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Submission Number: PET11-11-00282

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Submitted: March 15, 2011.

Group Contest with Internal Conflict and Power Inequality*

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This Draft: Mar 14. 2011

Abstract

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*We appreciate the useful comments of Roman Inderst, Martin Kolmar and the participants at the Cesifo Area Conference on Public Sector Economcis, the 2010 Young Researchers Workshop on Contests and Tournaments at the University of Magdeburg, the XXth Annual Conference of Jadavpur University, and the conference on "Tournaments, Contests and Relative Performance Evaluation" at North Carolina State University. Any remaining errors are our own.

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1 Introduction

This article studies the issue of simultaneous inter- and intra-group conflict in the shadow of power inequality between group members. There are instances in which groups engage in costly conflicts in order to win a reward while at the same time the group members confront each other in order to divide the reward among members of the group. When two or more individuals within a group work collectively toward certain goal, they often encounter conflict from another group which has a similar interest. At the same time, individuals often confront intra-group conflict regarding the division of the prize..[1]

There are many examples. Interest groups compete for rents from government policies while individuals within the interest group, who may have unequal powers within that group, contest for the spoils of the victory. Firms producing a system good as complements have to compete against another system and also have to divide profits among themselves. The same issue arises in joint R&D ventures. Countries in an alliance conflict against another alliance and they also have to decide how to share the burden of costs. The logic works in the same way for the parties within a political alliance. Even in nature, many species compete for limited resources within and between species simultaneously. One important application of the theory is sperm competition under polyandry. There are cases in which sperms from a male compete with each other in fertilizing the ovum, but at the same time they extract enzymes or take other collective actions that damage (at least the likelihood of success of) sperms of other males (Baker, 1996; Buckland-Nicks, 1998). In these examples, the nature of internal conflict simultaneously characterizes the shape of external conflict, in particular, through collective action between players within a group.[2]

[1]This area of literature dates back to Olson (1965) and later developed by Becker (1983), Palfrey and Rosenthal (1983), Katz et al. (1990), Hardin (1995) among others. This can also be interpreted as the collective action problem in two potentially important environments: competition between groups and internal conflict within a group. Please see Sandler (1992), Ostrom (2000) and Sandler and Hartley (2001) for the literature review.

[2]See Münster (2007) and Münster and Staal (2010, forthcoming) for more examples.

We analyze how the inter-group conflict interacts with the intra-group conflict in these environments with special emphases on the power inequality between the group members and complementarity in collective action. We use a standard Tullock (1980) type contest mechanism in each conflictive situation and include heterogeneity between group members in terms of power inequality. We define a stronger player in a group as the one who ex-ante has a higher probability of winning the internal conflict. We take into account the importance of complementarity in collective action by considering a Constant Elasticity of Substitution (CES) group impact function. Specifically, we consider two special and contrasting cases of the additive and the weakest-link group impact functions. Consequently, the interplay between internal and external conflict turns out to be a key feature in the analysis of the overall contest.

There are two contrasting conventional wisdoms about the question of how each group's internal conflict influences its chance of winning in external conflict. One view suggests that a group with less internal conflict has an advantage in external conflict against a rival group (Deutsch, 1949). The other view is that intra-group conflict is more conducive in eliciting efforts from group members for external conflict (Lüschen, 1970). We show that both views have some validity by clarifying the interaction between inter-group and intra-group contests. Furthermore, we ask the following questions. Does internal conflict matter in group members' collective action for external conflict? Since the players are heterogeneous in terms of their within-group power, which player is better off within a group? How significantly can the degree of power inequality change the total rent dissipation?

The severity of internal conflicts within groups is measured in terms of the rate of rent dissipation within intra-group conflicts. Not surprisingly, as group members have similar power, internal conflict is more severe. In this sense, a more (less) conflictive group is defined as one in which the power inequality is less (more). We find that a more (less) conflictive group expends more effort in the inter-group conflict, in particular when collective action requires complementary (substitutable) works between group members. This is because each member's incentive to contribute to collective action depends on one's equilibrium share of the prize in the internal conflict. Thus, when we compare the weaker individuals within groups, the individual in a more conflictive group is willing to contribute to collective action more than the one in a less conflictive group. The same logic holds for the stronger players. As a result, if individiuals' efforts are relatively complementary in impacting the collective action, a more conflictive group face a free-rider problem less severely in terms of not expending enough effort in collective action.

The answer to the question about whether a stronger player is better off than a weaker player within a group is not straightforward. Although the stronger player can dominate the weaker player in internal conflict and have a larger share of the prize, the stronger player has to contribute more to collective action for external conflict. This, in turn, results in a non-monotonic relationship between the power inequality and the players' payoffs. This has implications in the issues of coalition formation. The decision to join a coalition depends crucially on the expected payoff. In the case of possible inter- and intra-group conflicts, the relationship between the power asymmetry and the corresponding payoff can be the major driving force for coalition formation.

A similar outcome holds for the relationship between the power inequality and the rent dissipation. The total rent dissipation is also not monotone with the power inequality and, interestingly, is minimized when the players are symmetric. This result contrasts starkly to the standard contest models. As the players are more heterogeneous, the total rent dissipation gets smaller in a single contest model. However, when the players are engaged in multi-contest (inter- and intra-group contests), the total rent dissipation can increase with the heterogeneity of the players.

Most analyses in the group contest literature focus on issues related to contest design that maximizes overall contest effort by looking at different impact functions, cost structures and value distributions. This area of literature originated with Katz et al. (1990). The authors use a group impact function in which the group members' efforts are perfectly substitutable. The group effort is entered in a Tullock contest success function and the winner group is decided. They show that the equilibrium group rent dissipation is unique, however one faces multiple equilibria in terms of individual equilibrium efforts. Baik (1993, 2008) generalizes the analysis by introducing asymmetric valuation within group. He shows that the equilibrium rent dissipation by a group depends crucially on the distribution of prize valuation and not on the group size. However, Katz et al. (1990) and Baik (1993, 2008) do not model any internal conflict within the groups.[3]

Our model is closely related to the study by Münster (2007) in which a simultaneous inter- and intra-group contest with Tullock contest success function is analyzed. In this paper, the group impact function is modeled with a simplified CES function, and the players are resource constrained. The players can allocate their resources into production, inter-group conflict and intra-group conflict. The prize value is determined endogenously by total effort expended on production. The main finding of the paper is the group (or reverse group) cohesion effect which stands for a negative correlation between the intensity of inter-group conflict and that of intra-group conflict. Münster (2007) further studies the optimal group size and the optimal number of groups from a contest designer perspective. On the other hand, our paper addresses the very different issue of the impact of the heterogeneity within groups and complementarity on inter-group conflict.[4]

The remainder of the paper is organized as follows. Section 2 lays out the basic features of the model. Section 3 characterizes the equilibria for internal conflict within groups and for external conflict between groups. Then, in Sections 4 to 6 we analyze the effect of the power inequality on the probability of winning, the players' equilibrium payoffs, and the equilibrium rent dissipation. We conclude in Section 7 by discussing the possible extensions.

2 Model: Collective Action and Conflict Technologies

There are two groups, A and B, that contest for a prize whose common value is given by R. Each group G consists of two risk-neutral players, $G1$ and $G2$, where $G = A, B$. The way the prize is allocated between the two groups depends on the relative collective efforts put forth by each group. A group's share of the prize is further contested by the members of each group

[3] A series of analyses including Nitzan (1991), Katz and Tokatlidu (1996), Wärneryd (1998), Esteban and Ray (2001), Konrad (2004), Niou and Tan (2005), Münster (2007), Inderst et al. (2008), Cheikbossian (2008), Lee (2009), Kolmer and Rommeswinkel (2010) among many others, study the problem of group contests.

[4] Unlike Münster (2007), in our model, the concept of the group cohesion effect is not decisive and it depends crucially on the nature of the group impact function as well as the power distribution.

simultaneously. Thus, members of the same group have a common interest and cooperate in external contest against the rival group, but they are competitors against each other in the division of the spoils. Each player chooses two different non-negative effort: contributing to collective activity for inter-group conflict and contesting a given share of the prize within the group. A player i in group A (B) allocates a_i (b_i) units of effort toward internal conflict and α_i (β_i) units of effort for collective action toward external conflict. We assume complete information and that all players make their decisions simultaneously.

Internal Conflict. Contrary to a substantial part of the literature, we assume players within a group to be heterogeneous by ability or power, where power is defined in terms of advantage conferred in internal conflict.[5] Without any loss of generality, we designate player 1 of each group to be the one who has more power and thus has advantage in internal conflict. This advantage is embedded in the conflict technology. Let $p(x_1, x_2)$ be the probability that player 1 wins in the internal contest when x_1 and x_2 are the internal effort levels exerted by player 1 and 2. Under risk neutrality, it can also be interpreted as the share that player 1 receives. Then, internal conflict is resolved by a Tullock (1980) type contest. The contest success function in group G is given by

$$\begin{aligned} p(x_1, x_2; \theta_G) &= \frac{f(x_1)}{f(x_1) + \theta_G f(x_2)}, \text{ if } x_1 + x_2 \neq 0; \frac{1}{2}, \text{ otherwise} \\ \text{where } f(x_i) &= x_i^m, \ \theta_G \in [0,1], \text{ and } G = A, B. \end{aligned}$$

The probability that player 2 wins is simply $1 - p$. The parameter θ_G represents asymmetry in power distribution within group G, with a higher θ_G implying a more even power distribution.[6] One way to interpret this function is that player 1 has some advantage within the group in terms of education, experience, incumbency, technology etc. For instance, if θ_G

[5] Another way of incorporating asymmetry is the asymmetry in valuation. To our knowledge Baik (1993) is the first to analyze asymmetric valuation in a Tullock type group contest. However, this analysis does not consider internal conflict. Follow-up analyses of Baik (2008), Lee (2009) and Kolmer and Rommeswinkel (2010) also concentrate only on inter-group contest with asymmetric valuation under different impact functions. Konrad (2004) and Baik et al. (2001) analyze similar setting under an all-pay auction CSF.

[6] This was introduced by Gradstein (1995). Please see Skaperdas (1996) and especially Clark and Riis (1998) for axiomatization of this type of contest success function. We impose the condition $m \in (0, 2)$ to ensure the existence of equilibrium in pure strategies.

= 1, the power is evenly distributed between the two players, whereas if $\theta_G = 0$, all the power in internal conflict is possessed by player 1 with $p(x_1, x_2; \theta_G) = 1$. We refer to player 1 as the stronger player and player 2 as the weaker player. We also refer to an increase in θ_G as the dispersion of power and a decrease in θ_G as the concentration of power.

Collective Action. $F(y_1, y_2) : \boldsymbol{R}_+^2 \rightarrow \boldsymbol{R}_+$ is referred to as an impact function that represents collective action of a group in external conflict when the stronger and weaker players contribute y_1 and y_2, respectively. In the major part of the existing literature, collective action is assumed to be a sum of each individual's effort. This assumption boils down to a perfect substitute impact function and ignores any possibility of complementary effects in collective action. However, there are a wide variety of situations in which collective action cannot be treated as the sum of individual members' effort.[7] Lee (2009), Kolmer and Rommeswinkel (2010), and Chowdhury, Lee and Sheremeta (2011) are the studies that analyze different impact functions other than perfect substitutes. However, all of the studies concentrate on inter-group contest and do not endogenize the intra-group prize sharing rule.

To our knowledge, the only study that uses a generalized impact function and endogenizes within-group prize share rule in a simultaneous decision making setting is by Münster (2007). This is later axiomatized by Münster (2009). We also follow the axiomatic structure of Münster (2009), and extend collective action from the additive functional form to a general CES impact function.

$$F(y_i, y_j) = \left[y_i^r + y_j^r\right]^{\frac{1}{r}},$$

From the properties of a CES function, one can note that (i) $F(y_i, y_j)$ is concave, $F_i(y_1, y_2) \geq 0$, $F_{ii}(y_1, y_2) \leq 0$, and $F_{ij}(y_1, y_2) \geq 0$, where $i, j = 1, 2$ with $i \neq j$ and the subscripts indicate partial differentiation. Hence, collective action is increasing in each member's contribution,

[7] For example, Scully (1995) states "[p]layers interact with one another in team sports. The degree of interaction among player skills determines the nature of the production function." Also, in the early literature of voluntary contributions to a public good, Hirshleifer (1983) studies the case that the aggregate effort level can be the smallest contribution within a group, which is assuming the perfect complements between individuals' efforts. The paper acknowledges a possible complementary effect in collective action. However, the effect of complementary efforts between group members has not been thoroughly studied in a model of group contests. Borland (2007) also argues that while the production function in baseball is nearly additive in the sense that hitting and pitching are separate activities, players' efforts are almost perfect complements in American football. Please see Konrad (2009) chapters 5.5 and 6.3 for detailed discussion in this.

but at a diminishing rate. (ii) This impact function is designed to have a constant returns to scale. (iii) $r \in (-\infty, 1]$ represents the degree of complementarity between individuals' efforts.

External Conflict. The group impact function (external conflict technology) is also assumed to be driven by a Tullock (1980) type CSF. The crucial point here is that the group impact function depends on collective contributions by individual members of each group. Let $q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))$ denote the probability that group A wins in external conflict. Hence:

$$q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2)) = \frac{F(\alpha_1, \alpha_2)}{F(\alpha_1, \alpha_2) + F(\beta_1, \beta_2)}, \text{ if } F(\alpha_1, \alpha_2) + F(\beta_1, \beta_2) \neq 0; \frac{1}{2}, \text{ otherwise}$$

The probability that group B wins is simply $1 - q$. To economize on notation, we will often use $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2)$. For instance, $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta})) = \frac{F(\boldsymbol{\alpha})}{F(\boldsymbol{\alpha})+F(\boldsymbol{\beta})}$.

3 Equilibrium Analysis

3.1 Internal Conflict within Groups

The players in group A maximize the objective functions represented by

$$\begin{aligned} V_{A1} &= p(a_1, a_2; \theta_A) q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - a_1 - \alpha_1 \\ V_{A2} &= [1 - p(a_1, a_2; \theta_A)] q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - a_2 - \alpha_2. \end{aligned}$$

Our formulation assumes that each player makes a decision on his choice of effort in internal and external conflicts simultaneously. It is particularly useful if we interpret $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ as the probability that group A wins in a winner-take-all external contest. However, if we take the alternative, non-probabilistic interpretation of $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ as the share of A's contested resource, the analyses will still work. In the following we will interpret our results in terms of winner-take-all probability.

Similarly, the objective functions for the players in group B are given by

$$\begin{aligned} V_{B1} &= p(b_1, b_2; \theta_B)[1 - q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))]R - b_1 - \beta_1 \\ V_{B2} &= [1 - p(b_1, b_2; \theta_B)][1 - q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))]R - b_2 - \beta_2. \end{aligned}$$

We first derive an invariance result that each player's winning probability in their internal conflict $(p, 1-p)$ is independent of the level of their contributions to external conflict $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. The equilibrium probability of winning and losing in internal conflict is constant and depends only on the respective group's power distribution parameter θ_G. This result, summarized in the following Lemma, considerably simplifies our analysis.

Lemma 1 *In equilibrium, both the stronger and weaker players of group G choose the same level of efforts for internal conflict ($a_1^* = a_2^*$ and $b_1^* = b_2^*$). As a result, the winning probabilities for the stronger and weaker players depend only on θ_G; $p(a_1^*, a_2^*; \theta_A) = \frac{1}{1+\theta_A}$ and $p(b_1^*, b_2^*; \theta_B) = \frac{1}{1+\theta_B}$.*

Proof. The first order conditions with respect to internal conflict in group A are given by

$$\begin{aligned} \frac{\partial V_{A1}}{\partial a_1} &= \frac{\theta_A f'(a_1) f(a_2)}{[f(a_1) + \theta_A f(a_2)]^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))R - 1 = 0 \\ \frac{\partial V_{A2}}{\partial a_2} &= \frac{\theta_A f'(a_2) f(a_1)}{[f(a_1) + \theta_A f(a_2)]^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))R - 1 = 0. \end{aligned}$$

The first-order conditions can be summarized by

$$\frac{f(a_1)}{f'(a_1)} = \frac{f(a_2)}{f'(a_2)} = \frac{\theta_A}{(1+\theta_A)^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))R. \tag{1}$$

Since $\frac{f(x)}{f'(x)}$ is strictly increasing in x, condition (1) implies that both the players in group A choose the same level of efforts, $a_1^* = a_2^*$ for internal conflict regardless of their possibly different choice of α_1 and α_2 for external conflict. By proceeding in a similar manner, we can also derive that

$$\frac{f(b_1)}{f'(b_1)} = \frac{f(b_2)}{f'(b_2)} = \frac{\theta_B}{(1+\theta_B)^2} [1 - q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))]R \tag{2}$$

This implies that $b_1^* = b_2^*$, i.e., both the stronger and weaker players in each group exert the same level of effort for the internal conflicts. However, the total effort spent on internal conflict can be different for each group. The equilibrium conditions (1) and (2) lead us to the result that the stronger player's winning probabilities in Group A's internal conflict is $p(a_1^*, a_2^*; \theta_A) = \frac{1}{1+\theta_A}$ and the same for the weaker player is $1 - p(a_1^*, a_2^*; \theta_A) = \frac{\theta_A}{1+\theta_A}$. A similar result holds for group B internal conflict with $p(b_1^*, b_2^*; \theta_B) = \frac{1}{1+\theta_B}$. ■

To investigate the relationship between the rent dissipated (equilibrium effort expended) in internal conflict and the power distribution within each group, let us define

$$\lambda_A = \frac{a_1^* + a_2^*}{q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R} \text{ and } \lambda_B = \frac{b_1^* + b_2^*}{[1 - q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))]R}.$$

The denominator of λ_G represents the expected value of the collective prize for group G in the external conflict whereas the numerator of λ_G is the total effort expended in internal conflict. Thus, λ_G is the equilibrium rate of rent dissipation in internal conflict. It measures the level of resources used up for internal conflict *relative* to the expected value of collective prize for group G. The next lemma shows that the group with less power-inequality dissipates proportionately more rent out of their expected group prize in internal conflict. In this sense, the group with more even power distribution is more conflictive.

Lemma 2 $\lambda_A \gtreqless \lambda_B$ as $\theta_A \gtreqless \theta_B$.

Proof. Putting $\frac{f(x_i)}{f'(x_i)} = \frac{x_i}{m}$ into equation (1) and (2), we immediately obtain $\lambda_A = \frac{\theta_A}{(1+\theta_A)^2}\frac{2m}{w}$ and $\lambda_B = \frac{\theta_B}{(1+\theta_B)^2}\frac{2m}{w}$. Given $\theta_A \gtreqless \theta_B$, we must have $\frac{\theta_A}{(1+\theta_A)^2} \gtreqless \frac{\theta_B}{(1+\theta_B)^2}$. ■

Without loss of generality, for the rest of the paper, we assume $\theta_A \leq \theta_B$, i.e., the power is more asymmetrically distributed in group A than in group B. This implies that player 1 in group A has relatively more power than his counterpart in group B vis-a-vis their respective player 2's. If the power is more asymmetrically distributed in group A than in group B, then group B is more conflictive than group A.[8] The severity of internal conflict within a group

[8] This, however, does not necessarily mean that members in group B spend more resource for internal conflict. Since the total efforts depend on the size of contestable prize, people in group A may expend more efforts if group A's winning probability is much larger in the external conflict.

depends on the distribution of power across individual group members. As individual group members have similar power, they compete more aggressively.

3.2 External Conflict between Groups

Now let us study how the inter-group contest is shaped by the intensity of internal conflict and the distribution of power within each group. With the invariance result from *Lemma 1*, we can now state each player's objective function in relation to their contribution to external conflict.[9] For notational simplicity, we denote the equilibrium probability that player 1 wins in group G by $p(\theta_G)$. For group A members the payoff can be written as follows.

$$\begin{aligned} V_{A1} &= p(\theta_A)q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - a_1^* - \alpha_1 \\ V_{A2} &= (1 - p(\theta_A))q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - a_2^* - \alpha_2. \end{aligned}$$

For external conflict, player i in group A maximizes his payoff function V_{Ai} by choosing α_i, where $i = 1, 2$, given that all players act optimally. One can derive similar conditions for group B members who choose β_i; and the first-order conditions can be expressed as

$$\frac{F_1(\boldsymbol{\alpha})F(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}R = \frac{1}{p(\theta_A)} = (1 + \theta_A), \tag{3}$$

$$\frac{F_2(\boldsymbol{\alpha})F(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}R = \frac{1}{1 - p(\theta_A)} = (\frac{1 + \theta_A}{\theta_A}), \tag{4}$$

$$\frac{F(\boldsymbol{\alpha})F_1(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}R = \frac{1}{p(\theta_B)} = (1 + \theta_B), \text{ and} \tag{5}$$

$$\frac{F(\boldsymbol{\alpha})F_2(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}R = \frac{1}{1 - p(\theta_B)} = (\frac{1 + \theta_B}{\theta_B}). \tag{6}$$

They can be further manipulated and summarized in the following way.

[9] One may think that the role of power disparity in this model is providing the exogenous division rule of the prize. This is not true because the players' effort levels are important in our comparison of the equilibrium payoffs and the rent-dissipation.

$$\begin{aligned}
\frac{F_1(\boldsymbol{\alpha})}{F_2(\boldsymbol{\alpha})} &= \frac{1-p(\theta_A)}{p(\theta_A)} = \theta_A, & (7)\\
\frac{F_1(\boldsymbol{\beta})}{F_2(\boldsymbol{\beta})} &= \frac{1-p(\theta_B)}{p(\theta_B)} = \theta_B, & (8)\\
\frac{F_1(\boldsymbol{\alpha})}{F_1(\boldsymbol{\beta})}\frac{F(\boldsymbol{\beta})}{F(\boldsymbol{\alpha})} &= \frac{p(\theta_B)}{p(\theta_A)} = \left(\frac{1+\theta_A}{1+\theta_B}\right), \text{ and} & (9)\\
\frac{F_2(\boldsymbol{\alpha})}{F_2(\boldsymbol{\beta})}\frac{F(\boldsymbol{\beta})}{F(\boldsymbol{\alpha})} &= \frac{1-p(\theta_B)}{1-p(\theta_A)} = \left(\frac{\theta_B}{\theta_A}\right)\left(\frac{1+\theta_A}{1+\theta_B}\right) & (10)
\end{aligned}$$

Equations (7) and (8) tell us the relationship between the *marginal* contributions of players 1 and 2 in the generation of collective action in each group. In each group, the weaker player's equilibrium marginal contribution to the collective action is greater than the stronger player's. This is because the player with less internal power is expected to receive a smaller share of the prize in external contest.

This asymmetry in the relative marginal contributions of the two players translates into the asymmetry in the relative total contributions. Each player's incentive to contribute to collective action depends on one's equilibrium power in the internal conflict. The relative contribution of player 1 is greater in the less conflictive group A. This result leads us to the following result.

Proposition 1 $\frac{\alpha_1^*}{\alpha_2^*} \geq \frac{\beta_1^*}{\beta_2^*}$ *as* $\theta_A \leq \theta_B$ *i.e.,* $p(\theta_A) \geq p(\theta_B)$. *The stronger player's relative contribution to external conflict vis-a-vis the weaker player's is higher in group A where power distribution is relatively more asymmetric.*

Proof. $F(y_i, y_j)$ is a homothetic function. α_1^* (β_1^*) must have a linear relationship with α_2^* (β_2^*). This means that the slopes of the level sets of $F(y_i, y_j)$ are the same along rays coming from the origin. Let us define those as $s_A = \alpha_2^*/\alpha_1^*$ and $s_B = \beta_2^*/\beta_1^*$. Equations (7) and (8) can be written as

$$\frac{F_1(1,\alpha_2^*/\alpha_1^*)}{F_2(1,\alpha_2^*/\alpha_1^*)} = \frac{F_1(1,s_A)}{F_2(1,s_A)} \leq \frac{F_1(1,s_B)}{F_2(1,s_B)} = \frac{F_1(1,\beta_2^*/\beta_1^*)}{F_2(1,\beta_2^*/\beta_1^*)},$$

because $F_i(y_i, y_j)$ and $F_j(y_i, y_j)$ are homogeneous degree of 0. Note that $\frac{F_1(1,s_G)}{F_2(1,s_G)}$ is increasing in s_G under $F_{jj}(y_i, y_j) < 0$ and $F_{ij}(y_i, y_j) > 0$ as follows.

$$\frac{\partial}{\partial s_G}\left[\frac{F_1(1, s_G)}{F_2(1, s_G)}\right] = \frac{F_{12}(1, s_G)F_2(1, s_G) - F_1(1, s_G)F_{22}(1, s_G)}{[F_2(1, s_G)]^2} \geq 0.$$

Therefore we must have $s_A = \alpha_2^*/\alpha_1^* \leq \beta_2^*/\beta_1^* = s_B$. ■

One important implication of this result is that the two groups exhibit different patterns of inefficiency. Clearly, the generation of collective action in each group is inefficient, because efficiency requires that an individual is compensated with full marginal return of one's effort. It is easy to note that the inefficiency in terms of player 2 (player 1) is more pronounced for group A (B) in which the internal power distribution is more asymmetric (symmetric).

4 Win Probability in External Conflict

A basic, but unanswered, question is which group has a higher winning probability in external conflict. We can answer this question by figuring out whether $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ is greater than 1/2 or not. This is equivalent to whether $\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}$ is greater than 1 or not. Equations (9) and (10) together result in

$$\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} = \left(\frac{1+\theta_B}{1+\theta_A}\right)\left(\frac{\theta_A}{\theta_B - \theta_A}\right)\left[\frac{F_2(\alpha_1^*, \alpha_2^*)}{F_2(\beta_1^*, \beta_2^*)} - \frac{F_1(\alpha_1^*, \alpha_2^*)}{F_1(\beta_1^*, \beta_2^*)}\right]. \tag{11}$$

This shows that the answer hinges on the ratio of marginal contributions between stronger and weaker players in equilibrium. Thus, the way in which collective action is generated through individual contributions is crucial to predicting with group will win. In addition, it is worthwhile to study how each group's winning probability is changed by the distribution of power within a group.

An important factor in collective action is a possible complementarity between individual members' contributions. The degrees of complementarity can be measured by r in the CES group impact function. As is well-known, the elasticity of substitution is

$$\frac{d\ln(y_2/y_1)}{d\ln MRS} = \frac{1}{1-r},$$

which is a measure of the degree of complementarity or substitutability between individual members' contributions. As r increases, the contributions of the two players in the same group become less complementary (more substitutable).[10] In the next proposition we derive the relationship between the properties of the group impact function and the group winning probability.

Proposition 2 *When the group impact function is given as* $F(y_i, y_j) = (y_i^r + y_j^r)^{\frac{1}{r}}$, *then the ratio of collective action between the two groups is given by*

$$\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} = \left[\frac{p(\theta_A)^\rho + (1-p(\theta_A))^\rho}{p(\theta_B)^\rho + (1-p(\theta_B))^\rho}\right]^{\frac{1}{\rho}}, \text{ where } \rho = \frac{r}{1-r}.$$

Thus, $F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ *as* $r \gtreqless 1/2$. *If the individuals' contributions are relatively complementary in the generation of collective action, the winning probability of more conflictive group is greater, and vice versa.*

Proof. Equation (7), (8), (9), and (10) correspond to

$$\frac{\alpha_1}{\alpha_2} = \left(\frac{1-p(\theta_A)}{p(\theta_A)}\right)^{\frac{1}{r-1}}, \tag{5'}$$

$$\frac{\beta_1}{\beta_2} = \left(\frac{1-p(\theta_B)}{p(\theta_B)}\right)^{\frac{1}{r-1}}, \tag{6'}$$

$$\left(\frac{\alpha_1}{\beta_1}\right)^{r-1}\frac{\beta_1^r+\beta_2^r}{\alpha_1^r+\alpha_2^r} = \frac{p(\theta_B)}{p(\theta_A)} \text{ and} \tag{7'}$$

$$\left(\frac{\alpha_2}{\beta_2}\right)^{r-1}\frac{\beta_1^r+\beta_2^r}{\alpha_1^r+\alpha_2^r} = \frac{1-p(\theta_B)}{1-p(\theta_A)}. \tag{8'}$$

Putting equation (5') and (6') into (7'), we obtain $(\frac{\alpha_1}{\beta_1}) = \frac{p(\theta_B)}{p(\theta_A)} \cdot \frac{1+\left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1+\left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}$. Plugging this

[10] While we obtain the most popular, additive (perfect substitutes, i.e., no complementarity) impact function as r approaches 1, we obtain the weakest link (perfect complements) impact function as r approaches $-\infty$.

into (7') again, we get $\frac{\alpha_1^r+\alpha_2^r}{\beta_1^r+\beta_2^r} = \left(\frac{1+\left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1+\left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}\right)^{1-r} \left(\frac{p(\theta_A)}{p(\theta_B)}\right)^r$. Using this, we can further manipulate the equation as follows.

$$\begin{aligned}\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)} &= \left(\frac{\alpha_1^r+\alpha_2^r}{\beta_1^r+\beta_2^r}\right)^{\frac{1}{r}} = \left(\frac{1+\left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1+\left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}\right)^{\frac{1-r}{r}} \left(\frac{p(\theta_A)}{p(\theta_B)}\right) \\ &= \left(\frac{p(\theta_A)^{\frac{r}{1-r}}+(1-p(\theta_A))^{\frac{r}{1-r}}}{p(\theta_B)^{\frac{r}{1-r}}+(1-p(\theta_B))^{\frac{r}{1-r}}}\right)^{\frac{1-r}{r}}\end{aligned}$$

Now, $F(\alpha_1^*,\alpha_2^*) \gtreqless F(\beta_1^*,\beta_2^*)$ is comparable to $p(\theta_A)^\rho+(1-p(\theta_A))^\rho \gtreqless p(\theta_B)^\rho+(1-p(\theta_B))^\rho$. Let us define the function,

$$g(x) = x^\rho + (1-x)^\rho, \text{ where } x \geq 1/2.$$

This function is increasing in $\rho > 1$ and decreasing in $\rho < 1$, because $g'(x) = \rho(x^{\rho-1}-(1-x)^{\rho-1})$. Note that ρ must be greater than 0 for $r < 1$. Therefore, since $p(\theta_A) > p(\theta_B)$, $F(\alpha_1^*,\alpha_2^*) \gtreqless F(\beta_1^*,\beta_2^*)$ must correspond to $\rho \gtreqless 1$, which is again equivalent to $r \gtreqless 1/2$. ■

At first sight, the result in *Proposition 2* appears to be counter-intuitive. Under circumstances in which collective action requires complementary efforts, the individuals in the more conflictive group contribute to collective action more than in the less conflictive group. Conventional wisdom advises that conflict harms cooperation. However, our result implies that conflict and cooperation can coexist well, in particular, in situations of complementary collective action.

In the case of military alliances, the individuals' contributions are more likely to be substitutable. Thus, our model predicts that leadership in each alliances matters in external conflict. In fact, in the Cold War era, superpower dominance was at issue in the two military alliances, NATO and the Warsaw Pact. On the other hand, in the case of business alliances, the individuals' contributions are more likely to be complementary. For example, the complementarity between the distribution capability and the manufacturing skill is one of the most popular reasons for the strategic alliance. Thus, the size or market power of partners needs to be similar for a strong alliance to form.

A solution for $\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}$ enables us to conduct comparative statics in terms of power distribution to study whether the internal redistribution of power increase or decrease the group's winning probability. One famed argument by Olson (1965) in the context of public goods is that the redistribution of wealth in favor of inequality can make individuals contribute to collective action more, because an individual who gains a significant proportion of total benefits from public goods has more incentive to contribute. We, however, study this issue in terms of power distribution in a group contest. The following *Corollary* is derived immediately from the last *Proposition*.

Corollary $\frac{\partial}{\partial\theta_A}\left(\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}\right) \lesseqgtr 0$ *and* $\frac{\partial}{\partial\theta_B}\left(\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}\right) \gtreqless 0$ *as* $r \gtreqless 1/2$*; i.e., the dispersion of power inequality increases (decreases) the group's probability of winning when the individuals' contributions is relatively substitutable (complementary).*

This result confirms the intuition of Olson (1965) in a general setting. He argues that more inequality can facilitate collective action in a public good setting when collective action is defined as the sum of individuals' efforts ($r = 1$). We show that more inequality in terms of power can facilitate collective action even in a group contest setting and it is valid for any $r \geq 1/2$. In contrast, it should also be emphasized that the result can be sharply reversed if the individuals' contribution is relatively complementary as the case of $r < 1/2$.

Depending on the impact function, when individuals' efforts are relatively substitutable or the stronger player in the group plays a significant role in the collective action, the redistribution of power towards the stronger player facilitates collective action. This result is consistent with Olson's argument, because the driving force is that stronger individuals have more incentives to contribute to collective action. By contrast, this result is sharply reversed for the specific impact functions in which individuals' efforts are relatively complementary or the weaker player turns out to be the important player in generating collective action. Thus, in this case, a more equal distribution of power fosters collective action. In addition, the power distribution in a rival group gives the idea of a fierce or a milder conflict and this affects the amount of collective action in a similar way.

5 Who is better off: the Stronger or Weaker Player?

In this section, following the literature, we restrict our attention to the two relatively simple and contrasting cases, perfectly substitutable (additive effort) and perfectly complementary (weakest link effort) group impact functions to compare the equilibrium effort levels and payoffs. In the case of additive effort group impact function, cooperation is performed by the sum of individual group members' efforts, i.e., $F(y_i, y_j) = y_i + y_j$. In the case of weakest link group impact function, the minimum effort among individual group members establishes the level of collective action, i.e., $F(y_i, y_j) = \min\{y_i, y_j\}$. We can compute the equilibrium effort levels in external conflict as follows. The following results, summarized in the next proposition, holds under the needed participation constraints - i.e., the players earn non-negative payoff in the equilibria. The proof of this proposition comes directly from Baik (1993) and Lee (2009) and is not included here. For expositional simplicity, the results are represented in terms of both the winning probabilities and the power inequality parameter.

Proposition 3 *(1) Suppose $F(y_i, y_j) = y_i + y_j$. In this case, weaker players completely free-ride in contributing to collective action.*

$$\begin{aligned} \alpha_1^* &= \frac{p(\theta_A)^2 p(\theta_B)}{[p(\theta_A) + p(\theta_B)]^2} R = \frac{\theta_B}{[2 + \theta_B + \theta_B]^2} R \geq \\ \frac{p(\theta_A) p(\theta_B)^2}{[p(\theta_A) + p(\theta_B)]^2} R &= \frac{\theta_A}{[2 + \theta_B + \theta_B]^2} R = \beta_1^* \text{ and } \alpha_2^* = \beta_2^* = 0. \end{aligned}$$

(2) Suppose $F(y_i, y_j) = \min\{y_i, y_j\}$. It is well-known that there are multiple equilibria, but we focus on the most efficient outcome. Then, we obtain

$$\begin{aligned} \alpha_1^* &= \alpha_2^* = \frac{[1 - p(\theta_A)]^2 [1 - p(\theta_B)]}{[1 - p(\theta_A) + 1 - p(\theta_B)]^2} R = \frac{\theta_A^2 \theta_B (1 + \theta_B)}{[2\theta_A\theta_B + \theta_A + \theta_B]^2} R \\ &\leq \frac{[1 - p(\theta_A)] [1 - p(\theta_B)]^2}{[1 - p(\theta_A) + 1 - p(\theta_B)]^2} R = \frac{\theta_A {\theta_B}^2 (1 + \theta_A)}{[2\theta_A\theta_B + \theta_A + \theta_B]^2} R = \beta_1^* = \beta_2^*. \end{aligned}$$

In the case of additive effort, the winning probability in the external conflict depends only on the stronger players' effort levels. Thus, the less conflictive group's winning probability is always higher, i.e.,

$$\text{For } F(y_i, y_j) = y_i + y_j,\ q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)) = \frac{p(\theta_A)}{p(\theta_A) + p(\theta_B)} \geq 1/2.$$

In contrast, in the case of weakest-link, collective action is virtually determined by the weaker players, because the stronger players merely make the same effort as much as the weaker players in own group. In this case, the more conflictive group's winning probability is always higher, i.e.,

$$\text{For } F(y_i, y_j) = \min\{y_i, y_j\},\ q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)) = \frac{1 - p(\theta_A)}{1 - p(\theta_A) + 1 - p(\theta_B)} \leq 1/2.$$

In this sense, these two polar cases make our earlier argument in *Proposition 2* and the corresponding *Corollary* even clearer.

Now, let us compare the equilibrium payoffs of the two players within a group. The following Proposition shows that the stronger players earn higher payoff in the weakest link effort case. However, in the case of additive effort, the result is very different.

Proposition 4 *In the weakest-link case, $V^*_{G1} \geq V^*_{G2}$ for $G = A, B$ always. In contrast, in the additive effort case, we obtain*

$$\begin{cases} (1)\ \textit{if } \theta_B \geq \frac{{\theta_A}^2 + 2\theta_A - 1}{(1-\theta_A)} \textit{ and } \theta_A \geq \frac{{\theta_B}^2 + 2\theta_B - 1}{(1-\theta_B)},\ \textit{then } V^*_{A1} \geq V^*_{A2} \textit{ and } V^*_{B1} \geq V^*_{B2} \\ (2)\ \textit{if } \theta_B < \frac{{\theta_A}^2 + 2\theta_A - 1}{(1-\theta_A)} \textit{ and } \theta_A < \frac{{\theta_B}^2 + 2\theta_B - 1}{(1-\theta_B)},\ \textit{then } V^*_{A1} < V^*_{A2} \textit{ and } V^*_{B1} < V^*_{B2} \\ (3)\ \textit{if } \theta_B \geq \frac{{\theta_A}^2 + 2\theta_A - 1}{(1-\theta_A)} \textit{ and } \theta_A < \frac{{\theta_B}^2 + 2\theta_B - 1}{(1-\theta_B)},\ \textit{then } V^*_{A1} \geq V^*_{A2} \textit{ and } V^*_{B1} < V^*_{B2} \\ \qquad (3)\ \textit{otherwise},\ V^*_{A1} < V^*_{A2} \textit{ and } V^*_{B1} \geq V^*_{B2}. \end{cases}$$

Proof. The difference between the two players' equilibrium payoffs in group A is written as

$$V^*_{A1} - V^*_{A2} = (2p(\theta_A) - 1)q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - (\alpha^*_1 + a^*_1) + (\alpha^*_2 + a^*_2).$$

From *Lemma 1*, both the players exert the same level of effort in internal conflict, i.e., $a^*_1 = a^*_2$. In addition, from *Proposition 3*, in the case of weakest-link effort, they also contribute the same level of effort to collective action, i.e., $\alpha_1^* = \alpha_2^*$. Finally, by construction $\theta_A < 1$ i.e., $p(\theta_A) > 1/2$. As a result, we must have $V_{A1}^* \geq V_{A2}^*$. The same logic applies to the players in group B. Hence, the stronger player always has a higher payoff than the weaker player in the weakest-link case.

Applying the results from *Lemma 1* and *Proposition 3,* the difference between the two players' equilibrium payoffs in group A for the additive effort case boils down to

$$\begin{aligned} V_{A1}^* - V_{A2}^* &= (2p(\theta_A) - 1)q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R - \alpha_1^* \\ &= \frac{p(\theta_A)}{p(\theta_A) + p(\theta_B)} R \left[(2p(\theta_A) - 1) - \frac{p(\theta_A)p(\theta_B)}{p(\theta_A) + p(\theta_B)} \right] \end{aligned}$$

The stronger player's advantage is a higher winning probability in internal conflict. However, the stronger player's disadvantage is that only he has to contribute to external conflict because the weaker player free rides completely, $\alpha_2^* = 0$. As a result, the stronger player has a (weakly) higher payoff than the weaker player in group A in the weakest-link case only if the second part of the above given expression in non negative. This condition, after expressing the probabilities in terms of the inequality parameter becomes

$$\theta_B \geq \frac{{\theta_A}^2 + 2\theta_A - 1}{(1 - \theta_A)}$$

Similarly, in group B, the stronger player has a (weakly) higher payoff than the weaker player in group A in the weakest-link case only if

$$\theta_A \geq \frac{{\theta_B}^2 + 2\theta_B - 1}{(1 - \theta_B)}$$

Combining these two conditions we obtain the result. ■

This result is represented in Figure 1. Since we confine our attention to $\theta_B \geq \theta_A$, let us look at the area above 45 degree line. Loosely speaking, the result implies that when the power inequality among the individual group members is small enough (θ is high enough), then the weaker player's payoff is greater than the stronger player. This is because the weaker

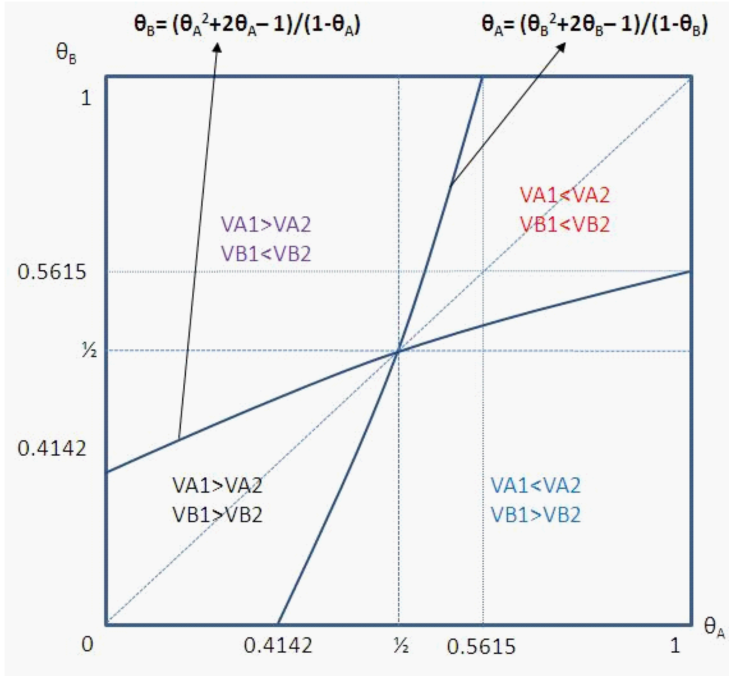

Figure 1. Within group equilibrium payoff for additive effort

player's free riding benefit is large despite his small share of the prize when the inequality is small enough. In addition, since the relative benefit of free riding is greater in the more conflictive group, the parameter range in which the weaker player's payoff is greater is larger in the more conflictive group, as seen in Figure 1. This becomes compeltely clear when we consider the polar case, $\theta = \theta_A = \theta_B$. In this case the result reduces to $V^*_{G1} \gtreqless V^*_{G2}$ as $\theta_G \lesseqgtr 1/2$.

6 Equilibrium Rent Dissipation

In this section, we compute the equilibrium rent dissipation, $(a^*_1 + a^*_2 + \alpha^*_1 + \alpha^*_2) + (b^*_1 + b^*_2 + \beta^*_1 + \beta^*_2)$, and analyze how this changes with the power inequality. We have already derived α^*_i and β^*_i in the previous section, here we find a^*_i and b^*_i for the two impact functions, respectively. Since we are interested in the effect of the power inequality on the total rent dissipation across the groups, we now assume symmetric power inequality across the groups, i.e., $\theta = \theta_A = \theta_B$ and $p(\theta) = p(\theta_A) = p(\theta_B) = 1/(1 + \theta)$. We first derive the following

Lemma.

Lemma 3 Under symmetric power inequality across the groups, the total rent dissipation in internal conflict is the same in the additive and the weakest link effort cases.

Proof. Inserting $q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$ into (1) and (2), we obtain

$$\begin{aligned}
\text{For } F(y_i, y_j) &= y_i + y_j, \begin{cases} a_1^* = a_2^* = \frac{p(\theta_A)^2(1-p(\theta_A))}{p(\theta_A)+p(\theta_B)} R \\ b_1^* = b_2^* = \frac{p(\theta_B)^2(1-p(\theta_B))}{p(\theta_A)+p(\theta_B)} R \end{cases} \\
\text{For } F(y_i, y_j) &= \min\{y_i, y_j\}, \begin{cases} a_1^* = a_2^* = \frac{p(\theta_A)(1-p(\theta_A))^2}{1-p(\theta_A)+1-p(\theta_B)} R \\ b_1^* = b_2^* = \frac{p(\theta_B)(1-p(\theta_B))^2}{1-p(\theta_A)+1-p(\theta_B)} R. \end{cases}
\end{aligned}$$

Imposing $\theta = \theta_A = \theta_B$ and $p(\theta) = p(\theta_A) = p(\theta_B) = 1/(1+\theta)$, the total rent dissipation in internal conflict for both the weakest link and the additive effort case turns out to be $a_1^* + a_2^* + b_1^* + b_2^* = 2p(\theta)(1 - p(\theta))R$. ■

This result allows us to pin down the total rent dissipation for the contest. It is summarized in the next Proposition.

Proposition 5 *Given $\theta = \theta_A = \theta_B$, in the weakest-link case, the total rent dissipation is monotonically increasing in θ. In contrast, in the additive effort case, the total rent dissipation $\frac{p(\theta)}{2}R + 2p(\theta)(1 - p(\theta))R$ is increasing for $\theta \in [0, 3/5]$ and decreasing for $\theta \in (3/5, 1]$.*

Proof. Using the results of Lemma 3, the following results can be immediately derived.

$$\begin{aligned}
\text{For } F(y_i, y_j) &= y_i + y_j, \begin{cases} \alpha_1^* + \alpha_2^* + \beta_1^* + \beta_2^* = \frac{p(\theta)}{2} R \\ a_1^* + a_2^* + b_1^* + b_2^* = 2p(\theta)(1 - p(\theta))R \end{cases} \\
\text{For } F(y_i, y_j) &= \min\{y_i, y_j\}, \begin{cases} \alpha_1^* + \alpha_2^* + \beta_1^* + \beta_2^* = (1 - p(\theta))R \\ a_1^* + a_2^* + b_1^* + b_2^* = 2p(\theta)(1 - p(\theta))R. \end{cases}
\end{aligned}$$

In the weakest-link case, The total rent dissipation is: $TR = (a_1^* + a_2^* + \alpha_1^* + \alpha_2^*) + (b_1^* + b_2^* + \beta_1^* + \beta_2^*) = (2p(\theta) + 1)(1 - p(\theta))R = \frac{\theta^2+3\theta}{(1+\theta)^2} R$.It is easy to show that $\frac{dTR}{d\theta} > 0$. Hence,

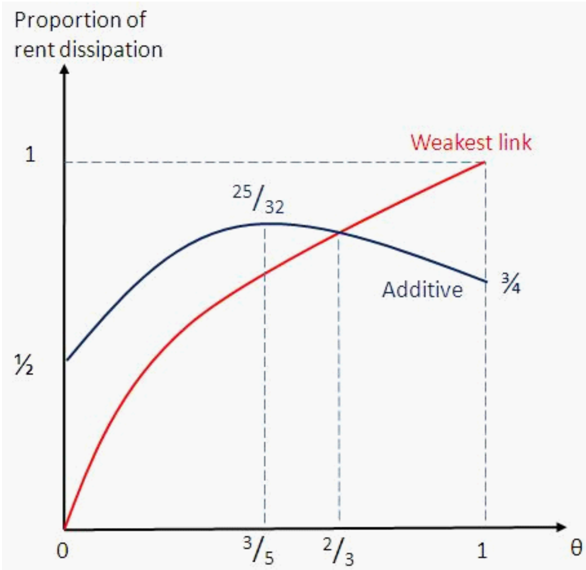

Figure 2. Rent dissipation against power inequality

total rent dissipation is decreasing in power inequality. In the additive effort case $TR = (a_1^* + a_2^* + \alpha_1^* + \alpha_2^*) + (b_1^* + b_2^* + \beta_1^* + \beta_2^*) = (\frac{5}{2}p(\theta) - 2p(\theta)^2)R = \frac{5\theta-1}{(1+\theta)^2}R$. It is easy to show from $\frac{dTR}{d\theta}$ that rent dissipation is $1/2$ when $\theta = 0$, is increasing and reaches its maximum at $\theta = 3/5$ then it declines to $3/4$ when $\theta = 1$. ■

In the weakest-link case, The rent dissipation is increasing in power inequality, i.e., decreasing in $p(\theta)$ both on external and internal conflict. This is quite intuitive. As the power inequality is smaller, the internal conflict becomes severe. In addition, since the weaker player determines the level of contribution to collective action, the external conflict becomes intense as well. Thus, the total rent dissipation $(2p(\theta)+1)(1-p(\theta))R$ is decreasing in $p(\theta)$ (increasing in θ).

In contrast, the additive effort case is more interesting. The rent dissipation on internal conflict is obviously decreasing in $p(\theta)$ (increasing in θ). However, note that the rent dissipation on external conflict is increasing in $p(\theta)$ (decreasing in θ). This means that more severe the external conflict more heterogeneous the players. It is because the free-rider problem is overshadowed as the stronger player's equilibrium share of the prize is larger. As a result, we find that the total rent dissipation does not behave monotonically with the power inequality as follows.

This result, represented in Figure 2, has several important implications. First, it is well-known that the rent dissipation is decreasing in the heterogeneity of the players in a single contest model. This is no longer true in simultaneous-contest. Second, our result suggests that the rent dissipation is underestimated in most papers based on the symmetric case single contest. In fact, when the group members are symmetric, the total rent dissipation is not maximized for the additive effort case. Finally, we show that simultaneous contest can be used to ensure full rent dissipation as in the weakest link egalitarian power case.[11]

7 Discussions

We develop a model of group contest in which simultaneous inter-group and intra-group conflict interplay with each other. We use power inequality within groups to analyze the impact of the heterogeneity within groups on inter-group conflict. We also analyze the equilibrium payoff and total effort levels in two contrasting cases. Here we conclude by discussing potential extensions of our model. There are many interesting ways this analysis can be further pursued, we mention only a couple of them.

Survival of the fittest: as a group or as an individual?: One interesting example is interspecific and intraspecific competition in ecology (Vandermeer, 1975). Competition within and between species arises for a limited amount of resources such as space, food, or mates. One important issue in the literature is to explain when different species can coexist or when one species becomes extinct. Let us interpret the prize R in our model as the given space for which two species are competing. Then, as a result of competition, each individual species occupy a portion of the space by $\frac{1}{1+\theta_A} q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$, $\frac{\theta_A}{1+\theta_A} q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$, $\frac{1}{1+\theta_B}(1 - q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)))$, and $\frac{\theta_B}{1+\theta_B}(1 - q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)))$.

Now, suppose that there is a minimum size of space for survival.[12] Then we can predict that the extinction of one species arises when $q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$ is sufficiently small or large.

[11] One may be interested in the asymmetric case of $\theta_A < \theta_B$. For example, we can conduct comparative statics of the total rent dissipation with respect to $t < 1$ when $\theta_A = t\theta_B$. We have a similar result: the rent dissipation on external competition is increasing in t, but that on internal competition is decreasing in t.

[12] This is common observation in biology and Ecology literature. See, for example Shaffer (1981).

That is to say, the outcome of interspecific competition should be extreme to the extent that one species cannot occupy a minimal space. This situation can arise when two groups are very heterogeneous in the sense that θ_B is sufficiently greater than θ_A, and the individual species efforts are either almost perfectly substitutable or complementary. Otherwise, two different species may be able to coexist with each species residing in a space of sufficient size for survival. For the case of coexistence, we can predict two different scenarios. If both θ_A and θ_B are relatively large, every individual may coexist. However, if both θ_A and θ_B are considerably small, only the superior individual in each species can survive and coexist.

Management of Internal conflict: Until now, we have been assuming that the prize is distributed within groups entirely by internal conflict. However, group members may be able to make binding commitments to share a portion of the prize on egalitarian grounds.[13] Then, the adjusted conflict technology is given by

$$p(x_1, x_2) = \frac{\phi_G}{2} + (1 - \phi_G)\frac{x_1}{x_1 + \theta_G x_2}, \text{ where } G = A, B.$$

ϕ_G represents the effectiveness of conflict management within a group. In other words, each group does not have to have internal conflict to divide this portion of the prize. On the other hand, they still have to contest for the other portion, $(1 - \phi_G)$, of the prize.

Another interesting interpretation is that a team organizer can control internal conflict in a way that the stronger and weaker players share a portion of the prize equally. In particular, while ϕ_G can be thought of as the portion of team rewards, $(1 - \phi_G)$ as the portion of the conflictive prize.

The share of the prize of the stronger player in each group is

$$p(\theta_G) = \frac{\phi_G}{2} + (1 - \phi_G)\frac{1}{1 + \theta_G}.$$

The winning probability of the stronger (weaker) player of group G is decreasing (increasing) in ϕ_G. Hence, the managerial problem would be to set the optimal commitment level ϕ_G^* to

[13] See Münster (2007) and Baik and Shogren (1995) for further discussions.

maximize the group's winning probability.

Given the symmetric CES function, we obtain $\phi_G^* = 0$ if $r > 1/2$ and $\phi_G^* = 1$ if $r \leq 1/2$. If group members' contributions are relatively complementary, higher ϕ_G increases the group's winning probability, and vice versa. Therefore, the team manager wants to distribute the prize equally within a team if collective action is relatively complementary. Otherwise, he would prefer to allow group members to fight over the prize.

There are other opportunities of extending our analyses in terms of both relaxing some of the assumption and modifying the structure to incorporate further real life applications. As we start our analysis with pre-specified groups, our analysis implicitly suggests that the heterogeneity of individuals will be an important factor in the study of the endogenous formation of groups. It would be an interesting exercise to extend our model to endogenize the group formation problem. Also, in our structure the two groups share the same group impact function, but this is not necessary in many cases. It will again be interesting to analyze the conflict between groups with different production functions. Recently, Clark and Konrad (2007) study the case where an attacker has the best-shot function and a defender has the weakest-link function. It would be worthwhile to extend their model to be a group contest. We assume symmetric (unit) marginal cost of internal and external conflict. A relaxation of this assumption may provide interesting comparative static analyses. Finally, in our model, the prize value is exogenously given. It will be interesting to study the case where the prize is endogenously determined. In other words, we can consider a model nesting the current analysis and that of Münster (2007) where asymmetric individuals allocate their resources between productive activity and conflictive activity.

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