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An Economic Mechanism to Regulate Multispecies Fisheries

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Abstract

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1 Introduction.

In the simplest case of fisheries inhabited by a single species, many instruments have been proposed to eliminate the "tragedy of the commons" problem. These include entry limitation, licensing, taxes on catches or individual transferable quotas. All are capable of implementing an optimal consumption path of the fish population (Clark, 1990).

The use of these instruments to regulate fisheries inhabited by several interacting species is less clear. On the one hand, the determination of the optimal policy itself is complicated by the biological interdependences within the ecosystem and the fishers' limited ability to alter the species composition of their catch (Squires et al., 1998). On the other hand, the data needed to determine and enforce the optimal policy, that is, statistics on fishing efforts, fish catches and/or stocks (depending on which instrument is used), renders the regulation impractible in most cases (Arnason, 1990).

Arnasson (1990) expounds one way out of this problem. He argues that all information required to determine the optimal policy is already available within the fishing industry. The fishing firms have knowledge about their own cost and harvesting function. Moreover, the competition within the industry stimulates an efficient use of the available biological data. All this suggests that fisheries management should largely rely on the fishers themselves.

This paper follows this line of reasoning. Assuming nonselective harvesting, we propose an economic mechanism capable of implementing an optimal policy in a multispecies fishery. Under this mechanism, each participant decides both his own fishing effort and that of the other participants. Individualized prices

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are set by the participants themselves. In equilibrium, the prevailing price system reflects the participants' expected future rents, at each point of time. Moreover, each participant pays his effort at a price equal to the sum of the others' individualized prices. Thus, in equilibrium, the participants internalize the external opportunity cost of their fishing effort. Finally, to ensure that the mechanism is balanced, each participant is paid his individualized price, on each unit of fishing effort by the others.

This paper contributes to the literature in three directions. First, as an illustration of our general model, we develop a variant of Clemhout and Wan (1985), by introducing harvest costs and nonselective harvesting technologies. This specification can be explicitly solved and, thus, serves as a benchmark example within the paper. Second, the paper shows that the economic mechanism constructed by Rouillon (2011), primarily designed to manage one species fisheries, also works for multispecies fisheries with nonselective harvesting. Third, and less importantly, this paper transcribes in continuous time the analysis in Rouillon (2011).

The rest of the paper is organized as follows. Section 2 sets the general biologic and economic model. Section 3 states the benchmark specification and caculates the open access and cooperative solutions. In section 4, we construct our economic mechanism to regulate the fishery and derive some of its properties. In section 5, we study the set of Nash equilibria of the associated difference game and show our implementation result.

2 The model.

Consider a I-fishers and J-species model of a common property fishery. There are I fishers $i = 1, 2, ..., I$. At each instant of time t, each fisher i chooses his catch effort rate $e_i(t) \in \mathbb{R}_+$. There are J species $j = 1, 2, ..., J$. The resource state at time t is described with a vector $\boldsymbol{x}(t) = (x_j(t))_{j=1}^J \in \mathbb{R}_+^J$. The initial state is a fixed constant $\boldsymbol{x}_0 = (x_{0j})_{j=1}^J \in \mathbb{R}_+^J$. At each instant t, the resource state evolves according to the ordinary differential equation

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{e}(t), \boldsymbol{x}(t)), \ \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{1}$$

where the function $\boldsymbol{f} = (f_j)_{j=1}^J$ is defined on $\mathbb{R}_+^I \times \mathbb{R}_+^J$ and has values in $\mathbb{R}^J$.

Each fisher i's objective is to maximize

$$\int_0^\infty u_i(\boldsymbol{e}(t), \boldsymbol{x}(t)) e^{-\delta_i t} dt, \tag{2}$$

where u_i is an instant utility function, defined on $\mathbb{R}_+^I \times \mathbb{R}_+^J$ and having values in $\mathbb{R}$, and δ_i is a rate of time preference.

Let S be the set of all functions defined on $\mathbb{R}_+^J$ and having values in $\mathbb{R}_+$. A stationary Makovian strategy for fisher i is a function $s_i \in S$. A vector

$\boldsymbol{s} = (s_i)_{i=1}^{I} \in S^I$ is called a strategic profile or a policy.

Remark 1. A strategic profile (policy) $\boldsymbol{s} \in S^I$ is said to be feasible if there exists a unique state trajectory $\boldsymbol{x}(t)$ satisfying (1), with $\boldsymbol{e}(t) = \boldsymbol{s}(\boldsymbol{x}(t))$, for all t, and if the corresponding fishers' objectives (2), for all i, are well defined. In the rest of the paper, only feasible strategic profiles (policies) are considered.

For all feasible strategic profile $\boldsymbol{s} = (s_i)_{i=1}^{I}$ and initial state $\boldsymbol{x}_0$, let $W^i(\boldsymbol{s}; \boldsymbol{x}_0)$ be defined by

$$\begin{aligned} &W^i(\boldsymbol{s}; \boldsymbol{x}_0) = \textstyle\int_0^\infty u_i(\boldsymbol{e}(t), \boldsymbol{x}(t)) e^{-\delta_i t} dt, \\ &\text{where:} \\ &\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{e}(t), \boldsymbol{x}(t)), \ \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ &\boldsymbol{e}(t) = \boldsymbol{s}(\boldsymbol{x}(t)). \end{aligned} \tag{3}$$

Definition 1. A stationary Markovian Nash equilibrium is a vector $\boldsymbol{s}^* = (s_i^*)_{i=1}^{I} \in S^I$ *such that, for all* i, s_i *and* $\boldsymbol{x}_0$, $W^i(\boldsymbol{s}; \boldsymbol{x}_0) \geq W^i((\boldsymbol{s}^*/s_i); \boldsymbol{x}_0)$, *where* $(\boldsymbol{s}^*/s_i) = \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_I^*\right)$.

For all feasible policy $\boldsymbol{s} = (s_i)_{i=1}^{I}$ and initial state $\boldsymbol{x}_0$, let

$$W(\boldsymbol{s}; \boldsymbol{x}_0) = \textstyle\sum_{i=1}^{I} W^i(\boldsymbol{s}; \boldsymbol{x}_0). \tag{4}$$

Definition 2. An optimal policy is a vector $\boldsymbol{s}^0 = \left(s_i^0\right)_{i=1}^{I} \in S^I$ *such that, for all* $\boldsymbol{s}$ *and* $\boldsymbol{x}_0$, $W\left(\boldsymbol{s}^0; \boldsymbol{x}_0\right) \geq W(\boldsymbol{s}; \boldsymbol{x}_0)$.

3 A benchmark specification.

In this section, as an illustration of the general model, we propose a tractable specification of our biologic and economic model. This framework allows us to explicitly characterize a stationary Markovian Nash equilibrium and the optimal policy, in Propositions 1 and 2 respectively.

We use a variant of Clemhout and Wan (1985), with two differences. First, we abandon their implicit assumption of a perfectly selective harvesting, and replace it by that of a nonselective harvesting technology. This assumption is more realistic in many fisheries (Squires and al., 1998). Second, we generalize the model by introducing a harvest cost, which was set equal to zero in Clemhout and Wan (1985).

Remark 2. Many models of multispecies fisheries in the literature, including the seminal contributions of Clemhout and Wan (1985) and Fischer and Mirman (1996), rely on the assumption of perfectly selective harvesting and zero harvesting costs. Models of multispecies fishery with nonselective harvesting and positive harvesting costs are Mesterton-Gibbons (1996) and Durohit and Chaudhuri (2004). However, none can be explicitly solved.

For simplicity, we assume identical fishers in terms of technologies (cost of effort and production function) and preferences (instant utility function and rate of time preference) and consider symmetric stationary Markovian Nash equilibria.

At each instant of time t, each fisher i's bears a cost c per unit of effort $e_i(t)$, with $c \geq 0$, harvests each specie j in quantity $h_{ij}(t) = q_j e_i(t) x_j(t)$, with $q_j \geq 0$, and derives a utility $a_j \ln(h_{ij}(t))$ from its consumption, with $a_j \geq 0$. Thus, for all i, the instant utility function is specified as

$$u_i(\boldsymbol{e}(t), \boldsymbol{x}(t)) = \textstyle\sum_{j=1}^{J} a_j \ln(h_{ij}(t)) - c e_i(t). \tag{5}$$

The common rate of time preference is $\delta_i = \delta$, for all i.

For all j, the dynamics of the resource is represented by

$$\dot{x}_j(t) = x_j(t)\left(\alpha_j - \textstyle\sum_{k=1}^{J} \beta_{jk} \ln(x_k(t))\right) - \textstyle\sum_{i=1}^{I} h_{ij}(t) \tag{6}$$

where, for all j, $\alpha_j \in \mathbb{R}$, $\beta_{jj} \geqslant 0$ and $\beta_{jk} \in \mathbb{R}$, for all k.

Remark 3. To analyse the resource dynamics, it is convenient to define $\boldsymbol{y} = (y_j)_{j=1}^{J} = (\ln(x_j))_{j=1}^{J}$. Then, an equilibrium is a state $\boldsymbol{y}^*$ such that $\sum_{k=1}^{J} \beta_{jk} y_k^* = \alpha_j$, fo all j. It is globally stable if, and only if, the eigenvalues of the matrix $\boldsymbol{\beta} = \left(\beta_{jk}\right)_{j,k=1}^{J}$ have negative real parts.

Proposition 1 below characterizes a stationary Markovian Nash equilibrium of the differential game.

Proposition 1. Assume that there exists $\mathbf{A}^* = \left(A_j^*\right)_{j=1}^{J} \in \mathbb{R}^J$ *such that* $\delta A_j^* + \sum_{k=1}^{J} A_k^* \beta_{kj} = a_j$*, for all* j*, and* $\sum_{j=1}^{J} A_j^* q_j + c > 0$*. Let* $e^* = \left(\sum_{j=1}^{J} a_j\right) / \left(\sum_{j=1}^{J} A_j^* q_j + c\right)$*. The strategic profile* $\boldsymbol{s}^* = (s_i^*)_{i=1}^{I}$*, where* $s_i^*(\boldsymbol{x}) = e^*$*, for all* i *and* $\boldsymbol{x}$*, defines a stationary Markovian Nash equilibrium of the differential game.*

Proof. We show that the policy $\boldsymbol{s}^* = (s_i^*)_{i=1}^{I}$, where $s_i^*(\boldsymbol{x}) = e^*$, for all i and $\boldsymbol{x}$, satisfies the HJB equation [1]

$$\delta V^i(\boldsymbol{x}) = \max_{e_i \in \mathbb{R}_+} \left\{ \begin{array}{c} \sum_{j=1}^{J} a_j \ln(q_j e_i x_j) - c e_i \\ + \sum_{j=1}^{J} V_j^i(\boldsymbol{x}) x_j \left(\alpha_j - \sum_{k=1}^{J} \beta_{jk} \ln(x_k) - q_j((I-1)e^* + e_i)\right) \end{array} \right\}. \tag{7}$$

[1] Here and below, $V_j^i(\boldsymbol{x})$ is the partial derivative of $V^i(\boldsymbol{x})$ with respect to x_j.

where

$$V^i(\boldsymbol{x}) = \sum_{j=1}^J A_j^* \ln(x_j) + B^*,$$
$$A_j^* = (1/\delta)\left[a_j - \sum_{k=1}^J A_k^* \beta_{kj}\right], \text{ for all } j,$$
$$B^* = (1/\delta)\sum_{j=1}^J \left[I a_j (\ln(q_j e^*) - 1) + A_j^* \alpha_j\right],$$
$$e^* = \left(\sum_{j=1}^J a_j\right) / \left(\sum_{j=1}^J A_j^* q_j + c\right).$$

Using $V_j^i(\boldsymbol{x}) = A_j^*/x_j$, for all j, and rearranging, (7) becomes

$$\delta V^i(\boldsymbol{x}) = \max_{e_i \in \mathbb{R}_+} \left\{ \begin{array}{c} \sum_{j=1}^J \left(a_j - \sum_{k=1}^J A_k^* \beta_{kj}\right) \ln(x_j) \\ \sum_{j=1}^J a_j \ln(e_i) - \left(\sum_{j=1}^J A_j^* q_j + c\right) e_i \\ + \sum_{j=1}^J a_j \ln(q_j) + \sum_{j=1}^J A_j^* \alpha_j - \sum_{j=1}^J A_j^* q_j (I-1) e^* \end{array} \right\}.$$

It is immediate to verify that the control $(e_i)_{i=1}^I = (s_i^*(\boldsymbol{x}))_{i=1}^I = (e^*)_{i=1}^I$ satisfies the first order conditions

$$\left(\sum_{j=1}^J a_j\right)/e_i - \left(\sum_{j=1}^J A_j^* q_j + c\right) = 0, \text{ for all } i,$$

and, thus, maximizes the RHS of (7). After substitution, (7) writes

$$\delta V^i(\boldsymbol{x}) = \sum_{j=1}^J \left[\left(a_j - \sum_{k=1}^J A_k^* \beta_{kj}\right) \ln(x_j) + a_j (\ln(q_j e^*) - 1) + A_j^* (\alpha_j - q_j (n-1) e^*)\right].$$

and, using

$$A_j^* = (1/\delta)\left[a_j - \sum_{k=1}^J A_k^* \beta_{kj}\right], \text{ for all } j,$$
$$B = (1/\delta)\sum_{j=1}^J \left[a_j (\ln(q_j e^*) - 1) + A_j^* (\alpha_j - q_j (n-1) e^*)\right],$$

we can confirm our conjecture that

$$V^i(\boldsymbol{x}) = \sum_{j=1}^J A_j^* \ln(x_j) + B^*.$$

■

Proposition 2 below characterizes the optimal policy.

Proposition 2. Assume that there exists $\mathbf{A}^0 = \left(A_j^0\right)_{j=1}^J \in \mathbb{R}^J$ *such that* $\delta A_j^0 + \sum_{k=1}^J A_k^0 \beta_{kj} = n a_j$*, for all* j*, and* $\sum_{j=1}^J A_j^0 q_j + c > 0$*. Let* $e^0 = \left(\sum_{j=1}^J a_j\right) / \left(\sum_{j=1}^J A_j^0 q_j + c\right)$*. An optimal policy is* $\boldsymbol{s}^0 = \left(s_i^0\right)_{i=1}^I$*, where* $s_i^0(\boldsymbol{x}) = e^0$*, for all* i *and* $\boldsymbol{x}$*.*

Proof. We show that the policy $\boldsymbol{s}^0 = \left(s_i^0\right)_{i=1}^I$, where $s_i^0(\boldsymbol{x}) = e^0$, for all i and

$\boldsymbol{x}$, satisfies the HJB equation [2]

$$\delta V(\boldsymbol{x}) = \max_{(e_i)_{i=1}^I \in \mathbb{R}_+^I} \left\{ \begin{array}{c} \sum_{i=1}^I \left[\sum_{j=1}^J a_j \ln(q_j e_i x_j) - c e_i \right] \\ + \sum_{j=1}^J V_j(\boldsymbol{x}) x_j \left(\alpha_j - \sum_{k=1}^J \beta_{jk} \ln(x_k) - q_j \sum_{i=1}^n e_i \right) \end{array} \right\}. \tag{8}$$

where

$$\begin{aligned} V(\boldsymbol{x}) &= \sum_{j=1}^J A_j^0 \ln(x_j) + B^0, \\ A_j^0 &= (1/\delta) \left[I a_j - \sum_{k=1}^J A_k^0 \beta_{kj} \right], \text{ for all } j, \\ B^0 &= (1/\delta) \sum_{j=1}^J \left[I a_j \left(\ln\left(q_j e^0\right) - 1 \right) + A_j^0 \alpha_j \right], \\ e^0 &= \left(\sum_{j=1}^J a_j \right) / \left(\sum_{j=1}^J A_j^0 q_j + c \right). \end{aligned}$$

Using $V_j(\boldsymbol{x}) = A_j^0 / x_j$, for all j, and rearranging, (8) becomes

$$\delta V(\boldsymbol{x}) = \max_{(e_i)_{i=1}^I \in \mathbb{R}_+^I} \left\{ \begin{array}{c} \sum_{j=1}^J \left(I a_j - \sum_{k=1}^J A_k^0 \beta_{kj} \right) \ln(x_j) \\ + \sum_{i=1}^I \left[\sum_{j=1}^J a_j \ln(e_i) - \left(\sum_{j=1}^J A_j^0 q_j + c \right) e_i \right] \\ + n \sum_{j=1}^J a_j \ln(q_j) + \sum_{j=1}^J A_j^0 \alpha_j \end{array} \right\}.$$

It is immediate to verify that the control $(e_i)_{i=1}^I = \left(s_i^0(\boldsymbol{x})\right)_{i=1}^I = \left(e^0\right)_{i=1}^I$ satisfies the first order conditions

$$\left(\sum_{j=1}^J a_j \right) / e_i - \left(\sum_{j=1}^J A_j q_j + c \right) = 0, \text{ for all } i,$$

and, thus, maximizes the RHS of (8). Subtituting, (8) writes

$$\delta V(\boldsymbol{x}) = \sum_{j=1}^J \left[\left(I a_j - \sum_{k=1}^J A_k^0 \beta_{kj} \right) \ln(x_j) + I a_j \left(\ln\left(q_j e^0\right) - 1 \right) + A_j^0 \alpha_j \right],$$

and, using

$$\begin{aligned} A_j^0 &= (1/\delta) \left[I a_j - \sum_{k=1}^J A_k^0 \beta_{kj} \right], \text{ for all } j, \\ B^0 &= (1/\delta) \sum_{j=1}^J \left[I a_j \left(\ln\left(q_j e^0\right) - 1 \right) + A_j^0 \alpha_j \right], \end{aligned}$$

we verify that

$$V(\boldsymbol{x}) = \sum_{j=1}^J A_j^0 \ln(x_j) + B^0.$$

■

The literature often identifies the stationary Markovian Nash equilibrium with the open access solution and the optimal policy with the cooperative solution (Levhari and Mirman, 1980; Fischer and Mirman, 1996). A well-known result is that open access leads to the "tragedy of the commons" (Gordon, 1954),

[2] Here and below, $V_j(\boldsymbol{x})$ is the partial derivative of $V(\boldsymbol{x})$ with respect to x_j.

i.e. to overfishing with respect to the cooperative solution. The following corollary of Propositions 1 and 2 displays a necessary and sufficient condition under which the "tragedy of the commons" arises within the framework considered here.

Corollary 1. There is overfishing under the open access solution, with respect to the cooperative solution (i.e., $e^* > e^0$*) if, and only if,* $\sum_{j=1}^{J}\left(A_j^* - A_j^0\right)q_j = (1-1/I)\sum_{j=1}^{J} A_j^* q_j > 0$.

Proof. Immediate, remarking that $A_j^* = (1/I)\,A_j^0$, for all j, and remembering that $e^* = \left(\sum_{j=1}^{J} a_j\right) / \left(\sum_{j=1}^{J} A_j^* q_j + c\right)$ and $e^0 = \left(\sum_{j=1}^{J} a_j\right) / \left(\sum_{j=1}^{J} A_j^0 q_j + c\right)$. ■

4 The economic mechanism.

A mechanism is a pair (M, ρ), consisting of a message space $M \equiv \times_{i=1}^{I} M_i$ and an outcome function ρ.

Under the mechanism, each participant i is asked to announce a message m_i in M_i. The outcome function ρ is a mapping from M into $\mathbb{R}_+^I \times \mathbb{R}^I$, which translates joint messages $\boldsymbol{m} = (m_i)_{i=1}^{I}$ into efforts $(E_i(\boldsymbol{m}))_{i=1}^{I}$ and transfers $(T_i(\boldsymbol{m}))_{i=1}^{I}$ to be implemented by the participants.

The specific mechanism used below is as follows.

We let $M_i \equiv \mathbb{R}^I \times \mathbb{R}_+^I$, for all i. A generic message of agent i is denoted $m_i = \left((E_{ik})_{k=1}^{I}, (P_{ik})_{k=1}^{I}\right)$.

The component E_{ik} is interpreted as a fishing effort agent i is willing for agent k. Likewise, the component E_{ii} is a fishing effort that agent i is willing for himself. The component P_{ik} is a compensatory price that agent i is proposing to pay to agent k per unit of his own fishing effort. Finally, P_{ii} is a compensatory price that agent i is willing to receive per unit of the other participants' fishing efforts.

Agent i's fishing effort is given by

$$E_i(\boldsymbol{m}) = (1/I)\max\left\{0, \sum_{k=1}^{I} E_{ki}\right\}. \tag{9}$$

In order to obtain the transfer to be paid by agent i, several steps are needed.

To begin with, for all k, rearrange the sequence $(P_{ik})_{i=1}^{I}$ in ascending order. In case where $P_{ik} = P_{jk}$, for some i and j, rearrange in ascending order of indexes. Then, define the agent k's personalized price $P_k(\boldsymbol{m})$ as the N-th term of the ordered sequence, with $N = I/2$, if I is even, and $N = (I+1)/2$, if I is odd.

Finally, agent i's transfer is given by

$$T_i(\boldsymbol{m}) = \sum_{j \neq i} P_j(\boldsymbol{m}) E_i(\boldsymbol{m}) - P_i(\boldsymbol{m}) \sum_{j \neq i} E_j(\boldsymbol{m}). \tag{10}$$

The following properties of the mechanism will prove to be useful below.

Property 1. For all $\boldsymbol{m} \in M$ and all $(e_k)_{k=1}^I \in \mathbb{R}_+^I$, each participant i can report a message m_i' such that $(E_k(\boldsymbol{m}/m_i'))_{k=1}^I = (e_k)_{k=1}^I$ and $(P_k(\boldsymbol{m}/m_i'))_{k=1}^I = (P_k(\boldsymbol{m}))_{k=1}^I$, where $(\boldsymbol{m}/m_i') = (m_1, ..., m_{i-1}, m_i', m_{i+1}, ..., m_I)$.

Property 1 means that under the mechanism, each participant is able to decide the efforts of everyone, without modifying the current system of individualized prices.

Proof. Pick $\boldsymbol{m} \in M$ and $(e_k)_{k=1}^I \in \mathbb{R}_+^I$. Consider any agent i. Let $m_i' = \left((E_{ik}')_{k=1}^I, (P_{ik}')_{k=1}^I\right)$ be such that, for all k, $E_{ik}' = Ie_k - \sum_{j \neq i} E_{jk}$ and $P_{ik}' = P_{ik}$. It is immediate that $E_k(\boldsymbol{m}/m_i') = (1/I) \max\left\{0, E_{ik}' + \sum_{j \neq i} E_{jk}\right\} = e_k$ and $P_k(\boldsymbol{m}/m_i') = P_k(\boldsymbol{m})$, for all k. ■

Property 2. Assume that $I \geq 3$. Given any $(p_k)_{k=1}^I \in \mathbb{R}_+^I$, let $\boldsymbol{m} \in M$ be any joint message such that $(P_{ik})_{k=1}^I = (p_k)_{k=1}^I$, for all i. Then, $(P_k(\boldsymbol{m}))_{k=1}^I = (P_k(\boldsymbol{m}/m_i'))_{k=1}^I = (p_k)_{k=1}^I$, for all i and $m_i' \in M_i$.

Property 2 states that, whenever all agents announce the same system of individualized prices, the mechanism implements it and no unilateral deviation by a single agent can modify it. (It is equivalent to say that, whenever all agents but one report the same price system, then the mechanism enforces it.)

Proof. Let $(p_k)_{k=1}^I \in \mathbb{R}_+^I$. Let $\boldsymbol{m} \in M$ be such that $(P_{ik})_{k=1}^I = (p_k)_{k=1}^I$, for all i.

By definition, for all k, $P_k(\boldsymbol{m})$ is the N-th term of the sequence $(P_{ik})_{i=1}^I$, rearranged in ascending order of values, and then of indexes. Since $(P_{ik})_{i=1}^I = (p_k, ..., p_k)$, we have $P_k(\boldsymbol{m}) = p_k$.

Now, consider any i and $m_i' \in M_i$. Let $(P_{ik}')_{k=1}^I$ be the associated vector of personalized prices announced by i. By definition, for all k, $P_k(\boldsymbol{m}/m_i')$ is the N-th term of the sequence $\left(P_{1k}, ..., P_{(i-1)k}, P_{ik}', P_{(i+1)k} ..., P_{Ik}\right)$, rearranged in ascending order of values, and then of indexes. The ordered sequence is:

$$\begin{aligned} (P_{ik}', p_k, ..., p_k), \text{ if } P_{ik}' &< p_k, \\ (p_k, ..., p_k), \text{ if } P_{ik}' &= p_k, \\ (p_k, ..., p_k, P_{ik}'), \text{ if } p_k &< P_{ik}'. \end{aligned}$$

In all cases, given that $I \geq 3$, we obtain $P_k(\boldsymbol{m}/m_i') = p_k$. ■

Property 3. For all $\boldsymbol{m} \in M$, $\sum_{i=1}^{I} T_i(\boldsymbol{m}) = 0$.

In other words, the mechanism (M, ρ) is balanced.

Proof. For all $\boldsymbol{m} \in M$, notice that the transfer $T_i(\boldsymbol{m})$ can also be written as:

$$T_i(\boldsymbol{m}) = \sum_{j=1}^{I} P_j(\boldsymbol{m}) E_i(\boldsymbol{m}) - P_i(\boldsymbol{m}) \sum_{j=1}^{I} E_j(\boldsymbol{m}).$$

Summing over i, one directly obtains:

$$\sum_{i=1}^{I} T_i(\boldsymbol{m}) = 0,$$

proving that the mechanism (M, ρ) is balanced. ■

5 Regulated Multispecies Fishery.

Suppose that the fishery is regulated by using repeatedly the mechanism defined above. With the dynamics of the resource state, this defines a differential game (Dockner and al., 2000), where the fishers' actions are reports of messages from their message space. Here, we define and analyse the stationary Markovian Nash equilibria of this differential game.

Consider the differential game induced by (M, ρ). A stationary Markovian strategy for fisher i is a function σ_i defined on $\mathbb{R}_+^J$ and having values in M_i. For all $\boldsymbol{\sigma} = (\sigma_i)_{i=1}^{I}$ and $\boldsymbol{x}_0$, define

$$\begin{array}{l} J^i(\boldsymbol{\sigma}; \boldsymbol{x}_0) = \int_0^{\infty} [u_i(\boldsymbol{e}(t), \boldsymbol{x}(t)) - t_i(t)] e^{-\delta_i t} dt \\ \text{subject to:} \\ \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{e}(t), \boldsymbol{x}(t)), \ \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ \boldsymbol{e}(t) = (E_i(\boldsymbol{\sigma}(\boldsymbol{x}(t))))_{i=1}^{I}, \\ t_i(t) = T_i(\boldsymbol{\sigma}(\boldsymbol{x}(t))) \end{array} \tag{11}$$

Definition 3 below restates Definition 1 using the appropriate notations, corresponding to the differential game associated with (M, ρ).

Definition 3. A stationary Markovian Nash equilibrium of the differential game induced by (M, ρ) *is a vector* $\boldsymbol{\sigma}^* = (\sigma_i^*)_{i=1}^{I}$ *such that, for all* i, σ_i *and* x_0, $J^i(\boldsymbol{\sigma}^*; \boldsymbol{x}_0) \geq J^i((\boldsymbol{\sigma}^*/\sigma_i); \boldsymbol{x}_0)$, *where* $(\boldsymbol{\sigma}^*/\sigma_i) = (\sigma_1^*, ..., \sigma_{i-1}^*, \sigma_i, \sigma_{i+1}^*, ..., \sigma_I^*)$.

5.1 Optimality.

Proposition 3 proves that a stationary Markovian Nash equilibrium of the differential game associated with the mechanism (M, ρ) induces an optimal utilization of the resource.

Proposition 3. If $\boldsymbol{\sigma}^*$ is a stationary Markovian Nash equilibrium of the differential game induced by (M, ρ), then the policy $\boldsymbol{s}^0 = \left(s_i^0\right)_{i=1}^I$, where $s_i^0(\boldsymbol{x}) = E_i(\boldsymbol{\sigma}^*(\boldsymbol{x}))$, for all i and $\boldsymbol{x}$, is an optimal policy.

Proof. Let $\boldsymbol{\sigma}^*$ be a stationary Markovian Nash equilibrium of the differential game induced by (M, ρ).

Assume, by way of contradiction, that there exists an initial state $\boldsymbol{x}_0$ and a feasible policy $\boldsymbol{s}$ such that

$$W(\boldsymbol{s}; \boldsymbol{x}_0) > \textstyle\sum_{i=1}^I J^i(\boldsymbol{\sigma}; \boldsymbol{x}_0). \tag{12}$$

Denote $\boldsymbol{e}(t)$ and $\boldsymbol{x}(t)$, for all t, the time paths of the fisher's efforts and resource stock, respectively, associated with the policy $\boldsymbol{s}$, starting from the initial state $\boldsymbol{x}_0$.

By property 1, used at each point $\boldsymbol{x}$, each fisher i can find a strategy σ_i such that, for all $\boldsymbol{x}$,

$$\left(E_k\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right)(\boldsymbol{x})\right)\right)_{k=1}^I = \left(s_k(\boldsymbol{x})\right)_{k=1}^I, \tag{13}$$

$$\left(P_k\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right)(\boldsymbol{x})\right)\right)_{k=1}^I = \left(P_k\left(\boldsymbol{\sigma}^*(\boldsymbol{x})\right)\right)_{k=1}^I. \tag{14}$$

From (13), it is clear that the strategic profile $(\boldsymbol{\sigma}^*/\sigma_i)$ implements the same time paths of the fisher's efforts and resource stock as the policy $\boldsymbol{s}$. Moreover, from (14), the associated time path of the price system is $\left(P_k\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right)(\boldsymbol{x}(t))\right)\right)_{k=1}^I = \left(P_k\left(\boldsymbol{\sigma}^*(\boldsymbol{x}(t))\right)\right)_{k=1}^I$, for all t. Thus, we have

$$J_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right), \boldsymbol{x}_0\right) = \int_0^\infty \left[u_i\left(\boldsymbol{e}(t), \boldsymbol{x}(t)\right) - t_i(t)\right] e^{-\delta_i t} dt,$$

where, for all t,

$$\begin{aligned} t_i(t) &= T_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right)(\boldsymbol{x}(t))\right), \\ &= \textstyle\sum_{j \neq i} P_j\left(\boldsymbol{\sigma}^*(\boldsymbol{x}(t))\right) c_i(t) - P_i\left(\boldsymbol{\sigma}^*(\boldsymbol{x}(t))\right) \sum_{j \neq i} c_j(t). \end{aligned}$$

Considering a similar unilateral deviation σ_i, by each player i in turn, and summing over i, we get, by property 3,

$$\textstyle\sum_{i=1}^I t_i(t) = 0,$$

and, therefore,

$$\begin{aligned} \textstyle\sum_{i=1}^I J_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right), \boldsymbol{x}_0\right) &= \textstyle\sum_{i=1}^I \displaystyle\int_0^\infty u_i\left(\boldsymbol{e}(t), \boldsymbol{x}(t)\right) e^{-\delta_i t} dt, \qquad (15) \\ &= W(\boldsymbol{s}; \boldsymbol{x}_0). \end{aligned}$$

Now, as $\boldsymbol{\sigma}^*$ is a Nash equilibrium, we have, for all i,

$$J_i\left(\boldsymbol{\sigma}^*, \boldsymbol{x}_0\right) \geq J_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right), \boldsymbol{x}_0\right),$$

which implies, by summation over i, that

$$\sum_{i=1}^{I} J_i\left(\boldsymbol{\sigma}^*, \boldsymbol{x}_0\right) \geq \sum_{i=1}^{I} J_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right), \boldsymbol{x}_0\right). \tag{16}$$

Together, (15) and (16) imply

$$\sum_{i=1}^{I} J_i\left(\boldsymbol{\sigma}^*, \boldsymbol{x}_0\right) \geq W\left(\boldsymbol{s}; \boldsymbol{x}_0\right),$$

which contradicts our assumption (12).

Finally, as (M, ρ) is balanced, by property 3, and $\sum_{i=1}^{I} J_i\left(\boldsymbol{\sigma}^*, \boldsymbol{x}_0\right) \geq W\left(\boldsymbol{s}; \boldsymbol{x}_0\right)$, for all $\boldsymbol{s}$ and $\boldsymbol{x}_0$, it follows that the policy $\boldsymbol{s}^0 = \left(s_i^0\right)_{i=1}^{I}$, where $s_i^0\left(\boldsymbol{x}\right) = E_i\left(\boldsymbol{\sigma}^*\left(\boldsymbol{x}\right)\right)$, for all i and $\boldsymbol{x}$, is an optimal policy. ■

5.2 Existence.

Considering the biologic and economic environment in Section 3, given by (5) and (6), Proposition 4 identifies a stationary Markovian Nash equilibrium to implement to the optimal policy, stated in proposition 2.

Proposition 4. Consider the biologic and economic environment described in Section 3. Assume that there exists $\mathbf{A}^0 = \left(A_j^0\right)_{j=1}^{J} \in \mathbb{R}^J$ *such that* $\delta A_j^0 + \sum_{k=1}^{J} A_k^0 \beta_{kj} = n a_j$*, for all* j*, and* $\sum_{j=1}^{J} A_j^0 q_j + c > 0$*. Let* $e^0 = \left(\sum_{j=1}^{J} a_j\right) / \left(\sum_{j=1}^{J} A_j^0 q_j + c\right)$ *and* $p^0 = (1/I) \sum_{j=1}^{J} A_j^0 q_j$*. The strategic profile* $\boldsymbol{\sigma}^* = \left(\sigma_i^*\right)_{i=1}^{I}$*, where* $\sigma_i^*\left(\boldsymbol{x}\right) = \left(\left(e^0\right)_{i=1}^{I}, \left(p^0\right)_{i=1}^{n}\right)$*, for all* i *and* $\boldsymbol{x}$*, defines a stationary Markovian Nash equilibrium of the differential game induced by* (M, ρ)*.*

Proof. Consider the strategic profile $\boldsymbol{\sigma}^* = \left(\sigma_i^*\right)_{i=1}^{I}$, where $\sigma_i^*\left(\boldsymbol{x}\right) = \left(\left(e^0\right)_{i=1}^{I}, \left(p^0\right)_{i=1}^{n}\right)$, for all i and $\boldsymbol{x}$.

By definition of (M, ρ), we have [3]

$$\begin{aligned} \left(E_i\left(\boldsymbol{m}^*\right)\right)_{i=1}^{I} &= \left(e^0\right)_{i=1}^{I}, \\ \left(P_i\left(\boldsymbol{m}^*\right)\right)_{i=1}^{I} &= \left(p^0\right)_{i=1}^{I}, \\ \left(T_i\left(\boldsymbol{m}^*\right)\right)_{i=1}^{I} &= (0)_{i=1}^{I}. \end{aligned}$$

We must show that fisher i's stationary Markovian strategy $\sigma_i^*\left(\boldsymbol{x}\right) = \left(\left(e^0\right)_{i=1}^{I}, \left(p^0\right)_{i=1}^{n}\right)$ satisfies the HJB equation, for all $\boldsymbol{x}$, [4]

$$\delta v\left(\boldsymbol{x}\right) = \max_{m_i \in M_i} \left\{ \begin{array}{c} \sum_{j=1}^{J} a_j \ln\left(q_j E_i\left(\boldsymbol{m}^*/m_i\right) x_j\right) - c E_i\left(\boldsymbol{m}^*/m_i\right) - T_i\left(\boldsymbol{m}^*/m_i\right) \\ + \sum_{j=1}^{J} v_j\left(\boldsymbol{x}\right) x_j \left(\alpha_j - \sum_{k=1}^{J} \beta_{jk} \ln\left(x_k\right) - q_j \sum_{k=1}^{I} E_k\left(\boldsymbol{m}^*/m_i\right)\right) \end{array} \right\},$$

[3] Here and below, we denote $\boldsymbol{m}^* = \boldsymbol{\sigma}^*\left(\boldsymbol{x}\right) = \left(\sigma_1^*\left(\boldsymbol{x}\right), \sigma_2^*\left(\boldsymbol{x}\right), ..., \sigma_I^*\left(\boldsymbol{x}\right)\right)$.

[4] Here and below, we denote $\left(\boldsymbol{m}^*/m_i\right) = \left(\sigma_1^*\left(\boldsymbol{x}\right), ..., \sigma_{i-1}^*\left(\boldsymbol{x}\right), m_i, \sigma_{i+1}^*\left(\boldsymbol{x}\right), ..., \sigma_I^*\left(\boldsymbol{x}\right)\right)$. $v_j\left(\boldsymbol{x}\right)$ is the partial derivative of $v\left(\boldsymbol{x}\right)$ with respect to x_j.

where

$$v(\boldsymbol{x}) = (1/I)\,V(\boldsymbol{x}) = (1/I)\left[\sum_{j=1}^{J} A_j^0 \ln(x_j) + B^0\right].$$

From property 1, fisher i can find m_i to attain any vector of efforts $(E_k(\boldsymbol{m}^*/m_i))_{k=1}^{I} = (e_k)_{k=1}^{I} \in \mathbb{R}_+^I$. From property 2, whatever the unilateral deviation m_i by fisher i, $(P_k(\boldsymbol{m}))_{k=1}^{I} = (P_k(\boldsymbol{m}/m_i'))_{k=1}^{I} = \left(p^0\right)_{k=1}^{I}$. Therefore, to prove that $\sigma_i^*(\boldsymbol{x})$ is fisher i's best-reply, it will be sufficient to show that $(e_k)_{k=1}^{I} = \left(e^0\right)_{k=1}^{I}$ satisfies, for all $\boldsymbol{x}$,

$$\delta v(\boldsymbol{x}) = \max_{(e_k)_{k=1}^{I} \in \mathbb{R}_+^I} \left\{ \begin{array}{c} \sum_{j=1}^{J} a_j \ln(q_j e_i x_j) - \left((I-1)\,p^0 + c\right) e_i + p^0 \sum_{k \neq i} e_k \\ + \sum_{j=1}^{J} v_j(\boldsymbol{x})\, x_j \left(\alpha_j - \sum_{k=1}^{J} \beta_{jk} \ln(x_k) - q_j \sum_{k=1}^{I} e_k\right) \end{array} \right\}.$$

Using $v_j(\boldsymbol{x}) = (1/I)\left(A_j^0/x_j\right)$ and $p^0 = (1/I)\sum_{j=1}^{J} A_j^0 q_j$, for all j, and rearranging, we get

$$\delta v(\boldsymbol{x}) = \max_{(e_k)_{k=1}^{I} \in \mathbb{R}_+^I} \left\{ \begin{array}{c} \sum_{j=1}^{J} \left(a_j - (1/I)\sum_{k=1}^{J} A_k^0 \beta_{kj}\right) \ln(x_j) \\ + \sum_{j=1}^{J} a_j \ln(e_i) - \left(\sum_{j=1}^{J} A_j^0 q_j + c\right) e_i \\ + \sum_{j=1}^{J} a_j \ln(q_j) + (1/I)\sum_{j=1}^{J} A_j^0 \alpha_j \end{array} \right\}. \qquad (17)$$

The control $e_k = e^0$, for $k = i$, satisfies the first order conditions

$$\left(\sum_{j=1}^{J} a_j\right)/e_i - \left(\sum_{j=1}^{J} A_j^0 q_j + c\right) = 0,$$

and, thus, maximizes the RHS of (17). Moreover, as e_k, for all $k \neq i$, vanished from the RHS of (17), $e_k = e^0$, for all $k \neq i$, trivially maximizes it.

Thus, substituting $(e_k)_{k=1}^{I} = \left(e^0\right)_{k=1}^{I}$ and rearranging, we get

$$\delta v(\boldsymbol{x}) = (1/I)\sum_{j=1}^{J}\left[\left(I a_j - \sum_{k=1}^{J} A_k^0 \beta_{kj}\right)\ln(x_j) + I a_j\left(\ln\left(q_j e^0\right) - 1\right) + A_j^0 \alpha_j\right].$$

Using

$$A_j^0 = (1/\delta)\left[I a_j - \sum_{k=1}^{J} A_k^0 \beta_{kj}\right], \text{ for all } j,$$
$$B^0 = (1/\delta)\sum_{j=1}^{J}\left[I a_j\left(\ln\left(q_j e^0\right) - 1\right) + A_j^0 \alpha_j\right],$$

we can confirm that

$$v(\boldsymbol{x}) = (1/I)\left[\sum_{j=1}^{J} A_j^0 \ln(x_j) + B^0\right].$$

■

6 Conclusion.

In progress.

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