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Ayse Mumcu
Bogazici University

Serhan Ogur
PJM Interconnection

Unal Zenginobuz
Bogazici University

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Cournot Competition in a Mixed Electricity Market with Transmission Constraints*

Ayşe Mumcu†, Serhan Oğur‡ and E. Ünal Zenginobuz§

December 27, 2010

Abstract

We study a model of mixed wholesale electricity market in which a public generator and a profit-maximizing private generator compete to serve consumers on the same transmission network. We allow for different objective functions that can be delegated to the public generator, ranging from pure consumers' surplus maximization to pure profit maximization. We consider a 3-node network configuration where transmission constraints leads to network externalities. The transmission network is operated by an Independent System Operator that uses nodal transmission prices to manage congestion. We characterize equilibria and study various issues involved, such as multiple congested equilibria and how to make a plausible choice among them. We also study the impact of the public generator's objective function on overall welfare, and characterize its optimal objective function. We show how the public generator's presence may be used to alleviate congestion on the network.

Keywords: Competition on electricity networks; network externalities; mixed oligopoly
JEL Classification: D43, L13, L32, L94

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†Department of Economics, Bogazici University, Bebek, Istanbul 34342, Turkey. mumcu@boun.edu.tr

‡PJM Interconnection, 955 Jefferson Avenue, Norristown, PA, USA 19403. ogurs@pjm.com

§Corresponding author. Department of Economics and Center for Economic Design, Bogazici University, 34342 Istanbul, Turkey. zenginob@boun.edu.tr

1 Introduction

In many countries, the wholesale generation segment of the electricity sector is characterized by the presence of both public and private generators. Since the end of 1980's quite a few countries have witnessed a thorough restructuring in their electricity sectors that also involved privatization of previously state owned generators (Chile, the U.K., Norway, Argentina, New Zealand, Sweden, Finland, Brazil, Spain, and Germany, to name a few). However, public ownership of generation assets and capacity is still considerable. In Europe there are quite a few countries with "mixed" wholesale electricity markets in which public generators compete with their private counterparts: EdF in France, ENEL in Italy, Statkraft in Norway, Vattenfall in Sweden, Fortum in Findland, CEZ in Czech Republic, and ESB in Ireland are all state-owned companies.[1] An important question in this context has been the optimal degree of state ownership in such companies, particularly so in Europe where many network industries have recently been opened to more private competition. For example, there has been recently a debate in France about the optimal extent of privatization of the public utilities Gaz de France and Electricité de France.

The mixed oligopoly literature analyzes equilibria in industries with competition between a small number of firms whose objectives differ. Until recently, most of the literature has focused on the particular case of public, welfare maximizing firm competing with a private, profit-maximizing firm. Having welfare maximization as the public firm's objective function typically leads to more output being observed in the equilibrium of the strategic game between the public firm and the private firm, which in turn ameliorates the inefficiency observed in an oligopoly due to restriction of output.[2] Recent papers allow for semi-public firms which maximize a convex combination of profit and welfare. Matsumura (1998) shows that some partial privatization is always preferable to both full nationalization and full privatization. Similarly, White (2002) shows that the equilibrium welfare in the industry would be higher if the public firm maximized some convex combination of welfare and profits rather than welfare alone.

In this paper we study a model of imperfectly competitive mixed wholesale electricity market in which a profit maximizing private generator and a public generator compete. In contrast to the mixed oligopoly studies mentioned above, we model the public generator as a firm that maximizes a weighted average of consumers' surplus and its own (short-run) profits. As in the case of welfare maximization, putting weight on consumers' surplus in the

[1] See Hall (1999) for a general account of publicly owned electricity companies in Europe.

[2] For the initial analyses of models in which a public firm is used as an instrument for the regulation of an industry in the way just described, see Merrill and Schneider (1966), Harris and Wiens (1980), Sertel (1988), De Fraja and Delbano (1989, 1990), and Cremer, Marchand, and Thisse (1989).

public firm's objective function will work towards increasing the industry output observed in equilibrium.

Compared to total welfare maximization, consumers' surplus maximization is significantly easier to instruct or implement institutionally, making it a more realistic modeling assumption on how public firms operate. For example, consumers' surplus can be made a part of the public firm's objective by appointing a number of consumer representatives to the board of managers or to the upper management. On the other hand, there does not seem to be an obvious way of representing the private firm's financial interests in the public firm's objective (which total welfare maximization would require), as it may look inappropriate or create a conflict of interest to place, say, a large shareholder of the private firm in the public firm's board or upper management.[3]

Note that our analysis is also applicable to electricity markets (e.g., those in the U.S.) in which one of the privately owned generators is subject to regulation while the other privately owned generators are pure profit maximizers.[4] Alternatively, one can think of a partially privatized public generator that follows an objective function reflecting the objectives of its public and private owners in proportion to their ownership shares. In that case, the partially privatized generator may be thought of as maximizing a weighted average of consumers' surplus (public owner's objective) and its profit (private owners' objective).

We model the working of the mixed duopolistic wholesale electricity market as a two stage game. In the first stage, the public authority (government or regulator) assigns an objective function to the public (or regulated) generator. This objective function takes the form of a weighted average of consumers' surplus and profits of the public generator. In the second stage the public generator engages in a Cournot competition (i.e. simultaneous quantity-setting game) with a private generator.[5] The private generator chooses its output to maximize its own profits, whereas the public generator makes its output decision to maximize the objective function assigned to it in the first stage, i.e. a weighted average of its own profits and consumers' surplus.

We study a three-node electricity network, which is the minimum configuration that allows

[3] To our knowledge, combination of consumers' surplus and its own profits as the objective function for the public firm has not been studied before in the mixed oligopoly literature.

[4] This would be reminiscent of the literature on strategic delegation of objectives other than profit maximization to the managers of private firms by their owners. Fershtman and Judd (1987) study the principal agent problem between profit maximizing owners and managers in an oligopolistic context and show that the optimal contract distorts the manager's incentives away from maximizing profits.

[5] Borenstein, Bushnell and Stoft (1998) use a model with Cournot competition in their analysis of power generation markets and provide a justification for this modelling choice. See Wolak and Patrick (1996) for evidence that exercise of market power in the U.K. has taken place through capacity withholding. Willems, Rumiantseva and Weight (2009) compare Cournot and Supply Function Equilibrium models of electricity competition and suggest use of Cournot models for short-term analysis.

us to analyze the effects of loop flows. Each pair of these nodes is connected by a transmission line with some fixed thermal capacity. The private and the public generators are located at two separate nodes and the consumers are located at the third node. There is no demand for power on nodes where the producers are located and there is no generation capacity available on the node where the consumers are located. The transmission network is subject to congestion due to capacity constraints on lines connecting the generators and consumers.

The transmission network over which the generators are connected and serve the customers is operated by an Independent System Operator (ISO) that utilizes a market-based congestion management system. Reliability of the transmission network is achieved via explicit nodal transmission price signals to the generators. Each generator is required to pay a nodal transmission congestion charge for each unit of injection and withdrawal of electricity on each node. The congestion charge on a node can be positive, zero or negative, depending on the impact of the injection (or withdrawal) on the transmission constraint. Nodal transmission prices are determined based on the principle that all participants pay proportionally for their contribution to a binding transmission line constraint the ISO has to control for.[6]

There are a large number of papers that study Cournot competition in electricity markets with transmission constraints. Both Hogan (1997) and Oren (1997) study a spatial model in which private firms engage in Cournot competition. Wei and Smeers (1999) use a variational inequality approach to compute the equilibrium on an electricity network including capacity expansion decisions. Borenstein, Bushnell and Stoft (1998) study the competitive effects of transmission capacity constraint when the two generators are located on different ends of the transmission line. Willems (2002) provides an assessment of various transmission capacity allocation rules under Cournot competition. The three-node model we use below is an adaptation of Joskow and Tirole (2000), who show how generators may benefit from a reduction in transmission capacity. Leautier (2001) also uses a similar simple three-node network to study the effects of transmission constraints on equilibrium outcomes.

Our paper differs from the studies just cited by focusing on a mixed electricity market in which a public and a private generator interact. In the particular environment studied, we drive closed-form solutions for the equilibrium outcomes of the strategic interaction involved, providing us an understanding of the underlying economic forces at play. We also study the impact of the public generator's objective function on overall welfare and characterize its

[6] Another (and equivalent) way to look at the same principle is that all participants pay (or get paid) for the externality they cause on the other participants, in terms of exacerbating or relieving the congestion on the constrained transmission facility. The transmission network model we employ is consistent with the congestion management and pricing method, called Locational Marginal Pricing (LMO), used in most U.S. ISO and Regional Transmission Organization (RTO) markets (PJM, Midwest ISO, ISO New England, New York ISO, California ISO).

optimal objective function. To our knowledge, our paper provides the first analysis of a mixed electricity generation market under a transmission capacity constraint.

The paper is organized as follows. In Section 2 we introduce the general features of the loop flow network model we study. In Section 3 we analyze the equilibria both when the transmission constraint is not binding (the "uncongested" equilibria) and when it is binding (the "congested" equilibria). In Section 4 we analyze the optimal choice of objective function for the public/regulated generator. In Section 5 we provide a summary of our results and offer some concluding remarks. Proofs of two of the propositions are relegated to an appendix at the end.

2 The Model

We consider a simple model of the electricity sector, where there are two generators supplying electricity to a single market.[7] One of these generators, denoted by P, is purely private and its objective is to maximize its profit. The other one is a public or regulated one, denoted by R, and it is assumed to maximize a weighted average of consumers' surplus and its own profits. That is, the public generator's objective function is assumed to be

$$\gamma CS(\cdot) + (1-\gamma)\Pi_R(\cdot) \tag{1}$$

where $CS(\cdot)$ is the total consumers' surplus, $\Pi_R(\cdot)$ is its own profit, and $\gamma \in [0,1]$ is the weight on consumers' surplus. Note that the case of $\gamma = 0$ refers to a pure private generator and $\gamma = 1$ to a generator concerned solely with maximizing consumers' surplus, while $\gamma = 1/2$ would have the public generator value consumers' surplus and its own profit equally.

In this paper we study the short-run output decisions of the generators and assume a constant returns to scale short-run production technology of $C_i(q_i) = c_i q_i$, where q_i is the output of generator $i = P, R$.

The consumers' demand for power is represented by an affine inverse demand function, $p(Q) = a - Q$, where $Q \equiv q_P + q_R$. Defining $\alpha_i = a - c_i$, $i = P, R$, as the grade of efficiency for generator i, we assume the following conditions on the demand and cost parameters:

Assumption 1: $\alpha_P > \alpha_R > 0$.

Assumption 2: $2\alpha_i - \alpha_j > 0, i, j \in \{P, R\}, i \neq j$.

Assumption 1 states that each generator finds it profitable to serve the whole market on its own and the private generator has a lower marginal cost. Assumption 2 posits that the

[7] The model of the electricity network is similar to the one studied in Joskow and Tirole (2000).

marginal cost differential between the two generators is not “too large” and it guarantees that when both generators are pure profit maximizers, the equilibrium is an interior one when the transmission capacity constraint is not binding.

Network Structure We assume that the network is a three-node network, which is the simplest model of electricity network that involves loop-flows in electricity transmission (see Figure 1). In this case electricity sent from one node to the other not only affects the flow on the line connecting these two nodes, but also the congestion on the other two lines. We study a simple three-node network with two generation nodes and one consumption node. The public generator is located on node 1, the private generator is located on node 2, and consumers are located on node 3.[8] There is no generation on node 3 and no consumption on node 1 or node 2.

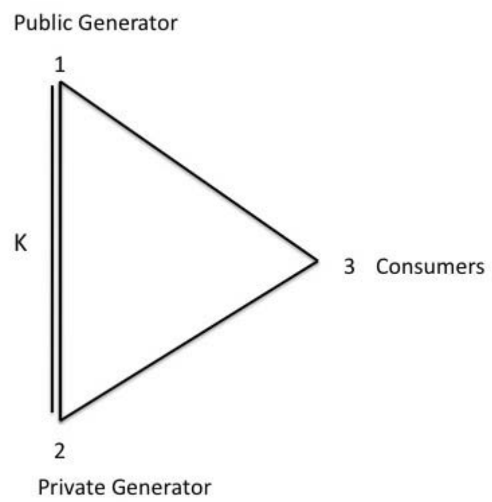

Figure 1: Transmission Network

The transmission line between two generation nodes is assumed to have a given capacity of K. The implication of this in our model, which is a consequence of electricity flowing from the generators to the consumers following the path of least resistance,[9] is a constraint on by

[8]There are a number of reasons for connecting the two generation nodes, even if this creates a loop flow. First, to increase the reliability of the network; in case of an outage of one of the lines connecting the generators directly to the consumers, both generators continue to supply electricity through the indirect line. In fact, a reliable operations dispatch procedure employed by all dispatchers (utility or ISO) called “n-1 contingency dispatch” is a reflection of this fact. Second, the market we are modeling can be interpreted as a sub-market in a larger interconnection with fixed available transmission capacities or transmission reservations on the lines, in which case the line connecting the two generators serves other transactions in other sub-markets and thus our ISO does not have the discretion to dismantle it.

[9]This is because electrons follow a unique path on an electrical transmission network determined by Kirchoff’s Law rather than the direction of trade.

q_P and q_R given by

$$| q_P - q_R | \leq 3K. \tag{2}$$

Transmission Market The grid is operated by an Independent System Operator (ISO) that is in charge of ensuring the safe and reliable utilization of the grid by auctioning transmission congestion rights, or Transmission Capacity Reservations (TCRs), as in Smeers and Wei (1997, 1999). The nodal transmission rights allow the generators to withdraw and inject up to a specific amount of electricity from and into the transmission network at a specified transmission node. As in Smeers and Wei (1997, 1999), it is assumed that transmission rights are actively traded at pre-dispatch time. Let $\lambda_i \in \mathbb{R}, i = 1, 2, 3$, be the price of the TCR at node i. That is, λ_i is the price of withdrawing a unit of electricity from node i. An entity would have to pay λ_i to withdraw a unit of electricity from (and pay $-\lambda_i$ to inject into) node i, in addition to the price of the unit of electricity. Without loss of generality, we normalize the TCR prices by setting $\lambda_1 = 0$.

Generators compete in two markets; the electricity generation market and the TCR market. In the electricity generation market generators are assumed to engage in Cournot competition, i.e. they compete by simultaneously choosing output levels. The quantities they choose are pre-dispatch quantities submitted to ISO. They also trade transmission rights at the TCR market. Both generators are assumed to take TCR prices as given in making their decisions, i.e., the TCR market is competitive.

With the above cost and demand specifications, the private generator's maximization problem becomes

$$\max_{q_P \geq 0} \Pi_P = [\alpha_P - (q_P + q_R)]\, q_P - (\lambda_3 - \lambda_2)q_P. \tag{3}$$

Note that the profits of the private generator involve a separate component, namely $(\lambda_3 - \lambda_2)q_P$, which arises from payments due to having to acquire TCRs for each unit of electricity generated and delivered. On the other hand, the public generator's maximization problem becomes

$$\max_{q_R \geq 0} \Phi_R = \gamma \left[\frac{1}{2}(q_P + q_R)^2 \right] + (1 - \gamma) \left\{ [\alpha_R - (q_P + q_R)]\, q_R - \lambda_3 q_R \right\}. \tag{4}$$

Note that the first term (weighted by γ) involves the effect of consumers' surplus on the public generator's objective function, while the second term (weighted by $(1 - \gamma)$) shows its profit arising from production and transmission.

3 Analysis of Equilibria

We first explain the equilibrium condition in the TCR market, which takes into account the externality each generator imposes on the other when transmitting a unit of electricity.

When the grid is not congested, production by a generator does not impose any positive or negative externality on the other, implying TCR prices of $\lambda_2 = \lambda_3 = 0$ in equilibrium. However, when the grid is congested, an additional unit of production by one generator creates a positive externality on the other by decongesting the line connecting them. The private generator, located on node 2, receives λ_2 at node 2 and pays λ_3 at node 3 for each unit of electricity it sends from node 2 to node 3. Hence the marginal transmission cost for the private generator is $\lambda_3 - \lambda_2$. Similarly, the public generator located at node 1 receives λ_1 ($\equiv 0$) at node 1 and pays λ_3 at node 3 for each unit of electricity it sends from node 1 to node 3, implying that the public generator is willing to pay up to λ_3 to benefit from the positive externality created by the additional unit of production by the private generator. In equilibrium we must have $\lambda_3 - \lambda_2 = -\lambda_3$ (marginal cost = marginal benefit), or

$$\lambda_2 - 2\lambda_3 = 0. \tag{5}$$

When this condition is not met, a trader can earn profits through pure arbitrage, by paying both generators the market price to ramp up their productions by a unit each and collecting the associated TCRs from both. This would continue to meet the line constraint and result in a net profit for the trader if the TCR prices do not meet the above condition. In equilibrium no generator wants to hold more or fewer TCRs than it already has.

Given the above description of equilibrium in the TCR market, equilibrium in the overall system is characterized by the following conditions:

- Equilibrium in the electricity generation market:

$$\begin{aligned} \gamma(q_P + q_R) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 \leq 0 \\ q_R \geq 0 \text{ and } q_R\left[\gamma(q_P + q_R) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3\right] = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 \leq 0 \\ q_P \geq 0 \text{ and } q_P\left[\alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2\right] = 0 \end{aligned} \tag{7}$$

- Feasibility in TCR market:

$$\mid q_P - q_R \mid \leq 3K \tag{8}$$

- Equilibrium in the TCR market:

$$\lambda_2 - 2\lambda_3 = 0 \tag{9}$$

Expressions (6) and (7) above are the first order conditions for the public and private generators, respectively, in the (constrained) maximization problem they face when they compete by choosing their quantities independently, while taking the TCR as given. As we show below, for a given γ and K, there may exist an "uncongested" equilibrium where the transmission capacity constraint (2) is not binding in equilibrium, as well as "congested" equilibrium where it is binding (each equilibrium involving different λ_2 and λ_3.)

3.1 Uncongested Equilibria

We first look at the case where the capacity of the line connecting the two generators, K, is sufficiently large so that the grid is not congested for any value of γ.[10] As discussed above, in this case we have $\lambda_2 = \lambda_3 = 0$ in equilibrium. The public generator's response function is

$$q_R(q_P;\gamma) = \begin{cases} \min\left\{\max\left\{0, \frac{(1-\gamma)\alpha_R-(1-2\gamma)q_P}{2-3\gamma}\right\}, a - q_P\right\} & \text{for} \quad q_P \in [0, a) \\ 0 & \text{for} \quad q_P \in [a, \infty) \end{cases} \tag{10}$$

while the private generator's response function becomes

$$q_P(q_R) = \begin{cases} \frac{\alpha_P - q_R}{2} & \text{for} \quad q_R \in [0, \alpha_P) \\ 0 & \text{for} \quad q_R \in [\alpha_R, \infty) \end{cases} \tag{11}$$

Note that the ("uncongested") response function of the public generator depends on γ. Figure 2 depicts the response functions of generators for different values of γ. In panel I, γ is relatively small, $\gamma \in [0, c_R/(a + c_R))$. In this case the public generator's and the private generator's outputs are strategic substitutes, as in a standard Cournot model. In uncongested equilibrium, the efficient private generator always produces more than the inefficient public generator.

[10] In fact, a sufficient condition for there to be no congestion for any γ is $K \geq a/3$, given that beyond total output level a market saturates.

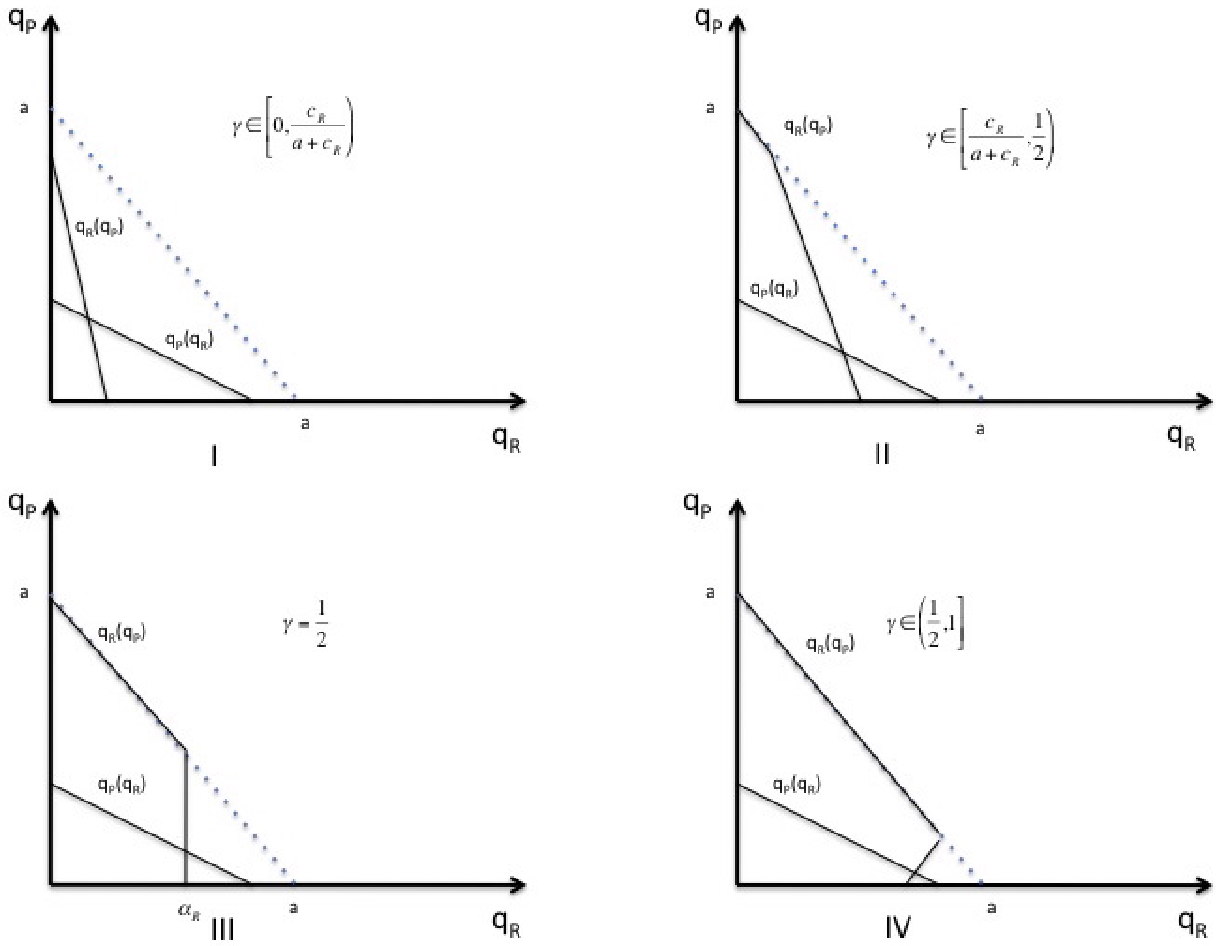

Figure 2: Response Functions of Generators

Panel II depicts a case where $\gamma \in [c_R/(a+c_R), 1/2)$. In this case the public generator's and the private generator's outputs are still strategic substitutes but there is a kink at $q_P = [(1-2-\gamma) - \alpha_R + (2-3-\gamma)c_R]/(1-\gamma)$ beyond which the slope of the public generator's response function changes to -1. When both γ and q_P are large enough, the public generator's best response is to produce just enough to complete the total output to a (at which point the market is saturated and consumers' surplus reaches its maximum value).

Panel III depicts the case of $\gamma = 1/2$. Here, the public generator's response function is vertical at $q_R = \alpha_R$ until $q_R + q_P = a$, at which point the slope changes to -1.

For $\gamma > 1/2$, which is illustrated in panel IV, public generator's response function is first positively sloped, and then the slope changes to -1. Hence, for small amounts of q_P, the public generator views private generator's output as a strategic complement. This is due to the fact that the public generator puts a relatively larger weight on consumer surplus than on its profits when $\gamma > 1/2$.[11]

Let $q_R^U(\gamma)$ and $q_P^U(\gamma)$ be the uncongested equilibrium output levels of the public and private generators, respectively, for a given γ. The (uncongested) equilibrium output choices by the public and private generators are

$$q_R^U(\gamma) = \begin{cases} \frac{2(1-\gamma)\alpha_R-(1-2\gamma)\alpha_P}{3-4\gamma} & \text{if} \quad 0 \le \gamma < \underline{\gamma}, \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if} \quad \underline{\gamma} \le \gamma < \overline{\gamma}, \\ a & \text{if} \quad \overline{\gamma} \le \gamma \le 1. \end{cases} \tag{12}$$

and

$$q_P^U(\gamma) = \begin{cases} \frac{(2-3\gamma)\alpha_P-(1-\gamma)\alpha_R}{3-4\gamma} & \text{if} \quad 0 \le \gamma < \underline{\gamma}, \\ 0 & \text{if} \quad \underline{\gamma} \le \gamma \le 1. \end{cases} \tag{13}$$

respectively, where

$$\underline{\gamma} = \frac{2\alpha_P - \alpha_R}{3\alpha_P - \alpha_R} \tag{14}$$

is the value of γ beyond which the private generator is ousted from the market and

$$\overline{\gamma} = \frac{2a - \alpha_R}{3a - \alpha_R} \tag{15}$$

is the value of beyond which the market is saturated by the public generator's output. The total output therefore is

$$Q^U(\gamma) = \begin{cases} \frac{(1-\gamma)(\alpha_P+\alpha_R)}{3-4\gamma} & \text{if} \quad 0 \le \gamma < \underline{\gamma}, \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if} \quad \underline{\gamma} \le \gamma < \overline{\gamma}, \\ a & \text{if} \quad \overline{\gamma} \le \gamma \le 1. \end{cases} \tag{16}$$

Equations (12) and (13) reveal that there are two types of uncongested equilibria. In the first case, both the public and the private generators produce a positive amount. This case corresponds to small values of γ, the weight on consumer's surplus in the public generator's objective function. When γ is large enough, only the public generator produces a positive amount.

As a direct result of Assumption 2, the public generator always produces a positive amount,

[11] The intuition behind the positive slope of public generator's response function is as follows. When the private generator increases its output at a given level of public generator's output, price decreases. This leads to an increase in the consumers' surplus and a decrease in the public generator's profit. The optimal response for the public generator is to increase its output until the increase in consumer surplus weighted by γ is just equal to the decrease in marginal profit weighted by $1-\gamma$. The fact that γ, the weight on consumer surplus, is greater than $1/2$ results in an increase rather than a decrease in public generator's output as an optimal response to an increase in private generator's output. On the other hand, a decrease in the private generator's output will lead to a decrease in the public generator's output as the optimal response for the same reason.

even when $\gamma = 0$, in an uncongested equilibrium. As shown below, the public generator's equilibrium output is increasing in γ in the uncongested case, therefore, there does not exist an uncongested equilibrium where the public generator produces zero output.

From (2) we know that whether there is congestion on the grid or not depends on the difference between the generators' output levels. To facilitate the characterization of the congestion equilibria, consider the difference between the uncongested equilibrium output levels of the two generators. Letting $\Delta q^U(\gamma) \equiv q_R^U(\gamma) - q_P^U(\gamma)$, this difference is

$$\Delta q^U(\gamma) = \begin{cases} \alpha_R - \alpha_P + \frac{\gamma(\alpha_P + \alpha_R)}{3-4\gamma} & \text{if} \quad 0 \leq \gamma < \underline{\gamma}, \\ \frac{(1-\gamma)\alpha_R}{2-3\gamma} & \text{if} \quad \underline{\gamma} \leq \gamma < \overline{\gamma}, \\ a & \text{if} \quad \overline{\gamma} \leq \gamma \leq 1. \end{cases} \tag{17}$$

To analyze the impact of γ on the nature of equilibria, define $\kappa(\gamma)$ as the capacity level that makes the capacity constraint at the uncongested equilibrium, characterized by (12) and (13), just binding for a given γ:

$$\kappa(\gamma) \equiv \frac{1}{3} \mid \Delta q^U(\gamma) \mid \tag{18}$$

It is easily shown that $\partial \Delta q^U(\gamma)/\partial\gamma \geq 0$ for all $\gamma \in [0, 1]$. Given $\alpha_R < \alpha_P$, $\Delta q^U(0) < 0$ and $\Delta q^U(1) = a > 0$. Since $\Delta q^U(\gamma)$ is continuous in γ in the relevant region, there must exist a $\hat{\gamma} \in [0, 1]$ such that for $\gamma \in [0, \hat{\gamma}]$, $\Delta q^U(\gamma) \leq 0$ and for $\gamma \in (\hat{\gamma}, 1]$, $\Delta q^U(\gamma) > 0$. Thus, this unique threshold level of γ is calculated as

$$\hat{\gamma} = \frac{3(\alpha_P - \alpha_R)}{5\alpha_P - 3\alpha_R}. \tag{19}$$

Note that $\hat{\gamma}$ is always less than 1/2. Thus, the relative magnitudes of the uncongested equilibrium output levels depend on the weight attached to the consumer surplus in the public generator's objective function. The private generator produces more for smaller levels of γ, while the public generator's output is higher as γ increases beyond $\hat{\gamma}$. Recall also that the private generator will be ousted from the market when γ further increases and reaches $\underline{\gamma}$ $(> \hat{\gamma})$. Therefore, as the weight attached to consumers' surplus in the objective of the public generator increases beyond a threshold, the public generator produces more than its private counterpart despite its cost inefficiency and it may even push the private generator outside the market.

Figure 3 shows $\kappa(\gamma)$ and the uncongested equilibria for given combinations of K and γ. The proposition below summarizes the results on uncongested equilibria.

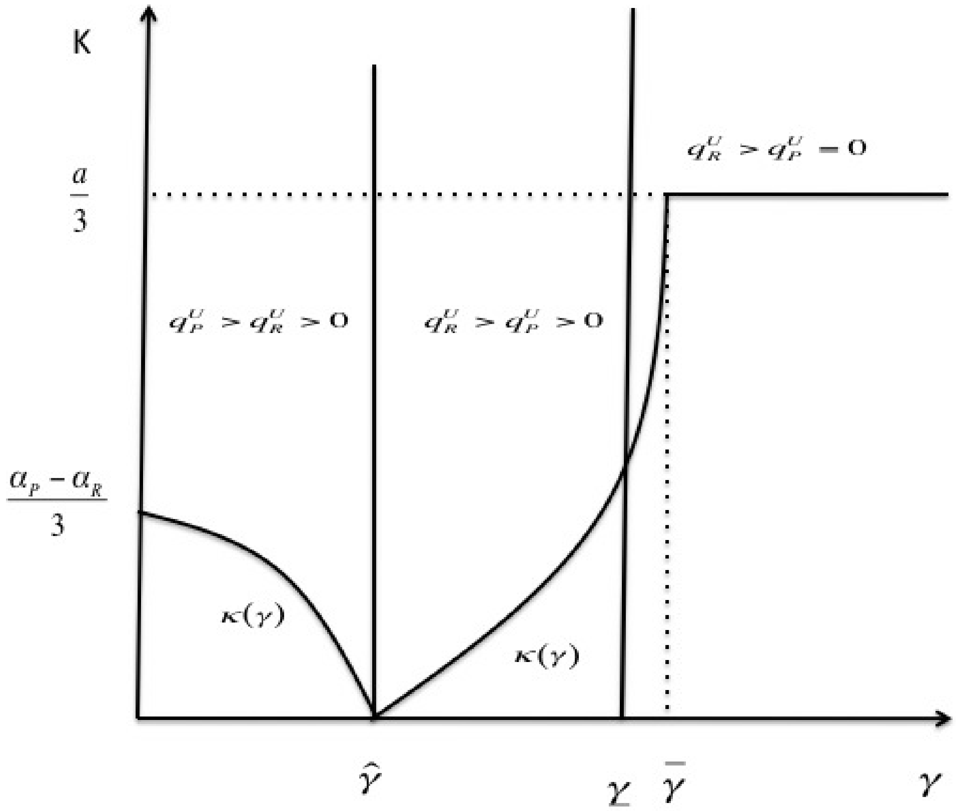

Figure 3: Uncongested Equilibria

Proposition 1 *For γ and K such that $K \geq \min\{\kappa(\gamma), \frac{a}{3}\}$, there is an uncongested equilibrium, where*

i. $q_P^U \geq q_R^U > 0$ *if* $\gamma \in [0, \hat{\gamma}]$,

ii. $q_R^U > q_P^U > 0$ *if* $\gamma \in (\hat{\gamma}, \underline{\gamma}]$,

iii. $q_R^U > q_P^U = 0$ *if* $\gamma \in (\underline{\gamma}, 1]$.

3.2 Multiple Congested Equilibria

For a given γ, if $K < \kappa(\gamma)$, then the equilibrium necessarily involves congestion on the grid. Since the capacity constraint involves the absolute value of the difference between output levels, there are potentially two equilibria for each level of K. That is, with congestion the TCR prices λ_2 and λ_3 are no longer zero, and there are two sets of λ_2 and λ_3 that satisfy (6)-(9).

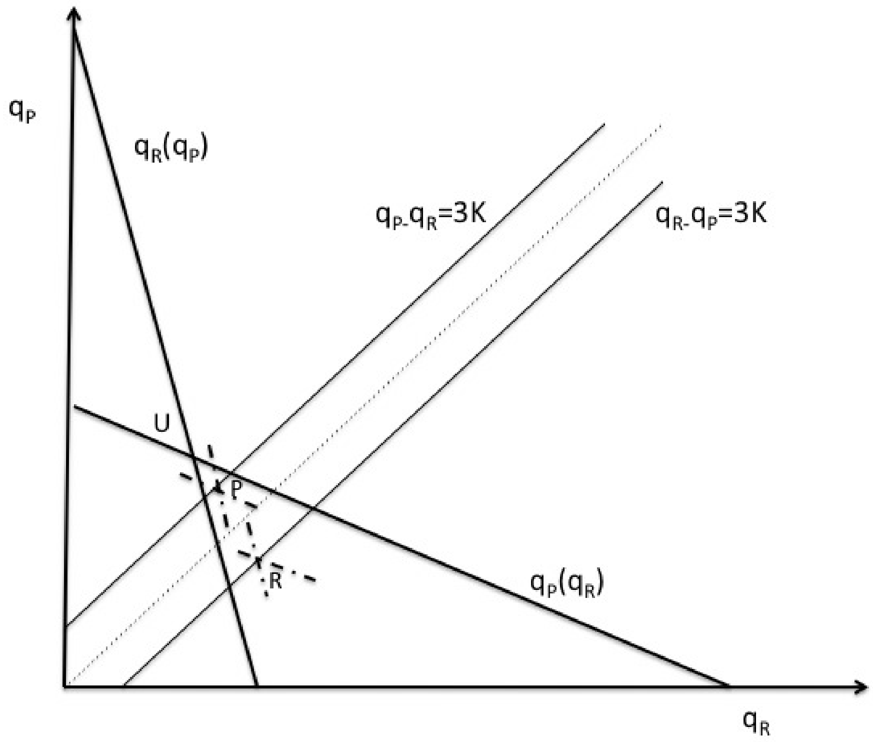

Figure 4: Response Functions at a Specific γ

Figure 4 displays the response functions of the generators for a specific $\gamma \in (\hat{\gamma}, \frac{1}{2})$. Note that this is a case where the private generator produces more than the public generator in the uncongested case. Hence point U in Figure 4 is an uncongested equilibrium for a capacity level $K \geq \kappa(\gamma)$. Take a $K < \kappa(\gamma)$. The lines implied by $\mid q_P - q_R \mid = 3K$ correspond to the capacity constraint in this case. In the TCR market nodal transmission rights are traded, and the equilibrium TCR prices shift the best response functions of the two generators such that equilibrium in the electricity market occurs at either point P, or point R, where the response functions (10) and (11), the TCR market equilibrium condition $\lambda_2 - 2\lambda_3 = 0$ and the capacity constraint lines $\mid q_P - q_R \mid = 3K$ are satisfied simultaneously at γ. In other words, for each (γ, K) pair that leads to congested equilibrium there will be multiple equilibria, each corresponding to a different set of TCRs. At point P, which we call $P - type$ equilibrium, the private generator produces more, and at point R, which we call $R - type$ equilibrium, the public generator produces more.

The example below illustrates the role of TCR's and the functioning of the nodal pricing scheme in resolving of the congestion on the constrained transmission line.

Example 1 *Consider the case where $\gamma = 0$. In this case, the unconstrained equilibrium output levels of the public and the private generators are*

$$q_R^U(0) = \frac{2\alpha_R - \alpha_P}{3} \tag{20}$$

and

$$q_P^U(0) = \frac{2\alpha_P - \alpha_R}{3} \tag{21}$$

respectively. Since $\gamma = 0$, at the uncongested equilibrium the private generator produces more than the public generator and $\Delta q^U(0) = \alpha_R - \alpha_P < 0$. Now consider K such that $\mid \alpha_R - \alpha_P \mid > 3K$, i.e. the uncongested equilibrium is not attainable. In order to bring production in line with the capacity constraint, either the effective cost of production to the public generator needs to be decreased or the effective cost of production to the private generator needs to be increased (or both). The constrained equilibria outcomes are

$$q_R^C = \frac{\alpha_P + \alpha_R \mp 9K}{6} \tag{22}$$

$$q_P^C = \frac{\alpha_P + \alpha_R \pm 9K}{6} \tag{23}$$

$$\lambda_2^C = (\alpha_R - \alpha_P) \pm 3K < 0 \tag{24}$$

$$\lambda_3^C = \frac{(\alpha_R - \alpha_P) \pm 3K}{2} < 0 \tag{25}$$

In one of the constrained equilibria the private generator produces more, and in the other it produces less. In both cases the public generator pays λ_3^C (a different negative amount in each case) at the margin for each unit of electricity transmitted, thus lowering its effective marginal cost to $c_R + \lambda_3^C < c_R$. The private generator, on the other hand, receives λ_2^C and pays λ_3^C for each unit of electricity it transmits, bringing its effective marginal cost to $c_P + \lambda_3^C - \lambda_2^C > c_P$ in each case. With the introduction of these congestion prices, the response function of each generator moves accordingly. One could also interpret this "adjustment" in terms of prices rather than costs. The effective price the public generator receives from the sale of a unit of electricity would then be $p^C - \lambda_3^C > p^C$, while the private generator would be selling the same good at an effective price of $p^C - \lambda_3^C + \lambda_2^C < p^C$.

Proposition 2 below gives the output produced by the generators and equilibrium TCR (nodal) prices at congested equilibria studied in this section.

Proposition 2 *For each* (γ, K)*, such that* $K < \min\left\{\kappa(\gamma), \frac{a}{3}\right\}$ *there exist multiple congested equilibria, where either the private or the public generator produces more. The equilibrium values for the variables are:*

$$q_R^C(\gamma, K) = \frac{\alpha_P + (1-\gamma)\alpha_R \mp 3K(3-2\gamma)}{6-5\gamma} \tag{26}$$

$$q_P^C(\gamma, K) = \frac{\alpha_P + (1-\gamma)\,\alpha_R \pm 9K(1-\gamma)}{6-5\gamma} \tag{27}$$

$$\lambda_3^C(\gamma, K) = \frac{3(1-\gamma)\alpha_R - (3-5\gamma)\alpha_P \pm 3K(3-4\gamma)}{6-5\gamma} \tag{28}$$

$$\lambda_2^C(\gamma, K) = 2\lambda_3^C \tag{29}$$

Proof. See Appendix. ■

Remark 2 Note that (26)-(29) characterize the congested equilibria where both generators produce strictly positive amounts. Congested equilibria where only one of the generators produces is also possible. In the proof of Proposition 2 we give the analysis of such cases as well.

We observe that each congested equilibrium will be associated with a different level of profit for ISO and a different total surplus (welfare). In the next section when we consider the optimal choice of γ, the parameter determining the objective function of the public generator, we will also study the implication of different equilibria on the profits of ISO.

4 Optimal γ: What Objective to Delegate to the Public Generator?

The equilibrium levels of production calculated above are for a given γ and K. Observe that γ, the weight given to consumers' surplus in the public generator's objective function, can be viewed as a policy tool. This brings out the question of choosing γ optimally. We take the total surplus

$$\begin{aligned} W(q_P, q_R; K, \gamma) = \int_0^{q_P+q_R} & P(Q)dQ - p(Q)Q + \Pi_P(q_R, Q_P) \\ & + \Pi_R(q_R, Q_P) + \Pi_{ISO}(q_R, Q_P) \end{aligned}$$

as the measure of welfare, which, given the specifications of our model, becomes

$$W(q_P, q_R; K, \gamma) = \alpha_R Q - \frac{Q^2}{2} - (\alpha_R - \alpha_P)q_P. \tag{30}$$

Recall that for a given γ and K, there will exist multiple congested equilibria. In order to carry out welfare analysis in that case, we have to deal with the multiplicity of equilibria. A plausible criterion to select among equilibria is to look at the profits of ISO.

4.1 Profits of ISO

The profits of ISO may be of concern for a number of reasons. ISO is in charge of administering the TCR market and operating the transmission network. The TCR market is assumed to operate much like a competitive market, each generator taking the transmission prices it faces as given and equilibrium prices being those that equate demand and supply for transmission rights at each node.

We have not ascribed a separate objective function to ISO other than perhaps allowing it to act like a "Walrasian auctioneer" in the TCR market, announcing the final prices that will drive the TCR market into equilibrium. In our model, as in the operation of any ISO that uses market-based congestion management, TCR pricing involves of transfers to and from ISO depending on the signs of λ_2 and λ_3, as well as the relative magnitudes of q_R and q_P. The profits of ISO are given by

$$\Pi_{ISO} = \lambda_3 q_R + (\lambda_3 - \lambda_2)q_P \tag{31}$$

and given that $\lambda_2 = 2\lambda_3$ in equilibrium, the equilibrium level of ISO profits will be

$$\Pi_{ISO} = \lambda_3(q_R - q_P). \tag{32}$$

Note that when the equilibrium is uncongested, we have $\lambda_2 = \lambda_3 = 0$, and hence $\Pi_{ISO} = 0$. However, in the case of congested equilibria, profits of ISO can be positive or negative, as indicated by (32) above.

It may very well be the case that the public authority (government) requires that ISO runs no losses. Recall also from our characterization of congested equilibria in Section 3.2 above that for each given pair of γ and K, there will be one set of λ_2 and λ_3 that corresponds to the constraint $q_R - q_P = 3K$ in equilibrium, and another that corresponds to $q_P - q_R = 3K$. It may be the case that ISO profits are positive for one case and negative for the other. We may then use the non-negative ISO profit requirement as an equilibrium selection criterion.

From (32), when the public generator produces more than the private generator in the congested equilibrium, profits of ISO is nonnegative if and only if $\lambda_3 \geq 0$. From (28), this can only be the case if and only if

$$3K \leq \left[\alpha_R - \alpha_P + \frac{\gamma(\alpha_R + \alpha_P)}{3 - 4\gamma}\right]. \tag{33}$$

Observe from (17) that the right hand side of the above inequality is equal to $\Delta q^U(\gamma)$ for $\gamma \in [0, \hat{\gamma}]$, i.e. the difference between the generators' output levels at the uncongested equilibrium for α_P, α_R, and γ that lead to a higher equilibrium output level for the private generator than that of the public generator. Given that we are considering capacity levels $K < \kappa(\gamma)$, the above condition will hold only if $\gamma > \hat{\gamma}$. This is the region the public generator produces more than the private generator at the uncongested equilibrium. Thus the profits of ISO will be nonegative in R-type congested equilibrium if and only if at the uncongested equilibrium, corresponding to the same set of α_P, α_R and γ, the public generator produces more.

When the private generator produces more than the public generator in the congested equilibrium, profits of ISO is nonnegative if and only if $\lambda_3 \leq 0$. From (28), this will be the case if and only if

$$3K \leq -\left[\alpha_R - \alpha_P + \frac{\gamma(\alpha_R + \alpha_P)}{3 - 4\gamma}\right]. \tag{34}$$

Similar to the argument above, the profits of ISO will be nonegative in P-type congested equilibrium if and only if at the uncongested equilibrium corresponding to the same set of α_P, α_R and γ it is the private generator that produces more.

4.2 Optimal γ

We will first look at the choice of optimal γ in the case where the total surplus maximization is attained at an uncongested (unconstrained) equilibrium. Unconstrained (uncongested) welfare optimum will be achieved when the transmission capacity constraint K is sufficiently large. As we will see below, the analysis of the unconstrained case will provide insight for the determination of optimal γ when K is small, i.e. when the transmission line will be congested at the welfare optimum.

Note from Figure 2 that as γ increases the reaction function of the public generator shifts, each time leading to a unique uncongested equilibrium with the corresponding values for q_P and q_R. We perform the welfare optimization in the (q_R, q_P) space. As γ increases, the equilibrium moves along the reaction function of the private generator up to $\underline{\gamma}$ (at which point the private generator ceases production). Beyond $\underline{\gamma}$, the equilibrium moves along $q_P = 0$ until $q_R = a$ (see Figure 2). Using (30) and substituting the private generator's reaction function, the total surplus can be expressed as

$$W(q_R;\gamma,K)=\begin{cases} -\frac{1}{8}q_R^2+(\alpha_R-\frac{3}{4}\alpha_P)q_R+\frac{3}{8}\alpha_P^2 & \text{if} \quad q_R\in[0,\alpha_P) \\ -\frac{1}{2}q_R^2+\alpha_R q_R & \text{if} \quad q_R\in[\alpha_P,a) \end{cases}. \tag{35}$$

Propositon 3 below characterizes the welfare maximizing values of γ when the transmission capacity K is sufficiently large so that an unconstrained welfare maximum is achievable.

Proposition 3 *Assume that the transmission capacity constraint K is sufficiently large so that the transmission line will never be congested ($K>\frac{\alpha_P}{3}$). Then the optimal objective to be delegated to the public generator involves putting a strictly positive weight on consumers' surplus if the efficiency deficit of the public generator is "small". Otherwise, it is optimal to put no weight on consumers' surplus, i.e. profit maximization is the welfare maximizing objective for the public generator. Specifically, the welfare maximizing γ for the uncongested case is given by*

$$\gamma_U^*=\begin{cases} \gamma_{U1}^*=\frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P} & \textit{if} \quad \alpha_R\in\left(\frac{4}{5}\alpha_P,\alpha_P\right) \\ 0 & \textit{if} \quad \alpha_R\in\left(\frac{1}{2}\alpha_P,\frac{4}{5}\alpha_P\right] \end{cases}. \tag{36}$$

Proof. See Appendix. ■

Propositon 3 replicates similar results of other studies on mixed oligopolies. If the cost differential between the private generator and the less efficient public generator is not too high, then instructing the public generator to put a positive weight on consumers' surplus ($\gamma>0$) in its objective function will be welfare enhancing in equilibrium. More aggressive behavior on the part of the public generator will increase the total electricity generated and sold in the market, and the positive welfare effect of this increase will more than make up for the relatively inefficient production by the public generator.

However, note that $\gamma_{U1}^*=\frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$ in γ_U^* is less than 1/2, the value that puts equal weight on consumers' surplus and profit maximization This reflects the tradeoff between allocative efficiency and productive efficiency when the less efficient public generator gets to increase its output with higher γ. After a point, it does not pay (in terms of total surplus) to have the less efficient public generator displace production by the more efficient private generator. The optimal γ never indicates at delegating consumers' surplus maximization as the sole objective for the public generator; in fact, consumers' surplus never gets equal weight with profit maximization. Moreover, if the relative cost inefficiency of the public generator is beyond a certain point then profit maximization ($\gamma=0$) is the optimal objective for the public generator, with the associated less aggressive output behavior and thus less (inefficiently produced) output by it in equilibrium.

We now turn to full characterization of optimal γ when the transmission capacity K is not necessarily high enough to allow implementing of the unconstrained welfare maximum using γ_U^*. When K is small welfare maximization may call for output levels that will congest the transmission line. As we will see below, in such cases it will be optimal to have the public generator produce more (than it would be asked to in the absence of capacity constraint) in order to alleviate the congestion on the network, thereby allowing more electricity to flow from producers to customers.

Recall that in the congested case there will be (γ, K) pairs that results in multiple congested equilibria. Using the analysis in Section 4.1 above, in searching for the optimal γ we will select the P-type congested equilibrium if $\gamma < \hat{\gamma}$, and R-type congested equilibrium otherwise ($\hat{\gamma}$ being the threshold γ that determines which generator would produce more at the uncongested equilibrium corresponding to the same set of α_P, α_R and γ).

Proposition 4 below characterizes the welfare maximizing values of γ for the general case, i.e. for K both large and small.

Proposition 4 *1. Let $\alpha_R \in \left(\frac{5}{6}\alpha_P, \alpha_P\right)$, i.e. the efficiency deficit of the public generator is "small". The welfare maximizing γ for this case is given by*

$$\gamma_C^* = \begin{cases} \gamma_{C1}^* = \frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K} > \gamma_{U1}^* & \text{if} \quad K \in \left(0, \frac{6\alpha_R-5\alpha_P}{3}\right] \\ \gamma_{U1}^* = \frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P} & \text{if} \quad K \in \left(\frac{6\alpha_R-5\alpha_P}{3}, \infty\right) \end{cases}.$$

2. Let $\alpha_R \in \left(\frac{4}{5}\alpha_P, \frac{5}{6}\alpha_P\right)$, i.e. the efficiency deficit of the public generator is in an intermediate range. The welfare maximizing γ for this case is given by

$$\gamma_C^* = \begin{cases} \gamma_{C2}^* = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K} \in (\gamma_{U1}^*, \gamma_{C1}^*) & \text{if} \quad K \in \left(0, \frac{5\alpha_P-6\alpha_R}{3}\right] \\ \gamma_{U1}^* = \frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P} & \text{if} \quad K \in \left(\frac{5\alpha_P-6\alpha_R}{3}, \infty\right) \end{cases}.$$

3. Let $\alpha_R \in \left(\frac{1}{2}\alpha_P, \frac{4}{5}\alpha_P\right)$, i.e. the efficiency deficit of the public generator is "large" (in the same region where the unconstrained welfare optimum would call for $\gamma = 0$). The welfare maximizing γ for this case is given by

$$\gamma_C^* = \begin{cases} \gamma_{C2}^* = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K} \in (\gamma_{U1}^*, \gamma_{C1}^*) & \text{if} \quad K \in \left(0, \frac{\alpha_P-\alpha_R}{3}\right] \\ 0 & \text{if} \quad K \in \left(\frac{\alpha_P-\alpha_R}{3}, \infty\right) \end{cases}.$$

Proof. *See Appendix.* ■

Figure 5 displays the welfare maximizing values of γ in (α_R, K) space. Proposition 4 shows that when the transmission capacity is limited, the public generator will be called to put positive weight on consumers' surplus even in cases where it is highly inefficient compared to the private generator. When the transmission constraint K is high enough so that an unconstrained equilibrium can be achieved, from Proposition 3 we know that profit maximization (i.e. less aggressive output behavior) is the optimal instruction for the public generator in cases where $\alpha_R \in \left(\frac{1}{2}\alpha_P, \frac{4}{5}\alpha_P\right]$. For the same interval of (in)efficiency, Part 3 of Proposition 4 reveals that when K is too small it becomes optimal to put a strictly positive weight on consumers' surplus and have the public generator behave more aggressively and produce more.

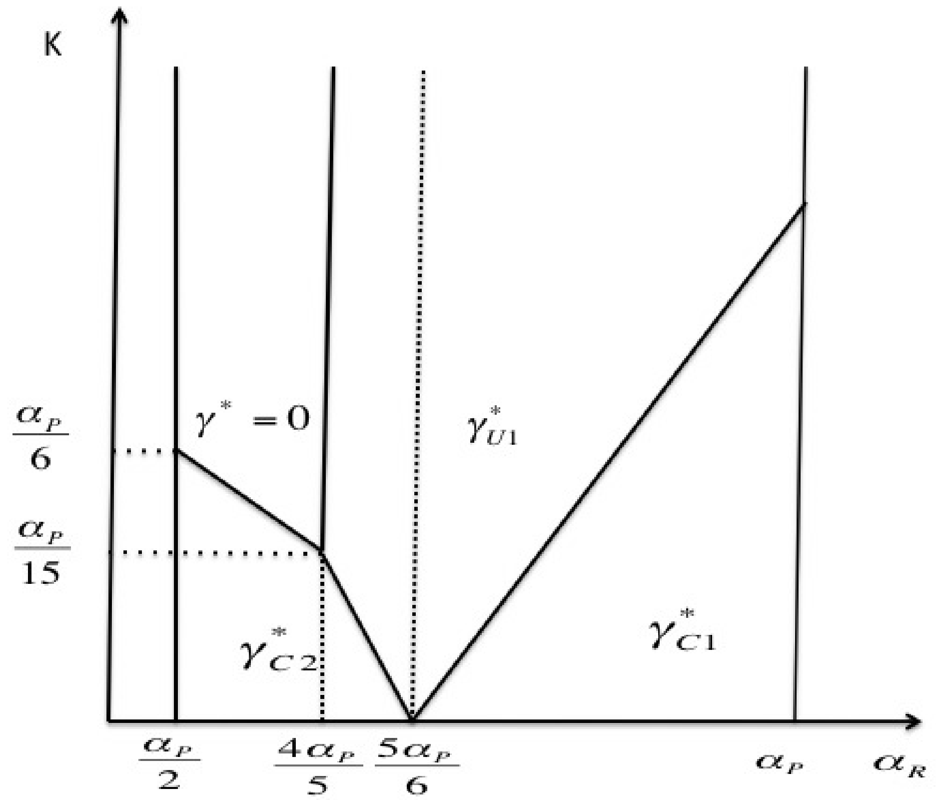

Figure 5: Optimal Objective Function for the Public Generator

Note also that when the public generator is sufficiently efficient, i.e. for $\alpha_R \in \left(\frac{4}{5}\alpha_P, \alpha_P\right)$, at all levels of K that will lead to a congested equilibrium under optimal delegation the weight to be put on consumers' surplus increases compared to the optimal weight in the corresponding interval for the uncongested case ($\gamma^*_{C1} > \gamma^*_{C2} > \gamma^*_{U1}$). As the efficiency deficit of the public generator becomes smaller, it becomes less costly to use production by the public generator to increase total surplus.

The intuition behind the results presented in Propositon 4 is that, when the transmission capacity is small, more production by the (less efficient) public generator relieves the transmission constraint and allows the more efficient private generator also produce more (recall that the capacity constraint depends on the difference between the generators' output levels). Thus, if the transmission capacity on the electricity network is fixed and cannot be increased (at least in the short run), the presence of a public generator (that produces more than its profit maximizing counterpart would have produced) allows welfare increasing overall output expansion in the electricity industry.

5 Discussion and Concluding Remarks

We studied a mixed electricity market in which a public generator and a private generator competed on a simple three-node looped network model with a transmission constraint. Both generators employed constant returns to scale technologies, and we assumed that the public generator was less efficient than the private generator. As the objective function for the public generator, we allowed for different convex combinations of consumers's surplus and its own profit. The transmission constraint led to network externalities that significantly complicated the nature of equilibria that emerged under different objective functions assigned to the public generator.

When the transmission capacity constraint K is sufficiently large, there is a unique uncongested equilibrium. In that case, the higher the weight of consumer surplus in the public generator's objective function (i.e. the higher γ is) the more it produces in equilibrium, displacing output produced by the private generator that ends up producing less. In fact, at high enough values for γ the private generator may even be completely ousted from the market, with public generator remaining as the sole producer. Given our assumption that the public generator is less efficient than the private generator, it is clear that this will not be desirable as far as total welfare is concerned if the cost differential between the two generators is sufficiently high. In fact, the welfare analysis we carry on later shows that it is never optimal to give equal weight to consumer surplus maximization as that of profit maximization in the public generator's objective function (i.e. we have $\gamma = 1/2$).[12]

[12] Note that there is mixed evidence on the relative efficiency of public versus private firms in the same industries, and it would also be plausible to assume a more efficient public generator. While Borcherding, Pommerehne and Schneider (1982), Megginson and Netter (2001), and Dewenter and Malatesta (1997) find that costs are often higher in public firms, Millward (1982) and Willner (2001) report evidence to the contrary. In the case of electricity generation the cost differential is typically a function of both organizational efficiency as well as differential access to inputs. For example, the public generator may be operating a hydroelectric power plant and the private one a natural gas fired power plant, in which case the marginal cost for the regulated generator will (in all likelihood) be lower. Note that with a transmission constraint on the network as in our model, production by the less efficient private generator could still be desirable, even in the case of constant marginal cost. In certain cases that would relieve the congestion on the transmission lines by creating a counter-flow on the congested transmission line, thereby allowing more production by the more efficient public generator. An earlier version of this paper contains an analysis with a more productive public

For small K, the transmission line becomes congested and multiple equilibria are possible under the competitive nodal pricing that resolves the congestion on the transmission network. For a given level of transmission capacity and a given objective function for the public generator, there is an equilibrium where the line is congested in one direction (with one generator, say the private one, producing more), as well as an equilibrium where the line is congested in the other direction (the other generator, say the public one, producing more). This complication is primarily due to the flexibility that the nodal pricing bring into the model, despite the fact that these prices cannot be set arbitrarily.

To carry out welfare analysis, which in our case amounts to choosing an optimal objective function for the public generator, the multiplicity of equilibrium that arose under congestion had to be resolved first. We used the profits of ISO as the criterion for equilibrium selection. The profits of ISO are simply the difference between what it collects and pays out in the operation of the transmission congestion rights market. We assumed that ISO operated under a no-loss constraint for its profits.

Our results indicate that the optimal regulatory policy, i.e. the weight of the consumers surplus in the public generators objective function, depends on the capacity of the transmission line as well as how inefficient the public generator is in relation to the private generator. The optimal choice never indicates maximizing consumers surplus as the sole objective for the public firm. In fact, if both the transmission line capacity and the efficiency gap between the two generators are high enough, then the prescription is to ignore consumers' surplus in favor of maximization of profits only. On the other hand, if the transmission line capacity is small enough, then, regardless of how inefficient the public generator is, consumers' surplus should always appear in public generator's objective function.

Finding the optimal objective function to delegate to the public generator for its competition with a private generator can be seen as analysis of optimal regulatory policy in the context of a mixed oligopolistic wholesale electricity market. The public generator plays a regulatory role by its sheer existence in the market with an objective function different than profit maximization. Then delegating an optimal objective function to the managers of the public generator amounts to an optimal regulatory policy.

Note that the analysis of the paper was carried out under the assumption that capacity of the transmission line K was fixed. It has been observed by many authors that transmission network not only transports electricity, but also promotes market efficiency as it allows generators to compete with each other.[13] So, expanding the transmission capacity would in many

generator (Mumcu, Oğur and Zenginobuz, 2007).

[13] Going back to Joskow and Schmalensee (1983) many studies examined the interaction between the availability and market power in generation and availability of transmission capacity. Wolfram (1998), Bushnell (1999), Bushnell and Wolak (1999), and Joskow and Tirole (2000) all show that generators benefit from a

instances be desirable from a welfare point of view. But, as noted by Willems (2002) as well, capacity of transmission lines cannot be expanded easily. Construction of transmission lines takes a long time, and their expansion has been met with increasing opposition by environmentalist groups. A related point is whether the private generation firms will have incentives to engage in transmission investments.[14] Our Proposition 4 indicates how the presence of a public generator can be used as an instrument to increase electricity supplied to the market in a welfare increasing manner in cases where a transmission line would be congested. A mixed generation sector would be the preferred industry structure under such conditions.

Note that optimal choice of objective function can also be viewed as a search for optimal level of privatization for the public generator. If one assumes that the objective of the public owners (consumers' surplus maximization) and that of the private owners (profit maximization) are represented in the objective function of the generator according to ownership shares, then the optimal objective function will indicate whether the generator should be left in public hands (and aim at maximizing consumers surplus), fully privatized (hence end up maximizing profit only), or should be partially privatized with less than 100% of shares in private hands. In our model with a public generator less efficient than its private counterpart, it is never optimal to have full public ownership nor is it optimal to fully privatize the industry. Partial privatization allows the possibility of alleviating output restricton that comes with imperfect competition even without the additional difficulty that tranmission line constraints bring about. With transmission line constraints, a more aggressive output behavior through more emphasis on cosumers' surplus has the added advantage of relieving the transmission line constraint, and thereby allowing the more efficient private generator also to produce more in equilibrium

Finally, we note that our analysis is carried out for an extremely simple environment, with a three-node network and a linear demand and cost structure. It has been aptly noted that Cournot models of electricity networks, which ours is an example of, are usually marred with the *devil is in the details* attribution, with their equilibria being highly sensitive to assumptions about market structure and the assumed behavior of players involved.[15] This is mainly due to inherent difficulties involved in modelling network industries where externalities abound and, hence, to a large extent inescapable. We believe that the main intution of our results should remain valid under different specifications for demand and cost, as well as network structure. An interesting extension would be to study the tradeoff between expanding transmission capacity and using the public generator's objective function on a fixed

reduction in transmission capacity.

[14] See Sauma and Oren (2009).

[15] Neuhoff et al. (2005).

transmission line to increase output. That would introduce a new dimension to benefit-cost analysis of infrastructure investments and analysis of privatization initiatives in the power industry.

Appendix: Omitted Proofs

Proof of Proposition 2

Proof. The congested equilibrium with $q_R^{C_R} > q_P^{C_R}$ is characterized by the simultaneous solution of the following equations:

$$\begin{aligned}\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 &= 0\\ \alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 &= 0\\ q_R - q_P &= 3K\\ \lambda_2 - 2\lambda_3 &= 0\end{aligned}$$

This results in the equilibrium quantities given by

$$q_R^{C_R}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R + 3K(3-2\gamma)}{6-5\gamma} \tag{37}$$

$$q_P^{C_R}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R - 9K(1-\gamma)}{6-5\gamma} \tag{38}$$

$$\lambda_3^{C_R}(\gamma) = \frac{3(1-\gamma)\alpha_R - (3-5\gamma)\alpha_P - 3K(3-4\gamma)}{6-5\gamma} \tag{39}$$

$$\lambda_2^{C_R}(\gamma) = 2\lambda_3^{C_R}(\gamma) \tag{40}$$

The set of equations that characterize the congested equilibrium with $q_P^{C_P} > q_R^{C_P}$ is

$$\gamma(q_R + q_P) + (1-\gamma)(\alpha_R - 2q_R - q_P) - \lambda_3 = 0 \tag{41}$$

$$\alpha_P - 2q_P - q_R - \lambda_3 + \lambda_2 = 0 \tag{42}$$

$$q_P - q_R = 3K \tag{43}$$

$$\lambda_2 - 2\lambda_3 = 0 \tag{44}$$

Their simultaneous solution leads to the following equilibrium quantities:

$$q_R^{C_P}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R - 3K(3-2\gamma)}{6-5\gamma} \tag{45}$$

$$q_P^{C_P}(\gamma) = \frac{\alpha_P + (1-\gamma)\alpha_R + 9K(1-\gamma)}{6-5\gamma} \tag{46}$$

$$\lambda_3^{C_P}(\gamma) = \frac{3(1-\gamma)\alpha_R - (3-5\gamma)\alpha_P + 3K(3-4\gamma)}{6-5\gamma} \tag{47}$$

$$\lambda_2^{C_P}(\gamma) = 2\lambda_3^{C_P}(\gamma) \tag{48}$$

There are also congested equilibria where only one of the generators produces positive level output and the other one shuts down. The congested equilibrium where only the public generator produces is characterized by the simultaneous solution to the following equations:

$$\gamma q_R + (1-\gamma)(\alpha_R - 2q_R) - \lambda_3 = 0 \tag{49}$$

$$\alpha_P - q_R - \lambda_3 + \lambda_2 \leq 0 \tag{50}$$

$$q_R = 3K \tag{51}$$

$$\lambda_2 - 2\lambda_3 = 0 \tag{52}$$

This results in the equilibrium quantities

$$q_R^{C_P}(\gamma) = 3K \tag{53}$$

$$q_P^{C_P}(\gamma) = 0 \tag{54}$$

$$\lambda_3^{C_P}(\gamma) = (1-\gamma)\alpha_R - 3K(2-3\gamma) \tag{55}$$

$$\lambda_2^{C_P}(\gamma) = 2\lambda_3^{C_P}(\gamma) \tag{56}$$

Finally, the set of equations that characterize the congested equilibrium where only the private generator produces is given by

$$\gamma q_P + (1-\gamma)(\alpha_R - q_P) - \lambda_3 \leq 0 \tag{57}$$

$$\alpha_P - 2q_P - \lambda_3 + \lambda_2 = 0 \tag{58}$$

$$q_P = 3K \tag{59}$$

$$\lambda_2 - 2\lambda_3 = 0 \tag{60}$$

and their simultaneous solution leads to the equilibrium quantities

$$q_R^{C_P}(\gamma) = 0 \tag{61}$$

$$q_P^{C_P}(\gamma) = 3K \tag{62}$$

$$\lambda_3^{C_P}(\gamma) = 6K - \alpha_P \tag{63}$$

$$\lambda_2^{C_P}(\gamma) = 2\lambda_3^{C_P}(\gamma) \tag{64}$$

■

Proof of Proposition 3

Proof. Differentiating (35) with respect to q_R we get

$$\frac{\partial W(q_R;\gamma,K)}{\partial q_R}=\begin{cases}\alpha_R-\frac{3}{4}\alpha_P-\frac{1}{4}q_R & \text{if} \quad q_R\in[0,\alpha_P)\\ \alpha_R-q_R & \text{if} \quad q_R\in[\alpha_P,a)\end{cases} \tag{65}$$

Noting that the second order condition is satisfied, the total surplus is maximized by setting either $q_R=4\alpha_R-3\alpha_P$ or $q_R=\alpha_R$. Given Assumption 2, simple calculations show that the total surplus with $q_R=4\alpha_R-3\alpha_P$ exceeds that with $q_R=\alpha_R$. Hence it is optimal to choose the γ that induces $q_R=4\alpha_R-3\alpha_P$ and the corresponding $q_P=2(\alpha_P-\alpha_R)$, which is equal to $\gamma^*_{U1}=\frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$, provided that it falls in the interval for γ allowed in the model, i.e. in $[0,1]$ .When it is non-negative, it can easily checked that $\gamma^*_{U1}<1$, given Assumption 2. Note, however, γ^*_{U1} can also be negative, with $\alpha_R\in\left(\frac{1}{2}\alpha_P,\alpha_P\right)$ as a result of Assumption 1 and Assumption 2.

If $\alpha_R\in\left(\frac{1}{2}\alpha_P,\frac{4}{5}\alpha_P\right]$ and $7\alpha_R>5\alpha_P$, then $\gamma^*_{U1}\leq 0$ and the total surplus is maximized at $\gamma^*_U=0$. For the case $7\alpha_R\leq 5\alpha_P$, we have $\gamma^*_{U1}\geq 0$, but in this case checking the relevant second order condition reveals that γ^*_{U1} is a global minimum and the total surplus is again maximized at $\gamma^*_U=0$.

If $\alpha_R\in\left(\frac{4}{5}\alpha_P,\alpha_P\right)$, then $\gamma^*_{U1}>0$ and checking the relevant second order condition reveals that it is indeed the global maximum.

Note that, using (18), it can be checked that the minimum transmission capacity that will allow an uncongested equilibrium at γ^*_{U1} can be calculated as $\left|\frac{6\alpha_R-5\alpha_P}{3}\right|$. When $\alpha_R\in\left(\frac{4}{5}\alpha_P,\frac{5}{6}\alpha_P\right)$, optimal γ will induce a P-type congested equilibrium and the minimum K that will allow an uncongested P-type equilibrium at γ^*_{U1} is $K=\frac{5\alpha_P-6\alpha_R}{3}$. When $\alpha_R\in\left(\frac{5}{6}\alpha_P,\alpha_P\right)$, optimal γ induces a R-type congested equilibrium and the minimum K that will allow an uncongested R-type equilibrium at γ^*_{U1} is $K=\frac{6\alpha_R-5\alpha_P}{3}$.Thus, a capacity constraint $K>\frac{\alpha_P}{3}$ will result in uncongested equilibrium at the optimal γ for all $\alpha_R\in\left(\frac{4}{5}\alpha_P,\alpha_P\right)$. ■

Proof of Proposition 4

Proof. We proceed by determining the optimal choice of q_p and q_R if the optimum surplus is to be attained at a congested equilibrium.

Case 1: We first analyze the case where $\alpha_R\in\left(\frac{4}{5}\alpha_P,\alpha_P\right)$, i.e. when $\gamma^*_{U1}=\frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$ is the total surplus maximizing value of γ in the uncongested equilibrium.

Observe that with $K=\left|(6\alpha_R-5\alpha_P)/3\right|$ the equilibrium induced by γ^*_{U1} is just binding. Hence for $K\in\left(\frac{|6\alpha_R-5\alpha_P|}{3},\infty\right)$ total surplus is maximized at γ^*_{U1}, since the choice set in the uncongested case included all possible output levels.

For $K \in \left(0, \frac{|6\alpha_R - 5\alpha_P|}{3}\right]$, we analyze the cases $\alpha_R \in \left(\frac{4}{5}\alpha_P, \frac{5}{6}\alpha_P\right]$ and $\alpha_R \in \left(\frac{5}{6}\alpha_P, \alpha_P\right)$ separately.

Case 1.a: Let $\alpha_R \in \left(\frac{4}{5}\alpha_P, \frac{5}{6}\alpha_P\right]$.

For each $K \in \left(0, \frac{5\alpha_P - 6\alpha_R}{3}\right)$ and $\gamma \in \left[0, \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}\right]$ the (P-type) congested equilibria move along $q_P - q_R = 3K$.[16] For $\gamma \in \left(\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}, \frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}\right]$ the equilibria will be uncongested, moving along the response function of the private generator. Finally, for $\gamma \in \left(\frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}, 1\right]$, the (R-type) congested equilibria move along $q_R - q_P = 3K$.

Maximizing the total surplus subject to $q_P - q_R = 3K$ yields $\frac{\alpha_P+\alpha_R+6K}{4}$ as the optimal output level for the private generator and the value of γ that induces this output level for the private generator is $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R-6K}$.

Note that P-type congested equilibrium will be obtained only when $\gamma \in \left[0, \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}\right]$. Thus the total surplus maximizing γ that induces the P-type congested equilibrium is $\gamma = \min\left\{\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R-6K}, \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}\right\}$.

Notice that $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R-6K}$ is increasing in K and $\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$ is decreasing in K. Both at $K=0$ and $K = \frac{5\alpha_P-6\alpha_R}{3}$ we have $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R-6K} > \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$. Thus for all $K \in \left(0, \frac{5\alpha_P-6\alpha_R}{3}\right)$ the total surplus maximizing γ is $\gamma^*_{C2} = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$.

Now consider increasing γ in the interval $\left(\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}, \frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}\right]$. With $\gamma \geq \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$ the equilibrium becomes uncongested. The question is whether it would be welfare improving to increase γ to de-congest the line. For any $K \in \left(0, \frac{5\alpha_P-6\alpha_R}{3}\right)$, $\frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P} < \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$, thus, we are already at the decreasing part of the total surplus function. Increasing γ beyond $\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$ yields a lower surplus. Thus, it is optimal to choose γ equal to $\gamma^*_{C2} = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$.

Now consider increasing γ in the interval $\left(\frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}, 1\right]$. Now the equilibria will be R-type congested equilibria and will move along $q_R - q_P = 3K$ line.

Maximizing the total surplus subject to $q_R - q_P = 3K$ yields $\frac{\alpha_P+\alpha_R-6K}{4}$ as the optimal output level for the private generator and the value of γ that induces this output level for the private generator is $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$.

Recall that an R-type congested equilibrium will be obtained only when $\gamma \in \left(\frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}, 1\right]$. Thus the total surplus maximizing γ that induces the R-type congested equilibrium is $\gamma = \max\left\{\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}, \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R+12K}\right\}$.

It can be checked that both $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$ and $\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R+12K}$ are increasing in K. Both at

[16] The threshold gamma, $\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$ is the value of gamma at which the (P-type) uncongested equilibrium will be just binding when the capacity constraint is K. Note that when $K=0$, this threshold is equal to $\widehat{\gamma}$, at which point the generators produce equal amounts. note also that when $K = \left|\frac{6\alpha_R-5\alpha_P}{3}\right|$, this threshold takes the value $\gamma^*_{U1} = \frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$.

$K = 0$ and $K = \frac{5\alpha_P - 6\alpha_R}{3}$ we have $\frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K} > \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R+12K}$. Thus for all $K \in \left(0, \frac{5\alpha_P-6\alpha_R}{3}\right)$ the total surplus maximizing γ is $\gamma^*_{C1} = \frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$.

Finally we need to check whether the value of total surplus at $\gamma = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$ is greater than the value of total surplus at $\gamma = \frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$. Recall that both gamma induce a congested equilibrium, the private generator producing more at the P-type equilibrium and the public generator producing more at the R-type equilibrium. It can easily be checked that the total output will be the same at both equilibria. For a given level of total output, the total surplus is increasing in the more efficient private generator's output. Thus, the total surplus maximizing γ is the one that induces the P-type congested equilibrium, namely $\gamma^*_{C2} = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$.

Case 1.b: Let $\alpha_R \in \left(\frac{5}{6}\alpha_P, \alpha_P\right]$.

In this case, the surplus maximizing uncongested equilibrium cannot be implemented by setting $\gamma = \frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$ due to capacity constraint.

Similar to Case 1.a, for each $K \in \left(0, \frac{6\alpha_R-5\alpha_P}{3}\right)$ and $\gamma \in \left[0, \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}\right]$ the (P-type) congested equilibria move along $q_P - q_R = 3K$; for $\gamma \in \left(\frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}, \frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}\right]$ the equilibria will be uncongested, moving along the response function of the private generator; and for $\gamma \in \left(\frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}, 1\right]$ the (R-type) congested equilibria move along $q_R - q_P = 3K$.

In this case the uncongested surplus maximizing value of γ, i.e. $\gamma^*_{U1} = \frac{5\alpha_R-4\alpha_P}{7\alpha_R-5\alpha_P}$ falls into the interval $\left[\frac{3(\alpha_R-\alpha_P+3K)}{3\alpha_R-5\alpha_P+12K}, 1\right]$.

Based on the analysis in Case 1.a., we can implement a P-type congested equilibrium at $\gamma = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$. However, the total surplus can be increased further if we increase γ to decongest the line. As γ reaches $\frac{3(\alpha_P-\alpha_R+3K)}{5\alpha_P-3\alpha_R+12K}$ the equilibrium will become an R-type congested equilibrium. We know from the discussion above that total surplus will be maximized at $\gamma = \frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$ at R-type congested equilibria. Thus, for $\alpha_R \in \left(\frac{5}{6}\alpha_P, \alpha_P\right]$, total surplus is maximized at $\gamma^*_{C1} = \frac{2(\alpha_P+\alpha_R)}{5\alpha_P+\alpha_R+6K}$

Case 2: We now analyze the case where $\alpha_R \in \left(\frac{1}{2}\alpha_P, \frac{4}{5}\alpha_P\right)$, i.e. when 0 is the total surplus maximizing value of γ in the uncongested equilibrium.

In this case, the minimum transmission capacity that will allow an uncongested equilibrium at $\gamma = 0$ is $K = \frac{\alpha_P-\alpha_R}{3}$.

The rest of the analysis is identical to the one in the previous case, except for the relevant intervals of K. For $K \in \left(\frac{\alpha_P-\alpha_R}{3}, \infty\right)$ it is optimal to set γ equal to 0, whereas for $K \in \left(0, \frac{\alpha_P-\alpha_R}{3}\right]$ it is optimal to set γ equal to $\gamma^*_{C2} = \frac{3(\alpha_P-\alpha_R-3K)}{5\alpha_P-3\alpha_R-12K}$. ■

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