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We would like to thank Gilat Levy, Ronny Razin, Felix Bierbrauer and Mark Le Quement for very fruitful discussions.
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# Are Close Races Best?

## The Impact of Electoral Competition on the Quality of Candidate Selection and Policy Choice.

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**Abstract**

We suggest a two-candidate signaling model of political competition to analyze the effect of the electoral mechanism on political outcomes. Candidates possess private information about their ability, and the electorate draws inferences about these abilities from the policy proposals during the electoral campaign. If voters are more responsive, the electoral process is more likely to bring the most able candidate into power. However, this comes at the cost of stronger distortions in politicians' behavior because low ability candidates will try to mimic their more able counterparts. This trade-off is shaped by key characteristics of the electoral mechanism. We show that two different limitations to political competition can sometimes be beneficial from a welfare perspective, namely increasing popularity gaps between both candidates and decreasing salience of the campaign issue.

**Keywords:** Electoral Competition, Signaling, Reputation

**JEL classification:** D72, D82, C72

We would like to thank Mark Le Quement, Felix Bierbrauer and Gilat Levy for very fruitful discussions.

# 1 Introduction

The crucial idea of representative democracy is to delegate political power from the people to political leaders that are supposed to pursuit social welfare. A major requirement for the success of this concept is an effective mechanism to recruit competent leaders – as a defining feature, representative democracies employ public elections between competing candidates. Voters generally prefer to have the most competent representatives in terms of their knowledge and understanding of political issues and their ability to implement adequate political instruments. Consequently, the political ability and competence of competing candidates is a major issue in any electoral campaign.

Elections do not only serve as a selection mechanism but are also meant to provide incentives for efficient policy implementation. If political ability is not observable, however, moral hazard is a crucial issue and induces a trade-off between the goals of effective selection and efficient policy choice. Voters have to cast their votes conditional on perceived abilities, and will use the candidates' campaign announcements to update these beliefs. Thus, there will be an incentive for office-motivated politicians with low competence to mimic the behavior of high-ability candidates, even if this implies the proposal of inefficient policies. On the one hand, effective selection of competent candidates is only possible if the vote result is strongly related to policy announcements. On the other hand, strong links between policy announcements and electoral prospects induce massively distorted policy proposals.

We introduce a simple framework with two competing candidates with heterogeneous competence and two available policies: the status quo policy and a political reform. Candidates are motivated both by welfare considerations and by the spoils of office. While the status quo provides a low, but certain level of welfare to the electorate, the welfare due to a political reform depends on the office-holder's competence. In the electoral campaign, each candidate makes a binding policy proposal. The voters prefer to have the more competent candidate, but are not able to observe competence directly. Instead, they use the candidate's policy proposals to form beliefs and vote accordingly.

We show that there is a unique political equilibrium for each parameter constellation. In general, there will be moral hazard: candidates will too often propose risky policies which allow them to establish high levels of perceived ability. As long as the office motivation of politicians is fairly moderate, there will be a separating equilibrium in which only the most competent candidates propose a political reform and win the election with high probability. In these situations, electoral competition indeed serves as effective selection device without completely destroying incentives for efficient policy implementation. If the politicians are mainly office-motivated in contrast, this will give rise to pooling equilibria in which all candidates propose a political reform regardless of their ability. Consequently, voters are not able to distinguish competent candidates from incompetent ones and elections fail to serve as effective recruitment devices.

Furthermore, we study the effect of different characteristics of the electoral mechanism, namely popularity gaps and salience. First, popularity gaps lower the degree of political competition, increasing the electoral prospects of one candidate independently of his campaign announcements and thereby lowering the selective potential of the electoral mechanism. Consequently, the favored candidate will behave more efficiently from an ex post perspective while the underdog will adopt more extreme (aggressive) strategies. The introduction of popularity gaps turns out to increase social welfare if candidates exhibit particularly high or low levels of office-motivation, but not in cases with intermediate levels. Second, if the campaign issue is more salient, the electorate will react more strongly in response to changes in the policy proposals. While this generally improves the efficiency of the selection mechanism, it also creates stronger policy distortions. The overall effect of salience is positive if politicians mainly care about social welfare, but negative in the case of high office-motivation.

After discussing related literature in the next section, we present the model in section 3. A benchmark for the analysis is set in section 5, in which we solve for the constrained optimal behavior. Thereafter, we analyze actual equilibrium behavior of both politicians in section 4. We proceed by examining the effects of the parameters of political competition in sections 6 to 7. Finally, we evaluate the performance of the electoral mechanism in comparison with a benevolent dictator in section 9 and conclude in section 10.

# 2 Related Literature

Beginning with Barro (1973) and Ferejohn (1986) reputational concerns of politicians and their effects on policy choice has drawn some attention in the literature. Two strands of this literature focus on the signaling of politicians' characteristics to the electorate. The first strand focuses on politicians' desire to build up reputation by strategically committing to policies that might ex post not be in the voters' best interest. In these models, voters typically draw inferences about politicians' abilities to conduct welfare enhancing reforms.Building on the work of Prendergast & Stole (1996), Majumdar & Mukand (2004) analyzed reform incentives when politicians are concerned with their perceived ability. Given the informational asymmetry, politicians start and continue reforms even when the expected return is negative. In contrast to our paper, the politician is already in office when choosing the policy. Closer to our work, but still distinct, is the model of Beniers & Dur (2007). It allows for strategic interaction between candidates insofar as it features two political incumbents. Again, proposed policies are always implemented. In line with this literature, candidates' signaling in our model also relates to their ability of implementing a welfare enhancing reform.[1] However, we focus on two candidates that commit to policies before elections. We also model the electoral mechanism in more detail than the above mentioned papers, and introduce the possibility of an electoral bias towards one candidate within the electoral campaign.

In the second strand, politicians cannot credibly commit to policies. Thus, their proposed policies serve as a signal to the policy they whish to implement after the election. Based on the spatial models of Hotelling (1929) and Downs (1957), Banks (1990) introduced imperfect commitment to policies. Politicians face costs

[1]We also relate to Fu & Li (2010) in this respect.

of lying when not announcing their preferred policies. Callander & Wilkie (2007) relax the assumption of homogenous lying costs and analyze the nature of political competition in this setup. They show that endogenous bounds to lying evolve in equilibrium. Valence advantages arise endogenously and are not the main focus of the study.[2] A common feature of these models is that the preference for a candidate is necessarily related to his expected behavior after the election. In a related paper, Kartik & McAfee (2007) model candidates that differ with respect to the strategic component in their policy choice. Voters prefer unstrategic candidates, such that strategic candidates have an incentive to mimic unstrategic ones. Also here, the equilibrium policies run counter the median voter theorem.

In our analysis of the behavior of candidates with different levels of ability, we also relate to papers that introduced a valence issue into the Hotelling-Downs model (among others, Ansolabehere & Snyder 2000, Aragones & Palfrey 2002, Groseclose 2001, Hollard & Rossignol 2008, Hummel 2010).[3] The general finding of this literature is that candidates with valence advantages choose moderate policies while disadvantaged candidates tend to the extremes. In this paper, we take a substantially different approach. A fundamental feature of our paper is the informational asymmetry between the electorate and political candidates. The electorate has homogenous preferences over political outcomes, but politicians posess private information about their capability in achieving these outcomes. This allows us to study how candidates' reputational concerns interact with the electoral mechanism.

# 3 The Model

There are three risk neutral agents, two candidates and a representative voter $V$. We will call one of these candidates the favorite $F$, the other one the underdog $U$. Both candidates run in an election which is conducted to determine the identity of the winner $w \in \{F, U\}$. The winner $w$ enters the public office and decides whether to implement a political reform, $x_w = 1$, or to maintain the status quo policy, $x_w = 0$. The voter's utility depends on the implemented policy. The political reform represents the risky alternative as it might fail. The voter benefits from a successful reform and suffers from a failed reform, as there is a reform cost of $c$. In contrast, there is no risk and no cost involved in maintaining the status quo. Candidates differ in their political ability, more precisely in the probability $a_i$ of a successful reform implementation.

The following utility function summarizes the voter's expected utility, conditional on the election winner's ability $a_w$:

$$W(x_w, a_w) = (a_w - c)x_w$$

Candidates care about winning the election (office motivation) as well as about the voter's utility (welfare motivation). This formulation suggests that the representative voter's utility can be interpreted as welfare function.[4] The utility function

[2] Bernhardt & Ingerman (1985) show a collapse of the median voter theorem in a similar setting.

[3] See Bruter et al. (2010) for a recent overview over this literature.

[4] These preferences can be motivated by the assumption that politicians are also citizens and,

of politician $i$ is given by

$$U_i(a_w, x_w) = e_i\theta + W(x_w, a_w)$$

where the indicator variable $e_i$ equals 1 if $w = i$ and 0 otherwise. Note that $\theta$ represents the relative weight of office motivation; changes in $\theta$ imply variations in the politician's motivation.

As a crucial feature of this model, each candidate's ability is his private information. In order to signal their ability, both candidates commit to a policy platform prior to the election, announcing either a political reform or the status quo. Voters use these announcements to update their beliefs about both candidates ability and vote accordingly. Overall, the game consists of three stages.

At the first stage, nature draws the ability of both agents $F$ and $U$ independently from a uniform distribution on the interval $[0, 1]$. The individual abilites are private knowledge to the respective candidates. As an example, the information set of a favorite with individual ability $a_i$ is given by $I_F(a_i) = \{(a_F, a_U)|\, a_F = a_i, a_U \in [0, 1]\}$.

At the second stage, both candidates simultaneously make binding policy announcements. We focus on pure strategies. Consequently, the strategy of politician $i$ will be a mapping $X_i : [0, 1] \to \{0, 1\}$.

At the third stage, the representative voter observes the policy announcements $x_F$, $x_U$, updates his beliefs about both candidates' abilities and casts his vote. We will denote the conditional expectations as $\hat{a_F}(x_F)$ and $\hat{a_U}(x_U)$. In order to describe voting behavior, we adopt an adjusted version of the probabilistic voting model (Lindbeck & Weibull 1987). Generally, the voter prefers to have the more able candidate in office. Additionally, the voting choice is influenced by the random variable $\beta$. We will assume that the voter elects $F$ if and only if $\hat{a_F}(x_F)+\beta > \hat{a_U}(x_U)$.

While this voting decision is deterministic for any realization of $\beta$, we assume in line with the probabilistic voting model that the candidates perceive $\beta$ to be a random variable with uniform distribution on the interval $\beta \sim u[-d + \delta, d + \delta]$. The range $d$ is related to the extent to which the representative voter cares about candidates' characteristics which are not related to their political abilities and not observable ex ante. In other words, $d$ represents an inverse measure of the salience or importance of the campaign issue for the electorate. The expectation $\delta \geq 0$ captures an ex ante existing and publicly known popularity gap in favor of candidate $F$. In line with the assumption $\delta \geq 0$, we will thus henceforth speak of candidate $F$ as the favorite and of candidate $U$ as the underdog. This popularity gap may result from institutional features of the democracy considered, from some form of asymmetric media coverage or from short-run preferences of the electorate. In any case, we assume that the electoral advantage $\beta$ has ex post no effect on the voter's utility. Nevertheless, both the popularity gap and the salience of the campaign issue are important determinants of the electoral mechanism and jointly determine the nature of competition between the candidates.

Given this voting function, the favorite's winning probability is given by

$$p(x_F, x_U) = prob\,[\beta > \hat{a}_U(x_U) - \hat{a}_F(x_F)] = \frac{d + \delta + \hat{a}_F(x_F) - \hat{a}_U(x_U)}{2d}. \tag{1}$$

consequently, are affected by the implemented policy in a similar way like the voters.

The equation for the underdog is obtained correspondingly. Note that the winning probabilities of both candidates are continuous in their perceived abilities. The following assumption ensures that both winning probabilities are properly defined, i. e. $p_i(x_i, x_{-i}) \in [0, 1]$ for all combinations of perceived abilities:

**Assumption 1.** $d > 2\delta + 1$.

To solve this game, we adopt the notion of Perfect Bayesian equilibrium. Thus, an equilibrium of this game consists of a strategy profile and a belief system such that (1) both candidates play mutually best responses at the announcement stage, correctly anticipating the winning probabilities for each vector $(x_F, x_U)$ which are implied by voter's beliefs, and (2) the voter's beliefs about expected abilities for each campaign announcement are derived from the candidates' strategies $X_F$, $X_U$ according to Bayes' rule everywhere on the equilibrium path.

# 4 Political equilibria

As a first step in solving for the equilibrium, we show that for every given belief system of the voters, candidates' strategies $X_i$ must satisfy the cut-off property. I.e. all candidates with ability equal to or exceeding the cut-off value $\alpha_i$ choose to reform, while candidates with lower ability remain with the status quo policy.

In general, a favorite with ability $a_F$ will choose to reform if and only if this provides him with a higher payoff than the status quo announcement, given the strategy adopted by his opponent and the voter's belief system:

$$E[U_F(a_w, x_w)|a_F, x_F = 1] \geq E[U_F(a_w, x_w)|a_F, x_F = 0]$$

Assume that the favorite anticipates some belief system and the implied winning probabilities for any combination of $x_F$ and $x_U$. We will argue that the favorite's best response on any strategy $\tilde{X}_U$ played by his opponent is always monotonous in $a_F$. Note that $a_F$, being unobservable to the electorate, does not affect the conditional winning probabilities. This implies directly that the favorite's expected payoff in case of announcing the status quo, $E[U_F(a_w, x_w)|a_F, x_F = 0]$ is constant in $a_F$, too. If the favorite proposes a reform, on the other hand, his ability matters if and only if he wins the election. By Assumption 1, his winning probability will always be strictly positive, so that the expected reform payoff $E[U_F(a_w, x_w)|a_F, x_F = 1]$ is strictly increasing in the favorite's ability. Lemma 1 establishes the monotonous structure of any optimal strategy.

**Lemma 1.** *Given any belief system held by the voters and any strategy adopted by his opponent, each candidate's optimal strategy exhibits the cut-off property. Thus, there is a unique cut-off $\alpha_i \in [0, 1]$ for each candidate $i \in \{F, U\}$ such that the optimal strategy is given by*

$$X_i(a_i) = \begin{cases} 1, & \text{if } a_i \geq \alpha_i \\ 0, & \text{if } a_i < \alpha_i \end{cases}$$

By Lemma 1, each candidate's strategies can be characterized entirely by the cut-off $\alpha_i$. Along the equilibrium path, the voter's beliefs must be derived from the correct strategies along the equilibrium path, and each candidate must play the best response on his opponent's equilibrium strategy. Consequently, the expected abilities in every separating equilibrium are given by

$$\hat{a}_i(x_i = 1) = \frac{1+\alpha_i}{2} \qquad \hat{a}_i(x_i = 0) = \frac{\alpha_i}{2} \tag{2}$$

In equilibrium, candidate $i$ correctly anticipates the winning probabilities implied by these beliefs. The same is true for the probabilities of a reform and status quo announcement by his opponent which are given by $prob(x_{-i} = 1) = 1 - \alpha_{-i}$ and $prob(x_{-i} = 0) = \alpha_{-i}$, respectively. Plugging in these expressions, we can express the favorite's payoff difference between both policy choices as a function of his ability $a_F$:

$$\begin{aligned} R_F(a_F, \alpha_F, \alpha_U) \;&=\; E[U_F(a_w, x_w)|x_F = 1, a_F, \alpha_F, \alpha_U] \\ &\quad -E[U_F(a_w, x_w)|x_F = 0, a_F, \alpha_F, \alpha_U] \\ &=\; (1-\alpha_U)\left\{E[U_F(a_w, x_w)|a_F, x_F = 1, x_U = 1, \alpha_F, \alpha_U]\right. \\ &\quad \left.-E[U_F(a_w, x_w)|a_F, x_F = 0, x_U = 1, \alpha_F, \alpha_U]\right\} \\ &\quad +\alpha_U\left\{E[U_F(a_w, x_w)|a_F, x_F = 1, x_U = 0, \alpha_F, \alpha_U]\right. \\ &\quad \left.-E[U_F(a_w, x_w)|a_F, x_F = 0, x_U = 0, \alpha_F, \alpha_U]\right\} \end{aligned}$$

We will henceforth refer to this payoff difference as the reform incentive function $R_F$ as it measures the desirability of a reform announcement conditional on the favorite's ability level. After inserting the conditional winning probabilities, it can be simplified to the following expression:

$$R_F(a_F, \alpha_F, \alpha_U) \;=\; \frac{\theta}{4d} + \underbrace{\frac{2d + 2\delta + \alpha_F}{4d}(a_F - c)}_{\Delta_1} - \underbrace{\frac{1-\alpha_U}{4d}\left(\frac{1+\alpha_U}{2} - c\right)}_{\Delta_2} \tag{3}$$

Both aspects of the politicians' preferences can easily be distinguished in this function. The first fraction represents the individual utility gain due to the boosted winning probability in case of a reform announcement which is always positive. In contrast, the following terms represents the induced welfare effect that can be split up into two parts. Expression $\Delta_1$ captures the expected welfare change that results if the cut-off agent actually enters the office and implements a reform, weighted by the probability of his electoral victory. Note that this term represents a welfare loss whenever the individual ability $a_F$ is below the reform cost $c$, and a welfare gain otherwise. The last term $\Delta_2$ reflects the competitive nature of political elections. Because the favorite's reform announcement decreases the winning probability of the underdog, it also reduces the part of expected welfare that an electoral victory of the underdog would provide. Welfare effects and effects on incentives for the politicians thus crucially depend on the exact location of the cut-offs and the parameter values in equilibrium.

Corresponding to the monotonicity of the candidates' optimal behavior (see Lemma 1), the reform incentive function is strictly increasing in $a_F$. Clearly, a favorite with ability $a_F$ will only choose to announce a reform if $R_F(a_F, \alpha_F, \alpha_U) > 0$. In deriving this expression, the voter's beliefs were represented by the cut-offs $\alpha_F$ and $\alpha_U$, which have to be consistent with actual strategies in equilibrium. By the definition of the cut-off ability, a favorite with ability $a_F = \alpha_F$ has to be indifferent between announcing the reform and the status quo. Consequently, the condition $R_F(\alpha_F, \alpha_F, \alpha_U) = 0$ has to be satisfied in any separating equilibrium.

Furthermore, the underdog's cut-off $\alpha_U$ has to fulfill a corresponding condition in equilibrium. Thus, any separating equilibrium with $\alpha_F > 0$ and $\alpha_U > 0$ of this game is characterized by the following two equations:

$$\theta + (2d + 2\delta + \alpha_F)(\alpha_F - c) - (1 - \alpha_U)\left(\frac{1 + \alpha_U}{2} - c\right) = 0 \quad (4)$$

$$\theta + (2d - 2\delta + \alpha_U)(\alpha_U - c) - (1 - \alpha_F)\left(\frac{1 + \alpha_F}{2} - c\right) = 0 \quad (5)$$

Note that a candidate with maximal ability $a_F = 1$ always has a strictly positive reform incentive, as he can never provide a lower welfare than his opponent.[5] Thus, the equilibrium cut-off must be strictly below unity. In contrast, it may happen that the reform incentives are positive even at the lowest ability level $a_i = 0$, for which case we get a cut-off level of zero. Thus, the actual cut-off levels are defined by the maximum of zero and the solution to the indifference condition. Taking this into account, the cut-off of the favorite is given by $\alpha_F(\alpha_U) = \max\{0; f_f(\alpha_U)\}$, where $f_f(\alpha_U)$ depicts the value of $\alpha_F$ that solves equation (4). Accordingly, we obtain the underdog's cut-off by $\alpha_U(\alpha_F) = \max\{0; f_u(\alpha_F)\}$ where $f_u(\alpha_F)$ captures the value of $\alpha_U$ that solves (5).

The political equilibrium is characterized by the values $\alpha_F^*$ and $\alpha_U^*$ that fulfill both reaction functions at the same time. Figure 1 on page 8 provides an illustration of the equilibrium, which is located at the intersection of the two reaction functions. Depending on the degree of office motivation, the reaction functions can intersect in three different ways, implying either separating or pooling equilibria. The following proposition establishes the existence and uniqueness of equilibria.

**Proposition 1.** *There exists a unique political equilibrium with $\alpha_F^* \in [0, 1)$, $\alpha_U^* \in [0, 1)$. More precisely, there is*

- *a separating equilibrium with $\alpha_F^* \in (0, 1)$ and $\alpha_U^* \in (0, 1)$ for low and moderate levels of office-motivation, $\theta < \theta_m$,*
- *a one-sided pooling equilibrium with $\alpha_F^* \in (0, 1)$ and $\alpha_U^* = 0$ for intermediate levels $\theta \in [\theta_m, \theta_h)$, and*

[5] It can be seen from (3), even the highest welfare loss due to a decreased winning probability of the opponent, which is present at the ex post efficient cut-off $\alpha_U = c$ with a loss of $\frac{1}{8d}(c-1)^2$, does not exceed the welfare gain from an own reform at $a_F = 1$, which is at least $\frac{(2d+1)}{2d}(1-c)$. The same argument also holds for the underdog. However, pooling equilibria without any reform announcement are sometimes possible for specific, implausible beliefs which give rise to $\hat{a}_i(xi = 1) < \hat{a}_i(xi = 0)$. We will not consider these equilibria in the following.

- *a pooling equilibrium with* $\alpha_F^* = \alpha_U^* = 0$ *for high levels of office-motivation,* $\theta \geq \theta_h$.

*The corresponding levels* $\theta_m$ *and* $\theta_h$ *are defined by* $\theta_m = \frac{1}{2} - c - 4(d+\delta)^2 - 4\delta c + 2(2d + 2\delta + c)\sqrt{(d+\delta)^2 + 2\delta c}$ *and* $\theta_h = \frac{1}{2} + (2d + 2\delta - 1)c$.

Intuitively, high office motivation will emphasize the reputational gain of proposing a reform so strongly that it is always favorable for both to reform. Subsequently lowering the importance of office motivation, first incentivizes the low ability favorites to step back from a reform. The favorites will react first since they have a stronger influence on expected welfare through their advantage in popularity. Second the political equilibrium shifts to an interior equilibrium, with strictly positive cut-off values for both player groups. This class of equilibrium is separating in the sense that only candidates with high ability will announce a reform while low-ability types stick to the status quo, preferring to bear the loss of the reputational gain for the sake of a higher expected welfare.

Investigating the separating equilibria closer, we find two possible constellations of both cut-offs and the reform cost. We label them according to the conflict of interest between the two candidates. If candidates are mostly interested in social welfare, they can be viewed as pursuing a common mission and their interests are mainly aligned. In contrast, if candidates are mostly office motivated, the conflict of interest is more dominant.

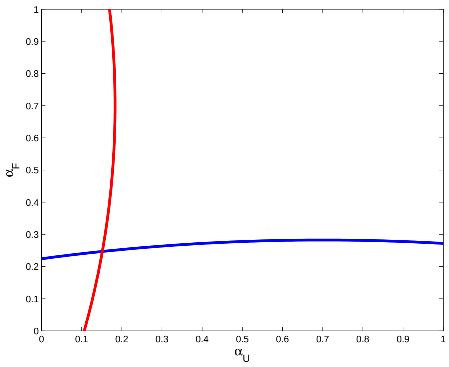


Figure 1: *The indifference reaction functions for* $c = 0.7, \delta = 0.2, d = 2$ *and* $\theta = 2$ *(separating equilibrium). The horizontal line represents the favorite's reaction function, the vertical line the underdog's reaction function.*

**Proposition 2.** *There are two types of separating equilibria, depending on the level of the reputation weight* $\theta$.

- *The mission case: For* $\theta < \theta_l = \frac{1}{2} + \frac{1}{2}c^2 - c$*, the cut-offs satisfy* $\alpha_U^* \geq \alpha_F^* > c$ *with strict inequality for* $\delta > 0$ *and equality for* $\delta = 0$.

- *The conflict case: For* $\theta \in (\theta_l, \theta_m)$*, the cut-offs satisfy* $\alpha_U^* \leq \alpha_F^* < c$ *with strict inequality for* $\delta > 0$ *and equality for* $\delta = 0$.

First, note that a general characteristic of political competition is that the underdog will employ a more extreme strategy than the favorite in the sense of a stronger deviation of the former from the ex post efficient cut-off $c$. This result is in line with the insights provided by a number of papers with focus on Downsian competition with valence differences (i. e. Ansolabehere & Snyder 2000, Aragones & Palfrey 2002). Throughout, these authors find that favored candidates stick closely to the median voter's position while the underdog adopts a more extreme policy platform. However, these authors do not account for heterogeneous abilities and signaling.

Furthermore, the behavior of both candidates features strong complementarity; both cut-off values are either located below or above the ex post efficient cut-off level $c$. As will become clear in the following, there are important differences between both types of separating equilibria.

In the conflict case with highly office motivated candidates, which we consider to be a realistic description of most democratic elections, underdogs announce political reforms more often than favorites. Moreover, there is an inefficiently high amount of political reforms in the sense that both competitors will even implement a reform with a negative expected payoff: $\alpha_U^* < \alpha_F^* < c$.

On the other hand, we will refer to the second class of equilibria with predominantly welfare oriented candidates as the mission case, inducing a behavior that resembles candidates who pursue a mission. In this case, both cut-offs are located above the cost ($\alpha_U^* > \alpha_F^* > c$), thereby being closer to the socially optimal cut-off levels as we will show in the following section.

# 5 The social optimum

In this model, social welfare is given by the overall probability of having a reforming politician of each kind in office, multiplied by the welfare that arises in this case. The overall probability of having a reforming politician of type $F$ in office is given by

$$\begin{aligned} prob(w = F, x_F = 1|\alpha_F, \alpha_U) &= prob(x_F = 1)[prob(x_U = 1)p(x_F = 1, x_U = 1) \\ &\quad + prob(x_U = 0)p(x_F = 1, x_U = 0)] \\ &= \frac{1}{4d}(1 - \alpha_F)(2d + 2\delta + \alpha_F) \end{aligned}$$

As the average welfare provided by the favorite is given by $\frac{1+\alpha_F}{2} - c$, we get the expected welfare contribution by the favorite as

$$\begin{aligned} W_F(\alpha_F) &= prob(w = F, x_F = 1|\alpha_F, \alpha_U)E[W(a_w, x_w)|w = F] \\ &= \frac{1}{4d}(1 - \alpha_F)(2d + 2\delta + \alpha_F)\left(\frac{1 + \alpha_F}{2} - c\right) \end{aligned}$$

The welfare contribution of the underdog follows similarly, and overall welfare is given by

$$\begin{aligned} W(\alpha_F, \alpha_U) &= W_F(\alpha_F) + W_U(\alpha_U) \\ &= \frac{1}{4d}(1-\alpha_F)(2d+2\delta+\alpha_F)\left(\frac{1+\alpha_F}{2}-c\right) \\ &\quad +\frac{1}{4d}(1-\alpha_U)(2d-2\delta+\alpha_U)\left(\frac{1+\alpha_U}{2}-c\right) \end{aligned} \tag{6}$$

Note that the resulting welfare function is additively separable in the cut-off abilities of both politicians which simplifies the analysis considerably. This directly implies that the welfare maximizing values of both cut-offs are to be determined independently. In the course of political competition, however, the equilibrium values are determined jointly, representing mutually best responses which induces an inefficient behavior of both politicians.

**Proposition 3.** *In the social optimum, both politicians propose to reform less often then optimal from an ex post perspective:*

$$\alpha_U^{SO} \geq \alpha_F^{SO} > c$$

*with equality if and only if* $\delta = 0$.

Intuitively, the possibility to compete with another high-ability politician introduces a social benefit of proposing a political reform less often. Think about a situation in which both politicians choose to reform whenever their ability exceeds the reform cost, that is $\alpha_F = \alpha_U = c$. If an agent with ability $a_I$ slightly above $c$ announces a reform, he will win the election with considerable probability and provide a small welfare benefit of $a_I - c$. But at the same time, his announcement of a reform reduces the chance that a reform-announcing opponent enters the political office. Such an opponent, however, would on average provide a clearly higher welfare of $(1+\alpha_U)/2-c$. Moreover, the voters could clearly prefer being able to distinguish politicians with small positive welfare contributions from candidates with large positive contributions, as this would allow them to promote the latter ones to office with much higher probability. Reform announcements by small-contribution agents eliminate this screening possibility. In equilibrium, the reputation of all reformers decreases which results in lower winning probabilities for the most able types. In other words, increasing the cut-off ability $\alpha_F$ from the ex post efficient level of $c$ implies a small welfare loss due to the drop out of some reforms with small positive welfare effect. But this effect is clearly dominated by a larger welfare gain due to the higher probability of reforms with large positive welfare contribution, which is induced through the higher winning probability of all politicians with ability above the new cut-off. Intuitively, more reluctance in the reforming behavior of politicians leads to a higher approval of all actually announced reforms by the electorate. At the social optimum, both effects exactly counterbalance each other, which can easily be seen in the defining functions of the efficient cut-offs:

$$\begin{aligned}(2d+2\delta+\alpha_F^{SO})(\alpha_F^{SO}-c) &= (1-\alpha_F^{SO})\left(\frac{1+\alpha_F^{SO}}{2}-c\right)\\(2d-2\delta+\alpha_U^{SO})(\alpha_U^{SO}-c) &= (1-\alpha_U^{SO})\left(\frac{1+\alpha_U^{SO}}{2}-c\right)\end{aligned}$$

In the following sections, we will show that both candidates' equilibrium conditions have a very similar structure. This similarity allows for a straightforward comparison between equilibrium outcomes and socially optimal behavior. Furthermore, we can use this structure to show the major benefit of political competition in contrast to the existence of an unchallenged politician. Generally speaking, electoral competition leads to an imperfect but nonetheless existing internalization of the adverse effects explained above, while such an internalization is completely missing in the one-politician case.

Slight rearrangements of the defining equations allow to see immediately why both cut-offs must be located above the cost of reform:

$$\begin{aligned}(2d+2\delta+2\alpha_F^{SO}-1)(\alpha_F^{SO}-c) &= \frac{1}{2}(1-\alpha_F^{SO})^2\\(2d-2\delta+2\alpha_U^{SO}-1)(\alpha_U^{SO}-c) &= \frac{1}{2}(1-\alpha_U^{SO})^2\end{aligned}$$

Furthermore, the underdog's cut-off will exceed the favorite's one whenever $\delta > 0$ as this decreases the left-hand side of the second equation but increases the left-hand side of the first equation. Restoring equality requires to lower the right-hand side of the second equation which implies that $\alpha_U^{SO}$ must be closer to unity than $\alpha_F^{SO}$. Intuitively, the lower winning probability of the opponent is compensated by a higher cut-off and thus a higher baseline reputation of the opponent. In the symmetric case, the socially optimal cut-offs coincide.

The following Lemma compares the socially optimal cut-offs with equilibrium behavior, thereby providing an important basis for the welfare analysis of changes in the competition parameters $d$ and $\delta$ in the next chapters.

**Lemma 2.** *Whenever $\theta > 0$ or $\delta > 0$, the equilibrium cut-offs $\alpha_F^*$, $\alpha_U^*$ differ from the socially optimal behavior. Specifically, the favorite always undertakes reforms too often: $\alpha_F^* < \alpha_F^{SO}$. In contrast, the underdog's cut-off may be too high or too low.*

# 6 Popularity gaps

In terms of the closeness of the race, democratic elections differ considerably internationally as well as over time. While presidential elections in the US typically represent head-to-head competitions between two candidates with similar winning probabilities ex ante, the electoral landscape in other countries is characterized by a predominant party who faces only weak competitors with negligible chances to enter the public office. A striking example for a country with extreme electoral asymmetry is represented by Japan where the liberal democratic party won all elections between 1955 and 2009 (except for a period of three years between 1993 and 1996). Other countries like Germany have experienced periods of alternating party dominance: while the conservative party succeeded in all federal elections between 1949 and 1966 as well as between 1982 and 1998, the social democrats clearly dominated the political landscape in the seventies. Empirical evidence seems to suggest that the initial position in terms of relative popularity has a strong effect on candidates' behavior and electoral campaigns. Electoral underdogs tend to fight harder and to suggest more as well as more radical political reforms, as they try to catch up and compensate for their initial popularity lag. Conversely, clear-cut favorites seem to announce fewer changes of their policy platform within the campaign and to stick to previous announcements, rather defending the status quo. In general, one would also expect the affiliation to the current government versus the opposition to play a role. However, our model suggests the empirically prevalent patterns of campaign behavior can also be explained as direct consequences of variations in the relative popularity, as this changes the reform incentives of electoral competitors significantly.

## 6.1 Comparative statics effects of popularity gaps

Increasing electoral asymmetry has two effects, a direct effect and an indirect effect which partially outbalance each other. The direct effect is straightforward: As a result of the augmented relative popularity of the favorite, his winning probability has increased ceteris paribus. Since the cut-off favorite provides negative expected welfare ($\alpha_F^* < c$) in case of entering the public office, this increases the idiosyncratic welfare change ($\Delta_1$). In order to restore equality of the first indifference condition, the favorite has to behave more efficiently by rising his cut-off ability $\alpha_F^*$. For the underdog, we have exactly the opposite argumentation. Increasing asymmetry lowers his winning probability and his idiosyncratic welfare loss. Thus, playing an inefficient strategy in terms of welfare is less costly to him and the cut-off $\alpha_U^*$ decreases.

Due to the interaction between both candidates, these induced changes of both cut-offs give rise to indirect competition effects. By the direct effect, the favorite now faces a smaller level of $\alpha_U^*$. Clearly, this implies that it becomes less attractive to have the underdog in office and increases the competition-related welfare loss $\Delta_2$. Thus, the favorite's reform incentives are strengthened. The indirect competition effect consequently mitigates the direct effect, though it never completely compensates it. Looking at the underdog, the indirect effect partially counterbalances the direct effect in a similar way, leaving an overall negative effect on $\alpha_U^*$.

A special case of this situation is represented by one-sided pooling equilibria in which all underdogs announce a political reform while the favorites cut-off is strictly positive. Consequently, the direct effects of asymmetry lead to incentives for more efficient behavior by the favorite and less efficient behavior by the underdog. The latter will not react by changing his cut-off, however, as he is already in a one-sided pooling equilibrium with $\alpha_U^* = 0$. Consequently, there is no indirect effect on the favorite's behavior and we see a generally stronger increase in $\alpha_F^*$.[6]

In the mission case, the comparative statics effects for both politicians are reversed. Note that in this situation, both cut-off values are located above the reform cost ($\alpha_U^* > \alpha_F^* > c$). Consequently, a reform proposal by the cut-off favorite is efficient from an idiosyncratic perspective, and the idiosyncratic welfare change $\Delta_1$ is a gain. The interpretation of $\Delta_2$ does not change, as the reform announcement still implies a reduction of the probability to have the welfare-providing underdog in office. In the mission case, increasing asymmetry still implies a higher winning probability to the favorite. As the importance of the welfare gain $\Delta_1$ has increased, the reform incentives for the favorite clearly go up. Consequently, the favorite starts to reform at lower levels of ability and the cut-off $\alpha_F^*$ falls. Like in the conflict case, the underdog's reform incentives change in the opposite direction, and he ends up with a higher cut-off ability. Additionally, we can once again identify indirect effects which mitigate the initial reaction of both cut-off values. The following proposition summarizes the comparative static results of variations in the relative popularity.

**Proposition 4.** *Increasing popularity gaps induce a stronger polarization of adopted strategies in all separating equilibria. The favorite's cut-off $\alpha_F^*$ always shifts towards the ex post efficient cut-off $c$, while the underdog's cut-off $\alpha_U^*$ moves in the opposite direction.*

Note that the derivatives of both cut-offs with respect to popularity gap $\delta$ depend on the type of separating equilibrium. In the conflict case, both candidates enact reforms to often from an ex post perspective: $\alpha_U^* \leq \alpha_F^* < c$. Proposition 4 consequently implies that the favorite's cut-off increases, i. e. that the favorite implements political reforms less often. In contrast, the underdog will behave even more extreme than before, proposing even more inefficient reforms. The sign of both derivatives is reversed in the mission case.

Intuitively, the exogenous boost of his relative popularity induces more efficient behavior by the favorite, because it makes him more certain to eventually enter the public office. Thus, the favorite faces a higher cost of ex post inefficient behavior while the gains in terms of increased reputation remain constant.

## 6.2 Welfare effect of popularity gaps

Popularity gaps are an element of the selection process which is completely independent of the expected competence levels, although only these should be relevant from a welfare perspective. At first sight, it seems as if this asymmetry interferes with the recruitment role of democratic elections. However, our comparative statics results clearly show that electoral asymmetries change politicians' reform incentives which might overall induce more efficient behavior. The total differential of the welfare function includes the direct effect of an increase in $\delta$ as well as the effects due to the

[6]For political equilibria with full pooling, there is obviously no effect on any candidate's cut-off ability, as long as we don't switch to a one-sided pooling equilibrium.

changes in the politicians' behavior:

$$\frac{dW}{d\delta} = \underbrace{\frac{\partial W}{\partial \delta}}_{>0} + \underbrace{\frac{\partial W}{\partial \alpha_F^*}}_{>0}\frac{d\alpha_F^*}{d\delta} + \frac{\partial W}{\partial \alpha_U^*}\frac{d\alpha_U^*}{d\delta} \tag{7}$$

Whenever there is a popularity gap, the direct effect will be positive. Since the favorite always behaves more efficiently from an idiosyncratic perspective, assigning him a higher winning probability is ceteris paribus beneficial. The welfare effects that result from the behavioral changes of both politicians will always have opposite signs, however. From lemma 2, we know that both equilibrium cut-offs will generally lie below the socially optimal levels. Consequently, the partial derivatives of $W$ with respect to $\alpha_F^*$ and $\alpha_U^*$ will be positive. By Proposition 4, the induced changes of both cut-off abilities will have different signs, implying a positive welfare effect due to the reaction of one politician and a negative welfare effect due to the reaction of his opponent.

In the special case of one-sided pooling, the analysis simplifies considerably because variations in the popularity gap only influence the favorite's behavior. The underdog's cut-off does not change because it is already at its lowest possible level $\alpha_U^* = 0$. Consequently, the third term in the derivative above drops out and we just have to look at the welfare change due to the favorite's reaction. From above, we know that any increase in $\delta$ induces more efficient behavior by the favorite: $\frac{d\alpha_F^*}{d\delta} > 0$. It follows that increasing electoral uncertainty is unambiguously beneficial in the case of one-sided pooling equilibria. In this special case, we can intuitively benefit from the virtues of asymmetry – namely the provision of more efficient incentives to the electoral favorite – without being harmed by the usual drawbacks – the less efficient reform incentives to the underdog.

For interior equilibria, the overall welfare effect rather depends on the relative size of the induced reactions in both cut-offs $\alpha_F^*$ and $\alpha_U^*$.

**Proposition 5.** *The welfare effect of increasing asymmetry is positive in one-sided pooling equilibria and in equilibria of the mission case, i. e. for high and low levels of office motivation. In contrast, it is strictly negative for separating equilibria of the conflict case, i. e. for moderate levels of office motivation,* $\theta \in (\theta_l, \theta_m)$.

In the conflict case, increasing asymmetry leads to less efficient behavior by the underdog which cannot completely be counterbalanced by the more efficient behavior of the favorite. Thus, overall welfare decreases as a result to any marginal increase in the electoral asymmetry. Intuitively, both politicians implement inefficiently many reforms in this situation ($\alpha_U^*, \alpha_F^* < c$), and increasing asymmetry cannot change this situation.

In contrast, the underdog's welfare contribution as well as social welfare can be increased by popularity gaps in the mission case. As long as $\alpha_U^*$ is still located below the socially optimal level[7] the increase in $\alpha_U^*$ will lead to a higher welfare contribution $W_U$. Furthermore, the direct effect and the positive effect because of the more efficient behavior of the underdog offset the detrimental shift in the cut-off

[7]The condition $\alpha_U^* < \alpha_U^{SO}$ will already be fulfilled for minimal levels of office-motivation.

for the favorites. Intuitively, in this case increasing asymetry allows us to combine an almost certain, idiosyncratically efficient policy implementation with the bonus of sometimes being able to identify and promote to office an extremely competent underdog.[8]

# 7 Salience of the campaign issue

Political reform proposals differ fundamentally in the degree of public interest they attract. Some are in the public focus and thus give the political candidates the chance to achieve reputational gains by proposing reforms in this area while others hardly influence election outcomes. Salience of political reforms is thus a major determinant of political behavior and competition. Obviously, higher salience (i.e. lower $d$) leads to more competitive behavior. In particular, the incentives that arise from the reputational gains are enlarged compared to the welfare component. On the one hand, being confronted with a decision that does not influence the public opinion, politicians will act ex post efficiently as long as they have slight interest in the welfare. On the other hand, a decision that has a large impact on the election leads to a further deviation from ex post efficient behavior. These insights are summarized in the following proposition:

**Proposition 6.** *In all separating equilibria, an increase in the salience of the reform, i.e. a decrease in $d$, induces both cut-offs $\alpha_F^*$ and $\alpha_U^*$ to shift away from the ex post efficient level $c$.*

First, note that the ex post efficient reform cut-off $c$ is always located below the socially optimal levels $\alpha_F^{SO}$ and $\alpha_U^{SO}$, because only the latter take into account the implications of policy announcement on selection quality. Moreover, the equilibrium values of $\alpha_F^*$ always lie beneath the corresponding social optimum and those of $\alpha_U^*$ do most of the time. Increasing salience is thus likely to induce more efficient behavior for the mission case and less efficient behavior for the conflict case. To analyze the overall welfare effects of salience, we have to consider this indirect channel through the behavior of the politicians and the direct effect of salience. Salience gives rise to a direct effect since it acts as a tool for the selection of the more able politician.

**Proposition 7.** *For any equilibrium in the conflict case with $\theta > \bar{\theta}(c, \delta)$, the welfare effect of increasing salience is negative beyond the optimal level $d^*(\theta, c, \delta) > 2\delta + 1$. In the mission case, the welfare effect of increasing salience is strictly positive.*[9]

We have seen before that the equilibrium cut-off levels resulting from political competition are smaller than the socially optimal cutoffs for almost all parameter values. This insight leads to the first part of the proposition. In the mission case salience leads to more efficient behavior since the cutoffs are increasing in the level

[8] The analysis performed herein is valid for small changes in the popularity gap that leaves the probabilites of being elected positive for both candidates. A complementing analysis that allows for one candidate to being elected with certainty is provided in section 9.

[9] Comment: Please note that the proof for the first part of this proposition is not yet complete. Currently, we can only proof it for the special case of $\delta = 0$ and show it numerically for the general case. The proof for the second part is complete.

of salience. Moreover the direct effect on welfare is also positive since higher salience leads to a more precise selection mechanism. Increasing $\theta$ subsequently, will first drive the indirect effects via the adaption of the cut-offs to zero and than lead to negative indirect effects.

The second part of the proposition guarantees that at some point the negative indirect effect will lead to an overall negative effect. For high levels of office-motivation, even candidates with very low ability announce a political reform so that electoral campaigns allow only for a very limited sorting of appropriate candidates - in the extreme case of polling equilibria, campaigns fail completely to provide information to the voters. If salience decreases in these situation, the positive effect of inducing more effective policy choice clearly dominates the negative effect of a diminished selection capability of elections. As further reductions in salience shift the equilibrium cut-offs closer and closer to their ex post efficient level of $c$, however, this positive incentive effects weakens and weakens. At the same time, further reductions in the selective capacity of the campaign get more harmful. Consequently, decreasing salience is beneficial only up to some optimal level $d^*(\theta, c, \delta)$. This optimal level is strictly increasing in the level of office-motivation.

# 8 Candidates' motivation

In this paper, we assume that politicians care about two different objectives, social welfare and about the spoils of public office. Clearly, the politicians will be more eager to win the election, the higher the office rewards are. It is straightforward to see that the reform incentive function $R_F(a_F, \alpha_F, \alpha_U)$ is strictly increasing in $\theta$, as the announcement of a political reform always increases the winning probability. Not surprisingly, this gives rise to unambiguous comparative statics effects.

**Proposition 8.** *In any separating equilibrium, rising office motivation induces a decrease in both cut-off ability values.*

In other words, the frequency of political reforms is strictly increasing in the importance of career-centered objectives of political candidates, as long as reforms are not undertaken by all agents anyhow. In both cases, the increase in $\theta$ will have a direct effect in terms of lowering both politicians' cut-offs. In the case of competition, however, this is accompanied by an indirect effect. Each candidate anticipates the more inefficient behavior by her opponent which implies that the competitor-related welfare loss of reform announcements is increased and reform proposals become even more attractive. Thus, there is a strong complementarity between both candidates' strategies.

In most instances, the increased discrepancy between the electorate's preferences and the politicians' objectives has a negative welfare effect. This can be seen most clearly by considering interior equilibria of the conflict case which result for values of $\theta$ in the interval $(\theta_l, \theta_m)$. Note that in any such equilibrium, both cut-offs $\alpha_F^*$ and $\alpha_U^*$ are located below the reform cost $c$. Consequently, even some per se detrimental reforms are implemented in equilibrium. Socially optimal behavior, however, implies cut-offs $\alpha_U^{SO} \geq \alpha_F^{SO} > c$ which are independent of $\theta$ and induce a considerably lower frequency of political reforms. By Proposition 8, any increase in the magnitude of office motivation leads to a further deviation from optimal behavior and lowers social welfare.

But this insight cannot be generalized to the mission case without qualifications. More precisely, we can show that the complete absence of office motivation is welfare maximizing if and only if we are in a symmetric setting ($\delta = 0$). Note also that $\theta = \delta = 0$ is the only parameter constellation for which equilibrium behavior coincides with socially optimal behavior. Whenever there is a popularity gap between both candidates, the optimal extent of office motivation is strictly positive.

**Proposition 9.** *The socially optimal level of office motivation is given by*

- $\theta^* = 0$ *for elections with equally popular candidates* ($\delta = 0$)*;*
- *a small but strictly positive level* $\theta^* > 0$ *whenever there is a popularity gap* ($\delta > 0$)*.*

For the first part, note that elections with equally popular candidates will always be characterized by symmetric cut-offs $\alpha_F^* = \alpha_U^*$, which are inefficiently low as long as the politicians are interested in the spoils of office. In the complete absence of office-related objectives, however, the equilibrium cut-offs coincide with their socially optimal values, and the social optimum can be realized.

The second part of the proposition results because the underdog does not behave optimally in the absence of office rewards. Note that his equilibrium value $\alpha_U^*$ exceeds its socially optimal level $\alpha_U^{SO}$ for extremely low values of $\theta$, i. e. for overwhelmingly welfare-oriented candidates (see section 5). This inefficient reluctance is a result of the interactive nature of political competition. When deciding about his policy announcement, the underdog internalizes the welfare effects $\Delta_1$ and $\Delta_2$. Comparing the equilibrium equation (5) with the defining equation for socially optimal behavior, we see that he should instead internalize a adjusted version of $\Delta_2$. Intuitively, the popularity gap induces the favorite to behave very efficiently from an ex post incentive. This in turn reduces the reform incentives of th underdog dramatically who is frightened to prevent the favorite's electoral victory.

Given such a situation, a limited extent of office motivation induces the underdog's cut-off to decrease, thereby approaching its socially optimal level. On the other hand, any increase in $\theta$ also causes less efficient behavior by the favorite. As the underdog's behavioral reaction is always stronger than the favorite's adjustment, the overall welfare effect is positive for very small levels of $\theta$. It can be shown that both effects exactly counterbalance each other at a strictly positive level which consequently represents the socially optimal level of office motivation. Necessarily, this constrained welfare maximum is still characterized by slightly too little reforms due to the underdog and too many reforms due to the favorite ($\alpha_U^*(\theta = 0) > \alpha_U^{SO}$, $\alpha_F^*(\theta = 0) < \alpha_F^{SO}$).

# 9 Comparison to a Benevolent Dictator

In this section, we examine whether democratic elections can improve upon the welfare generated by a benevolent dictator. We can do so by comparing the outcomes under two extreme conditions for the popularity gap. First, consider that the popularity gap is such that the favorite wins with certainty. Then, the favorite will only choose to announce a reform if this is ex-post efficient ($\alpha_F^* = c$), i.e. he will effectively behave as a benevolent dictator. Note that in this case, the politician's incentives are undistorted but the political process is not able to select the more able candidate. Expected welfare is merely the welfare generated by the favorite: $W(\alpha_F^*, \alpha_U^*)|_{\delta\to\infty} = (1-c)\left(\frac{1-c}{2}\right) = \frac{1}{2}(1-c)^2$.

However, if there is no popularity gap and the candidates are ex ante symmetric, selection of the better candidate is not impeded by non-ability related factors, but the reform incentives for the candidates are distorted. Also, both candidates feature the same cut-off such that welfare is given by $W(\alpha_F^*, \alpha_U^*)|_{\delta\to 0} = \frac{1}{2d}(1-\alpha_F^*)(2d + \alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right)$. Using these two equations we can conclude the following:

**Proposition 10.** *All equilibria in the mission case yield a higher welfare as compared to a benevolent dictator. In addition, there exist equilibria in the conflict case for which this is true as well.*

The first part of the proposition follows from noting that welfare is monotonically increasing in $\alpha_F^*$ and reaches its maximum at $\alpha_F^{SO} > c$ (cf. Proposition 3). At the same time, we know from Lemma 2 that $\alpha_F^*$ always stays below $\alpha_F^{SO}$. A lower bound for welfare in the mission case with $\delta = 0$ can thus be obtained by plugging in $\alpha_F^* = c$. We then get $W(c,c)|_{\delta\to 0} = \frac{1}{2}(1-c)^2\left(1 + \frac{c}{4d}\right)$ which is higher than welfare obtained under a benevolent dictator. From proposition 5 it follows directly that this result carries over to all mission case equilibria. Thus, the negative incentive effects do not dominate the selection possibility in the mission case.

This result carries over to a nonempty mass of equilibria in the conflict case. An increased office motivation worsens the selection quality as well as the incentives for efficient behavior. In the most extreme case of full pooling, eventually, $W(0,0)|_{\delta\to 0} = \frac{1}{2} - c$, which is clearly below $W(\alpha_F^*, \alpha_U^*)|_{\delta\to\infty}$. However, the change in welfare is continuous while the welfare advantage of political competition is positive at the border to the conflict case.

# 10 Conclusion

We presented a model in which the electorate draws inferences about the ability of political candidates when observing the reform proposals. In contrast to previous literature, candidates act strategically not only with respect to the electorates' updating but also with respect to the behavior of their opponents. The framework allows us to analyze the impacts of the following features of the electoral mechanism: popularity gaps, salience of the campaign issue, and politician's motivation.

Firstly, popularity gaps lead to a polarization of political platforms. While the favorite makes suggestions closer to the ex post efficient level, the underdog tends to the extremes. Since the welfare contribution of the favorite exceeds that of the underdog, the direct effect of a popularity gap on welfare is always positive. Taking into account both the altered selection among the candidates and the adaptation of the proposed policies, the total welfare effect is positive for low and high office motivation. Intermediate values of office motivation trigger a negative welfare effect. Secondly, an increased public interest in the relevant policy area leads to less efficient policy making from an ex post perspective. The improved selection of able candidates dominates the negative incentive effects when politicians are mainly welfare oriented. For higher values of office motivation, the reverse is true. Thirdly, politician's motivation exposes them to stronger reform incentives, which can be beneficial for low values of office motivation in which the underdog proposes reforms too seldom. We also find that for most parameter constellations, the benefits due to democratic selection dominate the negative incentive effects.

In general, politicians might not only try to signal their own ability but also try to change the perception of their opponent's ability. This component could be added to our basic model by introducing two different cost levels, that are unobservable by the electorate, which effectively makes the state of nature unobservable. Furthermore, an extension of the model to more than two politicians would allow for an analysis of the optimal number of candidates in an electoral system.

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# A Appendix

## A.1 Proof of Lemma 1

*Proof.* Consider the announcement decision of a favorite with ability $a_F$. He will choose to reform if and only if $E[U_F(a_w, x_w)|a_F, x_F = 1] \geq E[U_F(a_w, x_w)|a_F, x_F = 0]$, given the strategy adopted by his opponent and the voter's belief system. Consider some belief system and denote the implied winning probabilities for the favorite by the function $\hat{p}(x_F, x_U)$. Furthermore, assume that the underdog's strategy is given by some function $\bar{X}_U(a_U)$ and denote the implied expected welfare of having a reforming underdog in office by:

$$\begin{aligned} \bar{W}_U &= E[W(a_w, x_w)|w = U, x_U = 1] \\ &= \frac{1}{\int_0^1 \bar{X}_U(a_U)da_U} \int_0^1 (a_U - c)\bar{X}_U(a_U)da_U \end{aligned} \tag{8}$$

Then, agent $i$'s expected utility in case of a reform proposal will be given by

$$\begin{aligned} &E[U_F(a_w, x_w)|\bar{X}_U, \hat{p}, a_F, x_F = 1] = \\ &prob(x_U = 1)\left\{\hat{p}(x_F = 1, x_U = 1)(a_F - c + \theta) + [1 - \hat{p}(x_f = 1, x_u = 1)]\,\bar{W}_U\right\} \\ &+ prob(x_U = 0)\hat{p}(x_F = 1, x_U = 0)(a_F - c + \theta) \end{aligned}$$

Similarly, his expected utility in case of maintaining the status quo follows as

$$
\begin{aligned}
&E[U_F(a_w, x_w)|\bar{X}_U, \hat{p}, a_F, x_F = 0] = \\
&prob(x_U = 1)\left\{\hat{p}(x_F = 0, x_U = 1)\theta + [1 - \hat{p}(x_F = 0, x_U = 1)]\,\bar{W}_U\right\} \\
&+ prob(x_U = 0)\hat{p}(x_F = 0, x_U = 0)\theta
\end{aligned}
$$

Given these beliefs and strategy $\bar{X}_U$, agent $i$ will choose to announce a reform if and only if the former expectation is larger than the latter one. It can easily be seen that the expected utility of reforming is strictly monotonously increasing in the individual ability $a_F$ while the status quo expectation is independent of $a_F$. The same argument holds for the underdog. Thus, the optimal strategy of each candidate will always be of the cut-off type. □

## A.2 Proof of Proposition 1

*Proof.* The indifference conditions for both politicians can be expressed as $\alpha_F = \max\{0, f_F(\alpha_U)\}$ and $\alpha_U = \max\{0, f_U(\alpha_F)\}$, using the following continuous functions:

$$
\begin{aligned}
f_F(\alpha_U) &= \frac{c}{2} - d - \delta + \sqrt{\left(d + \delta + \frac{c}{2}\right)^2 - \frac{1}{2}\alpha_U^2 + \alpha_U c - c - \theta + \frac{1}{2}} \\
f_U(\alpha_F) &= \frac{c}{2} - d + \delta + \sqrt{\left(d - \delta + \frac{c}{2}\right)^2 - \frac{1}{2}\alpha_F^2 + \alpha_F c - c - \theta + \frac{1}{2}}
\end{aligned}
$$

In the following, it will be shown that the indifference reaction functions intersect exactly once in the interval $\alpha_F \in [0, 1)$, $\alpha_U \in [0, 1)$.Suppose there exists at least one intersection in the interval. We consider the first intersection $\alpha_U, \alpha_F$ in the interval. Taking the derivative of $f_F$ with respect to $\alpha_U$ and evaluating it at $\alpha_U, \alpha_F$ yields:

$$
\begin{aligned}
\frac{df_F}{d\alpha_U} = -\frac{\alpha_U - c}{2\alpha_F + 2d + 2\delta - c} &< 1 \\
c - \alpha_U &< 2\alpha_F + 2d + 2\delta - c \\
0 &< 2(d - c) + 2\delta + 2\alpha_F + \alpha_U
\end{aligned}
$$

Where the last equation holds because $d > c + 2\delta$.

Furthermore, $f_F$ is strictly concave in $\alpha_U$.

$$
\frac{d^2 f_F}{d\alpha_U^2} = -\frac{1}{2\alpha_F - c + 2d + 2\delta} - \frac{(c - \alpha_U)^2}{4\left(\alpha_F - \frac{c}{2} + d + \delta\right)^3} < 0
$$

Similarly, $f_U$ is strictly concave in $\alpha_F$. Note that $f_U$ has a unique maximum at $\alpha_F = c$ and is increasing for all $\alpha_F < c$ and decreasing for all $\alpha_F > c$. Hence, there exist two continuous inverse functions $f_{U1}^{-1}(\alpha_U)$ for $\alpha_F \leq c$ and $f_{U2}^{-1}(\alpha_U)$ for $\alpha_F > c$. From the inverse function theorem we know that $f_{U1}^{-1}(\alpha_U)$ is convex and $f_{U2}^{-1}(\alpha_U)$ is concave. To obtain uniqueness of a solution, we need to show that $f(\alpha_U)$ will intersect at most once with one of the inverse functions.

First, for $f_{U1}^{-1}(\alpha_U)$ the derivative at $\alpha_U, \alpha_F$ exceeds unity:

$$\frac{df_U^{-1}}{d\alpha_U} = -\frac{2\alpha_U + 2d - 2\delta - c}{\alpha_F - c} > 1$$
$$2\alpha_U + 2d - 2\delta - c > c - \alpha_F$$
$$2(d - \delta - c) > -\alpha_F - 2\alpha_U$$

Due to the curvature of the functions we get $\frac{df_U^{-1}}{d\alpha_U} > \frac{df_F}{d\alpha_U} \quad \forall \alpha_U > \alpha_U$. Hence the functions can intersect only once. Second suppose there is an intersection for $\alpha_F > c$. To ensure single crossing, we proove that the derivative of $f_F$ is always larger than the derivative of $f_{U2}^{-1}(\alpha_U)$.

$$\frac{df_{U2}^{-1}(\alpha_U)}{d\alpha_U} = \frac{2\alpha_U + 2d - 2\delta - c}{c - \alpha_F} < \frac{c - \alpha_U}{2\alpha_F + 2d + 2\delta - c}$$
$$\Leftrightarrow (2\alpha_U + 2d - 2\delta - c)(2\alpha_F + 2d + 2\delta - c) > (c - \alpha_U)(c - \alpha_F)$$
$$\Leftrightarrow (2\alpha_U + 2d - 2\delta - c)(\alpha_F + 2d + 2\delta) + (2\alpha_U + 2d - 2\delta - c)(\alpha_F - c) > (\alpha_U - c)(\alpha_F - c)$$
$$\Leftrightarrow (2\alpha_U + 2d - 2\delta - c)(\alpha_F + 2d + 2\delta) + (\alpha_U + 2d - 2\delta)(\alpha_F - c) > 0$$

This is the case since $d > 2\delta + c$.

The argument introduces that there can only be one intersection with either curve. To introduce uniqueness we also need to argue that it is not possible that there is exactly one intersection with both of them. Therefore, suppose that the intersection with $f_{U2}^{-1}(\alpha_U)$ is the first intersection. Form the preceding paragraphs we know that in this intersection the slope of $f_F(\alpha_U)$ is larger than the slope of $f_{U2}^{-1}(\alpha_U)$. Since, $f_{U2}^{-1}(\alpha_U)$ is clearly continuously attached to $f_{U1}^{-1}(\alpha_U)$ this means that there can not be another intersection with $f_{U1}^{-1}(\alpha_U)$ if there is only one with $f_{U2}^{-1}(\alpha_U)$. On the other hand if an intersection with $f_{U1}^{-1}(\alpha_U)$ is the first one we know that the slope of $f_F(\alpha_U)$ is smaller than of $f_{U1}^{-1}(\alpha_U)$ in this intersection. Due to the concavity of $f_F(\alpha_U)$ this means that another intersection with $f_{U2}^{-1}(\alpha_U)$ can never be possible. Combining these insights, both curves can intersect at most once. This introduces uniqueness.

Furthermore, we show that an intersection indeed exists for $\alpha_F \geq 0$, $\alpha_U \geq 0$. Look at the indifference conditions expressed as $\alpha_F(\alpha_U)$ and $\alpha_U(\alpha_F)$. Whenever $f_F(0) > f_{U1}^{-1}(0)$, there is an interior equilibrium. This is the case since $f_{U2}^{-1}(\alpha_U)$ is continuously attached to $f_{U1}^{-1}(\alpha_U)$ and $f_U(\alpha_F)$ has a second zero. This zero is always larger than $f_F(0)$. If instead $f_{U1}^{-1}(0) \geq f_F(0)$, there are two possible constellations. If we have $f_{U1}^{-1}(0) \geq f_F(0) > 0$, there is an intersection at $\alpha_F = f_F(0)$, $\alpha_U = 0$. These are one-sided pooling equilibria. If $f_{U1}^{-1}(0) \geq 0 \geq f_F(0)$, there is an intersection at $\alpha_F = \alpha_U = 0$.[10] Thus, there exists a unique equilibrium with $\alpha_F \geq 0$, $\alpha_U \geq 0$. We have,

Furthermore, it can be shown that the specific type of equilibrium depends on the values of the office motivation weight $\theta$ (relative to the other parameters). First, look

[10]In Proposition 2 we prove that for $\alpha_F < c \quad \Rightarrow \quad \alpha_U < c$. Hence, the constellation $0 > f_U^{-1}(0) \geq f_F(0)$ can never arise.

at the borderline case between full pooling and one-sided pooling equilibria. In this borderline case, the indifference condition for agent $i$ is fulfilled for the constellation $\alpha_F = \alpha_U = 0$ (while the underdog strictly prefers the reform independent of his ability, thus also playing $\alpha_U = 0$).

$$4dR_F(0,0,0) = (0)^2 + (2d+2\delta-c)0 + \frac{1}{2}(0)^2 - 0c - (2d+2\delta-1)c - \frac{1}{2} + \theta_h = 0$$
$$\Leftrightarrow \theta_h = \frac{1}{2} + (2d+2\delta-1)c$$

The second borderline value $\theta_m$, which separates one-sided pooling from separating equilibria, is defined similarly. However, in this case the indifference conditions must be satisfied (a) for some positive ability level $\hat{a} \geq 0$ and (b) the lowest-ability opponent $\alpha_U = 0$. Thus, we have two equations with two unknown variables:

$$\begin{aligned} 4dR_F(\hat{a},\hat{a},0) &= \hat{a}^2 + (2d+2\delta-c)\hat{a} - (2d+2\delta-1)c - \frac{1}{2} + \theta_l = 0 \\ 4dR_U(0,\hat{a},0) &= \frac{1}{2}\hat{a}^2 - \hat{a}c - (2d-2\delta-1)c - \frac{1}{2} + \theta_l = 0 \end{aligned}$$

We straightforwardly get the borderline equilibrium values for $\hat{a}$ and $\theta_m$:

$$\begin{aligned} \hat{a} &= -2(d+\delta) + 2\sqrt{(d+\delta)^2 + 2\delta c} \\ \theta_m &= \frac{1}{2} + (2d+2\delta-1)c + \hat{a}(c - \frac{1}{2}\hat{a}) \\ \theta_m &= \frac{1}{2} - c - 4(d+\delta)^2 - 4\delta c + 2(2d+2\delta+c)\sqrt{(d+\delta)^2 + 2\delta c} \end{aligned}$$

We have established the parameter ranges for all three types of equilibria as well as uniqueness and existence for $\alpha_F \geq 0$, $\alpha_U \geq 0$. As shown in the text, reform incentives are always positive for a candidate with highest ability. Thus, the equilibrium cut-offs are located below 1. □

## A.3 Proof of Proposition 2

*Proof.* In any interior equilibrium, the indifference conditions must be satisfied for both politicians. Subtracting equation (5) from (4) and multiplying by 2, we get an additional equilibrium condition:

$$\alpha_F^{*\,2} - \alpha_U^{*\,2} + 4\alpha_F^*(d+\delta) - 4\alpha_U^*(d-\delta) - 8\delta c = 0 \tag{9}$$

First, assume that $\alpha_F^* > c$ holds. Then, $\alpha_U^* > \alpha_F^* > c$ must be true. Rearranging (9) yields:

$$\begin{aligned} &\alpha_F^{*\,2} - \alpha_U^{*\,2} + 4\alpha_F^*(d-\delta) - 4\alpha_U^*(d-\delta) + 8\delta(\alpha_F^* - c) = 0 \\ &\qquad \Rightarrow \alpha_F^{*\,2} - \alpha_U^{*\,2} + 4(\alpha_F^* - \alpha_U^*)(d-\delta) < 0. \end{aligned}$$

As both $\alpha_F^*$ and $\alpha_U^*$ must be positive, the inequality can only be true if $\alpha_U^* > \alpha_F^*$. In a similar way, we can show that $\alpha_F^* < c$ implies $\alpha_U^* < \alpha_F^* < c$. Again rearranging (9):

$$\alpha_F^{*\,2} - \alpha_U^{*\,2} + 4\alpha_F^*(d-\delta) - 4\alpha_U^*(d-\delta) + 8\delta(\alpha_F^* - c) = 0$$
$$\alpha_F^{*\,2} - \alpha_U^{*\,2} + 4(\alpha_F^* - \alpha_U^*)(d-\delta) > 0.$$

Finally, $\alpha_F^* = c$ clearly implies $\alpha_U^* = \alpha_F^*$. Thus, there can not be any other interior equilibrium.

Plugging $\alpha_F^* = \alpha_U^* = c$ into the indifference condition for the underdog (5), we get:

$$4dR_U(c,c,c) = \theta_l + c^2 + (2d - 2\delta - c)c + \frac{1}{2}c^2 - c^2 - (2d - 2\delta - 1)c - \frac{1}{2} = 0$$
$$\theta_l = \frac{1}{2} + \frac{1}{2}c^2 - c$$

For the second part, assume that $\theta > \theta_l$. Using the underdog's indifference condition again, we get the following necessary condition:

$$\begin{aligned}
\theta + \alpha_U^{*\,2} + (2d - 2\delta - c)\alpha_U^* + \frac{1}{2}\alpha_F^{*\,2} - \alpha_F^* c - (2d - 2\delta - 1)c - \frac{1}{2} &= 0 \\
\theta_l + (\alpha_U^* + 2d - 2\delta)(\alpha_U^* - c) + \frac{1}{2}\alpha_F^{*\,2} - \alpha_F^* c + c - \frac{1}{2} &< 0 \\
(\alpha_U^* + 2d - 2\delta)(\alpha_U^* - c) + \frac{1}{2}(\alpha_F^* - c)^2 &< 0
\end{aligned}$$

As the second term is clearly positive, the inequality can only be fulfilled if the first term is negative. Thus, $\theta > \theta_l$ unambiguously implies $\alpha_U^* < c$, establishing the second part of the proposition.

For the first part, assume that $\theta < \theta_l$. Using the underdog's indifference condition, we get the same inequality like before, but this time with the opposite sign:

$$(\alpha_U^* + 2d - 2\delta)(\alpha_U^* - c) + \frac{1}{2}(\alpha_F^* - c)^2 > 0$$

By replacing $(\alpha_F^* - c)^2$ with $(\alpha_U^* - c)^2$, we can increase the left-hand side of this inequality. Thus, the condition above can only be fulfilled if the following is also true:

$$\begin{aligned}
(\alpha_U^* + 2d - 2\delta)(\alpha_U^* - c) + \frac{1}{2}(\alpha_U^* - c)^2 &> 0 \\
\left(\frac{3}{2}\alpha_U^* + 2d - 2\delta - \frac{1}{2}c\right)(\alpha_U^* - c) &> 0
\end{aligned}$$

By the assumption $d > \delta + c$, the first bracket is positive. Thus, we must have $\alpha_U^* > c$ whenever $\theta < \theta_l$ is true. $\square$

## A.4 Proof of Proposition 4

*Proof.* Considering one-sided pooling equilibria first, the favorite's cut-off ability $\alpha_F^*$ is defined by

$$\alpha_F^{*\,2} + (2d + 2\delta - c)\alpha_F^* - (2d + 2\delta - 1)c + \theta - \frac{1}{2} = 0$$

Taking the derivative of $\alpha_F^*$ with respect to $\delta$ gives us $d\alpha_F^*/d\delta = -2(\alpha_F^* - c)/(2\alpha_F^* + 2d + 2\delta - c) > 0$. The positive sign applies as we are in the conflict case with $\alpha_F^* < c$. For the underdog, there are strictly positive reform incentives even at the lowest ability level $\alpha_U^* = 0$. Thus, the corresponding equation actually is an inequality and will remain so following any marginal variation in $\delta$.

For separating equilibria, implicit differentiation of the equilibrium conditions (4) and (5) results in the following expressions:

$$\begin{aligned}
\frac{d\alpha_F^*}{d\delta} &= -2\frac{(c-\alpha_U^*)^2 - (c-\alpha_F^*)(2\alpha_U^* + 2d - 2\delta - c)}{(2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c)} \\
\frac{d\alpha_U^*}{d\delta} &= 2\frac{(c-\alpha_F^*)^2 - (c-\alpha_U^*)(2\alpha_F^* + 2d + 2\delta - c)}{(2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c)}
\end{aligned}$$

For both derivatives, the denominator is given by

$$D = (2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c) > 0$$

Under the assumption $2d > 2\delta + c + 1$, the first term strictly exceeds unity. In contrast, the second term is smaller than $c^2 < 1$ in the conflict case. For the mission case, the second term is strictly smaller than $1/4$, which can be derived from equilibrium conditions (4) and (5). For the underdog, we get for example:

$$\underbrace{(2d - 2\delta + \alpha_U^*)}_{>1}(\alpha_U^* - c) = (1 - \alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - \theta < \frac{1}{2} - \theta$$

Consider the conflict case with $\alpha_U^* < \alpha_F^* < c$. In this case, the numerators of both derivatives are negative. The sign of the numerator for the favorite's cut-off depends on the relative size of $(c - \alpha_F^*)$ and $(c - \alpha_U^*)$. Taking the difference between both equilibrium conditions, we can derive the following inequality:

$$c - \alpha_F^* = \frac{4d - 4\delta + \alpha_F^* + \alpha_U^*}{4d + 4\delta + \alpha_F^* + \alpha_U^*}(c - \alpha_U^*) \geq \frac{d-\delta}{d+\delta}(c - \alpha_U^*) \tag{10}$$

Inserting this expression, we can prove the negative sign of the numerator of $\frac{d\alpha_F^*}{d\delta}$ implying an altoghether positive derivative. Under the conditions $d > \delta + 1$ and $\delta < 1/2$, we get:

$$\begin{aligned}
(c-\alpha_U^*)^2 - (c-\alpha_F^*)(2\alpha_U^* + 2d - 2\delta - c) &\leq \frac{c-\alpha_U^*}{d+\delta}\left[(d+\delta)(c-\alpha_U^*) - (d-\delta)(2\alpha_U^* + 2d - 2\delta - c)\right] \\
&\leq \frac{c-\alpha_U^*}{d+\delta}\left[-(d-\delta)\underbrace{(2d - 2\delta + \alpha_U^*)}_{>2} + 2\delta\underbrace{(c-\alpha_U^*)}_{<1}\right] \\
&< 2\frac{c-\alpha_U^*}{d+\delta}(-d + 2\delta) < 0
\end{aligned}$$

Regarding the underdog's numerator, we can make use of the inequality $c-\alpha_F^* \leq c-\alpha_U^*$ (see Proposition 2) to prove the negative sign:

$$(c-\alpha_F^*)(c-\alpha_F^*) - (c-\alpha_U^*)(2\alpha_F^* + 2d + 2\delta - c) \leq -(c-\alpha_U^*)(2d + 2\delta + 3\alpha_F^* - 2c) < 0$$

Now, consider the mission case with $\alpha_U^* \geq \alpha_F^* > c$. Both numerators are clearly positive, which completes the proof. □

## A.5 Proof of Proposition 5

*Proof.* The derivative of the welfare function is given by equation (7). The partial derivatives of $\hat{W}(\alpha_F^*, \alpha_U^*) = 4dW(\alpha_F^*, \alpha_U^*)$ with respect to $\delta$, $\alpha_F^*$ and $\alpha_U^*$ are as follows:

$$\begin{aligned}
\frac{\partial \hat{W}}{\partial \delta} &= 2\left[(1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right)\right] \\
&= (\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c) > 0 \\
\frac{\partial \hat{W}}{\partial \alpha_F^*} &= (2d + 2\delta + \alpha_F^*)(c-\alpha_F^*) + (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) \\
&= \theta + (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) \\
&= \theta + \frac{1}{2}(\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c) > 0 \\
\frac{\partial \hat{W}}{\partial \alpha_U^*} &= (2d - 2\delta + \alpha_U^*)(c-\alpha_U^*) + (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) \\
&= \theta + (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) - (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) \\
&= \theta - \frac{1}{2}(\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c)
\end{aligned}$$

Plugging these partial derivatives in, equation (7) can be restructured in the following way:

$$\frac{d\hat{W}}{d\delta} = \theta\left(\frac{d\alpha_F^*}{d\delta} + \frac{d\alpha_U^*}{d\delta}\right) + \frac{1}{2}(\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c)\left(2 + \frac{d\alpha_F^*}{d\delta} - \frac{d\alpha_U^*}{d\delta}\right)$$

At $\delta = 0$, $\alpha_F^* = \alpha_U^*$ and the sum of $\frac{d\alpha_F^*}{d\delta}$ and $\frac{d\alpha_U^*}{d\delta}$ equals zero. This implies that $\frac{d\hat{W}}{d\delta} = 0$ at $\delta = 0$ in any interior equilibrium, i. e. in the mission case as well as the conflict case. For the general case $\delta > 0$, we get

$$\frac{d\alpha_F^*}{d\delta} + \frac{d\alpha_U^*}{d\delta} = \frac{2}{D}\left[(2d - 2\delta + 2\alpha_U^* - \alpha_F^*)(c-\alpha_F^*) - (2d + 2\delta + 2\alpha_F^* - \alpha_U^*)(c-\alpha_U^*)\right],$$

where $D > 0$ represents the common denominator of both derivatives (see equation (10)).

Rearrangement of the equilibrium condition gives us the following expressions for $(c-\alpha_F^*)$ and $(c-\alpha_U^*)$:

$$\begin{aligned}
c - \alpha_F^* &= \frac{4d - 4\delta + \alpha_F^* + \alpha_U^*}{8\delta}(\alpha_F^* - \alpha_U^*) \\
c - \alpha_U^* &= \frac{4d + 4\delta + \alpha_F^* + \alpha_U^*}{8\delta}(\alpha_F^* - \alpha_U^*)
\end{aligned}$$

Inserting these equations, the sum of derivatives can be simplified by simple algebra to get:

$$\begin{aligned}\frac{d\alpha_F^*}{d\delta}+\frac{d\alpha_U^*}{d\delta} &= \frac{\alpha_F^*-\alpha_U^*}{4\delta D}\left[(2d-2\delta+2\alpha_U^*-\alpha_F^*)(4d-4\delta+\alpha_F^*+\alpha_U^*)\right.\\ &\quad \left.-(2d+2\delta+2\alpha_F^*-\alpha_U^*)(4d+4\delta+\alpha_F^*+\alpha_U^*)\right]\\ &= \frac{\alpha_F^*-\alpha_U^*}{4\delta D}\underbrace{(3\alpha_U^*-3\alpha_F^*-8\delta)}_{<0}(4d+\alpha_F^*+\alpha_U^*)\end{aligned}$$

The negative sign of the first bracket follows directly for the conflict case ($\alpha_U^* < \alpha_F^*$). For the mission case, it can be established with a combination of both equilibrium functions, yielding (under the condition that $d > 1$):

$$(\alpha_U^*-\alpha_F^*)=\frac{4\delta}{4d+\alpha_F^*+\alpha_U^*}(\alpha_U^*+\alpha_F^*-2c)<\delta(\alpha_U^*+\alpha_F^*-2c)<\delta$$

Consequently, the sign of $\frac{d\alpha_F^*}{d\delta}+\frac{d\alpha_U^*}{d\delta}$ is always contrary to the one of $(\alpha_F^*-\alpha_U^*)$. In the conflict case, the former is negative while it is positive in the conflict case (for $\delta > 0$).

Similarly, we can simplify the expression $\left(2+\frac{d\alpha_F^*}{d\delta}-\frac{d\alpha_U^*}{d\delta}\right)$:

$$\begin{aligned}2+\frac{d\alpha_F^*}{d\delta}-\frac{d\alpha_U^*}{d\delta} &= \frac{2}{D}\left[(2\alpha_F^*+2d+2\delta-c)(2\alpha_U^*+2d-2\delta-c)-(\alpha_F^*-c)(\alpha_U^*-c)\right.\\ &\quad -(c-\alpha_U^*)^2+(c-\alpha_F^*)(2\alpha_U^*+2d-2\delta-c)-(c-\alpha_F^*)^2\\ &\quad \left.+(c-\alpha_U^*)(2\alpha_F^*+2d+2\delta-c)\right]\\ &= \frac{2}{D}\left[\underbrace{(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)}_{>1}-\underbrace{(2c-\alpha_F^*-\alpha_U^*)^2}_{<1}\right]>0\end{aligned}$$

Combining these expressions and using the equality $(\alpha_U^*+\alpha_F^*-2c)=\frac{4d+\alpha_F^*+\alpha_U^*}{4\delta}(\alpha_U^*-\alpha_F^*)$, we get the final expression for the welfare effect of $\delta$ (for $\delta > 0$):

$$\begin{aligned}\frac{d\hat{W}}{d\delta} &= \theta\left(\frac{d\alpha_F^*}{d\delta}+\frac{d\alpha_U^*}{d\delta}\right)+\frac{1}{2}(\alpha_U^*-\alpha_F^*)(\alpha_U^*+\alpha_F^*-2c)\left(2+\frac{d\alpha_F^*}{d\delta}-\frac{d\alpha_U^*}{d\delta}\right)\\ &= \theta\frac{\alpha_F^*-\alpha_U^*}{4\delta D}(3\alpha_U^*-3\alpha_F^*-8\delta)(4d+\alpha_F^*+\alpha_U^*)\\ &\quad +\frac{(\alpha_U^*-\alpha_F^*)(\alpha_U^*+\alpha_F^*-2c)}{D}\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right]\end{aligned}$$

For the mission case, the whole expression is unambiguously positive as long as $\delta > 0$. As the derivative equals zero at $\delta = 0$, this implies a global minimum of the welfare function at $\delta = 0$ whenever we are in the mission case and therefore establishes the last part of Proposition 5.

For the conflict case, inserting the relation between $(\alpha_U^*+\alpha_F^*-2c)$ and $(\alpha_U^*-\alpha_F^*)$

leads to the following expression:

$$
\begin{aligned}
\frac{d\hat{W}}{d\delta} &= \theta\frac{\alpha_F^*-\alpha_U^*}{4\delta D}(3\alpha_U^*-3\alpha_F^*-8\delta)(4d+\alpha_F^*+\alpha_U^*) \\
&\quad +\frac{(\alpha_F^*-\alpha_U^*)^2(4d+\alpha_F^*+\alpha_U^*)}{4\delta D}\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right] \\
&= \frac{(\alpha_F^*-\alpha_U^*)(4d+\alpha_F^*+\alpha_U^*)}{4\delta D}\left\{\underbrace{(3\alpha_U^*-3\alpha_F^*-8\delta)\theta}_{<0}\right. \\
&\quad \left.+(\alpha_F^*-\alpha_U^*)\underbrace{\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right]}_{>0}\right\}
\end{aligned}
$$

In order to establish the welfare effect in the conflict case, we have to show that the expression in the bracket is negative whenever $\delta > 0$. This is clearly true for $\theta_l = \frac{1}{2}+\frac{1}{2}c^2-c$, where we switch from the mission case to the conflict case. At this value of $\theta$, both cut-offs $\alpha_F^*$ and $\alpha_U^*$ coincide independently of $\delta$ (see Proposition 2). Consequently, all terms in the bracket except $-8\delta\theta < 0$ vanish.

For all values $\theta > \theta_l$, we can show that the term in brackets is further decreasing in $\theta$. Its derivative is given by

$$
\begin{aligned}
\frac{d\{\}}{d\theta} &= -3\theta\left(\frac{d\alpha_F^*}{d\theta}-\frac{d\alpha_U^*}{d\theta}\right)-(8\delta+3\alpha_F^*-3\alpha_U^*) \\
&\quad +\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right]\left(\frac{d\alpha_F^*}{d\theta}-\frac{d\alpha_U^*}{d\theta}\right) \\
&\quad +\frac{d\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right]}{d\theta}(\alpha_F^*-\alpha_U^*) \\
&< -(8\delta+3\alpha_F^*-3\alpha_U^*)+\left[(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2\right]\left(\frac{d\alpha_F^*}{d\theta}-\frac{d\alpha_U^*}{d\theta}\right)
\end{aligned}
$$

Plugging in the derivative of $(\alpha_F^*-\alpha_U^*)$, which is given by $\frac{d(\alpha_F^*-\alpha_U^*)}{d\theta} = \frac{4\delta+\alpha_F^*-\alpha_U^*}{D}$ (where $D$ is defined like in equation (10)), we get:

$$
\frac{d\{\}}{d\theta} < (4\delta+\alpha_F^*-\alpha_U^*)\left[\frac{(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2}{(2\alpha_F^*+2d+2\delta-c)(2\alpha_U^*+2d-2\delta-c)-(\alpha_F^*-c)(\alpha_U^*-c)}-2\right]
$$

Clearly, the sign of this expression depends on the sign of the bracket which can be shown to be negative:

$$
\frac{(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2}{(2\alpha_F^*+2d+2\delta-c)(2\alpha_U^*+2d-2\delta-c)-(\alpha_F^*-c)(\alpha_U^*-c)} < 2
$$

$$
(2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)-(2c-\alpha_F^*-\alpha_U^*)^2 < 2(2\alpha_F^*+2d+2\delta-c)(2\alpha_U^*+2d-2\delta-c)-2(\alpha_F^*-c)(\alpha_U^*-c)
$$

$$
(4d+4\delta+2\alpha_F^*+\alpha_U^*-2c)(c-\alpha_U^*)+(4d-4\delta+2\alpha_U^*+\alpha_F^*-2c)(c-\alpha_F^*) < (2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)
$$

$$
(2d+2\delta+\alpha_F^*)\underbrace{(2c-2\alpha_U^*)}_{<1}+(2d-2\delta+\alpha_U^*)\underbrace{(2c-2\alpha_F^*)}_{<1}+\underbrace{(\alpha_F^*+\alpha_U^*-2c)(2c-\alpha_F^*-\alpha_U^*)}_{<0} < (2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)
$$

$$
(2d+2\delta+\alpha_F^*)+(2d-2\delta+\alpha_U^*) < (2d+2\delta+\alpha_F^*)(2d-2\delta+\alpha_U^*)
$$

By the assumption $d > \delta+1$, we have $2d+2\delta+\alpha_F^* > 2d-2\delta+\alpha_U^* > 2$. Consequently, the last inequality is unambiguously true and the second part of the proposition is established for any $\delta > 0$. Recall again that the derivative equals zero at $\delta = 0$. Thus, we have a global maximum of the welfare function at $\delta = 0$ for the conflict case. □

## A.6 Proof of Proposition 6

*Proof.* Concerning one-sided pooling equilibria, the derivative of $\alpha_F^*$ with respect to the electoral risk parameter $d$ is identical to the derivative with respect to $\delta$ which has been proven to be strictly positive in the preceding proposition:

$$\frac{d\alpha_F^*}{dd} = \frac{d\alpha_F^*}{d\delta} = -1 + \frac{1}{2}\left(\frac{2(d+\delta-c/2)+2c}{\sqrt{(\frac{c}{2}-d-\delta)^2+(2d+2\delta-1)c+\frac{1}{2}-\theta}}\right) > 0$$

For separating equilibria, we use the same procedure as before. The denominator is positive for $\alpha_F^*$ and negative for $\alpha_U^*$. For $\frac{d\alpha_F^*}{dd}$ the numerator is given by:

$$\begin{aligned}
\frac{1}{2}N_{dI} &= \frac{1}{2}\left[\frac{\partial f_1}{\partial d}\frac{\partial f_2}{\partial \alpha_U^*} - \frac{\partial f_2}{\partial d}\frac{\partial f_1}{\partial \alpha_U^*}\right] \\
&= (\alpha_F^*-c)(2\alpha_U^*+2d-2\delta-c)-(\alpha_U^*-c)(\alpha_U^*-c) \\
&= (\alpha_F^*-\alpha_U^*)(\alpha_U^*-c)+(\alpha_F^*-c)(\alpha_U^*+2d-2\delta)
\end{aligned}$$

For the conflict case, this expression is unambiguously negative. Consequently, the derivative $\frac{d\alpha_F^*}{dd}$ is positive.

For the mission, we can again derive a relation between $(\alpha_F^*-c)$ and $(\alpha_U^*-c)$ from the indifference conditions:

$$\alpha_F^*-c = \frac{4d-4\delta+\alpha_F^*+\alpha_U^*}{4d+4\delta+\alpha_F^*+\alpha_U^*}(\alpha_U^*-c) > \frac{d-\delta}{d+\delta}(\alpha_U^*-c)$$

Plugging in this expression, we get:

$$\begin{aligned}
\frac{1}{2}N_{dI} &= (\alpha_F^*-c)(2\alpha_U^*+2d-2\delta-c)-(\alpha_U^*-c)(\alpha_U^*-c) \\
&> \frac{\alpha_U^*-c}{d+\delta}\left[(d-\delta)(2\alpha_U^*+2d-2\delta-c)-(d+\delta)(\alpha_U^*-c)\right] \\
&> \frac{\alpha_U^*-c}{d+\delta}\left[(d-\delta)(2d-2\delta-c)+c(d+\delta)+\alpha_U^*(d-3\delta)\right]
\end{aligned}$$

The numerator is positive if and only if the term in brackets is positive. Given the assumptions $d-2\delta-c > 0$ and $d \geq 1$, and making use of the upper bound for the cut-off ability ($\alpha_U^* \leq 1$), we can show that this condition is always satisfied.

$$\begin{aligned}
(d-\delta)(2d-2\delta-c)+c(d+\delta)+\alpha_U^*(d-3\delta) &> (d-\delta)d-\delta\alpha_U^* \\
&> \delta(d-\alpha_U^*)+cd > 0
\end{aligned}$$

For $\frac{d\alpha_U^*}{dd}$, the denominator is negative. The numerator is given by

$$\begin{aligned}\frac{1}{2}N_{dO} &= \frac{1}{2}\left[\frac{\partial f_1}{\partial d}\frac{\partial f_2}{\partial \alpha_F^*} - \frac{\partial f_2}{\partial d}\frac{\partial f_1}{\partial \alpha_F^*}\right] \\ &= (\alpha_F^* - c)(\alpha_F^* - c) - (\alpha_U^* - c)(2\alpha_F^* + 2d + 2\delta - c) \\ &= (\alpha_F^* - \alpha_U^*)(\alpha_F^* - c) - (\alpha_U^* - c)(\alpha_F^* + 2d + 2\delta)\end{aligned}$$

For the mission case, the numerator is negative, implying an overall negative derivative. For the conflict case, the numerator can be shown to be positive:

$$\begin{aligned}\frac{1}{2}N_{dO} &= (c - \alpha_U^*)(\alpha_F^* + 2d + 2\delta) - (c - \alpha_F^*)(\alpha_F^* - \alpha_U^*) \\ &> (c - \alpha_U^*)(\alpha_F^* + 2d + 2\delta) - (c - \alpha_U^*)(\alpha_F^* - \alpha_U^*) \\ &= (c - \alpha_U^*)(\alpha_U^* + 2d + 2\delta) > 0\end{aligned}$$

Thus, the derivative $\frac{d\alpha_U^*}{dd}$ is positive, too. □

## A.7 Proof of Proposition 7

The total differential of function W is given by:

$$\frac{dW}{dd} = \frac{\partial W}{\partial d} + \frac{\partial W}{\partial \alpha_F^*}\frac{\partial \alpha_F^*}{\partial d} + \frac{\partial W}{\partial \alpha_U^*}\frac{\partial \alpha_U^*}{\partial d}$$

with:

$$-4d^2\frac{\partial W}{\partial d} =$$
$$2\delta\left((1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right)\right) + \alpha_F^*(1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) + \alpha_U^*(1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right)$$

$$\begin{aligned}\frac{\partial W}{\partial \alpha_F^*} &= \frac{1}{4d}(2d + 2\delta + \alpha_F^*)(c - \alpha_F^*) + (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) \\ &= \theta + (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) \\ \frac{\partial \hat{W}}{\partial \alpha_U^*} &= \frac{1}{4d}(2d - 2\delta + \alpha_U^*)(c - \alpha_U^*) + (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) \\ &= \theta + (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right) - (1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right)\end{aligned}$$

the whole equation simplifies to:

$$\overbrace{\theta\left(\frac{\partial \alpha_F^*}{\partial d} + \frac{\partial \alpha_U^*}{\partial d}\right)}^{A} + \overbrace{\left((1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) - (1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right)\right)}^{B}\overbrace{\left(\frac{\partial \alpha_F^*}{\partial d} - \frac{\partial \alpha_U^*}{\partial d} - \frac{2\delta}{d}\right)}^{C}$$
$$\underbrace{-\frac{1}{d}\left[\alpha_F^*(1-\alpha_F^*)\left(\frac{1+\alpha_F^*}{2} - c\right) + \alpha_U^*(1-\alpha_U^*)\left(\frac{1+\alpha_U^*}{2} - c\right)\right]}_{D}$$

For the mission case we get from the comparative statics: $\frac{d\alpha_F^*}{dd} < 0$ and $\frac{d\alpha_U^*}{dd} < 0$. This induces directly that $A < 0$. Moreover:

$$\frac{\partial}{\partial a}(1-a)\left(\frac{1+a}{2}-c\right) = -\left(\frac{1+a}{2}-c\right) + (1-a)\frac{1}{2} = c - a$$

which is negative for the mission case. Thus $B > 0$. D is also negative since all terms in the brackets are positive because of the mission case. Consider now C:

$$\begin{aligned}
C = &\frac{2c - \alpha_F^* - \alpha_U^*}{4d + \alpha_F^* + \alpha_U^*} \frac{(4d - 4\delta + \alpha_F^* + \alpha_U^*)(\alpha_U^* - \alpha_F^* - 4\delta) - 8\delta(2\alpha_F^* + \alpha_U^* + 2d + 2\delta - 2c)}{D} - \frac{2\delta}{d} < 0 \\
\Leftrightarrow &\frac{d(\alpha_F^* + \alpha_U^* - 2c)((4d - 4\delta + \alpha_F^* + \alpha_U^*)(\alpha_F^* - \alpha_U^* + 4\delta) + 8\delta(2\alpha_F^* + \alpha_U^* + 2d + 2\delta - 2c))}{d(4d + \alpha_F^* + \alpha_U^*)D} \\
- &\frac{2\delta((4d + \alpha_F^* + \alpha_U^*)((2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c)))}{d(4d + \alpha_F^* + \alpha_U^*)D} < 0 \\
\Leftrightarrow &\ d(\alpha_F^* + \alpha_U^* - 2c)((4d - 4\delta + \alpha_F^* + \alpha_U^*)(\alpha_F^* - \alpha_U^* + 4\delta) + 8\delta(2\alpha_F^* + \alpha_U^* + 2d + 2\delta - 2c)) < \\
&2\delta((4d + \alpha_F^* + \alpha_U^*)((2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c))) \\
\Leftarrow &2d(\alpha_F^* + \alpha_U^* - 2c)(8d + 3\alpha_U^* + 5\alpha_F^* - 4c) < \\
&(4d + \alpha_F^* + \alpha_U^*)((\alpha_F^* - c)(\alpha_U^* + 2d - 2\delta) + (\alpha_U^* - c)(\alpha_F^* + 2d + 2\delta) + (\alpha_F^* + 2d + 2\delta)(\alpha_U^* + 2d - 2\delta)) \\
\Leftrightarrow &2d(\alpha_F^* + \alpha_U^* - 2c)(8d + 3\alpha_U^* + 5\alpha_F^* - 4c) < \\
&(4d + \alpha_F^* + \alpha_U^*)(2d(2\alpha_F^* - 2c + 2\alpha_U^* + 2d) + \underbrace{(\alpha_F^* - c)(\alpha_U^* - 2\delta) + (\alpha_U^* - c)(\alpha_F^* + 2\delta)}_{>0} + (\alpha_F^* + 2\delta)(\alpha_U^* - 2\delta)) \\
\Leftarrow &2d(\alpha_F^* + \alpha_U^* - 2c)(\alpha_U^* + 3\alpha_F^* - 4c) < \\
&(2d + 2c)2d(4d + \alpha_F^* + \alpha_U^*) + (4d + \alpha_F^* + \alpha_U^*)((\alpha_F^* + 2\delta)(\alpha_U^* - 2\delta)) \\
\Leftrightarrow &2d(\alpha_F^* + \alpha_U^* - 2c)\underbrace{(-2d + \alpha_U^* + 3\alpha_F^* - 6c)}_{<0} < \\
&(2d + 2c)2d(4d + 2c) + (4d + \alpha_F^* + \alpha_U^*)((\alpha_F^* + 2\delta)(\alpha_U^* - 2\delta)) \\
\Leftarrow &(2d + 2c)2d(4d + 2c) + (4d + \alpha_F^* + \alpha_U^*)(\alpha_F^* + 2\delta)(\alpha_U^* - 2\delta) > 0 \\
\Leftarrow &(2d + 2c)2d(4d + 2c) - 2\delta(4d + \alpha_F^* + \alpha_U^*)(\alpha_F^* + 2\delta) + (4d + \alpha_F^* + \alpha_U^*)(\alpha_F^* + 2\delta)\alpha_U^* > 0
\end{aligned}$$

which is true since the last part is positive in every single entry and the first part since $d > 2\delta + 1$. In total we get a negative sign of C which implies that the whole expression is negative. For the second part of the proposition, consider the transition from the full pooling to partial pooling. Clearly, the derivative of $\alpha_F^*$ is positive and the derivative of the welfare function also. On the other hand, the direct effect of $d$ is zero in the limiting case and since it is continuous in $\theta$ smaller than $\epsilon$ for $\theta$ close to $\theta_h$. Consequently, $\frac{dW}{dd} > 0$. This implies a negative welfare effect of salience.

## A.8 Welfare symmetrical case

In the symmetrical case, $\delta = 0$, both candidates' cut-off ability $\alpha$ is defined by:

$$\theta + (2d + \alpha)(\alpha - c) - (1 - \alpha)(\frac{1 + \alpha}{2} - c) = 0$$

There is a unique root in $[0, 1)$ as long as $d > c$ and $\theta \leq \frac{1}{2} + (2d - 1)c$. Implicit differentiation gives the derivative of $\alpha$ with respect to $d$:

$$\frac{d\alpha}{dd} = \frac{c - \alpha}{\frac{3}{2}\alpha + d - c}$$

As long as $\theta > \frac{1}{2}(1 - c)^2$, we are in the conflict case with $\alpha < c$ and the derivative above is positive, otherwise the derivative is negative.

The welfare function is given by

$$W = \frac{1}{2d}(2d+\alpha)(1-\alpha)(\frac{1+\alpha}{2}-c)$$

Its derivative with respect to $d$ follows as

$$\begin{aligned}\frac{dW}{dd} &= \frac{-\alpha(1-\alpha)(\frac{1+\alpha}{2}-c)}{2d^2} + \frac{1}{2d}\underbrace{[(1-\alpha)(\frac{1+\alpha}{2}-c)-(2d+\alpha)(\alpha-c)]}_{=\theta}\frac{d\alpha}{dd} \\ &= \frac{1}{2d^2}[\theta d\frac{c-\alpha}{\frac{3}{2}\alpha+d-c} - \alpha(1-\alpha)(\frac{1+\alpha}{2}-c)]\end{aligned}$$

We have to distinguish three cases:

(1) In the mission case, $\alpha > c$, both terms in the brackets are strictly negative. Thus, any increase in salience enhances welfare in the mission case.

(2) If $\alpha \leq 2c-1 < c$ which is only possible if $c > \frac{1}{2}$, the first term is strictly positive and the second term is non-negative. Thus, decreasing salience is always beneficial until $\alpha > 2c-1$.

(3) If $\alpha \in (2c-1, c)$, the first term is positive and the second term is negative. In the following, I will consider this case in more detail. Consider the term in brackets:

$$F(\theta,d) = \underbrace{\theta d\frac{c-\alpha}{\frac{3}{2}\alpha+d-c}}_{F_1} - \underbrace{\alpha(1-\alpha)(\frac{1+\alpha}{2}-c)}_{F_2}$$

First, note that there is a one-to-one relationship between $\alpha$ on the one hand and $d$ on the other hand: $\frac{d\alpha}{dd} > 0$.

Second, fix any level of $\theta$ and $c$. The function $F$ is monotonically decreasing in $d$:

$$\frac{dF}{dd} = \frac{\partial F_1}{\partial d} + \frac{\partial F_1}{\partial \alpha}\frac{d\alpha}{dd} - \frac{\partial F_2}{\partial \alpha}\frac{d\alpha}{dd}$$

The derivative of $F_1$ can be shown to be strictly negative:

$$\begin{aligned}\frac{\partial F_1}{\partial d} + \frac{\partial F_1}{\partial \alpha}\frac{d\alpha}{dd} &= \theta\left[\frac{(c-\alpha)(\frac{3}{2}\alpha-c)}{(\frac{3}{2}\alpha+d-c)^2} - d\frac{d+\frac{1}{2}c}{(\frac{3}{2}\alpha+d-c)^2}\frac{d\alpha}{dd}\right] \\ &= \frac{\theta}{(\frac{3}{2}\alpha+d-c)^2}\left[(c-\alpha)(\frac{3}{2}\alpha-c) - d(d+\frac{1}{2}c)\frac{c-\alpha}{\frac{3}{2}\alpha+d-c}\right] \\ &= \frac{\theta(c-\alpha)}{(\frac{3}{2}\alpha+d-c)^3}\left[\underbrace{(\frac{3}{2}\alpha-c)}_{<\frac{1}{2}c}\underbrace{(\frac{3}{2}\alpha+d-c)}_{<d+\frac{1}{2}c} - d(d+\frac{1}{2}c)\right] < 0\end{aligned}$$

In contrast, the derivative of $F_2$ with respect to $d$ is strictly positive:

$$\begin{aligned}\frac{\partial F_2}{\partial \alpha}\frac{d\alpha}{dd} &= [(1-\alpha)(\frac{1+\alpha}{2}-c)-\alpha(\frac{1+\alpha}{2}-c)+\alpha\frac{1-\alpha}{2}]\frac{d\alpha}{dd} \\ &= [(1-\alpha)(\frac{1+\alpha}{2}-c)+\alpha(c-\alpha)]\frac{d\alpha}{dd}>0\end{aligned}$$

Thus, we know that $\frac{dW}{dd}$ is monotonically decreasing in $d$. We continue by looking at a value of $d = d_0$ such that $\alpha(d_0, c, \theta) = 0$ in equilibrium (note that these cases only exist for sufficiently large $\theta$; a sufficient condition is given by $\theta \geq \frac{1}{2} + 2c^2 - c$). At $d_0$, we get a strictly positive welfare effect of increasing $d$:

$$\frac{dW}{dd}\mid_{d=d_0} = \frac{1}{2d^2}[\theta d\frac{c-\alpha}{\frac{3}{2}\alpha+d-c}-0]>0$$

On the other hand, note that $\alpha \to c$ for $d \to \infty$. Thus, we get

$$\lim_{d\to\infty}\frac{dW}{dd} = \frac{1}{2d^2}[0-\alpha(1-\alpha)(\frac{1+\alpha}{2}-c)]<0$$

Combining these insights, we know that for any $\theta \geq \frac{1}{2} + 2c^2 - c$, there will be a unique level of $d = d^*(c, \theta) > c > 0$ such that $\frac{dW}{dd}\mid_{d=d^*} = 0$. Thus, $d^*(c, \theta) > 0$ represents the optimal level of salience.

Finally, we can also show that the welfare effect of $d$ is monotonically increasing in $\theta$. Note that $\frac{d\alpha}{d\theta} < 0$ holds throughout. Differentiating the derivative $\frac{dW}{dd}$ with respect to $\theta$, we get:

$$\begin{aligned}\frac{d\frac{dW}{dd}}{d\theta} &= \frac{1}{2d^2}\left\{\underbrace{d\frac{c-\alpha}{\frac{3}{2}\alpha+d-c}}_{>0}-\underbrace{\left[\theta d\frac{2d+\frac{1}{2}c}{(\frac{3}{2}\alpha+d-c)^2}+(1-\alpha)(\frac{1+\alpha}{2}-c)+\alpha(c-\alpha)\right]}_{>0}\underbrace{\frac{d\alpha}{d\theta}}_{<0}\right\} \\ &> 0\end{aligned}$$

Consequently, the optimal level of salience $d^*(c, \theta)$ is strictly increasing in $\theta$. Define $\theta_d(c) > \theta_l = \frac{1}{2}(1-c)^2$ implicitly as the level of office motivation $\theta$ at which the optimal magnitude of salience is given by $d^*(c, \theta_d(c)) = \max\left\{c, \frac{1}{2}\right\}$. We consequently know that $\frac{dW}{dd}\mid_{d=c,\theta>\theta_d(c)} > 0$ which implies that $d^*(\theta, c) > \max\left\{c, \frac{1}{2}\right\}$ for all $\theta > \theta_d(c)$, and constraining electoral competition is socially optimal.

Note that for all values $d < c$, the equilibrium is not properly defined, and for $d < \frac{1}{2}$, the equilibrium winning probabilities are outside the interval $(0, 1)$, such that there is essentially no probabilistic voting and no limitation of political competition.

## A.9 Proof of Proposition 8

*Proof.* First, consider an interior equilibrium. The cut-off abilities $\alpha_F^*$ and $\alpha_U^*$ satisfy the equilibrium conditions (4) and (5). Implicit differentiation gives the following expressions:

$$\begin{aligned}
\frac{d\alpha_F^*}{d\theta} &= -\frac{2d - 2\delta + \alpha_U^*}{(2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c)} \\
\frac{d\alpha_U^*}{d\theta} &= -\frac{2d + 2\delta + \alpha_F^*}{(2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c)}
\end{aligned}$$

First, note that both derivatives have the same denominator $D$ which can be shown to be strictly positive by the assumption $d > \delta + 1$:

$$D = (2\alpha_F^* + 2d + 2\delta - c)(2\alpha_U^* + 2d - 2\delta - c) - (\alpha_F^* - c)(\alpha_U^* - c) > 0$$

Second, the same assumption guarantees the positive sign of both numerators. Consequently, both derivatives are strictly negative in any interior equilibrium.

Finally, consider one-sided pooling equilibria. The derivative of $\alpha_F^*$ with respect to $\theta$ is strictly negative:

$$\frac{d\alpha_F^*}{d\theta} = -\frac{1}{2}\frac{1}{\sqrt{(\frac{c}{2} - d - \delta)^2 + (2d + 2\delta - 1)c + \frac{1}{2} - \theta}} < 0$$

□

## A.10 Proof of Proposition 9

*Proof.* Variations of $\theta$ don't have any direct effect on social welfare, but an indirect effect via the induced changes in both cut-offs:

$$\frac{d\hat{W}}{d\theta} = \frac{\partial \hat{W}}{\partial \alpha_F^*}\frac{d\alpha_F^*}{d\theta} + \frac{\partial \hat{W}}{\partial \alpha_U^*}\frac{d\alpha_U^*}{d\theta}$$

The partial derivatives of $\hat{W}(\alpha_F^*, \alpha_U^*) = 4dW(\alpha_F^*, \alpha_U^*)$ with respect these $\alpha_F^*$ and $\alpha_U^*$ are given by:

$$\begin{aligned}
\frac{\partial \hat{W}}{\partial \alpha_F^*} &= (2d + 2\delta + \alpha_F^*)(c - \alpha_F^*) + (1 - \alpha_F^*)\left(\frac{1 + \alpha_F^*}{2} - c\right) \\
&= \theta + (1 - \alpha_F^*)\left(\frac{1 + \alpha_F^*}{2} - c\right) - (1 - \alpha_U^*)\left(\frac{1 + \alpha_U^*}{2} - c\right) \\
&= \theta + \frac{1}{2}(\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c) > 0 \\
\frac{\partial \hat{W}}{\partial \alpha_U^*} &= (2d - 2\delta + \alpha_U^*)(c - \alpha_U^*) + (1 - \alpha_U^*)\left(\frac{1 + \alpha_U^*}{2} - c\right) \\
&= \theta + (1 - \alpha_U^*)\left(\frac{1 + \alpha_U^*}{2} - c\right) - (1 - \alpha_F^*)\left(\frac{1 + \alpha_F^*}{2} - c\right) \\
&= \theta - \frac{1}{2}(\alpha_U^* - \alpha_F^*)(\alpha_U^* + \alpha_F^* - 2c)
\end{aligned}$$

For the conflict case, the partial derivatives of the welfare function with respect to both cut-offs are always strictly positive. As both cut-offs are strictly decreasing in $\theta$, we immediately get a negative welfare effect.

For the mission case, this negative sign is not generally true. Inserting the partial derivatives of $W$ as well as the derivatives of both cut-offs with respect to $\theta$, we get:

$$\begin{aligned}\frac{d\hat{W}}{d\theta} &= \theta\left(\frac{d\alpha_F^*}{d\theta}+\frac{d\alpha_U^*}{d\theta}\right)+\frac{1}{2}(\alpha_U^*-\alpha_F^*)(\alpha_U^*+\alpha_F^*-2c)\left(\frac{d\alpha_F^*}{d\theta}-\frac{d\alpha_U^*}{d\theta}\right)\\ &= -\theta\frac{4d+\alpha_F^*+\alpha_U^*}{D}+\frac{1}{2}(\alpha_U^*-\alpha_F^*)(\alpha_U^*+\alpha_F^*-2c)\frac{4\delta+\alpha_F^*-\alpha_U^*}{D}\end{aligned}$$

Making use of both equilibrium conditions, we can replace $\alpha_U^*+\alpha_F^*-2c$ by the term $\frac{(\alpha_U^*-\alpha_F^*)(4d+\alpha_F^*+\alpha_U^*)}{4\delta}$, yielding:

$$\frac{d\hat{W}}{d\theta} = \frac{4d+\alpha_F^*+\alpha_U^*}{2D}\left[(\alpha_U^*-\alpha_F^*)^2\frac{4\delta+\alpha_F^*-\alpha_U^*}{4\delta}-2\theta\right]$$

It is possible to show that $\delta+\alpha_F^*-\alpha_U^*>0$ is always satisfied, implying a positive sign of the first term in brackets. Thus, the whole expression must be strictly positive whenever $\theta=0$ and $\delta>0$. Furthermore, the complete term in brackets is decreasing in $\theta$:

$$\begin{aligned}\frac{d\,[\dots]}{d\theta} &= 2(\alpha_F^*-\alpha_U^*)\frac{d(\alpha_F^*-\alpha_U^*)}{d\theta}+\frac{3}{4}\frac{(\alpha_F^*-\alpha_U^*)^2}{\delta}\frac{d(\alpha_F^*-\alpha_U^*)}{d\theta}\\ &= \underbrace{(\alpha_F^*-\alpha_U^*)}_{<0}\underbrace{\frac{d(\alpha_F^*-\alpha_U^*)}{d\theta}}_{>0}\left[2+\frac{3}{4}\underbrace{\frac{\alpha_F^*-\alpha_U^*}{\delta}}_{>-1}\right]<0\end{aligned}$$

Thus, there is a unique $\tilde{\theta}(\delta)$ such that $\frac{d\hat{W}}{d\theta}>0$ if and only if $\theta<\tilde{\theta}$.

This beneficial effect of office motivation is due to the over-shooting of $\alpha_U^*$ for very low values of $\theta$, i. e. $\alpha_U^*>\alpha_U^{SO}$. Whenever we are in such a constellation, increasing office motivation induces a lower welfare contribution by the favorite, but a higher contribution by the underdog. It is easy to show that over-shooting results for if and only if $\theta<\bar{\theta}(\delta)$ where $\bar{\theta}(\delta)=\frac{1}{2}(\alpha_U^*-\alpha_F^*)(\alpha_F^*+\alpha_U^*-2c)=\frac{1}{2}(\alpha_U^*-\alpha_F^*)(4d+\alpha_F^*+\alpha_U^*)/(4\delta)>\tilde{\theta}(\delta)$.

This implies that we have $\frac{d\hat{W}}{d\theta}=0$ at a unique level $\tilde{\theta}(\delta)$ which consequently represents the socially optimal level of office motivation from a second-best perspective. The equilibrium induced by this value of $\theta$ necessarily features over-shooting by the underdog.

□