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Private certification and certifiers' objective: whose profit is this?

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Abstract

Private certification and labels are taking an ever growing importance in the world of responsible consumption. However, the objective of certifying organizations is still to be investigated. Considering a Bertrand-type duopoly, this paper considers which impact the objective of a private certifier may have on the market outcome. In a duopoly set up with firms bearing different costs with respect to quality provision, firms always opt for differentiation strategies: only one adopts the label. According to third party certifier's label quality choice, the label adoption can either be done by the low-cost firm or the high-cost firm. Certification programs created by the industry itself may be more quality improving than programs sets by independent and profit seeking firms.

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Private certification and labels are taking an ever growing importance in the world of responsible consumption. However, the objective of certifying organizations is still to be investigated. Considering a Bertrand-type duopoly, this paper considers which impact the objective of a private certifier may have on the market outcome.

Keywords: certification, labeling, product quality, Bertrand duopoly.

JEL classification: D21, D43, L30.

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1 Introduction

Responsible consumption has recently emerged as a major tool used by citizens to express their environmental and social preferences, and to consume accordingly to their political convictions. Among this tendency, labeling has acquired a key role in signaling to consumers unobservable attributes of the sold products. Overall, labels exist to signal environmental standards, food safety and quality, social practices. The existence of such schemes allows for alternative production and consumption markets, incorporating ethical practices into existing systems.

In this context, certifiers have a central role. Their main duty is to certify that a particular good has been produced accordingly to a set of standards. These organizations need independence, objectivity and transparency, in order to induce consumers' trust. Labels are adopted to overcome the informational problem when consumers are not able to induce the quality level of the product. In most cases, it concerns credence attributes of the product. Credence attributes are those that are unobservable either before or after purchase and use.¹ The information asymmetry is not necessarily solved as low quality firms might have possibilities and incentives to usurp the high quality firm. Dissuasion costs is a necessary condition but however not a sufficient condition for free-riding on the label (Mason (2006), Ibanez and Grolleau (2008)).

A large part of the economic literature concentrates on firms' strategies towards the adoption of labels. (Eco)Label models relate to frameworks of vertical product differentiation. In a duopoly set up, price competition is released through the adoption of an ecolabel by one of the firms. In general quality provision is costly. Amacher et al. (2004) consider the production technology to be endogenous. The cost of high quality provision features both variable and fixed components. The fixed component is related to audit cost, paid by the firm to obtain the ecolabel. The relative cost structure determines whether firms invest into green technologies as well as the quality level of the label. In general, the literature focuses on the market behavior of firms towards voluntary quality labels.

¹The issue of quality signaling can be better understood once different categories of goods are acknowledged. Nelson (1970) developed a useful categorization between search, experience and credence attributes.

However, in many cases, third-party organizations are those that set up the labeling schemes and define quality levels in order to promote public interests. Firms might adopt voluntarily the label against a certain cost in order to differentiate their products from competitors and thus release price competition. A important research issue is then: how do third-party certifiers settle the quality standard of the label, and then, what impact these quality level choices have on market behavior by firms?

To our knowledge, only Bottega et al. (2009) study the impact of the certifier's objective on the market outcome. They consider two different objectives of the certifier: maximizing global demand for the labeled product (wide public policy), or maximizing global quality of the market (global quality policy). In a duopoly set up with firms bearing different costs with respect to quality provision, firms always opt for differentiation strategies: only one adopts the label. However, the labeling firm is not necessarily the most efficient one. In the case of a wide public policy, the efficient firm will produce labeled products only if costs of labeling are sufficiently low. In the case of a global quality policy, the low cost firm will always push the high-cost firm into the labeling program.

This paper is a complement to Bottega et al. (2009). Indeed, we analyze the outcome related to two other certifier's objectives. First, the certifier may aim at maximizing the profit of the firm that chooses the certification scheme. This may be the case if the certifier is an organization financed by private firms of the concerned market. For instance, PEFC is applied by the forest industry itself, as a form of self-certification. Second, the certifier may aim at maximizing its own profit. This is the case if the certifier is a private profit-seeking firm.

The rest of the paper is organized as follows. In section 2, we present the model and look at the market equilibrium and firms' conditions to certify their production. In section 3, we investigate the impact of certifier's objective on the standard quality level and on firm's decision to adopt or not the label. Section 4 concludes.

2 Bertrand duopoly with certification system

We build on Bottega et al. (2009). The model consists of a Bertrand duopoly model with firms bearing different costs for quality provision. We consider the following game: in the

first stage, a third-party certifier fixes the quality level required for obtaining the label. In the second stage, firms choose sequentially whether to certify or not. In a third stage, they fix simultaneously prices. Finally, consumers make their consumption choices. We solve the model backwards and we begin by the description of consumer side.

2.1 Consumers with different tastes for quality

We consider a continuum of consumers indexed by θ . Parameter θ represents consumers' taste for quality and is uniformly distributed over $[0, 1]$. Consumers decide to buy one unit or zero of the good, which can be either certified (subscript c) or uncertified (subscript u). Quality may take several forms: environmental friendliness, social conditions, child labor, health considerations...

Quality of the consumed good is not observable to consumers. However, a labeling system allows consumers to perfectly induce the quality level \bar{q} of certified products. Thus, consumers expect a non-certified good to be of quality $q_u = 0$ and a certified good to be of quality $q_c = \bar{q}$.

Consumer j 's indirect utility function is:

$$v_j(p_i, q_i, \theta_j) = m - p_i + \theta_j q_i \quad \text{for } i = u, c \quad (1)$$

p_c and p_u represent the market prices for the certified and uncertified good, respectively. m denotes the consumers reservation price for an uncertified good. We consider that the market is fully covered, implying that the indirect utility function needs to be positive: $p_u \leq m$. With this restriction, we focus thus on current consumption goods with prices lower than consumers income. Finally, we are interested in situations in which certified demand is strictly positive. Thus we assume that condition $p_c \leq m + \theta_j \bar{q}$ is met for at least one consumer (i.e $\theta_j \leq 1$). This condition will impose a restriction on the value of the label standard $\bar{q} \leq q_{max}$ and on the value of the labeling cost denoted by k (see appendix A).

Consumer j prefers a certified good to an uncertified good whenever: $v_j(p_c, \bar{q}, \theta_j) \geq v_j(p_u, 0, \theta_j)$. The indifferent consumer between the certified and uncertified good is thus defined by:

$$\tilde{\theta} = \frac{p_c - p_u}{\bar{q}} \quad (2)$$

As consumers are uniformly distributed over θ , demand for certified (D_c) and uncertified (D_u) goods are given by:

$$\begin{cases} D_c = 1 - \tilde{\theta} = \frac{\bar{q} + p_u - p_c}{\bar{q}} \\ D_u = \tilde{\theta} = \frac{p_c - p_u}{\bar{q}} \end{cases} \quad (3)$$

For certified demand to be positive, $p_c \leq p_u + \bar{q}$ has to hold.²

2.2 Bertrand duopoly with difference in costs of quality

We consider 2 firms h and l sharing the market. They differ in their costs of producing quality. The firms cost function depends on their cost efficiency and their choice of labeling their production or not:

$$C_{zi}(q_i, p_i, k_i, c_z) = (c_z q_i^2 + k_i) D_{zi}, \quad \text{for } i = c, u \quad \text{and } z = h, l \quad (4)$$

$k_u = 0$ and $k_c = k$. c_z is the cost of providing quality for firm z . We define: $c_l = c$ and $c_h = \delta c$, with $\delta > 1$. Thus, l is the low-cost firm, and h is the high-cost firm. k is the unit cost of certifying, paid to the certifying organization. D_{zi} is the demand perceived by firm z , when playing strategy i . Firms h and l choose whether to certify or not the good they produced, and then set products price. The choice of certifying is sequential, while price strategies are simultaneous.

Firm z profit function is therefore given by:

$$\Pi_{zi}(q_i, p_i, k_i, c_z) = (p_i - c_z q_i^2 - k_i) D_{zi}, \quad \text{for } i = c, u \quad \text{and } z = h, l \quad (5)$$

At this point four types of market outcome can be considered.

No firm certify: If no firm choose to certify, the classic Bertrand game applies. The cost of providing a low-quality good ($q_u = 0$) being null, the price of the uncertified good in this

²Implications of this assumption are given in appendix A.

case is $p_u = 0$, which provides zero profit for both firms: $\Pi_{zu} = 0, \forall z = h, l$. Obviously, certified demand is null here: $D_c = 0, D_u = 1$.

Both firms certify: To make positive profits, firm h needs to sell the good at a price greater than or equal to: $p_c^{min} = c_h \bar{q}^2 + k$. In that case, firm l can fix a price slightly below firm h zero-profit price, to capture all the market demand and to make positive profit:

$$p_c = c_h \bar{q}^2 + k - \epsilon \quad \text{with} \quad \epsilon \rightarrow 0^+ \quad (6)$$

Non-certified demand is necessarily null $D_u = 0$. Certified demand is thus: $D_c = 1$. As demand is totally captured by firm l , firm h profit is null. Firm l profit is thus:

$$\begin{cases} \Pi_{lc} = \bar{q}^2 c(\delta - 1) \\ \Pi_{hc} = 0 \end{cases} \quad (7)$$

Only one firm certifies: When only one firm chooses to certify its production, both firms have some market power due to product differentiation. Firm z profit maximization programme when choosing to certify (considering that firm $z' \neq z$ does not certify) is:

$$\max_{p_c} \Pi_{zc} = (p_c - \bar{q}^2 c_z - k) \frac{\bar{q} + p_u - p_c}{\bar{q}} \quad (8)$$

Conversely, firm z' profit maximization programme is (considering that firm z certifies):

$$\max_{p_u} \Pi_{z'u} = p_u \frac{p_c - p_u}{\bar{q}} \quad (9)$$

First order conditions give equilibrium prices, demands and profit. Equilibrium prices are:

$$\begin{cases} p_c = \frac{2}{3}(\bar{q}(1 + \bar{q}c_z) + k) \\ p_u = \frac{1}{3}(\bar{q}(1 + \bar{q}c_z) + k) \end{cases} \quad (10)$$

Certified and uncertified demands are in this context:

$$\begin{cases} D_{zc} = \frac{\bar{q}(2 - \bar{q}c_z) - k}{3\bar{q}} \\ D_{z'u} = \frac{\bar{q}(1 + \bar{q}c_z) + k}{3\bar{q}} \end{cases} \quad (11)$$

Table 1: **Equilibrium values when only one firm certifies**

Demand	$D_{z'u}$	$\frac{\bar{q}(1+\bar{q}c_z)+k}{3\bar{q}}$
	D_{zc}	$\frac{\bar{q}(2-\bar{q}c_z)-k}{3\bar{q}}$
Market Prices	p_u	$\frac{1}{3}(\bar{q}(1+\bar{q}c_z)+k)$
	p_c	$\frac{2}{3}(\bar{q}(1+\bar{q}c_z)+k)$
Firms Profit	$\Pi_{z'u}$	$\frac{(\bar{q}(1+\bar{q}c_z)+k)^2}{9\bar{q}}$
	Π_{zc}	$\frac{(\bar{q}(2-\bar{q}c_z)-k)^2}{9\bar{q}}$
Global Quality	GQ	$\frac{\bar{q}(2-\bar{q}c_z)-k}{3}$

Respective profits of firm z and z' are:

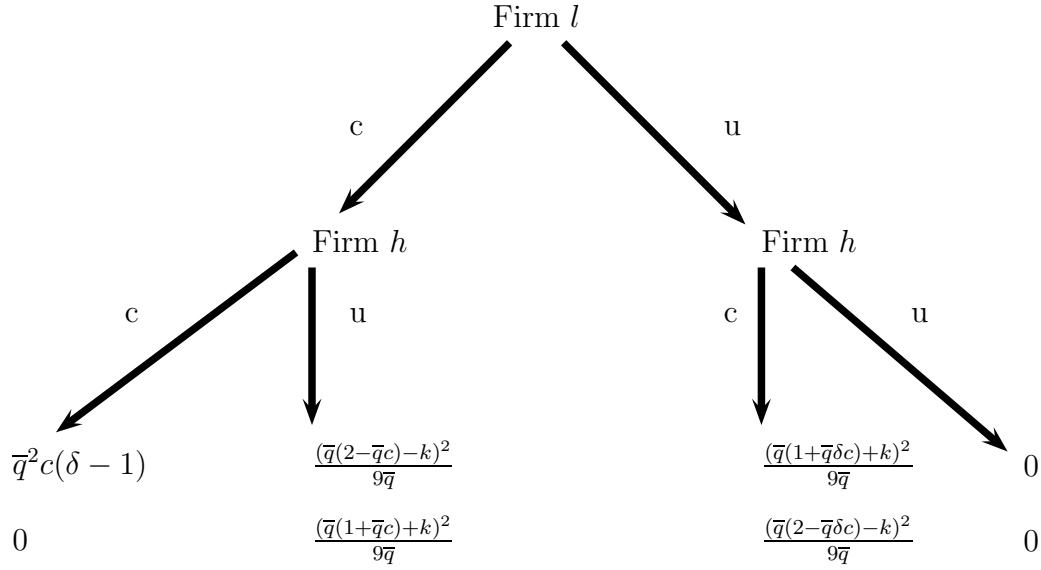
$$\begin{cases} \Pi_{z'u} = \frac{(\bar{q}(1+\bar{q}c_z)+k)^2}{9\bar{q}} \\ \Pi_{zc} = \frac{(\bar{q}(2-\bar{q}c_z)-k)^2}{9\bar{q}} \end{cases} \quad (12)$$

Table 1 gives a summary of the potential outcomes if only one firm z certifies its product (Global market quality is defined as $GQ = \bar{q}D_{zc}$).

2.3 Certifying decision

The choice of labeling is assumed to be sequential. Indeed, real world examples frequently show labeling processes have usually relatively slow-motion patterns, as only few firms adopt labels in the earlier stages of implementation. This statement may be explained by the fact that those processes require firms to adapt their production modes and are often costly to the firm. This implies that observation of ones opponents is essential, which justify the sequential choice. When choosing to certify or not, we consider that the low-cost firm may have a first mover advantage. Indeed, it is straightforward that the first objective of the less efficient firm is to avoid symmetric strategies (see figure 1): the high-cost firm is in a position where it can be excluded out of the market by the low-cost firm. So the objective for the less efficient firm is to get an equilibrium with product differentiation (whatever the product quality). It thus seems natural that this firm has an advantage to wait for its opponent

Figure 1: **The certification game**



strategy, in order to be sure that an asymmetric outcome will come out.³ Finally, once both firms have chosen to certify or not, they fix prices simultaneously.

Firm h choice: It is trivial to see that firm h always chooses firm l 's opposite strategy. Indeed, firm h gets zero profit whenever it imitates firm l 's strategy, while its profit is positive when choosing the opposite strategy. Thus, the market always splits, with a firm certifying its production, while the other does not.

Proposition 1 : *In the case of a Bertrand duopoly, the existence of a label always splits the market, when the choice of labeling is sequential and the low-cost firm chooses first.*

Proof : firm h 's profit is always positive when choosing firm l 's opposite strategy: $\frac{(\bar{q}(2-\bar{q}\delta c)-k)^2}{9\bar{q}} > 0$, and $\frac{(\bar{q}(1+\bar{q}c)+k)^2}{9\bar{q}} > 0$.

Indeed, the existence of a label allows firm h to get some market power, and thus to make positive profit. If firm l chooses not to certify its production, firm h can capture consumers

³The case in which firm h is the leader and firm l the follower has also been considered. The main difference results from the threat of the low-cost firm to exclude the leading high-cost firm if it opts for certification. Computations for the high-cost firm leader case are available upon request to the authors.

with the highest taste for quality. Conversely, if firm l certifies its production, firm h captures low taste for quality consumers.

At this stage, assumption of positive certified demand $p_c \leq p_u + \bar{q}$ imposes some restrictions on k and \bar{q} (see appendix A).

Firm l choice: From proposition 1, firm l knows that firm h always chooses firm l 's opposite strategy. Thus, its choice is made between two potential outcomes.

Firm l chooses to certify its production if:

$$\frac{(\bar{q}(2 - \bar{q}c) - k)^2}{9\bar{q}} \geq \frac{(\bar{q}(1 + \bar{q}\delta c) + k)^2}{9\bar{q}} \quad (13)$$

This condition is met for:

$$\begin{cases} \bar{q} \in [q_1; q_2] \\ q_1 = \frac{1 - (1 - 8(\delta + 1)kc)^{1/2}}{2(\delta + 1)c} \\ q_2 = \frac{1 + (1 - 8(\delta + 1)kc)^{1/2}}{2(\delta + 1)c} \end{cases} \quad (14)$$

We can define the interval in which firm l chooses to certify:

$$I = q_2 - q_1 = \frac{(1 - 8(\delta + 1)kc)^{1/2}}{(\delta + 1)c} \quad (15)$$

Note that this interval only exists for small labeling prices (or small costs of improving quality for both firms): $k \leq \frac{1}{8(\delta + 1)c} \equiv \bar{k}$. Assuming this condition is met, the interval is decreasing in the cost of certifying, the cost of quality, and the cost differential between the two firms: $\frac{\partial I}{\partial k} < 0$, $\frac{\partial I}{\partial c} < 0$ and $\frac{\partial I}{\partial \delta} < 0$. If this condition is not met, that is for high cost of certification, the low-cost firm never certifies its production and the equilibrium is: $[l, h] = [u, c]$.

The leading firm will take the high quality advantage only if it allows to gain higher profits (Shaked and Sutton, 1982). The high quality advantage does not always exist as labeling is costly. The firm that certifies faces both higher production costs as well as labeling costs. Then, if the label program sets the quality standard at a low level, the low-cost firm who has the leader advantage prefers not to certify and thus let the high quality advantage to the high-cost firm. The reason for this behavior can be explained as follows: as the differentiation between products is small, the low-cost firm prefers to accentuate differentiation by inducing

Table 2: **Certification decision according to certification requirement**

Labeling Cost	k	$\leq \bar{k}$		$> \bar{k}$
Label Quality	\bar{q}	$< q_1$	$[q_1; q_2]$	$> q_2$
Market Outcome	$[l; h]$	$[u; c]$	$[c; u]$	$[u; c]$

the high-cost firm into labeling. By pushing the high-cost firm to certify, obliging the firm to set higher prices as it faces higher costs, the low-cost firm creates space to capture consumers without increasing its production costs.

Conversely, if the labeling scheme sets the quality standard at a high level, the certifying firm will bear a high increase of its production costs. A direct consequence will be a high price for the certified product and a low demand. The low-cost firm might then prefer to sell low quality products. The advantage of obliging the high-cost firm to adopt certification, is an even higher price for certified products and thus a higher demand for uncertified products. In other words, the leading firm will only choose the high quality option (adopt labeling) if the quality level set by the label program is in between q_1 and q_2 . In all other cases, the low-cost firm will constraint the rival and less efficient firm to opt for labeling. Potential outcomes can thus be described in table 2.

Proposition 2 : *In the case of a Bertrand duopoly, if the choice of certifying is sequential, the most efficient firm tends to certify its production when the label standard takes intermediate values and the labeling cost is not too high. Conversely, the most efficient firm will let the less efficient firm certify its production when the label standards takes extreme values.*

Proof: firm l chooses to certify its production if $\frac{(\bar{q}(2-\bar{q}c)-k)^2}{9\bar{q}} \geq \frac{(\bar{q}(1+\bar{q}\delta c)+k)^2}{9\bar{q}} \iff \bar{q}(3+\bar{q}c(\delta-1))(\bar{q}(1-\bar{q}c(1+\delta))-2k) \geq 0$. As $\delta > 1$, this condition is satisfied whenever $\bar{q}(1-\bar{q}c(1+\delta))-2k \geq 0$. This is only true when $k \leq \bar{k}$ and $\bar{q} \in [q_1; q_2]$

Now that we have defined the different equilibrium patterns, we can focus on the certifier's objective and its consequences.

3 Market impact of the private certifier's objective

We consider now that the third-party certifier may have diverse objectives when implementing the certification scheme. First, it may aim at maximizing the profit of the firm that chooses the certification scheme. This may be the case if the certifier is an organization created by the business industry as a tool to develop the market. Second, it may aim at maximizing its own profit. This may be the case if the certifier is a private firm aiming at filling a gap on the labeling market.

Certification implementation consists of setting a standard for the label quality \bar{q} and a unit price of certification k . We consider a constant unit cost of certification a born by the certifier.

3.1 Maximization of the certified firm's profit

The certifying organization may aim at maximizing the certified firm's profit. The certifier's programme is:

$$\begin{aligned} \max_{\{\bar{q}, k\}} \Pi_{zc}(\bar{q}, k) &= \frac{(\bar{q}(2 - \bar{q}c_z) - k)^2}{9\bar{q}} \\ \text{s.t. } k &\geq a \end{aligned} \quad (16)$$

First order conditions are:

$$\frac{\partial \Pi_{zc}(\bar{q}, k)}{\partial k} = 0 \Leftrightarrow -2(\bar{q}(2 - \bar{q}c_z) - k) = 0 \Rightarrow k = \bar{q}(2 - \bar{q}c_z) \quad (17)$$

$$\frac{\partial \Pi_{zc}(\bar{q}, k)}{\partial \bar{q}} = 0 \Leftrightarrow (\bar{q}(2 - \bar{q}c_z) - k) (-3\bar{q}^2c_z + 2\bar{q} + k) = 0 \quad (18)$$

The maximum profit is reached at (see appendix B):

$$k = a \quad (19)$$

$$\bar{q} = \frac{1 + \sqrt{1 + 3c_z k}}{3c_z} \quad (20)$$

One can see that $\Pi_{zc} < \Pi_{Zu}$ for $k = a$ and $\bar{q} = \frac{1 + \sqrt{1 + 3c_z k}}{3c_z}$. It follows that the most efficient firm always choose not to certify and to let the less efficient certify, leading to an outcome of the type: $[l, h] = [u, c]$.

Proposition 3 : *In the case of a Bertrand duopoly, if the choice of certifying is sequential, if the certifier aims at maximizing the certified firm's profit, the most efficient firm chooses not to certify. This way, it obliges the less efficient firm to certify and get larger profit.*

This proposition is quite counter intuitive. The certifier's objective is to improve the certified firm's situation. However, the most efficient firm prefers not to certify. Indeed, this type of objective has two components: first, maximizing the total producer surplus, which is beneficial to both firms; second, maximizing the share of this surplus received by the certified firm, which is only beneficial to the certified firm. In total, the first mechanism dominates the second one, and the firm that has the first mover advantage is the one that benefits the most from this type of certifier. It is interesting noting that this outcome is the same as when the certifier chooses to maximize global market quality, as shown in Bottega et al. (2009).

3.2 Certifier's profit maximization

The certifier may aim at maximizing its own profit:

$$\max_{\bar{q}, k} Y(\bar{q}, k) = (k - a)D_{zc} \quad (21)$$

Which can be written:

$$\max_{\bar{q}, k} Y(\bar{q}, k) = (k - a) \frac{\bar{q}(2 - \bar{q}c_z) - k}{3\bar{q}} \quad (22)$$

First order conditions give:

$$\begin{cases} \bar{q} = \left(\frac{k}{c_z}\right)^{1/2} = \frac{2 - (ac_z)^{1/2}}{3c_z} \\ k = \frac{(2 - (ac_z)^{1/2})^2}{9c_z} \end{cases} \quad (23)$$

The equilibrium is of type $[l, h] = [c, u]$ if the label value is consistent with equation (14), which implies: $(ac)^{1/2} \in \left[\frac{2\delta+3}{\delta+3}; 2\right]$.

If the certifier chooses the label policy in order to maximize its own profit, such a policy implies an equilibrium where the low cost firm certifies if the cost of certification takes intermediate values.

Proposition 4 : *In the case of a Bertrand duopoly, if the choice of certifying is sequential, if the certifier aims at maximizing its own profit, the most efficient firm chooses to certify only for intermediate cost of monitoring a and cost of providing quality c . Note that the interval increases when the cost differential δ increases. Finally, the certification scheme is less demanding than when the certifier aims at maximizing the certified firms' profit.*

4 Discussion

Bottega et al. (2009) showed that the motivations behind the certifier's action of setting a scheme was important in the final market outcome. This paper is a follow up, assessing the impact of the certifier's action when it is trying to maximize the certified firm's profit and its own profit.

We show that if the certifier aims at maximizing its own profit, it sets a price of certification larger than its marginal cost. In this case, the most efficient firm chooses the certification scheme is the monitoring cost and costs of improving quality take intermediate values. Indeed, if those costs are too high, it is less profitable to join the scheme. If they are too low, the benefit from differentiating through certification is smaller.

If the certifier aims at maximizing the certified firm's profit, this surprisingly lead to an equilibrium in which the most efficient firm does not join the scheme and let the less efficient firm participate. Moreover, the scheme is more demanding than in the case in which the certifier maximizes its own profit.

It follows that certification programs created by the industry itself may be more quality improving than programs sets by independent and profit seeking firms.

Appendix A: implications of the full market coverage and positive certified demand

We have assumed that $p_u \leq m$ and $p_c \leq p_u + \bar{q}$, so that the market is fully covered and certified demand is strictly positive. This assumption implies restrictions on the label standard and the cost of labeling:

$$\begin{cases} \bar{q} \in [q_{\min}, q_{\max}] \\ q_{\min} = \frac{1 - \sqrt{1 - c_z k}}{c_z} \\ q_{\max} = \frac{1 + \sqrt{1 - c_z k}}{c_z} \\ k \leq \frac{1}{c_z} \end{cases}$$

Note here that the label standard related to the wide public policy $\bar{q} = (k/c_z)^{1/2}$ and the global quality policy $\bar{q} = 1/c_z$ both unambiguously hold in this interval. Considering the numerical example, we assume: $m = 20$, $c = 0.05$, $\delta = 1.4$ and $k = 0.8$.

Market Outcome	$[l; h]$	$[c; c]$	$[u; c]$	$[c; u]$
Label Standard	$q_{covered}$	$(\frac{m-k}{\delta c})^{1/2}$	$\frac{(1-4\delta c(k-\frac{3k}{2}))^{1/2}-1}{2\delta c}$	$\frac{(1-4c(k-\frac{3k}{2}))^{1/2}-1}{2c}$
Value		≤ 16.56	≤ 14.49	≤ 16.15

Appendix B: maximizing the certified firm's profit

We have four possible solutions to equation (16):

$$\begin{aligned} \bar{q}_1 &= -\frac{1 + \sqrt{1 + c_z k}}{c_z} \\ \bar{q}_2 &= \frac{-1 + \sqrt{1 + c_z k}}{c_z} \\ \bar{q}_3 &= \frac{1 - \sqrt{1 + 3c_z k}}{3c_z} \\ \bar{q}_4 &= \frac{1 + \sqrt{1 + 3c_z k}}{3c_z} \end{aligned}$$

Note that \bar{q}_1 and \bar{q}_3 are negative and cannot be solutions to our problem. The space of solutions thus reduces to $\{\bar{q}_2, \bar{q}_4\}$. We now have to consider different cases.

We now have to consider different cases. First, let us have a look to the first order condition with respect to k : $\tilde{k} = \bar{q}(2 - \bar{q}c)$. The certifying firm's profit decreases with k when $k \leq \tilde{k}$

and increases when $k \geq \tilde{k}$. The profit is minimum and equal to zero for $k = \tilde{k}$. Furthermore, $\tilde{k} \leq \frac{1}{c_z}$.⁴ To summarize, $a \leq \tilde{k} \leq \frac{1}{c_z}$ and \tilde{k} is the value minimizing certified firm's profit. We first look at interior solutions and then turn to border solutions.

Interior solution: For $k = \bar{q}(2 - \bar{q}c_z) = \tilde{k}$, the profit is always equal to zero. We thus have an infinite number of solutions for \bar{q} . Whatever the value of \bar{q} , the profit is zero. However, note that $\Pi_{zc}(\bar{q}, \tilde{k})$ describes a minimum. We then have to look at border solutions.

Border solutions: We have two possible border solutions: $k = a$ and $k = \frac{1}{c_z}$. For these values of k , we have two possible solutions for \bar{q} : $\bar{q}_2 = \frac{-1 + \sqrt{1 + c_z k}}{c_z}$ and $\bar{q}_4 = \frac{1 + \sqrt{1 + 3c_z k}}{3c_z}$.

First, we have $\bar{q}_4 < \bar{q}_2$. $\bar{q}_4 - \bar{q}_2 = \frac{4 + \sqrt{1 + 3c_z k} - 3\sqrt{1 + c_z k}}{3c_z}$. The difference equals zero for $k = \frac{8}{c_z}$ and is negative for $k < \frac{8}{c_z}$. As, by assumption, $k < \frac{1}{c_z}$, we have $\bar{q}_4 < \bar{q}_2$.

Moreover, $\bar{q}_4 > q_{\min}$. $\bar{q}_4 - q_{\min} = \frac{-2 + 3\sqrt{1 - c_z k} + \sqrt{1 + 3c_z k}}{3c_z}$. $\bar{q}_4 - q_{\min}$ equals zero for $k = \frac{1}{c_z}$ and that it is positive for $k < \frac{1}{c_z}$. As, by assumption, $k < \frac{1}{c_z}$, we have that $\bar{q}_4 > q_{\min}$.

Finally, $q_{\max} - \bar{q}_2 > 0$. $q_{\max} - \bar{q}_2 = \frac{2 + \sqrt{1 - c_z k} - \sqrt{1 + c_z k}}{c_z}$. $q_{\max} - \bar{q}_2 > 0$ if $2 + \sqrt{1 - c_z k} - \sqrt{1 + c_z k} > 0 \Leftrightarrow (2 + \sqrt{1 - c_z k})^2 - (\sqrt{1 + c_z k})^2 = 4 - 2c_z k + 4\sqrt{1 - c_z k} > 0$ which is always true $\forall k < \frac{1}{c_z}$.

We can thus establish the following ranking: $q_{\min} < \bar{q}_4 < \bar{q}_2 < q_{\max}$.

If now we study the sign of the equation(18) (first order condition with respect to \bar{q}):

\bar{q}	$[q_{\min}, \bar{q}_4]$	$[\bar{q}_4, \bar{q}_2]$	$[\bar{q}_2, q_{\max}]$
$(-3\bar{q}^2 c_z + 2\bar{q} + k)$	+	-	-
$(\bar{q}(2 - \bar{q}c_z) - k)$	+	+	-
$\frac{\partial \Pi_{zc}(\bar{q}, k)}{\partial \bar{q}} = (\bar{q}(2 - \bar{q}c_z) - k)(-3\bar{q}^2 c_z + 2\bar{q} + k)$	+	-	+

The maximum is then reached for $\bar{q}_4 = \frac{1 + \sqrt{1 + 3c_z k}}{3c_z}$.⁵

⁴To see this: $\tilde{k} - \frac{1}{c_z} = \frac{\bar{q}c_z(2 - \bar{q}c_z) - 1}{c_z} < 0 \Leftrightarrow \bar{q}c_z(2 - \bar{q}c_z) - 1 < 0 \Leftrightarrow (\bar{q}c_z - 1)^2 > 0$ which is always true. ($k \leq \frac{1}{c_z}$ is one of condition guaranteeing that the market is fully covered and that certified demand is strictly positive).

⁵Note that we have a minimum for $q = \bar{q}_2$ and $\Pi_{zc}(\bar{q}_2, k) = 0$. ($\lim_{\bar{q} \rightarrow q_{\min}} \Pi_{zc}(\bar{q}, k) = 0$). Furthermore we have a local maximum for $q = \bar{q}_4$. But a maximum can also be reached for q_{\max} . We compute $\Pi_{zc}(\bar{q}_4, k) - \Pi_{zc}(q_{\max}, k) = \frac{16(1 + \sqrt{1 + 3c_z k} + 33c_z k(-3 + \sqrt{1 + 3c_z k}))}{243c_z k}$. We look at the sign of $1 + \sqrt{1 + 3c_z k} + 33c_z k(-3 + \sqrt{1 + 3c_z k})$. $1 + \sqrt{1 + 3c_z k} + 33c_z k(-3 + \sqrt{1 + 3c_z k}) = 0$ for $k = \frac{1}{c_z}$. Furthermore it is easy to see that $\forall k \leq \frac{1}{c_z}$, $1 + \sqrt{1 + 3c_z k} + 33c_z k(-3 + \sqrt{1 + 3c_z k}) \geq 0$. Thus $\forall k \leq \frac{1}{c_z}$, we have that $\Pi_{zc}(\bar{q}_4, k) - \Pi_{zc}(q_{\max}, k) \geq 0$

For the potential values of k , the solution for \bar{q} is $\bar{q}_4 = \frac{1+\sqrt{1+3c_z k}}{3c_z}$

However, $\Pi_{zc}(\bar{q}_4, a) - \Pi_{zc}(\bar{q}_4, \frac{1}{c_z}) = \frac{16(1-3ac_z+\sqrt{1+3ac_z})^2}{243c_z(1+\sqrt{1+3ac_z})} > 0$

Thus, whatever the value of a , $\Pi_{zc}(\bar{q}_4, a) > \Pi_{zc}(\bar{q}_4, \frac{1}{c_z})$

Indeed $\Pi_{zc}(\bar{q}_4, \frac{1}{c_z}) = 0$ and $\forall a \neq \frac{1}{c_z}$, $\Pi_{zc}(\bar{q}_4, a) > 0$

It follows that the solution to our problem is:

$$\begin{aligned} k &= a \\ \bar{q} &= \frac{1 + \sqrt{1 + 3c_z k}}{3c_z} \end{aligned}$$

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