Submission Number: PET11-11-00287

Commuting-induced spillovers and the case for efficiency-enhancing local wage taxes

João Filipe
NOVA School of Business and Economics

Abstract

This paper presents a model of a duo-centric linear city where agents choose in which jurisdiction they want to work. Jurisdictions are unequally productive and local governments use a head tax and, possibly, a source-based wage tax, to finance a local public good. Each agent derives utility from both the local public good of the jurisdiction where he works and where he lives; thus, interjurisdictional commuting generates endogenous spillovers. We analyze the tax competition equilibrium when local governments only use the head tax or both the head and the wage tax and compare it to the utilitarian benchmark. We show that local public goods are always underprovided in the most productive jurisdiction, but may be overprovided in the least productive one. We also show that distortive source-based wage taxation may improve upon the equilibrium with residence taxes alone, as it allows to charge commuters with part of the cost of the public good they enjoy.

I am grateful to Susana Peralta for precious comments and suggestions. Financial support from Fundação para a Ciência e a Tecnologia, Portugal, is gratefully acknowledged.
Submitted: March 15, 2011.

Commuting-induced spillovers and the case for efficiency-enhancing local wage taxes*

João Filipe†
NOVA School of Business and Economics

March 2011

Abstract

This paper presents a model of a duo-centric linear city where agents choose in which jurisdiction they want to work. Jurisdictions are unequally productive and local governments use a head tax and, possibly, a source-baseed wage tax, to finance a local public good. Each agent derives utility from both the local public good of the jurisdiction where he works and where he lives; thus, interjurisdictional commuting generates endogenous spillovers. We analyze the tax competition equilibrium when local governments only use the head tax or both the head and the wage tax and compare it to the utilitarian benchmark. We show that local public goods are always underprovided in the most productive jurisdiction, but may be overprovided in the least productive one. We also show that distortive source-based wage taxation may improve upon the equilibrium with residence taxes alone, as it allows to charge commuters with part of the cost of the public good they enjoy.

Keywords: Tax competition, Commuting, Local Public Good Spillover, Median Voter Equilibria.

JEL Classification: H23, H71, H73, R23, R5.

---

*I am grateful to Susana Peralta for precious comments and suggestions. Financial support from Fundação para a Ciência e a Tecnologia, Portugal, is gratefully acknowledged.

†Address: NOVA School of Business and Economics, Campus de Campolide, 1099-032 Lisboa, Portugal; Phone: + 351 21 380 16 00; Fax: + 351 21 388 60 73. E-mail: *jfilipe@novasbe.pt*

1 Introduction

In this paper we look at the impact of local level fiscal decentralization on the provision of public goods in a framework where agents commute and local governments provide public goods (or publicly provided private goods) with an endogenous spillover effect. The reality we are trying to model is clearly pointed by Fisher (1996, p.6) when he writes that "Many individuals live in one city, work in another, and do most of their shopping at stores or a shopping mall in still another locality". The spillovers we want to analyze are due to the fact that agents reside in one place but can work in a different one and therefore can be subject to two different local governments.

This framework encompasses a large variety of possible forms of local governments, from different jurisdictions in one metropolitan area to neighbooring cities or even states with common borders as long as it makes sense to have agents commuting from one to the other. As stated in Peralta (2007) "there is extensive evidence of the increasing importance of inter-jurisdictional commuting, possibly fostered by the improvement in transportation technologies". Such increasing importance is documented for example in Shields and Swenson (2000), Glaeser et al. (2001) and Renkow (2003) using US data, by Van Ommeren et al. (1999) for The Netherlands or Cameron and Muellbauer (1998) for Great Britain. In all this papers we can find clear evidence that both the number of commuters and the commuting distance has been increasing in the last 40 or 50 years.

In this scenario of increased interaction between local governments due to daily commuting of agents it is important to look at the impact of the fiscal policy and decision-making process in the utility and choices of the agents. This commuting generates endogenous spillovers since agents are exposed to multiple local governments which provide public goods and charge taxes. The object of this paper is precisely the interaction between agents and different authorities, and the resulting spillovers.

The literature usually treats spillovers as exogenous, i.e., agents "automatically" get utility from the public goods provided in other jurisdictions.[1] However, in a commuting setup spillovers actually arise in a quite natural and endogenous way, namely by travelling on a daily basis across jurisdictions the agents enjoy public goods in both the municipality they live and in that where they work.

If we think about the type of public goods usually provided by municipalities or jurisdictions we notice that most of them are only consumed by

[1] This is the Oate's tradition spillover that we can find for example in Besley and Coate (2003).

agents if they actually go to that municipality. It makes no sense to consider that an individual experiences an increase in utility just because the nearby town (where he never goes) now offers, for example, a better garbage collection service. The residents of that city will see their utility increase due to cleaner streets, but it is difficult to argue that someone with no contact with that city will now be better off.

Looking at real world facts we can see that agents spend most of their time in the place where they live and where they work, so it is natural to consider that agents consume the public goods provided in those places. However, among the several types of public goods provided by municipalities or jurisdictions we can find some that are more used by the inhabitants of the municipality (such as garbage collection, gas supply, parks or monuments with free entrance for locals, etc.) while others are used by both the inhabitants and the commuters that work there (road construction and maintenance, free parks, public transportation, street lighting, etc.). Our purpose was precisely to find a way that better reflects these facts.

Local governments worldwide have different levels of autonomy[2], namely when it comes to tax collection, and access to different kinds of taxes. Such taxes include residence-based wealth taxes, pure residence-based income taxes, pure source-based income taxes, or "hybrid" ones. These taxes are usually combined with grants or transfers from the central governments to form the total local budget.

Examples of residence-based wealth taxes are mostly residential and business property taxes which, in the United States, "are the most important source of local government tax revenue" (Braid, 2005). Pure residence-based income taxes charged by local governments can be found in Baltimore (according to Braid, 2009) or in Portugal.

As stated in Braid (2009) U.S. cities like San Francisco, Los Angeles, Newark (New Jersey) and Birmingham (Alabama) have pure source-based wage taxes or payroll taxes that apply uniformly to citizens depending only on their workplace: "a central-city's wage tax applies at the same rate to central-city and suburban residents working in the central city, but not to central-city and suburban residents working in the suburbs" (Braid, 2009).

Examples of cities using "hybrid" income taxes are also presented in Braid (2009). In these cases all central city residents are taxed at a rate, irrespective of where they work, while residents in the suburbs who work in the central city can be taxed at a different rate. Kansas City, St. Louis, Wilmington, Detroit, New York City and Philadelphia are the provided examples. But

[2]For a thorough analysis of the fiscal autonomy of local governments please check the OECD (2009) study.

the use of wage or income taxes by local governments is not confined to the U.S. As we can read in Peralta (2007) Mexico and "several OECD countries have payroll taxes at the state or local level: Australia, Austria, France and Greece". Also Korea has source based income taxes (Chu and Norregaard, 1997). Besides these, Braid (2005) points the use of such taxes also on Sweden, Denmark, France, Germany, Japan and Spain.

When we think about the fiscal autonomy of local governments the problem of centralized vs. decentralized decision immediately arises. We traveled a long way since the pioneering work of Oates (1972) who formalized the standard approach for this question and reached the *Oates's Decentralization Theorem* that states that decentralization is preferred in the absence of spillover effects while otherwise there is a trade-off due to the incapability of the central government to follow different public policies in different regions. This assumption of uniformity of the centralized policy is used in many other papers on fiscal federalism to impose a cost on centralization[3]. The arguments in favor of local governments are usually justified by some kind of informational advantage on the features of their regions (they are "closer to the people", which allows to better respond to the agents' needs) but the decentralization comes to a cost due to the failure to internalize tax and expenditure spillover effects (Oates, 1999).

Our purpose is to analyze the majority voting decentralized equilibrium against the benchmark of a first-best benevolent social planner solution.[4]

Our model introduces public goods with an endogenous spillover effect in the framework of a linear city used by, e.g., Peralta (2007) and Braid (2000) to tackle interjurisdictional tax spillovers. The city is divided into two jurisdictions and agents choose where they want to work. Productivity, and thus wages, differ across regions and individuals trade-off the advantages (i.e., wage and working conditions) of a given job against travel costs (distance, time, and money) when choosing their work place. Our main contribution is to allow individuals to to enjoy public goods in the work place. We do not, however, model the residence choice of agents assuming that residence and working choices are independent, as argued by Wildasin (1986) and supported by empirical evidence provided by Rouwendal and Meijer (2001), Glaeser et al. (2001) and Zax (1991 and 1994). For a recent analysis of the residence decision refer to Wrede (2009) where land is included and agents

[3]For example in Alesina and Spolaore (1997) when studying the size of nations or in Bolton and Roland (1997) analysing the threat of secession.

[4]The use of such equilibrium in tax competition scenarios can also be found in Fuest and Hubber (2001) and Grazzini and van Ypersele (2003) who show that centralized decision regarding capital taxes can make the median voter worse off.

can choose their residence location according to a *bid-rent function*.[5]

The contribution of this paper is twofold. On the one hand, it introduces a public good spillovers on a linear city tax competition model with commuting in the line of Peralta (2007). On the other hand, it introduces a distortive wage tax on a model with spillovers.

With this reality in mind, we need a representative agent who derives utility from both the public good provided in the residence location and in the working place. This means that agents only get utility from the public good supplied in the other jurisdiction if they choose to work there. Otherwise they only get utility from the one provided in their own jurisdiction. This formulation allow us to have an endogenous spillover effect instead of the traditional exogenous one.

Naturally agents might enjoy the public goods in other jurisdictions if they go there for leisure or shopping and therefore use the public goods provided even without working there. However, such use is occasional and most of the goods and services from which individual get utility in those cases are privately provided ones (hotels, restaurants, leisure facilities, shopping malls, theaters, etc.). As such, we chose to disregard these situations and concentrate on the commuters for work case.

We prove that in the tax competition equilibrium the public good provided in the most productive region is always underprovided, while that of the less productive region can be under or overprovided. We also show that tax competition leads to a less than efficient number of commuters in some cases. Interestingly, we show that the introduction of a distortive wage tax improves the provision of the public goods, when compared to a situation where local governments only use a lump sum residence tax. The use of the distortive wage tax is therefore, a *second-best result*, as it partially offsets the distortion generated by the endogenous spillover of the public goods.

This paper is organized as follows. In Section 2 we present the model. Section 3 computes the first best which is then used as a benchmark to compare the results obtained in Sections 4 and 5 as the tax competition equilibrium where only a lump sum tax is used and where both a lump sum and a distortive tax are used, respectively. Section 6 compares the two tax competition equilibria found before, and Section 7 concludes.

2 The Model

We consider a linear city divided into two jurisdictions with the same size. Each jurisdiction has an employment center where agents can work. The total

[5] An approach similar to the one used in Fernandez (2004) based on Wheaton (1977).

number of residents of the city is normalized to 1, as well as the city size, with extreme points of the segment -1/2 and 1/2. Inhabitants are uniformly distributed across the city and cannot choose their residence location. Each agent is indexed by his residence place, x.

Let $n(x)$ and $N(x)$ denote the density and distribution function, respectively, so that

$$n(x) = 1 \quad \text{and} \quad N(x) = x + \tfrac{1}{2}$$

Since the two jurisdictions have the same size and residents are uniformly distributed, both have the same number of inhabitants, $\bar{N} = 1/2$. The median resident of each jurisdiction coincides with the geographic center of the jurisdiction, i.e., $m_H = -1/4$ and $m_L = 1/4$. The employment centers are assumed to be symmetrically located in γ and $-\gamma$ and located outwards from the median resident ($\gamma > 1/4$). This opens up the possibility for a majority of residents of one jurisdiction to commute to the other one.

Firms located at the employment centers produce an homogeneous good according to a linear technology $Y_i = \alpha_i N_i$, where Y_i is the output and N_i is the number of workers in jurisdiction i.[6] The two jurisdictions have unequal productivities. We use H to denote the high-productivity jurisdiction and L for the low-productivity one, with $\alpha_H > \alpha_L$.

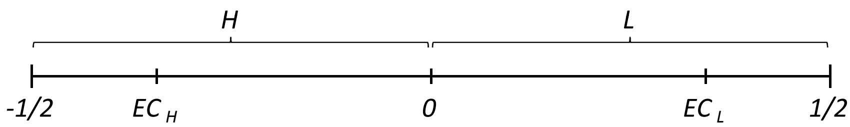

Figure 1: The City

The government of each jurisdiction collects a head tax (T_i) paid by all its residents and, possibly, an ad-valorem source-based tax on wages (τ_i) paid by all workers in the employment center of jurisdiction i to finance a public good budget G_i.[7]

The local government budget constraint is therefore

$$G_i = T_i \bar{N} + \alpha_i \tau_i N_i$$

where α_i is the gross wage earned by workers in the employment center of jurisdiction i and N_i is the number of workers in that jurisdiction.

[6]As stated in Peralta (2007) the assumption of a linear technology is not essential and the obtained results would remain unchanged if we introduce perfectly mobile capital in the model with a constant returns to scale production function.

[7]Note that in our setup the head tax T_i can be seen as land or residential property tax with fixed house size; since residence place is not chosen by agents this is a lump-sum tax.

Agents support a per-mile commuting cost c and can choose to which employment center they want to commute (i.e., where they want to work). Commuting to the jurisdiction where they do not live is, therefore, more costly than commuting to the one where they live since the distance they must travel is higher. Each individual provides one unit of labor and pays a wage tax at the source so that the net wage earned by an individual working in j is $\omega_j = \alpha_j(1-\tau_j)$. All agents have a revenue W from other sources which is assumed to be high enough such that everyone can always pay his tax bill. Agents get utility both from private consumption and from the public good provided.

We follow Peralta (2007) and Braid (2000) and assume a quasi-linear utility function; however, differently from that author, we allow the individuals to enjoy booth the public goods of their residence and work places. The utility enjoyed by individual x, who lives in i and works in j is given by:

$$u_{ij}(x;\tau;G_i;G_j) = \omega_j - T_i + W - c|x - EC_j| + (1-k)\upsilon(G_i) + k\upsilon(G_j) \quad (1)$$

$$i, j = H, L$$

where EC_j is the location of the employment center where the agent chooses to work (γ or $-\gamma$), G_i is the public good provided in the jurisdiction where he lives, G_j is the public good provided in the jurisdiction where he works and $\upsilon(G)$ is an increasing concave function. We will sometimes use the function $\sqrt{(G)}$ to illustrate some of our results. The intensity of the spillover effect due to having individuals deriving utility from the public goods provided in both jurisdictions is measured by the constant k, where $0 \leq k \leq 1$. When $k \leq 1/2$, G_i is more important than G_j, i.e., agents care more for the public good provided in the jurisdiction where they live than for the one provided in the jurisdiction where they work.[8]

Again, notice that this is not the standard spillover effect we can find on the literature. In our case agents only get utility from the public good provided in the other jurisdiction if they decide to work there, i.e., the spillover is endogenous. When they decide the working location they are also choosing the public good mix they want to consume.

[8] This is what is considered, for example, in Besley and Coate (2003) and would fit our model since we argue that agents are able to get utility from a wider variety of public goods provided in their residence place. However, the assumption of these boundaries for k is not necessary to reach the results of this paper so we choose not to impose them and leave the problem as general as possible.

2.1 The choice of the workplace

An agent will work in the jurisdiction where he lives if $u_{ii}(x;\tau;G_i;G_H) - u_{ij}(x;\tau;G_i;G_L) \geq 0$ and will commute to the other jurisdiction otherwise.

Looking at this utility difference we can calculate the marginal interjurisdictional commuter, denoted $\hat{x}$. From (1) we can see that the difference between the utility obtained working in H and the one obtained by working in L is:

$$u_{iH} - u_{iL} = \begin{cases} \omega_H - \omega_L + 2\gamma c + k[\upsilon(G_H) - \upsilon(G_L)] & \text{if } x \leq -\gamma \\ \omega_H - \omega_L + 2xc + k[\upsilon(G_H) - \upsilon(G_L)] & \text{if } -\gamma < x < \gamma \\ \omega_H - \omega_L - 2\gamma c + k[\upsilon(G_H) - \upsilon(G_L)] & \text{if } x \geq \gamma \end{cases}$$

If $u_{iH}(x;\tau) - u_{iL}(x;\tau)$ is positive the agent will choose to work in H, otherwise he chooses to work in L. Note that for $|x| > \gamma$ the utility difference is independent from x which means that if one agents that lives between the employment center of a jurisdiction and its outer limit wants to commute to the other one, every agent will want to do the same. We assume away such non-interesting cases and focus on the situation where $-\gamma < x < \gamma$. The marginal ij-commuter $\hat{x}$ will be the one indifferent between working in H or L, therefore

$$\hat{x} = \frac{\omega_H - \omega_L + k[\upsilon(G_H) - \upsilon(G_L)]}{2c} \tag{2}$$

This marginal interjurisdictional commuter $\hat{x}$ defines a *commuting equilibrium* where all $x < \hat{x}$ work in H and all $x > \hat{x}$ work in L.

3 First Best

We now compute the utilitarian first best to use as a benchmark for the tax competition equilibrium analysis, i.e., the decision of a benevolent social planner that chooses the wage taxes, the residence taxes, the level of public good provided in each jurisdiction and allocates workers to an employment center so that overall utility is maximized.

The planner thus faces an overall budget constraint such that the provision of public goods must be fully paid by the wage and head taxes, i.e.,

$$G_H + G_L = \tau_H \alpha_H \left(\bar{N} + \hat{x}\right) + \tau_L \alpha_L \left(\bar{N} - \hat{x}\right) + \bar{N}\left(T_H + t_L\right) \tag{3}$$

The problem faced by the social planner is therefore

$$\max_{\hat{x},G_H,G_L,\tau_H,\tau_L,T_H,T_L} U = U_H + U_L$$
$$s.t.\ G_H + G_L = \tau_H \alpha_H \left(\bar{N} + \hat{x}\right) + \tau_L \alpha_L \left(\bar{N} - \hat{x}\right) + \bar{N}\left(T_H + t_L\right)$$

where U is the overall utility of the population, equal to the sum of the utility of all inhabitants of jurisdiction H (U_H) and of all inhabitants of jurisdiction L (U_L). Note that it will never be optimal to have H-residents commuting to L since their commuting cost will be higher than if they work in H and their productivity will be lower. Therefore, we can only have L residents commuting to H, i.e., $\hat{x} \geq 0$, which allow us to calculate U_H and U_L as:

$$U_H = \int_{-\frac{1}{2}}^{0} u_{HH} dx \tag{4}$$

$$U_L = \int_{0}^{\hat{x}} u_{LH} dx + \int_{\hat{x}}^{\frac{1}{2}} u_{LL} dx \tag{5}$$

Denoting by C_i the total commuting costs of all the residents of jurisdiction i, we have

$$C_H = c\left[\int_{-\frac{1}{2}}^{-\gamma} (-\gamma - x)dx + \int_{-\gamma}^{0} (x+\gamma)dx\right] = c\left(\frac{1}{8} + \gamma^2 - \frac{\gamma}{2}\right) \tag{6}$$

$$C_L = c\left[\int_{0}^{-\hat{x}} (x+\gamma)dx + \int_{\hat{x}}^{\gamma} (\gamma - x)dx + \int_{\gamma}^{\frac{1}{2}} (x-\gamma)dx\right] = C_H + c\left(\hat{x}^2\right) \tag{7}$$

where the last term in C_L is the increase in commuting costs due to the interjurisdictional commuters which must travel a longer distance.

Total utility in each jurisdiction is therefore given by:

$$U_H = \bar{N}\left[\omega_H - T_H + W + v(G_H)\right] - C_H \tag{8}$$

$$U_L = \bar{N}\left[\omega_L - T_L + W + v(G_L)\right] - C_H + \hat{x}\left[\omega_H - \omega_L + k\Delta(v)\right] - c\left(\hat{x}^2\right) \tag{9}$$

where $\Delta(v) = v(G_H) - v(G_L)$ and $k\Delta(v)$ is the impact on utility of the consumption of the public good provided in jurisdiction H rather the one provided in L to interjurisdictional commuters.

Note that the two last terms of U_L are the gain to L of having interjurisdictional commuters. The novelty of our analysis is reflected on the term $\Delta(\upsilon) = \upsilon(G_H) - \upsilon(G_L)$ generated by the spillover effect of the public goods: agents near the border of jurisdiction L now have two effects on utility when commuting to H: the difference in wage and the difference in the level of public goods provided (weighted by k since they always get utility $(1-k)$ from G_L, the public good provided in the jurisdiction where they live).

Solving the social planner problem formalized previously we can easily see that the planner is indifferent between using the wage or the head tax since he can allocate the workers to any of the employment centers. Therefore the choice of τ_H, τ_L, T_H and T_L is irrelevant for our analysis. The only thing that must be ensured is that the budget constraint is satisfied with these taxes. We can then assume $\tau_i = 0$ and finance the public goods exclusively with the head (lump-sum) taxes. This has the merit of not using a distortive tax and ensuring that we are not implicitly performing any type of interjurisdictional transfers. Remember that the purpose of the calculation of the first best is to use it as a benchmark to compare with the tax competition equilibrium and so we want to keep it as neutral as possible. The use of the distortive wage tax could be seen as a form of interjurisdictional transfer[9] and we do not want to allow for such possibility in this framework.

The relevant first order conditions are therefore:

$$\frac{\partial()}{\partial \hat{x}} = 0 \Leftrightarrow \hat{x}^o = \frac{\alpha_H - \alpha_L + k\Delta(\upsilon)}{2c} \tag{10}$$

$$\frac{\partial()}{\partial G_H} = 0 \Leftrightarrow \upsilon'(G_H^o)\left(\frac{1}{2} + \hat{x}k\right) = 1 \tag{11}$$

$$\frac{\partial()}{\partial G_L} = 0 \Leftrightarrow \upsilon'(G_L^o)\left(\frac{1}{2} - \hat{x}k\right) = 1 \tag{12}$$

Equation (10) gives the optimal interjurisdictional commuter $\hat{x}^o$, which results from the trade-off between commuting costs and productivity gains and the public good level.[10]

Equations (11) and (12) express the Samuelson condition for the optimal provision of public goods. Since G_H provides k-weighted utility also to $\hat{x}$ residents of L, the marginal benefit of G_H is higher than without the spillover

[9] As pointed in Peralta (2007).

[10] Comparing this condition with the one obtained in Peralta (2007) we can see that the difference lies exactly on the presence of the term $k\Delta(\upsilon)$; the spillover makes agents consider the difference in public goods provision when deciding the work place since their utility depend on G_j.

(reflected by the term $\hat{x}k$) while the inverse applies to G_L.

4 The Tax Competition Equilibrium with Residence Taxes

Having calculated the conditions that define the first best, we can now compute the tax competition equilibrium and compare it to the utilitarian optimum. In this section we will assume that a government elected by majority rule in each jurisdiction decides the taxes and public goods levels. The elected policy will then be the one preferred by the median voter of each jurisdiction which in our model coincide with the median resident, i.e., $mH = -1/4$ and $mL = 1/4$.

In this section we compute the tax competition equilibrium when local governments only have access to the residence tax, T_i. We can then use this equilibrium to compare with the one resulting from the use of a distortive wage tax combined with a lump-sum tax, which is computed in the next section.

Each local government maximizes the utility of the median voter subject to the commuting equilibrium, $\hat{x}$, and to the budget constraint of the jurisdiction when $\tau = 0$:

$$\begin{aligned} &\max_{G_i,T_i} u_{m_i} \\ &s.t.\ \hat{x} = \frac{\alpha_H - \alpha_L + k\Delta(\upsilon)}{2c} \\ &\qquad G_i = \bar{N}T_i \end{aligned}$$

For the utility of the median voter we must separate the case where he commutes to the other jurisdiction from the case where he commutes to the employment center of his own jurisdiction. We present two cases, depending on whether the median voter of L works in L or H. While we cannot ensure in general that H does not commute to region L, we show in the appendix that this can never happen with $\upsilon(G) = \sqrt{(}G)$. The intuition for this resides on the fact that the gross wage earned in L is lower and the traveled distance by m_H is much higher than if he decides to work in the employment center of H. For the median voter of L it can make sense to commute to H thanks to the increase in productivity.

The median voter of H thus enjoys an utility of:

$$u_{m_H} = \alpha_H + W - T_H - c\left(-\frac{1}{4} + \gamma\right) + \upsilon(G_H)$$

For the median voter of L we must separate the case where he works in L from the case where he commutes to work in H. In the former case, since the only public good he consumes is G_L, we can compute his utility as:

$$u_{m_L} = \alpha_L + W - T_L - c\left(\gamma - \frac{1}{4}\right) + \upsilon(G_L)$$

However, if he decides to work in H he will get utility both from G_L (weighted by $1-k$) and G_H (weighted by k) and his utility is, therefore, given by:

$$u_{m_L} = \alpha_H + W - T_L - c\left(\frac{1}{4} + \gamma\right) + (1-k)\upsilon(G_L) + k\upsilon(G_H)$$

4.1 Median voter of L works in L

Let us first assume that the median voter of L works in the employment center of L, which happens when $\hat{x} < 1/4$. Solving the utility maximization problem for m_H and m_L we have the equilibrium levels of G_H and G_L implicitly defined by:

$$\upsilon'(G_H^*) = 2 \quad (13)$$

$$\upsilon'(G_L^*) = 2 \quad (14)$$

These conditions express the usual equality between marginal benefit and marginal cost. Since the population mass of each jurisdiction is 1/2, the marginal cost borne by the median voter to provide an additional unit of public good is 2.

If we compare the tax competition equilibrium obtained with the first-best we can reach the following proposition:

Proposition 1: *In the tax competition equilibrium where only the residence tax is available and both median voters work in their own jurisdictions:*

(i) The local public good in jurisdiction H is underprovided and the one in jurisdiction L is overprovided;

(ii) There is undercommuting of agents.

Proof. See appendix. □

The median voter of H does not take into account the spillover effect of the public good provided in H on the L-residents that commute to his jurisdiction and, therefore, considers a lower marginal benefit of G_H when compared to the first-best. This leads to a situation of underprovision of this public good. Similarly, the median voter of L does not consider that a fringe $\hat{x}$ of the residents of L commute to H and, thus, get utility from G_L weighted by k, leading to overprovision of G_L.

Since only the residence tax is being used agents decide their work place considering the gross wage earned and the public good provided in each jurisdiction. Knowing that G_H is underprovided, jurisdiction H is less attractive than in the first-best solution, while jurisdiction L is more attractive due to the overprovision of G_L. Therefore, the number of agents commuting from L to H will be lower than in the first best case.

Finally, we check that the equilibrium obtained respects the condition $\hat{x} < 1/4$, i.e., the median voter of L works in L. Since we are unable to provide general conditions for this, we choose to illustrate it with a particular utility function, $\upsilon(G) = \sqrt{(}G)$, thereby showing that this equilibrium is possible:

$$\upsilon'(G_i) = \frac{1}{2\sqrt{G_i}}$$

From the first order conditions (13) and (14) we get the equilibrium levels of public goods:

$$\frac{1}{2\sqrt{G_H^*}} = \frac{1}{2\sqrt{G_L^*}} = 2 \Leftrightarrow G_H^* = G_L^* = \frac{1}{16}$$

Thus, the marginal interjurisdictional commuter is

$$\hat{x^*} = \frac{\alpha_H - \alpha_L + k\left[\sqrt{\frac{1}{16}} - \sqrt{\frac{1}{16}}\right]}{2c} = \frac{\alpha_H - \alpha_L}{2c}$$

Since the median voter of L works in L,

$$\hat{x^*} = \frac{\alpha_H - \alpha_L}{2c} < \frac{1}{4} \Leftrightarrow \alpha_H - \alpha_L < \frac{c}{2}$$

Therefore, this is the condition that guarantees that, when $\upsilon(G) = \sqrt{G}$, the equilibrium exists.

4.2 Median voter of L works in H

If the median voter of L works in H he now earns wage α_H and gets utility from public goods provided by both jurisdictions. For the median voter of H everything remains the same, thus the G_H implicitly defined by equation (13) is still valid. The key change for m_L is that the utility enjoyed thanks to the public good provided in his own jurisdiction is now weighted by $(1-k)$. This results in an equilibrium level of G_L implicitly defined by:

$$(1-k)\upsilon'(G_L^*) = 2 \tag{15}$$

The level of public good provided in jurisdiction L will therefore be lower than in the previous case since the marginal benefit of G_L to m_L is smaller.

Performing the comparison with the first best we can find out the following:

Proposition 2: *In the tax competition equilibrium where only the residence tax is available and the median voter of H works in his own jurisdiction while the median voter of L is an interjurisdictional commuter, both local public goods are underprovided.*

Proof. See appendix. □

As in the previous case, the median voter of H does not take into account the spillover effect of the public good provided in H on the L-residents that commute to his jurisdiction, which leads to the underprovision of G_H. Regarding G_L, since the median voter of L works in H he does not take into consideration that part of the residents in L get utility $\upsilon(G_L)$ from it instead of $(1-k)\upsilon(G_L)$, which results in the underprovision of this public good.

In this case, we may have both under or overcommuting at the tax competition equilibrium. This stems from the fact that both jurisdiction are less attractive than they are in the first best solution due to the underprovision of both public goods.

We now check that the equilibrium obtained respects the condition $\hat{x} > 1/4$, i.e., the median voter of L commutes to H. As in the previous section we need to compute the equilibrium levels of G_H and G_L and we do it for the case where $\upsilon(G_i) = \sqrt{G_i}$.

Since the problem solved by the local government in H is the same as before, we have $G_H^* = 1/16$. For G_L we must satisfy the (15):

$$\frac{1}{2\sqrt{G_L^*}} = \frac{2}{1-k} \Leftrightarrow G_L^* = \frac{(1-k)^2}{16}$$

Thus, the marginal interjurisdictional commuter is

$$\hat{x^*} = \frac{\alpha_H - \alpha_L + k\left[\sqrt{\frac{1}{16}} - \sqrt{\frac{(1-k)^2}{16}}\right]}{2c} = \frac{\alpha_H - \alpha_L + \frac{k^2}{4}}{2c}$$

Since the median voter of L works in H,

$$\hat{x^*} = \frac{\alpha_H - \alpha_L + \frac{k^2}{4}}{2c} > \frac{1}{4} \Leftrightarrow \alpha_H - \alpha_L + \frac{k^2}{4} < \frac{c}{2}$$

Therefore, this is the condition that guarantees that, when $v(G) = \sqrt{G}$, the equilibrium exists.

5 The Tax Competition Equilibrium with Residence and Wage Taxes

We now focus on the tax competition equilibrium attained when local governments can use both the residence (lump-sum) and the wage (distortive) tax.

As a matter of fact, agents are now concerned with the net wage they earn in each employment center rather than the gross wage dictated by their productivity. This means that local governments, when deciding the wage tax level, face a trade-off between financing the public good and reducing the number of interjurisdictional commuters due to the reduction of the net wage in the jurisdiction.

As in the previous framework, each local government maximizes the utility of the median voter subject to the commuting equilibrium $\hat{x}$ and to the budget constraint of the jurisdiction:

$$\begin{aligned}
&\max_{G_i,\tau_i,T_i} u_{m_i} \\
&s.t.\ \hat{x} = \frac{(1-\tau_H)\alpha_H - (1-\tau_L)\alpha_L + k\Delta(v)}{2c} \\
&\quad G_i = \bar{N}T_i + \tau_i \alpha_i N_i
\end{aligned}$$

Remember that N_i is the number of agents working in the employment center of jurisdiction i so that $N_H = \frac{1}{2} + \hat{x}$ and $N_L = \frac{1}{2} - \hat{x}$.

Again, for the utility of the median voter we must separate the case where he commutes to the other jurisdiction from the case where he commutes to the employment center of his own jurisdiction. For the median voter of H, and as we did in the previous section, we assume that he will never commute to jurisdiction L since the gross wage is lower and the traveled distance is much higher than if he decides to work in the employment center of H. For the median voter of L it can make sense to commute to H thanks to the increase in productivity. Therefore, the median voter of H enjoys an utility of:

$$u_{m_H} = (1 - \tau_H)\,\alpha_H + W - T_H - c\left(-\frac{1}{4} + \gamma\right) + \upsilon(G_H)$$

For the median voter of L, his utility when he works in his own jurisdiction is given by:

$$u_{m_L} = (1 - \tau_L)\,\alpha_L + W - T_L - c\left(\gamma - \frac{1}{4}\right) + \upsilon(G_L)$$

If he decides to work in H he will get utility both from G_L (weighted by $1-k$) and G_H (weighted by k) and his utility is, therefore, given by:

$$u_{m_L} = (1 - \tau_H)\,\alpha_H + W - T_L - c\left(\frac{1}{4} + \gamma\right) + (1-k)\upsilon(G_L) + k\upsilon(G_H)$$

5.1 Median voter of L works in L

Let us first assume that the median voter of L works in the employment center of L, which happens when $\hat{x} < 1/4$. Solving the utility maximization problem for m_H and m_L we have the equilibrium levels of G_H and G_L implicitly defined by:

$$\upsilon'(G_H^{**}) = 2\left[1 - \tau_H \alpha_H \frac{k}{2c}\upsilon'(G_H^{**})\right] \tag{16}$$

$$\upsilon'(G_L^{**}) = 2\left[1 - \tau_L \alpha_L \frac{k}{2c}\upsilon'(G_L^{**})\right] \tag{17}$$

These conditions express the usual equality between marginal benefit and marginal cost. Note that the marginal cost is affected by the term $\tau_i\alpha_i(k/2c)\upsilon'(G_i)$, which is the impact of the level of public good on the government budget due to interjurisdictional commuters, whose choice of working place is driven by public good provision. This means that increasing the provision of the public good increases the number of workers subject to the wage tax and this affects the cost borne by the median voter.

If we now look at the first order conditions that define reaction functions on τ_H and τ_L and combine them we obtain the equilibrium levels of the wage taxes given by:

$$\tau_H^{**} = \frac{\alpha_H - \alpha_L + k\Delta^{**}(\upsilon)}{3\alpha_H}$$

$$\tau_L^{**} = \frac{-(\alpha_H - \alpha_L) - k\Delta^{**}(\upsilon)}{3\alpha_L}$$

which yields the equilibrium marginal interjurisdictional commuter:

$$\hat{x}^{**} = \frac{\alpha_H - \alpha_L + k\Delta^{**}(\upsilon)}{6c}$$

With these expression we can show that, in equilibrium, $\tau_H^{**}\alpha_H > \tau_L^{**}\alpha_L$ and $G_H^{**} > G_L^{**}$ since the opposite relations are ruled-out by the condition $\hat{x}^{**} > 0$.[11]

The characterization of the tax competition equilibrium is provided in the following proposition:

Proposition 3: *In the tax competition equilibrium where both the residence and the wage taxes are available and both median voters work in their own jurisdictions:*

(i) The wage is taxed in H and subsidized in L;

(ii) The local public good in jurisdiction H is underprovided while the one in jurisdiction L is overprovided;

(iii) There is undercommuting of agents.

Proof. See appendix. □

The result that region H taxes wages while region L subsidizes them is not also obtained by Peralta(2007): H residents are exporting part of their tax burden to the interjurisdictional commuters from region L using the wage tax and since the median voter of L works in L, he uses the head tax to impose a higher tax burden to the interjurisdictional commuters, which will not receive the wage subsidy. What we are seeing is a transfer of income from the interjurisdictional commuters to everyone else.

As for the provision of public goods, agents in H have a marginal cost of G_H lower than those in L. Since both the median voters of H and L are exporting part of the tax burden to the L interjurisdictional commuters we have two effects: for H residents, G_H is less expensive due to the tax export and due to the fact that by increasing G_H the number of such commuters increase, which makes it even less expensive; for the median voter of L increasing G_L decreases the number of commuters, which increases the marginal cost.

Comparing the levels of public good provided in each jurisdiction with the first-best solution calculated previously we reach an intuitive underprovision of the public good of jurisdiction H and overprovision of the one of jurisdiction L: the median voter of H does not take into consideration the spillover produced by G_H to the L residents that commute to H and the median voter

[11]The proof can be found in the appendix.

of L does not take into consideration the fact that part of the L population (with mass $\hat{x}$) only takes utility $(1-k)$ from G_L.

Looking at the marginal interjurisdictional commuter, we can see that there is undercommuting if compared to the first-best. This is easily explained by the fact that now jurisdiction H is less attractive while jurisdiction L is more attractive than in the first best case. A lower net wage earned in the employment center of H (due to the positive wage tax τ_H) and a lower level of G_H make jurisdiction H not so appealing while the opposite happens for L (with subsidized wages and higher provision of G_L).

5.2 Median voter of L works in H

We shall now analyse the Nash equilibrium where the median voter of L works in the employment center of H, i.e., he ij-commutes. Note that the problem for the median voter of H remains unchanged, thus the previous first order conditions for this problem are still valid and the G_H is implicitly defined by equation (16). However, for m_L the problem is now different, since his utility is now given by:

$$u_{m_L} = (1-\tau_H)\,\alpha_H + W - T_L - c\left(\frac{1}{4}+\gamma\right) + (1-k)\upsilon(G_L) + k\upsilon(G_H)$$

Recall that the difference to the previous case is that the median voter of L now gets $(1-k)$-weighted utility from G_L and k-weighted utility from G_H, the public good provided where he works.

The first order condition that implicitly defines G_H is the same as before while for G_L we now have:

$$(1-k)\upsilon'(G_L^{**}) = 2\left[1 - \tau_L \alpha_L \frac{k}{2c}\upsilon'(G_L^{**})\right]$$

The marginal benefit of G_L for the median voter of L is now weighted by $(1-k)$ instead of 1, since he now works in H and therefore gets k-weighted utility from G_H. The expression for the marginal cost is the same as before.

Regarding the wage taxes we must now combine again the first order conditions from the problems of m_H and m_L which lead us to the following equilibrium expressions:

$$\tau_H^{**} = \frac{\alpha_H - \alpha_L + c + k\Delta^{**}(\upsilon)}{3\alpha_H}$$

$$\tau_L^{**} = \frac{2c - (\alpha_H - \alpha_L) - k\Delta^{**}(\upsilon)}{3\alpha_L}$$

which yields the equilibrium marginal interjurisdictional commuter:

$$\hat{x}^{**} = \frac{\alpha_H - \alpha_L + k\Delta^{**}(\upsilon)}{6c} + \frac{1}{6}$$

The next proposition characterizes the tax competition equilibrium:

Proposition 4: *In the tax competition equilibrium where both the residence and the wage taxes are available and the median voter of H works in his own jurisdiction while the median voter of L is an interjurisdictional commuter:*

(i) The wages are taxed in H and in L;

(ii) The public good in jurisdiction H is underprovided;

(iii) If G_L is overprovided, there is undercommuting of agents;

(iv) If there is overcommuting of agents, G_L is underprovided.

Proof. See appendix. □

In this situation no jurisdiction is willing to subsidize wages. The median voter of L is not willing to subsidize the wage in L due to the fact that he is not working in that jurisdiction. Since he is now one of the interjurisdictional commuters he wants to use τ_L to finance the budget of L because he is not subject to such tax.

As for the provision of public goods the intuition is basically the same as in the previous case, with the additional fact that on the choice of G_L the marginal benefit for m_L is now smaller since it is weighted by $(1-k)$.

We can still show that G_H is underprovided, but regarding the public good of jurisdiction L and the number of commuters we cannot be sure how the equilibrium levels compare with the first best. All we can say is that if we have overprovision of G_L we will have undercommuting (jurisdiction L is more attractive than it should) and if we have overcommuting we will certainly have underprovision of G_L. However, the opposite implications are not valid.

The intuition for the underprovision of G_H is the same as before, but now for G_L all we know is that, comparing to the case where the median voter of L worked in L and we were able to say that it was being overprovided, the marginal benefit is now lower due to the $(1-k)$ weight. This implies that the G_L level chosen by m_L will now be lower, but we cannot be sure if this reduction is such that it is no longer overprovided: it can still be above the first best or it can now be bellow the first best. This uncertainty about the under or overprovision of G_L extends to the commuting level since it is a determinant of the desirability of working in L.

6 Only Residence Tax vs. Residence and Wage Taxes

In this section we compare the tax competition equilibrium obtained when local governments only use the lump-sum head tax to the one when both the lump-sum head tax and the distortive wage tax are used.

Following the structure of the previous sections, we first focus on the case where the median voter of L works in L. Comparing the two tax competition equilibria we achieve a *second-best result* induced by the use of the distortive tax:

Proposition 5: *When both median voters work in their own jurisdictions, the use of the distortive tax enhances the provision of the public goods vis-a-vis the case where only the lump-sum tax is used.*

Proof. See appendix. □

As a matter of fact, the proof shows that:

$$G_H^O > G_H^{**} > G_H^*$$

$$G_L^O < G_L^{**} < G_L^*$$

The distortion introduced by the wage tax partially offsets the inefficiency created by the tax competition equilibrium due to the spillover effect of the public goods to the interjurisdictional commuters. This is a typical *second-best result* where the introduction of two distortions (the wage tax and the inter-jurisdictional externalities) improves upon the case where only one distortion is present. The tax export generated by the wage tax on H reduces the marginal cost to the policy-maker in H, thus leading him to provide a higher level of G_H, thus getting closer to the optimal provision. The reverse applies to L where the overprovision is reduced by the introduction of the wage subsidy that increases the cost of provision to m_L.

When we look at the case where the median voter of L works in H the achieved result is not so strong:

Proposition 6: *When the median voter of H works in H while the median voter of L is an interjurisdictional commuter, the use of the distortive tax increases the level of public goods provided in both jurisdictions vis-a-vis the case where only the lump-sum tax is used.*

Proof. See appendix. □

The proof shows that:

$$G_H^O > G_H^{**} > G_H^*$$

$$G_L^{**} > G_L^*$$

Note that we can no longer say for sure that the provision of both public goods is enhanced with the introduction of the distortive wage tax. We can be sure of such enhancement regarding G_H, but when we look at G_L we can be facing an increase which changes the situation of underprovision into an overprovision one since we are not sure of the comparison between G_L^{**} and G_L^o as seen in the previous section.

7 Conclusion

This paper introduces commuting-related spillovers in a duo-centric linear city where local governments provide public goods and agents choose in which region they want to work.

We show that in the tax competition equilibrium the public goods provided in the most productive region is always underprovided and the one provided in the less productive region can be under or overprovided. Furthermore, we showed that the use of the distortive tax tends to be preferred to the single use of a lump sum tax in terms of the provision of the public goods as ir partially offsets the distortion introduced by the endogenous spillover effect.

Since the results were obtained using very general assumptions they are quite robust since they do not depend on explicit functional forms for, e.g., $\upsilon(G_i)$. The two kinds of taxes used are also currently used in real world countries, such as U.S. states as referred in the introduction and their application is, therefore, reasonable and feasible.

Appendix

Proof of **Proposition 1**:

(i) $\upsilon'(G_H^o) - \upsilon'(G_H^*) = \frac{2}{1+2\hat{x}^o k} - \frac{2}{1}$
Since $1 + 2\hat{x}^o k > 1 \Rightarrow \upsilon'(G_H^o) - \upsilon'(G_H^*) < 0 \Leftrightarrow G_H^o > G_H^*$
$\upsilon'(G_L^o) - \upsilon'(G_L^*) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1}$
Since $1 - 2\hat{x}^o k < 1 \Rightarrow \upsilon'(G_L^o) - \upsilon'(G_L^*) > 0 \Leftrightarrow G_L^o < G_L^*$

(ii) $\hat{x}^o - \hat{x}^* = \frac{\alpha_H - \alpha_L + k\left[\upsilon(G_H^o) - \upsilon(G_L^o)\right]}{2c} - \frac{\alpha_H - \alpha_L + k\left[\upsilon(G_H^*) - \upsilon(G_L^*)\right]}{2c}$
Since $G_H^o > G_H^*$ and $G_L^o < G_L^* \Rightarrow \hat{x}^o - \hat{x}^* > 0 \Leftrightarrow \hat{x}^o > \hat{x}^*$

$\square$

Proof of **Proposition 2**:

For G_H please check the proof of proposition 1 as the problem is the same.
$\upsilon'(G_L^o) - \upsilon'(G_L^*) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1-k}$
Since $\hat{x}^o \in \left(0; \frac{1}{2}\right) \Rightarrow 1 - k < 1 - 2\hat{x}^o k \Leftrightarrow \upsilon'(G_L^o) - \upsilon'(G_L^*) < 0 \Rightarrow G_L^o > G_L^*$

$\square$

Proof of **Proposition 3**:

(i) $\tau_H^{**} = \frac{\alpha_H - \alpha_L + k\left[\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right]}{3\alpha_H}$
Since $\alpha_H > \alpha_L$ and $G_H^{**} > G_L^{**} \Rightarrow \tau_H^{**} > 0$
$\tau_L^{**} = \frac{-(\alpha_H - \alpha_L) - k\left[\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right]}{3\alpha_H}$
Since $\alpha_H > \alpha_L$ and $G_H^{**} > G_L^{**} \Rightarrow \tau_L^{**} < 0$

(ii) $\upsilon'(G_H^o) - \upsilon'(G_H^* *) = \frac{2}{1+2\hat{x}^o k} - \frac{2}{1+2\hat{x}^{**} k}$
Since $\hat{x}^o > \hat{x}^{**} \Rightarrow \upsilon'(G_H^o) - \upsilon'(G_H^{**}) < 0 \Leftrightarrow G_H^o > G_H^{**}$
$\upsilon'(G_L^o) - \upsilon'(G_L^{**}) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1-2\hat{x}^{**} k}$
Since $\hat{x}^o > \hat{x}^{**} \Rightarrow \upsilon'(G_L^o) - \upsilon'(G_L^{**}) > 0 \Leftrightarrow G_L^o < G_L^{**}$

(iii) $\hat{x}^o - \hat{x}^{**} = \frac{\alpha_H - \alpha_L + k\left[\upsilon(G_H^o) - \upsilon(G_L^o)\right]}{2c} - \frac{\alpha_H - \alpha_L + k\left[\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right]}{6c}$
Since $G_H^o > G_H^{**}$ and $G_L^o < G_L^{**} \Rightarrow \hat{x}^o - \hat{x}^{**} > 0 \Leftrightarrow \hat{x}^o > \hat{x}^{**}$

$\square$

Proof of **Proposition 4**:

(i) $\tau_H^{**}\alpha_H^{**} = \frac{c}{3} + 2c\left(\hat{x}^{**} - \frac{1}{6}\right)$
Since $\hat{x}^{**} \in \left(\frac{1}{4}; \frac{1}{2}\right) \Rightarrow \tau_H^{**} > 0$
$\tau_L^{**}\alpha_L^{**} = \frac{2}{3}c - 2c\left(\hat{x}^{**} - \frac{1}{6}\right)$
Since $\hat{x}^{**} \in \left(\frac{1}{4}; \frac{1}{2}\right) \Rightarrow \tau_L^{**}\alpha_L^{**} \in \left(0; \frac{1}{2}\right) \Rightarrow \tau_L^{**} > 0$

(ii) For G_H please check the proof of proposition 3 as the problem is the same.

(iii) $\upsilon'(G_L^o) - \upsilon'(G_L^{**}) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1-2\hat{x}^{**}k-\frac{2}{3}k}$
Since the numerators of both fractions are the same we can compare just the denominators:
$[1-2\hat{x}^o k] - \left[1-2\hat{x}^{**}k - \frac{2}{3}k\right] = 2k\left[\frac{1}{3} - (\hat{x}^o - \hat{x}^{**})\right]$
Overprovision of $G_L \Leftrightarrow G_L^o < G_L^{**} \Leftrightarrow \upsilon'(G_L^o) > \upsilon'(G_L^{**}) \Leftrightarrow$
$\Leftrightarrow 2k\left[\frac{1}{3} - (\hat{x}^o - \hat{x}^{**})\right] < 0 \Rightarrow \hat{x}^o > \hat{x}^{**}$

(iv) Overcommuting of agents $\Leftrightarrow \hat{x}^o < \hat{x}^{**} \Rightarrow 2k\left[\frac{1}{3} - (\hat{x}^o - \hat{x}^{**})\right] > 0 \Leftrightarrow$
$\Leftrightarrow \upsilon'(G_L^o) < \upsilon'(G_L^{**}) \Leftrightarrow G_L^o > G_L^{**}$

□

Proof of **Proposition 5**:

$\upsilon'(G_H^*) - \upsilon'(G_H^{**}) = \frac{2}{1} - \frac{2}{1+2\hat{x}^{**}}$
Since $1+2\hat{x}^{**} > 1 \Rightarrow \upsilon'(G_H^*) - \upsilon'(G_H^{**}) > 0 \Leftrightarrow G_H^* < G_H^{**}$
$\upsilon'(G_L^*) - \upsilon'(G_L^{**}) = \frac{2}{1} - \frac{2}{1-2\hat{x}^{**}}$
Since $1-2\hat{x}^{**} < 1 \Rightarrow \upsilon'(G_L^*) - \upsilon'(G_L^{**}) < 0 \Leftrightarrow G_L^* > G_L^{**}$

□

Proof of **Proposition 6**:

$\upsilon'(G_H^*) - \upsilon'(G_H^{**}) = \frac{2}{1} - \frac{2}{1+k\left(2\hat{x}^{**}-\frac{1}{3}\right)}$
Since $\hat{x}^{**} \in \left(\frac{1}{4};\frac{1}{2}\right) \Rightarrow 1+k\left(2\hat{x}^{**}-\frac{1}{3}\right) > 1 \Rightarrow \upsilon'(G_H^*) - \upsilon'(G_H^{**}) > 0 \Leftrightarrow$
$\Leftrightarrow G_H^* < G_H^{**}$
$\upsilon'(G_L^*) - \upsilon'(G_L^{**}) = \frac{2}{1-k} - \frac{2}{1+k\left(\frac{4}{3}-2\hat{x}^{**}\right)}$
Since $\hat{x} \in \left(\frac{1}{4};\frac{1}{2}\right) \Rightarrow 1+k\left(\frac{4}{3}-2\hat{x}^{**}\right) > 1-k \Rightarrow \upsilon'(G_L^*) - \upsilon'(G_L^{**}) > 0 \Leftrightarrow$
$\Leftrightarrow G_L^* < G_L^{**}$

□

The median voter of H is not willing to work in L in section 4

If the median voter of H commutes to L, $\hat{x} < -1/4$ and the local government in H will:

$$\max_{G_i,T_i} u_{m_H} = \alpha_L + W - T_H - c\left(\frac{1}{4}+\gamma\right) + (1-k)\sqrt{G_H} + k\sqrt{G_L}$$
$$s.t.\ \hat{x} = \frac{\alpha_H - \alpha_L + k\left(\sqrt{G_H} - \sqrt{G_L}\right)}{2c}$$
$$G_H = \frac{1}{2}T_H$$

FOC:

$$\frac{1}{2\sqrt{G_H}} = \frac{2}{1-k} \Leftrightarrow \sqrt{G_H^*} = \frac{1-k}{4}$$

The first order condition in G_L is the same as (14) which leads to $\sqrt{G_L} = 1/4$. The commuting equilibrium is therefore defined by:

$$\hat{x} = \frac{\alpha_H - \alpha_L + k\left(\frac{1-k}{4} - \frac{1}{4}\right)}{2c}$$

and since $k \in (0;1)$

$$\hat{x} \in \left(\frac{\alpha_H - \alpha_L - \frac{1}{4}}{2c}; \frac{\alpha_H - \alpha_L}{2c}\right) > -\frac{1}{4}$$

Thus, and as expected, it is impossible to have the median voter of H commuting to L since he would bear a higher commuting cost and earn a lower wage.

$\square$

Proof that $\tau_H^{**}\alpha_H > \tau_L^{**}\alpha_L$ and $G_H^{**} > G_L^{**}$ in 5.1

$\tau_H^{**}\alpha_H = \frac{\alpha_H - \alpha_L + k\left(\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right)}{3}$
$\tau_L^{**}\alpha_L = \frac{-(\alpha_H - \alpha_L) - k\left(\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right)}{3}$
$\tau_H^{**}\alpha_H - \tau_L^{**}\alpha_L = \frac{2}{3}\left(\alpha_H - \alpha_L + k\left[\upsilon(G_H^{**}) - \upsilon(G_L^{**})\right]\right)$
If $\tau_H^{**}\alpha_H > \tau_L^{**}\alpha_L \Rightarrow G_H^{**} > G_L^{**} \Rightarrow \tau_H^{**}\alpha_H > \tau_L^{**}\alpha_L$ and $\hat{x^{**}} > 0$
If $\tau_H^{**}\alpha_H < \tau_L^{**}\alpha_L \Rightarrow k[\upsilon(G_L^{**}) - \upsilon(G_H^{**}) > \alpha_H - \alpha_L \Rightarrow \hat{x^{**}} < 0$
which is impossible.

$\square$

References

Alexina, A. and E. Spolaore, 1997, On the number and size of nations, The Quarterly Journal of Economics 112(4), 1027-1056.

Besley, T. and S. Coate, 2003, Centralized versus decentralized provision of local public goods: a political economy approach, Journal of Public Economics 87(12), 2611-2637.

Bolton, P. and G. Roland, The breakup of nations: a political economy analysis, The Quarterly Journal of Economics 112(4), 1057-1090.

Braid, R., 2000, A spatial model of tax competition with multiple tax instruments, Journal of Urban Economics 47, 88-114.

Braid, R., 2005, Tax competition, tax exporting and higher-government choice of tax instruments for local governments, Journal of Public Economics 89, 1789-1821.

Braid, R., 2009, The employment effects of a central city's source-based wage tax or hybrid wage tax, Regional Science and Urban Economics 39, 512521.

Cameron, G. and J. Muelbauer, 1998, The housing market and regional commuting and migration choices, Scottish Journal of Political Economy 45(4), 420-446.

Chu, K. and J. Norregaard, 1997, Korea, *in* Ter-Minassian, Teresa (ed.), Fiscal Federalism in Theory and Practice, IMF.

Fernandez, G., 2004, Tax increment financing: interaction between two overlapping jurisdictions, Journal of Urban Economics 55, 151164.

Fisher, R.S., 1996, State and local public finance, 2nd. ed., Irwin.

Fuest, C. and B. Huber, 2001, Why is there so little tax coordination? The role of majority voting and international tax evasion, Regional Science and Urban Economics 31, 299-317.

Glaeser, E., J. Kolko and A. Saiz, 2001, The consumer city, Journal of Economic Geography 1(1), 27-50.

Grazzini, L. and T. van Ypersele, 2003, Fiscal coordination and political competition, Journal of Public Economic Theory 5(2), 305-325.

Oates, W., 1972, Fiscal Federalism, Harcourt Brace Jovanovich.

Oates, W., 1999, An Essay on Fiscal Federalism, Journal of Economic Literature 37(3), 1120-1149.

OECD, 2009, The fiscal autonomy of sub-central governments: an update.

Peralta, S., 2007, Political support for tax decentralization, Journal of Public Economic Theory 9, 1013-1030.

Renkow, M., 2003, Employment growth, worker mobility and rural economic development, American Journal of Agricultural Economics 85(2), 503-513.

Rouwendal, J. and E. Meijer, 2001, Preferences for housing, jobs and commuting: a mixed logit analysis, Journal of Regional Science 41(3), 475-505.

Shields, M. and D. Swenson, 2000, Regional labour markets: the relationship between industry level employment and in-commuting in pennsylvania counties, The Journal of Regional Analysis and Policy 30(2), 81-94.

Van Ommeren, J., P. Rietveld and P. Nijkamp, 1999, Job moving, residential moving and commuting: a search perspective, Journal of Urban Economics 46, 230-253.

Wheaton, M.C., 1977, A bid-rent approach to housing demand, Journal of Urban Economics 4, 200217.

Wildasin, D.E., 1986, Spatial variations of the marginal utility of income and unequal treatment of equals, Journal of Urban Economics 19, 125-129.

Wrede, M., 2009, A distortive wage tax and a countervailing commuting subsidy, Journal of Public Economic Theory 11(2), 297-310.

Zax, J., 1991, The substitution between quits and moves, The Economic Journal 101(409), 1510 - 1521.

Zax, J., 1994, When is a move a migration?, Regional Science and Urban Economics 24, 341-360.