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Education Choice and Endogenous Economic Growth

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Abstract

In this paper we analyze the interaction between public and private education choice and economic growth pattern by overlapping-generations model in the model mainly based on Cardak(2004, Economica). In his model public education expenditure is financed by income tax, however, the determination of the tax rate is not clear. And human capital production function when private education is provided becomes linear. This result is influenced by his assumption that income is equal to human capital. Moreover, the evolution is determined by the parent's human capital and education expenditure. But in this paper we assume that there is a production of goods and income is determined by wage rate and human capital which depends on Galor and Tsiddon(1997). That is, income is not equal to human capital. The dynamical system is determined by the parent's human capital, education expenditure and the individual's own devoting. Moreover, we assume that income tax rate is determined by the agent's preference which depends on Glomm and Ravikumar(1992). We extend the analysis of Cardak(2004, Economica) and suggest the case that the human capital production function when private education is provided becomes concave.

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Key words: human capital, economic growth, overlapping-generations, public education, education choice

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1 INTRODUCTION

In this paper we analyze the interaction between public and private education choice and economic growth pattern by overlapping-generations model in the model mainly based on Cardak(2004, Economica). Typically the studies of public and private education focus on comparisons of separate public and private education models, such as in Glomm and Ravikumar (1992), Gradstein and Justman (1997) and Saint Paul and Verdier (1993). Another common theme found in the literature is the case where public and private education are complements in human capital production, as in Benabou (1996), Eckstein and Zilcha (1994) and Kaganovich and Zilcha (1999). In Cardak(2004, Economica), he departs from the literature by analysing the case where public and private education coexist as mutually exclusive alternatives, rather than comparing public with private education.

In this paper we extend the analysis in Cardak(2004, Economica). In his model public education expenditure is financed by income tax, however, the determination of the tax rate is not clear. And human capital production function when private education is provided becomes linear. This result is influenced by his assumption that income is equal to human capital. Moreover, human capital accumulation are determined by the parent's human capital level and education expenditure. But in this paper we assume that there exists a production of goods and income is determined by wage rate and human capital which depends on Galor and Tsiddon(1997). That is, income is not equal to human capital. Human capital accumulation is determined by the parent's human capital, education expenditure and individual's own devoting. Moreover, we assume that income tax rate is determined by the agent's preference which depends on Glomm and Ravikumar(1992). We extend the analysis of Cardak(2004, Economica) and suggest the case that human capital production function when private education is provided becomes concave.

In this paper, we survey the basic model in section 2. And in section 3, we solve individual's optimaizations and agent's optimization, and consider the interation between the education choice and economic growth. The main results of this paper are shown in section 3.

2 The Model

Consider a small open overlapping-generations economy that operates in a perfectly competitive world in which economic activity extends over an infinite discrete time. Individuals of generation t live two periods which we call t and $t+1$.[1]

2-a. The Goods Market

Production occurs within a period according to a constant-return-to-scale.[2] The output (Y_t) produced at time t is

[1] We assume that the children of individuals in a generation are born in the second period.

[2] therefore,

$$F\left(zK_t, z\lambda H_t\right) = zF\left(K_t, \lambda H_t\right); \quad z > 0$$

$$Y_t = F\left(K_t, \lambda H_t\right); \quad k_t \equiv \frac{K_t}{\lambda H_t}$$

where K_t and H_t are the quantities of capital and efficiency-labor employed in the production at time t respectively and λ is the technological coefficient (parameter). The output per producer $(f\left(k_t\right))$ at time t is defined as follow. [3]

$$f(k_t) \equiv \frac{Y_t}{\lambda H_t}$$

Producers operate in a perfectly competitive environment. Given, the wage rate (w_t) and the rate of return to capital (r_t) at time t respectively. Producer's profit is as follow.

$$\Pi\left(K_t, \lambda H_t\right) = F\left(K_t, \lambda H_t\right) - w_t H_t - r_t K_t = \lambda H_t f\left(k_t\right) - w_t H_t - r_t \lambda H_t k_t$$

Producers choose the level of employment of capital, K_t, and labor, H_t, so as to maximize profits.

$$\frac{\partial \Pi}{\partial k_t} = \lambda H_t f'\left(k_t\right) - r_t \lambda H_t = 0 \Longrightarrow f'\left(k_t\right) = r_t$$
$$\frac{\partial \Pi}{\partial H_t} = \lambda f\left(k_t\right) - w_t - r_t \lambda k_t = 0 \Longrightarrow w_t = \lambda\left[f\left(k_t\right) - f'\left(k_t\right) k_t\right]$$

[3] $f(k_t)$ is strictly monotonic increasing, strictly concave satisfying the neoclassical boundary conditions.

$f(k_t) - f'(k_t)k_t$ shows real wage, and following equation and difinition are obtained.

$$
\begin{aligned}
w_t &= \lambda_t \left[f(k_t) - f'(k_t)k_t \right] \equiv \lambda_t w(k_t) \\
r_t &= \overline{r}
\end{aligned}
$$

By difinition w_t is given as follows.

$$w_t \equiv \lambda w(\overline{k}) \equiv \lambda \overline{w} \tag{1}$$

2-b. Human Capital Accumulations

An individual i of generation t is born to a parent with h_t^i units of human capital. His parent invests him q_t^i units of education expenditure, and he allocates n_t units of his endowment toward leisure at time t and devotes the remaining $1 - n_t$ units toward human capital accumulation. He acquires h_{t+1}^i units of human capital.

$$h_{t+1}^i = (1 - n_t)^{\beta} \left(q_t^i \right)^{\gamma} \left(h_t^i \right)^{\delta}; \; \beta, \gamma, \delta \in (0,1) \tag{2}$$

q_t^i is determined by the respective education,

$$q_t^i = \left\{ \begin{array}{l} E_t \quad if \quad e_t^i = 0 \cdots public \;\; education \;\; chosen \\ \\ e_t^i \quad if \quad e_t^i > 0 \cdots private \;\; education \;\; chosen \end{array} \right\}$$

where E_t and e_t^i are the parent's education expenditures of an individual i at time t under public education and private education. E_t is defined as follows,

$$E_t = \frac{\tau_t h_t}{P_t} = \frac{\tau_t \int_0^\infty h_t^i \cdot g_t\left(h_t^i\right) dh_t^i}{P_t}$$

where τ_t, h_t, and P_t are the income tax rate, the average human capital level, and the proportion of the population in public education at time t. In this paper we show population distributions using the density function. In this paper we make the population size a standard to 1.

2-c. Equilibrium

The labor income (I_{t+1}^i) generated by an individual i from generation t at time $t+1$ is the wage rate per efficiency-labor (w_{t+1}) at time $t+1$ multiplied by the number of efficiency units (h_{t+1}^i) supplied by the individual. From equation (1) and (2),

$$I_{t+1}^i = w_{t+1} h_{t+1}^i = \overline{w}\lambda \left(1-n_t\right)^\beta \left(q_t^i\right)^\gamma \left(h_t^i\right)^\delta$$

In this paper we assume that there are no inheritances. That is, the consumption (c_{t+1}^i) of an individual i at time $t+1$ is determined as follows.

$$c_{t+1}^i = \left\{ \begin{array}{ll} \left(1-\tau_{t+1}\right) I_{t+1}^i & \cdots e_{t+1}^i = 0 \cdots \textit{public education chosen} \\ \left(1-\tau_{t+1}\right) I_{t+1}^i - e_{t+1}^i & \cdots e_{t+1}^i > 0 \cdots \textit{private education chosen} \end{array} \right\} \tag{3}$$

2-c-(1). Public Education

An individual i of generation t chooses n_t and c_{t+1}^i so as to maximize the utility (U^{PU}) in the whole two periods under public education.

$$\underset{n_t, c_{t+1}^i}{Maximize} \quad U^{PU} = (1-\alpha_1-\alpha_2)\log n_t + \alpha_1 \log c_{t+1}^i + \alpha_2 \log E_{t+1}$$

$$subject \quad to \quad c_{t+1}^i = I_{t+1}^i, \quad I_{t+1}^i = \overline{w}\lambda h_{t+1}^i, \quad h_{t+1}^i = (1-n_t)^\beta (E_t)^\gamma \left(h_t^i\right)^\delta$$

The optimal time allocated to human capital accumulation $(1-n_t^{PU})$ by the individual born under public education is as below.[4]

$$1-n_t^{PU} = \frac{\beta\alpha_1}{1+(\beta-1)\alpha_1-\alpha_2} \tag{4}$$

Here, we assume that the agent chooses the optimal income tax rate so as to maximize the utility(U^G) at time $t+1$.

$$\underset{\tau_{t+1}}{Maximize} \quad U^G = \eta \log\left[(1-\tau_{t+1})\,\overline{w}\lambda h_{t+1}^i\right] + (1-\eta)\log\left[\tau_{t+1}\overline{w}\lambda h_{t+1}^i\right]$$

The optimal income tax rate (τ_{t+1}) at time $t+1$ is as follow.[5]

$$\tau_{t+1} = 1-\eta \tag{5}$$

From equation (3), (4) and (5), the optimal consumption (c_{t+1}^{PU}) at time $t+1$ is determied as follow.

[4] Equation (4) is proved in the Appendix A.

[5] Equation (5) is proved in the Appendix B.

$$c_{t+1}^{PU}=\eta\overline{w}\lambda\left\{\frac{\beta\alpha_1}{1+(\beta-1)\,\alpha_1-\alpha_2}\right\}^{\beta}(E_t)^{\gamma}\left(h_t^i\right)^{\delta}$$

2-c-(2). Private Education

An individual i of generation t chooses n_t, c_{t+1}^i, and e_{t+1}^i so as to maximize the utility (U^{PR}) in the whole two periods under private education.

$$\underset{n_t,c_{t+1}^i,e_{t+1}^i}{Maximize}\quad U^{PR}=(1-\alpha_1-\alpha_2)\log n_t+\alpha_1\log c_{t+1}^i+\alpha_2\log e_{t+1}^i$$

$$subject\quad to\quad c_{t+1}^i=I_{t+1}^i-e_{t+1}^i,\quad I_{t+1}^i=\overline{w}\lambda h_{t+1}^i,\quad h_{t+1}^i=(1-n_t)^{\beta}\left(e_t^i\right)^{\gamma}\left(h_t^i\right)^{\delta}$$

The optimal consumption (c_t^{PR}) and education expenditure (e_{t+1}^{PR}) under private education are as follows.[6]

$$c_t^{PR}=\frac{\alpha_1\,(1-\tau_{t+1})\,I_{t+1}^i}{\alpha_1+\alpha_2},\quad e_{t+1}^{PR}=\frac{\alpha_2\,(1-\tau_{t+1})\,I_{t+1}^i}{\alpha_1+\alpha_2}\tag{6}$$

The optimal time allocated to human capital accumulation ($1-n_t^{PR}$) by the individual born at time t under private education is as follow.[7]

$$1-n_t^{PR}=\frac{\beta\,(\alpha_1+\alpha_2)}{1+(\beta-1)\,(\alpha_1+\alpha_2)}\tag{7}$$

[6] Equation (6) is proved in the Appendix C.

[7] Equation (7) is proved in the Appendix D.

3 EDUCATION CHOICE AND ECONOMIC GROWTH

Public education and private education choice is determined by the comparison of the utilities. The human capital level (h^*_{t+1}) which satisfies $U^{PU} = U^{PR}$ at time $t+1$ is as follow.

$$h^*_{t+1} = \left[\frac{1+(\beta-1)(\alpha_1+\alpha_2)}{1+(\beta-1)\alpha_1-\alpha_2}\right]^{\frac{1-\alpha_1-\alpha_2}{\alpha_2}} \left[\frac{\alpha_1+\alpha_2}{\alpha_1}\right]^{\frac{\alpha_1}{\alpha_2}} \left[\frac{E^*_{t+1}(\alpha_1+\alpha_2)}{\alpha_2\eta\overline{w}\lambda}\right] \tag{8}$$

All individuals with human capital $h^i_{t+1} \leq h^*_{t+1}$ will prefer to provide public education and with human capital $h^*_{t+1} < h^i_{t+1}$ will prefer to privide private education for their children at time $t+1$. All individual's human capital under pulic education converges to the steady state (h^u_s), and human capital under private education converges to the steady state (h^r_s).

$$h^u_s = \left[\frac{\beta(\alpha_1+\alpha_2)}{1+(\beta-1)(\alpha_1+\alpha_2)}\right]^{\frac{\beta}{1-\delta}} \left[\frac{(1-\eta)h_t}{P_t}\right]^{\frac{\gamma}{1-\delta}} \tag{9}$$

$$h^r_s = \left[\frac{\beta\alpha_1}{1+(\beta-1)\alpha_1-\alpha_2}\right]^{\frac{\beta}{1-\gamma-\delta}} \left[\frac{\alpha_2\eta\overline{w}\lambda}{\alpha_1+\alpha_2}\right]^{\frac{\gamma}{1-\gamma-\delta}} \tag{10}$$

In this paper we assume that $h^u_s < h^r_s$. This basis is that participation in private education is voluntary and it must provide greater education expenditures and thereby higher steady state human capital level in order to be optimal, depends on Cardak(2004, Oxford Economic Papers). In Cardak(2004, Economica) the human capital production function under private education is linear, but in this paper that is concave and is similar to the function under public education. The human capital production functions coexist and cross when the human capital level at time t is (h_t^{**}).

$$h_t^{**} = \left[\frac{(\alpha_1+\alpha_2)\{1+(\beta-1)\alpha_1-\alpha_2\}}{\alpha_1\{1+(\beta-1)(\alpha_1+\alpha_2)\}}\right]^{\frac{\beta}{\gamma}}\left[\frac{(\alpha_1+\alpha_2)}{\alpha_1\eta\overline{w}\lambda}\right]\frac{(1-\eta)h_t}{P_t} \tag{11}$$

From equation (8), (9), and (10), the evolutions of human capital under public education and private education are depicted as figure 1.

[Insert Figure 1 around here.]

Moreover,

$$P_t = \int_0^{h_t^*} g_t\left(h_t^i\right)dh_t^i \tag{12}$$

where, h_t^* is the human capital level which satisfies $U^{PU} = U^{PR}$ at time t. From equation (11), when h_t^* is large value, the proportion of the population in public education increases and the education expenditure under public education per capita decrease. That is not desirable for economic growth. That is, it is desirable that h_t^* is low value for ecnomic growth.

4 CONCLUDING REMARKS

The main conclusions and the contribution to the recent researches are as follows.

(a) In Cardak(2004, Economica), the private education production function is linear but in this paper it becomes concave function which is similar to the model of Cardak(2004, Economica).

(b) We consider not only the optimal consumption and the education expenditure, but also the optimal time to allocate to human capital accumulation.

(c) In Cardak(2004, Economica), the analysis of optimal income tax rate is not enough, but in this paper we can show it clearly by the agent's optimaization depends on Glomm and Ravikumar(1992).

(d) It is desiable that the standard value of human capital level in education choice is low value for economic growth.

In this paper we extend the analysis of Cardak(2004, Economica) and suggest the realistic case about the model of education choice.

APPENDIX A

The utility function is rewritten as follow.

$$U^{PU} = (1-\alpha_1-\alpha_2)\log n_t + \alpha_1 \log\left[(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta}\,(E_t)^{\gamma}\left(h_t^i\right)^{\delta}\right] + \alpha_2 \log E_{t+1}$$

The optimal time allocated to human capital accumulation is derived as follows.

$$\begin{aligned}
\frac{\partial U^{PU}}{\partial c_{t+1}^i} &= \frac{1-\alpha_1-\alpha_2}{n_t} - \frac{\beta\alpha_1(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta-1}(E_t)^{\gamma}(h_t^i)^{\delta}}{(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta}(E_t)^{\gamma}(h_t^i)^{\delta}} \\
&= \frac{1-\alpha_1-\alpha_2}{n_t} - \frac{\beta\alpha_1}{1-n_t} = 0 \\
n_t^{PU} &= \frac{1-\alpha_1-\alpha_2}{1+(\beta-1)\alpha_1-\alpha_2} \quad \Longrightarrow \quad 1-n_t^{PU} = \frac{\beta\alpha_1}{1+(\beta-1)\alpha_1-\alpha_2}
\end{aligned}$$

APPENDIX B

The optimal income tax rate is derived as follows.

$$
\begin{aligned}
\frac{\partial U^G}{\partial \tau_{t+1}} &= -\eta \frac{I_{t+1}^i}{(1-\tau_{t+1}) I_{t+1}^i} + (1-\eta) \frac{I_{t+1}^i}{\tau_{t+1} I_{t+1}^i} \\
&= -\frac{\eta}{1-\tau_{t+1}} + \frac{1-\eta}{\tau_{t+1}} = 0 \\
\tau_{t+1} &= 1-\eta
\end{aligned}
$$

APPENDIX C

The utility function is rewritten as follow.

$$
U^{PR} = (1-\alpha_1-\alpha_2) \log n_t + \alpha_1 \log c_{t+1}^i + \alpha_2 \log \left[(1-\tau_{t+1}) I_{t+1}^i - c_{t+1}^i \right]
$$

The optimal consumption is derived as follows.

$$
\begin{aligned}
\frac{\partial U^{PR}}{\partial c_{t+1}^i} &= \frac{\alpha_1}{c_{t+1}^i} - \frac{\alpha_2}{(1-\tau_{t+1}) I_{t+1}^i - c_{t+1}^i} = 0 \\
c_{t+1}^{PR} &= \frac{\alpha_1 (1-\tau_{t+1}) I_{t+1}^i}{\alpha_1 + \alpha_2}
\end{aligned}
$$

In the same way, the optimal education expenditure is derived as follows.

$$
\begin{aligned}
U^{PR} &= (1-\alpha_1-\alpha_2) \log n_t + \alpha_1 \log \left[(1-\tau_{t+1}) I_{t+1}^i - e_{t+1}^i \right] + \alpha_2 \log e_{t+1}^i \\
\frac{\partial U^{PR}}{\partial e_{t+1}^i} &= -\frac{\alpha_1}{(1-\tau_{t+1}) I_{t+1}^i - e_{t+1}^i} + \frac{\alpha_2}{e_{t+1}^i} = 0 \\
e_{t+1}^{PR} &= \frac{\alpha_2 (1-\tau_{t+1}) I_{t+1}^i}{\alpha_1 + \alpha_2}
\end{aligned}
$$

APPENDIX D

The utility function is rewritten as follow.

$$U^{PR} = (1-\alpha_1-\alpha_2)\log n_t + \alpha_1 \log\left[(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta}\,(E_t)^{\gamma}\left(h_t^i\right)^{\delta} - e_{t+1}^i\right] + \alpha_2 \log e_{t+1}^i$$

The optimal time allocated to human capital accumulation is derived as follows.

$$\begin{aligned}
\frac{\partial U^{PR}}{\partial n_t} &= \frac{1-\alpha_1-\alpha_2}{n_t} - \frac{\alpha_1\beta\,(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta-1}\,(E_t)^{\gamma}\,(h_t^i)^{\delta}}{(1-\tau_{t+1})\,\overline{w}\lambda\,(1-n_t)^{\beta}\,(E_t)^{\gamma}\,(h_t^i)^{\delta} - e_{t+1}^i} \\
&= \frac{1-\alpha_1-\alpha_2}{n_t} - \frac{\alpha_1\beta\eta\overline{w}\lambda\,(1-n_t)^{\beta-1}\,(E_t)^{\gamma}\,(h_t^i)^{\delta}}{\eta\overline{w}\lambda\,(1-n_t)^{\beta}\,(E_t)^{\gamma}\,(h_t^i)^{\delta} - \frac{\alpha_2\eta\overline{w}\lambda(1-n_t)^{\beta}(E_t)^{\gamma}\left(h_t^i\right)^{\delta}}{\alpha_1+\alpha_2}} = 0 \\
n_t^{PR} &= \frac{1-\alpha_1-\alpha_2}{1+(\beta-1)(\alpha_1+\alpha_2)} \quad \Longrightarrow \quad 1-n_t^{PR} = \frac{\beta\,(\alpha_1+\alpha_2)}{1+(\beta-1)(\alpha_1+\alpha_2)}
\end{aligned}$$

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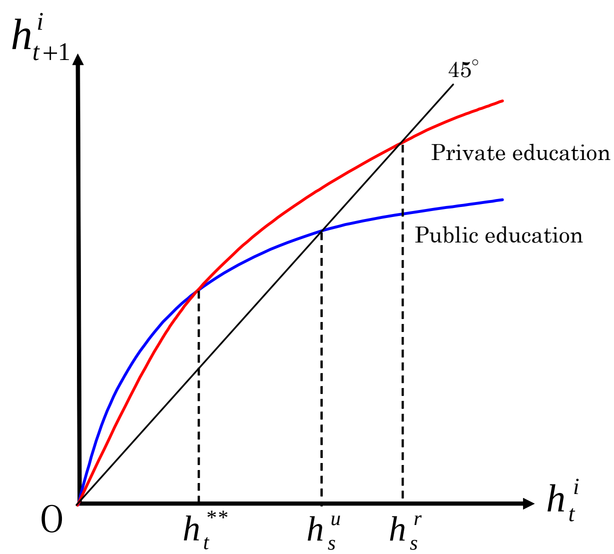

Fig. 1. The evolutions of human capital