and only candidate i correctly learns own type θ_i. Then, given θ_i, each candidate simultaneously proposes a policy $x_i \in X$. Only the medium can correctly observe the pair of proposed policies $x \in X^2$.

At the information disclosure stage, given observed pair x, the medium sends a message $m \in M(x)$. After observing the message, the voter takes an action $y \in Y$. Then, the policy announced by the winning candidate is implemented.

The players' strategies are defined as follows. Candidate i's strategy $\alpha_i : \Theta \to X$ is a function from his own type to a policy for $i \in \{1, 2\}$.[6] The medium's strategy $\beta : X^2 \to M$ is a function from an observed policy pair to a message. The voter's strategy $\gamma : M \to \Delta(Y)$ is a function from an observed message to a probability distribution over the voter's action set Y. The voter's strategy is represented by $\gamma(m) = (q(m), 1 - q(m))$, where $q(m)$ represents the probability that the voter casts the ballot for candidate 1 when he observes message m. Let $\mathcal{P} : M \to \Delta(X^2)$ represent the voter's posterior belief, which is a function from an observed message to a probability distribution over the set of possible policy pairs X^2.

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept. Because the voter knows that only the medium who observes policy pair x' can send message $m = x'$, we put the following requirement as an restriction to off the equilibrium path beliefs.

Requirement 1 *For any $x' \in X^2$, if the voter observes a message $m = x'$, then the voter's posterior belief satisfies $\mathcal{P}(x = x'|m = x') = 1$.*

Definition 1 ***Perfect Bayesian Equilibrium.***
A five-tuple $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions;

(i) $\alpha_i^*(\theta_i) = \arg\max_{x_i \in X} \mathbb{E}[u_i(\theta_i, x_i, \gamma^*(\beta^*(x_i, \alpha_{j*}(\theta_j))))]$ $\forall \theta_i \in \Theta$, $\forall i, j \in \{1, 2\}$ *with* $j \neq i$,

(ii) $\beta^*(x) = \arg\max_{m \in M(x)} w(x, \gamma^*(m), b)$ $\forall x \in X^2$,

(iii) $\gamma^*(m) = \arg\max_{q \in \Delta(Y)} \mathbb{E}[v(x, q)|m]$ $\forall m \in M$, *and*

[6] A mixed strategy of candidate i is represented by, with abuse of notation, $\alpha_i(\theta_i) = (\alpha_i^0, \alpha_i^a, 1 - \alpha_i^0 - \alpha_i^a)$ where α_i^j represents the probability that candidate i proposes policy j.

(iv) *$\mathcal{P}^*$ is derived by α^{1*}, α^{2*} and β^* consistently with Bayes' rule whenever it is possible. Otherwise, $\mathcal{P}^*$ is an any probability distribution satisfying Requirement 1.*

In addition, we assume the following tie-breaking rules; one is for the voter, and the other is for the medium. Then, we focus on PBEs satisfying the tie-breaking rules in the subsequent analysis.

Requirement 2 ***Tie-breaking Rules.***

(i) *Given the voter's posterior belief $\mathcal{P}$, if y_1 and y_2 are indifferent for the voter, then he votes each candidate with probability $\frac{1}{2}$.*

(ii) *Given a policy pair x such that $x_1 = x_2$. Then, the medium discloses the information.*

In the subsequent analysis, we consider whether the median voter theorems holds as a reference point. We define the median voter theorems in this context as follows.

Definition 2 ***Median Voter Theorem.***

(i) *We say that the strict median voter theorem holds if there exists the unique PBE in which $\alpha_1^*(\theta_N) = \alpha_2^*(\theta_N) = 0$.*

(ii) *We say that the weak median voter theorem holds if there exists a PBE in which $\alpha_1^*(\theta_N) = \alpha_2^*(\theta_N) = 0$.*

That is, we require the existence of a PBE in which both the non-policy type candidates propose the voter's ideal policy. The strict median voter theorem requires the uniqueness of equilibrium, but the weak median voter theorem does not require the uniqueness.

3 Benchmark Model: No Media

In this section, we analyze the model without the medium as a benchmark model; that is, the voter can directly observe the proposed policies. In the benchmark model, the median

probability	(θ_1, θ_2)	proposed policy pair	winner	equilibrium policy
p^2	(θ_N, θ_N)	$(0, 0)$	1 or 2	0
$p(1-p)$	(θ_N, θ_P)	$(0, 2a)$	1	0
$(1-p)p$	(θ_P, θ_N)	$(a, 0)$	2	0
$(1-p)^2$	(θ_P, θ_P)	$(a, 2a)$	1	a

Table 1: Equilibrium outcomes in the benchmark model

voter theorem holds, and then, we can say that the voter's ideal policy is supported as the equilibrium outcome unless both candidates are the policy type.

The voter's equilibrium strategy is straightforward. Because the voter can directly observe the proposed policies, the voter can cast the ballot for the candidate whose policy is closer to his ideal point 0 for certain. That is, the voter chooses y_1 for certain if $|x_1| < |x_2|$, y_2 for certain if $|x_1| > |x_2|$, and equally likely pick up y_1 and y_2 if $|x_1| = |x_2|$.

Next, given the voter's strategy, consider the candidates' best responses. It is worthwhile to remark that if the candidate is the policy type, then proposing his preferred policy weakly dominates other actions. In other words, if candidate 1 is the policy type, then proposing $x_1 = a$ is weakly better than the others. Similarly, if candidate 2 is the policy type, then proposing $x_2 = 2a$ is weakly better than the others.[7] This property is independent from the existence of the media, so we can just restrict our attention to behaviors of the non-policy type candidates throughout this paper. To simplify descriptions, an equilibrium in which the non-policy type candidates propose x_1 and x_2 for certain is called (x_1, x_2)-*equilibrium.*

The argument for the non-policy type candidates is same to the standard Downsian models. That is, because the voter can directly observe policy pair x, proposing the voter's

[7] Because we can say that the policy type candidate takes the weakly dominant action on the equilibrium path, the policy type candidate can be interpreted as a behavioral type who always proposes the preferred policy.

ideal policy, i.e., $x_i = 0$, is the dominant action for each non-policy type candidate. In other words, $(0,0)$-equilibrium is the unique equilibrium. The equilibrium outcomes are summarized in Table 1. We can observe that the voter's ideal policy can be supported as the equilibrium policy unless both candidates are the policy type. The following proposition summarizes the results in the benchmark model.

Proposition 1 *Consider the benchmark model. Then,*

(i) $(0,0)$-*equilibrium is the unique equilibrium, i.e., the strict median voter theorem holds.*

(ii) *The voter's ideal policy is supported as the equilibrium outcome unless both candidates are the policy type.*

Proof. All proofs are in Appendix A. ■

4 Manipulated News Model

Now, we move back to the model involving the medium. We call this model *manipulated news model.* First, we show that the medium can conceal a part of unfavorable information when the media bias is not small. Then, we demonstrate how media manipulation of information distorts equilibrium outcomes compared with the benchmark model. In other words, we show that even the weak median voter theorem could fail in the manipulated news model.

4.1 Information disclosure stage

In this subsection, we analyze a persuasion game between the medium and the voter given the candidates' proposed policies. First, it is worthwhile to make clear the voter's uncertainty at the beginning of the information disclosure stage. The voter faces uncertainty about the proposed policy pair because of the uncertainty about the candidates' types. For example, suppose that $\alpha_1^*(\theta_N) = \alpha_2^*(\theta_N) = 0$ are the non-policy type candidates' equilibrium behaviors. The voter knows that either one of the pairs, $(0,0), (0,2a), (a,0)$ and $(a,2a)$ is proposed in the equilibrium, but he cannot specify which policy pair is actually proposed. This is the voter's uncertainty at the beginning of the information disclosure stage.[8] Therefore, the news from the medium is crucial for the voter to choose the correct candidate in the manipulated news model.

Next, we define, and characterize a *full disclosure equilibrium* as a reference point. Let $Z(\alpha_1^*, \alpha_2^*)$ be the support of the voter's equilibrium prior, i.e., the set of possible policy pairs from the view of the voter given the equilibrium strategies α_1^* and α_2^*. It is defined by $Z(\alpha_1^*, \alpha_2^*) \equiv \{x' \in X^2 | Pr.(x = x' | \alpha_1^*, \alpha_2^*) > 0\}$. Let $y^v(x)$ be the voter's correct decision-making from the view of ex-post defined by:

$$y^v(x) = \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2| \end{cases} \tag{5}$$

Then, we define a *full disclosure equilibrium* as follows;

Definition 3 ***Full Disclosure Equilibrium.***

A PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *is a full disclosure equilibrium if* $\gamma^*(\beta^*(x)) = y^v(x), \forall x \in Z(\alpha_1^*, \alpha_2^*)$.

That is, the full disclosure equilibrium is a PBE where the voter chooses the preferred candidate for certain on the equilibrium path. In other words, we require that the necessary information for correct decision-making is completely transmitted to the voter on the equilibrium path.

[8] In other words, as long as we use Nash concepts, players correctly expect the others' strategies in equilibrium. That is, no one faces strategic uncertainty in equilibrium. In the manipulated news model, proposed policies are the strategies of the candidates, and then, the voter correctly expects the candidates' strategies in equilibrium. However, because the voter does not know the types of candidates, she faces uncertainty about proposed policy pair.

There are remarks about the full disclosure equilibrium. The definition of the full disclosure equilibrium only requires that the voter's decision-making is correct on the equilibrium path. Hence, we do not care about the correctness of the voter's decision-making off the equilibrium path. In other words, even in the full disclosure equilibrium, the voter's decision-making could be incorrect off the equilibrium path. Moreover, we also do not care about the medium's behaviors. If the medium "directly" discloses the information, i.e., $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$, then this is obviously the full disclosure equilibrium. However, even if the medium withholds the information, then this behavior of the medium could support the full disclosure equilibrium. For instance, if there exists a policy pair $x' \in Z(\alpha_1^*, \alpha_2^*)$ such that the medium withholds the information on the equilibrium path only when the medium observes policy pair x'. In this example, withholding the information itself is a signal about policy pair x', and then, the information about policy pair x' is "indirectly" disclosed. Therefore, the medium's such behavior could support the full disclosure equilibrium.

Now, we begin to characterize the voter's and medium's equilibrium strategies. The voter's equilibrium behavior is straightforward. If the media sends a message $m = x$, then the voter completely learns the proposed policies. Hence, the voter's decision-making is correct. On the other hand, if the media sends a message $m = \phi$, then the voter's uncertainty about the policy pair could not be resolved. The voter's decision-making is based on the posterior belief $\mathcal{P}(\cdot|\phi)$. Therefore, the voter's best response to message $m = \phi$ is characterized as follows:

$$\gamma^*(\phi) = \begin{cases} (1,0) & \text{if } \mathbb{E}[|x_1||\phi] < \mathbb{E}[|x_2||\phi] \\ (\frac{1}{2},\frac{1}{2}) & \text{if } \mathbb{E}[|x_1||\phi] = \mathbb{E}[|x_2||\phi] \\ (0,1) & \text{if } \mathbb{E}[|x_1||\phi] > \mathbb{E}[|x_2||\phi] \end{cases} \tag{6}$$

Given the voter's equilibrium strategy, consider the medium's strategy. Withholding the information is the weak dominant action for the medium whose preference is disagreed with that of the voter. Because we define the voter and the medium's preferences as (1)

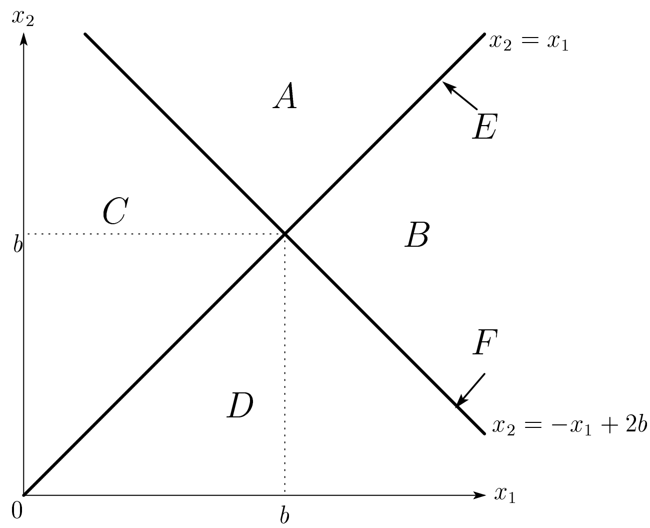

Figure 1: Distribution of preferences

and (2), given a policy pair x, the voter prefers y_1 to y_2 if and only if $|x_1| \leq |x_2|$, and the medium prefers y_1 to y_2 if and only if $|x_1 - b| \leq |x_2 - b|$. Hence, the space $\mathbb{R}^2_+$ is divided into the following six regions, as shown in Figure 1:

$$A \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | x_2 > x_1 \text{ and } x_2 > -x_1 + 2b\} \tag{7}$$

$$B \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | -x_1 + 2b < x_2 < x_1\} \tag{8}$$

$$C \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | x_1 < x_2 < -x_1 + 2b\} \tag{9}$$

$$D \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | x_2 < x_1 \text{ and } x_2 < -x_1 + 2b\} \tag{10}$$

$$E \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | x_2 = x_1\} \tag{11}$$

$$F \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ | x_2 = -x_1 + 2b\} \tag{12}$$

We call regions A, B, E and F *agreement regions*, and regions C and D *disagreement regions*. If a proposed policy pair lies in the agreement regions, then the voter's and medium's preferences do not conflict. In other words, the preferences completely agree, or one of the players is indifferent from the result of the election. In region A, both the voter and the medium strictly prefers y_1 to y_2, and in region B, they agree with strictly preferring y_2 to

y_1. In region E, both the voter and the medium are indifferent between y_1 and y_2. In region F, the voter has a strictly preference, but the medium is indifferent. On the other hand, if a proposed pair lies in the disagreement regions, then the voter's and medium's preferences strictly conflict. In region C, the voter strictly prefers y_1 to y_2, but the medium strictly prefers y_2 to y_1. Similarly, in region D, the voter strictly prefers y_2 to y_1, but the medium strictly prefers y_1 to y_2.

If the medium observes a policy pair in the agreement regions, then disclosing the information is one of the best response because the medium's preference is aligned to that of the voter. On the other hand, if the medium observes a policy pair in the disagreement regions, then the withholding is weakly better than the disclosing for the medium; while the disclosing induces the medium's unfavorable candidate for certain, the withholding could induce her preferred candidate with positive probability. Hence, one of the medium's equilibrium strategies is characterized as follows:

$$\beta^*(x) = \begin{cases} x & \text{if } x \in (A \cup B \cup E \cup F) \cap X^2 \\ \phi & \text{of } x \in (C \cup D) \cap X^2 \end{cases} \tag{13}$$

Hereafter, we focus on equilibria in which satisfying (13) when we construct equilibria.[9]

The full disclosure equilibrium is characterized as follows;

Proposition 2 *Consider the manipulated news model. There exists the full disclosure equilibrium* $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *if and only if either* $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ *or* $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$.

Intuitively, whether there exists the full disclosure equilibrium depends on whether the voter can correctly infer the medium's motivation behind the withholding. For example, suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. That is, at the beginning of the information

[9] Of course, (13) is not the unique best response of the media. Then, if we show an impossibility theorem, then we do not restrict the medium's strategy to (13).

disclosure stage, the voter can infer that the medium wants to conceal the information only about policy pairs in disagreement region C. Given the prior belief of the voter, the withholding itself is a signal showing that the proposed policy pair lies in disagreement region C. Then, the voter chooses candidates 1 if he observes the withholding. Because the voter can correctly infer the medium's motivation of the withholding, the medium cannot conceal the unfavorable information on the equilibrium path, and then, full information disclosure is possible.

However, if the voter is ambiguous about the medium's motivation behind the withholding, then the medium can conceal a part of unfavorable information on the equilibrium path. Suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. In the voter's prior belief, there are two explanations for the withholding. The voter cannot distinguish whether the medium tried to conceal the policy pair in disagreement region C or disagreement region D from observing the withholding. Due to this indeterminacy, the voter's decision-making is incorrect with positive probability on the equilibrium path. That is, full information disclosure is impossible.[10]

As the corollary of Proposition 2, we obtain the following result.

Corollary 1 *Consider the manipulated news model. Suppose that* $b > \frac{1}{2}a$. *Then, in any equilibrium, there exists, at least, one policy pair* $x \in X^2$ *such that* $\gamma^*(\beta^*(x)) \neq y^v(x)$.

That is, the existence of the medium certainly distorts the voter's decision-making for some policy pair. If the full disclosure equilibrium does not exist, then this claim is obvious; the voter chooses the unfavorable candidate with positive probability on the equilibrium path. However, this claim is also true for the full disclosure equilibria. In any full disclosure equilibrium, the voter's decision-making for some off the equilibrium path policy pair must

[10] In the terminology of the persuasion games, if $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then there does not exist the *worst case inference* for message $m = \phi$. Therefore, the *unraveling argument* is failed.

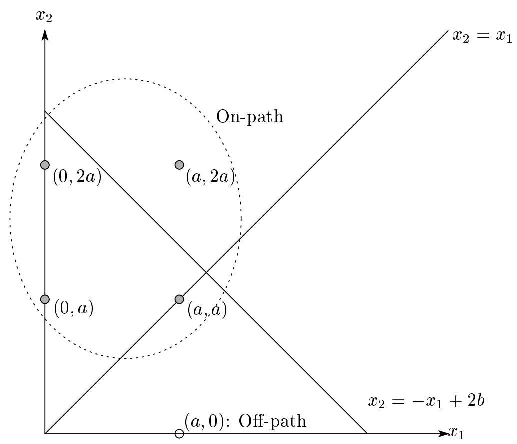

Figure 2: Incorrect decision-making for off the equilibrium path policy.

be incorrect. In other words, the full information disclosure on the equilibrium path is supported by the voter's off the equilibrium path incorrect decision-making.

Suppose, for example, that $\alpha_1^*(\theta_N) = 0$ and $\alpha_2^*(\theta_N) = a$. By Proposition 2, in this scenario, the full disclosure equilibrium exists because $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ as shown in Figure 2. To support the full disclosure equilibrium, the voter's response to the withholding must be $\gamma^*(\phi) = (1, 0)$. In the full disclosure equilibrium, policy pair $x = (a, 0)$ is off the equilibrium path, and the voter's and medium's preferences about that policy pair are disagreed; the voter prefers candidate 2, but the medium prefers candidate 1. Thus, given the voter's response to the withholding $\gamma^*(\phi) = (1, 0)$, the medium who observes policy pair $x = (a, 0)$ withholds the information, and then candidate 1 wins for certain. However, from the view of ex-post, the voter's decision-making is incorrect.

In summary, the medium can conceal a part of unfavorable information when the media bias is not small. Due to media manipulation of information, the voter's decision-making is certainly distorted at some policy pair. In the next subsection, we will see how the voter's incorrect decision-making affects the behaviors of the non-policy type candidates.

4.2 Policy setting stage

In this subsection, we analyze how the non-policy type candidates behave in the policy setting stage. Depending on the magnitude of the media bias, consider the following cases. Then, we demonstrate that $(0,0)$-equilibrium does not exist, i.e., the weak median voter theorem fails if the media bias is large enough.

4.2.1 Case 1: $0 < b < \frac{1}{2}a$

When the media bias is sufficiently small, the medium has no incentive to withhold the information. That is, the voter can correctly learn what is the proposed policy pair through the news from the medium. In other words, this case is equivalent to the benchmark model. Then, $(0,0)$-equilibrium is the unique equilibrium. Therefore, the median voter theorem still holds in the manipulated news model when the media bias is sufficiently small;

Proposition 3 *Consider the manipulated news model with* $0 < b < \frac{1}{2}a$. *Then,* $(0,0)$*-equilibrium is the unique equilibrium. In other words, the strict median voter theorem holds.*

4.2.2 Case 2: $\frac{1}{2}a \leq b \leq a$

In this case, the voter and the medium conflict only over policies 0 and a. Hence, only policy pairs $(0,a)$ and $(a,0)$ are in the disagreement regions.

Proposition 4 *Consider the manipulated news model with* $\frac{1}{2}a \leq b \leq a$.

(i) *The weak median voter theorem holds if and only if* $p \leq \frac{1}{2}$.[11]

(ii) *If* $p \leq \frac{1}{2}$, *then there exists* $(0,a)$ *and* (a,a)*-equilibria.*

(iii) *If* $p > \frac{1}{2}$, *then there exists* (a,a)*-equilibrium.*

[11] This is the necessary and sufficient condition under the tie-braking rules specified in Requirement 2.

(iv) *There exists a mixed strategy equilibrium in which the non-policy type candidates randomize policies* $x_i = 0$ *and* a *for* $i = 1, 2$ *if* $p \geq \frac{1}{2}$.

In the manipulated news model with not small media bias, there are several contrasts with the benchmark model. First, there exist multiple equilibria. In addition to the policy convergence to the voter's ideal policy, we can observe policy divergence, policy convergence to the media's ideal policy and policy randomization in equilibria. In other words, the strict median voter theorem fails. Second, even the weak median voter theorem does not always hold. We need the condition that the policy type candidates are more likely than the non-policy type candidates.

The main reason of these contrasts is that the benefit for the non-policy type candidates from proposing the voter's ideal policy is discounted due to the media manipulation. That is, appealing to the voter becomes less attractive to the candidates. In the benchmark model, proposing other than the voter's ideal policy is dominated by proposing the voter's ideal policy. Because the voter can correctly observe proposed policy pair, proposing the voter's ideal policy makes the candidate more attractive to the voter than proposing other policies. However, in the manipulated news model, the voter could not correctly recognize the attractiveness of the candidate who proposes the voter's ideal policy due to media manipulation of information. As the result, proposing other than the voter's ideal policy could not be dominated by proposing the voter's ideal policy.

For example, suppose that the voter believes that policy pair $x = (0, a)$ and $(a, 0)$ are equally likely when he observes the withholding. That is, the voter's posterior is balanced. Given the posterior, the voter chooses each candidate with equally likely when he observes the withholding. Suppose, in addition, that candidate 1 proposes policy $x_1 = a$. In this scenario, if candidate 2 proposes $x_2 = 0$, then his winning probability is $\frac{1}{2}$ due to media manipulation. However, if the information goes through to the voter without manipulation, then the candidate 2 wins for certain by proposing $x_2 = 0$. That is, the benefit from proposing $x_2 = 0$ is discounted. Particularly, under that posterior of the voter, proposing $x_i = 0$ and a are indifferent for each candidate. Therefore, (a, a)- and the mixed strategy equilibria exist.[12] [13]

In addition to appealing to the voter becoming less attractive to the non-policy type candidates, they have incentives to influence the medium's behaviors when the voter's posterior after the withholding is biased. Suppose, for example, that the voter believes that policy pair $x = (0, a)$ is more likely than policy pair $x = (a, 0)$ after the withholding. Given this posterior, the voter chooses candidate 1 for certain as the best response to the withholding. Hence, given the voter's response, candidate 1 strictly prefers media manipulation, but candidate 2 strictly dislikes it. Then, candidate 1 has a strict incentive to lead media manipulation, and, on the other hand, candidate 2 has a strict incentive to avoid the manipulation. In other words, the candidates choose policies to control the medium's behaviors. We call these incentives of the candidates *media-controlling incentives.*

The media-controlling incentives are exhibited in $(0, 0)$- and $(0, a)$-equilibria, and these incentives, associated with the candidates' uncertainty about the types of opponents, put restrictions on the parameter to hold these equilibria. For instance, in $(0, 0)$-equilibrium, candidate 2 has an incentive to lead media manipulation. That is, candidate 2 would like to induce either policy pair $x = (0, a)$ or $(a, 0)$. However, candidate 2 does not know whether candidate 1 is the non-policy type. Hence, if candidate 1 is more likely to be the

[12]In Case 2, policy $2a$ is the worst policy for both the voter and the medium. Then, if one of the candidates proposes $x_i = 2a$, then the induced policy pair is always disclosed by the medium, and the voter never chooses that candidate. That is, proposing $x_i = 2a$ is strictly dominated by $x_i = 0$ and a.

[13]In (a, a)-equilibrium, policy pairs $x = (0, a)$ and $(a, 0)$ are off the equilibrium path. Hence, any posterior belief given $m = \phi$ is fine. In the mixed strategy equilibrium, candidate 1 and 2 randomize such that the on the equilibrium path belief satisfies $\mathcal{P}^*((0, a)|\phi) = \mathcal{P}^*((a, 0)|\phi)$. The condition $p \geq \frac{1}{2}$ is needed to well define the randomization.

policy type, i.e., more likely to propose $x_1 = a$, then candidate 2's best response is proposing $x_2 = 0$. On the other hand, if candidate 1 is more likely to be the non-policy type, i.e., more likely to propose $x_1 = 0$, then proposing $x_2 = a$ is candidate 2's best response. This is the reason why $(0, 0)$-equilibrium does not always exist; if the non-policy type candidate is more likely than the policy type candidate, then the media-controlling incentive of candidate 2 destroys $(0, 0)$-equilibrium. Similarly, in order to compatible the media-controlling incentive of candidate 2 in $(0, a)$-equilibrium, the same restriction is needed.[14]

4.2.3 Case 3: $a < b \leq \frac{3}{2}a$

In this case, conflict between the voter and the medium becomes more severe. They agree that policy a is weakly better than policy $2a$, but they disagree with other possibilities. That is, policy pairs $(0, a), (0, 2a), (a, 0)$ and $(2a, 0)$ lie in the disagreement regions.

Proposition 5 *Consider the manipulated news model with $a < b \leq \frac{3}{2}a$.*

(i) *The weak median voter theorem fails.*

(ii) *If $p \leq \frac{1}{2}$, then there exists $(0, a), (a, 0)$ and (a, a)-equilibria.*

(iii) *If $\frac{1}{2} < p \leq \frac{2}{3}$, then there exists $(a, 0)$ and (a, a)-equilibria.*

(iv) *If $p > \frac{2}{3}$, then there exists (a, a)-equilibrium.*

(v) *There exists a mixed strategy equilibrium in which the non-policy type candidates randomize $x_i = 0$ and a for $i = 1, 2$ if $p \geq \frac{1}{2}$.*

There are two differences compared with Case 2. First, we can observe new policy divergence, i.e., $(a, 0)$-equilibrium. Second, the weak median voter theorem never holds.[15]

[14] In both equilibria, proposing the voter's ideal policy is compatible with the media-controlling incentive of candidate 1 without any restriction.

[15] This impossibility result depends on the tie-braking rules specified in Requirement 2. We will revisit this point in Section 5.3.

These are the consequence of the larger media bias. The larger media bias implies that the medium more frequently withholds the information, and then, more information can be successfully concealed. The more distorted information transmission due to the larger media bias has two effects. First, the benefit for the non-policy type candidates from appealing to the voter is more discounted. Second, the media-controlling incentives of the non-policy type candidates could be conversed even if the candidates adopt the same strategies in Case 2.

The first effect is straightforward. Because more policy pair is withheld, appealing to the voter by proposing $x_i = 0$ is more difficult. Suppose, for example, that $\alpha_1^*(\theta_N) = a$ and $\alpha_2^*(\theta_N) = 0$. In this case, candidate 1 has the media-controlling incentive for avoiding media manipulation because the voter's posterior after the withholding is biased to "candidate 2 is better". Because policy pair $x = (0, 2a)$ is disclosed in Case 2, proposing $x_1 = a$ is not compatible to candidate 1's media-controlling incentive. In other words, deviating to $x_1 = 0$ is strictly beneficial for candidate 1 because he can avoid media manipulation more frequently than proposing $x_1 = a$. However, in Case 3, the media withholds the information about policy pair $x = (0, 2a)$, and then the advantage of proposing $x_1 = 0$ is limited. That is, as long as $p \leq \frac{2}{3}$, proposing $x_1 = a$ is compatible to candidate 1's media-controlling incentive.

The second effect is more tricky. Because more policy pair is withheld, the voter's posterior after the withholding could be substantially different. For example, suppose that $\alpha_1^*(\theta_N) = \alpha_2^*(\theta_N) = 0$. In Case 2, the voter's posterior after the withholding puts probability 1 to policy pair $x = (a, 0)$, so the voter chooses candidate 2 for certain after the withholding. Hence, candidate 1 has the media-controlling incentive for avoiding media manipulation. However, in Case 3, the voter 's posterior puts probability $\frac{1}{2}$ to policy pairs $x = (0, 2a)$ and $(a, 0)$. Because the voter chooses candidate 1 for certain given this posterior, candidate 1 has the media-controlling incentive for leading media manipulation. In other words, candidate 1's motivation of controlling the medium becomes completely different. Then, proposing $x_1 = 0$ is no longer desirable to candidate 1 under the media-controlling incentive for leading media manipulation. Therefore, $(0, 0)$-equilibrium never exist.

4.2.4 Case 4: $b > \frac{3}{2}a$

In this case, the voter's and medium's preferences are completely disagreed except for the cases of policy convergence. That is, only policy pairs $x = (0, 0), (a, a)$ and $(2a, 2a)$ exist in the agreement regions, and the others exist in the disagreement regions. It is worthwhile to remark that because conflict between the voter and the medium is severe, withholding all information is the weakly dominant strategy of the medium. However, this strategy does not satisfy the tie-braking rule specified in Requirement 2. Hence, for a while, we ignore the possibility that the medium uses the weakly dominant strategy, but we will revisit this point in Section 5.3.

Proposition 6 *Consider the manipulated news model with $b > \frac{3}{2}a$.*

(i) *The weak median voter theorem fails.*

(ii) *If $p < \frac{1}{3}$, then there exists $(0, a)$-equilibrium.*

(iii) *If $\frac{1}{3} \leq p < \frac{1}{2}$, then there exists $(0, a)$- and $(2a, 0)$-equilibria.*

(iv) *If $p = \frac{1}{2}$, then there exists $(0, a)$-, $(2a, 0)$-, $(a, 0)$- and $(2a, a)$-equilibria.*

(v) *There exist mixed strategy equilibria in which the non-policy type candidates randomize*

- **1.** *policies $x_i = 0$ and a for $i = 1, 2$ when $\frac{1}{2} \leq p \leq \frac{2}{3}$.*
- **2.** *policies $x_i = 0$ and $2a$, or policies $x_i = a$ and $2a$ for $i = 1, 2$ when $p \geq \frac{1}{2}$.*
- **3.** *all policies when $p \geq \frac{2}{3}$.*

In this case, we can observe more variety of policy divergence in equilibrium. The reason of the variety of policy divergence is that proposing $x_i = 2a$ is not dominated by the other actions because the media bias is so large that the medium always withholds the information whenever the proposed policy pair is divergent. In Cases 2 and 3, proposing $x_i = 2a$ is risky in the sense that if the proposed policies are divergent and the medium discloses the information, then the candidate who proposes $x_i = 2a$ loses for certain. On the other hand, proposing $x_i = a$ could provide the positive winning probability when the proposed policies are divergent and this information is disclosed. In other words, proposing $x_i = a$ dominates proposing $x_i = 2a$ when the media bias is not so large.

However, if the media bias is sufficiently large, we need not worry about the case where the proposed policies are divergent and this information is disclosed. In Case 4, any divergent policy pair is withheld. As the result of more frequent media manipulation, the advantage of proposing $x_i = a$ over proposing $x_i = 2a$ is disappeared. In other words, in addition to the discount of the benefit for the non-policy type candidates from proposing $x_i = 0$, the benefit from proposing $x_i = a$ is also discounted. Therefore, any policy could be supported in some equilibrium. This is the source of the variety of policy divergence.

It is worthwhile to mention that, in Case 4, there exist only pure strategy equilibria when $p < \frac{1}{2}$, but only mixed strategy equilibria exist when $p > \frac{1}{2}$. This phenomenon is due to the game structure. Because the media bias is sufficiently large, the medium withholds the information whenever the proposed policies are divergent. Associated with the asymmetry between the candidates, the voter's posterior is always biased after the withholding. Then, when the media bias is sufficiently large, for the non-policy type candidates, the media-controlling incentives are stronger than the incentives to appeal to the voter. Therefore, the game structure of the electoral competition when the both candidates are the non-policy type is "matching penny". In other words, the candidate who wants to lead media manipulation tries to induce policy divergence, but the candidate who dislikes the manipulation tries to induce policy convergence. Because of this game structure, only mixed strategy equilibria exist if the candidate are more likely to be the non-policy type.

In summary, media manipulation of information distorts behaviors of the non-policy type candidates through the discount of the benefit from appealing to the voter and the media-controlling incentives. With the growth of the media bias, appealing to the voter becomes less attractive to the non-policy type candidates, but the media-controlling incentives become stronger. Then, the weak median voter theorem fails when the media bias is not small. Finally, if the media bias is sufficiently large, then the non-policy type candidates focus on controlling behaviors of the media, not appealing to the voter, and, therefore, a variety of randomization can be observed in equilibrium.

4.3 Comparison of equilibrium outcomes

Given the analysis so far, we compare equilibrium outcomes of the manipulated news model with those of the benchmark model. As we have already shown that the strict median voter theorem does not hold when the media bias is not small, and then, equilibrium outcomes are definitely distorted. In this subsection, we consider a question of how equilibrium outcomes are distorted due to media manipulation.

We focus on the mixed strategy equilibrium in which the non-policy type candidates randomize policies $x_i = 0$ and a when the media bias is $a < b \leq \frac{3}{2}a$, which is specified in Proposition 5-(v). The equilibrium outcomes are summarized in Table 2. We can observe two kinds of distortions in this equilibrium. The first distortion comes from the distortions in the candidates' behaviors, and the second one comes from the distortions in the voter's behaviors. The first is called *indirect distortion*, and the second is called *direct distortion*.

In the indirect distortion, equilibrium outcomes are distorted compared with those of the benchmark model through the distortion in the candidates' behaviors. As we have

probability	proposed policy pair	media	winner	equilibrium policy
$\frac{1}{4}p$	$(0,0)$	discloses	1 or 2	0
$\frac{1}{4}p$	$(0,a)$	withholds	1	0
$\frac{1}{2}(1-p)$	$(0,2a)$	withholds	1	0
$\frac{1}{4}p$	$(a,0)$	withholds	1	a
$\frac{1}{4}p$	(a,a)	discloses	1 or 2	a
$\frac{1}{2}(1-p)$	$(a,2a)$	discloses	1	a

Table 2: Equilibrium outcomes in the mixed strategy equilibrium

already mentioned, the non-policy type candidates have incentives to propose other than the voter's ideal policy due to media manipulation. As the result of the distortions in the candidates' behaviors, the policy pairs proposed on the equilibrium path are changed, and then, the winning policy is also changed. This is the indirect distortion. In the mixed strategy equilibrium, the indirect distortion is appeared in the fifth row of Table 2. As shown in Table 1, policy pair $x = (a, a)$ is never proposed on the equilibrium path in the benchmark model. However, because the non-policy type candidates randomize policies, that policy pair can be proposed on the equilibrium path. Therefore, policy a becomes the winning policy even if candidate 2 is the non-policy type.

On the other hand, the direct distortion is the distortion through the distortion in the voter's behaviors. As shown in Proposition 2, the voter's decision-making could be incorrect on the equilibrium path. That is, due to the media manipulation, the voter cannot obtain enough information, and then, he chooses the unfavorable candidate with positive probability. As the result of the incorrect decision-making, the winning policy is different from that of the benchmark model. This is the direct distortion. In the mixed strategy equilibrium, we can observe the direct distortion in the fourth row of Table 2. Policy pair $x = (a, 0)$ is proposed on the equilibrium path in both the benchmark and the manipulated news models. As shown in Table 1, policy 0 is the winning policy in the benchmark model. However, the winning policy is a in the mixed strategy equilibrium due to the voter's incorrect choice.

We can observe either one of the above distortions in all equilibria except for $(0, 0)$-equilibrium. Then, the voter's ex-ante expected utility in the manipulated news model is less than that of the benchmark model; the winning policy is distorted to the media's ideal policy with positive probability. Therefore, we can conclude that if we measure the social welfare by the voter's ex-ante expected utility, then the existence of the medium reduces the social welfare.

5 Discussion

Our analysis so far is based on several assumptions, for example, the asymmetry between the candidates, directions of the policy type candidates' preferred policies, the tie-braking rules, and the simplified message structure. In this section, we study how the results change once the assumptions are relaxed, and briefly discuss the robustness of our results.

5.1 Asymmetry between the candidates

We have assumed that the candidates are asymmetric in the sense that the preferred policies of the policy type candidates are different; the policy type candidate 1 prefers proposing $x_1 = a$, but the policy type candidate 2 prefers proposing $x_2 = 2a$. In order to consider the importance of the asymmetry, we modify the model as follows; the von Neumann-Morgenstern utility function of candidate 2 is defined by:

$$u_2(\theta_2, x_2, y) = \begin{cases} 1 & \text{if } \theta_2 = \theta_N \text{ and } y = y_2 \\ 1 & \text{if } \theta_2 = \theta_P \text{ and } x_2 = a \text{ and } y = y_2 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

That is, the policy type candidate 2 also prefers proposing $x_2 = a$. In other words, the candidates are completely symmetric. Except for this modification, the model setup is equivalent to so far.

In contrast with the asymmetric setup, $(0, 0)$-equilibrium exists without any restrictions on the parameters.[16] That is, the weak median voter theorem is persistent in the symmetric setup. The reason for this persistence is that the voter's posterior after the withholding tends to be balanced due to the symmetry between the candidates. That is, choosing candidate 1 and 2 tend to be indifferent for the voter after observing the withholding. Similar to the asymmetric setup, it is less attractive to the non-policy type candidates to appeal to the voter because of media manipulation, and then, there exist multiple equilibria.[17] However, if the voter has the balanced posterior, then the non-policy type candidates do not have the media-controlling incentives, which are the main force for destroying $(0, 0)$-equilibrium in the asymmetric setup. Therefore, $(0, 0)$-equilibrium can be more easily supported in the symmetric setup. In addition to $(0, 0)$-equilibrium, (a, a)- and $(2a, 2a)$-equilibria are more persistent in the symmetric setup compared with the asymmetric setup because of the same reason.[18] That is, we can say that the symmetry between the candidates is crucial for supporting policy convergent equilibria.

However, the persistence of $(0, 0)$-equilibrium is not robust because the symmetry between the candidates is a demanding condition. Incorporating a slightly difference into the preferred policies of the non-policy type candidates is enough for destroying this persistent result; once introducing a small difference, the model moves back to the asymmetric setup.

[16]The formal statement and the proof are in Appendix B.

[17]There exists $(0, a)$- and $(a, 0)$-equilibria when $p \leq \frac{1}{2}$, and the mixed strategy equilibrium where the non-policy type candidates randomize policies $x_i = 0$ and a when $p \geq \frac{1}{2}$. The formal proof is available from the author upon requests.

[18]While (a, a)-equilibrium exists without any restrictions on the parameters, we need $b > \frac{3}{2}a$ to hold $(2a, 2a)$-equilibrium.

5.2 Directions of the preferred policies of the policy type candidates

We have assumed that the directions of the preferred policies of the policy type candidates are same to the media bias. That is, the medium's ideal point b and the policy type candidates' preferred policies a and $2a$ are all in the right side of the voter's ideal point 0. In this subsection, instead, we assume that the voter's ideal policy is in the middle of the policy type candidates' preferred policies, as widely assumed in the literature. We modify the setup as follows. The policy space is $X \equiv \{-a, 0, a\}$ with $a > 0$, and candidate 2's von Neumann-Morgenstern utility function is defined by:

$$u_2(\theta_2, x_2, y) = \begin{cases} 1 & \text{if } \theta_2 = \theta_N \text{ and } y = y_2 \\ 1 & \text{if } \theta_2 = \theta_P \text{ and } x_2 = -a \text{ and } y = y_2 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

To make easy the reference, we call this setup *two-sided setup*, and the original setup *one-sided setup.*

The results of the two-sided setup are similar to those of the one-sided setup. The strict median voter theorem holds when the media bias is small enough, and there exist multiple equilibria when the media bias is not small.[19] However, there is a difference in the persistence of $(0, 0)$-equilibrium. In the two-sided setup, $(0, 0)$-equilibrium could exist even when the media bias is large enough. The reason of the persistence is that there is less room for the media manipulation in the two-sided setup even if the media bias is large.

The structure of $(0, 0)$-equilibrium in the two-sided setup is equivalent to Case 2 in the one-sided setup. Due to the biased posterior after the withholding, the non-policy type candidates have the media-controlling incentives; candidate 1 dislikes media manipulation, but candidate 2 prefers it. In the one-sided setup, this structure of the media-controlling

[19] There exists $(0, 0)$- and $(0, a)$-equilibria when $p \leq \frac{1}{2}$, and only the mixed strategy equilibrium in which the non-policy type candidates randomize policies $x_i = 0$ and $x_i = a$ when $p \geq \frac{1}{2}$. The formal proof is available from the author upon requests.

incentives is conversed with the growth of the media bias. With the growth of the media bias, more policy pairs become to be withheld on the equilibrium path, and then, the voter's posterior after the withholding becomes to be biased in the opposite direction. As the result, the structure of the media-controlling incentives is conversed. Then, the weak median voter theorem fails in the case of large media bias, as mentioned in section 4.2.3.

Contrary to the one-sided setup, the structure of the media-controlling incentives is robust to the growth of the media bias in the two-sided setup because the larger media bias does not guarantee more manipulation. Since the media bias is fixed in the positive direction, the medium never withholds policy pair $x = (0, -a)$, which is proposed on $(0, 0)$-equilibrium path, irrelevant to the magnitude of the media bias. In other words, the information transmitted on the equilibrium path is no more distorted even if the media bias becomes larger. Therefore, the structure of the media-controlling incentives is independent of further growth of the media bias. This is the reason of the persistence.

However, we replicate the exactly same results to the one-sided setup in the two-sided setup by admitting the medium's preference to each candidate. For example, we modify the medium's preference as follows:

$$w(x, y, b) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1 \\ -|x_2 + b| & \text{if } y = y_2. \end{cases} \tag{16}$$

One interpretation of this preference is that the medium evaluates the winning policy with the character of the winning candidate. The medium's preference to candidate 1's policy is right-biased, but her preference to candidate 2's policy is left-biased.

5.3 Tie-breaking rules

The tie-breaking rules specified in Requirement 2 is crucial to the results. While the tie-breaking rules for the voter is well accepted in the literature of voting, for the medium seems to be more controversial. We assume that the medium discloses the information whenever the proposed policies are convergent. However, there is no absolute reason for the disclosure of convergent policies. For example, when the media bias is large enough, i.e., $b > \frac{3}{2}a$, withholding all policy pair is medium's weak dominant strategy. It is reasonable to focus on the straightforward strategy, but, obviously, this dominant strategy violates the tie-breaking rule. In this sub-section, we consider the manipulated news model without requiring the tie-breaking rule for the medium.

Once the tie-breaking rule is relaxed, there exist more equilibria. Suppose, for example, that $b > \frac{3}{2}a$ and the medium always withholds the information. In this scenario, candidates' any strategies can be supported as equilibrium strategies. Because the medium sticks to withhold the information, the non-policy type candidates have no incentive to deviate for changing the medium's behaviors. In other words, because the medium always withholds the information, the voter's decision-making is unchanged even if the proposed policy pair is changed. Then, any deviations of the candidates cannot improve their own winning probabilities. Therefore, any policy pair can be supported as an equilibrium policy pair.

Although this multiplicity is serious problem, most of the equilibria are not robust to small perturbations in the medium's behaviors. Instead of assuming full disclosure or full withholding, we assume that the medium discloses the information about convergent policies with probability $\epsilon \in (0, 1)$. That is, the medium who observes the convergent policy pairs randomizes disclosure and withholding.[20] To make easy the reference, we call this tie-breaking rule *ϵ-randomization rule*, and the original *disclosure rule*. We can show that even if the probability of disclosure ϵ is small enough, then the set of equilibrium policy pairs under the ϵ-randomization rule is equivalent to that under the disclosure rule.[21] Similarly, once we relax the disclosure rule, then there exists $(0, 0)$-equilibrium without any restrictions

[20]Because the result of the election is indifferent for the medium when the proposed policy is convergent, such a randomization can be supported as one of the best response of the medium.

[21]The formal statements and the proof are in Appendix B.

when $\frac{1}{2}a < b \leq \frac{3}{2}a$. However, this result is also not robust to the ϵ-randomization. Even if the disclosure probability ϵ is sufficiently small, the result about the existence of $(0,0)$-equilibrium is equivalent to that under the disclosure rule. Therefore, we can justify that focusing on equilibria satisfying the disclosure rule from the view of the robustness.

5.4 Simplified message structure

We have assumed that the medium takes only full disclosure or full withholding. It is not difficult to consider richer message space, for example, $M(x) \equiv \{S \subseteq X^2 | x \in S\}$; that is the medium sends any message containing the truth. The same setup except for the modification of the message space derives the similar results. We can observe more equilibrium, but the persistence of $(0,0)$-equilibrium is never improved.[22] Because of the rich message space, the medium could conceal more unfavorable information, and then, the voter's decision-making is more distorted. That is, appealing to the voter is less attractive to the non-policy type candidates compared with the simplified message space case. Therefore, more variation of policy distributions can be observed in equilibrium.

However, the rich message space never improves information transmission. According to Miura (2010), the equilibria focused in Section 4 are associated with the most informative ones in the more general persuasion game. With the asymmetry between the candidates, the rich message space does not guarantee the existence of an equilibrium in which the structure of the media-controlling incentives is different from that in the simplified message space setup. Then, the persistence of $(0,0)$-equilibrium is not improved. Thus, we can say that the simplified message structure is not essential to the results.

[22] The formal statements and the proof are in Appendix B.

6 Conclusion

This paper analyzes a Downsian voting model including one mass medium, and explicitly describes the strategic aspects of the medium's reporting behavior. We show that the medium can conceal a part of unfavorable information in any equilibrium when the media bias is not small. Because of this media manipulation of information, appealing to the voter becomes more difficult than the model without mass medium. Then, we can observe a variety of policy distribution in equilibrium. Moreover, the non-policy type candidates additionally have the media-controlling incentives. With the growth of the media bias, the media-controlling incentives dominate the incentives of appealing to the voter. That is, the candidates choose policies in order to control the medium's behavior, not to appealing to the voter when the media bias is large enough. Then, the weak median voter theorem fails. As the result, equilibrium outcomes in the manipulated news model are distorted compared with the benchmark model; the medium's preferred policy is supported in equilibrium with higher probability than the benchmark model. We can observe two channels of the distortion. In the indirect distortion, equilibrium outcomes are distorted through the distortion in the candidates' behaviors. On the other hand, in the direct distortion, equilibrium outcomes are distorted through the distortion in the voter's behaviors.

As the conclusion of this paper, we briefly discuss extensions. First, we should incorporate media competitions in this framework. In this paper, there exists only one mass medium, and the medium's preference is fully policy-motivated. However, in the real world, there exist multiple media, and they are also interested in profits (e.g. Mullainathan and Shleifer (2005), Barron (2006), and Gentzkow and Shapiro. (2006)). Hence, it seems to be interesting to consider how the results change if we introduce competitions among mass media who have policy and profit motivated preferences.

Second, while we assume that the winning candidate is fully committed to implement the proposed policy, we can relax this assumption. That is, the winning candidate may not implement the proposed policy (e.g. Banks (1991) and Harrington (1992).). This phenomenon is sometimes observed in the real world. If the winning candidates could change proposed policy with positive probability, then, in our conjecture, media manipulations could become more severe because the possibility expands the set of possible policy outcomes. However, the result of manipulation would not be straightforward.

Appendix A: Proofs of Propositions

Proof of Proposition 1

(i) we show that the following is a PBE:

$$\alpha_1^*(\theta_1) = \begin{cases} 0 & \text{if } \theta_1 = \theta_N \\ a & \text{if } \theta_1 = \theta_P \end{cases} \tag{17}$$

$$\alpha_2^*(\theta_2) = \begin{cases} 0 & \text{if } \theta_2 = \theta_N \\ 2a & \text{if } \theta_2 = \theta_P \end{cases} \tag{18}$$

$$\gamma^* = \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2},\frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2|. \end{cases} \tag{19}$$

As we have already mentioned in the body of the paper, γ^* and $\alpha_i^*(\theta_P)$ are the voter and the policy type candidates' equilibrium behaviors. It is easily shown that for the non-policy type candidates, proposing $x_i = 0$ is a dominant action; for the non-policy type candidate 1, it is the weakly dominant action, and for the non-policy type candidate 2, it is the strictly dominant action. Therefore, policy pair $x = (0, 0)$ is the unique equilibrium policy by the non-policy type candidates. That is, the median voter theorem holds. (ii) It is obvious from Table 1. ■

Proof of Proposition 2

(Necessity) Suppose, in contrast, that there exists the full disclosure equilibrium when $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Pick $x' \in C \cap Z(\alpha_1^*, \alpha_2^*)$ and $x'' \in D \cap Z(\alpha_1^*, \alpha_2^*)$, arbitrarily. Then, $\gamma^*(\beta^*(x')) = (1, 0)$ and $\gamma^*(\beta^*(x'')) = (0, 1)$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$. That is, at least, one of the medium who observes policy pair either x' or x'' discloses the information. Without loss of generality, assume that $\beta^*(x') = x'$.

(i) $\beta^*(x'') = x''$**.** In this scenario, $m = \phi$ is an off the equilibrium path message. Let $\gamma^*(\phi) = (q, 1-q)$ be the voter's response to off the equilibrium path message $m = \phi$, where $q \in [0, 1]$. Because the medium who observes policy pair x'' choses $m = x''$ on the equilibrium path, $q = 0$; otherwise the medium has an incentive to deviate from $m = x''$ to $m = \phi$. However, given $\gamma^*(\phi) = (0, 1)$, the medium who observes policy pair x' deviates from $m = x'$ to $m = \phi$, a contradiction.

(ii) $\beta^*(x'') = \phi$**.** By the hypothesis, $\gamma^*(\phi) = (0, 1)$. However, given the voter's best response, the medium who observes policy pair x' deviates to $m = \phi$, a contradiction.

Therefore, if there exists the full disclosure equilibrium, then either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*)$.

(Sufficiency) Suppose that either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. Let $S(\mathcal{P}^*(\cdot|\phi))$ be the support of the voter's posterior after observing $m = \phi$. Then, we show the there exists the full disclosure equilibrium supported by the medium's strategy specified by (13).

Case 1: $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$**.** Because any point in $Z(\alpha_1^*, \alpha_2^*)$ is in the agreement regions, $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full disclosure equilibrium because the voter can chooses the preferred candidate for certain on the equilibrium path.

Case 2: $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. Given the medium's equilibrium strategy specified by (13), $S(\mathcal{P}^*(\cdot|\phi)) \subset C$. Hence, $\gamma^*(\beta^*(x)) = (1,0)$ for any $x \in C \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, because the medium discloses the information about policy pairs in the agreement regions, this is the full disclosure equilibrium.

Case 3: $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Similar to Case 2, $S(\mathcal{P}^*(\cdot|\phi)) \subset D$ given the medium's equilibrium strategy specified by (13), and then $\gamma^*(\beta^*(x)) = (0,1)$ for all $x \in D \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full disclosure equilibrium. ■

Proof of Corollary 1

Suppose that $b > \frac{1}{2}a$. Then, policy pair $(0,a)$ lies in disagreement region C, and policy pair $(a,0)$ lies in disagreement region D. By Requirement 2, the voter's best response to $m = \phi$ must be either $\gamma^*(\phi) = (1,0), (\frac{1}{2}, \frac{1}{2})$ or $(0,1)$. If $\gamma^*(\phi) = (1,0)$, then the medium who observes policy pair $(a,0)$ withholds the information, and then $\gamma^*(\beta^*((a,0))) \neq y^v((a,0))$. Similarly, if $\gamma^*(\phi) = (0,1)$, then $\gamma^*(\beta^*((0,a))) \neq y^v((0,a))$. If $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$, then $\gamma^*(\beta^*(x)) \neq y^v(x)$ for $x = (0,a)$ and $(a,0)$ because the medium who observes policy pair $(0,a)$ or $(a,0)$ strictly prefers the withholding. ■

Proof of Proposition 3

Suppose that $0 < b < \frac{1}{2}a$. Then, and policy pair $x \in X^2$ lies in the agreement regions. That is, whatever policy pair is observed, the medium's preference is completely aligned with that of the voter. Hence, the medium has no incentive to withhold the information. In other words, the voter faces no uncertainty about what is the actually proposed policy pair. Therefore, this scenario is equivalent to the benchmark model. By Proposition 1, $(0,0)$ is the unique equilibrium policy pair by the non-policy type candidates. This equilibrium exists for any $p \in (0,1)$. ■

Proof of Proposition 4

(i) (Sufficiency) Suppose that $p \leq \frac{1}{2}$. We show that the following is a PBE. Note that only policy pairs $x = (0, a)$ and $(a, 0)$ lie in the disagreement regions because $\frac{1}{2}a \leq b \leq a$.

$$\begin{aligned}
\alpha_1^*(\theta_1) &= \begin{cases} 0 & \text{if } \theta_1 = \theta_N \\ a & \text{if } \theta_1 = \theta_P \end{cases} \\
\alpha_2^*(\theta_2) &= \begin{cases} 0 & \text{if } \theta_2 = \theta_N \\ 2a & \text{if } \theta_2 = \theta_P \end{cases} \\
\beta^*(x) &= \begin{cases} \phi & \text{if } x = (0, a) \text{ or } (a, 0) \\ x & \text{otherwise} \end{cases} \\
\gamma^*(m) &= \begin{cases} (1, 0) & \text{if } m = (0, a), (0, 2a) \text{ or } (a, 2a) \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } m = (0, 0), (a, a) \text{ or } (2a, 2a) \\ (0, 1) & \text{if } m = (a, 0), (2a, 0), (2a, a) \text{ or } \phi \end{cases} \\
\mathcal{P}^*(x|m) &= \begin{cases} 1 & \text{if } [m = x' \text{ and } x = x' \text{ for any } x' \in X^2] \text{ or } [m = \phi \text{ and } x = (a, 0)] \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{20}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ is the voter and the medium's best responses. For candidate 1 the winning probabilities from proposing $x_1 = 0, a$ and $2a$ are $1 - \frac{1}{2}p, 1 - p$ and $\frac{1}{2}(1 - p)$, respectively. Obviously, $x_1 = 0$ dominates the others, so candidate 1 does not deviate from $x_1 = 0$. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $1 - \frac{1}{2}p$, $\frac{1}{2}(1+p)$ and 0, respectively. Because $p \leq \frac{1}{2}$, candidate 2 does not deviate from $x_2 = 0$. Obviously, $\mathcal{P}^*(\cdot)$ is consistent with Bayes' rule on the equilibrium path. Hence, this is a PBE.

(Necessity) Suppose, in contrast, that there exists $(0, 0)$-equilibrium when $p > \frac{1}{2}$. Because $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, 2a), (a, 0), (a, 2a)\}$, the following two scenarios are possible.

(1) Full disclosure scenario. By Proposition 2, the full disclosure equilibrium is possible; that is, the voter's decision-making is always correct on the equilibrium path. To support the full disclosure equilibrium, $\gamma^*(\phi) = (0, 1)$ is needed; otherwise, the medium who observes policy pair $x = (a, 0)$ deviates. Hence, the medium sends $m = \phi$ when she observes policy pair $x = (0, a)$; this is off the equilibrium path. Given the voter and the medium's strategies, the winning probabilities of candidates are same to the equilibrium characterized in the sufficiency part, so if $p > \frac{1}{2}$, then candidate 2 deviates to $x_2 = a$, a contradiction.

(2) Withholding scenario. Suppose that the medium who observes policy pair $x = (a, 0)$ is pooling with the medium who observes the policy pair either $x = (a, 2a)$ or $(0, 2a)$ by sending $m = \phi$. Because $(a, 2a)$ and $(0, 2a)$ are in agreement region A, $\gamma^*(\phi) = (1, 0)$ is needed to hold this equilibrium; otherwise, the medium who observes either $x = (a, 2a)$ or $(0, 2a)$ deviates. Given the voter and the medium's strategies, candidate 1's winning probability from $x_1 = 0$ is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = 1$ is 1. Hence, candidate 1 deviates to $x_1 = a$, a contradiction.

Therefore, to hold $(0, 0)$-equilibrium, $p \leq \frac{1}{2}$ is needed.

(ii), (iii), (iv) We show that the followings are PBEs. We focus on equilibria in which the medium's equilibrium strategy is equivalent to that specified in (20). In addition, we restrict our attention to the case in which the policy type candidates always propose the preferred candidates. Moreover, the voter's posterior and best response to the disclosure message is uniquely determined in any equilibrium; they are same to that specified in (20). To avoid repetition, we, hereafter, omit the descriptions of the medium's equilibrium strategy, the policy type candidates' equilibrium behaviors, and the posterior and best response of the voter after observing the disclosure message.

(1) $(0,a)$**-equilibrium.** Suppose that $p \leq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= 0 \\
\alpha_2^*(\theta_N) &= a \\
\gamma^*(\phi) &= (1,0) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} 1 & \text{if } x = (0,a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{21}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ are the best responses of the voter and the medium, respectively. For candidate 1, the winning probability from $x_1 = 0$ is 1, so candidate 1 has no incentive to deviate. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{2}p, \frac{1}{2}(1-p)$ and 0, respectively. Because $p \leq \frac{1}{2}$, candidate 2 does not deviate from $x_2 = 0$. Obviously, the belief is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. □

(2) (a,a)**-equilibrium.** For any $p \in (0,1)$:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= \alpha_2^*(\theta_N) = a \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0,a) \text{ or } (a,0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{22}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ are the best responses of the voter and the medium, respectively. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. Hence, candidate 1 does not deviate from $x_1 = a$. For candidate 2, the winning probabilities from $x_1 = 0, a$ and $2a$ are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Hence, candidate 2 does not deviate from $x_1 = a$. Obviously, $\mathcal{P}^*(\cdot|\cdot)$ is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. □

(3) Mixed strategy equilibrium. Suppose that $p \geq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (\frac{1}{2p}, 1 - \frac{1}{2p}, 0) \\
\alpha_2^*(\theta_N) &= (\frac{1}{2}, \frac{1}{2}, 0) \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0, a) \text{ or } (a, 0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{23}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ are the best responses of the voter and the medium, respectively. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. Hence, randomizing $x_1 = 0$ and a are one of his best response. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Hence, randomizing $x_2 = 0$ and a are one of his best response. Obviously, the posterior is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. ■

Proof of Proposition 5

Lemma 1 *Fix an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $(C \cup D) \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$, arbitrarily. Then, for any policy pairs $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*(x')) = \gamma^*(\beta^*(x''))$.*

Proof of Lemma 1. Suppose, in contrast, that there exists an equilibrium such that for some policy pairs $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$.

Case 1: $x', x'' \in C$ or $x', x'' \in D$. Without loss of generality, assume that $x', x'' \in C$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. If $\beta^*(x') = x'$ and $\beta^*(x'') = x''$, then $\gamma^*(x') = \gamma^*(x'') = (1, 0)$. Hence, exactly one of either x' or x'' must send $m = \phi$. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$.

Then, $\gamma^*(x') = (1, 0)$. In addition, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the medium who observes policy pair x' has an incentive to deviate to $m = \phi$, a contradiction.

Case 2: $x' \in C$ **and** $x'' \in D$**.** Again, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. From Proposition 2, the medium's full disclosure behavior cannot supported in the equilibrium. Then, the medium must withhold the information for exactly one of the policy pair x' or x''. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$. Because $\gamma^*(x') = (1, 0)$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the medium who observes policy pair x' deviates to $m = \phi$, a contradiction. ■

Proof of Proposition 5. (i) Suppose, by contrast that there exists $(0, 0)$-equilibrium. That is, $\alpha_1^*(\theta_N) = 0$ and $\alpha_2^*(\theta_N) = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, 2a), (a, 0), (a, 2a)\}$. By Proposition 2, full disclosure is impossible. Then, consider the following two cases;

(1) $(a, 0)$ and $(a, 2a)$ are separating. Suppose, by contrast, that $\beta^*((0, 2a)) \neq \beta^*((a, 0))$. Because policy pair $x = (a, 0)$ is not pooling with any policy pairs in $Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*((a, 0))) = (0, 1)$. By Lemma 1, $\gamma^*(\beta^*((a, 0))) = \gamma^*(\beta^*((0, 2a))) = (0, 1)$. Then, $\beta^*((0, 2a)) = \beta^*((a, 2a)) = \phi$; otherwise, $\gamma^*(\beta^*((0, 2a))) = (1, 0)$. However, if $\gamma^*(\phi) = (0, 1)$, then the medium who observes policy pair $x = (a, 2a)$ deviates to $m = (a, 2a)$, a contradiction. Therefore, in this sub-case, $\beta^*((0, 2a)) = \beta^*((a, 0)) = \phi$ and $\beta^*((a, 2a)) = (a, 2a)$ must hold. The voter's posterior after observing $m = \phi$ is $\mathcal{P}^*((0, 2a)|\phi) = \mathcal{P}^*((a, 0)|\phi) = \frac{1}{2}$. Because $2\mathcal{P}^*((0, 2a)|\phi) > \mathcal{P}^*((a, 0)|\phi)$, $\gamma^*(\phi) = (1, 0)$. Given the voter and the medium's strategies, for candidate 1, the winning probability from $x_1 = 0$ is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = a$ is 1. Hence, candidate 1 deviates to $x_1 = a$, a contradiction.

(2) $(a, 0)$ and $(a, 2a)$ are pooling. That is, $\beta^*((a, 0)) = \beta^*((a, 2a)) = \phi$. To hold this equilibrium, $\gamma^*(\phi) = (1, 0)$ must hold; otherwise, the medium who observes policy pair $x = (a, 2a)$ deviates. However, if $\gamma^*(\phi) = (1, 0)$, then candidate 1 deviates to $x_1 = a$, as shown in sub-case (1), a contradiction.

Therefore, $(0, 0)$-equilibrium does not exist.

(ii), (iii), (iv), (v) We show that, for each case, there exists an equilibrium in which the medium's equilibrium strategy is characterized as follows:

$$\beta^*(x) = \begin{cases} \phi & \text{if } x = (0, a), (0, 2a), (a, 0) \text{ or } (2a, 0) \\ x & \text{otherwise} \end{cases} \tag{24}$$

Because of the same reasons mentioned in the proof of Proposition 4, we also omit to describe $\alpha_i^*(\theta_P), \gamma^*(x)$ and $\mathcal{P}^*(\cdot|x)$, hereafter.

(1) $(0, a)$**-equilibrium.** Suppose that $p \leq \frac{1}{2}$. Then:

$$\begin{aligned} \alpha_1^*(\theta_N) &= 0 \\ \alpha_2^*(\theta_N) &= a \\ \gamma^*(\phi) &= (1, 0) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} p & \text{if } x = (0, a) \\ 1 - p & \text{if } x = (0, 2a) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{25}$$

Given the posterior, $\gamma^*(\phi) = (1, 0)$ is the voter's best response. Given the voter's strategy, (24) is obviously the medium's best response. For candidate 1, proposing $x_1 = a$ induces the winning probability 1, so he has no incentive to deviate. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{2}p, \frac{1}{2}(1 - p)$ and 0, respectively. Hence, candidate 2 has no incentive to deviate because $p \leq \frac{1}{2}$. Obviously, the posterior is consistent with Bayes' rule. Thus, this is a PBE. □

(2) $(a, 0)$**-equilibrium.** Suppose that $p \leq \frac{2}{3}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= a \\
\alpha_2^*(\theta_N) &= 0 \\
\gamma^*(\phi) &= (0,1) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} 1 & \text{if } x = (a,0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{26}$$

Given the posterior, $\gamma^*(\phi) = (1, 0)$ is the voter's best response. Given the voter's strategy, (24) is obviously the medium's best response. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $\frac{1}{2}p, 1-p$ and $\frac{1}{2}(1-p)$, respectively. Hence, because $p \leq \frac{2}{3}$, candidate 1 does not deviate from $x_1 = a$. For candidate 2, the winning probability from $x_2 = 0$ is 1, so he has no incentive to deviate. The posterior is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE.[23] □

(3) (a, a)**-equilibrium.** For any $p \in (0, 1)$:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= \alpha_2^*(\theta_N) = a \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{4} & \text{if } x = (0,a), (0,2a), (a,0) \text{ or } (2a,0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{27}$$

Given the posterior, $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$ is the voter's best response. Given the voter's strategy, (24) is obviously the medium's best response. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $\frac{1}{2}, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. Hence, candidate 1 does not deviate from $x_1 = a$. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Hence, candidate 2 does not deviate

[23] $\beta^*(x) = \phi$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$ with $\gamma^*(\phi) = (1, 0)$ is possible. However, in this scenario, candidate 2 deviates to $x_2 = a$.

from $x_2 = a$. Obviously, the posterior is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. $\square$

(4) Mixed strategy equilibrium. Suppose that $p \geq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (\frac{1}{2p}, 1 - \frac{1}{2p}, 0) \\
\alpha_2^*(\theta_N) &= (\frac{1}{2}, \frac{1}{2}, 0) \\
\gamma^*(\phi) &= (1, 0) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} 1-p & \text{if } x = (0, 2a) \\ \frac{1}{2}p & \text{if } x = (0, a) \text{ or } (a, 0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{28}$$

Given the voter's posterior, $\gamma^*(\phi) = (1, 0)$ is the voter's best response because $2\mathcal{P}^*((0, 2a)|\phi)+$ $\mathcal{P}^*((0, a)|\phi) > \mathcal{P}^*((a, 0)|\phi) + 2\mathcal{P}^*((2a, 0)|\phi)$. Given the voter's strategy, (24) is obviously the medium's best response. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $1 - \frac{1}{4}p, 1 - \frac{1}{4}p$ and $\frac{1}{2}$, respectively. Hence, any randomizing between $x_1 = 0$ and a is one of the candidate 1's best response. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{4}, \frac{1}{4}$ and 0, respectively. Hence, any randomization between $x_2 = 0$ and a is one of the candidate 2's best response. Therefore, $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule given $\alpha_1^*(\cdot)$ and $\alpha_2^*(\cdot)$. Thus, this is a PBE. $\blacksquare$

Proof of Proposition 6

(i) Suppose, in contrast, that there exists $(0, 0)$-equilibrium. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, 2a), (a, 0), (a, 2a)\}$. By Proposition 2, the full disclosure equilibrium never exists. In addition, by Lemma 1, $\gamma^*(\beta^*((0, 2a))) = \gamma^*(\beta^*((a, 2a))) = \gamma^*(\beta^*((a, 0)))$ must hold. Hence, there are the following sub-cases.

(1) $\beta^*(x) = \phi$ **for** $x = (0, 2a), (a, 0)$ **and** $(a, 2a)$**.** Given the candidates and the medium's strategies, the voter's consistent belief after the withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{p}{1+p} & \text{if } x = (0, 2a) \text{ or } (a, 0) \\ \frac{1-p}{1+p} & \text{if } x = (a, 2a) \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

Given the posterior, the voter's best response to the withholding is $\gamma^*(\phi) = (1, 0)$. Then, for candidate 1, his winning probability from proposing $x_1 = 0$ is $1 - \frac{1}{2}p$. However, if candidate 1 proposes $x_1 = a$, then his winning probability is 1. Then, candidate 1 has an incentive to deviate, a contradiction.

(2) $\beta^*((a, 2a)) = \beta^*((a, 0)) = \phi$ **and** $\beta^*((0, 2a)) = (0, 2a)$**.** Because $\gamma^*(\beta^*((0, 2a))) = (1, 0)$, by Lemma 1, $\gamma^*(\phi) = (1, 0)$ is needed. However, given $\gamma^*(\phi) = (1, 0)$, candidate 1 has an incentive to deviate to $x_1 = a$ as shown in (1), a contradiction.

(3) $\beta^*((0, 2a)) = \beta^*((a, 0)) = \phi$ **and** $\beta^*((a, 2a)) = (a, 2a)$**.** We can derive a contradiction by the same argument in (2).

Therefore, $(0, 0)$-equilibrium never exists.

(ii), (iii), (iv) We show that, for each case, there exists an equilibrium in which the medium's equilibrium strategy is characterized as follows:

$$\beta^*(x) \quad = \quad \begin{cases} x & \text{if } x = (0, 0), (a, a) \text{ or } (2a, 2a) \\ \phi & \text{otherwise} \end{cases} \tag{30}$$

Because of the same reasons mentioned in the proof of Proposition 4, we also omit to describe $\alpha_i^*(\theta_P), \gamma^*(x)$ and $\mathcal{P}^*(\cdot|x)$, hereafter.

(1) $(0,a)$**-equilibrium.** Suppose that $p \leq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= 0 \\
\alpha_2^*(\theta_N) &= a \\
\gamma^*(\phi) &= (1,0) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{p^2}{1-p+p^2} & \text{if } x=(0,a) \\ \frac{p(1-p)}{1-p+p^2} & \text{if } x=(0,2a) \\ \frac{(1-p)^2}{1-p+p^2} & \text{if } x=(a,2a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{31}$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ represent the voter and the medium's best responses. For candidate 1, the winning probability from $x_1 = a$ is 1, so he has no incentive to deviate. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{2}p$, $\frac{1}{2}(1-p)$, and 0, respectively. Then, as long as $p \leq \frac{1}{2}$, candidate 2 has no incentive to deviate. It is also obvious that the voter's belief $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Therefore, this is a PBE. □

(2) $(2a,0)$**-equilibrium.** Suppose that $\frac{1}{3} \leq p \leq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= 2a \\
\alpha_2^*(\theta_N) &= 0 \\
\gamma^*(\phi) &= (0,1) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{p^2}{1-p+p^2} & \text{if } x=(2a,0) \\ \frac{p(1-p)}{1-p+p^2} & \text{if } x=(a,0) \\ \frac{(1-p)^2}{1-p+p^2} & \text{if } x=(a,2a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{32}$$

Given the posterior $\mathcal{P}^*(\cdot|\phi)$, the voter weakly prefers y_1 to y_2 is equivalent to $p \leq \frac{1}{3}$. Then, because $p > \frac{1}{3}$, the voter's best response to the withholding is $\gamma^*(\phi) = (0,1)$. Given the voter's strategy, it is obvious that $\beta^*(\cdot)$ represent the medium's best responses. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $\frac{1}{2}p, 0$ and $\frac{1}{2}(1-p)$, respectively. Hence, if $p \leq \frac{1}{2}$, candidate 1 has no incentive to deviate. For candidate 2, $x_2 = 0$ induces the winning probability 1. Then, candidate 2 has no incentive to deviate. It is also obvious that $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Therefore, this is a PBE.[24] □

(3) $(a, 0)$**-equilibrium.** Suppose that $p = \frac{1}{2}$. Then:

$$\begin{aligned} \alpha_1^*(\theta_N) &= a \\ \alpha_2^*(\theta_N) &= 0 \\ \gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (a,0) \text{ or } (a, 2a) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{33}$$

Given the posterior $\mathcal{P}^*(\cdot|\phi)$, the voter is indifferent between y_1 and y_2. Then, by Requirement 2, the voter's best response is $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Given the voter's strategy, $\beta^*(\cdot)$ is obviously the medium's best response. It is worthwhile to note that given the voter and the medium's strategies, any policy is indifferent for each the non-policy type candidate. That is, when the proposed policies are convergent, then the winning probability for each candidate is $\frac{1}{2}$, and moreover, even if the proposed policies are divergent, the winning probability is still $\frac{1}{2}$. Thus, each candidate has no incentive to deviate. Obviously, the posterior $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Therefore, this is a PBE. □

[24] When $p = \frac{1}{3}$, it is indifferent for the voter to choose y_1 and y_2. Then, by Requirement 2, the voter's equilibrium strategy is changed from (32).

(4) (2a, a)-equilibrium. Suppose that $p = \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= 2a \\
\alpha_2^*(\theta_N) &= a \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (a, 2a) \text{ or } (2a, a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{34}$$

Similar to $(a, 0)$-equilibrium, it is indifferent for the voter to choose y_1 and y_2 after the withholding. Then, $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Because the medium withholds the information whenever the policies are divergent, proposing any policy is indifferent for each candidate. Obviously, the posterior $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Therefore, this is a PBE. $\square$

(v) Similar to the previous cases, we show that there exist mixed strategy equilibria in which the medium's strategy is (30), and omit to describe repetition.

(1) Mixing $x_i = 0$ and a for $i = 1, 2$. Suppose that $\frac{1}{2} \leq p \leq \frac{2}{3}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (\frac{1}{2p}, 1 - \frac{1}{2p}, 0) \\
\alpha_2^*(\theta_N) &= (\frac{1}{2}, \frac{1}{2}, 0) \\
\gamma^*(\phi) &= (1, 0) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1-p}{2-p} & \text{if } x = (0, 2a) \text{ or } (a, 2a) \\ \frac{p}{4-2p} & \text{if } x = (0, a) \text{ or } (a, 0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{35}$$

Given the posterior $\mathcal{P}^*(\cdot|\phi)$, $\gamma^*(\phi) = (1, 0)$ is the voter's best response for any p. It is obvious that $\beta^*(\cdot)$ is the medium's best response. For candidate 1, the winning probabilities from $x_1 = 0, a$ and $2a$ are $1 - \frac{1}{4}p, 1 - \frac{1}{4}p$ and $\frac{1}{2}(1+p)$, respectively. As long as $p \leq \frac{2}{3}$, proposing $x_1 = 0$ and a dominate proposing $x_1 = 2a$. Then, $\alpha_1^*(\theta_N)$ is one of the best response of candidate 1, and this is well defined when $p \geq \frac{1}{2}$. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $\frac{1}{4}, \frac{1}{4}$ and 0, respectively. Then, $\alpha_2^*(\theta_N)$ is one of the best response of candidate 2. Obviously, the posterior $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Thus, this is a PBE. □

(2) Mixing $x_i = 0$ and $2a$ for $i = 1, 2$. Suppose that $p \geq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (\frac{1}{2}, 0, \frac{1}{2}) \\
\alpha_2^*(\theta_N) &= (\frac{1}{2p}, 0, 1 - \frac{1}{2p}) \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1-p}{2-p} & \text{if } x = (a, 0) \text{ or } (a, 2a) \\ \frac{p}{4-2p} & \text{if } x = (0, 2a) \text{ or } (2a, 0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{36}$$

Given the posterior $\mathcal{P}^*(\cdot|\phi)$, it is indifferent for the voter to choose y_1 and y_2. Then, by Requirement 2, the voter's best response is $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Given the voter's strategy, $\beta^*(\cdot)$ is obviously the medium's best response. Given the voter and the medium's strategies, proposing any policy is indifferent for each candidate. Thus, $\alpha_i^*(\theta_N)$ is one of the best response of candidate i for $i = 1, 2$; they are well defined if $p \geq \frac{1}{2}$. The posterior $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule. Thus, this is a PBE. □

(3) Mixing $x_i = a$ and $2a$ for $i = 1, 2$. Suppose that $p \geq \frac{1}{2}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (0, 1 - \frac{1}{2p}, \frac{1}{2p}) \\
\alpha_2^*(\theta_N) &= (0, \frac{1}{2p}, 1 - \frac{1}{2p}) \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (a, 2a) \text{ or } (2a, a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{37}$$

We can show that this is a PBE by the same argument in (2). □

(4) Mixing all policies. Suppose that $p \geq \frac{2}{3}$. Then:

$$\begin{aligned}
\alpha_1^*(\theta_N) &= (\frac{1}{3p}, 1 - \frac{2}{3p}, \frac{1}{3p}) \\
\alpha_2^*(\theta_N) &= (\frac{1}{3p}, \frac{1}{3p}, 1 - \frac{2}{3p}) \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{6} & \text{if } x = (0,a), (0,2a), (a,0), (a,2a), (2a,0) \text{ or } (2a,a) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{38}$$

We can show that this is a PBE by the same argument in (2). $p \geq \frac{2}{3}$ is needed to be well defined the mixed strategies. ■

Appendix: B Formal Statements and Proofs in Section 5.

Asymmetry between the candidates

Proposition 7 *Consider the manipulated news model with symmetric candidates. Then, the weak median voter theorem always holds; that is,* $(0,0)$*-equilibrium always exists.*

Proof. When $0 < b < \frac{1}{2}a$, the strict median voter theorem holds because the medium never withholds the information. Then, obviously, the weak median voter theorem also holds. When $b = \frac{1}{2}a$, there exist multiple equilibria, but similar to the previous case, withholding all information is one of the medium's best response, and then, $(0,0)$-equilibrium exists. So, the weak median voter theorem holds. Hence, hereafter, we assume that $b > \frac{1}{2}a$. Suppose that $\alpha_1^*(\theta_N) = \alpha_2^*(\theta_N) = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0,0), (0,a), (a,0), (a,a)\}$. We also focus on the medium's strategy specified by (13). Because $b > \frac{1}{2}a$, policy pair $x = (a,0)$ is in disagreement region C and policy pair $x = (0,a)$ is in disagreement region D. Then, $\beta^*((a,0)) = \beta^*((0,a)) = \phi$. Given the candidates and the medium's strategy, the voter's consistent belief after the withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{1}{2} & \text{if } x = (a, 0) \text{ or } (0, a) \\ 0 & \text{otherwise} \end{cases} \tag{39}$$

Given the consistent posterior, the voter's best response to the withholding is $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Given the medium and the voter's strategies, proposing $x_1 = 0$ and a are indifferent for the non-policy type candidate 1 because $x_1 = 0$ and a provide the winning probability $\frac{1}{2}$. The winning probability from $x_1 = 2a$ varies depending on the media bias b; the winning probabilities from $x_1 = 2a$ when $\frac{1}{2}a < b \leq a$, $a < b \leq \frac{3}{2}a$ and $b > \frac{3}{2}a$ are $0, \frac{1}{2}p$ and $\frac{1}{2}$, respectively. Therefore, candidate 1 has no incentive to deviate from $x_1 = 0$. By the symmetry between the candidates, candidate 2 also never deviates from $x_2 = 0$. Therefore, $(0, 0)$-equilibrium always exists without any restrictions. ■

Tie-breaking rules

First, note that the disclosure rule is just 1-randomization rule. Let us introduce additional notations. Let EP^ϵ be the set of equilibrium behaviors of the non-policy type candidates under the ϵ-randomization rule.[25] Let CP the set of convergent policy pairs; that is, $CP \equiv \{(0,0), (a,a), (2a, 2a)\}$. Let $q(\alpha_1, \alpha_2)$ be the probability that proposed policies are convergent when non-policy type candidate i's behavior is $\alpha_i \in \Delta(X)$ for $i = 1, 2$.[26] Let β^ϵ be the generic notation of the medium's strategy satisfying the ϵ-randomization rule. Especially, with abuse of notation, $\beta^\epsilon(x) = (r, 1-r)$ represents that the medium who observes policy pair x discloses the information with probability $r \in [0,1]$, and withholds with probability $1-r$. Let $\mathcal{P}^\epsilon(\cdot|\cdot; \alpha_1, \alpha_2, \beta^\epsilon)$ be the voter's posterior belief derived by α_1, α_2

[25]Formally, $EP^\epsilon \equiv \{(\alpha_1^*(\theta_N), \alpha_2^*(\theta_N)) \in (\Delta(X))^2|$ there exists$\beta^*, \gamma^*, \mathcal{P}^*$ s.t. $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE, where β^* satisfies the ϵ-randomization rule.$\}$.

[26]Because we can fix the equilibrium behaviors of the policy type candidates, with abuse of notations, we represent the behaviors of non-policy type candidate i is by α_i.

and β^ϵ consistently with Bayes' rule whenever it is possible. Let $\gamma^\epsilon(\cdot;\alpha_1,\alpha_2,\beta^\epsilon)$ be the voter's best response given the posterior $\mathcal{P}^\epsilon(\cdot|\cdot;\alpha_1,\alpha_2,\beta^\epsilon)$.

Proposition 8 *Consider the manipulated news model with $b > \frac{3}{2}a$. Then, for any $\epsilon \in (0,1)$, $EP^1 = EP^\epsilon$.*

Proof. Fix $\epsilon \in (0,1)$, arbitrarily. Because $b > \frac{3}{2}a$, for any behaviors by the non-policy type candidates α_1 and α_2, $(C \cup D) \cap Z(\alpha_1,\alpha_2) \neq \emptyset$. First, show that $EP^1 \subseteq EP^\epsilon$. Take $(\alpha_1,\alpha_2) \in EP^1$, arbitrarily. That is, there exists $\beta^1, \gamma^1(\cdot;\alpha_1,\alpha_2,\beta^1)$ and $\mathcal{P}^1(\cdot|\cdot;\alpha_1,\alpha_2,\beta^1)$ such that $(\alpha_1,\alpha_2,\beta^1,\gamma^1;\mathcal{P}^1)$ is a PBE. Consider the following cases.

(i) $\gamma^1(\phi;\alpha_1,\alpha_2,\beta^1) = (\frac{1}{2},\frac{1}{2})$**.** Then, $C \cap Z(\alpha_1,\alpha_2) \neq \emptyset$ and $D \cap Z(\alpha_1,\alpha_2) \neq \emptyset$, and, by Lemma 1, the medium who observes policy pairs in the disagreement regions withholds the information for certain. That is, the medium's equilibrium strategy under the disclosure rule β^1 is characterized as follows:

$$\beta^1(x) = \begin{cases} (0,1) & \text{if } x \notin CP \\ (1,0) & \text{if otherwise} \end{cases} \tag{40}$$

Define β^ϵ as follows:

$$\beta^\epsilon(x) = \begin{cases} (0,1) & \text{if } x \notin CP \\ (\epsilon,1-\epsilon) & \text{if otherwise} \end{cases} \tag{41}$$

Because $\gamma^1(\phi;\alpha_1,\alpha_2,\beta^1) = (\frac{1}{2},\frac{1}{2})$, by Requirement 2-(i),

$$\begin{aligned} & \sum_{x\in Z(\alpha_1,\alpha_2)} x_1\mathcal{P}^1(x|\alpha_1,\alpha_2,\beta^1) = \sum_{x\in Z(\alpha_1,\alpha_2)} x_2\mathcal{P}^1(x|\alpha_1,\alpha_2,\beta^1) \\ \iff & \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_1 Pr.(x|\alpha_1,\alpha_2) = \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_2 Pr.(x|\alpha_1,\alpha_2) \\ \iff & \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_1 Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon)\sum_{x\in Z(\alpha_1,\alpha_2)\cap CP} x_1 Pr.(x|\alpha_1,\alpha_2) \\ = & \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_2 Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon)\sum_{x\in Z(\alpha_1,\alpha_2)\cap CP} x_2 Pr.(x|\alpha_1,\alpha_2) \\ \iff & \sum_{x\in Z(\alpha_1,\alpha_2)} x_1\mathcal{P}^\epsilon(x|\alpha_1,\alpha_2,\beta^\epsilon) = \sum_{x\in Z(\alpha_1,\alpha_2)} x_2\mathcal{P}^\epsilon(x|\alpha_1,\alpha_2,\beta^\epsilon) \end{aligned} \tag{42}$$

That is, by Requirement 2-(i), $\gamma^\epsilon(\phi;\alpha_1,\alpha_2,\beta^\epsilon) = (\frac{1}{2},\frac{1}{2})$. Because $b > \frac{3}{2}a$, any divergent policy pairs lie in the disagreement regions. Then, by the definition of β^ϵ, if the proposed policies are divergent, then the medium withholds the information. Therefore, proposing any policy is indifferent for the both non-policy type candidates. That is, because none of the non-policy type candidate has an incentive to deviate from α_i, $(\alpha_1,\alpha_2,\beta^\epsilon,\gamma^\epsilon;\mathcal{P}^\epsilon)$ is a PBE. Thus, $(\alpha_1,\alpha_2) \in EP^\epsilon$.

(ii) $\gamma^1(\phi;\alpha_1,\alpha_2,\beta^1) \neq (\frac{1}{2},\frac{1}{2})$**.** Without loss of generality assume that $\gamma^1(\phi;\alpha_1,\alpha_2,\beta^1) = (1,0)$. That is, for policy pair $x \notin CP$, candidate 1 wins for certain, and for policy pair $x \in CP$, each candidate wins with probability $\frac{1}{2}$. Hence, because $(\alpha_1,\alpha_2) \in EP^1$:

$$q(\alpha_1,\alpha_2) \leq q(\alpha_1',\alpha_2)\forall\alpha_1' \in \Delta(X) \tag{43}$$

$$q(\alpha_1,\alpha_2) \geq q(\alpha_1,\alpha_2')\forall\alpha_2' \in \Delta(X) \tag{44}$$

That is, candidate 1 minimizes the probability that policy convergence happens, and candidate 2 maximizes that probability in the equilibrium. Define β^ϵ as follows:

$$\beta^\epsilon = \begin{cases} \beta^1(x) & \text{if } x \notin CP \\ (\epsilon, 1-\epsilon) & \text{otherwise} \end{cases} \tag{45}$$

Because $\gamma^1(\phi;\alpha_1,\alpha_2,\beta^1) = (1,0)$:

$$\begin{aligned}
& \sum_{x\in Z(\alpha_1,\alpha_2)} x_1\mathcal{P}^1(x|\phi;\alpha_1,\alpha_2,\beta^1) < \sum_{x\in Z(\alpha_1,\alpha_2)} x_2\mathcal{P}^1(x|\phi;\alpha_1,\alpha_2,\beta^1) \\
\iff & \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_1 Pr.(\beta^1(x)=\phi|x,\alpha_1,\alpha_2)Pr.(x|\alpha_1,\alpha_2) \\
& < \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_2 Pr.(\beta^1(x)=\phi|x,\alpha_1,\alpha_2)Pr.(x|\alpha_1,\alpha_2) \\
\iff & \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_1 Pr.(\beta^\epsilon(x)=\phi|x,\alpha_1,\alpha_2)Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon)\sum_{x\in Z(\alpha_1,\alpha_2)\cap CP} x_1 Pr.(x|\alpha_1,\alpha_2) \\
& < \sum_{x\in Z(\alpha_1,\alpha_2)\setminus CP} x_2 Pr.(\beta^\epsilon(x)=\phi|x,\alpha_1,\alpha_2)Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon)\sum_{x\in Z(\alpha_1,\alpha_2)\cap CP} x_2 Pr.(x|\alpha_1,\alpha_2) \\
\iff & \sum_{x\in Z(\alpha_1,\alpha_2)} x_1\mathcal{P}^\epsilon(x|\phi;\alpha_1,\alpha_2,\beta^\epsilon) < \sum_{x\in Z(\alpha_1,\alpha_2)} x_2\mathcal{P}^\epsilon(x|\phi;\alpha_1,\alpha_2,\beta^\epsilon)
\end{aligned} \tag{46}$$

That is, $\gamma^{\epsilon}(\phi;\alpha_1,\alpha_2,\beta^{\epsilon}) = (1,0)$. Hence, for any policy pair $x \notin CP$, candidate 1 wins for certain, and for $x \in CP$, candidate 1 wins with probability $1-\frac{1}{2}\epsilon$ and candidate 2 wins with probability $\frac{1}{2}\epsilon$. Put differently, candidate 1 wants to minimize the probability that policy convergence occurs, and candidate 2 wants to maximizes that probability. By (43) and (44), we can say that α_1 and α_2 are the best response of the non-policy type candidates. Therefore, $(\alpha_1,\alpha_2,\beta^{\epsilon},\gamma^{\epsilon};\mathcal{P}^{\epsilon})$ is a PBE. Thus, $(\alpha_1,\alpha_2) \in EP^{\epsilon}$.

Because $(\alpha_1,\alpha_2) \in EP^1$ is arbitrary, $EP^1 \subseteq EP^{\epsilon}$. We can show $EP^{\epsilon} \subseteq EP^1$, similarly. Therefore, $EP^1 = EP^{\epsilon}$. ■

Simplified message structure

First, we demonstrate that the following new equilibrium exists under the richest message structure. Suppose that $a < b \leq \frac{3}{2}a$ and $\frac{1}{3} \leq p < \frac{1}{2}$. Then:

$$
\begin{aligned}
\alpha_1^*(\theta_N) &= 2a \\
\alpha_2^*(\theta_N) &= 0 \\
\beta^*(x) &= \begin{cases} m' \equiv \big((a,2a),(2a,0)\big) & \text{if } x=(a,2a) \text{ or } (2a,0) \\ X^2 & \text{if } x=(0,a) \text{ or } (0,2a) \\ x & \text{otherwise} \end{cases} \\
\gamma^*(x) &= \begin{cases} (1,0) & \text{if } m=m' \\ (0,1) & \text{if } (a,0)\in m \\ (1,0) & \text{if } m\neq m' \text{ and } (a,0)\notin m \end{cases} \\
\mathcal{P}^*(m) &= \begin{cases} \frac{(1-p)^2}{1-2p-2p^2} & \text{if } m=m' \text{ and } x=(a,2a) \\ \frac{p^2}{1-2p-2p^2} & \text{if } m=m' \text{ and } x=(2a,0) \\ 1 & \text{if } (a,0)\in m \text{ and } x=(a,0) \\ \frac{1}{2} & \text{if } m\neq m' \text{ and } (a,0)\notin m \text{ and } [x=(0,a) \text{ or } (0,2a)] \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{47}
$$

Obviously, the posterior $\mathcal{P}^*$ is consistent with Bayes' rule on the equilibrium path, and γ^* is the voter's best response given $\mathcal{P}^*$ as long as $p < \frac{1}{2}$.[27] In addition, β^* is the medium's best response. For candidate 1, the winning probabilities from proposing $x_1 = 0, a$ and $2a$ are $\frac{1}{2}a, 1-p$ and $\frac{1}{2}(1+p)$, respectively. Because $p \geq \frac{1}{3}$, candidate 1 does not deviate. For candidate 2, the winning probabilities from $x_2 = 0, a$ and $2a$ are $1 - \frac{1}{2}p, \frac{1}{2}(1+p)$ and p, respectively. Because $p < \frac{1}{2}$, candidate 2 does not deviate. Thus, this is a PBE. ■

Proposition 9 *Consider the manipulated news model with the rich message space. Then, the persistence of* $(0,0)$*-equilibrium is equivalent to that under the simplified message space.*

Proof. Depending on the magnitude of the media bias, consider the following cases.

Case 1: $0 < b < \frac{1}{2}a$**.** The medium has no incentive to send vague messages. Then, by Proposition 3, $(0,0)$-equilibrium is the unique equilibrium.

Case 2: $\frac{1}{2}a \leq b \leq a$**.** We can construct $(0,0)$-equilibrium with same structure specified in Proposition 4 by letting $\mathcal{P}^*((a,0)|m) = 1$ if $(a,0) \in m$. On the other hand, suppose, in contrast, that there exists $(0,0)$-equilibrium in which the medium who observes policy pair $x = (a,0)$ is pooling with either $x = (a,2a)$ or $(0,2a)$. To support this equilibrium, the voter must choose candidate 1 for certain after that pooling message. However, given the medium and the voter's strategies, candidate 1 deviates to $x_1 = a$ because the winning probability from $x_1 = 0$ is $1 - \frac{1}{2}p$, but that from $x_1 = a$ is 1, a contradiction. Hence, such $(0,0)$-equilibrium does not exists.

Case 3: $a < b \leq \frac{3}{2}a$**.** Suppose, in contrast, that there exists $(0,0)$-equilibrium.

(i) $x = (a,0)$ **is separating.** That is, $\gamma^*(\beta^*((a,0))) = (0,1)$ and $\gamma^*(\beta^*((0,2a))) = (1,0)$. By Lemma 2 of Miura (2010), there exists an off the equilibrium path

[27] We omit to describe the voter's best response to the disclosure message.

message m' such that $(a,0),(0,2a) \in m'$. However, whatever response to that off the equilibrium path message, either one of the medium who observes policy pair $x=(a,0)$ or $(0,2a)$ deviates to sending message m', a contradiction.

(ii) $x=(a,0)$ **is pooling only with** $x=(0,2a)$**.** Define $m' \equiv \beta^*((a,0)) = \beta^*((0,2a))$. The consistent belief is $\mathcal{P}^*((a,0)|m') = \mathcal{P}^*((0,2a)|m') = \frac{1}{2}$. Then, $\gamma^*(m') = (1,0)$. Note that $\gamma^*(\beta^*((a,2a))) = (1,0)$ must hold; otherwise, the medium deviates to the full disclosure message. Because policy pair $x=(0,0)$ is disclosed, the winning probability of candidate 1 from proposing $x_1 = 0$ is less than 1. However, the winning probability from $x_1 = a$ is 1. Hence, candidate 1 deviates, a contradiction.

(iii) $x=(a,0)$ **is pooling with** $x=(a,2a)$**.** Define $m' \equiv \beta^*((a,0)) = \beta^*((a,2a))$. To hold this equilibrium, $\gamma^*(m') = (0,1)$ must hold. Because of the same reason to case (ii), candidate 1 deviates to $x_1 = a$, a contradiction.

Therefore, there exists no $(0,0)$-equilibrium in Case 3.

Case 4: $b > \frac{3}{2}a$**.** Suppose, in contrast, that there exists $(0,0)$-equilibrium.

(i) $x=(a,0)$ **is separating.** Because of the same reason in Case 3-(i), we can derive a contradiction.

(ii) $x=(a,0)$ **is pooling with** $x=(0,2a)$ **and** $(a,2a)$**.** Define $m' \equiv \beta^*((a,0)) = \beta^*((0,2a)) = \beta^*((a,2a))$. The posterior after observing message m' is equivalent to (29). Then, because $\gamma^*(m') = (1,0)$, candidate 1 deviates to $x_1 = a$, a contradiction.

(iii) $x=(a,0)$ **is pooling only with** $x=(0,2a)$**.** Then, $\gamma^*(\beta^*((a,2a))) = (1,0)$. Define $m' \equiv \beta^*((a,0)) = \beta^*((0,2a))$. Because $\mathcal{P}^*((a,0)|m') = \mathcal{P}^*((0,2a)|m') = \frac{1}{2}$, the voter's best response is $\gamma^*(m') = (1,0)$. Therefore, because deviation to

$x_1 = a$ gives candidate 1 his winning probability 1, this is a profitable deviation to candidate 1, a contradiction.

(iv) $x = (a, 0)$ **is pooling only with** $x = (a, 2a)$**.** Define $m' \equiv \beta^*((a, 0)) = \beta^*((a, 2a))$. The posterior after observing message m' is $\mathcal{P}^*((a, 0)|m') = p$ and $\mathcal{P}^*((a, 2a)|m') = 1 - p$. Suppose that $p < \frac{1}{2}$. Then, the voter's best response is $\gamma^*(m') = (1, 0)$. Because of the same reason to Case 4-(iii), candidate 1 deviates to $x_1 = a$, a contradiction. Suppose that $p > \frac{1}{2}$. Then, $\gamma^*(m') = (0, 1)$. To support this equilibrium, $\gamma^*(m) = (0, 1)$ for any message m containing $(a, 0)$. Because of the construction, $\gamma^*(\beta^*((0, 2a))) = (1, 0)$ must hold in the equilibrium. That is, $(a, 0) \notin \beta^*((0, 2a))$. However, the medium who observes policy pair $x = (0, 2a)$ has an incentive to deviate to sending a message containing $(a, 0)$, a contradiction. In the case of $p = \frac{1}{2}$, we can derive a contradiction similar to the case of $p > \frac{1}{2}$.

Therefore, there exists no $(0, 0)$-equilibrium in Case 4. ■

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