Submission Number: PET11-11-00239

Polls and incumbency advantages in political campaigns: a dynamic contest model

Philipp Denter
University of California, Irvine, and University of St.Gallen

Dana Sisak
UC Berkeley and University of St. Gallen

Abstract

In political campaigns candidates spend resources over a period of time to increase their chances of coming out ahead. Along the way they frequently receive feedback regarding their current popularity through polls. We analyze the effect polls have on candidates' campaign spending and their prospects. We develop a two-person two-period contest model, in which an incumbent and his rival decide how much to spend in the campaign at each point in time. After the first round the candidates may receive a common signal about their current popularity through a poll. The incumbent's incentives to spend are higher than his rival's if there is a poll, and identical otherwise. Hence, polls increase the incumbent's chances to prevail and have a self-fulfilling feature. Concerning the distribution of campaign spending over time, polls lead to either higher expected spending at the beginning or at the end of the campaign, while absent polls the profile of spending is more balanced over time. Further, if the campaign race is close at start, polls increase candidates' expected aggregate campaign spending.

Submitted: March 14, 2011.

Polls and Incumbency Advantages in Political Campaigns: A Dynamic Contest Model*

Philipp Denter† Dana Sisak‡

March 14, 2011

Abstract

In political campaigns candidates spend resources over a period of time to increase their chances of coming out ahead. Along the way they frequently receive feedback regarding their current popularity through polls. We analyze the effect polls have on candidates' campaign spending and their prospects. We develop a two-person two-period contest model, in which an incumbent and his rival decide how much to spend in the campaign at each point in time. After the first round the candidates may receive a common signal about their current popularity through a poll. The incumbent's incentives to spend are higher than his rival's if there is a poll, and identical otherwise. Hence, polls increase the incumbent's chances to prevail and have a self-fulfilling feature. Concerning the distribution of campaign spending over time, polls lead to either higher expected spending at the beginning or at the end of the campaign, while absent polls the profile of spending is more balanced over time. Further, if the campaign race is close at start, polls increase candidates' expected aggregate campaign spending.

*Preliminary version. We would like to thank Johann Bauer, Stefan Bühler, Martin Kolmar, Michael McBride, Giovanni Mellace, Hendrik Rommeswinkel, Philip Schuster, Stergios Skaperdas. We are also grateful for comments made by participants of the 2009 SSES meeting in Geneva as well as by seminar participant at University of St.Gallen and University of Tübingen for valuable comments and suggestions. Al errors remain our own. Both authors gratefully acknowledge the financial support of the Swiss National Science Foundation.

†University of California, Irvine, Department of Economics, and University of St.Gallen, School of Economics and Political Science. eMail: philipp.denter@unisg.ch.

‡University of California, Berkeley, Haas School of Business, and University of St.Gallen, School of Economics and Political Science. eMail: dana.sisak@unisg.ch.

1 Introduction

During political campaigns candidates spend large amounts of resources to increase their chances of coming out ahead. For example, total campaign spending during the 2008 Presidential Campaign in the United States amounted to more than USD 1.6 billion, according to the Federal Election Commission.[1] While candidates spend resources in the campaign they frequently receive feedback about their current popularity with the voters, provided by pollsters such as Gallup or the National Opinion Research Center (NORC) in the United States, or Forsa and the Allensbach Institute in Germany. Intuitively, this feedback is valuable for candidates because it allows to adapt better to the voters' sentiment. For example, a candidate far trailing behind is likely to surrender, if he gets to know his popularity. In this study we analyze the impact polls have on campaign spending. We identify that polls tend to strengthen the position of incumbents, respectively strong candidates, thereby unbalance the campaign. Polls also lead to an imbalanced distribution of campaigning efforts over time, relative to a situation without polls. Expected spending is either high in the beginning and low at the end or the other way around. Further, if the race is close polls increase the fierceness of the competition and therefore increase aggregate campaign spending.

Political scientists and economists have long tried to figure out the impact polls have on elections. Early contribution to this literature are Margolis (1984) or Lang and Lang (1984). For example, Margolis argued that the availability of scientific polls led politicians to become more opportunistic in their campaigning strategies. Because voters are usually not very good informed about all politically important issues, politicians opportunistic behavior may lead to suboptimal or bad outcomes. Nadeau, Cloutier, and Guay (1993) show that there is evidence for the existence of a bandwagon effect in opinion formation processes. Therefore, polls may be self-enforcing. More recently, Goeree and Großer (2007) studied the effect of polls on voters behavior when preferences are correlated. Interestingly, they show that polls may be self-defeating, thus contradicting Nadeau, Cloutier, and Guay (1993). Because preferences are correlated, voters who belief to belong to a majority vote less often, since they might free ride on other voters. This is unlikely to be the case for minority preferences, so that polls close to an election decrease the probability that the majority group in the population actually wins in the election. Another recent article finding experimental evidence that polls may actually welfare reducing is

[1] Data on individual spending can be found on the commission's home page: `http://www.fec.gov/DisclosureSearch/MapAppRefreshCandList.do?d-16544-p=1&d-16544-s=4&d-16544-o=1`.

Klor and Winter (2007). Another related study is that ofMorgan and Stocken (2008), which is concerned with analyzing information transmission and revelation via polling.

Our focus differs from that of the above mentioned papers in that instead of focussing of a poll's impact on voter behavior, we are interested in the incentive effects of polls for the campaigners to spend resources in the competition. To analyze the effect of polls we construct a game theoretic model in which candidates may invest in campaign advertising to woo for voters. We follow the approach taken by for example Snyder (1989) or Skaperdas and Grofman (1995) and model the campaign as an advertising competition, however using a different specification which is due to Lazear and Rosen (1981). The competition is inherently dynamic, meaning candidates may spend resources over time to reach their goals. Along the way they may or may not receive feedback concerning their current popularity through polls. We model this as the realization of a random variable which candidates may observe or not. Only if candidates observe the realization they are able to adjust their strategies conditional on the realization of their popularity. We find that an incumbent, respectively a stronger candidate, spends more resources during the campaign than his rival if there are polls. Therefore, polls tend to increase a candidate's advantage by providing stronger investment incentives. In this respect polls have a self-fulfilling feature, what is contrary to the findings of Goeree and Großer (2007), but in line with for example Nadeau, Cloutier, and Guay (1993). However, the explanation is different. A candidate enjoying an advantage over his rival has in the presence of polls stronger incentives to invest for the following reasons: First, spending more today increases his expected advantage tomorrow, thereby decreasing the expected fierceness of the competition, what benefits him. Similarly, the trailing candidate's spending incentives are diluted for a related reason. Spending more today decreases the expected future advantage of his rival, leading to increased expected competition and higher expected costs. With respect to the timing of campaign spending, if the campaign is close polls lead to increasing campaign spending over time; in a lopsided campaign spending decreases. Without a poll spending remains constant over time or increases slightly, depending on the discount rate of the candidates. The intuition for this finding is as follows: If there is a poll there are two effects determining when effort will be spend: First, in the second stage there is less noise ahead and a signal about the current state is more informative than the same signal in period 1. Therefore, if the race is close the marginal impact of effort in the second stage is higher than in the first stage. The opposite is true in a lopsided campaign. Second, if there is a poll and one player has an advantage in stage 1, and this player has a stronger incentive to invest, the advantage is likely to increase over time. Further, if the race is close aggregate campaign spending is higher if there is a poll, and lower else.

Another literature this paper adds to is concerned with the analysis of dynamic contest. Papers related to ours are Yildirim (2005), Gershkov and Perry (2009), or Gürtler and Münster (2010). Yildirim (2005) looks at contestants' incentives to spend effort over rounds during a dynamic competition for a rent. Gershkov and Perry (2009) analyze the optimal design of a dynamic contest in which an organizer has the goal to maximize total spending. Specifically, analyze the optimal decision rule to declare the winner. Gürtler and Münster (2010) analyze players' incentives to sabotage their rivals in a dynamic competition. Articles analyzing candidate behavior during a dynamic campaign are for example Gurian (1993) or again Klumpp and Polborn (2006).

The paper is organized as follows: We describe the models set up in the next section. In Section 1 we analyze the benchmark without a poll, before we turn to the analysis of a model with a poll in Section 4. In 5 we compare the models and identify the effect of polls. Section 6 concludes.

2 The Model

Each player $i = I, R$, which are mnemonics for incumbent and rival respectively, can spend effort $x_i^t \geq 0$ in stage $t = 1, 2$. Effort costs are determined by the cost function $C(x)$, which is strictly increasing and convex, $C' > 0$ and $C'' > 0$. Both players share the same cost function and effort costs do not vary over stages, $C(x^1) = C(x^2)$ if $x^1 = x^2$. Effort translates into effective effort through the following technology:

$$e_i^t = x_i^t + \eta_i^t.$$

η_i^t is a random variable and reflects that luck plays a role. Let $\epsilon^t = \eta_R^t - \eta_I^t$, and assume this is distributed according to G with density g and support $\mathcal{S}$. We make the following assumption on g:

Assumption 1. *Let ϵ be a random variable with support $\mathcal{S}$ and $g(\epsilon)$ its density. We assume*

1. *$g(\epsilon)$ has zero mean and is symmetric around zero,*

2. *$g(\epsilon)$ is unimodal (without local maxima),*

3. *$g(\epsilon)$ is differentiable on $\mathcal{S}$.*

Parts 1 is always fulfilled if η_I^t and η_R^t are i.i.d. Part 2 implies that the density is weakly decreasing when the absolute value of the random variable ϵ^t increases. Equivalently, we may say $g(\epsilon)$ is quasiconcave. A lot of standard distribution function fulfill these assumptions. As an example we analyze the uniform distribution later in the paper.

We now define a state variable that summarizes all information relevant for a player's decision in each stage t of the game:

$$d^t = e_I^{t-1} - e_R^{t-1} + d^{t-1} = x_I^{t-1} - x_R^{t-1} + \epsilon^{t-1} + d^{t-1}.$$

That is, the state in each stage is the sum of all effective efforts from previous stages plus the head start d^0. Without loss of generality we assume $d^0 \geq 0$. If $d^0 > 0$ I enjoys an incumbency advantage. If $d^0 = 0$ both players are identical before stage 1.

The campaign ends after stage 2 and one candidate is declared the winner. Both candidates value winning the competition by V. The winner is determined by a contest success function which is fully discriminative with respect to the state after stage 1, d^2. Therefore, the probability that I wins is

$$p_I(d^2) = \begin{cases} 1 & \text{if } d^2 > 0 \\ 1/2 & \text{if } d^2 = 0 \\ 0 & \text{if } d^2 < 0 \end{cases}.$$

R wins with probability $p_R = 1 - p_I$.

In the next section we discuss the open-loop equilibrium of the game, which corresponds to a campaign without polls. In Section 4 we look at the feedback (or closed-loop) equilibrium, which corresponds to a campaign with a poll after the first period.[2]

3 Open Loop Equilibrium

Here we determine now the open loop equilibrium of the game. Open loop means the players do not observe the state after stage 1, d^1, because there is no poll, and thus cannot react to the random shock. A strategy for player i is $x_i = \{x_i^1, x_i^2\}$. An open loop equilibrium is a pair of strategies $\{x_1, x_2\}$ which are mutually best responses.

[2] The concept open-loop equilibrium refers to a situation in which a player remains ignorant of the current state of the world throughout the game, and thus cannot condition his action in a given period on the state of the world. In a feedback equilibrium this is different and players are able to condition on the state. For a discussion see Fudenberg and Tirole (1991).

In an open loop equilibrium all players specify the actions they choose in each stage beforehand, unconditional on the realization of future states. The only state players can condition on is d^0. While maximizing they have to take into account the future noise. Given the structure of the game the total shock is $\epsilon = \epsilon^1 + \epsilon^2$, which is distributed according to $\tilde{G}$ with density $\tilde{g}$, which is the convolution $g * g$ of g with itself.[3] For example, if G was normal with mean zero and variance σ^2 the cumulative distribution would also be normal with mean zero and variance $2\sigma^2$. For the case of g being uniform the cumulative density is the triangular distribution, as we show later. See for example Casella and Berger (2002), section 5.2, for reference.

In an open-loop game players cannot update their information or condition their actions on the realization of the state during the game. Such structures can be found in games, in which players do not observe their opponents past actions and the realization of noise over time. The structure is then similar to the game discussed in the seminal article by Lazear and Rosen (1981) where one player has a handicap, respectively the other player has a head start.

The probability that I comes out ahead is

$$Pr[x_I^1 + x_I^2 + d^0 > x_R^1 + x_R^2 + \epsilon] = Pr[x_I^1 + x_I^2 + d^0 - x_R^1 - x_R^2 > \epsilon] = \tilde{G}\left(\sum_{t=1}^{2}(x_I^t - x_R^t) + d^0\right).$$

Therefore, we can write player I's expected utility in stage 1 as

$$EU_I = \tilde{G}\left(\sum_{t=1}^{2}(x_I^t - x_R^t) + d^0\right) V - \sum_{t=1}^{2} C(x_I^t), \tag{1}$$

while player R's expected utility is

$$EU_R = \left[1 - \tilde{G}\left(\sum_{k=1}^{2}(x_I^t - x_R^t) + d^0\right)\right] V - \sum_{t=1}^{2} C(x_R^t). \tag{2}$$

Then, player i's first order condition with respect to effort in period t is

$$\frac{\partial EU_i}{\partial x_i^t} = \tilde{g}\left(\sum_{k=1}^{2}(x_I^t - x_R^t) + d^0\right) V - C'(x_i^t) \geq 0. \tag{3}$$

It is easily observed that this condition is identical over stages and across players.[4] As a con-

[3] Of course, there is no economic reason to limit the analysis to i.i.d. shocks over time. However, for simplicity we constrain the analysis to this case.

[4] The first order condition is only applicable to find the equilibrium when the second order condition is also fulfilled. This is not trivially given in our model. The second order condition is fulfilled for both players if

sequence, the equilibrium is symmetric and $x_I^1 = x_I^2 = x_R^1 = x_R^2 = x^+$. This is also intuitive, because from the point of view of each candidate effort in any stage is a perfect substitute to effort in another stage, if he only looks at his winning probability, so that he is indifferent between all allocations of total effort $\bar{x} = x^1 + x^2$ among stages. However, due to the convexity of the cost function he prefers to split efforts evenly over rounds. Moreover, because of g being symmetric around zero this is also true for $\tilde{g}$, and as a consequence $\tilde{g}(d^0) = \tilde{g}(-d^0)$. Therefore, both candidates incentives are completely identical and hence the same holds true for equilibrium efforts. Using this in (3) it is easily verified that

$$\tilde{g}\left(d^0\right) V - C'(x^+) \geq 0 \tag{4}$$

must be satisfied in an interior equilibrium. Simple manipulation then reveal the equilibrium, which is highlighted in the following proposition:

Proposition 1. *If the open-loop game has an interior equilibrium, equilibrium efforts are constant over candidates and stages, i.e.* $x_I^1 = x_I^2 = x_R^1 = x_R^2 = x^+$, *and given by*

$$x^+ = C'^{-1}\left(\tilde{g}\left(d^0\right) V\right).$$

Proof. See appendix. □

Equilibrium effort is increasing in V and in $\tilde{g}(d^0)$. This is quite intuitive, since a higher rent V implies there is more to fight about and as a consequence the marginal returns of campaigning effort are higher. Moreover, efforts are decreasing in $|d^0|$, i.e., as the campaign becomes more lopsided candidates invest less. This is something we know already from other contest models (for example, see Konrad (2009), chapter 2). Intuitively, as the contestants become more similar the fierceness of competition increases. It is also easy to see that in the formulas: Because $\tilde{g}$ decreases when moving away from its mean, a higher $|d^0|$ implies a lower density and so the marginal impact of effort decreases. Finally, effort is also decreasing in marginal costs of effort C', what is also very intuitive.

$C''(x^+) \geq |\tilde{g}'(d^0)|$, i.e. if the slope of the marginal cost function is sufficiently steep relative to the absolute value of the slope of the density at d^0. For example, if $\tilde{G}$ is uniform this condition is always fulfilled. Henceforth, we assume this condition holds.

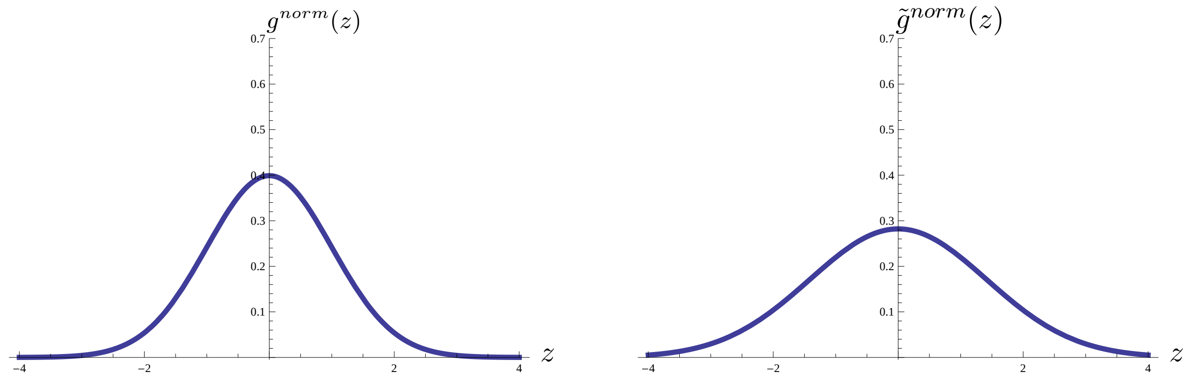

Figure 1: Distribution of per period shock (left panel) and cumulative shock (right panel) if $\epsilon \sim N(0,1)$.

Equilibrium expected utility is

$$EU_i^* = Pr[e_i^2 > e_j^2]\, V - 2C\left(C'^{-1}\left(\tilde{g}(d^0)V\right)\right). \tag{5}$$

Normal Distribution If the shock in each round is standard normal, i.e. the mean is zero and the variance is one. The convoluted distribution is then another normal with mean zero and variance two (see Figure 1). For the cost function we assume $C(x_i^t) = \frac{(x_i^t)^2}{2}$. We shall use these assumptions in all the examples to follow.

The density of the convoluted random variable in the case of normal shocks in each period is given by

$$\tilde{g}^{(norm)}(z) = \frac{e^{-\frac{z^2}{4\sigma^2}}}{2\sqrt{\pi}\sigma}.$$

The effort choice in the symmetric equilibrium is given by

$$x^{+(norm)} = \frac{e^{-\frac{(d^0)^2}{4}} V}{2\sqrt{\pi}}.$$

Effort is strictly positive for all finite d^0, which is due to the fact that the density of the normal distribution is then also strictly positive, implying the marginal impact of effort is positive. Effort is decreasing in the state d^0, and increasing in the value of the rent V. Note that the second order conditions are always fulfilled in this example.

4 Closed Loop Equilibrium

We now analyze the closed-loop version of the game. A strategy for candidate i is a tuple $x_i = \{x_i^1(d^0), x_i^2(d^1)\}$. The equilibrium concept we employ is subgame perfect Nash equilibrium.
.

4.1 Stage 2

We solve the game by using backwards induction. Therefore, we need to solve the second stage first and than "reason back" to stage 1. The structure in the second stage is very similar to the structure of the open-loop game. The only difference is that instead of a cumulative shock, candidates now take into account only the shock in stage 2. That means that uncertainty about the environment and the campaign outcome is ceteris paribus lower. As a consequence, it is plausible to conjecture that the effort of each candidate is now higher than in the open-loop game, if d^1 is relatively small in absolute terms, because candidates now have a larger impact on the outcome. Or, to put it differently, because the smaller variance implies the density is now higher around the mean, the marginal impact of effort in this region is higher. However, if $|d^1|$ is relatively high in absolute terms, what implies one candidate is enjoying a high lead, we should expect efforts to be lower. This is again very intuitive. First, a candidate trailing behind is now discouraged to compete at all, because it is very costly to catch up and winning by luck is relatively unlikely, because of the lower variance of the shock. In other words, the lower variance decreases the density at the tails of the distribution and therefore dilutes the candidates' marginal incentives to invest.

The following Proposition summarizes the second stage result:

Proposition 2. *If the second stage of the closed-loop game has a unique interior equilibrium in pure strategies, both candidates spend identical amounts of effort, denoted by x^{**}, regardless of the state variable d^1. This effort is given by:*

$$x^{**} = C'^{-1}\left(g\left(d^1\right)V\right). \tag{6}$$

Proof. See appendix. □

Effort is again increasing in V and decreasing in marginal costs, the state d^1 and the density at d^1. Accordingly, a more dispersed shock leads to higher efforts if the game is very lopsided and less efforts else.[5]

Corollary 1. *Equilibrium expected utility is*

$$EU_I^*(d^1) = G(d^1)\,V - C\left(C'^{-1}\left(g(d^1)V\right)\right), \tag{7}$$

$$EU_R^*(d^1) = (1 - G(d^1))\,V - C\left(C'^{-1}\left(g(d^1)V\right)\right). \tag{8}$$

Proof. Follows immediately from the discussion. □

Because past effort is sunk only the state d^1 and the equilibrium strategies are relevant for equilibrium utility. It is easily verified that the candidate in the lead enjoys higher utility in equilibrium, since costs are identical but he wins with a higher probability.

The equilibrium looks very similar to the equilibrium in the open-loop game. The only structural differences are the reduced number of rounds and the structure of the noise variable. This last part, however, might make a big difference, as can be seen easily by looking at the example of a uniformly distributed shock in each period. We discuss the implications in more detail in Section 5.

Normal Distribution Again, we assume the shock is standard normal as in the previous section. Then the density looks like

$$g^{(norm)}(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}},$$

what implies efforts are given by

$$x^{**(norm)} = \frac{V\,e^{-\frac{(d^1)^2}{2}}}{\sqrt{2\pi}}.$$

4.2 Stage 1

In stage 1 both candidates have the opportunity to spend effort. While doing that they look forward to stage 2 and take into account the effect their stage 1 decision has on their stage 2

[5]The second order condition is fulfilled for both candidates if $C''(x^{**}) \geq |g'(d^1)|$, i.e. if the slope of the marginal cost function is sufficiently steep relative to the absolute value of the slope of the density at d^1. For example, if G is uniform this condition is always fulfilled. Henceforth, we assume this condition holds.

expected utility, which is defined in Corollary 1. Note that $d^1 = d^0 + e_I^1 - e_R^1$. Both candidates take the expectation of this stage 2 utility with respect to the realization of the shock in stage 1, ϵ^1. Therefore, we can write down the problem of candidate I as

$$\max_{x_i^1 \geq 0} \int_{\mathcal{S}} EU_i^*(d^0 + x_I^1 - x_R^1 + \epsilon^1)\, g(\epsilon^1)\, d\epsilon^1 - C(x_I^1).$$

and similarly for candidate R. Taking the derivative with respect to x_i^1 yields after some manipulations the following system of first order conditions[6]:

$$\underbrace{E_{\epsilon^1}\left[g\left(\cdot\right)\right]V}_{(i.)} - \underbrace{E_{\epsilon^1}\left[\frac{g(d^0 + x_I^* - x_R^* + \epsilon^1)}{C''(C'^{-1}(g(\cdot)))}\frac{\partial g}{\partial d^0}\right]V}_{(ii.)} - \underbrace{C'\left(x_I^*\right)}_{(iii.)} \geq 0, \tag{9}$$

$$\underbrace{E_{\epsilon^1}\left[g\left(\cdot\right)\right]V}_{(i.)} + \underbrace{E_{\epsilon^1}\left[\frac{g(d^0 + x_I^* - x_R^* + \epsilon^1)}{C''(C'^{-1}(g(\cdot)))}\frac{\partial g}{\partial d^0}\right]V}_{(ii.)} - \underbrace{C'\left(x_R^*\right)}_{(iii.)} \geq 0. \tag{10}$$

We can identify three different effects: (i) the marginal benefit of effort, (iii) the direct marginal costs of effort, and (ii) an indirect cost effect of effort. (i) and (iii) are identical for both candidates. As we can see easily, (ii) is also identical for both in absolute value, but the sign is different. As a consequence, (ii) can be identical for both only if it is zero. The reason is that the impact of I's effort on the density is always identical in magnitude to R's effort's impact, but the sign is always the opposite. Let

$$\xi(d^0) \equiv E_{\epsilon^1}\left[\frac{g(d^0 + x_I^1 - x_R^1 + \epsilon^1)}{C''(C'^{-1}(g(d^0 + x_I^1 - x_R^1 + \epsilon^1)))}\frac{\partial g(d^0 + x_I^1 - x_R^1 + \epsilon^1)}{\partial d^0}\right]. \tag{11}$$

Because of $\xi(d^0)$ the candidates' incentives might differ, depending on the state variable d^0. An immediate conjecture is that given payers are perfectly symmetric there also exists a symmetric equilibrium. It is however, not completely clear what happens if $d^0 \neq 0$. In the following lemma we show how incentives are related to the state d^0.

Lemma 1. *Assume candidates did not yet spend effort, $x_I^1 = x_R^2 = 0$. Then $\xi(0) = 0$, $\xi(+) < 0$ and $\xi(-) > 0$.*

Proof. See appendix. □

[6]For a derivation of these equations see the appendix.

This result is very important and shows how stage 1 incentives depend on d^0. As is intuitive, if the candidates are symmetric in stage 1, both share identical incentives to spend effort. However, if one candidate has an advantage because he is in the lead, this candidate has a larger incentive to spend than the candidate trailing behind. This contradicts somehow the common wisdom of the candidate trailing behind having now an extra incentive to try very hard in order to win. Nevertheless, the result is very intuitive, once we take into account the effect of effort in stage 1 on stage 2. As we have shown in (9) and (10), there are three different effects a candidate's effort has for himself. Spending more increases the own probability to win and increases direct costs or effort. Those two effects are identical for both candidates. The only difference is ξ, which is an indirect expected effect of effort now on costs in stage 2. Stage 2 effort is identical for both candidate, as we have seen before, and is maximal if $d^1 = 0$. It decreases monotonically in the absolute value of d^1. Now, if I is in the lead and invests he not only influences his current costs, but also his expected future costs. Ceteris paribus, higher effort in stage 1 implies a higher lead in stage 2. This, in turn, decreases his future costs, because a higher lead of some candidate leads to less spending of both in stage 2. Therefore, the candidate in the lead has an additional incentive to spend effort in stage 1. The opposite is true for the candidate trailing behind. By spending more he ceteris paribus decreases the lead and so increases his own costs. This dampens his incentive to invest.

We now sum up the results obtained for stage 1 in the following proposition:

Proposition 3. *Assume the first order conditions in (9) and (10) characterize the equilibrium.*

1. *If $d^0 = 0$ there exists an equilibrium in which both candidates choose the same amount of effort, $x_I^1 = x_R^1 = x^*$, which is given by*

$$x^* = C'^{-1}(E_{\epsilon^1}\left[g(\epsilon^1)\right] V).$$

2. *If $d^0 > 0$ candidate I spends (weakly) more in equilibrium, $x_I^{1*} \geq x_R^{1*}$. The inequality is strict if the noise distribution is not uniform.*

Proof. See appendix. □

It is now interesting to look at the evolution of campaign efforts over time. While we have seen that without a poll after the first period players remain ignorant with regard to the current state, or their current popularity, and therefore campaign spending is constant over time, this is now unlikely to be the case. For such a comparison to be meaningful we need to keep everything constant except the stage, so we assume $d^0 = d^1 = d$. We look at $d^0 = 0$ first. Intuitively, campaign spending in stage 1 should ceteris paribus be lower than in stage 2. The intuition is simple: In stage two both candidates have a higher expected marginal return of effort, because there is less noise. In contest theoretic terms this means the discriminatory power of the contest is lower. The following corollary shows that this intuition is correct:

Corollary 2. *Let $d^0 = d^1 = 0$. Then both candidates' campaign spending in stage 1 is identical within each period. Second period spending is always higher than first period spending, unless the noise distribution is uniform. In this case campaign spending is identical across candidates and periods.*

Proof. See appendix. □

If $d^0 = d^1 \neq 0$ it is not trivial to show equilibrium spending. By a continuity argument we can say that for low but positive values of d^0 period 2 spending are still higher than in period 1. Out above intuition, that spending in period 2 is higher for $d^0 = 0$, is also here valid. However, if we increase d^0 this may change at some point. Unfortunately, the complexity of ξ renders this analysis almost untractable. We are, however, able to look what happens when d^0 gets very large. If this is the case both g and g' become very small, and their product $g \times g'$ vanishes. Thus, for high values we can neglect ξ in the analysis and may say $x_I - x_R \approx 0$ (thereby keeping in mind that x_I is in fact marginally larger than x_R). Given this the above intuition is valid, and period 2 spending is higher if the race is close, whereas period 1 spending is larger otherwise.

Corollary 3. *Let $d^0 = d^1 > 0$, such that I has an advantage. Then aggregate campaign spending is higher in period 2 if the race is close, and higher in period 1 otherwise.*

Proof. See appendix. □

As an example look at the case of normally distributed noise, $\epsilon \sim N(0, s)$. Here we can identify a threshold level of the state $\bar{d} = s\sqrt{2\ln[2]}$. If I's advantage in either stage is higher campaign spending in period 1 is higher, otherwise campaign spending in period 2.

5 The Effect of Polls

In this section we now analyze how polls influence candidate behavior. We therefore compare the open-loop to the closed-loop equilibrium of the game. This enables us to identify the differences and so to asses the impact polls have on candidates' campaign contributions.

We first look at period 1:

Proposition 4. *1. If $d^0 = 0$, whether or not there is a poll does not have an influence on equilibrium campaign spending in stage 1. Also, both candidates' spending is identical under both policies.*

2. If there is an incumbency advantage, $d^0 > 0$, I spends more in period 1 than R if there is a poll. If there is no poll, both players' campaign spending in period 1 is again identical.

Proof. For part 1 see the appendix. Part 2 follows immediately from Propositions 1 and 3. □

Introducing a poll increases the incumbent's spending incentives relative to the rival's in stage 1. As a consequence, I's advantage is not only preserved due to polling but increased. The intuition is as follows: If there is a poll I has higher incentives to spend effort in the campaign than R because this allows him to increase his expected advantage in the next period, what decreases his costs. R, to the contrary, faces opposite incentives: if he catches up he increases his expected costs in the next period, what dilutes his incentives. However, if there is no poll, both players are not able to condition their period 2 effort on their popularity in this stage, and hence this effect vanishes.

We now compare stage 2 spending of both candidates to be able to assess how a poll influences a candidates chances to come out ahead.

Corollary 4. *In period 2 both candidates spend identical campaigning effort under both policies, no matter what is the state of the game.*

Proof. This follows immediately from Propositions 1 and 2. □

Similar to the race without a poll stage 2 spending is identical for both players. This is also very intuitive: Because in stage 2 there is no poll ahead, the players' incentives are identical. A marginal change in effort changes both the winning probabilities and costs in the same way.

Given the above discussion the following corollary is immediate:

Corollary 5. *Polling provides stronger incentives to spend resources during a campaign for the incumbent than for her rival. Hence, when there is polling it is less likely that an incumbent loses and therefore turnover decreases.*

Proof. This follows from the above discussion. □

We now turn to analyze how a poll affects expected total campaign spending.

Proposition 5. *The expected total campaign spending of both players over both stages is higher if there is a poll and the incumbency advantage is small. If the advantage is large campaign spending is higher absent a poll.*

Proof. Tbd. □

6 Concluding Remarks

We analyzed the effects of polls on campaign spending. We found that polls increase an incumbency advantage. With respect to total campaign spending the effects of a poll are not clear. Depending on whether or not there is an incumbency advantage spending may be both higher and lower.

At the moment we are working on two extensions. First, what is the impact of polls on platform choices of candidates. Second, we are collecting data to test the implications of the model empirically.

A Mathematical appendix

A.1 Derivation of first order conditions in (9) and (10)

Remember the expected utility of I and R conditional on being in state d^1 in the second stage is

$$\begin{aligned} EU_I^*(d^1) &= G(d^1)\,V - C\left(C'^{-1}\left(g(d^1)V\right)\right), \\ EU_R^*(d^1) &= (1-G(d^1))\,V - C\left(C'^{-1}\left(g(d^1)V\right)\right), \end{aligned}$$

as we have shown in Corollary 1. Note that $d^1 = d^0 + x_I^1 - x_R^1 + \epsilon^1$. Then we can write the optimization problem of the candidates as:

$$\begin{aligned}
&\max_{x_I^1 \geq 0} \quad \int_{\mathcal{S}} G(d^0 + x_I^1 - x_R^1 + \epsilon^1)\, R - C\left(C'^{-1}\left(g(d^0 + x_I^1 - x_R^1 + \epsilon^1)\right)\right) g(\epsilon^1) d\epsilon^1 - C(x_I^1),\\
&\max_{x_R^1 \geq 0} \quad \int_{\mathcal{S}} (1 - G(d^0 + x_I^1 - x_R^1 + \epsilon^1))\, R - C\left(C'^{-1}\left(g(d^0 + x_I^1 - x_R^1 + \epsilon^1)\right)\right) g(\epsilon^1) d\epsilon^1 - C(x_R^1).
\end{aligned}$$

Taking the derivative with respect to the respective own strategy yields

$$\begin{aligned}
&\int_{\mathcal{S}} \left(g(\cdot)\, V - \left[C'(C'^{-1}(\cdot)) \frac{\partial C'^{-1}(\cdot)}{\partial g(\cdot)} \frac{\partial g(\cdot)}{\partial x_I^1} \right] \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_I^1),\\
&\int_{\mathcal{S}} \left(g(\cdot)\, V - \left[C'(C'^{-1}(\cdot)) \frac{\partial C'^{-1}(\cdot)}{\partial g(\cdot)} \frac{\partial g(\cdot)}{\partial x_R^1} \right] \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_R^1).
\end{aligned}$$

Making use of the fact that $C'(C'^{-1}(z)) = z$ and $\partial g(\cdot)/\partial x_I^1 = \partial g(\cdot)/\partial d^0 = -\partial g(\cdot)/\partial x_R^1$ these equations simplify to

$$\begin{aligned}
&\int_{\mathcal{S}} \left(g(\cdot)\, V - \left[g(\cdot) \frac{\partial C'^{-1}(\cdot)}{\partial g(\cdot)} \frac{\partial g(\cdot)}{\partial d^0} \right] \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_I^1)\\
&\int_{\mathcal{S}} \left(g(\cdot)\, V + \left[g(\cdot) \frac{\partial C'^{-1}(\cdot)}{\partial g(\cdot)} \frac{\partial g(\cdot)}{\partial d^0} \right] \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_R^1)
\end{aligned}$$

Using $\partial C'^{-1}(g(\cdot))/\partial g(\cdot) = 1/(C''(C'^{-1}(g(\cdot)))$ we get

$$\begin{aligned}
&\int_{\mathcal{S}} \left(g(\cdot)\, V - \left[\frac{g(\cdot)}{C''(C'^{-1}(g(\cdot)))} \frac{\partial g(\cdot)}{\partial d^0} \right] V \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_I^1),\\
&\int_{\mathcal{S}} \left(g(\cdot)\, V + \left[\frac{g(\cdot)}{C''(C'^{-1}(g(\cdot)))} \frac{\partial g(\cdot)}{\partial d^0} \right] V \right) g(\epsilon^1)\, d\epsilon^1 - C'(x_R^1),
\end{aligned}$$

which is identical to the equations in (9) and (10).

A.2 Proof of Lemma 1

Proof. Before getting started with proving the lemma, recall that

$$\xi(d^0) = E_{\epsilon^1} \left[\frac{g(d^0 + x_I^1 - x_R^1 + \epsilon^1)}{C''(C'^{-1}(g(d^0 + x_I^1 - x_R^1 + \epsilon^1)))} \frac{\partial g(d^0 + x_I^1 - x_R^1 + \epsilon^1)}{\partial d^0} \right].$$

We are interested in a situation in which both candidates did not yet spend anything, because this allows us to disentangle the effect of the state d^0 from the effect of efforts on incentives to invest. Thus let $x_I^1 = x_R^1 = 0$. Now define

$$\omega(d^0) \equiv \frac{g(d^0+\epsilon^1)}{C''(C'^{-1}(g(d^0+\epsilon^1)))} \frac{\partial g(d^0+\epsilon^1)}{\partial d^0},$$

which is the function we want to take the expectation of at exactly this point. Now remember that by adding an arbitrary constant a to the argument of a function, that implies that the graph of a function is shifted horizontally by $-a$. Therefore, if $g(\epsilon^1)$ is axis-symmetric across zero, $g(d^0+\epsilon^1)$ is axis-symmetric across $-d^0$. As a consequence the same holds true also for functions of this function, like $g(d^0+\epsilon^1)V$, $C'^{-1}(g(d^0+\epsilon^1)V)$, $C''(C'^{-1}(g(d^0+\epsilon^1)V))$, and $\frac{g(d^0+\epsilon^1)}{C''(C'^{-1}(g(d^0+\epsilon^1)V))}$. Because of $g(\epsilon^1)$ being axis-symmetric across zero, its derivative is point-symmetric across zero. By a similar argument as above it also holds that $\frac{\partial g(d^0+\epsilon^1)}{\partial d^0}$ is point-symmetric across $-d^0$. Therefore, it holds true that also the product of an axis-symmetric function across $-d^0$ and a point-symmetric function across this point, in our case this function is $\omega(d^0)$, is point-symmetric across $-d^0$.

We first show that $\xi(0) = 0$.

Let $f(z)$ be a function which is axis-symmetric across zero and let $h(z)$ be another function which is point-symmetric across zero. Both functions share the same support $\mathcal{K}$. Then, if we want to find $\int_{\mathcal{K}} f(z)h(z)dz$, we can split the integral into two parts:

$$\int_{\mathcal{K}} f(z)h(z)dz = \int_{\{z\in\mathcal{K}:z\leq 0\}} f(z)h(z)dz + \int_{\{z\in\mathcal{K}:z>0\}} f(z)h(z)dz.$$

Because of the symmetry properties $f(z) = f(-z)$ and $h(z) = -h(-z)$ we can rewrite the second term as

$$\int_{\{z\in\mathcal{K}:z>0\}} f(z)h(z)dz = -\int_{\{z\in\mathcal{K}:z\leq 0\}} f(z)h(z)dz.$$

Using this substitution it is easily verified that

$$\int_{\mathcal{K}} f(z)h(z)dz = \int_{\{z\in\mathcal{K}:z\leq 0\}} f(z)h(z)dz - \int_{\{z\in\mathcal{K}:z\leq 0\}} f(z)h(z)dz = 0.$$

Now let $f(z) = \omega(\epsilon^1)$ and $h(z) = g(\epsilon^1)$ and observe that the integral we want to calculate is the expectation of $\omega(\epsilon^1)$ and therefore equal to $\xi(0)$ to complete this part of the proof.

ω is shifted to the left and is point-symmetric across $-d^0$. Now note two things: First, to the left of $-d^0$ the values of ω are positive, to the rights the values are negative. Second, for any shock ϕ leading to a realization $\omega(d^0+\phi) = m$ there exists exactly one other shock ϕ', which leads to a realization $\omega(d^0+\phi') = -m$ and is an inversion of the former point across

$(-d^0, 0)$. Moreover, this holds true for any point in the graph of ω. Accordingly, we can define the whole graph as pairs of inversion points. Now observe, that the probability of an outcome $-m$ is always weakly larger than the probability of outcome m for all $m \geq 0$. To see this note that a shock generating m must be of size $-d^0 - c$, while the shock generating $-m$ must be $-d^0 + c$, for some constant $c \geq 0$. But then the shock ϕ that produces outcome m is in absolute value weakly larger than ϕ'. As a consequence, because shocks are distributed symmetrically around zero, the density of ϕ' is weakly larger than the density of ϕ, $g(\phi') \geq g(\phi)$. Note that this must hold for all m, ϕ and ϕ', and accordingly the expectation of ω must be negative. □

A.3 Proof of Proposition ??

Proof. The proof follows from Lemma 1. For $d^0 = 0$ the incentives of both candidates are identical and the symmetric equilibrium exists. For $d^0 > 0$ we showed in Lemma 1 that $\xi(d^0) < 0$, implying I's expected marginal utility from spending is higher than R's. Because both have identical marginal costs, I spends more in equilibrium. □

A.4 Proof of Corollary 3

Proof. Recall from Proposition 2 and Proposition **??** that in case of $d^0 = d^1 = 0$ equilibrium efforts in stage 1 and 2 are

$$x^*(0) = C'^{-1}(E_{\epsilon^1}[g(\epsilon^1)]V)$$

and

$$x^{**}(0) = C'^{-1}(g(0)V)$$

respectively. Because C'^{-1} is a monotonically increasing function, in order to prove the claim it suffices to show that $E_{\epsilon^1}[g(\epsilon^1)] \leq g(0)$. By Assumption 1, the maximum value $g(\epsilon)$ can take on is $g(0)$. As a consequence, the expectation of $g(\epsilon)$ cannot be higher than $g(0)$, and is lower than that value unless $g(\epsilon)$ is constant, which is true only for the uniform distribution. □

A.5 Proof of Proposition 4

Proof. We need to show that $\tilde{g}(0) = E_{\epsilon^1}[g(\epsilon^1)]$. Recall that $\tilde{g}(0)$ is the convolution $g * g$. The convolution is defined as

$$(g * g)(z) = \int_{\mathcal{S}} g(y)g(z-y)dy.$$

We want to find $(g * g)(0)$, so this is

$$(g * g)(0) = \int_{\mathcal{S}} g(y)g(-y)dy.$$

Because of the symmetry of g it holds that $g(y) = g(-y)$. Using this in the convolution formula we get

$$(g * g)(0) = \int_{\mathcal{S}} g(y)g(y)dy = E_y[g(y)].$$

Substituting ϵ^1 for y we see that this is exactly what we found in the equilibrium of stage 1 of the closed-loop game. □

A.6 Proof of Proposition ??

Proof. The first part follows from Propositions 1 and 2. The second part tba. □

References

Casella, G., and R. Berger (2002): *Statistical Inference*. Duxbury Thomson Learning.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. The MIT Press.

Gershkov, A., and M. Perry (2009): "Tournaments with midterm reviews," *Games and Economic Behavior*, 66(1), 162–190.

Goeree, J., and J. Grosser (2007): "Welfare Reducing Polls," *Economic Theory*, 31(1), 51–68.

Gürtler, O., and J. Münster (2010): "Sabotage in dynamic tournaments," *Journal of Mathematical Economics*, 46(2), 179–190.

Gurian, P. H. (1993): "Candidate Behavior in Presidential Nomination Campaigns: A Dynamic Model," *The Journal of Politics*, 55(1), 115–139.

Klor, E., and E. Winter (2007): "The welfare effects of public opinion polls," *International Journal of Game Theory*, 35, 379–394, 10.1007/s00182-006-0050-5.

Klumpp, T., and M. K. Polborn (2006): "Primaries and the New Hampshire Effect," *Journal of Public Economics*, 90(6-7), 1073–1114.

Konrad, K. A. (2009): *Strategy and Dynamics in Contests*. Oxford University Press.

Lang, K., and G. E. Lang (1984): "The Impact of Polls on Public Opinion," *Annals of the American Academy of Political and Social Science*, 472, pp. 129–142.

Lazear, E., and S. Rosen (1981): "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, 89(5), 841–864.

Margolis, M. (1984): "Public Opinion, Polling, and Political Behavior," *Annals of the American Academy of Political and Social Science*, 472, pp. 61–71.

Morgan, J., and P. C. Stocken (2008): "Information aggregation in polls," *American Economic Review*, 98(3), 864–96.

Nadeau, R., E. Cloutier, and J.-H. Guay (1993): "New Evidence about the Existence of a Bandwagon Effect in the Opinion Formation Process," *International Political Science Review*, 14(2), pp. 203–213.

Skaperdas, S., and B. Grofman (1995): "Modeling Negative Campaigning," *The American Political Science Review*, 89(1), 49–61.

Snyder, J. M. (1989): "Election Goals and the Allocation of Campaign Resources," *Econometrica*, 57(3), pp. 637–660.

Yildirim, H. (2005): "Contests with multiple rounds," *Games and Economic Behavior*, 51(1), 213–227.