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Abstract

We examine a firm that can transfer its technological advantage to a rival in exchange for receiving monetary transfers when its rivals are heterogeneous. Even though a partial technology transfer can reduce the joint profit and in a duopoly a complete transfer can reduce the joint profit (Katz and Shapiro 1985), under weakly concave demand, a complete transfer always increases joint profit if there are at least three firms. We observe that the joint-profit-maximizing licensee is neither too efficient nor too inefficient. Though jointly profitable transfers between sufficiently inefficient firms reduce welfare, a transfer from the most efficient firm always increases welfare. In the latter half of the paper, we consider two license auction games by the most efficient firm under complete information: a simple auction game in which potential licensees simply bid for technology transfer, and a menu auction game by Bernheim and Whinston (1986). With natural refinements of Nash

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equilibria, we show that the resulting licensees are ordered by their efficiency: menu auction, simple auction, and joint-profit maximizing licensees in (weakly) descending order.

JEL:D4, L24, L4

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1 Introduction

In this paper we consider the standard model of a firm licensing its production technology to its rivals in a product market, but relax the assumption that the rivals are homogenous. Specifically, we examine firms that compete in Cournot competition and differ in their constant marginal cost of production. A technology transfer then reduces a licensee's marginal cost to the level of the licensor. We assume as in, for example, Jehiel et al (1996) and Jehiel and Moldovanu (2000) that the licensor licenses only one license and that the production decisions of the firms remain independent with any transfer agreement. That is, we focus on the direct gains from the licensing and so abstract from any possible benefits from collusion, as is standard in the licensing literature. We will first analyze the gains (in joint profit) from licensing for the licensor and a licensee, then social welfare gains. Then, we consider two auction games to determine the licensee, and investigate how efficient the resulting licensee is.

We begin, following the seminal work by Katz and Shapiro (1985), by analyzing whether such a transfer is always jointly profitable. Katz and Shapiro (1985) have shown that licensing could reduce joint profit in a duopoly if the licensor has a near-monopoly position because then the transfer would reduce the licensor's near monopoly profit and so joint profit. For a small technology transfer this result can generalize to a non-duopoly market. Despite this we are able to show that a complete technology transfer (so that the licensee has the same cost as the innovator) is always profitable so long as the demand curve is weakly concave and there are at least three firms in the market (Theorem 1). That is, as long as the transfer is complete (and we have an interior condition), a transfer is always profitable no matter its absolute size. The licensor does not have to be the most efficient firm for this result to hold.

We then focus on which partner would maximize joint profit. We find that for weakly concave demand, it is neither a very inefficient nor a very efficient rival that maximizes joint profit (Observation 1). Intuitively, with a complete transfer the less efficient the licensee, the greater is the transfer. One might at first glance expect then that this implies that the least efficient rival must be the licensee that maximizes joint profit. This is not necessarily true because with very inefficient firms the decrease in profit from being a little less efficient is small – profits are convex in cost – so the marginal gain from choosing a slightly less efficient firm is small. However, the marginal cost to joint profits from choosing a slightly less efficient firm – the reduction in the market price as a result of the transfer, which harms both the licensor and the licensee – is not. So, the licensor chooses a partner who is neither too efficient nor too inefficient.

Turning to the welfare effects of transfers, Theorem 1 has a corollary given the known result that making an inefficient firm more efficient can reduce welfare (Lahiri and Ono 1988): jointly profitable transfers are welfare reducing if both the licensor and licensee are sufficiently similar and inefficient. This is in contrast to Katz and Shapiro (1985) who found that profitable transfers are never welfare reducing in a duopoly, hence, the importance of considering non-duopoly markets. This is also in contrast to Katz and Shapiro (1986) and Sen and Tauman (2007) who find that with homogenous firms, licensing always raises welfare and so heterogeneity is also important in evaluating the welfare implications of licensing. On the other hand, when the most efficient firm makes a complete transfer, then social welfare always increases under general demand (Theorem 2). We are also able to show that this licensor would choose a more efficient partner than the one that would maximize welfare (Observation 3). The conclusion for a policy maker whose objective is to maximize social welfare is that efficient firms should not be discouraged from licensing their technology and the efficient firm should be encouraged to pick less efficient partners than it normally would.

One natural question is to ask which firm would wins the right to use the technology and how much would the licensor collected from licensing. We consider the licensor uses first-price auction mechanisms to sell the right to use technology. In the first auction method (a simple auction game), each potential licensee submit a bid and the winner only pays for the bid. Since there are many Nash equilibria and some of them are less plausible, we refine the set of Nash equilibrium by requesting that nonlicensees would not be worse-off if the licensor happens to choose it: truthful Nash equilibrium (TNE in simple auction). In this refined set of Nash equilibrium, licensing fee can be pinned down and the licensee would be partner that maximizing the joint profit of licensee, licesnor and any other potential rival. Given complex negative externalities in technology transfer among various potential licensee, we also consider menu auction, proposed by Bernheim and Whinston (1986), where each potential licensee submit a menu that states the contingent payments for all possible transfers by the licensor. While licensing must occur in a Nash equilibrium in a simple auction game, it is not true in a menu auction game. Similar to the simple auction, we refine the set of Nash equilibrium by truthful Nash equilibrium (TNE in menu auction).[1] We show that a simple auction licensee is at least as efficient as the joint-profit-maximizing partner, and a menu auction licensee is at least as efficient as simple auction licensee (Theorem 3). Furthermore, if only the menu auction licensee pays in a menu auction game, then the same licensee also win the license in simple auction (Proposition 6).

In the next section we introduce the basic modeling assumptions. Section 3 examines the effect the amount of technology transferred has on profit while section 4 examines the effect of the type of partner. Section 5 identifies which firm will get the right to use technology in license auction games. Section 6 contains the welfare analysis and section 7 concludes.

2 The Model

We consider the basic Cournot market structure. There is a commodity besides a numeraire good, and its inverse demand is a continuously function $P(Q)$ in $[0, \bar{Q}]$ that is twice continuously differentiable with $P'(Q) < 0$ for all $Q \in (0, \bar{Q})$ and $P(\bar{Q}) = 0$. There are K firms in the market with no fixed cost of production. In the main analysis, we will consider the equilibria in which all K firms remain active, i.e., the licensing is not potentially drastic. Later, we will allow that some existing firms exit (with or without fixed costs).

Firms differ in their constant marginal cost c_k, and firms $1, 2, 3, \ldots$, and K are ordered in such a way that $c_1 \leq c_2 \leq ... \leq c_K$. That is, firms are indexed as $k \in \{1, ..., K\}$ with $k = 1$ being the most efficient firm. With a little abuse of notation let the set $\{1, 2, \ldots, K\}$ be denoted by K as well.

Each firm k's production level is denoted by q_k. Firm i's profit function is written as

$$\pi_i(q_i, q_{-i}) = (P(Q) - c_i)\, q_i,$$

where $Q = \sum_{k \in K} q_k$. The first order condition for profit maximization (assuming interior solution) is

$$P'(Q)q_i + P(Q) - c_i = 0.$$

[1] Truthful Nash equilibria in simple auction and in menu auction appear to be similar in their definitions, but their implications are somewhat different. In simple auction, TNE is a rather innocuous refinement of Nash equilibrium, while in menu auction, TNE has an implication for communication-based refinement (Bernheim and Whinston 1986).

This implies

$$q_i = \frac{(P(Q) - c_i)}{-P'(Q)},$$

and firm i's profit is written as

$$\pi_i(q_i, q_{-i}) = \frac{(P(Q) - c_i)^2}{-P'(Q)}.$$

We assume the **strategic substitutability** condition throughout the paper: for all $i \in K$:

$$P''(Q)q_i + P'(Q) \leq 0.$$

Note that the second order condition for profit maximization ($P''(Q)q_i + 2P'(Q) \leq 0$) is guaranteed by the strategic substitutability. The strategic substitutability is weaker than requiring that the inverse demand is **weakly concave** $P''(Q) \leq 0$.[2] In proving some of our main results, we strengthen the strategic substitutability by the weak concavity of inverse demand.

The strategic substitutability condition guarantees the uniqueness of equilibrium of this game. Let $C = \sum_{k \in K} c_k$ denote the aggregate marginal cost. With this we can establish a standard result, whose derivation will be useful for later analysis.

Lemma 1. Under the strategic substitute condition, equilibrium is unique. Moreover, assuming that all firms are active ($q_k > 0$), equilibrium total output level Q is a decreasing function of aggregate marginal cost C.

Proof. Since equilibrium output of firm k, q_k, is expressed only by the equilibrium total output level Q, if we can show that Q is unique, then we are done. First, summing up the first order conditions for profit maximization over all firms, we obtain

$$P'(Q)Q + KP(Q) = C.$$

That is, the aggregate equilibrium output is a solution of the above equation. Differentiating the LHS of the above equation, we obtain

$$\frac{d\,(LHS)}{dQ} = P''(Q)Q + (K+1)\,P'(Q)$$

[2] That is, the weak concavity of inverse demand implies the second order condition for profit maximization.

Now, summing the strategic substitutability conditions up over all firms, we obtain

$$P''(Q)Q + KP'(Q) \leq 0.$$

This implies that the LHS of the aggregated first order condition is decreasing in Q since $P'(Q) < 0$. This implies that equilibrium aggregate output Q is uniquely determined by aggregate marginal cost C, and is decreasing in Q.$\square$

3 Production Technologies and Transfers

Each firm k has its own technology of producing the commodity (the marginal cost of production is c_i), and it has the property right to its own technology (e.g., it holds a patent). We focus on a firm that has a single unit of technology to transfer and assume that the output decisions remain independent after any transfer as the independence of production decisions is usually a condition imposed by competition authorities as well as being the standard assumption in the literature. Firm i can license its technology with an exclusive usage agreement to another firm through a licensing agreement. As standard in the literature (Katz and Shapiro 1986, etc.), the other firm by obtaining this technology reduces its marginal cost to that of firm i. That is, if firms i and j have technologies with marginal costs c_i and c_j, respectively (assume $c_i < c_j$ without loss of generality), then firm j can reduce its marginal cost of production to c_i by adopting firm i's technology through licensing or some agreement. Following Katz and Shapiro (1985), we focus on how technology transfers affect the joint profit of firms i and j.

The first question is whether such a transfer is always jointly profitable. That is, could such a transfer reduce joint profits? For example, if the licensee is sufficiently inefficient then it is well known (Lahiri and Ono 1988) that small cost reductions reduce producer surplus (and welfare). Furthermore, Katz and Shapiro (1985) show that for a duopoly such a transfer could reduce joint profits. Thus it is possible that joint profits could be harmed by such a transfer. To give this question context we begin our analysis, by extending the analysis of Katz and Shapiro (1985) of partial transfers to non-duopoly markets. Specifically assume that firm i can license some fraction of the cost difference T_{ij}. That is, firm j can reduce its marginal cost of production by $T_{ij} \in (0, c_j - c_i]$ by adopting firm i's technology.

By the technology transfer firm j's marginal cost decreases. This reduces C (aggregate marginal cost), and there will be negative externalities to other firms through the lower price since K is fixed. The sum of profit for firms and is

$$\Pi_{ij}^J(T_{ij}) = \pi_i + \pi_j = \frac{(P(Q) - c_i)^2}{-P'(Q)} + \frac{(P(Q) - c_i)^2}{-P'(Q)}.$$

Recalling that $c_i < c_j$, and treating T_{ij} as a continuous variable (the amount of technology that is transferred from i to j), that is, the cost reduction for firm j can be made continuously, it is easy to see from the proof of Lemma 1 that the equilibrium aggregate output Q given a transfer T_{ij} is determined by

$$P'(Q(T_{ij}))Q(T_{ij}) + KP'(Q(T_{ij})) = C - (T_{ij}).$$

Totally differentiating this equation, we obtain

$$\frac{dQ}{dT_{ij}} = \frac{1}{-P''(Q) - (K+1)P'} > 0.$$

Thus, with a little algebra it can be shown that the change in joint profit from a *small technology transfer* (i.e., evaluated at $T_{ij} = 0$) is:

$$\begin{aligned} \left.\frac{d\Pi_{ij}^J}{dT_{ij}}\right|_{T_{ij}=0} &= \frac{1}{-P''(Q) - (K+1)P'} [2\left\{-(P - c_i) + K(P - c_j)\right\} \\ &\quad + \left\{(P - c_i)^2 + (P - c_j)^2 + 2(P - c_j)(P'Q)\right\} \frac{P''}{(-P')^2}\Bigg]. \end{aligned}$$

Since the sign of the coefficient of the bracket is positive, if the sign of the contents of the bracket are positive, then we can say that joint profit increases as technology transfer T_{ij} increases. We summarize this as a lemma.

Lemma 2. A small technology transfer from firm i to firm j improves their joint profit if and only if the following condition holds:

$$2\left\{-(P - c_i) + K(P - c_j)\right\} + \left\{(P - c_i)^2 + (P - c_j)^2 + 2(P - c_j)(P'Q)\right\} \frac{P''}{(-P')^2} > 0.$$

From the above formula, we see that the impact of a small technology transfer on joint profit need not be positive. First, it is affected by the shape of demand function, i.e., the sign of P''. Further, the contents of the first and second braces can also take either sign. Since the contents of the first brace can take either sign, then even if demand is linear ($P''(Q) = 0$) the above condition can be violated: the marginal impact of the technology transfer on joint profit can be negative.

3.1 Complete Technology Transfer and Joint Profit

We now return to the question of the profitability of a complete technology transfer. Somewhat surprisingly given that the marginal impact of a technology transfer can be negative, we can show that under weakly concave demand (which includes linear demand) a complete technology transfer is always profitable as long as there is a third firm. Due to the fact that a small transfer may reduce joint profit, we cannot simply rely on comparative statics on technology transfers: we need to utilize an artificial economy to prove the theorem. The proof is involved, and found in the appendix.

Theorem 1. Assume that all firms produce positive outputs even after the technology transfer. If demand is weakly concave and $K \geq 3$, then for any two firms i and j with $c_i < c_j$, a complete technology transfer from firm i to firm j is joint profit improving.

This result is also surprising given that Katz and Shapiro (1985) obtain conditions for a complete technology transfer to reduce joint profits. However, they examine a duopoly while our condition requires that there be at least three firms in the market. The existence of at least a third firm drives the theorem as part of the gain to the licensee comes from lost profits of the non-licensor firm(s). Thus, while the licensor's profits decrease from the transfer, the licensee's gain, which partly comes from the licensor's and other rivals' loss, is sufficient to offset the loss to the licensor. However, since a partial technology transfer could reduce joint profits, one may wonder how it can be guaranteed that a complete transfer does not joint profits. To intuitively see the reason, consider what happens when a partial technology transfer would reduce joint profits if, instead, the licensor increased the licensee's cost (thereby raising joint profits) until the licensee is driven out of the market. Joint profits have now increased. At this point we note from the divisionalization literature (Baye, et al 1996) that if the licensee could create a second, identical division then its profits increase.

3.2 Goldilocks: the Joint-Profit-Maximizing Partner

While in the previous section we considered the effect that the amount of technology transferred has on joint profit given some partner, in this section we consider which partner would maximize joint profit. That is, for firm i, which firm j would create the greatest increase in joint profit from a technology transfer? Note that the licensee by choosing a less efficient partner leads to a larger technology transfer.

Recall that $Q(T_{ij})$ denotes the equilibrium aggregate output when technology is transferred from firm i to firm j by $T_{ij} \in [0, c_j - c_i]$. Therefore, $Q(0)$ and $Q(c_j - c_i)$ are the equilibrium aggregate outputs before and after firm i makes a complete transfer to firm j. Assuming that $i < j < k$ ($c_i < c_j < c_k$), and consider the choice between making a complete technology transfer to j or k. The increases in the joint profits of the technology giver and recipient is respectively described by

$$\Pi_{ij}^J(c_j - c_i) - \Pi_{ij}^J(0) = \frac{2\left(P(Q(c_j - c_i)) - c_i\right)^2}{-P'(Q(c_j - c_i))} - \frac{\left(P(Q(0)) - c_i\right)^2}{-P'(Q(0))} - \frac{\left(P(Q(0)) - c_j\right)^2}{-P'(Q(0))},$$

and

$$\Pi_{ik}^J(c_k - c_i) - \Pi_{ik}^J(0) = \frac{2\left(P(Q(c_k - c_i)) - c_i\right)^2}{-P'(Q(c_k - c_i))} - \frac{\left(P(Q(0)) - c_i\right)^2}{-P'(Q(0))} - \frac{\left(P(Q(0)) - c_k\right)^2}{-P'(Q(0))}.$$

The increase in total joint profit from choosing k instead of j is

$$\begin{aligned}
& \left(\Pi_{ik}^J(c_k - c_i) - \Pi_{ik}^J(0)\right) - \left(\Pi_{ij}^J(c_j - c_i) - \Pi_{ij}^J(0)\right) \\
= \; & \frac{2\left(P(Q(c_k - c_i)) - c_i\right)^2}{-P'(Q(c_k - c_i))} - \frac{\left(P(Q(0)) - c_k\right)^2}{-P'(Q(0))} - \frac{2\left(P(Q(c_j - c_i)) - c_i\right)^2}{-P'(Q(c_j - c_i))} + \frac{\left(P(Q(0)) - c_j\right)^2}{-P'(Q(0))} \\
= \; & \frac{\left(2P(Q(0)) - (c_k + c_j)\right)(c_k - c_j)}{-P'(Q(0))} + 2\left[\frac{\left(P(Q(c_k - c_i)) - c_i\right)^2}{-P'(Q(c_k - c_i))} - \frac{2\left(P(Q(c_j - c_i)) - c_i\right)^2}{-P'(Q(c_j - c_i))}\right]
\end{aligned}$$

where the first bracket represents difference of the gains in the change in cost of production (direct effect) and second term representing the difference of losses in the price drop due to the industry as a whole become more efficient (indirect effect). Keeping c_j and c_i constant, It is easy to observe that the first term is increasing in c_k but the second term is increasing in c_k. To see it clearly, let us for the moment restrict to linear demand case $P(Q) = \alpha - \beta Q$.

We have

$$
\begin{aligned}
& \left(\Pi_{ik}^{J}(c_k - c_i) - \Pi_{ik}^{J}(0)\right) - \left(\Pi_{ij}^{J}(c_j - c_i) - \Pi_{ij}^{J}(0)\right) \\
= & \frac{\left(2\left(\alpha + C\right) - (c_k + c_j)(1+K)\right)\left(c_k - c_j\right)}{\beta\left(1+K\right)} \\
& + \frac{2\left[\left(2\left(\alpha + C\right) + 2c_i - c_k - c_j - 2c_i(1+K)\right)\left(c_j - c_k\right)\right]}{\beta\left(1+K\right)} \\
= & \frac{c_k - c_j}{\beta\left(1+K\right)}\left[-2\left(\alpha + C\right) - (c_k + c_j)(K-1) + 4c_i K\right] \\
= & \frac{c_k - c_j}{\beta\left(1+K\right)}\left[-2\left(\alpha + C_{-i,j,k}\right) - (c_k + c_j)(K+2) + \left(4K-2\right)c_i\right]
\end{aligned}
$$

which is quadratic function in c_k where the leading coefficient is negative. Hence, this implies that if the gain is highest when c_k cannot be too big or too small.

This 'goldilocks' condition is intuitive: you cannot make a rival who is efficient that much more efficient. Thus, there is a benefit from picking less efficient rivals as there is a greater increase in profit from the transfer. However, you can pick too inefficient of a rival. The reason is that as you pick a more inefficient rival the price falls more, harming you as well as the rival. At the same time, when considering sufficiently inefficient firms, a slightly more inefficient firm does not have that much less profit (since its output is approaching zero, i.e., marginal cost is approaching the price) and the gain from selecting a slightly more inefficient rival approaches zero.

Observation 1. With a complete transfer, the joint-profit maximizing partner for a firm is neither too efficient nor too inefficient relative to the firm under weakly concave demand.

A simple example example illustrates this observation. Consider a market with four firms with costs $c_1 = 0$, $c_2 = .1$, $c_3 = .2$ and $c_4 = .3$. In this case the most jointly profitable partner for the most efficient firm is to select is the intermediate cost rival (firm 3) with marginal cost .2. Interestingly, even though a small technology transfer to the least efficient rival (firm 4) would reduce joint profit, a complete transfer increases joint profit more than a transfer to the most efficient firm (firm 2).

4 Welfare Effects

We now investigate the effect of technology transfers on social welfare, which we define as the sum of the firms' profit and consumer surplus. Since technology transfers reduce production cost, social welfare tends to increase in the amount of technology transferred. Indeed, Katz and Shapiro (1985) show that with a duopoly, licensing that increases joint profit always increases welfare (and welfare decreasing licensing always decreases joint profit). Likewise Sen and Tauman (2007) find licensing to be welfare improving under general licensing schemes. Here we extend the welfare analysis to when firms are heterogeneous.

Despite previous results, profitable licensing could reduce welfare. This possibility arises because if a very inefficient firm obtains a technology transfer that reduces its cost only slightly, then social welfare is reduced because its resulting increase in production will displace the production of more efficient firms. This result has already been observed by Lahiri and Ono (1988). The question here is whether this implies that jointly profitable licensing can reduce welfare contrary to previous results. By the use of Theorem 1 combined with Lahiri and Ono's result we are able to state that the previous results do not generalize to when there are more than two firms and firms are heterogeneous: profitable licensing can be welfare reducing licensing.

Given this result one may wonder if there are conditions that guarantee that a technology transfer raises welfare. We then show that if the most efficient firm makes a complete technology transfer, then welfare increases. The policy implications of these results appear straightforward: competition authorities should be scrutinous of technology transfers (through licensing, joint venture, or merger) between marginal firms (in the technological efficiency sense) in an industry, especially small transfers. On the other hand, the most efficient firm within an industry should not be discouraged from making a technology transfer to a rival.

4.1 Welfare-reducing profitable licensing

We begin by presenting Lahiri and Ono's condition for when an improvement in the marginal cost of an inefficient firm reduces social welfare.

Observation 2. (Lahiri and Ono 1988): When firm j's marginal cost (c_j) decreases, social welfare decreases if c_j is sufficiently high, though consumer welfare (surplus) increases.

From Lemma 6 there is an immediate corollary to Theorem 1 that yields a result contrary to previous ones in the literature: there are profitable technology transfers that reduce total welfare though benefiting consumers.

Corollary 1. Suppose that demand is weakly concave and that there are more than two firms. Then, if firm j has sufficiently high marginal cost (c_j) and firm i's marginal cost is sufficiently close to firm j's, then welfare decreases though consumer welfare (surplus) increases by a profitable licensing between i and j.

The previous example (for Observation 1) can be used to illuminate when this can happen. Recall that in that example there are four firms with costs $c_1 = 0$, $c_2 = .1$, $c_3 = .2$ and $c_4 = .3$. In this case, a complete technology transfers between firm 3 and 4 is jointly profitable and welfare reducing. As a second example consider a market with five firms with costs $c_1 = 0$, $c_2 = .075$, $c_3 = .15$, $c_4 = .225$ and $c_5 = .29$. In this case, a complete technology transfer from firm 3 (or firm 4) to the least efficient firm (firm 5) is jointly profitable and welfare reducing.

Though this result is different to the licensing literature, there are previous results in the literature that may at first glance appear to be similar even though they are quite distinct. First, Katz and Shapiro (1985) have shown that in a duopoly a technology transfer can reduce welfare, but only when it reduces joint profit. Hence, such transfers would never actually occur. In contrast, here there can be technology transfers that reduce welfare, but increase joint profit. Second, Faulí-Oller and Sandonís (2002) have shown that in a duopoly that profitable licensing can reduce welfare, but this requires the use of a royalty (raising the recipient's marginal cost) and only occurs in price competition. As they note, "the royalty works as a collusive device" and so reduces welfare. More generally, licensing contracts can reduce welfare through their collusive effects (Shapiro 1985 and others), which do not exist here.

4.2 Welfare-improving profitable licensing

Since technology transfers between inefficient firms can reduce welfare, the next question is whether there are conditions for transfers to increase welfare.

Indeed, we can show that if firm i is the most efficient firm, then a complete transfer is always welfare increasing. For this result, we need no condition on demand function (see the appendix for the proof).

Theorem 2. Suppose that the most efficient firm (firm 1) makes a complete transfer to firm j ($c_1 \leq c_2 \leq ... \leq c_j \leq ... \leq c_K$ and $c_1 < c_j$). Then, the social welfare improves.

Thus, a complete technology transfer by the most efficient firm always raises welfare.

We next consider which partner for firm 1 would maximize welfare. Recall that when considering which partner would maximize the increase in joint profit, the choice of the partner affects the joint profit before the transfer (a less efficient partner chosen means smaller joint profit before the transfer). However, when considering which partner would maximize the increase in welfare, welfare before the transfer occurs is not affected by the choice of partner since welfare includes the sum of all firms' operating cost. For this reason, when considering welfare, the choice of a partner is equivalent to the choice of amount of technology transferred. As we know (Lemma 6), if the licensor is sufficiently inefficient then derivative with respect to c_j is initially negative: welfare can decrease with a complete technology transfer when both firms are sufficiently inefficient. However, from the proof to Theorem 2 in the appendix, it is clear that a complete technology transfer to the least efficient firm from the most efficient firm achieves the highest social welfare gain. However, when considering instead private incentives, we saw that for joint profit maximization there was an interior solution (Observation 1): the best partner was not too inefficient. Thus, the most efficient firms may choose an overly efficient firm as a partner.

Observation 3. The profit maximizing partner for the most efficient firm to make a complete transfer to is more efficient than the welfare maximizing partner, since the latter is the least efficient firm.

We close this section by noting that a little bit more can be said about partner selection if we restrict ourselves to linear demand. In particular, it is straightforward to show that if firm i is sufficiently efficient, but not necessarily the most efficient firm ($c_i \approx c_1$), then it is also welfare optimal that this firm i chooses the least efficient firm in contrast to the private incentives (Proposition 1). The results in the section then suggest that policy makers may want to encourage the dominant firm in an industry to license its superiority and moreover to license it to a partner less efficient than the firm would find most profitable.

5 Choosing a Licensee Through Auctions

In the previous section, we investigated which partner makes the joint profit between the licensor and the licensee. In this section, we will try to identify which firm wins the right to use the technology for how much. The method of selling the right to use technology is by an auction. We consider two types of auction methods: The first way is that all potential licensees bid for the right to use the technology and the winner only pays the license fee according to its bid (simple auction). The second way is that each potential licensee offer a menu that describe how much it is willing to pay for each of potential licensees gets the technology, and the licensor firm takes the sum of the bids among all potential licensees according to their bids (menu auction). These two license auctions have advantages and disadvantages. A simple auction can be considered as a natural auction, since thw winner of the license auction only pays for the license. However, there are externalities among potential licensees. If an undesirable for a firm gets the license, the firm may suffer a lot and it may prefer a more desirable rival to get the license: it might as well support a more desirable firm to get a license. Given this, a menu auction also makes sense in a licensing market, although it is less natural at the first glance.

We will assume that the licensor is the most efficient firm, firm 1, in the rest of the paper. This is a natural setup for the licensing problem, and as Theorem 2 assures, such licensing will certainly improve the welfare.

5.1 Simple Auction

Firm 1 is the licensor, which has a superior technology than others: $c_1 < c_2 \leq c_3 \leq ... \leq c_K$. For $k, j = 1, .., K$, let $\pi_k(j)$ be firm k's equilibrium profit when firm j gets the technology of firm 1 (the licensor). When $j = 1$, firm 1's technology is not transferred to any other firm. We consider the following game. A **simple auction** is a game played by firms $2, 3, ..., K$, in which each firm $k \in \{2, .., K\}$ simultaneously offers $T_k \geq 0$ to be the unique licensee to the licensor who chooses a firm (say, firm j) as a licensee that maximizes the sum of firm 1's profit and T_j: i.e., $j \in M(T) \equiv \arg\max_{k\in\{1,...,K\}} (\pi_1(k) + T_k)$, where $T = (T_1, T_2, ..., T_K)$. Knowing this, each firm in $\{2, ..., K\}$ chooses its bid T_k. In a simple auction, an outcome (j^*, T^*) is a **Nash equilibrium** if $j^* \in M(T^*)$ and there is no $k \in \{2, ..., K\}$ such that $T_k \geq 0$ such that $k \in M\left(T_k, T^*_{-k}\right)$ and $U_k(k, T) > U_k(j^*, T)$, where $U_k(j, T) = \pi_k(k) - T_k$ if $j = k$. We first characterize the set of Nash equilibria and then prove the existence of a Nash equilibrium. The proof is relegated to the appendix. Although firm 1 is not a bidder, we let $T_1 \equiv 0$ for notational convenience.

Lemma 3. In a simple auction, an outcome (j^*, T^*) is a Nash equilibrium in a simple auction if and only if

(a) $\pi_1(j^*) + T^*_{j^*} \geq \pi_1(j) + T^*_j$ for all j.

(b) If $j^* > 1$, then $\pi_1(j^*) + \pi_j(j^*) + T^*_{j^*} \geq \pi_1(j) + \pi_j(j)$ for all $j \neq j^*$.

(c) If $j^* > 1$ and $T^*_{j^*} > 0$, then $\pi_1(j^*) + T^*_{j^*} = \pi_1(j) + T^*_j$ for some $j \neq j^*$ and $\pi_1(j^*) + T^*_{j^*} = \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}}$ implies $\pi_{j^*}(j^*) - T^*_{j^*} \geq \pi_{j^*}\left(\tilde{j}\right)$.

Moreover, there exists a Nash equilibrium for every simple auction game.

The following Lemma shows that no licensing is a Nash equilibrium outcome if and only if no licensing leads to better joint profit with any potential partner.

Lemma 4. If an outcome $(1, T^*)$ is a Nash equilibrium, then $\pi_1(j) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ for all $j \neq 1$. Conversely, if $\pi_1(j) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ for all $j \neq 1$, there there exists an outcome $(1, T^*)$ is a Nash equilibrium for some T^*.

Proof. Suppose $(1, T^*)$ is a Nash equilibrium but $\pi_1(1) + \pi_j(1) < \pi_1(j) + \pi_j(j)$ for some j. Then j can deviate to $\tilde{T}_j = \pi_j(j) - \pi_j(1) - \varepsilon$ for some $\varepsilon > 0$ so that $U_j(j, \tilde{T}_j, T^*_{-j}) \geq U_j(1, T^*)$ and $U_1(j, \tilde{T}_j, T^*_{-j}) \geq U_1(k, \tilde{T}_j, T^*_{-j})$ for all k. Now suppose $\pi_1(j) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ for all $j \neq 1$. Let $T^*_j = \pi_j(j) - \pi_j(1)$ for all $j \neq 1$, then $\pi_1(1) \geq \pi_1(j) + T^*_j$ for all $j \neq 1$ which is sufficient condition for $(1, T^*)$ to be a Nash equilibrium.$\square$

Under weakly concave demand, as Theorem 1 shows that transfer is always joint-profit improving, licensing must occur in a Nash equilibrium. Then, which firm could be the licensee and how much should the licensee pay? As Lemma 3 suggested, usually more than one firm can be a licensee in the set of Nash equilibria because the willingness to pay for a licensee is not only the value of the licensee but also to prevent negative externalities from other firm becoming a licensee: there may be a less efficient firm claiming to match the payment but such claim need not be credible. Even if there is unique licensee in the set of Nash equilibrium, the license fee is still indeterminate, since any rival can offer a big fee knowing that it would not win the technology. As long as the licensee is willing to match the offer, it can be a Nash equilibrium. Thus, other firms can control the licensee's payment without being affected. In the light of this, we consider a reasonable refinement of Nash equilibrium is a version of truthful equilibrium. The idea is vaguely related to trembling-hand argument for the licensor. The licensor may make a slight mistake in choosing a potential licensee. Hence, each firms would make a weakly dominant offer relative to the equilibrium outcome. For firm $j \in K\backslash\{1\}$, a strategy T_j is said to be **truthful relative to** j^* if and only if either (i) $U_j(j,T) = U_j(j^*,T)$ or (ii) $U_j(j,T) < U_j(j^*,T)$ and $T_j(j) = 0$. A **truthful Nash equilibrium (TNE)** is a Nash equilibrium (j^*, T^*) such that each firm chooses a truthful strategy relative to j^*. With this refinement, we can pin down the equilibrium license fee and characterize the equilibrium.

Proposition 1. No licensing is a TNE if and only if $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$. If an outcome (j^*, T^*) is a TNE with licensing $(j^* > 1)$, then $T^*_{j^*} = \max_{j \in K\backslash\{1\}}\{\pi_1(j) - \pi_1(j^*) + \pi_j(j) - \pi_j(j^*)\}$ and $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$. Conversely, if $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$, then there exists a TNE outcome (j^*, T^*).

Proof. First suppose that no licensing is a TNE. Then, $\pi_1(j) + T^*_j \leq \pi_1(1)$ and $\pi_j(j) = \pi_j(1) + T^*_j$ holds for all $j \neq 1$. Thus, $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ holds. Conversely, if $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ holds for all $j \neq 1$, then $\pi_1(j) + T^*_j \leq \pi_1(1)$ and $\pi_j(j) = \pi_j(1) + T^*_j$ holds.

Second, we consider the case with licensing. Let (j^*, T^*) be a TNE. In a TNE, we have $T^*_j = \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$. From condition (a) of a Nash equilibrium, we have $\pi_1(j^*) + T^*_{j^*} \geq \pi_1(j) + T^*_j$ for all $j \neq 1$ so that $T^*_{j^*} = \max_{j \in K\backslash\{1,j^*\}}\{\pi_1(j) + \pi_j(j) - \pi_j(j^*) - \pi_1(j^*)\}$. By condition (c) of a Nash equilibrium, we have $\tilde{j} \neq j^*$ such that $\pi_1(j^*) + T^*_j = \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}}$ and $\pi_{j^*}(j^*) - T^*_{j^*} \geq \pi_{j^*}\left(\tilde{j}\right)$. Hence, we have $\pi_1(j^*) + \pi_{j^*}(j^*) - \pi_{j^*}\left(\tilde{j}\right) \geq \pi_1\left(\tilde{j}\right) + \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*)$. Since $\pi_1\left(\tilde{j}\right) + \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*) \geq \pi_1(j) + \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$, we have $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$.

Consider $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$.

Define $T_j^* = \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$ and $T_{j^*}^* = \max_{j \in K \setminus \{1\}} \{\pi_1(j) + \pi_j(j) - \pi_j(j^*)\} - \pi_1(j^*)$. It is easy to check all conditions in a Nash equilibrium are satisfied.□

As a corollary of the first part of Proposition 1 and Theorem 1, we can state the following.

Corollary 2. Under weakly concave demand, no-licensing is not a TNE of simple auction game.

We call the licensee in a truthful Nash equilibrium outcome as a **simple auction licensee**. Without negative externality, the joint-profit-maximizing partner would be the simple auction licensee. Hence, it should not be to surprising that a firm is simple auction licensee if and only if such a transfer maximizes the joint profits of the licensor, the licensee and any one firm.[3] Then it is natural to compare a simple auction licensee and the joint-profit-maximizing partner. It turns out that the simple auction licensee, if exists, is at least as efficient as the joint-profit-maximizing partner.

Proposition 2. Under weakly concave demand, the simple auction licensee (if exist) is at least as efficient as the joint-profit-maximizing partner.

Proof. Let $j^* \in \arg\max_{j \in K} [\pi_1(j) + \pi_j(j)] - [\pi_1(1) + \pi_j(1)]$ be the joint-profit-maximizing partner. Suppose there exists $k > j^*$ such that $\pi_1(k) + \pi_k(k) + \pi_{j^*}(k) > \pi_1(j^*) + \pi_k(j^*) + \pi_{j^*}(j^*)$. Since we have $\pi_1(j^*) + \pi_{j^*}(j^*) - \pi_{j^*}(1) > \pi_1(k) + \pi_k(k) - \pi_k(1)$, it is easy to see $\pi_{j^*}(k) - \pi_k(j^*) > \pi_{j^*}(1) - \pi_k(1)$. However, we have

$$\begin{aligned} \pi_{j^*}(k) - \pi_k(j^*) &= \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_k)} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \\ &< \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_{j^*})} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \end{aligned}$$

[3] It is interesting that it is somewhat related to the potential function in a potential game.

since $-P'(Q_{j^*}) < -P'(Q_k)$. Then we have

$$
\begin{aligned}
& \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_{j^*})} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \\
= & \frac{(P(Q_k) + P(Q_{j^*}) - c_{j^*} - c_k)(P(Q_k) - P(Q_{j^*}) + c_k - c_{j^*})}{-P'(Q_{j^*})} \\
< & \frac{(2P(Q_1) - c_{j^*} - c_k)(c_k - c_{j^*})}{-P'(Q_1)} = \pi_{j^*}(1) - \pi_k(1)
\end{aligned}
$$

since $2P(Q_1) \geq P(Q_k) + P(Q_{j^*})$, $-P'(Q_1) < -P'(Q_{j^*})$ and from equilibrium conditions we have $c_k - c_{j^*} = [-P'(Q_k)]Q_k - [-P'(Q_{j^*})]Q_{j^*} + K[P(Q_{j^*}) - P(Q_k)]$ so that $0 \leq P(Q_k) - P(Q_{j^*}) + c_k - c_{j^*} \leq c_k - c_{j^*}$. Hence, we have $\pi_{j^*}(k) - \pi_k(j^*) < \pi_{j^*}(1) - \pi_k(1)$, which is a contradiction.□

As is seen from the characterization of TNE (Proposition 1), it is easy to see that a Nash equilibrium in pure strategy must satisfy many inequalities. Although we are not able to show the existence of a simple auction licensee under weakly concave demand but we can show it always exists under linear demand assumption.

Proposition 3. Under linear demand, there exists a TNE in a simple auction game.

5.2 Menu Auction

We consider the firm 1 (the most efficient firm) is making decision to license technology to some firms (licensees) $L \subseteq N = K \backslash \{1\} = \{2, \ldots, K\}$. Since there is negative externality in technology transfer,[4] other firms without transfer (non-lincesees $N \backslash L$) would like to influence the licensing decision and may be willing to offer firm 0 not to license to those licensees L. We try to capture such strategic interaction using the menu auction framework proposed by Bernheim and Whinston (1986).

A **menu auction game** Γ is described by $(N+2)$ tuples:

$$\Gamma \equiv \left\{A, (V_k)_{k \in N \cup \{1\}}\right\},$$

where A is the set of actions, $V_k : A \to \mathbb{R}$ is k's (quasi-linear) payoff function, 0 denotes the agent, and N is the set of principals. In the extensive form

[4] Note that the transfer need not be complete but partial.

of the game the principals simultaneously offer contingent payments to the agent who subsequently chooses an action that maximizes her total payoff. A strategy for each principal $k \in N$ is a function $T_k : A \to [b_k, \infty)$, which is a monetary reward (or punishment) of $T_k(a)$ to the agent for selecting a, where b_k is the lower bound for payment from principal k. For each action a, principal k receives a net payoff:

$$U_k(a, T) = V_k(a) - T_k(a),$$

where $T = (T_{k'})_{k' \in N}$ is a strategy profile. The agent chooses an action that maximizes her total payoff: the agent selects an action in the set $M(T)$ with:

$$M(T) \equiv \arg\max_{a \in A} \left[V_0(a) + \sum_{k \in N} T_k(a) \right].$$

The menu auction game is merely a game among principals, although, strictly speaking, a tie-breaking rule among $M(T)$ needs to be specified for the agent.

An outcome of a menu auction game Γ is (T, a). An outcome (a^*, T^*) is a **Nash equilibrium** if $a^* \in M(T^*)$ and there is no $k \in N$ such that $T_k : A \to \mathbb{R}_+$ and $a \in M\left(T_k, T^*_{-k}\right)$ such that $U_k(a, T) > U_k(a^*, T)$. Bernheim and Whinston (1986) characterize the sufficient and necessary condition for Nash equilibrium:

Theorem (Bernheim and Whinston, 1986). An outcome (a^*, T) is a Nash equilibrium if and only if

1. $T_i(a) \geq 0$ for all $a \in A$ and all $i \in N$.
2. $a^* \in \arg\max_{a \in A} \left[\sum_{i=1}^{n} T_i(a) + V_0(a)\right] \equiv M(T)$.
3. $[V_i(a^*) - V_i(a)] + [V_0(a^*) - V_0(a)] \geq \left[\sum_{-i} T_i(a) - \sum_{-i} T_i(a^*)\right]$ for all $i \in N$ and $a \in A$.
4. For all $i \in N$, there exists $a_i \in M(T)$ such that $T_i(a_i) = 0$.

As the above theorem suggests, there are usually numerous of Nash equilibria in a menu auction game. It is not too surprising that no licensing can be a Nash equilibrium, different from simple auction games. For example, consider a market with three firms where $c_1 = 0$, $c_2 = 0.1$ and $c_3 = 0.3$. It easy to see that when $T_2(1) = 0.02625$, $T_2(2) = T_2(3) = 0$ and $T_3(1) = T_3(2) = 0$ and $T_3(3) = 0.073125$, then $(1, T)$ is a Nash equilibrium in menu auction.

To get reasonable suggestion, Bernheim and Whinston (1986) consider a reasonable refinement on the set of Nash equilibria and they argue that "truthful strategies" are quite crucial in menu auction. A strategy T_k is said to be **truthful relative to** $\bar{a}$ if and only if for all $a \in A$ either (i) $U_k(a, T) = U_k(\bar{a}, T)$ or (ii) $U_k(a, T) < U_k(\bar{a}, T)$ and $T_k(a) = b_k$. An outcome (a^*, T^*) is a **truthful Nash equilibrium (TNE)** if and only if it is a Nash equilibrium, and T_k^* is truthful relative to a^* for all $k \in N$. They show that in menu auction games, the set of truthful Nash equilibria (TNE) and the set of coalition-proof Nash equilibria (CPNE) are equivalent in utility space, and CPNE and TNE are only different with respect to off-equilibrium strategies. Bernheim and Whinston (1986) show that efficient action is chosen by the agent in every TNE outcome in a menu auction: if (a^*, T^*) is a TNE, then we have $a^* \in \text{argmax}_{a \in A}[\sum_{i \in N} V_i(a) + V_0(a)]$. We call the licensee in a TNE of menu auction as a **menu auction licensee**.

Even though no licensing can be a Nash equilibrium, licensing must occur in a TNE under linear demand. The proof is relegated to appendix.

Proposition 4. If demand is linear and $K \geq 3$, then licensing must occur in a truthful Nash equilibrium in menu auction.

The following example illustrates that joint-profit-maximizing partner, simple auction licensee and menu auction licensee are in general different. Under linear demand with five firms with $c_1 = 0$, $c_2 = 0.05$, $c_3 = 0.1$, $c_4 = 0.14$, and $c_5 = 0.2$, it is easy to see that firm 3 is the menu auction licensee, firm 4 is the simple auction licensee, and firm 5 is the joint-profit-maximizing partner. In this example, the menu auction licensee is at least as efficient as the simple auction licensee. The following proposition shows that it holds in general.

Proposition 5. A menu auction licensee is at least as efficient as a simple auction licensee.

Proof. Denote j^S and j^M simple auction licensee and menu auction licensee. By property of a TNE in menu auction, we have $\sum_{h \in K} \pi_h\left(j^M\right) \geq \sum_{h \in K} \pi_h\left(j^S\right)$. By Proposition 1, we have $\pi_1\left(j^S\right) + \pi_{j^M}\left(j^S\right) + \pi_{j^S}\left(j^S\right) \geq$

$\pi_1\left(j^M\right)+\pi_{j^S}\left(j^M\right)+\pi_{j^M}\left(j^M\right)$. Hence, we have $\sum_{h\in K\setminus\{1,j^S,j^M\}}\pi_h\left(j^M\right)\geq\sum_{h\in K\setminus\{1,j^S,j^M\}}\pi_h\left(j^S\right)$.This implies that $j^M\leq j^S$.□

The underlying intuition of this proposition is that as menu auction licensee is a industry-profit-maximizing partner and simple auction licensee is a three-firm-profit-maximizing firm, the negative externality of the technology transfer would make the profit-maximizing firm more efficient, which counteract the effect of negative externality. Propositions 2 and 5 can be summarized as the licensing partners' efficiency ranking among different regimes in the following Theorem.

Theorem 3. Suppose that firm 1 is licensing technology to another firm. Under weakly concave demand, the licensing partner that maximizes the gains in their joint profit is weakly less efficient than the partner determined in a simple auction, and the latter is weakly less efficient than the partner determined by a menu auction: i.e.,

menu auction licensee $\leq$ simple auction licensee $\leq$ joint-profit-maximizing partner,

where firms are ordered by its efficiency in a descending manner.

Finally, we provide a sufficient condition for the licensees in simple and menu auctions to coincide. In a menu auction, non-licensing firms may be paying for licensee to prevent a more inefficient firm obtain the license. It is natural to conjecture that when the licensee be the only paying firm, the licensee is also a simple auction licensee. The proof is in appendix.

Proposition 6. If firm j^* is a menu auction licensee and only j^* is paying for the license, then j^* is a simple auction licensee.

6 Conclusion

We explore technology transfers (through licensing or joint venture agreements) in a market with firms heterogeneous in cost. We find that under weakly concave demand, any complete technology transfer between firms increases joint profit so long as there is at least one other firm in the market. We then consider which partner a firm would choose to license its technology. It turns out that the optimal partner is neither too close to the firm in terms of efficiency nor too inefficient.

In a companion paper (Creane and Konishi 2009), we analyze the effects of partial technology transfers using the same model. The licensor can choose how much technology as well as who to transfer. We show that if there is a joint-profit-improving technology transfer, then the joint-profit maximizing technology transfer is a complete technology transfer under some condition on demand function (including linear demand). This result justifies our analysis on complete technology transfers.

Appendix

Theorem 1. Assume that interior solutions (no firm chooses zero production before and after a technology transfer). If demand is weakly concave ($P''(Q) \leq 0$) and $K \geq 3$, then for any two firms i and j with $c_i < c_j$, a complete technology transfer from firm i to firm j is joint profit improving.

Proof. The proof utilizes an artificial market. This device is useful by observing the fact that transferring technology partially can reduce the joint profit. Instead, we replace firm j with an artificial (public: not profit-maximizing) firm i' with marginal cost c_i, but we control its output level so that the joint profit between firms i and i' increases monotonically. After that, we go back to the original economy. This is the strategy to prove the theorem.

Consider an artificial market parametrized by $\alpha \in [0,1]$, in which firm j ($c_i < c_j$) is replaced by an artificial firm i' that satisfies (i) $c_{i'} = c_i$, (ii) $q_{i'}(\alpha) = \alpha q_i(\alpha)$, and (iii) for all $k \neq i'$, $q_k(\alpha) = \frac{P(Q(\alpha))-c_k}{-P'(Q(\alpha))}$ holds where $Q(\alpha) = \sum_{k\neq i'} q_k(\alpha) + \alpha q_i(\alpha)$. That is, although the output decision by firm i' is linked with that of firm i, firms $k \neq i'$ do not use this information by choosing the best response to $Q_{-k}(\alpha) = \sum_{\ell\neq k} q_\ell(\alpha)$ (the standard Cournot behavior: not the Stackelberg one).

When $\alpha = 1$, we have

$$P'(Q(1))Q(1) + KP(Q(1)) - (C_{-i'} + c_i) = 0,$$

which describes the aggregate Cournot equilibrium output $Q(1)$ after the complete technology transfer from firm i to firm j, since the best response by firm i' is identical to the one by firm i when $\alpha = 1$.

We first show that in this artificial market, the joint profit of firms i and i', $\Pi^J(\alpha) = (1+\alpha)\pi_i(\alpha) = \frac{(1+\alpha)(P(Q(\alpha))-c_i)^2}{-P'(Q(\alpha))}$, increases monotonically as α

goes up. The best response by firm $k \neq i'$ is described by

$$P'(Q(\alpha))q_k(\alpha) + P(Q(\alpha)) - c_k = 0.$$

Specifically, we have

$$P'(Q(\alpha))q_i(\alpha) + P(Q(\alpha)) - c_i = 0,$$

thus we can write

$$P'(Q(\alpha))q_{i'}(\alpha) + \alpha\left(P(Q(\alpha)) - c_i\right) = 0.$$

Summing up these equations, we have

$$P'(Q(\alpha))Q(\alpha) + (K - 1 + \alpha)\, P(Q(\alpha)) - (C_{-i} + \alpha c_i) = 0.$$

Totally differentiating the above, we have

$$\left(P''Q + P' + (K - 1 + \alpha)\, P'\right) dQ + (P - c_i)d\alpha = 0$$

$$\frac{dQ}{d\alpha} = \frac{P(Q(\alpha)) - c_i}{-P''(Q(\alpha))Q(\alpha) - (K + \alpha)\, P'(Q(\alpha))}.$$

Now, we show $\Pi^J(\alpha) = \frac{(1+\alpha)(P(Q(\alpha))-c_i)^2}{-P'(Q(\alpha))}$ changes as α increases.

$$\begin{aligned}
\frac{d\Pi^J}{d\alpha} &= \frac{(P - c_i)^2}{-P'} + (1 + \alpha) \times \frac{2(P - c_i)P'(-P') + P''\,(P - c_i)^2}{(-P')^2} \times \frac{P - c_i}{-P''Q - (K + \alpha)\, P'} \\
&= A \times \left[(-P')(-P''Q - (K + \alpha)\, P') + (1 + \alpha)\left\{-2\,(-P')^2 + P''\,(P - c_i)\right\}\right] \\
&= A \times \left[(-P')(-P''Q - (K + \alpha)\, P') + (1 + \alpha)\left\{-2\,(-P')^2 + P''\,(P - c_i)\right\}\right] \\
&= A \times \left[\{(K + \alpha) - 2(1 + \alpha)\}\,(-P')^2 + (-P'')\,\{-P'Q - (1 + \alpha)\,(P - c_i)\}\right] \\
&= A \times \left[(K - 2 - \alpha)\,(-P')^2 + (-P'')\,\{(-P')\,(Q - (1 + \alpha)q_i) - (1 + \alpha)\,(P'q_i + P - c_i)\}\right]
\end{aligned}$$

where $A = \frac{(P-c_i)^2}{(-P')^2(-P''Q-(K+\alpha)P')} > 0$. We can determine the sign of $\frac{d\Pi^J}{d\alpha}$. Note that $P' < 0$ and $P'' \leq 0$. Since $K \geq 3$, $K - 2 - \alpha \geq 0$ must follows, and the first term in the bracket of the last line is nonnegative for all $\alpha \in [0, 1]$. Since $K \geq 3$ with interior solution, we have $Q > (1 + \alpha)q_i$, and $P'q_i + P - c_i = 0$ holds by firm i's first order condition. This implies that the second term is positive. Thus, we can conclude that $\frac{d\Pi^J}{d\alpha} > 0$ holds for all $\alpha \in [0, 1]$.

Now, we show that the equilibrium allocation with firm j is mimicked by an equilibrium allocation in our artificial market at a certain $\hat{\alpha} \in (0,1)$. Let $(\hat{P}, (\hat{q}_k)_{k=1}^K)$ be the Cournot equilibrium allocation before firm j received a complete technology transfer. Let $\hat{\alpha} = \frac{\hat{q}_j}{\hat{q}_i}$. Since $c_j > c_i$ and we assume an interior solution, we have $\hat{q}_i > \hat{q}_j > 0$ and $0 < \hat{\alpha} < 1$. Thus, $(\hat{P}, (\hat{q}_k)_{k=1}^K) = (P(\hat{\alpha}), (q_k(\hat{\alpha}))_{k=1}^K)$ holds, and the initial equilibrium allocation is mimicked by the equilibrium in an artificial market with $\alpha = \hat{\alpha}$. Since $\hat{q}_j = \hat{\alpha}\hat{q}_i = \hat{\alpha} q_i(\hat{\alpha})$, we have

$$\begin{aligned}
\hat{\pi}_i + \hat{\pi}_j &= \left(\hat{P} - c_i\right) \hat{q}_i + \left(\hat{P} - c_j\right) \hat{q}_j \\
&= \left(P(\hat{\alpha}) - c_i\right) q_i(\hat{\alpha}) + \left(P(\hat{\alpha}) - c_j\right) \hat{\alpha} q_i(\hat{\alpha}) \\
&< \left(P(\hat{\alpha}) - c_i\right) q_i(\hat{\alpha}) + \left(P(\hat{\alpha}) - c_i\right) \hat{\alpha} q_i(\hat{\alpha}) \\
&= \Pi^J(\hat{\alpha}).
\end{aligned}$$

Since $\Pi^J(\alpha)$ is monotonically increasing in α, we have $\Pi^J(\hat{\alpha}) < \Pi^J(1)$. Since $\Pi^J(1)$ is the same as the joint profit by firms i and j after the complete technology transfer from firm i to firm j, we can conclude that the joint profit by firms i and j must increase after the complete technology transfer.□

Theorem 2. Suppose that the most efficient firm (firm 1) makes a complete transfer to firm j ($c_1 \leq c_2 \leq ... \leq c_j \leq ... \leq c_K$ and $c_1 < c_j$). Then, the social welfare improves.

Proof. By Lemma 1, we know that if the aggregate marginal cost C decreases, the equilibrium total output Q increases. Now, consider firm k. If C decreases keeping c_k constant, Q increases while q_k shrinks. We can write the relationship between Q and q_k (through changes in C behind) as follows:

$$q_k(Q) = \frac{P(Q) - c_k}{-P'(Q)}.$$

Let us denote the original (before transfer) equilibrium by "hat," and the new equilibrium by "tilde." Since firm j's marginal cost c_j only goes down from $\hat{c}_j = c_j$ to $\tilde{c}_j = c_i$ keeping all other marginal costs constant, we have $\hat{Q} < \tilde{Q}$ and $\hat{q}_k > \tilde{q}_k$ for all $k \neq j$. Then, we necessarily have $\hat{q}_j < \tilde{q}_j$ and $\tilde{q}_j - \hat{q}_j > \tilde{Q} - \hat{Q}$.

The social welfare is written as

$$\begin{aligned} SW &= (\text{total benefit}) - (\text{total cost}) \\ &= \int_0^Q P(Q')dQ' - \sum_{k=1}^{K} c_k q_k. \end{aligned}$$

Therefore, we have

$$\begin{aligned} \widetilde{SW} &= \int_0^{\tilde{Q}} P(Q')dQ' - \sum_{k=1}^{K} c_k \tilde{q}_k \\ &= \int_0^{\hat{Q}} P(Q')dQ' + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1 \tilde{q}_j \\ &= \int_0^{\hat{Q}} P(Q')dQ' + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{Q} - \hat{Q}) - c_1 \left(\tilde{q}_j - (\tilde{Q} - \hat{Q}) \right) \\ &= \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1 \left(\tilde{q}_j - (\tilde{Q} - \hat{Q}) \right) \\ &\quad + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - c_1(\tilde{Q} - \hat{Q}). \end{aligned}$$

The last two terms are obviously positive since $P(\tilde{Q}) > c_1$. Thus, we have

$$
\begin{aligned}
\widetilde{SW} - \widehat{SW} &> \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k\neq j} c_k \tilde{q}_k - c_1 \left(\tilde{q}_j - (\tilde{Q} - \hat{Q})\right) - \widehat{SW} \\
&= \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k\neq j} c_k \tilde{q}_k - c_1 \left(\tilde{q}_j - (\tilde{Q} - \hat{Q})\right) - \int_0^{\hat{Q}} P(Q')dQ' + \sum_{k=1}^{K} c_k \hat{q}_k \\
&= \sum_{k=1}^{K} c_k \hat{q}_k - \sum_{k\neq j} c_k \tilde{q}_k - c_1 \left(\tilde{q}_j - (\tilde{Q} - \hat{Q})\right) \\
&= \sum_{k\neq j} c_k \left(\hat{q}_k - \tilde{q}_k\right) + c_j \hat{q}_j - c_1 \left(\tilde{q}_j - \sum_{k=1}^{K} (\tilde{q}_k - \hat{q}_k)\right) \\
&= \sum_{k\neq j} c_k \left(\hat{q}_k - \tilde{q}_k\right) + c_j \hat{q}_j - c_1 \left(\hat{q}_j - \sum_{k\neq j} (\tilde{q}_k - \hat{q}_k)\right) \\
&= \sum_{k\neq j} \left(c_k - c_1\right) \left(\hat{q}_k - \tilde{q}_k\right) + \left(c_j - c_1\right) \hat{q}_j > 0.
\end{aligned}
$$

Hence, we conclude $\widetilde{SW} > \widehat{SW}$.□

Lemma 3. In a simple auction, an outcome (j^*, T^*) is a Nash equilibrium in a simple auction if and only if

(a) $\pi_1(j^*) + T^*_{j^*} \geq \pi_1(j) + T^*_j$ for all j.

(b) If $j^* > 1$, then $\pi_1(j^*) + \pi_j(j^*) + T^*_{j^*} \geq \pi_1(j) + \pi_j(j)$ for all $j \neq j^*$.

(c) If $j^* > 1$ and $T^*_{j^*} > 0$, then $\pi_1(j^*) + T^*_{j^*} = \pi_1(j) + T^*_j$ for some $j \neq j^*$ and $\pi_1(j^*) + T^*_{j^*} = \pi_1(\tilde{j}) + T^*_{\tilde{j}}$ implies $\pi_{j^*}(j^*) - T^*_{j^*} \geq \pi_{j^*}(\tilde{j})$.

Moreover, there exists a Nash equilibrium for every simple auction game.

Proof. We first characterize the set of Nash equilibria and then we show the existence.

Consider (j^*, T^*) is a Nash equilibrium outcome. Condition (a) is obvious from the structure of the game. For (b), suppose we have some $j \neq j^*$ such that $\pi_j(j^*) < \pi_j(j) - \left[\left(\pi_1(j^*) + T^*_{j^*}\right) - \pi_1(j)\right]$, then firm j can offer $\tilde{T}_j = \pi_1(j^*) + T^*_{j^*} - \pi_1(j) + \varepsilon_j$ for some $\varepsilon_j > 0$ so that $U_j\left(j, \tilde{T}_j, T^*_{-j}\right) \geq U_j(j^*, T^*)$ and $U_1\left(j, \tilde{T}_j, T^*_{-j}\right) \geq U_1\left(k, \tilde{T}_j, T^*_{-j}\right)$ for all k. What remains is condition (c). If there is no j such that $\pi_1(j^*) + T^*_{j^*} = \pi_1(j) + T^*_j$, then from condition (a), we have $\pi_1(j^*) + T^*_{j^*} > \pi_1(j) + T^*_j$ for all j. Then firm j^* can offer $\tilde{T}_{j^*} = T^*_{j^*} - \varepsilon_{j^*}$ for some $\varepsilon_{j^*} > 0$ so that $U_{j^*}\left(j^*, \tilde{T}_{j^*}, T^*_{-j^*}\right) \geq U_j(j^*, T^*)$ and $U_1\left(j^*, \tilde{T}_{j^*}, T^*_{-j^*}\right) \geq U_1\left(k, \tilde{T}_{j^*}, T^*_{-j^*}\right)$ for all k. If there is some $\tilde{j}$ with $\pi_1(j^*) + T^*_{j^*} = \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}}$ such that $\pi_{j^*}(j^*) - T^*_{j^*} < \pi_{j^*}\left(\tilde{j}\right)$, then firm j^* can deviate to $\tilde{T}_{j^*} = 0$ so that $U_{j^*}\left(\tilde{j}, \tilde{T}_{j^*}, T^*_{-j^*}\right) \geq U_j(j^*, T^*)$ and $U_1\left(\tilde{j}, \tilde{T}_{j^*}, T^*_{-j^*}\right) \geq U_1\left(k, \tilde{T}_{j^*}, T^*_{-j^*}\right)$ for all k. Therefore, any Nash equilibrium satisfies all three conditions.

Suppose tothe contrary that an outcome satisfies the three conditions but (j^*, T^*) is not a Nash equilibrium. Condition (a) implies that firm 1 selects firm j^*. First, consider the case that j^* has incentive to deviate from $T^*_{j^*}$ to $\tilde{T}_{j^*}$. It is clear that $\tilde{T}_{j^*} < T^*_{j^*}$ because $\tilde{T}_{j^*} \geq T^*_{j^*}$ would still make firm j^* be the licensee with no less payment. However, condition (c) implies that when j^* reduces payment, there exists $\tilde{j} \neq j^*$, $\pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}} = \pi_1(j^*) + T^*_{j^*}$ with $\pi_{j^*}(j^*) - T_{j^*}(j^*) \geq \pi_{j^*}\left(\tilde{j}\right)$ to be chosen as the licensee, which violates the condition that j^* will deviate. Now consider $j \neq j^*$ deviates from T^*_j to $\tilde{T}_j$. Then we have $\tilde{T}_j \geq 0$ such that $\pi_1(j) + \tilde{T}_j \geq \pi_1(k) + T^*_k$ for all $k \neq j$ and $\pi_j(j) - \tilde{T}_j > \pi_j(j^*)$. From condition (b), we have $\pi_1(j^*) + \pi^*_{j^*}(j^*) + T^*_{j^*} \geq \pi_1(j) + \pi_j(j^*)$. Hence, we have $\pi_1(j^*) + T^*_{j^*} - \tilde{T}_j > \pi_1(j)$. From condition (c), we have for some $\tilde{j} \neq j^*$, $\pi_1(j^*) + T^*_{j^*} = \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}}$, so that $\pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}} - \tilde{T}_j > \pi_1(j)$ which contradicts the conditions that j deviates.

No we are ready to show the existence of a Nash equilibrium. If $\pi_1(1) \geq \pi_1(j) + \pi_j(j) - \pi_j(1)$ for all j, then $(1, T)$ where $T_j = 0$ for all j is clearly a NE. For all $S \subseteq N\backslash\{1\}$ with $|S| = 2$, define $v(S) = \max_{h \in S}\{\pi_1(h) + \sum_{k \in S} \pi_k(h)\}$ and $s^*(S) \in \text{argmax}_{h \in S}\{\pi_1(h) + \sum_{k \in S} \pi_k(h)\}$. Let $S^* \in \text{argmax}_{S \subseteq N\backslash\{1\}, |S|=2}\, v(S)$, $j^* = s^*(S^*)$ and $\tilde{j} \in \text{argmax}_{h \in K\backslash\{1, j^*\}}\{\pi_1(h) + \pi_h(h) - \pi_h(j^*)\}$. Define $T^*_{j^*} = \pi_1\left(\tilde{j}\right) + \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*) - \pi_1(j^*)$, $T^*_{\tilde{j}} = \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*)$ and $T^*_j = 0$ for all $j \neq j^*$. It is easy to check (j^*, T^*) is indeed a Nash equilibrium outcome. Since we have $\pi_1(j^*) + T^*_{j^*} = \pi_1\left(\tilde{j}\right) + \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*) \geq \pi_1(j) + T^*_j$ for all j, condition (a) is true. Now consider condition (c). For all $j \neq j^*$, $\pi_1(j^*) + \pi_j(j^*) + T^*_{j^*} = \pi_1(j^*) + \pi_j(j^*) + \pi_1(j) + \pi_j(j) - \pi_j(j^*) - \pi_1(j^*) \geq \pi_1(j) + \pi_j(j)$. Condition (d) is obvious because $\pi_1(j^*) + T^*_{j^*} = \pi_1\left(\tilde{j}\right) + \pi_{\tilde{j}}\left(\tilde{j}\right) - \pi_{\tilde{j}}(j^*) = \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}}$ and $\pi_{j^*}(j^*) - T^*_{j^*} = \pi_{j^*}(j^*) - \pi_1\left(\tilde{j}\right) + T^*_{\tilde{j}} - \pi_1(j^*) = \pi_{j^*}\left(\tilde{j}\right) \geq \pi_{j^*}\left(\tilde{j}\right)$.□

Lemma 5. For any distinct i, j and k, define $\Pi(i,j,k) = [\pi_i(j) - \pi_k(j)] - [\pi_j(i) - \pi_k(i)] - [\pi_i(k) - \pi_j(k)]$. Under linear demand, we have $\Pi(i,j,k) = 0$ for $i < j < k$.

Proof. Define Q_k be the industry equilibrium output if the licensee is firm k. First, we have

$$\begin{aligned}\Pi(i,j,k) &= \left[\frac{(P(Q_j)-c_i)^2}{-P'(Q_j)} - \frac{(P(Q_j)-c_k)^2}{-P'(Q_j)}\right] - \left[\frac{(P(Q_i)-c_j)^2}{-P'(Q_i)} - \frac{(P(Q_i)-c_k)^2}{-P'(Q_i)}\right] \\ &\quad - \left[\frac{(P(Q_k)-c_i)^2}{-P'(Q_k)} - \frac{(P(Q_k)-c_j)^2}{-P'(Q_k)}\right] \\ &= \frac{(2P(Q_j)-c_k-c_i)(c_k-c_i)}{-P'(Q_j)} - \frac{(2P(Q_i)-c_k-c_j)(c_k-c_j)}{-P'(Q_i)} \\ &\quad - \frac{(2P(Q_k)-c_i-c_j)(c_j-c_i)}{-P'(Q_k)}\end{aligned}$$

Let the inverse demand function be $P(Q) = \alpha - \beta Q$ where $\alpha, \beta > 0$. Thus, we have $-P'(Q_h) = -P'(Q_j) = -P'(Q_k) = \beta$ and $P(Q_h) = (\alpha - C - c_1 + c_h)/(1+K)$ for all $h \neq 1$. Hence, we have

$$\begin{aligned}\Pi(i,j,k) &= \left(2\frac{\alpha - C - c_1 + c_j}{1+K} - c_k - c_i\right)\frac{c_k - c_i}{\beta} - \left(2\frac{\alpha - C - c_1 + c_i}{1+K} - c_k - c_j\right)\frac{c_k - c_j}{\beta} \\ &\quad - \left(2\frac{\alpha - C - c_1 + c_k}{1+K} - c_j - c_i\right)\frac{c_j - c_i}{\beta} \\ &= \frac{2}{\beta(1+K)}\left[-c_j(c_k - c_i) + c_i(c_k - c_j) + c_k(c_j - c_i)\right] = 0.\square\end{aligned}$$

Proposition 3. Under linear demand, there exists a TNE in a simple auction.

Proof. Let $j_1 = \arg\max_{j \in K}[\pi_1(j) + \pi_j(j)] - [\pi_1(1) + \pi_j(1)]$ be the joint-profit-maximizing partner. If $j_1 < K$, then Proposition 2 has already shown that $\pi_1(j_1) + \pi_k(j_1) + \pi_{j_1}(j_1) > \pi_1(k) + \pi_k(k) + \pi_{j_1}(k)$ for all $j_1 < k$. If we have $\pi_1(j_1) + \pi_k(j_1) + \pi_{j_1}(j_1) > \pi_1(k) + \pi_k(k) + \pi_{j_1}(k)$ for all $k < j_1$, then we are done. Suppose not. Define $j_2 = \max\{j < j_2 : \pi_1(j) + \pi_j(j) + \pi_{j_2}(j) > \pi_1(j_1) + \pi_j(j_1) + \pi_{j_2}(j_2)\}$. We are going to show we have

$\pi_1(j_2) + \pi_k(j_2) + \pi_{j_2}(j_2) > \pi_1(k) + \pi_k(k) + \pi_{j_2}(k)$ for all $k > j_2$: First, we will show $\pi_1(j_2) + \pi_k(j_2) + \pi_{j_2}(j_2) > \pi_1(k) + \pi_k(k) + \pi_{j_2}(k)$ for all $k > j_1$. It is trivial if $j_1 = K$. Consider $j_1 < K$. We have

$$\begin{aligned}
& \pi_1(j_2) + \pi_{j_2}(j_2) + \pi_k(j_2) \\
= \; & \pi_1(j_2) + \pi_{j_2}(j_2) + \pi_k(j_2) - \Pi(j_2, j_1, k) \\
= \; & \pi_1(j_2) + \pi_{j_2}(j_2) + \pi_k(j_2) - [\pi_{j_2}(j_1) - \pi_k(j_1) - \pi_{j_1}(j_2) + \pi_k(j_2) - \pi_i(k) + \pi_{j_2}(k)] \\
= \; & \pi_1(j_2) + \pi_{j_2}(j_2) + \pi_{j_1}(j_2) - \pi_{j_2}(j_1) + \pi_k(j_1) + \pi_{j_2}(k) - \pi_{j_1}(k) \\
> \; & \pi_1(j_1) + \pi_{j_2}(j_1) + \pi_{j_1}(j_1) - \pi_{j_2}(j_1) + \pi_k(j_1) + \pi_{j_2}(k) - \pi_{j_1}(k) \\
= \; & \pi_1(j_1) + \pi_{j_1}(j_1) + \pi_k(j_1) + \pi_{j_2}(k) - \pi_{j_1}(k) \\
\geq \; & \pi_1(k) + \pi_{j_1}(k) + \pi_k(k) + \pi_{j_2}(k) - \pi_{j_1}(k) \\
= \; & \pi_1(k) + \pi_{j_2}(k) + \pi_k(k)
\end{aligned}$$

What remains is to show we have $\pi_1(j_2)+\pi_h(j_2)+\pi_{j_2}(j_2) > \pi_1(h)+\pi_h(h)+\pi_{j_2}(h)$ for all h such that $j_2 < h < j_1$. (This step is trivial if $j_2 = j_1 - 1$.) By construction, we have $\pi_1(j_1) + \pi_{j_1}(j_1) + \pi_h(j_1) \geq \pi_1(h) + \pi_{j_1}(h) + \pi_h(h)$. Hence, we have

$$\begin{aligned}
& \pi_1(j_2) + \pi_h(j_2) + \pi_{j_2}(j_2) \\
= \; & \pi_1(j_2) + \pi_h(j_2) + \pi_{j_2}(j_2) + \Pi(j_2, h, j_1) \\
= \; & \pi_1(j_2) + \pi_h(j_2) + \pi_{j_2}(j_2) + [\pi_{j_2}(h) - \pi_{j_1}(h) - \pi_h(j_2) + \pi_{j_1}(j_2) - \pi_{j_2}(j_1) + \pi_h(j_1)] \\
= \; & \pi_1(j_2) + \pi_{j_1}(j_2) + \pi_{j_2}(j_2) + \pi_{j_2}(h) - \pi_{j_1}(h) - \pi_{j_2}(j_1) + \pi_h(j_1) \\
\geq \; & \pi_1(j_1) + \pi_{j_1}(j_1) + \pi_{j_2}(j_1) + \pi_{j_2}(h) - \pi_{j_1}(h) - \pi_{j_2}(j_1) + \pi_h(j_1) \\
= \; & \pi_1(j_1) + \pi_{j_1}(j_1) + \pi_{j_1}(j_1) + \pi_{j_2}(h) - \pi_{j_1}(h) \\
\geq \; & \pi_1(h) + \pi_h(h) + \pi_{j_1}(h) + \pi_{j_2}(h) - \pi_{j_1}(h) \\
= \; & \pi_1(h) + \pi_h(h) + \pi_{j_2}(h)
\end{aligned}$$

Therefore, we have $\pi_1(j_2) + \pi_k(j_2) + \pi_{j_2}(j_2) > \pi_1(k) + \pi_k(k) + \pi_{j_2}(k)$ for all $k > j_2$. If we have $\pi_1(j_2) + \pi_k(j_2) + \pi_{j_2}(j_2) > \pi_1(k) + \pi_k(k) + \pi_{j_2}(k)$ for all $k < j_2$, then we are done. Otherwise, we can inductively define $j_{n+1} = \max\{j < j_n : \pi_1(j) + \pi_j(j) + \pi_{j_n}(j) > \pi_1(j_1) + \pi_j(j_n) + \pi_{j_n}(j_n)\}$ and repeat the argument to show $\pi_1(j_{n+1})+\pi_k(j_{n+1})+\pi_{j_{n+1}}(j_{n+1}) > \pi_1(k)+\pi_k(k) + \pi_{j_{n+1}}(k)$ for all $k > j_{n+1}$. Since j_n is strictly decreasing and $j_n \geq 2$, the process must end in finite steps. Then, there exists some firm j^* with $2 \leq j^* \leq j_1$ such that $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_{j^*}(k) > \pi_1(k) + \pi_{j^*}(k) + \pi_k(k)$ for all $k \neq j^*$.□

Proposition 4. If demand is linear and $K \geq 3$, then licensing must occur in a truthful Nash equilibrium in menu auction.

Proof. Since a TNE in menu auction always achieves the most efficient action, it suffices to show that transfer to firm 2 always leads to higher total industry profit.

We are going to show the linear case first. Let the inverse demand function be $P(Q) = \alpha - \beta Q$. The industry total profit would be

$$\Pi = \sum_{i \in K} \frac{1}{\beta} \left[\frac{\alpha + \sum_{j \in K} c_j}{(1+K)} - c_i \right]^2$$

So that if transfer from firm 1 to firm 2, then the industry total profit is

$$\hat{\Pi} = \sum_{i \in K \backslash \{2\}} \frac{1}{\beta} \left[\frac{\alpha + \sum_{j \in K} c_j + c_1 - c_2}{(1+K)} - c_i \right]^2 + \frac{1}{\beta} \left[\frac{\alpha + \sum_{j \in K} c_j + c_1 - c_2}{(1+K)} - c_1 \right]^2$$

the change in profit would then be

$$
\begin{aligned}
&\hat{\Pi}-\Pi \\
= \;& \frac{1}{\beta}\sum_{i\in K\backslash\{2\}}\left[\left(\frac{\alpha+\sum_{j\in K}c_j+c_1-c_2}{1+K}-c_i\right)^2-\left(\frac{\alpha+\sum_{j\in K}c_j}{1+K}-c_i\right)^2\right] \\
& +\frac{1}{\beta}\left[\left(\frac{\alpha+\sum_{j\in K}c_j+c_1-c_2}{(1+K)}-c_1\right)^2-\left(\frac{\alpha+\sum_{j\in K}c_j}{(1+K)}-c_2\right)^2\right] \\
= \;& \frac{1}{\beta}\sum_{i\in K\backslash\{2\}}\left(\frac{2\alpha+2\sum_{j\in K}c_j+c_1-c_2}{1+K}-2c_i\right)\left(\frac{c_1-c_2}{1+K}\right) \\
& +\frac{1}{\beta}\left(\frac{2\alpha+2\sum_{j\in K}c_j+c_1-c_2}{1+K}-c_1-c_2\right)\left(\frac{c_1-c_2}{1+K}-c_1+c_2\right) \\
= \;& \frac{1}{\beta}\frac{c_1-c_2}{1+K}\left((K-1)\,\frac{2\alpha+2\sum_{j\in K}c_j+c_1-c_2}{1+K}-2\sum_{i\in K\backslash\{2\}}c_i\right) \\
& +\frac{K}{\beta}\frac{c_2-c_1}{1+K}\left(\frac{2\alpha+2\sum_{j\in K}c_j+c_1-c_2}{1+K}-c_1-c_2\right) \\
= \;& \frac{1}{\beta}\frac{c_2-c_1}{1+K}\left(\frac{2\alpha+2\sum_{j\in K}c_j+c_1-c_2}{1+K}+2\sum_{i\in K\backslash\{2\}}c_i-Kc_1-Kc_2\right) \\
= \;& \frac{1}{\beta}\frac{c_2-c_1}{(1+K)^2}\left(2\alpha+2\sum_{j\in K}c_j+c_1-c_2+2\,(1+K)\sum_{i\in K\backslash\{2\}}c_i-K\,(1+K)\,c_1-K\,(1+K)\,c_2\right) \\
= \;& \frac{1}{\beta}\frac{c_2-c_1}{(1+K)^2}\left(2\alpha+2\,(2+K)\sum_{j\in K}c_j-\left(K^2+K-1\right)c_1-\left(K^2+3K+3\right)c_2\right)
\end{aligned}
$$

Since each firm is having positive output after transfer, price must be higher than c_K. Hence, we have $\frac{1}{1+K}\left(1+\sum_{j\in K}c_j-c_2\right)-c_3>0$ which implies $\alpha + \sum_{j\in K} c_j > (K+2)\, c_2$ if $K \geq 3$. Then, we have

$$\begin{aligned}
& \hat{\Pi} - \Pi \\
> \; & \frac{1}{\beta}\frac{c_2 - c_1}{(1+K)^2}\left(2(K+2)\,c_2 + 2(K+1)\sum_{j\in K} c_j - \left(K^2+K-1\right)c_1 - \left(K^2+3K+3\right)c_2\right) \\
= \; & \frac{1}{\beta}\frac{c_2 - c_1}{(1+K)^2}\left(2(K+1)\sum_{j\in K} c_j - \left(K^2+K-1\right)(c_1+c_2)\right) \\
= \; & \frac{1}{\beta}\frac{c_2 - c_1}{(1+K)^2}\left(2(K+1)\sum_{i>2} c_i - \left(K^2-K-3\right)(c_1+c_2)\right) \\
\geq \; & \frac{1}{\beta}\frac{c_2 - c_1}{(1+K)^2}\left(2(K+1)(K-2)\,c_2 - \left(K^2-K-3\right)(c_1+c_2)\right) \geq 0
\end{aligned}$$

Hence, transfer to firm 2 always generate strictly higher industry total profit than no licensing and as TNE in menu auction always achieves the most efficient action, no licensing cannot be a TNE outcome of menu auction game.□

Proposition 6. If firm j^* is a menu auction licensee and only j^* is paying for the license, then j^* is a simple auction licensee.

Proof. Clearly $j^* = 1$, then $\pi_1(1) \geq \pi_j(j) + \sum_{h\in K} T_h(j)$ for all $j \in K$. By truthful strategies, we have $T_j(j) \geq \pi_j(j) - \pi_h(1)$ for all $j \in K$. Therefore, we have, $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j) + \sum_{h\in K} T_h(j) \geq \pi_1(j) + \pi_j(j)$ for all $j \neq 1$. Hence, by Lemma 5, it is a TNE in simple auction.

Consider $j^* > 1$. Define $T_h = \sum_{k\in K} T_h^*(k)$. It is easy to check that (j^*, T) is a NE. What remains is to show that (j^*, T) is a TNE. Since (j^*, T^*) is a TNE in menu auction, we have $\pi_1(j^*) + T_{j^*}^*(j^*) \geq \pi_1(k) + \sum_{h\in K} T_h^*(k)$ for all k. Bernheim and Whinston (1986) shows that there exists a TNE $(j^*, \tilde{T})$ such that $\pi_j(j^*) - \tilde{T}_j(j^*) = \pi_j(j) - \tilde{T}_j(j)$ if $\pi_{j^*}(j^*) - \tilde{T}_{j^*}(j^*) \geq \pi_j(j)$ and $\tilde{T}_j(j) = 0$ if $\pi_{j^*}(j^*) - \tilde{T}_{j^*}(j^*) < \pi_j(j)$. Hence, $\pi_1(j^*) + \pi_{j^*}(j^*) - \pi_{j^*}(j) \geq \pi_1(j) + \sum_{h\in P}[\pi_h(j) - \pi_h(j^*)]$ for all j where $P = \{j \in K \backslash \{j^*\} : \pi_j(j^*) \leq \pi_j(j)\}$. By rearranging, we have $\pi_1(j^*) + \pi_{j^*}(j^*) + \sum_{h\in P} \pi_h(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \sum_{h\in P} \pi_h(j)$. For all $h \in P$, we have $\pi_h(h) \geq \pi_h(j^*)$, which implies $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_k(j^*) \geq \pi_1(k) + \pi_{j^*}(k) + \pi_k(k)$. By Proposition 3, we know (j^*, T) is a TNE in simple auction.□

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