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The Essentiality of Money in Environments with Centralized Trade*

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Abstract

A major concern in modern monetary theory is the construction of tractable models where money is essential. Lagos and Wright (2005) pioneered a class of models that address this concern. The key ingredient of the Lagos–Wright framework is that trade alternates between centralized and decentralized markets. In this paper, unlike the previous literature, we explicitly model the process of exchange in the centralized market. We show that there exists a tension between the essentiality of money and centralized trading and characterize conditions under which money is essential.

Key Words: Money, Centralized Markets, Essentiality.

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1 Introduction

Modern monetary theory is based on the notion that one must be explicit about the frictions that render money essential. Two frictions are considered to be particularly relevant for essentiality: limited commitment and limited record-keeping. In fact, it is commonly agreed that limited commitment and limited record-keeping are necessary for the essentiality of money.¹ A less established view is that as long as the distribution of preferences and technologies in the economy prevents the recurrence of double-coincidences, the absence of both commitment and record-keeping suffices to render money essential. In particular, according to this view, the essentiality of money does not hinge on whether exchange takes place in decentralized or centralized markets. This belief has granted much flexibility in the recent effort towards building models where money is essential and yet substantive issues can be analyzed in a tractable manner. Lagos and Wright (2005) (henceforth LW) is at the center of this effort. The main contribution of LW is constructing an environment where, unlike in the search models of money in the tradition of Kiyotaki and Wright (1989), money is divisible and the distribution of money holdings is degenerate. The key element of LW is that trade alternates between centralized and decentralized markets.

In this paper we examine whether in fact centralized trading has no implications for the essentiality of money. We are not the first ones to pose this question. Aliprantis, Camera and Puzzello (2007) (henceforth ACP) show that money can fail to be essential if individual actions are observable in a centralized market. Lagos and Wright (2008) point out that in LW agents only observe prices in the centralized market, and thus ACP's critique does not apply. Unlike ACP, we assume that agents only observe prices in the centralized market and examine whether this indeed renders money essential.

The starting point of our analysis is the framework of LW modified in two ways. First, while LW assumes a continuum of agents, we consider large but finite populations. In any model, the assumption of a continuum of agents is made for tractability and is only justifiable if it has no substantive economic implications. To put it differently: "The rationale for using

¹For the role of limited commitment in the essentiality of money, see Huggett and Krasa (1996) and Kiyotaki and Moore (2002). For the role of limited record-keeping, see Ostroy (1973), Aiyagari and Wallace (1991), and Kocherlakota and Wallace (1998).

the continuum-of-agents model is that it is a useful idealization of a situation with a large finite number of agents, but if equilibria in the continuum model are radically different from equilibria in the model with a finite number of agents, then this idealization makes little sense” (Levine and Pesendorfer (1995), p. 1160). We want to ensure that the essentiality of money is not driven by the continuum population assumption.

The second departure from the LW framework is that we explicitly model the process of exchange in the centralized market. In LW, the centralized market is a Walrasian market, where agents do not trade with each other, but trade only against their budget constraints. This specification is parsimonious and allows for analytical simplicity. However, it does not describe how exchange in the centralized market actually takes place, a requirement that is called for if the main objective is to understand the conditions under which money is essential as a medium of exchange.²

We model the process of exchange in the centralized market as a non-cooperative game, more precisely, as a strategic market game along the lines of Shapley and Shubik (1977).³ There are two reasons for this. First, strategic market games are a natural departure from Walrasian markets. In fact, it is well known that in static settings, equilibrium outcomes of strategic market games converge to Walrasian outcomes as the number of agents increases to infinity (Mas-Colell (1982)). Second, strategic market games allow for a mapping between the formation of prices and the trading decisions of agents, while retaining the idea of centralized markets as large anonymous markets where agents only observe prices.⁴

The first result of the paper is that if agents are patient enough, there exists a non-monetary equilibrium that implements the first-best regardless of the population size. The intuition for this result is simple. When the number of agents is finite, individual actions in the centralized market have an impact on prices regardless of the population size. Thus,

²More generally, modeling exchange in the centralized market explicitly is in line with the emphasis that modern monetary theory places on taking seriously the process of exchange.

³The classic reference on strategic market games is Shubik (1973). See also Dubey and Shubik (1978) and Postlewaite and Schmeidler (1978). Alonso (2001), Green and Zhou (2005), Hayashi and Matsui (1996), and Howitt (2005) are examples of applications of market games in the context of monetary theory.

⁴A third reason for the non-cooperative approach is that if one wants to assess the conditions under which money is essential, one must consider whether agents have the incentive to follow alternative credit-like arrangements. The standard Walrasian market does not specify how payoffs are defined off the equilibrium path and thus it is ill-suited to check the feasibility of competing mechanisms of exchange.

prices can convey information about individual deviations and this can be used to sustain cooperative behavior. This shows that modeling centralized trade as a Walrasian market, where the market power of agents is exogenously set to zero, misses an important aspect of dynamic trading environments. Namely, that even if an agent’s action has a small impact on current aggregate outcomes, it may still have a sizable impact on future aggregate outcomes.⁵

The fact that prices reflect individual behavior in a precise way plays an important role in our non-essentiality result. This suggests that this result may not survive the introduction of noise in the price formation process, especially when the population is large. It turns out that this need not be the case. It is true that the presence of noise reduces the ability of prices to convey information about individual behavior when there are many agents. However, we show that whether agents become informationally negligible or not in large economies critically depends on the ratio between the number of agents who participate in trade and the number of goods that are traded in the centralized market, that is, on how “thick” the centralized market is. Thus, our results highlight a nontrivial tension between the essentiality of money and centralized trading, and provide conditions under which specifying the centralized market as a Walrasian market is justified in monetary models.

The paper is organized as follows. We introduce our framework in the next section. We establish the non-essentiality of money in Section 3 and examine its robustness to the introduction of noise in the price formation process in Section 4. Section 5 concludes and the Appendix contains omitted proofs.

2 The Model

We first describe the environment and preferences. Then we describe the economy as an infinitely repeated game.

⁵Green (1980) makes a similar point in the context of repeated Cournot competition.

2.1 Environment and Preferences

Time is discrete and indexed by $t \geq 1$. There are two stages at each date and preferences are additively separable across dates and stages. The population consists of a finite number N of infinitely lived agents. Agents do not discount payoffs between stages in a period and have a common discount factor $\delta \in (0, 1)$ across periods. The two stages of a period differ in terms of the matching process, preferences, and technology. In the first stage, agents are randomly and anonymously matched in pairs. In the second stage, trade takes place in a centralized market.

Agents trade a divisible special good in the decentralized market and a divisible general good in the centralized market. Both the special good and the general good are perishable across stages and dates. There are $S \geq 3$ types of the special good and $G \geq 2$ types of the general good. Each agent is characterized by a pair $(s, g) \in \{1, \dots, S\} \times \{1, \dots, G\}$. An agent of type (s, g) can only produce a special good of type s and a general good of type g , and only likes to consume a special good of type $s + 1 \pmod{S}$ and a general good of type $g + 1 \pmod{G}$. There is an equal number of agents of each type. In particular, $N \geq SG$ and the probability that an agent is a consumer in a meeting in the decentralized market, which is also the probability that he is a producer, is $N/(S(N - 1))$.

An agent who consumes $q \geq 0$ units of the special good he likes enjoys utility $u(q)$, whereas an agent who produces q units of the special good incurs disutility $c(q)$. The functions u and c are strictly increasing and differentiable, with u strictly concave and c convex. Moreover, $u(0) = c(0) = 0$, $u'(0) > c'(0)$, and there exists $\bar{q} > 0$ such that $u(\bar{q}) = c(\bar{q})$. Let $q^* > 0$ be the unique solution to $u'(q) = c'(q)$. Welfare is maximized in a single-coincidence match when the producer transfers q^* units of the special good to the consumer.

The centralized market has one trading post for each type of general good. We describe how the trading posts operate in the next subsection. Production is as follows. For each agent, one unit of effort generates one unit of the general good. There exists an upper bound $\bar{x} > 0$ on the amount of effort an agent can exert in a period. An agent who consumes $x \geq 0$ units of the general good he likes obtains utility $U(x)$, while an agent who produces x units of the general good incurs disutility x . The function U is differentiable, strictly increasing

and strictly concave, with $U(0) = 0$, $U'(0) > 1$, and $\lim_{x \rightarrow \infty} U'(x) = 0$. Let $x^* > 0$ be the unique solution to $U'(x^*) = 1$. Since the production cost is linear, ex-ante welfare in the centralized market is maximized when all agents consume x^* units of the general good they like. This requires that total production of each type of good is $(N/G)x^*$.

2.2 The Game

We now describe the economy as an infinitely repeated game. We begin with the description of the stage game.

Stage Game

The stage game consists of one round of trade in the decentralized market followed by one round of trade in the centralized market. Trade in the decentralized market takes place as follows. First, in every single-coincidence meeting the agents simultaneously and independently choose from $\{\text{yes}, \text{no}\}$ after learning whether they are consumers or producers.⁶ If either agent in a match says no, the match is dissolved with no trade occurring. If both agents in a match say yes, that is, if both agents agree to trade, the producer transfers q^* units of the good to the consumer, after which the match is dissolved.⁷

Trade in the centralized market takes place as follows. There are G trading posts, one for each (type of) general good. In every period t each agent $j \in \{1, \dots, N\}$ simultaneously and independently chooses: (i) the quantity y_t^j of the good he contributes to the post that trades the good he can produce; (ii) the vector $b_t^j = (b_t^{j,1}, \dots, b_t^{j,G})$ of bids he submits to the trading posts. We assume that $\sum_{g=1}^G b_t^{j,g} \leq y_t^j$, that is, the sum of an agent's bids cannot exceed the total amount of effort he contributes to the trading post. This assumption, which is similar to the assumption in Shapley–Shubik (1977) that agents cannot bid more than their endowments, does not matter for our results and is made for expositional simplicity.⁸

⁶For simplicity, we assume that agents cannot take actions in the decentralized market if they are in a no-coincidence meeting. We obtain the same results without this assumption.

⁷The same results obtain if the producer can choose the quantity q he transfers to the consumer. The approach we follow simplifies the description of strategies considerably.

⁸Drawing a parallel with Shapley and Shubik (1977), one could think that effort has a tangible and perishable counterpart that can be used to make bids. It is possible to show that the results in our paper are unchanged if we assume that there exists an exogenous upper bound on how much an agent can bid in every period.

Let $\mathcal{N}_p(g)$ be the set of agents who can produce the general good g . The price of the general good g in period t is

$$p_t^g = \frac{\sum_{j=1}^N b_t^{j,g}}{\sum_{j \in \mathcal{N}_p(g)} y_t^j},$$

where we adopt the convention that $0/0 = 0$. The quantity of the general good g that agent j obtains in period t is then given by $x_t^{j,g} = b_t^{j,g}/p_t^g$. Note that

$$\sum_{j \in \mathcal{N}_p(g)} y_t^j = \frac{1}{p_t^g} \sum_{j=1}^N b_t^{j,g} = \sum_{j=1}^N \frac{b_t^{j,g}}{p_t^g} = \sum_{j=1}^N x_t^{j,g}$$

for all $g \in \{1, \dots, G\}$, so that aggregate supply is always equal to aggregate demand in each trading post.

Observe that in our environment there are gains from trade in the centralized market. This is in contrast to LW, where all agents produce and consume the same good in the centralized market, and so only the exchange of goods for money is beneficial. Since our focus is on non monetary equilibria, it is natural to assume that there are gains from trade in the centralized market. Lagos and Wright (2003a) point out that the assumption of no gains from trade in the centralized market is only meant to simplify the analysis, and that one can naturally modify the LW environment to introduce gains from trade.⁹

It is easy to see that the stage game admits multiple Nash equilibria in which no trade takes place in the decentralized market.

The Repeated Game

The game consists of infinite repetitions of the stage game. We describe strategies in the repeated game by means of automata.¹⁰ For this, let $A_1 = \{\text{yes}, \text{no}\}$ be the action set of an agent in a single-coincidence meeting in the decentralized market and

$$A_2 = \left\{ a_2 = (y, (b^1, \dots, b^G)) : y \leq \bar{x} \text{ and } \sum_{g=1}^G b^g \leq y \right\}$$

be the action set of an agent in the centralized market. An automaton for an agent is a list $\{W, w^0, (f_1, f_2), (\tau_1, \tau_2)\}$ where: (i) W is the set of states; (ii) $w^0 \in W$ is the initial state; (iii)

⁹Indeed, in Lagos and Wright (2003b) the general good comes in many varieties and is subject to a double-coincidence of wants problem.

¹⁰See Section 2.3 in Mailath and Samuelson (2006) for a discussion of the use of automata to describe behavior in repeated games.

$f_1 : W \times \{1, \dots, S\} \times \{1, \dots, G\} \rightarrow A_1$ and $f_2 : W \rightarrow A_2$ are decision rules in the decentralized and centralized markets, respectively; (iv) $\tau_1 : W \times \{1, \dots, S\} \times \{1, \dots, G\} \times A_1^2 \rightarrow W$ and $\tau_2 : W \times A_2 \times \mathbb{R}_+^g \rightarrow W$ are transition rules in the decentralized and centralized markets, respectively. If the decision rules for an agent are given by the pair (f_1, f_2) , the agent behaves as follows when his state is w : (i) he chooses $f_1(w, s', g')$ in a single-coincidence meeting in the decentralized market if his partner's type is (s', g') ; (ii) he chooses $f_2(w)$ in the centralized market. If the transition rules for an agent are given by the pair (τ_1, τ_2) , then: (i) $\tau_1(w, s', g', a_1, a'_1)$ is the agent's new state when he enters the decentralized market in state w if he chooses a_1 , and his partner is of type (s', g') and chooses a'_1 ; (ii) $\tau_2(w, a_2, p)$ is the agent's new state when he enters the centralized market in state w , chooses a_2 , and observes the vector of prices p .¹¹ We restrict attention to strategy profiles where the set of states is the same for all agents.

Given a strategy profile σ , a profile of states for an agent is a map $\pi : W \times \{1, \dots, S\} \times \{1, \dots, G\} \rightarrow \{1, \dots, N-1\}$ such that $\pi(w, s, g)$ is the number of agents of type (s, g) in the rest of the population who are in state w . Denote the set of all state profiles by Π . A belief for an agent is a map $p : \Pi \rightarrow [0, 1]$ such that $\sum_{\pi \in \Pi} p(\pi) = 1$, where $p(\pi)$ is the probability the agent assigns to the event that the profile of states is π . Let Δ be the set of all possible beliefs. A belief system for an agent is a map $\mu : W \rightarrow \Delta$. In an abuse of notation, we use μ to denote the profile of belief systems where all agents have the same belief system μ . We can simplify the description of Π when σ is symmetric, that is, when σ is such that any two agents in the same state behave in the same way. In this case, a profile of states for an agent is simply a map $\pi : W \rightarrow \{1, \dots, N-1\}$ such that $\pi(w)$ is the number of agents in the rest of the population who are in state w .

We consider sequential equilibria of the repeated game. The first-best in the repeated game is achieved when in each period trade takes place in all single-coincidence meetings in the decentralized market and all agents consume x^* units of the general good they like in the centralized market.

¹¹For simplicity, we only define transition rules for agents in the decentralized market if they participate in a single-coincidence meeting. In what follows, we always assume that an agent's state does not change if he participates in a non-coincidence meeting.

3 The Non-Essentiality of Money

In this section we construct an equilibrium that sustains the first-best if agents are patient enough. For this, let: (i) e_g , with $g \in \{1, \dots, G\}$, be the vector with all entries equal to zero except the g th entry, which is equal to one; (ii) e be the vector with all entries equal to one. For convenience, we use $g + 1$ as a shorthand for $g + 1 \pmod{G}$ in the remainder of the paper. This implies that $g + 1 = 1$ if $g = G$.

Define σ^* to be the strategy profile where an agent of type g , that is, an agent of type (s, g) for some $s \in \{1, \dots, S\}$, behaves according to the following automaton. The set of states is $W = \{C, D, A\}$ and the initial state is C . The decision rules are given by

$$f_1(w, s', g') = \begin{cases} \text{yes} & \text{if } w \in \{C, D\} \\ \text{no} & \text{if } w = A \end{cases} \quad \text{and} \quad f_2(w) = \begin{cases} (x^*, x^* e_{g+1}) & \text{if } w = C \\ (\bar{x}, 0) & \text{if } w = D \\ (0, 0) & \text{if } w = A \end{cases}.$$

For instance, an agent in state C behaves as follows. In the decentralized market, he agrees to trade regardless of his partner's type. In the centralized market he contributes x^* to the trading post g and bids x^* at the trading post $g + 1$. The transition rules are given by

$$\tau_1(w, s', g', a_1, a'_1) = \begin{cases} C & \text{if } w = C \text{ and } (a_1, a'_1) = (\text{yes}, \text{yes}) \\ D & \text{if } w = C \text{ and } (a_1, a'_1) \neq (\text{yes}, \text{yes}) \\ w & \text{if } w \in \{D, A\} \end{cases}$$

and

$$\tau_2(w, a_2, p) = \begin{cases} C & \text{if } w \in \{C, D\} \text{ and } p \in \{e\} \cup \mathcal{P} \\ A & \text{if } w \in \{C, D\} \text{ and } p \notin \{e\} \cup \mathcal{P} \text{ or } w = A \end{cases},$$

where \mathcal{P} is the set of possible price vectors in the centralized market when $N - 2$ agents are in state C and the two remaining agents are in state D . For instance, an agent in state C in a single-coincidence meeting in the decentralized market remains in C only if there is trade in his match, otherwise he moves to state D . Likewise, an agent in state C in the centralized market stays in C if the price he observes belongs to $\{e\} \cup \mathcal{P}$, otherwise he moves to state A . Observe that σ^* implements the first-best.

To finish, consider the belief system μ^* where: (i) an agent in state C believes that all other agents are in state C ; (ii) an agent in state A believes that all other agents are in

state A ; (iii) an agent in state D believes that there is one other agent in state D and the remaining agents are in state C . Clearly, (σ^*, μ^*) is a consistent assessment. We have the following result.

Proposition 1. *Suppose that $\bar{x} \geq c(q^*)$. There exists $\delta' \in (0, 1)$ independent of N and G such that (σ^*, μ^*) is a sequential equilibrium for all $\delta \geq \delta'$.*

Proof: Let V_C^{DM} and V_C^{CM} be the (discounted) lifetime payoffs to an agent in state C before he enters the decentralized and centralized markets, respectively. Then,

$$V_C^{\text{DM}} = \frac{1}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U(x^*) - x^* \right\} \quad \text{and} \quad V_C^{\text{CM}} = U(x^*) - x^* + \delta V_C^{\text{DM}}.$$

Now let V_A be the lifetime payoff to an agent in state A . It is easy to see that $V_A = 0$. Finally, let V_D be the lifetime payoff to an agent in state D before he enters the centralized market. Since an agent in state D in the centralized market believes that there are $N-2$ agents in state C and one other agent in state D , he believes that the price vector in the centralized market will lie in the set \mathcal{P} . Thus,

$$V_D = -\bar{x} + \delta V_C^{\text{DM}}.$$

We start with incentives in state C . An agent in state C in the decentralized market has no profitable one-shot deviation if

$$-c(q^*) + V_C^{\text{CM}} = -c(q^*) + U(x^*) - x^* + \delta V_C^{\text{DM}} \geq V_D = -\bar{x} + \delta V_C^{\text{DM}},$$

which is satisfied since $\bar{x} \geq c(q^*)$. Consider then an agent in state C in the centralized market. Without loss of generality, we can assume that the agent's type is $g = 1$. Let $a_2 = (y, (b^1, \dots, b^G)) \neq (x^*, x^* e_2)$ be the agent's action in the centralized market and denote the corresponding vector of prices by $p = (p^1, \dots, p^G)$. We first show that there is no profitable one-shot deviation by the agent when a_2 is such that $p = e$. It is immediate to see that $p^g = 1$ for $g > 1$ if, and only if, $b^2 = x^*$ and $b^g = 0$ for $g > 2$. Moreover, $p^1 = 1$ if, and only if, $b^1 = y - x^*$. Thus, if $a_2 \neq (x^*, x^* e_2)$ and $p = e$, the agent's flow payoff is $U(x^*) - y$ with $y > x^*$, which is smaller than $U(x^*) - x^*$. The desired result follows from the fact V_C^{DM} is the highest continuation payoff possible for the agent.

Now, we show that there is no profitable one-shot deviation by the agent when a_2 is such that $p \neq e$. First note that the agent's flow payoff from a_2 is $U(b^2/p^2) - y$, where

$$p^2 = \frac{(\frac{N}{G} - 1)x^* + b^2}{\frac{N}{G}x^*}.$$

It is easy to see that b^2/p^2 is maximized when $b^2 = y - b^1$. Thus, the highest flow payoff gain possible for the agent given the choice of y in a_2 is

$$\Delta(y) = U\left(y \frac{\frac{N}{G}x^*}{(\frac{N}{G} - 1)x^* + y}\right) - y - [U(x^*) - x^*].$$

Since U is strictly concave and $U'(x^*) = 1$, we have that

$$\Delta(y) \leq U'(x^*) \left\{ y \frac{\frac{N}{G}x^*}{(\frac{N}{G} - 1)x^* + y} - x^* \right\} - (y - x^*) = y \frac{x^* - y}{(\frac{N}{G} - 1)x^* + y} \leq \frac{x^*}{2},$$

where we use the fact that $N \geq SG$ and $S \geq 3$. Consequently, the highest flow payoff gain from a one-shot deviation is bounded above by a constant that is independent of the population size and the number of types of the general good.

The next step is to show that any one-shot deviation by the agent necessarily leads to a price vector $p \notin \mathcal{P}$. Recall that \mathcal{P} is the set of all price vectors consistent with 2 agents in state D and the remaining agents in state C . By construction, a necessary condition for a price vector to be an element of \mathcal{P} is that the total production by the agents exceeds the sum of their bids by $2\bar{x}$. However, the total production by the other agents in the population is equal to the sum of their bids. Thus, since the agent can at most produce \bar{x} , there is no choice of a_2 such that $p \in \mathcal{P}$. Therefore, any one-shot deviation by the agent with $p \neq e$ leads to state A , which implies a loss of continuation payoffs equal to V_C^{DM} . Since $\Delta(y) < x^*/2$ for all $y \geq 0$, it is easy to see that there exists $\delta^1 \in (0, 1)$ independent of N and G for which no such one-shot deviation is profitable if $\delta \geq \delta^1$.

Consider incentives in state D . No agent is ever in state D in the decentralized market. Consider then an agent in state D in the centralized market. Once again, we can assume without loss of generality that the agent's type is $g = 1$. Let $a_2 = (y, (b^1, \dots, b^G)) \neq (\bar{x}, 0)$ be the agent's action in the centralized market and denote the corresponding vector of prices by $p = (p^1, \dots, p^G)$. In order to find an upper bound for the agent's flow payoff gain, suppose

the agent can place a bid $b^2 = \bar{x}$ in the post $g = 2$ without having to produce for the post $g = 1$. Since total production of the good $g = 2$ is at most $(N/G - 1)x^* + \bar{x}$ (when the other agent in state D is of type $g = 2$), total bids for this good by the other agents are at least $(N/G - 2)x^*$, and $N/G \geq S \geq 3$, an upper bound for the flow payoff the agent can obtain is

$$U\left(\bar{x} \frac{\left(\frac{N}{G} - 1\right)x^* + \bar{x}}{\left(\frac{N}{G} - 2\right)x^* + \bar{x}}\right) = U\left(\bar{x} \left\{1 + \frac{x^*}{\left(\frac{N}{G} - 2\right)x^* + \bar{x}}\right\}\right) \leq U(2\bar{x}).$$

Now observe that any one-shot deviation by the agent necessarily leads to a price vector $p \notin \{e\} \cup \mathcal{P}$. In fact, since there is one other agent in state D and $N - 2$ agents in state C and an agent cannot bid more than what he produces, any one-shot deviation by the agent implies that total production differs from the sum of bids by an amount $\eta \in [\bar{x}, 2\bar{x})$. Therefore, any one-shot deviation by the agent leads to state A , which implies a loss of continuation payoffs equal to V_C^{DM} . It is easy to see that this implies that there exists $\delta^2 \in (0, 1)$ independent of N and G such that no one-shot deviation is profitable if $\delta \geq \delta^2$.

To finish note that since state A is absorbing and involves no trade in both markets, it is immediate to see that no one-shot deviation is profitable in this state. We can then conclude that (σ^*, μ^*) is an equilibrium as long as $\delta \geq \max\{\delta^1, \delta^2\}$. \square

Our strategy of proof is quite different from the strategy of proof in ACP. Their environment is very much like a repeated prisoner's dilemma in the sense that communicating a defection to the population in the centralized market involves taking an action that is myopically optimal. In our setting, communicating a defection is costly in terms of flow payoffs. What sustains the threat of punishment is that if an agent deviates off the path of play, this leads to an even greater punishment.

Note that Proposition 1 is true without the restriction that $c(q^*) \leq \bar{x}$. In our candidate equilibrium, since cooperation is restored after agents observe a price in \mathcal{P} , the only punishment for an agent who defects in the decentralized market is his payoff loss in the subsequent round of trading in the centralized market. In order for such a punishment to be effective, it must be that $c(q^*)$, the cost of cooperating in the decentralized market, is small enough. The condition $c(q^*) \leq \bar{x}$ can be dropped if a defection in the decentralized market were to lead to a greater punishment, as it would be the case if a price in \mathcal{P} led to a number of periods of no trade in both markets.

Our non-essentiality result highlights the fact that in dynamic trading environments, lack of market power means that an agent's action not only has a small impact on current aggregate outcomes, but also a small impact on future aggregate outcomes. The presence of a centralized market implies that agents retain their market power as the population size increases, despite the fact that their ability to affect current aggregate outcomes disappears in this process. The reason for this is that centralized trading implies that no matter the population size, agents are informationally relevant in the sense that their actions have a noticeable impact on prices, which can then be used to coordinate future behavior.

4 Noisy Prices

The equilibrium construction in the proof of Proposition 1 uses the fact that the mapping between actions in the centralized market and prices is deterministic. This raises the question of whether our non-essentiality result remains valid in large populations when there is noise in the price formation process. Indeed, if prices are a noisy function of actions in the centralized market, using prices to coordinate behavior becomes more difficult when there are many agents.¹² In this section we show that whether agents are informationally irrelevant in large economies, and thus whether money is essential in large economies, depends on the structure of the centralized market.

There are many different ways in which one could introduce noise in the price formation process. The channel we choose, aggregate production shocks, is natural. In addition, as we shall see, it has the desirable property that regardless of the realized shocks, aggregate supply is always equal to aggregate demand in each trading post. This ensures that lack of market clearing (rationing) is not an additional channel through which agents can communicate deviations in the decentralized market to the entire population.

The environment is the same as in Sections 2 and 3 except that now in every period t and in each trading post g , the total effort $Y_t^g = \sum_{j \in \mathcal{N}_p(g)} y_t^j$ directed to production for

¹²More generally, when prices in the centralized market are a random function of individual behavior, our framework becomes a repeated game with noisy observations. Results from Green (1980), Sabourian (1990), and Al-Najjar and Smorodinsky (2001) would then suggest that our non-essentiality result does not hold in large populations.

exchange in the post g yields $\theta_t^g Y_t^g$ units of the general good g , where θ_t^g is a stochastic shock to production in the post g . Recall that $\mathcal{N}_p(g)$ is the set of agents who can produce the general good g . The price p_t^g in the trading post g in period t is now given by

$$p_t^g = \frac{\sum_{j \in N} b_t^{j,g}}{\theta_t^g \sum_{j \in \mathcal{N}_p(g)} y_t^j},$$

where once again we adopt the convention that $0/0 = 0$. Since

$$\theta_t^g \sum_{j \in \mathcal{N}_p(g)} y_t^j = \frac{1}{p_t^g} \sum_{j \in N} b_t^{j,g} = \sum_{j \in N} \frac{b_t^{j,g}}{p_t^g},$$

we still have that aggregate supply is always equal to aggregate demand in each trading post.

We assume that the shocks θ_t^g are independently and identically distributed over time and across posts according to a cdf Ω with $\mathbb{E}[\Omega] = 1$ and support in some interval $[\theta_{\min}, \theta_{\max}]$, where $0 < \theta_{\min} < \theta_{\max} < \infty$. The assumption that $\theta_{\min} > 0$ and $\theta_{\max} < \infty$ is natural. If either $\theta_{\min} = 0$ or $\theta_{\max} = \infty$, it can be the case that for a given realization $(\theta^1, \dots, \theta^G)$ of the shocks to production, the output in a trading post where only one agent exerts effort is greater than the output in another trading post where all agents exert maximum effort, no matter the population size. We also assume that Ω is differentiable and that there exists $\underline{\lambda} > 0$ such that $\Omega'(\theta) \geq \underline{\lambda}$ for all $\theta \in [\theta_{\min}, \theta_{\max}]$. This last condition, which is satisfied if Ω is the uniform distribution, is not necessary for the results that follow. However, it does simplify the analysis to some extent.

Let $U_\Omega : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be given by

$$U_\Omega(x) = \int U(\theta x) d\Omega(\theta).$$

It is immediate to see that U_Ω is strictly concave. Thus, the problem $\max_{x \geq 0} U_\Omega(x) - x$ has a unique solution, that we denote by x^{**} . Note that $U'_\Omega(0) = U'(0)E[\Omega] > 1$, and so $x^{**} > 0$. We assume that $x^{**} < \bar{x}$.¹³ Ex-ante welfare in the centralized market is maximized when for each realization $(\theta^1, \dots, \theta^G)$ of the shocks to production, the agents of type g consume $\theta^g x^{**}$ units of the general good they like. This requires that total effort for the production of each type of general good is $(N/G)x^{**}$. The first-best in the repeated game is achieved if in each

¹³Note that it need not be the case that $x^{**} \leq x^*$. A sufficient condition for $x^{**} \leq x^*$ is that U' is convex.

period trade takes place in all single-coincidence meetings in the decentralized market and ex-ante welfare is maximized in the centralized market.

In the presence of noisy prices, the map between an agent's actions in the centralized market and prices is no longer deterministic. Hence, if in each trading post total effort and bids by the other agents are large, the chance that an agent's decisions can affect prices in a noticeable way is small. Then, a natural conjecture is that an agent is informationally relevant, and thus the first-best can be sustained, if total activity in each trading post is not too large. A sufficient condition for this is that the number of trading posts is not small relative to the population size, i.e., the centralized market is not thick. In what follows we show that market thickness, as measured by the ratio N/G , is indeed a key determinant of the agents' informational relevance.

Our first result is that if G is fixed, so that market thickness increases with N , then the first-best is not an equilibrium outcome when the population is large enough no matter how patient agents are.

Proposition 2. *Fix $G > 1$. For every strategy profile σ that implements the first-best and for all $\delta \in (0, 1)$, there exists $N' \geq 1$ such that σ is not a Nash equilibrium if $N \geq N'$.*

The proof of Proposition 2 is in Appendix A. The idea of the proof is as follows. Consider a strategy profile σ that implements the first-best. First observe that σ must be such that in every period total effort in each trading post is $(N/G)x^{**}$. It turns out that in order for σ to be a Nash equilibrium in large populations, all agents must always bid x^{**} for the general good they like. However, when total effort and bids in each trading post are $(N/G)x^{**}$, an agent's impact on the distribution of prices disappears as N increases. This implies that deviations in the decentralized market from the behavior prescribed by σ go undetected when the population is large. To put it differently, in large populations efficiency in the centralized market is incompatible with efficiency in the decentralized market.

A natural question to ask is whether some trade in the decentralized market can be sustained as an equilibrium outcome in large populations when the number of trading posts is fixed. As the discussion in the previous paragraph suggests, this may be possible if one sacrifices efficiency in the centralized market by keeping the volume of trade in some

trading posts small enough to make agents informationally relevant. In fact, it is possible to show that there exists $\delta' \in (0, 1)$ such that efficient trade in the decentralized market is an equilibrium outcome for all $\delta \geq \delta'$ regardless of the population size.

A key element in the proof of Proposition 2 is that when all agents exert effort x^{**} and bid x^{**} for the good they like, the set of possible price vectors one can observe in the centralized market has a nonempty interior. This is the case if in each period the shocks to production are independent across posts, as we have assumed. However, this is also the case if the shocks to production in each trading post are the sum of a common component and an idiosyncratic component. Thus, Proposition 2 is valid under a more general specification of the shocks to production than we assumed.

The preceding discussion suggests that if N/G is bounded above, so that the centralized market does not become infinitely thick as N increases, then as long as agents are patient enough, the first-best is an equilibrium outcome regardless of the population size. It turns out that this is indeed the case.

Proposition 3. *Suppose that $\lim_{N \rightarrow \infty} N/G < \infty$. There exists $\delta'' \in (0, 1)$ independent of N such that the first-best is an equilibrium outcome for all $\delta \geq \delta''$.*

5 Concluding Remarks

In this paper we show that there is a nontrivial tension between the essentiality of money and centralized trading. Our non-essentiality result stands in contrast to Araujo (2004), who shows that in the Kiyotaki–Wright environment (Kiyotaki and Wright (1993)), autarky is the only non-monetary equilibrium outcome when the population is large enough, no matter how patient agents are. This fundamental difference between environments where trade is fully decentralized and environments where trade occurs both in centralized and decentralized markets follows from the fact that centralized trading can substitute memory as a monitoring device.

A key feature of our analysis is that we explicitly model the process of exchange in the centralized market. Doing so means that one has to introduce a map between individual actions in the centralized market and prices. Even though we restrict attention to a par-

ticular map, the one derived from a strategic market game, our message is quite general.¹⁴ Namely, that the non-essentiality of money is tied to the informational relevance of agents, which depends on the market structure. In particular, modeling the centralized market as a Walrasian market, where agents are informationally irrelevant by assumption, is not always justified in the context of monetary models. Our model provides conditions under which centralized trading is consistent with the essentiality of money.

Appendix A: Proof of Proposition 2

A necessary condition for the ex-ante welfare to be maximized in the centralized market is that for each $g \in \{1, \dots, G\}$, all agents of type $g - 1$ submit the same bid $b > 0$ and total effort for the production of good g is $(N/G)x^{**}$. Consider a strategy profile σ that implements the first-best and let $b_t^g > 0$ be the (on the path of play) bid that the agents of type $g - 1$ submit to the trading post g in period t . Since agents cannot bid more than their effort, $b_t^g > x^{**}$ implies that total effort for the production of good $g - 1$ in period t is greater than $(N/G)x^{**}$, a contradiction. Hence, $b_t^g \leq x^{**}$ for all $t \geq 1$ and $g \in \{1, \dots, G\}$. We claim that if $b_t^g < x^{**}$ for some $t \geq 1$ and $g \in \{1, \dots, G\}$, then for each $\delta \in (0, 1)$, there exists $N' \geq 1$ such that σ is not a Nash equilibrium if $N \geq N'$.

Suppose that $b_t^g < x^{**}$. Since total effort for the production of good g in period t is $(N/G)x^{**}$, at least one agent of type $g - 1$ exerts effort x^{**} or more in period t . Consider one such agent and suppose he deviates by increasing his bid from b_t^g to x^{**} . The agent's flow payoff gain from this deviation is

$$\begin{aligned} \Delta &= U_\Omega \left(x^{**} \frac{\frac{N}{G}x^{**}}{(\frac{N}{G} - 1)b_t^g + x^{**}} \right) - U_\Omega(x^{**}) = U_\Omega \left(x^{**} \left\{ 1 + \frac{(\frac{N}{G} - 1)(x^{**} - b_t^g)}{(\frac{N}{G} - 1)b_t^g + x^{**}} \right\} \right) - U_\Omega(x^{**}) \\ &\geq U_\Omega \left(x^{**} \left\{ 1 + \frac{(S - 1)(x^{**} - b_t^g)}{(S - 1)b_t^g + x^{**}} \right\} \right) - U_\Omega(x^{**}), \end{aligned}$$

since $N/G \geq S$. In particular, Δ is positive regardless of the population size (but it does depend on b_t^g).

Now observe that if the realized value of the period- t shock to production in the post g is θ^g , then the (on the path of play) price of good g is $b_t^g/\theta^g x^{**}$. Hence, the deviation under consideration leads to a punishment only if

$$\frac{(\frac{N}{G} - 1)b_t^g + x^{**}}{\theta^g \frac{N}{G}x^{**}} > \frac{b_t^g}{\theta_{\min}x^{**}} \Leftrightarrow \theta^g < \theta_{\min} \left(1 + \frac{x^{**} - b_t^g}{\frac{N}{G}b_t^g} \right).$$

Given that the greatest punishment possible for an agent is permanent autarky, that is, no trade in both markets in all subsequent periods, an upper bound for the agent's payoff loss

¹⁴Another trading mechanism would be a double auction. Large double auctions have also been used to provide non-cooperative foundations for competitive markets. See Rustichini et al. (1994) and Cripps and Swinkels (2006).

after his deviation is

$$\frac{\lambda\delta}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U_\Omega(x^{**}) - x^{**} \right\}, \quad (1)$$

where

$$\lambda = \Pr \left\{ \theta^g \in \left[\theta_{\min}, \theta_{\min} \left(1 + \frac{x^{**} - b_t^g}{\frac{N}{G} b_t^g} \right) \right] \right\}.$$

Since $\lim_{N \rightarrow \infty} \lambda = 0$, we can then conclude that there exists $N' \geq 1$ such that Δ is greater than (1) for all $N \geq N'$. This establishes the desired result.

Let now σ be a strategy profile implementing the first-best such that $b_t^g = x^{**}$ for all $t \geq 1$ and $g \in \{1, \dots, G\}$. Note that in order for σ to implement the first-best, it must be that on the path of play all agents always exert effort x^{**} in the centralized market. We claim that for each $\delta \in (0, 1)$, there exists $N' \geq 1$ such that σ is not a Nash equilibrium if $N \geq N'$. We divide the argument in two parts.

Part I:

Suppose that $M < N/G$ agents in the centralized market deviate from the behavior that σ prescribes on the path of play, i.e., they either do not exert effort x^{**} or submit a bid for the good they like that is different from x^{**} . By relabeling the agents if necessary, we can assume that the agents under consideration are the agents 1 to M . Let $b^{j,g}$ be the bid of agent $j \in \{1, \dots, M\}$ in the post g and $y^{j,g}$ be the effort that j exerts for the production of good g . Note that $y^{j,g} = 0$ if j is not of type g . Moreover, let m_g be the number of agents in $\{1, \dots, M\}$ who are of type g . Given a realization $\theta = (\theta^1, \dots, \theta^G)$ of the shocks to production, the vector of prices in the centralized market is then given by $\hat{p}(\theta) = ((1/\theta_1)\hat{p}_1, \dots, (1/\theta_G)\hat{p}_G)$, where

$$\hat{p}_g = \frac{(N/G - m_{g-1})x^{**} + \sum_{j=1}^M b^{j,g-1}}{(N/G - m_g)x^{**} + \sum_{j=1}^M y^{j,g}}.$$

Since $|(m_g - m_{g-1})x^{**} + \sum_{j=1}^M b^{j,g-1} - \sum_{j=1}^M y^{j,g}| \leq M(x^{**} + \bar{x})$, it is easy to see that

$$|\hat{p}_g - 1| \leq \kappa_N(M) = \frac{M(x^{**} + \bar{x})}{(N/G - M)x^{**}} \quad (2)$$

for all $g \in \{1, \dots, G\}$. Note that $\kappa_N(M)$ is increasing in M and $\lim_{N \rightarrow \infty} \kappa_N(M) = 0$. In particular, for each $M \geq 1$, \hat{p}_g converges uniformly to 1 as N increases to infinity.

By construction, $\hat{p}(\theta)$ does not belong to the set $\mathcal{P}_{\text{path}}$ of price vectors one can observe on the path of play if, and only if, there exists $g \in \{1, \dots, G\}$ such that

$$(1/\theta_g)\hat{p}_g \notin [1/\theta_{\max}, 1/\theta_{\min}]. \quad (3)$$

If $\hat{p}_g < 1$, the probability that (3) does not happen is the probability that $\theta^g \leq \hat{p}^g \theta_{\max}$, which is $\Omega(\hat{p}_g \theta_{\max})$. If $\hat{p}_g > 1$, the probability that (3) does not happen is the probability that

$\theta^g \geq \widehat{p}_g \theta_{\min}$, which is $1 - \Omega(\widehat{p}_g \theta_{\min})$. Hence, the probability that $\widehat{p}(\theta)$ does not belong to the set $\mathcal{P}_{\text{path}}$ is

$$\begin{aligned} 1 - \prod_{g=1}^G [\Omega(\widehat{p}_g \theta_{\max}) \mathbb{I}\{\widehat{p}_g < 1\} + [1 - \Omega(\widehat{p}_g \theta_{\min})] \mathbb{I}\{\widehat{p}_g \geq 1\}] \\ \leq 1 - \prod_{g=1}^G \min \{ \Omega([1 - \kappa_N(M)] \theta_{\max}), 1 - \Omega([1 + \kappa_N(M)] \theta_{\min}) \} = \pi_N(M), \end{aligned} \quad (4)$$

where \mathbb{I} is the indicator function and the inequality follows from (2). Notice that $\pi_N(M)$ is increasing M and such that $\lim_{N \rightarrow \infty} \pi_N(M) = 0$.

To finish, observe that

$$\begin{aligned} U_{\Omega}(x^{**}) &= \int U(\theta x^{**}) d\Omega(\theta) \leq \int U(\theta x^{**} / \widehat{p}_g) d\Omega(\theta) + \frac{x^{**}(1 - \widehat{p}_g)}{\widehat{p}_g} \int \theta U'(\theta x^{**} / \widehat{p}_g) d\Omega(\theta) \\ &\leq U_{\Omega}(x^{**} / \widehat{p}_g) + \frac{x^{**}|1 - \widehat{p}_g|}{\widehat{p}_g} \int \theta U'(\theta x^{**} / \widehat{p}_g) d\Omega(\theta), \end{aligned}$$

where the first inequality follows from the strict concavity of U . From (2) and the fact that $\int \theta U'(\theta x^{**}) d\Omega(\theta) = 1$, it is easy to see that for each $M \geq 1$, there exists $N_1 \geq 1$ such that

$$U_{\Omega}(x^{**}) \leq U_{\Omega}(x^{**} / \widehat{p}_g) + \frac{2\kappa_N(M)}{1 - \kappa_N(M)} \quad (5)$$

for all $N \geq N_1$. Note that $U_{\Omega}(x^{**} / \widehat{p}_g) - x^{**}$ is the payoff in the centralized market to an agent of type $g - 1$ who exerts effort x^{**} and bids x^{**} for the good he likes when M other agents deviate from the behavior that σ prescribes on the path of play.

Part II:

Consider the following deviation: (i) in the decentralized market, never agree to trade if a producer and always agree to trade if a consumer; (ii) in the centralized market, always exert effort x^{**} and always bid x^{**} for the good one likes. In what follows we show that there exists $N' \geq 1$ such that this deviation is profitable if $N \geq N'$, which establishes the desired result.

First, let T be such that

$$\delta^T \left\{ \frac{1}{S} c(q^*) + U_{\Omega}(x^{**}) - x^{**} \right\} < c(q^*).$$

It is easy to see that there exist $\bar{\varepsilon} > 0$ and $N_2 \geq 1$ such that if $\varepsilon < \bar{\varepsilon}$ and $N \geq N_2$, then

$$\begin{aligned} \frac{(1 - \varepsilon)(1 - \delta^T)}{1 - \delta} \left\{ (1 - \varepsilon) \frac{N}{S(N - 1)} u(q^*) + U_{\Omega}(x^{**}) - \varepsilon - x^{**} \right\} \\ > \frac{1}{1 - \delta} \left\{ \frac{N}{S(N - 1)} [u(q^*) - c(q^*)] + U_{\Omega}(x^{**}) - x^{**} \right\}. \end{aligned}$$

Consider now an agent who follows the deviation described above and let \mathcal{O}_t be the event that up to (but not including) period t the price vectors in the centralized market are all in

$\mathcal{P}_{\text{path}}$, in which case no more than $2^t - 1$ agents in t deviate from the behavior prescribed by σ on the path of play. Then, conditional on \mathcal{O}_t , we have that: (i) the probability that the agent's partner in a single-coincidence meeting in period t does not agree to trade is bounded above by $(2^t - 1)/N$; (ii) the probability that the price vector in the centralized market in period t does not belong to $\mathcal{P}_{\text{path}}$ is bounded above by $\pi_N(2^t - 1)$. Note that in (ii) we used the fact that $\pi_N(M)$ is increasing in M . Therefore, since the right side of (5) is also increasing in M , a lower bound for the agent's payoff is

$$\frac{1 - \delta^T}{1 - \delta} [1 - \pi_N(2^T - 1)]^T \left\{ \frac{N}{S(N-1)} \left(1 - \frac{2^T - 1}{N - 1} \right) u(q^*) \right. \\ \left. + U_\Omega(x^{**}) - \frac{2x^{**}\kappa_N(2^T - 1)}{1 - \kappa_N(2^T - 1)} - x^{**} \right\}$$

as long as $N \geq N_1$. Given that $\lim_{N \rightarrow \infty} \kappa_N(2^T - 1) = \lim_{N \rightarrow \infty} \pi_N(2^T - 1) = 0$, it is easy to see that there exists $N' \geq \max\{N_1, N_2\}$ such that the deviation is profitable if $N \geq N'$.

Appendix B: Proof of Proposition 3

As in the proof of Proposition 1, for simplicity we consider a strategy profile in which the only punishment for an agent who defects in the decentralized market is his payoff loss in the subsequent round of trading in the centralized market. As before, in order for such a punishment to be effective, it must be that $c(q^*)$ is small enough. More precisely, in what follows, we assume that there exists $0 < \kappa < \min\{x^{**}, \bar{x} - x^{**}\}$ such that

$$-c(q^*) + U_\Omega(x^{**}) - x^{**} \geq \max\{U_\Omega(x^{**} + \kappa) - (x^{**} + \kappa), U_\Omega(x^{**} - \kappa) - (x^{**} - \kappa)\}.$$

This assumption can be dropped if a defection in the decentralized market were to lead to a greater expected punishment.

Define σ^{**} to be the strategy profile where an agent of type g behaves according to the following automaton. The set of states is $W^g = \{C, D_{-1}^g, D_0^g, \{D_{g'}^g\}_{g' \neq g}, A\}$ and the initial state is C . The decision rules are

$$f_1(w, s', g') = \begin{cases} \text{yes} & \text{if } w \neq A \\ \text{no} & \text{if } w = A \end{cases} \quad \text{and} \quad f_2(w) = \begin{cases} (x^{**}, x^{**}e_{g+1}) & \text{if } w = C \\ (x^{**} - \kappa, (x^{**} - \kappa)e_{g+1}) & \text{if } w = D_{-1}^g \\ (x^{**} + \kappa, (x^{**} + \kappa)e_{g+1}) & \text{if } w = D_0^g \\ (x^{**}, \varepsilon e_{g+1} + (x^{**} - \varepsilon)e_{g'+1}) & \text{if } w = D_{g'}^g \\ (0, 0) & \text{if } w = A \end{cases},$$

where $\varepsilon > 0$ is small enough that $U_\Omega(\varepsilon) - x^{**} < 0$. The transition rules are

$$\tau_1(w, s', g', a_1, a'_1) = \begin{cases} C & \text{if } w = C \text{ and } (a_1, a'_1) \in \{(\text{yes}, \text{yes}), (\text{no}, \text{no})\} \\ D_{g'}^g & \text{if } w = C, (a_1, a'_1) \in \{(\text{yes}, \text{no}), (\text{no}, \text{yes})\}, \text{ and } g' \neq g \\ D_{-1}^g & \text{if } w = C, (a_1, a'_1) = (\text{no}, \text{yes}), \text{ and } g' = g \\ D_0^g & \text{if } w = C, (a_1, a'_1) = (\text{yes}, \text{no}), \text{ and } g' = g \\ w & \text{if } w \neq C \end{cases}$$

and

$$\tau_2(w, a_2, p) = \begin{cases} C & \text{if } w \neq A \text{ and } p \in \tilde{\mathcal{P}} \\ A & \text{if } w \neq A \text{ and } p \notin \tilde{\mathcal{P}} \text{ or } w = A \end{cases},$$

where $\tilde{\mathcal{P}} = \{p \in \mathbb{R}_+^G : p = ((1/\theta^1), \dots, (1/\theta^G)) \text{ with } (\theta^1, \dots, \theta^G) \in [\theta_{\min}, \theta_{\max}]^G\}$.¹⁵ By construction, the profile σ^{**} implements the first-best.

Now let μ^{**} be the belief system where: (i) an agent in state C believes that all other agents are in state C ; (ii) an agent in state A believes that all other agents are in state A ; (iii) an agent of type g in state D_{-1}^g believes that there is one agent of type g in state D_0^g and the remaining agents are in state C ; (iv) an agent of type g in state D_0^g believes that there is one agent of type g in state D_{-1}^g and the remaining agents are in state C ; (v) an agent of type g in state $D_{g'}^g$, with $g' \neq g$, believes that there is one agent of type g' in state $D_g^{g'}$ and the remaining agents are in state C . Clearly, (σ^{**}, μ^{**}) is a consistent assessment. In what follows we show that there exists $\delta' \in (0, 1)$ independent of N such that (σ^{**}, μ^{**}) is a sequential equilibrium when $\delta \geq \delta'$.

Let V_C^{DM} and V_C^{CM} be the lifetime payoffs to an agent in state C before he enters the decentralized market and the centralized market, respectively. Then,

$$V_C^{\text{DM}} = \frac{1}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U_\Omega(x^{**}) - x^{**} \right\}$$

and

$$V_C^{\text{CM}} = U_\Omega(x^{**}) - x^{**} + \delta V_C^{\text{DM}}.$$

Now let V_D^g be the lifetime payoff to an agent of type g in state $D \in \{D_{-1}^g, D_0^g, \{D_{g'}^g\}_{g' \neq g}\}$ before he enters the centralized market. Since such an agent believes that the vector of prices will lie in the set $\tilde{\mathcal{P}}$, we have that

$$V_D^g = \begin{cases} U_\Omega(x^{**} - \kappa) - (x^{**} - \kappa) + \delta V_C^{\text{DM}} & \text{if } D = D_{-1}^g \\ U_\Omega(x^{**} + \kappa) - (x^{**} + \kappa) + \delta V_C^{\text{DM}} & \text{if } D = D_0^g \\ U_\Omega(\varepsilon) - x^{**} + \delta V_C^{\text{DM}} & \text{if } D = D_{g'}^g \end{cases}.$$

Note that $U_\Omega(\varepsilon) - x^{**} < U_\Omega(x^{**} - \kappa) - (x^{**} - \kappa)$ by construction. Finally, observe that the lifetime payoff to an agent in state A is $V_A = 0$.

It is immediate to see that no one-shot deviation is profitable in state A . Let us start with incentives in state C then. An agent in the decentralized market has no profitable one-shot deviation if

$$\begin{aligned} & -c(q^*) + U_\Omega(x^{**}) - x^{**} + \delta V_C^{\text{DM}} \\ & \geq \max\{U_\Omega(x^{**} + \kappa) - (x^{**} + \kappa), U_\Omega(x^{**} - \kappa) - (x^{**} - \kappa)\} + \delta V_C^{\text{DM}}, \end{aligned}$$

¹⁵The definition of automata presented in Section 2 assumes that the set of states is the same regardless of the agent's type in the centralized market. We can extend the definition of σ^{**} to accommodate this requirement as follows. The set of states is $W = \{A, C\} \cup_{g \in \{1, \dots, G\}} \{D_{-1}^g, D_0^g, \{D_{g'}^g\}_{g' \neq g}\}$. The decision rules f_1 and f_2 for an agent of type g are such that $f_1(w, s', g') = \text{yes}$ and $f_2(w) = (x^{**}, x^{**}e_{g+1})$ if $w \notin W^g$. The transition rules τ_1 and τ_2 for an agent of type g are such that $\tau_1(w, s', g', a_1, a'_1) = \tau_2(w, a_2, p) = w$ if $w \notin W^g$. Since an agent of type g is never on a state $w \notin W^g$ there is no need to check for one-shot deviations in such states.

which is satisfied by construction. Consider now an agent in the centralized market and assume, without loss of generality, that his type is $g = 1$. Let $a_2 = (y, (b^1, \dots, b^G)) \neq (x^{**}, x^{**}e_2)$ be the agent's action. There are two possible types of one-shot deviations. One that leads to a price vector in $\tilde{\mathcal{P}}$ with probability one and one that does not. The first type of one-shot deviation involves $b^2 = x^{**}$, $b^1 = y - x^{**}$, and $y > x^{**}$. It is easy to see that this reduces the agent's flow payoff, and so is not optimal (given that V_C^{DM} is the highest continuation payoff possible for the agent).

Consider then a one-shot deviation that leads to state A with positive probability. Since setting $b^g > 0$ for some $g \geq 3$ reduces flow payoffs and does not increase continuation payoffs, we can assume that $b^g = 0$ for $g \geq 3$. Now observe that the agent's flow payoff from a_2 is $U_\Omega(b^2/\tilde{p}^2) - y$, where

$$\tilde{p}^2 = \frac{(\frac{N}{G} - 1)x^{**} + b^2}{\frac{N}{G}x^{**}}.$$

It is easy to see that b^2/\tilde{p}^2 is maximized when $b^2 = y - b^1$. Thus, the highest flow payoff gain possible for the agent given the choice of y in a_2 is

$$\Delta(y) = U_\Omega\left(y \frac{\frac{N}{G}x^{**}}{(\frac{N}{G} - 1)x^{**} + y}\right) - y - [U_\Omega(x^{**}) - x^{**}].$$

Since U_Ω is strictly concave and $U'_\Omega(x^{**}) = 1$, we have that

$$\Delta(y) \leq y \frac{x^{**} - y}{(\frac{N}{G} - 1)x^{**} + y}.$$

Note that $\Delta(y) > 0$ only if $y < x^{**}$. Suppose then that $y < x^{**}$. This implies that the one-shot deviation leads to state A if the realized value θ^2 of the shock to production in the post $g = 2$ is such that

$$\frac{\tilde{p}^2}{\theta^2} < \frac{1}{\theta_{\max}} \Leftrightarrow \theta^2 > \theta_{\max} \left(1 - \frac{x^{**} - b^2}{\frac{N}{G}x^{**}}\right).$$

Since $b^2 \leq y$ and $\Omega'(\theta)$ is bounded below by $\underline{\lambda} > 0$, a lower bound on the expected continuation payoff loss from the one-shot deviation is

$$\underline{\lambda}\theta_{\max} \frac{x^{**} - y}{\frac{N}{G}x^{**}} \delta V_C^{\text{DM}}.$$

Given that

$$\frac{N}{G}x^{**}\Delta(y) \leq y(x^{**} - y) \frac{\frac{N}{G}x^{**}}{(\frac{N}{G} - 1)x^{**} + y} \leq \frac{3}{2}y(x^{**} - y),$$

we can then conclude that the one-shot deviation is not profitable if

$$\underline{\lambda}\theta_{\max}\delta V_C^{\text{DM}} \geq \underline{\lambda}\theta_{\max} \frac{\delta}{1 - \delta} \left\{ \frac{1}{S} [u(q^*) - c(q^*)] + U_\Omega(x^{**}) - x^{**} \right\} > \frac{3}{2}x^{**}.$$

It is easy to see from the last condition that there exists $\delta^1 \in (0, 1)$ independent of N such that no one-shot deviation in state C in the centralized market is profitable if $\delta \geq \delta^1$.

To finish, consider incentives in states $D \in \{D_{-1}^g, D_0^g, \{D_{g'}^g\}_{g' \neq g}\}$. No agent can be in such state in the decentralized market. Consider then an agent in state $D \in \{D_{-1}^g, D_0^g, \{D_{g'}^g\}_{g' \neq g}\}$ in the centralized market. Once again, we assume, without loss of generality, that the agent's type is $g = 1$. We only consider the case in which $D = D_{g'}^1$ for some $g' \neq 1$. The analysis in the other cases is very similar. Let $a_2 = (y, (b^1, \dots, b^G)) \neq (x^{**}, \varepsilon e_2 + (x^{**} - \varepsilon)e_{g'+1})$ be the agent's action. Note that $b^g > 0$ for $g \notin \{2, g' + 1\}$ is never optimal for it reduces the agent's flow payoff. Also note that we can restrict attention to one-shot deviations where $y = b^2 + b^{g'+1}$. In fact, if $y > x^{**}$ and $y > b^2 + b^{g'+1}$, the agent can increase his flow payoff and (weakly) reduce the probability that the state changes to A by reducing y while keeping b^2 and $b^{g'+1}$ the same. If $y \leq x^{**}$ and $y > b^2 + b^{g'+1}$, the agent can reduce the probability that the state changes to A by either increasing $b^{g'+1}$ or increasing b^2 . Now observe that the agent's flow payoff from a_2 is $U_\Omega(b^2/\hat{p}^2) - y$, where

$$\hat{p}^2 = \frac{\frac{N}{G}x^{**} - \varepsilon + b^2}{\frac{N}{G}x^{**}}.$$

Thus, the flow payoff gain for the agent given a choice of y and b^2 in a_2 is

$$\Delta(y, b^2) = U_\Omega\left(b^2 \frac{\frac{N}{G}x^{**}}{\frac{N}{G}x^{**} - \varepsilon + b^2}\right) - y - [U_\Omega(\varepsilon) - x^{**}].$$

There are two types of one-shot deviations that we need to consider: (i) the choice of y in a_2 is $y \geq x^{**}$; (ii) the choice of y in a_2 is $y < x^{**}$. Consider case (i) first. In this case, only an increase in b^2 is profitable. Suppose then that $b^2 > \varepsilon$. This implies that a one-shot deviation leads to state A if the realized value θ^2 of the shock to production in the trading post $g = 2$ is such that

$$\frac{\hat{p}^2}{\theta^2} > \frac{1}{\theta_{\min}} \Leftrightarrow \theta^2 < \theta_{\min} \left(1 + \frac{b^2 - \varepsilon}{\frac{N}{G}x^{**}}\right).$$

The expected continuation payoff loss from the deviation is then at least

$$\underline{\lambda} \theta_{\min} \frac{b^2 - \varepsilon}{\frac{N}{G}x^{**}} \delta V_C^{\text{DM}}.$$

Now note that

$$\Delta(x^{**}, b^2) = U_\Omega\left(b^2 \frac{\frac{N}{G}x^{**}}{\frac{N}{G}x^{**} - \varepsilon + b^2}\right) - U_\Omega(\varepsilon) \leq U'_\Omega(b^2 - \varepsilon),$$

and that for any $b^2 > \varepsilon$, $\Delta(y, b^2) \leq \Delta(x^{**}, b^2)$ for all $y \geq x^{**}$. Thus, the one-shot deviation is not profitable if

$$\underline{\lambda} \theta_{\min} \frac{1}{\frac{N}{G}x^{**}} \delta V_C^{\text{DM}} \geq U'_\Omega(\varepsilon). \quad (6)$$

Since $\lim_{N \rightarrow \infty} N/G < \infty$, there exists Γ such that $N/G \leq \Gamma$ for all N . Hence, there exists $\delta^2 \in (0, 1)$ independent of N and G (but dependent on Γ) such that (6) holds for all $\delta \geq \delta^2$.

Consider now case (ii). In this case, since the agent increases his flow payoff by reducing his disutility of production, a one-shot deviation can involve either an increase or a decrease in $b^2 - \varepsilon$. If $b^2 - \varepsilon > 0$, the one-shot deviation leads to state A when the realized values θ^1 and θ^2 of the shocks to production in the trading posts $g = 1$ and $g = 2$ are such that

$$\frac{\frac{N}{G}x^{**} - \varepsilon + b^2}{\theta^2 \frac{N}{G}x^{**}} > \frac{1}{\theta_{\min}} \quad \text{or} \quad \frac{\frac{N}{G}x^{**}}{\theta^1 \left[\left(\frac{N}{G} - 1 \right) x^{**} + y \right]} > \frac{1}{\theta_{\min}}.$$

A lower bound for the probability of this event is

$$\begin{aligned} \underline{\lambda} \theta_{\min} \frac{b^2 - \varepsilon}{\frac{N}{G}x^{**}} + \underline{\lambda} \theta_{\min} \frac{x^{**} - y}{\left(\frac{N}{G} - 1 \right) x^{**} + y} - (\underline{\lambda} \theta_{\min})^2 \frac{b^2 - \varepsilon}{\frac{N}{G}x^{**}} \frac{x^{**} - y}{\left(\frac{N}{G} - 1 \right) x^{**} + y} \\ \geq \underline{\lambda} \theta_{\min} \frac{1}{\frac{N}{G}x^{**}} \max \{ b^2 - \varepsilon, x^{**} - y \}. \end{aligned}$$

If $b^2 - \varepsilon < 0$, the one-shot deviation leads to state A when the realized values θ^1 and θ^2 of the shocks to production in the trading posts $g = 1$ and $g = 2$ are such that

$$\frac{\frac{N}{G}x^{**} - \varepsilon + b^2}{\theta^2 \frac{N}{G}x^{**}} < \frac{1}{\theta_{\max}} \quad \text{or} \quad \frac{\frac{N}{G}x^{**}}{\theta^1 \left[\left(\frac{N}{G} - 1 \right) x^{**} + y \right]} > \frac{1}{\theta_{\min}}.$$

The probability of this event is bounded below by

$$\begin{aligned} \underline{\lambda} \theta_{\max} \frac{\varepsilon - b^2}{\frac{N}{G}x^{**}} + \underline{\lambda} \theta_{\min} \frac{x^{**} - y}{\left(\frac{N}{G} - 1 \right) x^{**} + y} - \underline{\lambda}^2 \theta_{\min} \theta_{\max} \frac{b^2 - \varepsilon}{\frac{N}{G}x^{**}} \frac{x^{**} - y}{\left(\frac{N}{G} - 1 \right) x^{**} + y} \\ \geq \underline{\lambda} \theta_{\min} \frac{1}{\frac{N}{G}x^{**}} \max \{ b^2 - \varepsilon, x^{**} - y \}. \end{aligned}$$

Hence, the expected continuation payoff loss from the deviation is at least

$$\underline{\lambda} \theta_{\min} \frac{1}{\frac{N}{G}x^{**}} \max \{ |b^2 - \varepsilon|, x^{**} - y \} \delta V_C^{\text{DM}}.$$

To finish, observe that

$$\Delta(y, b^2) = x^{**} - y + \Delta(x^{**}, b^2) \leq [1 + U'_\Omega(\varepsilon)] \max \{ b^2 - \varepsilon, x^{**} - y \},$$

and so the one-shot deviation is not profitable if

$$\underline{\lambda} \theta_{\min} \frac{1}{\frac{N}{G}x^{**}} \max \{ |b^2 - \varepsilon|, x^{**} - y \} \delta V_C^{\text{DM}} \geq [1 + U'_\Omega(\varepsilon)] \max \{ b^2 - \varepsilon, x^{**} - y \}. \quad (7)$$

Since there exists $\Gamma > 0$ such that $N/G \leq \Gamma$ for all N , it is straightforward to see that there exists $\delta^3 \in (0, 1)$ independent of N and G such that (7) holds for all $\delta \geq \delta^3$. This establishes the desired result.

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