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Mixed-member electoral systems and the scope of government

Christian Pfeil
Walter-Eucken-Institute, Freiburg, Germany

Abstract

The paper tries to theoretically answer the question how mixed-member electoral systems affect the scope of government in terms of the provision of a local public good versus redistribution (transfers). However, it currently represents an early stage of research. Based on the literature on pure electoral systems and their impact on the scope of government, I extend a model of electoral competition as well as a model of strategic delegation. The extension of the former model shows that a MMM system leads to a rather tough competition for the marginal district, no matter what the initial pure formula is. The extension of the latter model is in progress.

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Mixed-Member Electoral Systems and the Scope of Government

Christian F. Pfeil*

March 2011

(Work in Progress)

Abstract The paper tries to theoretically answer the question how mixed-member electoral systems affect the scope of government in terms of the provision of a local public good versus redistribution (transfers). However, it currently represents an early stage of research. Based on the literature on pure electoral systems and their impact on the scope of government, I extend a model of electoral competition as well as a model of strategic delegation. The extension of the former model shows that a MMM system leads to a rather tough competition for the marginal district, no matter what the initial pure formula is. The extension of the latter model is in progress.

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1 Prelude

The motivation to deal with mixed electoral systems is quite simple: a number of countries like Germany, New Zealand, Japan, Mexico or Venezuela use mixed electoral systems but there is no comprehensive literature providing insights into the effects that such systems have on policy outcomes like government spending. Furthermore it is far from clear that the policy outcome of a mixed system is lying somewhere inbetween the policy outcomes of the pure systems just as mixed systems are an institutional combination of the pure systems. Thus this paper deals with the effect of mixed systems on the composition of government spending. For this I can build on a good deal of theoretical as well as empirical **literature** of pure electoral systems

*Walter-Eucken-Institute Freiburg & Freiburg University, pfeil@walter-eucken-institut.de

that differs between plurality rule and proportional representation. Persson and Tabellini (1999 [3], 2000 [4]) present a probabilistic voting model of electoral competition which also models the politician's motivation to run in terms of an endogenous rent. The theory indicates that less public goods (illustrating the general interests) are provided under a majoritarian election. Party programmes in proportional elections, however, tend to redistribution (illustriating narrow interests). The model of Milesi-Ferretti et al. (2002 [2]) focuses on the delegation process of voters. The propositions, however, are similar: Majoritarian systems provide more local public goods relative to redistribution. Empirical evidence provided by Persson and Tabellini (1999 [3]) uses the sum of expenditures on transportation, education, order/safety and health as measure of the scope of government. The results are in line with the theory but not robust. In another study the same authors use social security and welfare spending as measure with the results being statistically significant (Persson and Tabellini, 2004 [5]).

Basically there is a vast heterogeneity of electoral systems and thus it is not easy establishing a **classification of mixed systems**. But nevertheless electoral systems can be divided along the two basic electoral formulas plurality rule (PL) and proportional representation (PR). The former awards office to one candidate which leads to considerable political stability with the candidate having a definite relation to the constituencies' affairs. However, narrow interests are of nearly no chance to get represented in parliament because only big parties have a viable chance of winning the majority. This is what the latter formula ensures albeit at the expense of political stability with no direct candidate's accountability to the constituency as there are party lists. The intension of mixing both systems from a political viewpoint is thus to have a system that helps the local interests to get represented with simultaneously ensuring a minimum of political stability. Consequently the first feature of mixed systems is their combining these both pure electoral formulas in the election of one chamber. Again there is a considerable heterogeneity among the mixed systems. Shugart and Wattenberg (2003 [6]) classify an electoral system as mixed if it uses two tiers for each pure type. This means that there is one district set for each subelection which overlap because they are used simultaneously in one election. A system that uses one formula in one part of the country and the other formula in the remainder of the jurisdiction in the election of one chamber ist thus not considered being a mixed system. This directly implies that voters cast two votes, one vote for each tier. The tier conducting the PL election is called the nominal tier. Here typically single-seat districts are used. The other tier is called the list tier. Here typically closed party lists are used. In differentiating between a PL tier and a PR tier this classification is very useful with respect to my distiction between broad and narrow interests.

Although mixed systems combine both pure electoral formulas they tend to either of the formulas. Thus an additional **differentiation among mixed systems** is necessary. If in a mixed system the two tiers are totally independent of each other Shugart and Wattenberg (2003 [6]) call it a mixed-member majoritarian (MMM) election[1]. Seats gained in either of the two subelections independently contribute to the overall number of seats allocated to a party.

[1]also called parallel system

That is not the case with respect to a mixed-member proportional (MMP) election. Here the number of seats a party takes from the list depends on the number of seats the party received in the PL election. Finally the number of seats is in line with proportionality. Put another way the party receives a certain number of seats from the PL election and takes further seats from the list until the entire amount of seats matches with proportionality. Thus the number of list seats is a residuum.

To my knowledge literature on mixed systems is scarce. Thames and Edwards (2006 [7]) provide an empirical analysis testing the relevance of the distinction between MMM and MMP. They can show that the former is associated with a lower level of (public good) spending devoted to social support programmes[2]. Furthermore they uncover that the difference is the larger the more seats in the MMM election are assigned in the nominal tier. However, they do not provide a theoretical argument.

The next section presents the electoral competition model and its extension. This model is in accordance with Persson and Tabellini (2000 [4]) and Brocas et al. (2000 [1]). Section 3 then presents the strategic delegation model of Milesi-Ferretti et al. (2002 [2]). Both models differ between redistribution signifying the general interest and a local public good.

2 The Model of Persson/Tabellini

Consider an economy with a large number of **citizens**. This citizens differ in their probability of being employed in such a way that all persons belonging to one of the $k = h, m, l$ types face the same probability n^k of being employed. One can imagine this to be high, medium and low qualified persons. The average value of n^k in the population is n. This also represents the fraction of employed individuals in the economy. If a person is employed it draws utility from private consumption $c = y(1-\tau)$ with y being the income and τ being the nondistorting tax rate. Income can be assumed to be the same for alle individuals since it is not a determining variable of subgroups of the population in this model. Otherwise it could be seen as average income over all individuals. Unemployed persons receive an unemployment subsidy f. Additionally citizens can also be clustered with respect to the region they live in. Assume that there are three of it, $r = 1, 2, 3$. Besides private consumption citizens also benefit from local public good consumption with g^r describing local public consumption per capita in region r. Thus one citizen of type k living in region r faces preferences

$$w^{k,r} = n^k U(c) + (1 - n^k)U(f) + H(g^r). \tag{1}$$

The utility from private consumption $U(.)$ as well as from public good consumption $H(g^r)$ is assumed to be concave and monotonically increasing. At the time of the election voters care about economic policy, that is the policy platform q, but they also evaluate ideological or personal attributes of the parties. Thus they additionally have a bias towards one of the

[2]programmes for e.g. sickness, disability, old age, unemployment

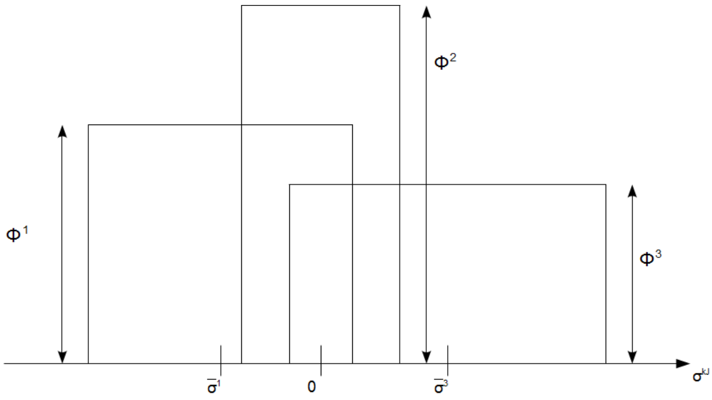

Figure 1: density functions of one k-type in the three regions

parties. A voter of type k in region r prefers party A if

$$W^{k,r}(q_A) > W^{k,r}(q_B) + \sigma^{k,r} + \delta$$

with $W^{k,r}(q)$ being the indirect utility function of a voter of type k in region r as a function of the policy vector $q = q(\tau, f, g^r)$. The parameter $\sigma^{k,r}$ depicts the bias of a k-type voter in region r towards party B if it is positive. Otherwise the voter has a bias towards party A. If $\sigma^{k,r} = 0$ the person is ideologically neutral, so he/she cares only about economic policy. Besides the individual bias towards a party there is an average (relative) popularity δ of candidate B in the whole population. It can also take on positive as well as negative values and has a uniform distribution on

$$\left[-\frac{1}{2\psi^r}, \frac{1}{2\psi^r}.\right]$$

The individual bias $\sigma^{k,r}$ also has a uniform distribution but with respect to the PL election it is necessary to assume that the regions differ in the mean of the distribution with the means of $\sigma^{k,1}$ and $\sigma^{k,3}$ being sufficiently different from zero. The parameter thus has a uniform distribution on

$$\left[-\frac{1}{2\phi^{k,r}} + \bar{\sigma}^{k,r}, \frac{1}{2\phi^{k,r}} + \bar{\sigma}^{k,r}\right]$$

with density $\phi^{k,r}$ that can be interpreted as the number of k-voters having a certain bias σ. Since σ and δ can not be influenced by the candidates or parties the election outcome is uncertain to them. Figure 1 depicts the density functions of type k-voters over all three regions. It shows that let's say the group of high-qualified persons in region 1 tends to party A and in region 3 it tends to party B. As this holds for all k-types this means that all citizens in region 1 tend to party A whatever their level of qualification is. Furthermore the figure shows that each group has an ideologically neutral (individual unbiased) voter with $\sigma^{k,r} = 0$ and that each k-type cum region has a continuum of voters with unit mass. A voter is indifferent between party A and party B and thus called a swing voter, if

$$W^{k,r}(q_A) = W^{k,r}(q_B) + \sigma^{k,r} + \delta. \tag{2}$$

The equation indicates that a swing voter not necessarily needs to be ideologically neutral. As can be seen in the figure it is also assumed that region 2 has the highest density what simply means that region 2 has the relative highest number of residents.

On the other side there are two **parties (or candidates)** A and B who offer a party programme q_A/q_B in order to become elected. Their basic motivation to run lies in receiving an exogenous rent from holding office which is not modeled here. The politicans are opportunistic and simply want to hold office. So they do not care which policy is implemented. Ahead of the election the two candidates or parties commit to their policy platforms q_A and q_B and thus electoral competition basically consists of the parties' platform choice. They act simultaneously and do not cooperate. The winning party's platform is implemented. As mentioned the election outcome is uncertain when platforms are chosen. The tax financing of the unemployment insurance and the local public goods offered in the party programmes must hold the budget constraint

$$ny\tau = (1-n)f + \frac{1}{3}\sum_r g^r.$$

Now presume that both parties offer the same policy vector $q_A = q_B$. Party A now considers a deviation from q_A by reducing the tax rate or offering more local public goods. This changes $W^{k,r}(q_A)$ in equation 2 and hence the number of votes party A can expect to get. More precisely former swing voters in all regions now assuredly vote for one of the parties and former definite voters are now swing voters. Offering more local public goods for region 1 financed by less public goods for region 3 for example shifts the swing voter in region 1 to the right and in region 3 to the left by the same distance. As there are more voters in region 1 than in region 3 because of $\phi^1 > \phi^3$ this leads to a net increase in votes for party A. Figure 1 now indicates that parties focus on the swing voter. This because all voters on his right definitely vote for party A and on his left all individuals definitely vote for party B. So for both parties there are no additional votes to obtain.

In a **majoritarian election** parties place their candidates in all single-seat districts. To win the parliamentary election and thus being entitled to set its policy a party needs to get the majority of votes in the majority of districts. Remember that the model consists of three districts. A party now wins the electoral competition by winning two districts out of three. If a party surely wins two or three districts then there is no competition anymore. So electoral competition only takes place if each party has one district for sure with one district being marginal. Thus a party in this models targets the swing voter in the marginal district. The objective of party A in a majoritarian election thus is

$$\max \sum_k \phi^2 W^{k,2}(q_A) \tag{3}$$

Differentiating the objective funktion with respect to τ_A and f and using $n = \frac{\sum n^k}{3}$ leads to

$$\frac{\delta U}{\delta c} = \frac{\delta U}{\delta f}$$

which means that the marginal utility of consumption from income is equal to the marginal utility of consumption from the subsidy. Both functions are assumed to be concave which not necessarily requires them to be identical. Nevertheless the party programme offers full insurance such that the marginal utilities are equal. Differentiating the objective function with respect to f and g^2 and using the definition of n leads to

$$\frac{\delta H}{\delta g^2} = \frac{1}{3} \cdot \frac{\delta U}{\delta f}. \tag{4}$$

To get this result it needs to be assumed that the group-specific means $\bar{\sigma}^1$ and $\bar{\sigma}^3$ are sufficiently distant from zero. If this is true a change in a party's policy vector shifts the swing voter in the marginal district but the shift of the swing voter by the same distance in the safe district does not abolish the safe district's majority. So a party can win votes in the marginal district by not losing the majority in the safe district.

The objective function of a party under **proportional election** is different. Here parties need the majority of the whole electorate i.e. over all districts. More precisely it is assumed that there is only one national district that encompasses all voters. In this model this means that party targets the swing voters of all districts. Since the model abstracts from percentage thresholds, perfect proportionality is given. The model additionally disregards coalition building so the party that wins more than 50 % of the seats is authorised to implement its previously announced party programme. In contrast to the basic model it is assumed here, that $\phi^{k,r} = \phi^r$ for all k which means that the number voters with a high, a medium and a low qualification is the same within a district. The objective of party A is

$$\max \sum_k \sum_r \phi^r W^{k,r}(q_A) \tag{5}$$

Differentiating the objective function with respect to f and g^2 and using the definition of n leads to

$$\frac{\phi^2}{\sum_r \phi^r} \cdot \frac{\delta H}{\delta g^2} = \frac{1}{3} \cdot \frac{\delta U}{\delta f} \tag{6}$$

with the result also holding for district g^1 and g^3. This shows that under PR election all districts receive the local public good. Furthermore it shows that the amount of g^2 is smaller under PR than under PL. This is because of the term $\phi^2/\sum_r \phi^2$. As this takes values between 0 and 1 the marginal utility of the local public goods here needs to be larger than in equation (4) to hold. As the function $H(.)$ is assumed to be concave a larger marginal utility implies a smaller amount of the local public good being provided. Thus the composition of government shifts towards redistribution. Full insurance is in place again.

As mentioned above in a **mixed-member majoritarian election** both tiers are conducted independently of each other and winning votes in each subelection directly provides the party with seats in parliament. Thus there is a serious incentive to run in both tiers. However, the different types show different incentives for the parties as explained in the previous sections. Given this the objective of party A is

$$\max \sum_k \phi^2 W^{k,2}(q_A) + \sum_k \sum_r \phi^r W^{k,r}(q_A) \tag{7}$$

Differentiating the objective function with respect to τ_A and f and using the definition of n shows that also in a mixed system there is full insurance in this framework. Differentiating the objective function with respect to f and g^1 and using the definition of n leads to

$$\frac{\phi^1}{\phi^2 + \sum_r \phi^r} \cdot \frac{\delta H}{\delta g^1} = \frac{1}{3} \cdot \frac{\delta U}{\delta f} \tag{8}$$

with the result also holding for district g^3. With respect to g^2 it leads to

$$\frac{\phi^2}{\phi^2 + \sum_r \phi^r} \cdot \frac{\delta H}{\delta g^2} = \frac{1}{6} \cdot \frac{\delta U}{\delta f} \tag{9}$$

which apparently means that the result is not symmetric for all districts and thus a differentiation between the definite constituencies 1 and 3 and the marginal district 2 is in place. Consider first the districts 1 and 3. Compared to PR there is an additional ϕ^2 in the denominator of the left-hand side. Following the same argumentation as above this means that the amount of the local public good is smaller than in equation (6). Taken separately this is a little surprising as a majoritarian election is added to a proportional one. However, these are the definite constituencies. The result with respect to district 2 can be seen by directly comparing equation (9) with equation (6). Again there is the additional ϕ^2 in the denominator which tells us a smaller amount of the local public good compared to the PR election. However, the right hand side is cut in half. This forces the amount of the local public good to be larger than under PR election. The question now is which effect will force through. I guess that the additional ϕ^2 can not outweigh the halving of the right hand side. Thus I conclude the amount of the local public good to be larger than under the PR regime. As the local good provision in the safe districts is smaller than under PR it seems that the larger amount in district 2 is at the expense of the districts 1 and 3. When MMM is compared to PL it comes up that the safe districts are now also provided with the local good. Additionally the amount of the local good in the marginal district is in fact larger than under PL. The argumentation here is similiar to that stated above: the halfing of the right-hand side overweighs the additional fraction with values between 0 and 1 of the left-hand side. This can be seen by directly comparing equation (9) with (4). To sum up the composition of government is in favour of the local public good under MMM. However, the combination of the pure formulas in MMM seems to result in a rather tough competition for the marginal district no matter what the initial formula is. Once the larger amount of the local good for the marginal district is at the expense of the safe districts. Another time it is reflected in a overproportional amount of the local good for the marginal district. Thus it seems that the existing PR election backs up the tough competition for the marginal district. In this sense the mix of the pure electoral formulas does not simply produce policy outcomes "inbetween".

3 The Model of Milesi-Ferretti/Perotti/Rostagno

In this model the country also consists of a large number of citizens i with unit mass. Like in the former model the population is devided twofold. There are three groups $g = A, B, C$ with sizes μ_A, μ_B, and μ_C. With respect to the PR election it is necessary to assume that a group does not consists of more than 50 % and less than 25 % of the population. Additionally one group is assumed to be larger than the other two for this simplifies the PL election analysis. Besides that there are three geographical regions $r = 1, 2, 3$. And there are also two categories of spending: purchases for goods and/or services (local public good) and transfers. The former is related to the regions whereas the latter is related to social groups. Regarding the social groups it is assumed that members of one group g only benefit from transfer s_g but not from transfers assigned to the other groups. Individuals living in region r, of course, only benefit from the local publig good g_r provided to their region. Here the eligibility criteria to receive a transfer is exogenous. This is that an external institution decides on the eligibility to receive a transfer. So e.g. an unemployment subsidy is provided to unemployed individuals with a work history. The utility of an individual i of group g living in region r is

$$U_{i,g,r} = (1-t)^{\alpha_i \beta_i} \; s_g^{\alpha_i(1-\beta_i)} \; g_r^{1-\alpha_i} \tag{10}$$

with t being the proportional tax rate and the income being normalized at 1. The individuals have Cobb-Douglas preferences α on income versus the local public good as well as Cobb-Douglas preferences β on income versus transfer income. The parameters α and β are distributed uniformly within each group with $0 < \alpha < 1$ and $0 < \beta < 1$.

The individuals just described now act as voters and assign one from their midst to be the representative in accordance with the effective electoral system. In a PL system each region is a separate district and elects one representative following plurality. Under the PR system there is one national district encompassing all three regions. Under MMM rule there is a PL election in each district with a simultaneous PR election in one national district. Under MMP rule ... All elected deputies then form the government. This government formation proceeds as follows: One of the elected deputies is randomly chosen to build the government. He also randomly offers government membership to the other deputies until enough members are gathered. If not enough members can be found, the government does not come into existence and hence no spending is being authorised at all. As the government members maximise their utility function they receive a utility of zero if the government does not come into existence.

Government spending is confined by the requirement of a balanced budget. As government members originate from the electorate they also only benefit from the transfer s_g targeted to their group g. Finally the government decides on taxes t, transfers s_g and local public goods g_r. The model is solved backward. At first it is modelled which policies the government members will implement at the second stage and then it is analysed how a group designate its representative at the first stage. Milesi-Ferretti et al. (2002 [2]) show that the individual with median values of α and β is the decisive voter when the deputy is elected within the group.

Under a **majoritarian system** each group in each region elects its representative following plurality rule. Since there are three districts, three seats are assigned. Furthermore it is assumed that one group, let´s say group B, is larger than the other groups and because the population composition is the same in all regions, the three elected representatives belong to group B but come from different regions. Assume that deputies from region 1 and 2 are the government members. Using logs, the government then maximises the utility

$$\begin{aligned} V^{PL} &= (\alpha^e_{r_1}\beta^e_{r_1} + \alpha^e_{r_2}\beta^e_{r_2})\log(1-t) + (\alpha^e_{r_1}(1-\beta^e_{r_1}) + \alpha^e_{r_2}(1-\beta^e_{r_2}))\log s_B \\ &\quad + (1-\alpha^e_{r_1})\log g_1 + (1-\alpha^e_{r_2})\log g_2 \end{aligned} \tag{11}$$

of all its members. The index e indicates an elected deputy. The budget constraint in this setting is

$$t = \mu_B s_B + g_1 + g_2.$$

As can be seen both deputies espouse the transfer s_B but different public goods. Whereas one public good (g_3) is not advocated because of the government formation process two transfers (s_A, s_C) are missing because of the electoral system. Differentiating the objective function with respect to t, s_B, g_1 and g_2 and using $\mu_B s_B = \bar{s}_B$ leads to

$$t^{PL} = \frac{2 - (\alpha^e_{r_1}\beta^e_{r_1} + \alpha^e_{r_2}\beta^e_{r_2})}{2} \tag{12}$$

$$\bar{s}^{PL}_B = \bar{s}^{PL} = \frac{\alpha^e_{r_1}(1-\beta^e_{r_1}) + \alpha^e_{r_2}(1-\beta^e_{r_2})}{2} \tag{13}$$

$$g^{PL}_1 = \frac{1-\alpha^e_{r_1}}{2} \tag{14}$$

$$g^{PL}_2 = \frac{1-\alpha^e_{r_2}}{2} \tag{15}$$

$$g^{PL} = g^{PL}_1 + g^{PL}_2 = \frac{2-\alpha^e_{r_1}-\alpha^e_{r_2}}{2} \tag{16}$$

with r_1, r_2 being the region where the representatives have been elected. The total transfer spending under PL consists only of transfers group B. At the first stage each group simultaneously chooses its representative among its members using majority rule. As the median voter is the decisive one the median of group B in region 1 maximises the utility function

$$\begin{aligned} E_{B,1} = \sum_{n=2}^{3} [&\alpha_m \beta_m \log(1 - t^{PL}(r_1, r_n)) + \alpha_m (1 - \beta_m) \log \bar{s}_B^{PL}(r_1, r_n) \\ &+ (1 - \alpha_m) \log g_1^{PL}(r_1, r_n)] \end{aligned} \tag{17}$$

with respect to α_{r1}^e and β_{r1}^e. The variables t^{PL}, $\bar{s}_B^{PL}$ and g_1^{PL} are given by the equations (12), (13) and (14). This means that the median voter maximises his utility in case of a deputy from region 1 and 2 or in case of deputies from region 1 and 3 being in government. So this utility function covers all possible government compositions. Using the first-order conditions and forcing $\alpha_{r_2}/\alpha_{r_3}$ to equal α_{r_1} and β_{r_2}/β_{r_3} to equal β_{r_1} results in

$$\alpha_{r_1}^e = \frac{\alpha_m}{2 - \alpha_m} \tag{18}$$

and

$$\beta_{r_1}^e = \beta_m \tag{19}$$

which means that the median voter chooses the median value of β but a value of α smaller than the median. Since all potential members of the government will espouse his transfer B he does not need to care about this. With respect to the local good, however, he preferes a representative with a preference toward the local good. Substituting (18) and (19) into (12), (13) and (16) finally leads to

$$t^{PL} = 1 - \frac{\alpha_m \beta_m}{2 - \alpha_m} \tag{20}$$

$$\bar{s}^{PL} = \frac{\alpha_m (1 - \beta_m)}{2 - \alpha_m} \tag{21}$$

and

$$g^{PL} = \frac{2(1 - \alpha_m)}{2 - \alpha_m}. \tag{22}$$

Under a **proportional system** there is one national district in which three representatives are elected. Since representatives get elected depending on their vote share in this national district and because of the assumption that a group has more than 25% but less than 50% of total population each group is represented by an elected lawmaker. Under this rule it is irrelevant from which region they come. The local public good is provided equally in all regions. Following the same government formation process, assuming that deputies from group A and B are government members and using logs, the government then maximises the utility

$$\begin{aligned} V^{PR} = &(\alpha_A^e \beta_A^e + \alpha_B^e \beta_B^e) \log(1 - t) + \alpha_A^e (1 - \beta_A^e) \log s_A + \alpha_B^e (1 - \beta_B^e) \log s_B \\ &+ (2 - \alpha_A^e - \alpha_B^e) \log(g/3) \end{aligned} \tag{23}$$

of all its members. The budget constraint in this setting is

$$t = \mu_A s_A + \mu_B s_B + g.$$

Here the transfer s_C is missing because of the government formation process and the public good is provided equally in the national district. Differentiating the objective function with respect to t, s_A, s_B and g and using $\mu_A s_A = \bar{s}_A$ as well as $\mu_B s_B = \bar{s}_B$ leads to

$$t^{PR} = \frac{2 - (\alpha_A^e \beta_A^e + \alpha_B^e \beta_B^e)}{2} \tag{24}$$

$$\bar{s}_A^{PR} = \frac{\alpha_A^e (1 - \beta_A^e)}{2} \tag{25}$$

$$\bar{s}_B^{PR} = \frac{\alpha_B^e (1 - \beta_B^e)}{2} \tag{26}$$

$$\bar{s}^{PR} = \bar{s}_A^{PR} + \bar{s}_B^{PR} = \frac{\alpha_A^e (1 - \beta_A^e) + \alpha_B^e (1 - \beta_B^e)}{2} \tag{27}$$

$$g^{PR} = \frac{2 - \alpha_A^e - \alpha_B^e}{2} \tag{28}$$

with A and B being the group to which the delegates belong to. As can be seen by comparing the results total spending on public goods and transfers is equal under both regimes if $\alpha_{r_1} = \alpha_A$, $\beta_{r_1} = \beta_A$, $\alpha_{r_2} = \alpha_B$ and $\beta_{r_2} = \beta_B$. Nevertheless the optimal choices of α and β by the median voter are different. The median voter of group A maximises the utility function

$$\begin{aligned} E_A = \sum_{n=B}^{C} [&\alpha_m \beta_m \log(1 - t^{PR}(g_A, g_n)) + \alpha_m (1 - \beta_m) \log \bar{s}_A^{PR}(g_A, g_n) \\ &+ (1 - \alpha_m) \log g^{PR}(g_A, g_n)] \end{aligned} \tag{29}$$

with respect to α_A^e and β_A^e. The variables t^{PR}, $\bar{s}_A^{PR}$ and g^{PR} are given by the equations (24), (25) and (28). Here it means that the median voter maximises his utility in case of a deputy from group A and B or in case of deputies from group A and C being in government. Here also all possible government compositions are encompassed. Using the first-order conditions and forcing α_B/α_C to equal α_A and β_B/β_C to equal β_A results in

$$\alpha_A^e = \frac{\alpha_m (2 - \beta_m)}{1 + \alpha_m (1 - \beta_m)} \tag{30}$$

and

$$\beta_A^e = \frac{\beta_m}{2 - \beta_m} \tag{31}$$

which means that the median voter preferes a deputy with α higher than the median value and a β smaller than the median value. The prefered delegate on the one hand has a preference towards income relative to the local publig good. On the other hand the prefered deputy has a preference toward the transfer relative to income. The final results

$$t^{PR} = \frac{1 + \alpha_m(1 - 2\beta_m)}{1 + \alpha_m(1 - \beta_m)} \tag{32}$$

$$\bar{s}^{PR} = \frac{2\alpha_m(1 - \beta_m)}{1 + \alpha_m(1 - \beta_m)} \tag{33}$$

$$g^{PR} = \frac{1 - \alpha_m}{1 + \alpha_m(1 - \beta_m)} \tag{34}$$

can be achieved by substituting (30) and (31) in (24), (27) and (28). Under the assumption of constant median voters' preferences a direct comparison with the PL results in the equations (20), (21) and (22) shows that spending on goods and services is higher under PL rule whereas spending on transfers is higher under PR rule.

Under a **multi-member majoritarian system** a PL subelection as well as a PR subelection are conducted simultaneously. The voters face a dual ballot. So with respect to the basic model there are some changes necessary here. Firstly I allow for six deputies to get elected as well as four representatives to enter the government. Secondly I need to refrain from a government formation process such that one deputy randomly chooses the other government members. This assumption was made to ensure that the government formation process does not influence the policy outcome. In the mixed cases this assumption precisely would imply an influence as there are two types of deputies in the parliament and the random process could manipulate this composition of this types. Whereas one type comprises the deputies taking a stand for their local good running in the PL subelection the other type gathers all who espouse the transfer running in the PR subelection. Thus I force the government to consist of the two types in the same relation of their appearance in the parliament for the government formation process should not influence policy outcomes. The consequence of differing between the two types is that thirdly the representatives now do not need to be a candidate for both types of spending anymore since the PL subelection represents the provision of the local good and the PR subelection represents the provision of the transfer. Thus the utility function of a candidate running in the PL subelection changes to

$$U = (1 - t)^{\alpha} g_r^{1-\alpha} \tag{35}$$

and the utility function of a candidate running in the PR subelection changes to

$$U = (1 - t)^{\beta} s_g^{1-\beta}. \tag{36}$$

According to the pure systems the elected representatives in the PL subelection all belong to group B and espouse their local good. Thus it is assumed that deputies from region 1 and 2 are in government. In the PR subelection one deputy of each group gets elected. Thus it is assumed that deputies from group A and B are in government. Using logs, the government then maximises the utility

$$V^{MMM} = (\alpha_1^e + \alpha_2^e + \beta_A^e + \beta_B^e)\log(1-t) + (1-\beta_A^e)\log s_A + (1-\beta_B^e)\log s_B \\ + (1-\alpha_1^e)\log g_1 + (1-\alpha_2^e)\log g_2 \tag{37}$$

of all its members. The budget constraint in this setting is

$$t = \mu_A s_A + \mu_B s_B + g_1 + g_2.$$

4 Concluding Remarks

In this paper I present an extension of an electoral competition model with respect to a multi-member majoritarian electoral system so far. It suggests that combining the pure formulas in a MMM election results in a rather tough competition for the marginal district, no matter what the initial formula is. In this sense the mix of the pure electoral formulas does not simply produce policy outcomes "inbetween" like the originators of mixed systems had in mind with respect to political variables like political stability or the degree of representation.

The very next step is the extension of the model of Milesi-Ferretti which currently is in progress. I am full of confidence that this will allow an appropriate modeling of a MMP election.

The subsequent paper will conduct an empirical testing of the theoretical results.

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Appendix 1 - Persson/Tabellini model - Pure electoral systems

$$L = \sum_k \phi^2 \left[n^k U(y(1-\tau_A)) + (1-n^k)U(f) + H(g^2)\right] + \lambda \left[ny\tau_A - (1-n)f - \frac{1}{3}\sum_r g^r\right]$$

$$L = \sum_k \sum_r \phi^r \left[n^k U(y(1-\tau_A)) + (1-n^k)U(f) + H(g^r)\right] + \lambda \left[ny\tau_A - (1-n)f - \frac{1}{3}\sum_r g^r\right]$$

Appendix 2 - Persson/Tabellini model - Mixed-member election

$$\begin{aligned} L = &\sum_k \phi^2 \left[n^k U(y(1-\tau_A)) + (1-n^k)U(f) + H(g^2)\right] \\ &+ \sum_k \sum_r \phi^r \left[n^k U(y(1-\tau_A)) + (1-n^k)U(f) + H(g^r)\right] \\ &+ \lambda \left[ny\tau_A - (1-n)f - \frac{1}{3}\sum_r g^r\right] \end{aligned}$$

$$\frac{\delta L}{\delta \tau_A} \quad \rightarrow \quad \lambda = \frac{1}{n} \cdot \frac{\delta U}{\delta c} \cdot \left(\phi^2 \cdot \sum n^k + \sum \phi^r \cdot \sum n^k \right)$$

mit $n = \frac{\sum n^k}{3}$ folgt

$$\lambda = \frac{\delta U}{\delta c} \cdot (3\phi^2 + 3 \sum \phi^r)$$

$$\frac{\delta L}{\delta f} \quad \rightarrow \quad \lambda = \frac{1}{(1-n)} \cdot \frac{\delta U}{\delta f} \cdot \left(\phi^2 \cdot \sum (1-n^k) + \sum \phi^r \cdot \sum (1-n^k) \right)$$

mit $n = \frac{\sum n^k}{3}$ und $\sum(1-n^k) = 3 - \sum n^k$ folgt

$$\lambda = \frac{\delta U}{\delta f} \cdot (3\phi^2 + 3 \sum \phi^r)$$

$$\frac{\delta L}{\delta g^1} \quad \rightarrow \quad \lambda = 9 \cdot \phi^1 \cdot \frac{\delta H}{\delta g^1}$$

$$\frac{\delta L}{\delta g^2} \quad \rightarrow \quad \lambda = 18 \cdot \phi^2 \cdot \frac{\delta H}{\delta g^2}$$

$$\frac{\delta L}{\delta g^3} \quad \rightarrow \quad \lambda = 9 \cdot \phi^3 \cdot \frac{\delta H}{\delta g^3}$$

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