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Social ideology and taxes in a differentiated candidates framework

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Abstract

How does ideological polarization on non-economic matters influence the size of government? We analyze this question using a differentiated candidates framework: Two office-motivated candidates differ in their (fixed) ideological position and their production function for public goods, and choose which tax rate to propose. We provide conditions under which a unique equilibrium exists. In equilibrium, candidates propose different tax rates, and the extent of economic differentiation is influenced by the distribution and intensity of non-economic preferences in the electorate. In turn, the extent of economic differentiation influences whether parties divide the electorate primarily along economic or social lines.

Keywords: Differentiated candidates, policy divergence, ideology.

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1 Introduction

It is well known that the two major political parties in the U.S. differ significantly in their positions on both social issues as well as economic issues. As a consequence, both economic preferences as well as "ideology", which we take to mean preferences on cultural issues, influence voting behavior.

For example, Table 1 displays information from California voter exit polls in the 2008 elections.[1] Lines correspond to information on how a voter voted on Proposition 8, a ballot measure whose objective it was to outlaw gay marriage (so yes-votes are by voters who are "social conservatives"). Columns correspond to a voter's household income in the 2007. Entries in the cells are Obama's share of the two party vote for President (i.e., $\frac{\text{votes for Obama}}{\text{votes for Obama or McCain}} \times 100\%$).

	income < \$50000	≥ \$ 50000
YES on Prop. 8	41%	36%
NO on Prop. 8	90%	86%

Table 1: Cultural and economic determinants of voting behavior

The attitude toward gay marriage is a useful proxy for preferences on social policy only, as the economic effect of Proposition 8 is very limited. Household income is a plausible proxy for preferences on economic policy and the scope of government. Table 1 indicates very clearly that both economic and ideological factors influence a person's vote for an office such as the presidency that combines a role in economic policy with a strong influence on social issues (for example, via judicial appointments). Social conservatives (i.e., yes-votes on Proposition 8) are substantially less likely to vote for Obama than social liberals (by about 50 percentage points), and poorer voters are more likely to vote for Obama than richer ones (by about 5 percentage points).[2]

This voter behavior is plausible if parties/candidates differ in both the economic and the social-cultural policies that they would implement if elected. If parties are completely identical (both on economic and social issues), then voter behavior should be completely random. If parties differed only in one policy area, (e.g., the social position as is the case in the probabilistic voting model,

[1] Data from National Election Pool state exit poll for California, available from the Roper Center (http://www.ropercenter.uconn.edu/elections/common/state_exitpolls.html)

[2] Of course, neither category can be expected to be a perfect measure of preferences in the respective policy area: There are several other social policy questions such as abortion or gun rights on which the two parties differ substantially and which may influence a voter's ideological preference for one of the parties' social policy positions. Similarly, a voter's economic interests in an election are not only determined by household income in any given year in the past, but also by his expectations about future income, household size and composition (as this influences both how much taxes a voter has to pay and, presumably, his consumption of public goods) and age.

where both candidates choose identical economic platforms) voter choices should be completely determined by their preferences on that area, but independent of their preferences on the other policy. If, for example, the social and economic preferences are correlated (e.g., wealthier voters are more likely to be socially conservative) then voting behavior is correlated although there is no causal relation. However, once we condition on both income and social preference, as we do in Table 1, then the remaining differences must be based on causation rather than correlation.[3]

More generally, the extent of the policy difference between the parties in each policy area will influence how strongly different preferences on a dimension translate into different voting behavior. For example, in "What's the matter with Kansas", Frank (2004) argues that Democrats' economic policy has become very similar to Republican economic policy, causing many voters who would be "natural" Democratic partisans instead follow their culturally conservative leanings and vote Republican. Similarly, the exit poll data in Table 1 appear to indicate that cultural determinants of voting behavior have a quantitatively stronger effect than economic ones.

From an economic point of view, this raises an important question: How does ideological polarization on social issues affect economic policy? While both economic and ideological factors interact in determining a voter's choice between candidates, the standard models in political economy are ill-equipped to analyze these questions. If the simple one-dimensional policy model is interpreted as one of economic policy, there is, by definition, no ideological dimension, and voters split according to their economic preferences (even if there is only slight differentiation between the economic platforms preposed by the candidates).

The probabilistic voting model accomodates both an "economic" dimension on which candidates choose a policy and an "ideological" dimension which is a pure shock to the utility of voters and can be thought of as arising from social-policy issues on which the candidates' positions differ. However, in the equilibrium of the standard probabilistic voting model, both candidates always propose the same economic policy, and thus the voters' preference for one of the candidates is *only* determined by their ideological position and not by their economic characteristics. The data reported in Table 1 suggest that this prediction is not entirely correct, and clearly, the reason is that real life Democratic and Republican candidates differ not only in ideological positions, but also in economic policy platforms.

The main question of this paper is how ideological polarization affects political platforms and

[3]In fact income and the position on Proposition 8 are correlated: 44.7% of voters with income less than $50,000 voted in favor of Proposition 8, while the percentage increased to 56.2% among those with incomes exceeding $50,000, indicating that wealthier voters are more socially conservative. However, the numbers given in table 1 are conditioned on both income *and* social preferences, and are therefore unaffected by correlation between the two.

economic decision making. We approach this issue in a framework where candidates have positions that consist of fixed and flexible positions. We think of the fixed position as reflecting ideological differences that the candidates do not want to or cannot credibly compromise on, just like in the citizen-candidate model. However, just like the Downsian model, candidates in our model are office motivated, and choose position on economic policy to maximize their winning probability.

The advantage of our framework is twofold. First, since both candidates' immutable positions on social issues and their equilibrium platforms on economic issues differ, voters choose their preferred candidate based on both economic and ideological issues: Social conservatives who happen to be sufficiently keen on government spending may vote for the Democrat, and social liberals who are sufficiently opposed to high taxation may vote for the Republican. Second, candidates compete for voter support by choosing economic platforms, taking as given their ideological differences and the preference distribution in the population. Within our framework we can think of *polarization* as a measure of preference intensity on the ideological component. We analyze how increasing ideological polarization translates into changes of economic policy. In addition, we can consider the effects of shifts in the ideological composition of the electorate (say, an increase in the number of social conservatives), as well as changes in the economic preference distribution (either allowing for an on average higher demand for public goods, or for more polarization of economic preferences).

Our main results are as follows. We first show that an equilibrium is characterized by two cutoff voter types, one for each ideological type. Cutoff voters are indifferent between candidates and therefore need to strictly prefer the economic platform of the candidate whose ideological position they dislike. Specifically, the socially liberal cutoff voter is in favor of more government spending than the socially conservative cutoff voter. Note that this is only true for the *cutoff* voter. It may well be the case that, on average, social conservatives prefer lower tax rates than social liberals. However, what matters for the position choice of candidates are the potentially swingable cutoff voters.

In equilibrium, candidates propose tax rates that are intermediate between the rate preferred by the social liberal cutoff voter and the one preferred by the social conservative cutoff voter. A candidate who marginally increases his proposed tax rate would gain more support from social conservatives, but would lose some liberals. In equilibrium, those gains and loses from increasing tax rates exactly balance (for each candidate).

More generally, taking as given the opponent's tax rate, varying a candidate's tax rate generates a curve of cutoff voter pairs in a two-dimensional space, and a candidate can be seen as choosing the best cutoff voter pair from this curve. We show that, in an equilibrium, the two candidates' curves are tangent to each other at the equilibrium-induced cutoff voter pair. They are also tangent to an isoprobability curve, i.e. a curve that connects all those cutoff voter combinations that lead to the same winning probability for the Democrat.

We provide sufficient conditions for an equilibrium to exist and to be unique. The graphical characterization of the equilibrium described above can be used to study the comparative statics properties of the equilibrium, because it is relatively easy to characterize how parameter changes affect the described curves. We show that if there are more socially conservative voters, then both candidates propose more government spending, but the small-government candidate's winning probability increases. We also provide comparative static results about how an increase in ideological preference intensity and in economic polarization affect equilibrium policies.

2 Related literature

Our model is based on the general differentiated candidates framework developped in Krasa and Polborn (2009, 2010b, 2010a), in which two office-motivated candidates compete for office. Candidates have some characteristics that cannot be changed, but choose a position (or "policy") in order to maximize their respective probability of winning. Voters' utility depends on both fixed characteristics and flexible policies. In this model, candidates are differentiated with respect to ideology and their ability to provide public goods, with one of the candidates having an advantage in providing a large quantity of public goods, while his opponent has an advantage in providing a lean government. Both candidates choose a tax rate in order to maximize their respective winning probability.

The advantage of the differentiated candidates framework relative to a standard probabilistic voting model (PVM) is that there is complete policy convergence in the PVM (i.e., in any equilibrium, both candidates choose the same economic policy), and thus, voting behavior is determined only by the voters' position on the "ideological" dimension in which candidates are exogenously fixed. Any observed influence of economic factors on voting behavior would have to stem from ideologically fixed positions that influence the utilities of rich and poor voters differentially.

The advantage of our model relative to a citizen-candidate model (in which candidates are fixed to their "ideal position" in every policy area) is that there is a unique equilibrium in our model, and that we can relate changes in ideological polarization of the electorate to changes in the economic policies proposed by the candidates.[4]

[4]The citizen-candidate framework can handle multidimensional policy spaces without fundamental difficulties (Osborne and Slivinski 1996, Besley and Coate 1997). However, there are generally very many equilibria that only share the property that the equilibrium leads to a split vote (i.e., both candidates always receive the same number of

In a standard one-dimensional spatial model, equilibrium policy depends only on the ideal policy position of the median voter, but is independent of the higher-order moments of the distribution of voter preferences. There are a number of papers that use different variations on a one-dimensional framework in order to analyze how increasing diversity of voter preferences affects the size of government. Fernández and Levy (2008) develop a model with a non-monotonic relationship between preference diversity and redistrution. Austen-Smith and Wallerstein (2006)

The topic of economic and social polarization has attracted considerable interest in political science. For example, McCarty, Poole, and Rosenthal (2006) show that there is considerable correlation between the development of economic inequality in the U.S. as measured by the Gini coefficient and a measure of polarization between Democrats and Republicans in the U.S. Congress. Lindqvist and Östling (2010)

3 Model

3.1 Description of the model

Two candidates, $j = D, R$, compete in an election. There are two major components of policy, which Stokes (1963) calls "position issues" and "valence issues". Position issues are ideological issues such as abortion or gun control, and candidates are exogenously committed to differentiated positions; due to their own history or their party label, they cannot credibly change this position. Voters have different ideal positions on the position issue. In contrast, the valence issue is related to the management of public good provision by the office holder, and all voters prefer ceteris paribus (i.e., if costs of implementing the policy are not taken into account) a higher provision level. Candidates propose a tax rate and will then use the tax revenue to provide a public good. Candidates differ is their production function, so that, on top of the tax rate, the identity of the office holder also matters for the quantity of public goods produced.

The modeling of the valence issue follows the application detailed in Krasa and Polborn (2009), Section 6. Candidate j proposes a tax rate t_j, which is applied to the average income of the population, $\bar{m}$, normalized to 1. Thus, tax revenue if candidate j is elected is t_j and is used to pay for government fixed cost and for the provision of a public good g. The ability to provide the public good differs among candidates, and is given by $f_j : \mathbb{R}_+ \to \mathbb{R}$. We assume an affine linear production function, i.e., $g_j = f_j(t_j) = a_j t_j - b_j$, and analyze situations in which candidate R has

votes). Just like in the one-dimensional setup, the citizen candidate model imposes few restrictions on which policies can arise in equilibrium. Thus, no useful comparative static analysis with respect to social polarization is possible in that framework.

an advantage with respect to fixed cost b, while his opponent D has a higher marginal product in public good provision. Formally,

Assumption 1. *Candidate R's has lower fixed costs but also a lower marginal product, i.e.,* $a_R < a_D$ *and* $b_R < b_D$*.*

The candidates' positions on the ideological position issue are fixed. Because there are only two candidates, we can, without loss of generality, assume that $q \in \{L, C\}$, where $q = L$ ("liberal") for the Democrat and $q = C$("conservative") for the Republican.

Individual voters' preferences depend on public good consumption g, their private good consumption x (determined by the tax rate), and the ideological position of the elected candidate. Formally, the utility function of a voter of type $\tau = (\eta, m, p) \in T$ is $u_\tau(x, g, q) = x + \eta w(g) + v_\tau(p, q)$, where x is the voter's private consumption.g is public consumption; w is increasing, strictly concave and differentiable, and satisfies $\lim_{x\to 0} w'(x) = \infty$ and $\lim_{x\to\infty} w'(x) = 0$; and $v_\tau(p, q)m$ is a measure of the ideological (dis)utility. Specifically, we assume that the voter's utility if candidate j is elected is

$$u_\tau((1 - t_j)m, g_j, q_j) = (1 - t_j)m + \eta w(g_j) + \begin{cases} \delta m & \text{if } p = L,\ q = D \\ \rho m & \text{if } p = C,\ q = R\ . \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Note that, while all individuals have the same function $w(g)$, different preferences over public good consumption are reflected in the parameter $\eta \in \mathbb{R}$, with high $\eta-$types having a stronger preference for public goods.

The last term in (1) captures the voter's ideology as δ is the ideological benefit, expressed as a percentage of income, that liberals get if the Democratic candidate (rather than the Republican) is elected, and ρ is the same for conservatives if the Republican candidate is elected. Note that the assumption that $v(L, R) = v(C, D) = 0$ is without loss of generality because for each voter type $p = L, C$ we have one free normalization.[5]

At the time when candidates choose their platforms, they are uncertain about the distribution of types τ. Specifically, there is a state of the world $\omega \in \Omega$, distributed according to the cumulative

[5]Equation (1) can be derived from a spatial representation $v_\tau(p, q) = -|p - q|m$ (i.e., a voter's utility decreases linearly with the distance between the voter's ideal position p and the candidate's position q, and the equivalent variation for having one's ideologically favorite candidate elected is a constant fraction of the voter's income) by normalizing accordingly. Note that $\delta \neq \rho$ corresponds to cases where the distance between the Republican candidate's ideological position and the ideal position of conservatives is different from the distance between the Democrat's ideological position and the ideal position of liberals.

distribution function β. Given ω, type $\tau \in T$ is distributed according to ν_ω. Let S_j be the set of all types who vote for candidate j. Then, candidate j's winning probability is given by

$$\Pi_j = \int_\Omega \int_T \xi(\nu_\omega(S_j))\, d\beta(\omega),$$

where

$$\xi(x) = \begin{cases} 0 & \text{if } x < 0.5; \\ 0.5 & \text{if } x = 0.5; \\ 1 & \text{if } x > 0.5. \end{cases}$$

The timing of events is as follows:

Stage 1 Candidates $j = D, R$ simultaneously announce tax rates $t_j \in [0, 1]$. Candidates are office-motivated (they receive utility 1 if elected, and utility 0 otherwise, independent of the implemented policy), so that their objective is to maximize their respective winning probabilities Π_D and Π_R.

Stage 2 Nature draws ω, which determines the distribution of voter preferences τ in the electorate. Each citizen votes for his preferred candidate, or abstains when indifferent.[6] The candidate with a majority of votes wins, collects taxes and provides the public good.

3.2 Discussion of modeling choices

Differential candidate capabilities. A key departure of the differentiated candidates model is that candidates have differential abilities, with one candidate better at providing limited government, while the other candidate is better than his competitor for large expenditures. While non-standard, this assumption appears eminently reasonable. Economists agree that workers or firms differ in their productivities, and this fact is evident as output can easily be measured in many private sector occupations. In contrast, the "output" of politicians in terms of public good production is significantly more difficult to measure, and thus it is tempting to use expenditures on inputs as a proxy measure for the quantity of the public good supplied. However, in reality, citizens derive utility, for example, from the quality of education in state schools and not *per se* from the money spent on education. Thus, when two competing candidates propose to spend the same amount of money on schools, this does not mean that both of them would produce the same quality of service for citizens if elected. Our model formalizes this notion.

[6] If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If a voter is indifferent, he could in principle vote for any candidate or abstain, and we assume that he abstains.

There are several different interpretations of the candidates' differentiated production possibilities. First, there is a widespread notion that Republicans have an advantage when it comes to running a small government. For example, Egan (2008) demonstrates that Republicans have a long-run public opinion advantage over Democrats on the issue of "taxes", while simultaneously a majority of people say that they trust Democrats more than Republicans on large expenditure issues such as education and health care. Of course, it is not straightforward to interpret what these opinion poll results actually mean, as revenues and expenditures are two sides of the same coin.[7] Our preferred interpretation of these opinion polls is therefore that (many) people think that the advantage of a Republican government is that it is better in taking care of taxpayer dollars by trimming government spending to a minimum, a task in which Democrats may be hampered, for example by their connections to unions of government workers. On the other hand, Democrats are preferable for delivering a high level of public good service.

A difference between political parties can also arise as a consequence of specialization on different policy areas: Republicans may be specialized in the efficient provision of services such as law enforcement that are "basic" in the sense that every government – whether Democratic or Republican – has to provide them, while the Democrats' efficiency advantage lies in the provision of "optional" services (i.e., services that could, but need not be provided by the government) such as, for example, government provision of health care.

Alternatively, suppose that learning-by-doing increases the incumbent's marginal productivity over his challenger's one. However, incumbency also leads to entrenchment, so if the next office holder were charged with reducing bureaucracy and government spending, it may well be the case that the challenger is better able to achieve this objective.

Ideology. Economists tend to focus on economic issues as the central field of conflict in political competition. Specifically, in most political economy models, candidates choose a policy that is interpreted as a tax rate, and voters split over candidates according to their economic preferences. In our opinion, this view is only half-right.

We agree that economic issues are the main flexible position for candidates: While it may be very difficult for a candidate to credibly change a position on a position issue such as abortion, the death penalty or gun control, there are no comparable constraints that prevent a politician who favored a 5 percent sales tax in a previous campaign to credibly advocate a 6 percent or a 4 percent

[7]Possibly, (some) people just want to say that they would most like to have a Democratic (i.e., large) level of spending on issues such as education and health care, while being only lightly taxed (as under Republicans). Of course, such a "can I have my cake and eat it, too"-attitude would not be a meaningful political preference in a world of limited resources.

rate in the current campaign. A reason for this difference is also that the optimal economic policy (for any preference type) depends on the state of the economy and thus naturally changes over time, while one's view of the desirability of gay marriage or abortion restrictions is more likely to be fairly constant over time.

The economic policy platforms of Republican and Democratic candidates usually differ in a non-trivial way, but, while economic positions clearly influence the voting choice of some voters, economic interests are far from being a perfect predictor of voting behavior. For example, according to the exit polls of the 2008 U.S. presidential election,[8] voters making less than $100000 went 55-43 for Obama over McCain, while they split voters making more than $100000 49-49. This is a significant, but not overwhelmingly large difference. Non-economic social issues play a role for voting choices that is at least as important, and probably more important than economic interests. Whether a voter regularly goes to church (a proxy for attitudes towards social issues) is an extremely strong predictor of voting intentions. For example, according to the exit polls of the 2008 U.S. presidential election, voters who attended church weekly went for McCain 55-43, while occasional church-goers went for Obama 57-42, and those who never go to church went for Obama 67-30.

These results indicate that we need a theory of candidate competition and voting that accommodates the strong role of non-economic issues on voting behavior, and helps us understand how ideological issues influence the positions that candidates take on economic issues.

Uncertainty about the voter preference distribution. Including uncertainty about the voter distribution has two objectives. First, it appears quite realistic to assume that the preference distribution in the electorate is not precisely known and that candidates have to make their choices under some uncertainty. Second, the assumption helps us to refine the set of equilibria. If, in a model where the distribution of voters is known with certainty, candidate payoffs depend only on whether they win (rather than vote share), then, generically, there are many equilibria. The reason is that one candidate usually wins for sure, and thus, the policy choice of his opponent is indeterminate. Also, the better candidate can win with a whole set of policies. Therefore, many strategies could be part of an equilibrium when candidates care only about the probability of winning in a model with a given voter distribution. This is the reason why assuming uncertainty about the voter preference distribution is useful.

[8] Available at http://www.cnn.com/ELECTION/2008/results/polls/#USP00p1.

4 Equilibrium

Substituting candidate j's proposed tax rate t_j into the utility function of a type τ voter, we get indirect utility $u_\tau((1-t_j)m, g_j, q_j) = (1-t_j)m + \theta w(g_j) + v_\tau(p, q_j)$. Dividing by m, we have

$$\frac{u_\tau((1-t_j)m, g_j, q_j)}{m} = (1-t_j) + \theta w(g_j) + \begin{cases} \delta & \text{if } p = L,\ q = D \\ \rho & \text{if } p = C,\ q = R \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $\theta = \eta/m$. With utility written in this form, the relevant type space is $\Theta \times \{L, C\}$. Distribution Φ_ω defines a cumulative distribution function $G_\omega(\theta_L, \theta_C)$.

We now show that a candidate's supporters are drawn from θ-types below or above a cutoff, where the location of the cutoff depends on the ideological type. Formally, the sets of a candidate's supporters are of the form $\{L\} \times (-\infty, \theta_L] \cup \{C\} \times (-\infty, \theta_C]$, or $\{L\} \times [\theta_L, \infty] \cup \{C\} \times [\theta_C, \infty)$. In the following, denote the amount of public goods provided by the two candidates by $g_D = f_D(t_D)$ and $g_R = f_R(t_R)$, respectively. Furthermore, let $v(p,q) \equiv v_\tau(p,q)/m$ denote the last term in (2). A voter of type (θ, p) prefers candidate D over candidate R if and only if

$$(1-t_D) + \theta w(g_D) + v(p, D) \geq (1-t_R) + \theta w(g_R) + v(p, R). \tag{3}$$

(3) is equivalent to

$$\theta \geq \frac{t_D - t_R + v(p,R) - v(p,D)}{w(g_D) - w(g_R)}, \tag{4}$$

if $w(g_D) - w(g_R) > 0$, and the inequality changes its sign if $w(g_D) - w(g_R) < 0$.

Recall that $\delta = v(L, D)$ and $\rho = v(C, R)$. Then

$$\theta_L^* = \frac{(t_D - t_R) - \delta}{w(g_D) - w(g_R)}, \tag{5}$$

$$\theta_C^* = \frac{(t_D - t_R) + \rho}{w(g_D) - w(g_R)} \tag{6}$$

are the voter types that are indifferent between the candidates. Higher types vote for the candidate who offers more public good production, while lower types support the candidate who provides less public good. More formally, if $g_D > g_R$ then $w(g_D) > w(g_R)$ and candidate R receives the votes of all liberal voters with $\theta \leq \theta_L$ and of all conservative voters with $\theta \leq \theta_C$. Candidate R' winning probability is derived by integrating $\xi(G_\omega(\theta_L, \theta_C))$ with respect to the distribution of the state of the world ω, i.e.,

$$G(\theta_L, \theta_C) = \int_\Omega \xi(G_\omega(\theta_L, \theta_C))\, d\beta(\omega). \tag{7}$$

If, instead, $g_D < g_R$ then candidate R receives the support of all voters (θ, P) where $\theta > \theta_p^*$, $p = L, C$, and the winning probability is $1 - G(\theta_L^*, \theta_C^*)$. The situation is reversed for candidate D, i.e. if $g_D > g_R$ then D's winning probability is $1 - G(\theta_L^*, \theta_C^*)$, else if $g_D < g_R$ then D's winning probability is $G(\theta_L^*, \theta_C^*)$.

Finally, suppose that $g_D = g_R$. Then (3) simplifies to $v(p, D) - t_D \geq v(p, R) - t_R$, i.e., the equation is independent of θ. As a consequence, all voters with ideology p either vote for the same candidate, or if $v(p, D) - t_D = v(p, R) - t_R$ then they are indifferent between candidates.

Before we proceed with the analysis, consider as a benchmark the case in which ideology does not matter for voters ($\delta = \rho = 0$). Evidently, the cutoffs θ_L^* and θ_C^* are equal in this case, and in an equilibrium, the candidate who is supported by low types choose his policy optimally such that the cutoff is maximized, while the candidate who is supported by high types choose his policy optimally such that the cutoff is minimized. One can also show that the Democrat chooses a higher tax rate than the Republican in an equilibrium, and therefore high θ-types support the Democrat, while low types support the Republican. For a detailed derivation of this equilibrium, including conditions under which such an equilibrium exists and is unique, see Krasa and Polborn (2009).

Consider now the equilibrium of our model when ideological preferences matter for voters. For the moment, let us primarily consider the case that the Democrat provides more public goods than the Republican.[9] Within each ideological group, the highest types will then vote for the Democrat, while the lowest types vote Republican. Because ideological partisans get an additional payoff from the election of their closer candidate, the cutoff voter type among conservatives, θ_C, is larger than the cutoff voter type among liberals, θ_L. This follows directly from (5) and (6). Intuitively, the social conservative who is indifferent between the Democrat and the Republican candidate is so because he prefers the economic platform of the Democrat sufficiently to just counterbalance his cultural preference for the Republican; but a voter who prefers the Democrat's economic platform is someone with a preference for high public good provision (i.e., a voter with a relatively high θ). By analogous arguments, the culturally liberal cutoff voter is economically quite conservative (i.e., a low $\theta-$type).

Candidates can affect the cutoffs among both liberals and conservatives through their choice of equilibrium platform. How does an increase of a candidate's proposed tax rate affect the cutoff types θ_L and θ_C? If a candidate proposes a tax rate that is higher than the preferred rate of both the liberal and the conservative cutoff voter, then decreasing the proposed rate makes both cutoff voters better off and hence increases the sets of both conservatives and liberals who vote for this

[9] In the case without ideological differences between candidates, this is always satisfied in a pure strategy equilibrium. In the model with ideology, this is not necessarily the case, but at a minimum, this is a useful benchmark.

candidate. Similarly, if a candidate proposes a tax rate that is lower than the preferred rate of both the liberal and the conservative cutoff voter, then increasing the proposed rate makes both cutoff voters better off and hence increase the sets of both conservatives and liberals who vote for this candidate. Finally, if a candidate proposes a tax rate that is higher than the preferred tax rate of the liberal cutoff voter, but lower than the preferred tax rate of the conservative cutoff voter,[10] then an increase in the tax rate would increase the candidate's set of conservative supporters and decrease his set of liberal supporters. Clearly, in an equilibrium, both candidates' proposed tax rates must be in the latter region where changing them creates a trade-off between gaining liberal and losing conservative support, or vice versa.

For a more formal analysis, we need the derivatives of θ_L^* and θ_C^* with respect to t_D and t_R, taking into account that g_D and g_R are functions of t_D and t_R, respectively.

$$\frac{\partial \theta_L^*}{\partial t_D} = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_D w'(g_D)}{(w(g_D) - w(g_R))^2}, \tag{8}$$

$$\frac{\partial \theta_C^*}{\partial t_D} = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho) a_D w'(g_D)}{(w(g_D) - w(g_R))^2}, \tag{9}$$

$$\frac{\partial \theta_L^*}{\partial t_R} = -\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta) a_R w'(g_R)}{(w(g_D) - w(g_R))^2}, \tag{10}$$

$$\frac{\partial \theta_C^*}{\partial t_R} = -\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho) a_R w'(g_R)}{(w(g_D) - w(g_R))^2}, \tag{11}$$

A comparison between (8) and (9), and between (10) and (11) shows that $\frac{\partial \theta_L^*}{\partial t_D} > \frac{\partial \theta_C^*}{\partial t_D}$ and $\frac{\partial \theta_L^*}{\partial t_R} < \frac{\partial \theta_C^*}{\partial t_R}$. That is, a tax increase has a less favorable effect on the liberal cutoff voter than on the conservative cutoff voter.[11]

It is useful to think how the tax rates proposed by the candidates affect the cutoff types in a $\theta_L - \theta_C$ diagram. We first define functions k_D and k_R that map the respective candidate's tax rate proposal into a curve of the cutoff points θ_L^* and θ_C^*, taking as given the tax rate of the opponent (which we suppress in the notation). Thus, k_D describes the feasible set of cutoff voter combinations that the Democratic candidate can implement for any tax rate between 0 and 1, and k_R is the same curve for the Republican.

An important characteristic of these curves is their *signed curvature*. In general, the curvature of a two-dimensional curve $(x_1(t), x_2(t))$ is defined as

$$\kappa = \frac{x_1' x_2'' - x_2' x_1''}{({x_1'}^2 + {x_2'}^2)^{3/2}}. \tag{12}$$

[10] Remember that the *socially* liberal cutoff voter is economically more conservative than the socially conservative cutoff voter.

[11] Remember that an upward shift of a cutoff voter is favorable for the Republican and detrimental for the Democrat.

The absolute value of κ at a particular point is the inverse of the radius of the circle that approximates the curve in this point; thus, a small value of κ corresponds to an almost linear curve, while a large value of κ is a strongly bent curve. A positive value of κ indicates that, as t increases, the cutoff point moves through the curve in a clockwise direction (and vice versa).[12]

The following Lemma 1 characterizes the curves k_D and k_R, drawn in Figure 1.

Lemma 1.

1. *The function* $k_R\colon [0,1] \to \mathbb{R}^2$ *defined by* $t_R \mapsto (\theta_L^*(t_R), \theta_C^*(t_R))$ *has signed curvature of*
$$\kappa_R = -\frac{(\rho+\delta)a_R^2 w''(g_R)}{w(g_D)-w(g_R)} \left(\left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_R}\right)^2 \right)^{-3/2}. \tag{13}$$

2. *The function* $k_D\colon [0,1] \to \mathbb{R}^2$ *defined by* $t_D \mapsto (\theta_L^*(t_D), \theta_C^*(t_D))$ *has signed curvature of*
$$\kappa_D = -\frac{(\rho+\delta)a_D^2 w''(g_D)}{w(g_D)-w(g_R)} \left(\left(\frac{\partial \theta_L^*}{\partial t_D}\right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_D}\right)^2 \right)^{-3/2}. \tag{14}$$

Lemma 1 implies that the sign of κ_D and κ_R equals the sign of the term in the denominator (because $w'' < 0$, and all the other terms are positive). Thus, if $w(g_D) > w(g_R)$, both curves rotate in a counterclockwise direction, and vice versa if instead $w(g_D) < w(g_R)$.

We next show that if the slope of k_R is negative then the curve is either strictly concave or strictly convex depending on whether $\theta_C^* > \theta_L^*$ or $\theta_C^* < \theta_L^*$ (that is, depending on whether we are above of below the 45-degree line in Figure 1). An analogous relationship holds for the curve k_D.

Lemma 2. *Suppose that* $\rho + \delta > 0$. *Then*

1. $k_R(t_R)$ *is strictly concave toward the origin for all* t_R *where the slope is negative and for which* $\theta_C^* > \theta_L^*$.

2. $k_R(t_R)$ *is strictly convex toward the origin for all* t_R *where the slope is negative and for which* $\theta_C^* < \theta_L^*$.

3. $k_D(t_D)$ *is strictly convex toward the origin for all* t_D *where the slope is negative and for which* $\theta_C^* > \theta_L^*$.

4. $k_D(t_D)$ *is strictly concave toward the origin for all* t_D *where the slope is negative and for which* $\theta_C^* < \theta_L^*$.

[12] For example, consider $t \mapsto (r\sin(t), r\cos(t))$. This is a circle with radius r, and has curvature $\kappa = -1/r$. The negative sign indicates that as we raise t, the curve is drawn clockwise. In contrast, the curvature of $t \mapsto (r\cos(t), r\sin(t))$ is $\kappa = 1/r$. The positive sign means that the rotation (as t increases) is counterclockwise.

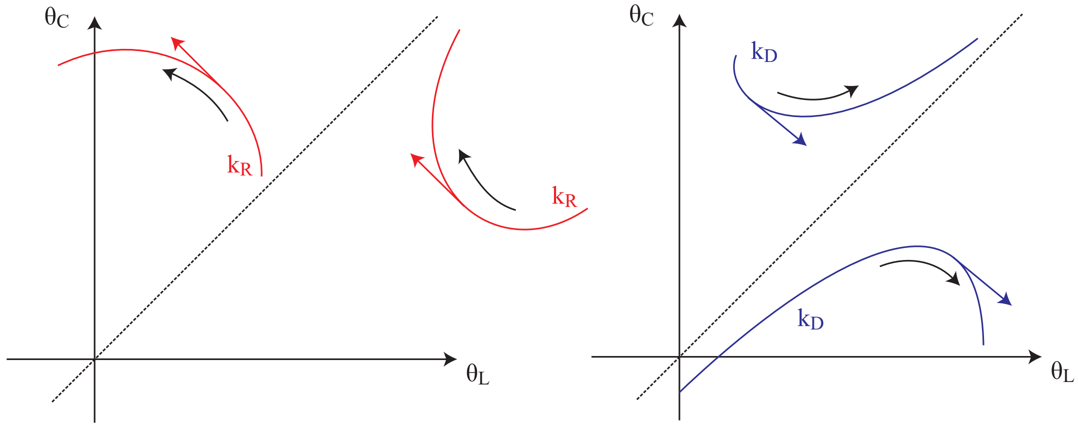

Figure 1: The curves $k_R\colon t_R \mapsto (\theta_L^*, \theta_C^*)$ and $k_D\colon t_D \mapsto (\theta_L^*, \theta_C^*)$.

Figure 1 illustrates Lemma 2. The left panel shows the curve $k_R(t_R)$. (10) and (11) imply that $\frac{\partial\theta_C^*}{\partial t_R} > \frac{\partial\theta_L^*}{\partial t_R}$. If the slope of the curve is negative, then $\frac{\partial\theta_L^*}{\partial t_R}$ and $\frac{\partial\theta_C^*}{\partial t_R}$ must have opposing signs. Thus, $\frac{\partial\theta_C^*}{\partial t_R} > 0$ while $\frac{\partial\theta_L^*}{\partial t_R} < 0$. The tangent vector therefore points toward the northwest.

Consider first the k_R-curve above the 45 degree line (i.e., $\theta_C^* > \theta_L^*$). (5) and (6) imply that $w(g_D) > w(g_R)$, so that candidate D provides more public good than candidate R. Thus, all low θ types vote for R and all high θ types vote for D. (5) and (6) imply that $w(g_D) > w(g_R)$. Lemma 1 implies that $\kappa_R > 0$, so that the curve rotates counterclockwise. Since the tangent vector for negative slopes points northwest, it follows that the curve is concave. Analogous arguments show that k_R is convex below the 45-degree line, and that k_D is convex (concave) above (below) the 45-degree line, as shown in Figure 1.

Given the other candidate's tax rate, candidate i chooses his equilibrium tax rate to maximize his probability of winning. In order to characterize the equilibrium and to determine necessary conditions for its existence, it is useful to define *isoprobability curves*, i.e., curves comprising all combinations of cutoff voter types that lead to the same winning probability. Formally, an isoprobability curve is a set of (θ_L, θ_C) that fulfill an equation of the form $G(\theta_L, \theta_C) = \bar{k}$, where $\bar{k}$ is a constant. Such isoprobability curves are depicted in Figure 2. Clearly, any isoprobability curve must have a negative slope, as an increase in θ_L must be offset by a decrease in θ_C in order to keep the candidates' winning probabilities unaffected.

In the left panel of Figure 2, consider point (θ_L, θ_C), the cutoffs implied by some tax rates (t_D, t_R). Is (t_D, t_R) an equilibrium? Note that candidate D, who can move along the convex curve k_D by changing t_D, could increase his utility only if he can get to a point below the solid isoprobability curve, as this would increase his winning probability. However, this is not possible

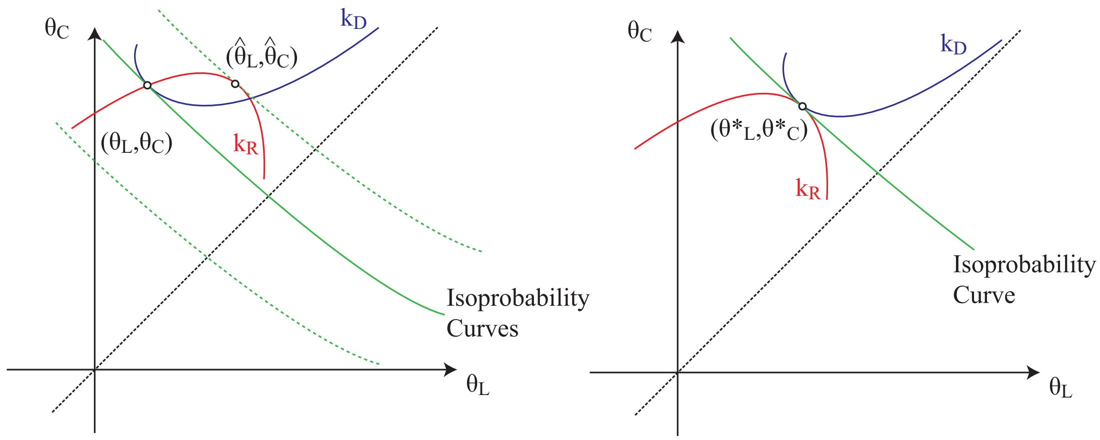

Figure 2: Necessary conditions for an equilibrium.

here because k_D is tangent to the isoprobability curve at (θ_L, θ_C). In contrast, k_R is not tangent, so that candidate R can increase his winning probability by moving to any point above the solid isoprobability curve, for example to $(\hat{\theta}_L, \hat{\theta}_C)$, which is, in fact, his optimal deviation. As indicated in Figure 1, curve k_R rotates in a counter-clockwise direction as t_R increases, so to reach $(\hat{\theta_L}, \hat{\theta_C})$ requires a decrease in t_R. Since candidate R can improve by deviating, (θ_L, θ_C) cannot be an equilibrium.

In order for (θ_L^*, θ_C^*) to be an equilibrium, we must have a situation as depicted in the right panel. Now both k_D and k_R are tangent to the isoprobability curve at (θ_L^*, θ_C^*). Hence any "small" deviation makes the deviating candidate worse off. By a small deviation, we mean any deviation that leads to cutoff types $(\hat{\theta}_L, \hat{\theta}_C)$ above the 45 degree line, i.e. that does not change the *structure* of voter support in the sense that, for both ideological groups, it is still relatively poor people who vote for the Democrat and relatively rich people who vote for the Republican.

Lemma 3 formally summarizes the necessary tangency condition for an equilibrium. Note that this result does not depend on Assumption 1.

Lemma 3. *Let (t_D^*, t_R^*) be an equilibrium with $f(t_D^*) \neq f(t_R^*)$ and $0 < t_D^*, t_R^* < 1$. Then the curves k_R and k_D are tangent at t_D^*, t_R^* to the isoprobability curve through $(\theta_L^*, \theta_R^*) = k_R(t_R^*) = k_D(t_D^*)$,*

$$a_D w'(f_D(t_D)) = a_R w'(f_R(t_R)). \tag{15}$$

and $D_{t_R} k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = D_{t_D} k_R(t_D^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0$.

We next investigate sufficient conditions for existence of an equilibrium. In addition to a standard (global) Nash equilibrium, we also consider "local" equilibria. In a local equilibrium small deviations from the equilibrium strategies cannot make a candidate strictly better off. There are unmodeled, but plausible considerations that make the notion of local equilibrium particularly relevant to our model of candidate competition. Implicitly, we assume in our model that candidates can commit to *any* tax rate (as long as it is sufficiently large to pay for the candidate-specific fixed costs of bureaucracy), and that voters believe that the candidate will carry out whatever promise he makes in the campaign. However, in practice, some promises may be more credible than others. Suppose, for example, that voters ex-ante “expect” that the Democrat will announce a tax rate of 10 percent, and the Republican one of 8 percent. For this configuration to be stable as an equilibrium, it appears highly desirable that deviating to any other tax rate between, say 9 and 11 percent is not profitable for the Democrat, and similarly that small deviations are not profitable for the Republican. In contrast, even if the Democrat could in principle gain by deviating to, say, a tax rate of 5 percent (assuming the Republican stays at 8 percent), this may not be sufficiently credible to convince low θ types (i.e., rich voters) to vote for the Democrat. The notion of a local equilibrium captures this intuition that “big” deviations from expected behavior may not be feasible for candidates.

We also consider the notion of a semi-global equilibrium. Consider a situation in which candidate R receives the support of all types below the cutoffs θ_C^* and θ_L^*, and L the support of everyone above. A strategy profile is a semi-global equilibrium if it is robust against all deviations that do not change the qualitative structure of the candidates' support, i.e., after the deviation, R still gets the support of all sufficiently low types, and D those of sufficiently high types. Since small deviations do not change the qualitative structure of the candidates' support, a semi-global equilibrium is also a local equilibrium, but not necessarily the other way around.

Definition 1.
1. *(t_R^*, t_D^*) is a local equilibrium if there exist open neighborhoods $U(t_R^*)$, $U(t_D^*)$ of t_R^* and t_D^*, respectively, such that (t_R^*, t_D^*) is a Nash equilibrium if the candidates' strategies are restricted to $U(t_R^*)$, and $U(t_D^*)$, respectively.*

2. *(t_R^*, t_D^*) is a semi-global equilibrium if and only if (t_R^*, t_D^*) is a Nash equilibrium when candidate strategies are restricted to the set $\{t_R | f_R(t_R) < f_D(t_D^*)\}$ and $\{t_D | f_D(t_D) > f_R(t_R^*)\}$.*

Consider again the right panel of Figure 2. The necessary conditions for an equilibrium are satisfied at (θ_L^*, θ_C^*). It is also clear that at least any local deviation (in the sense of Definition 1 cannot increase the winning probability of a candidate: Any point on k_R is below, and any point on k_D is above the isoprobability curve. The reason is that the curvature of k_R and k_D exceed that of the isoprobability curve.

The left panel of Figure 3 shows a situation in which the necessary conditions are satisfied at

(θ_L^*, θ_C^*), but where this is not a local equilibrium. The isoprobability curve has a strictly higher curvature at (θ_L^*, θ_C^*) than k_D. Thus, candidate D can increase his winning probability by increasing t_D in order to move to a point below the isoprobability curve such as $(\hat{\theta}_L, \hat{\theta}_C)$.

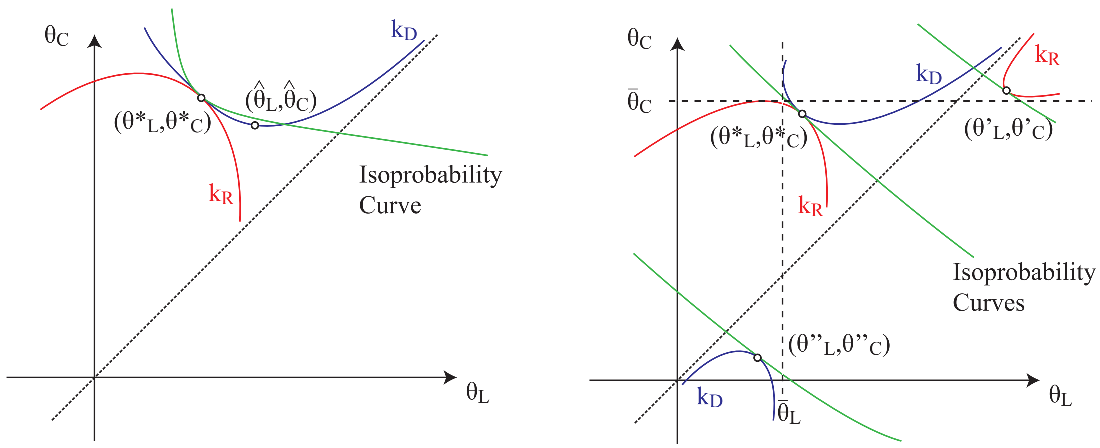

Figure 3: Necessary and Sufficient Conditions for an equilibrium.

As a consequence, a necessary condition for a local equilibrium is that the curvature κ_G of the isoprobability curve at (θ_L^*, θ_C^*) is strictly less than those of k_R and k_D. Theorem 1 formally states necessary and sufficient conditions for an equilibrium. First, at any local equilibrium, both k_D and k_R must be tangent to the isoprobability curve and have a greater curvature than the isoprobability curve. The second statement shows that these conditions are not only necessary, but also sufficient for a local equilibrium. Point 3 shows that if, in addition, k_D and k_R have a greater curvature than the isoprobability curve for all points (θ_L, θ_C) that are on the same side of the 45 degree line as (θ_L^*, θ_C^*), then the local equilibrium is also semi-global.

Theorem 1.

1. *Suppose that* (t_D^*, t_R^*) *is a local equilibrium with* $f(t_D^*) \neq f(t_R^*)$ *and* $0 < t_D^*, t_R^* < 1$. *Then* $t_D^* = t_D(t_R^*)$;
$$D_{t_R} K_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0; \tag{16}$$
and $|\kappa_R(t_R^*)| \geq -\kappa_G(\theta_L^*, \theta_R^*)$, $|\kappa_D(t_D^*)| \geq \kappa_G(\theta_L^*, \theta_R^*)$.

2. *Let* t_R^* *solve* (16). *Let* $t_D^* = t_D(t_R)$. *Suppose that* $0 < t_D^*, t_R^* < 1$ *and* $|\kappa_R(t_R^*)| > -\kappa_G(\theta_L^*, \theta_C^*)$, $|\kappa_D(t_D^*)| > \kappa_G(\theta_L^*, \theta_C^*)$. *Then* t_R^*, t_D^* *is a local equilibrium.*

3. *Let* t_R^* *solve* (16). *Let* $t_D^* = t_D(t_R^*)$. *Suppose that* $0 < t_D^*, t_R^* < 1$ *and* $|\kappa_R(t_R)| > -\kappa_G(\theta_L, \theta_C(\theta_L))$, $|\kappa_D(t_D)| > \kappa_G(\theta_L, \theta_C(\theta_L))$, *for all* t_R, t_D *with* $\operatorname{sign}(f_D(t_D) - f_R(t_R)) = \operatorname{sign}(f_D(t_D^*) - f_R(t_R^*))$, *and all* θ_L *with* $\operatorname{sign}(\theta_L - \theta_C(\theta_L)) = \operatorname{sign}(\theta_L^* - \theta_C^*)$. *Then* t_R^*, t_D^* *is a semi-global equilibrium.*

Proof. See Appendix. □

We next analyze under which conditions a semi-global equilibrium is global. By Assumption 1, candidate D has higher fixed costs but lower marginal costs. Consider t_R^*, t_D^* that satisfy the first-order condition, and for which candidate D provides more of the public good. Then the associated (θ_L^*, θ_C^*) is above the 45 degree axis. If the equilibrium is semi-global, then any deviation that remains above the 45 degree line cannot increase a candidate's probability of winning the election.

Thus, consider deviations to points below the 45-degree line. If the deviation is by candidate D, then it involves a tax rate at which he provides strictly less of the public good than candidate R. In the right panel of Figure 3, the optimal such deviation implements (θ_L'', θ_C'') — note that, for $g_D < g_R$, the k_D-curve becomes convex and D is supported by all voters *below* the cutoff (θ_L'', θ_C''). Thus, he maximizes his winning probability by moving to the highest possible isoprobability curve. Theorem 2 below show that $(\theta_L'', \theta_C'') < (\bar{\theta}_L, \bar{\theta})$. Candidate D's winning probability in the original equilibrium is $1-G(\theta_L^*, \theta_C^*)$, since he receives the support of all voters above θ_L^*, and θ_C^*, respectively. After the deviation, he receives the support of all voters below θ_L^*, and θ_C*, and his winning probability is $G(\theta_L'', \theta_C'')$. Since isoprobability curves have strictly negative slope, $G(\theta_L'', \theta_C'') < G(\theta_L^*, \theta_C^*)$. Thus, a sufficient condition for candidate D's deviation not be to optimal is $G(\bar{\theta}_L, \bar{\theta}) < 1 - G(\theta_L^*, \theta_C^*)$. An analogous argument shows that deviations by candidate R are not profitable if $G(\theta_L^*, \theta_C^*) > 1 - G(\bar{\theta}_C, \bar{\theta}_C)$. Note that both inequalities are satisfied if the election is competitive, i.e. if the winning probabilities of the candidates are not too far from 1/2.

For the formal statement of this result in Theorem 2, we need to guarantee that the branches of k_R and k_D above the 45 degree line resemble those in Figure 3, that is, cutoffs $\bar{\theta}_C$ and $\bar{\theta}_L$ exists. The following assumption is sufficient for this.

Assumption 2. *Let $\hat{t}_R$ be defined by $f_R(\hat{t}_R) = f_D(t_D^*)$, and $\hat{t}_D$ by $f_R(t_R^*) = f_D(\hat{t}_D)$. Then $\hat{t}_R > t_D^* + \rho$ and $\hat{t}_D > t_R^* + \delta$.*

Note that $\hat{t}_R > t_D^*$ is the tax rate that the Republican would have to charge in order to provide the same amount of public goods as the Democrat does in equilibrium, so that candidates differ only in tax rate and ideology. Thus, all voters of the same ideology (irrespective of their θ) have the same preference over candidates. To avoid corner solutions and indeterminateness of equilibrium tax rates, Assumption 2 requires that deviating to $\hat{t}_R$ is unattractive for the Republican (he would even lose all conservatives). The analogous condition for the Democratic candidate is $\hat{t}_D > t_R^* + \delta$. Clearly, Assumption 2 restricts the size of ρ and δ, because if ideology overwhelms all economic considerations, then all conservatives vote for candidate R and all liberals for candidate D. Further, note that ρ and δ can be larger if the difference between f_D and f_R increases, because this raises

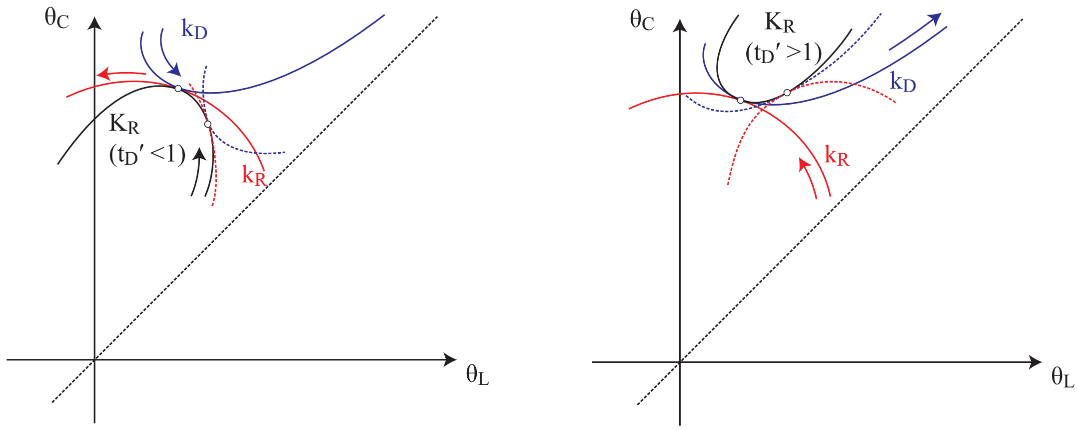

Figure 4: The curve K of tangency points.

both $\hat{t}_R$ and $\hat{t}_D$. Intuitively, increasing the economic ability difference between candidates reduces the relative impact of ideology parameters ρ and δ.

Theorem 2. *Let (t_R^*, t_D^*) be a semi-global equilibrium with $f_R(t_R^*) < f_D(t_D^*)$, and let*

$$\bar{\theta}_C = \max_{\{t_R | f_R(t_R) < f_D(t_D^*)\}} \theta_C^*(t_R), \text{ and } \bar{\theta}_L = \min_{\{t_D | f_D(t_D) > f_R(t_R^*)\}} \theta_L^*(t_R). \tag{17}$$

Suppose that Assumption 2 is satisfied. Then

1. *$\bar{\theta}_C$ and $\bar{\theta}_D$ defined in (17) exists and $(\bar{\theta}_C, \bar{\theta}_C) > (\theta_L^*, \theta_C^*) > (\bar{\theta}_L, \bar{\theta}_L)$.*

2. *If $1 - G(\bar{\theta}_L, \bar{\theta}_L) > G(\theta_L^*, \theta_C^*) > 1 - G(\bar{\theta}_C, \bar{\theta}_C)$. then (t_D^*, t_R^*) is a (global) Nash equilibrium.*

5 Comparative Statics

From the previous section, we know that, in equilibrium, k_D and k_R are both tangent to an isoprobability curve and thus to each other. For the comparative static analysis, it is therefore useful to define a new curve $K(t_R)$ that connects all points at which k_D and k_R are tangent to each other. This curve is displayed in Figure 4, and Lemma 4 below analyzes the key properties of $K(t_R)$.

Lemma 4.

1. *The curves k_D and k_R are tangent at (t_D, t_R) if and only if*

$$a_D w'(f_D(t_D)) = a_R w'(f_R(t_R)). \tag{18}$$

2. *Let* $t_D(t_R)$ *be the solution of the equation* (18). *Then the curve of all tangency points* $K_R\colon [0,1] \to \mathbb{R}^2$ *defined by* $t_R \mapsto (\theta_L^*(t_R, t_D(t_R)), \theta_C^*(t_R, t_D(t_R))$ *has a signed curvature of*

$$\hat{\kappa} = \frac{\kappa_R}{|t_D'(t_R) - 1|}. \tag{19}$$

3. $D_{t_R} K_R(t_R) = (1 - t_D'(t_R)) D_{t_R} k_r(t_R)$

Since a marginal increase in taxes by one unit allows for the provision of a_j additional units of the public good if candidate j is in charge, the left-hand side of (18) is the marginal utility (gross of taxes) from an increase of t_D, divided by the voter's θ. The right-hand side measures the same effect for an increase in t_R. The first part of Lemma 4 shows that k_D and k_R are tangent if and only if these terms are equal. Note that, for any value of t_R, there exists a unique value of t_D such that (18) holds because $w(\cdot)$ is assumed to be strictly concave and takes all values between 0 and ∞. Further, if assumption 1 is satisfied, then $t_D(t_R) > t_R$. As a consequence, the K_R curve is located above the 45 degree line. From Lemma 1, we know that $\kappa_R > 0$ whenever $\theta_C > \theta_L$. Thus, the second point of Lemma 4 implies that the signed curvature of $K_R(t_R)$, is strictly positive, and therefore K_R turns counter-clockwise.

The third point determines whether the curve is convex or concave toward the origin. Consider the left panel in Figure 4, which considers the case where $t_D'(t_R) < 1$. The graph depicts a tangency point of k_R and k_D for some value of t_D and t_R. Since $1 - t_D'(t_R) > 0$, the derivative of K_R at this point must point in the same direction as the derivative of k_R. Thus, K_R is concave toward the origin, just like k_R. However, (19) implies that K_R has a strictly higher curvature than k_R. Note that for different tangency values of t_R and t_D, we get different k_R and k_D curves that are again tangent at K_R (see the dashed curves in Figure 4).

The right panel shows a situation in which $1 < t_D'(t_R) < 2$. Now, the tangent vector of K_R points in the opposite direction of the tangent vector of k_R since $t_D'(t_R) > 1$. As a consequence, K_R is convex toward the origin. Since $t_D'(t_R) < 2$ the curvature of K_R is larger than that of k_R. If $t_D'(t_R) > 2$, then K_R would curve less than k_R.

What determines the sign of $1 - t_D'(t_R)$? If $w(g) = -e^{-ag}$ then $t_D'(t_R) \equiv a_R/a_D < 1$. As another example, suppose that utility is of the form $w(x) = x^{1-s}/(1-s)$. Then (18) implies that

$$t_D = \frac{b_D}{a_D} - a_D^{\frac{1-s}{s}} a_R^{-\frac{1}{s}} b_R + \left(\frac{a_R}{a_D}\right)^{\frac{s-1}{s}} t_R. \tag{20}$$

Thus, $t_D'(t_R) = (a_R/a_D)^{\frac{s-1}{s}}$. Since $a_R < a_D$, this implies that $t_D' > 1$ if $s < 1$ and $t_D' < 1$ if $s > 1$.

Note that for $s = 1$ we have log utility, in which case t_D increases one-to-one in t_R so that the difference $t_D - t_R$ is constant at all points where k_D and k_R are tangent to each other. Furthermore,

(19) implies that, for $s = 1$, K_R has an infinite curvature so that the "curve" K_R is condensed into a single point. That is, only one particular pair of cutoff values θ_L^* and θ_C^* are consistent with equilibrium. This holds independent of the distribution G. In particular, changes in the distribution of preference types in the electorate (say, a higher percentage of liberals or a higher preference of all voters for public good provision) do not affect equilibrium cutoffs.

Finally, it is worthwhile to point out one general comparative static result that follows from the fact that $t_D(\cdot)$ is an increasing function of t_R: There are no exogenous change that lead to an increase in the equilibrium value of t_R and, at the same time, to a decrease in t_D; whenever a parameter change affects equilibrium platforms, the change goes in the same direction. However, in contrast to the classical median voter model, or the probabilistic voting model in which candidate platforms move in parallel, a parameter change may lead to more or less differentiation in economic platforms in our model. For example, if $t_D' < 1$ and equilibrium tax rates increase as a reaction to a parameter change, then $t_D - T_R$ decreases, so that party platforms become more similar to each other.

We can now start with the comparative static analysis. Our main objective is to analyze the effects of "polarization", which we can conceptualize in two different ways. First, an increase in δ and ρ corresponds to an increase in how intensively voters care about the ideological differences between candidates. We call this *ideological polarization*. Second, a spread in the distribution of θ for both ideological voter types would correspond to *economic polarization*, i.e., an increase of the number of people who either want a very strong or a very limited government spending.

In order to do comparative statics, we introduce a parametric class of distributions of voter types.

Assumption 3.

1. *There is a fraction p of liberals, and $1 - p$ of conservatives in the population.*

2. *For each ideology type, θ is normally distributed with mean $\mu - \omega$, and standard deviation σ.*

If we denote by F the cdf of a normal distribution with mean μ and standard deviation σ, then the distribution of voter types given ω is therefore $F(\theta - \omega)$.

We now construct the isoprobability curves $G(\theta_L, \theta_R)$ by determining the collection of all θ_L, θ_R at which the election ends in a tie, for a given $\bar{\omega}$, i.e.

$$pF(\theta_L - \bar{\omega}) + (1 - p)F(\theta_C - \bar{\omega}) = 0.5, \tag{21}$$

Thus, if the cutoff types (θ_L, θ_C) are on this curve, then the candidate who gets the support of all low θ types, wins in all states $\omega < \bar{\omega}$, loses when $\omega > \bar{\omega}$, and the election ends in a tie if $\omega = \bar{\omega}$. Lemma 5 summarizes how the shape of the isoprobability curves depends on parameters.

Lemma 5. *Suppose that Assumption 3 holds.*

1. *If $p = 1/2$, then all isoprobability curves are straight lines with curvature 0.*

2. *The curvature κ_G is continuous in p. In particular, for p close to $1/2$, κ_G is close to 0.*

3. *On the 45-degree line ($\theta_L = \theta_C$), $\kappa_G = 0$ for any p.*

4. *If $p > 0.5$ ($p < 0.5$), then isoprobability curves are strictly convex (strictly concave) above the 45 degree line, and strictly concave (strictly convex) below the 45 degree line.*

5. *The slope of the isoprobability curve through (θ_L, θ_C) is given by*

$$\theta_C'(\theta_L) = -\frac{p}{1-p} e^{\frac{\theta_C - \theta_L}{2\sigma^2}\left[\frac{\theta_L + \theta_C}{2} - (\mu + \bar{\omega})\right]} \tag{22}$$

   The slope $\theta_C'(\theta_L)$, is negative and decreases in p (i.e., becomes steeper), is constant in μ, and increases (decreases) in σ if $p < 1/2$ ($p > 1/2$). Moreover, $\lim_{\sigma\to\infty} \theta_C' = -p/(1-p)$.

Proof. See Appendix. □

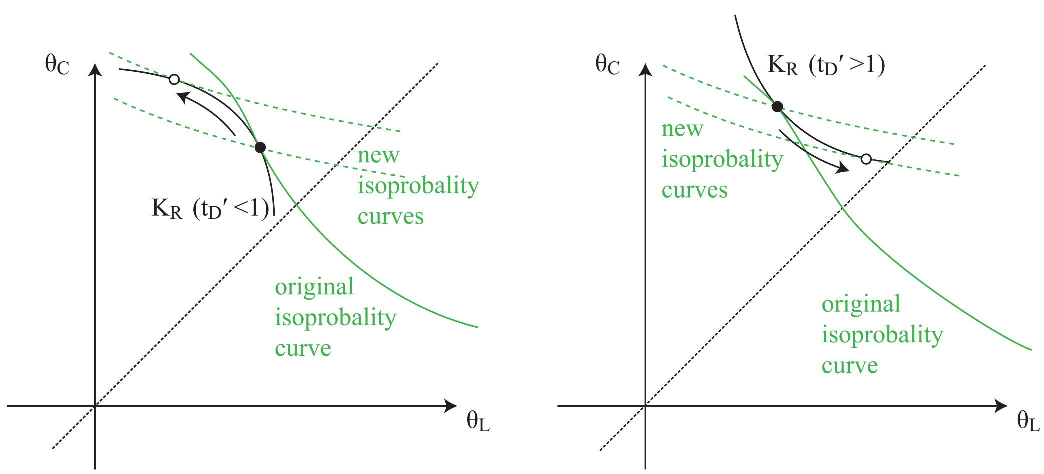

Figure 5: Comparative Statics

Equilibria are points where an isoprobability curve is tangent to the curve K_R. This fact allows for a relatively straightforward comparative static analysis of the equilibrium. First, if the mean μ of the distribution changes, then, for a given point (θ_L, θ_C), (21) remains satisfied if $\bar{\omega}$ adjusts in a way that exactly offsets the change in μ. As a consequence, the isoprobability curves remain the same, but each of them corresponds to a lower realization of ω. Thus, the equilibrium does not change since the K_R curve remains tangent to the isoprobability curve, but the winning probability of candidate D increases. This result implies a remarkable rigidity of the candidates' economic policy proposals. Remember that an increase in μ means that, for any given level of ω, all voters would like to have more public goods than before. Yet, candidates do not change their equilibrium policy platforms. We discuss the reasons for this result in more detail below, after Theorem 3.

Now consider a decrease in p, i.e., an increase of the proportion of (social) conservatives in the electorate. Lemma 5 implies that the slope of the isoprobability curve increases (i.e., becomes flatter). In Figure 5, the original equilibrium is the tangency point of K_R and the solid isoprobability curve. The new, flatter, isoprobability curves are indicated by the dashed lines, and the black circle marks the new tangency point. In both cases (the left panel with $t'_D(t_R) < 1$ and the right panel with $t'_D(t_R) > 1$), the new equilibrium moves in the direction of the rotation of the curves, i.e., both t_R and t_D increase. However, in the left panel θ_L^* decreases, while θ_C^* increases, and the reverse is true in the right panel.

Now consider an increase in σ, the standard deviation of the distribution of θ. Since $\theta = \eta/m$, where η is the preference for public good provision and m the individual's income, an increase in σ can be caused by an increase in income inequality or an increase in polarization of η.

If the number of liberals and conservative are the same, i.e., $p = 0.5$, then Lemma 5 implies that equilibrium platforms and winning probabilities do not change. Now suppose that there are more conservatives than liberals, i.e., $p < 0.5$. Then Lemma 5 implies that increasing σ results in flatter isoprobability curves. Thus, the results are qualitatively the same that we obtained for decreasing p. If $p > 0.5$ than increasing σ results in steeper isoprobability curves and all effects are therefore reversed.

The following Theorem 3 summarizes these results.

Theorem 3. *Suppose that Assumptions 1 and 3 hold, and that either $t'_D(t_R) < 1$ for all t_R or $t'_D(t_R) > 1$ for all t_R.*

1. *If μ increases, then the equilibrium policies are unaffected, but the winning probability of candidate D increases.*

2. *If p, the percentage of social liberals in the electorate, decreases, then the equilibrium tax rates t_D and t_R increase and the winning probability of candidate D decreases. If $t'_D(t_R) < 1$ then cutoff θ_L^* decreases and θ_C^* increases, while the reverse is true when $t'_D(t_R) > 1$.*

3. *An increase of σ has the following effects.*

(a) *If* $p < 0.5$*, then equilibrium tax rates* t_D *and* t_R *increase. If* $t'_D(t_R) < 1$*, then cutoff* θ_L^* *decreases while* θ_C^* *increases, and the reverse is true for* $t'_D(t_R) > 1$*.*

(b) *If* $p = 0.5$*, then equilibrium policies, cutoffs* θ_L^**,* θ_C^* *and winning probabilities are unaffected.*

(c) *If* $p > 0.5$*, then equilibrium tax rates* t_D *and* t_R *decrease. If* $t'_D(t_R) < 1$*, then* θ_L^* *increases while* θ_C^* *decreases, and the reverse is true for* $t'_D(t_R) > 1$*.*

The effects here highlight the significant differences between our differentiated candidates model and previous literature. The first result in Theorem 3 shows that a change of the median of the voters' economic preference distribution have no effect on equilibrium policies, but instead change the candidates' winning probabilities. To understand this result, note first that in a standard spatial setup, both candidates can appeal equally to all voters. For this reason, both candidates cater to the median voter, the voter type who is decisive for the outcome of the election (or the expected median voter, if there is uncertainty about the distribution of voter types). This implies that, if the median voter's preferences change, the candidates' positions exactly reflect this change and will adapt to the median's new preferred position.

In contrast, candidates in our model have exogenous advantages and disadvantages in appealing to particular voter types. Cutoff types are determined as those voter types to whom candidates *can* appeal equally. Remember that, in equilibrium, both candidates choose positions that appeal to some type located between the liberal and the conservative cutoff voter. Thus, in principle, each candidate could expand his set of liberal supporters relative to the equilibrium, but only at the expense of his conservative support. This trade-off is the same for both candidates, and it does not change as the likely preference distribution changes. Specifically, an increase in μ by $\Delta\mu$ increases the *expected* average value of θ in the population and thus, for a given value of ω, increases the vote share of the Democrat. Thus, the critical state of the world in which the candidates receive a vote share of exactly one-half decreases by $\Delta\mu$, and consequently, candidates face exactly the same preference distribution in the critical state. Thus, both candidates continue to maximize their respective voter support with unchanged platforms.

The second result shows that an increase in the number of social conservatives (i.e., of voters with a *cultural* bias for the small government party) leads both candidates to an *increase* their proposed tax rate. This result may appear surprising, but the logic behind it is quite straightforward. Remember that candidates compete for the support of cutoff voters, and that cutoff voters are torn between their economic and cultural-ideological preferences in that they like the economic position of one candidate and the cultural position of his competitor. In particular, the socially-conservative cutoff voter prefers a higher level of government spending than is provided by both candidates, while the socially liberal cutoff voter would prefer a smaller level of government spending. An increase in the number of social conservatives makes it attractive for both candidates to put more weight on the economic preferences of the conservative cutoff voter, and thus to increase the provision of public goods. Note that, while the effect of the ideology shift on both candidates' platforms is clear, the effect on the *expected policy* is ambiguous: An increase in the proportion of social conservatives leads to a larger size of government proposed by both the Democrat and the Republican, but it also makes it more likely that the Republican wins who proposes lower taxes than the Democrat.

The shape of the preferences for public goods, $w(g)$ determines whether the decrease of the fraction of liberals leads to more or less similarity between the candidates' economic platforms. For example, if w corresponds to exponential utility or if $w(g) = g^{1-s}/(1-s)$, with $s > 1$ (i.e., more curvature than logarithmic utility), then we have shown that $t'_D < 1$. In this case, the difference between tax rates $t_D - t_R$ decreases as p is decreased. This, in turn, means that citizens' votes reflect more their ideology rather than economic preferences. In other words, since θ^*_L decreases, and θ^*_C increases, a larger fraction of social liberals vote for the Democrat and a larger fraction of social conservatives for the Republican.

Finally, the third result in Theorem 3 establishes a relation between economic polarization and equilibrium economic platforms. For example, suppose that the income distribution becomes more unequal, which result in a larger standard deviation of θ. Consider the case where $p < 1/2$ (which appears plausible in the U.S. where self-described conservatives usually outnumber self-described liberals by a substantial margin), then an increase in economic polarization (i.e., σ) increases the equilibrium spending by both candidates. Qualitatively, this result is the same as when p is decreased. With exponential utility, or a utility function for which the marginal utility of public good consumption decreases sufficiently fast, citizens' votes will reflect primarily their social rather than their economic preferences, in response to an increase of the fraction of conservatives.

This is the same result as in Meltzer and Richard (1981), but it is clearly based on a different fundamental mechanism. In Meltzer and Richard (1981), economic polarization is interpreted as an increase in the difference between the income of the median voter and the average income in the economy, so that the median voter benefits more from redistribution. In contrast, we assume that the distribution of types is symmetric, so that both mean and median are equal to μ, independently of σ. Thus, our effect is not based on a change in the relation between median and average income. Instead, the intuition is as follows. If $p < 1/2$, then the conservative cutoff voter is closer to the mean of the distribution of θ.[13] As σ increases, the cutoffs adjust in a way that both the liberal

[13]To see this, consider what happens when $p \to 0$. Then, the conservative cutoff voter in the critical state of the world must be very close to the mean of the distribution, because only then can the Democrat and the Republican receive the same number of votes. Since the position of the liberal cutoff voter is always substantially different from

and the conservative cutoff decrease, and this increases the candidates equilibrium tax rate.

Next, we investigate how the equilibrium is affected by changes in ρ and δ, which can be interpreted as changes in the social policy partisanship of conservatives and liberals, respectively, another measure of polarization.

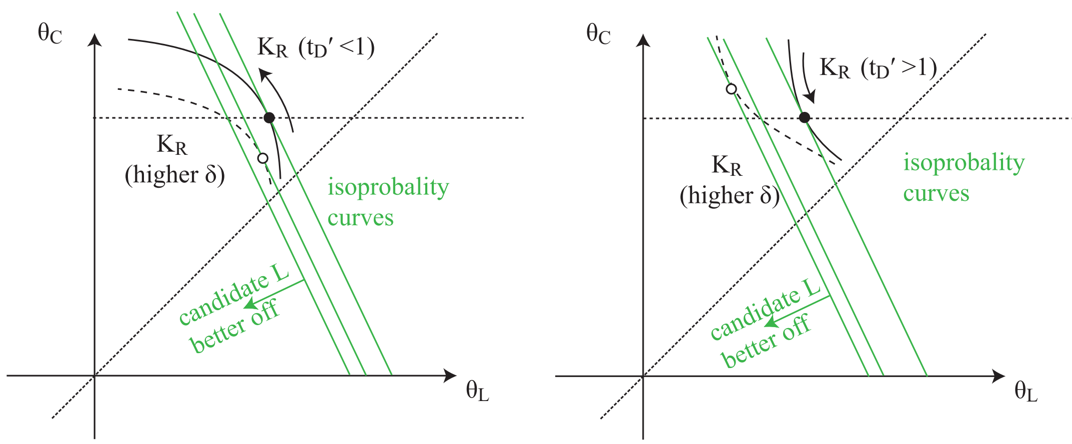

Figure 6: Comparative Statics: Increase of δ for linear isoprobability curves

To develop an intuition for the results, we first consider the case where isoprobability curves are straight lines. This would be the case for example if $p = 0.5$, or if σ is sufficiently large such that the distributions starts to resemble a uniform distribution over θ on the relevant range.

Figure 6 shows the effect of increasing δ. The solid curves represent the orginal K_R curves (for $t'_D < 1$ in the left panel and for $t'_D > 1$ in the right panel). The dashed curves show K_R after δ increases. Note that $t_D(t_R)$ is independent of δ. Thus, (5) implies that θ_L^* decreases, while (6) implies that θ_C remains unchanged. Therefore points on the solid and dashed K_R lines that correspond to the same tax rate t_R are aligned horizontally. The curved arrow along K_R indicates the direction of movement when t_R is increased.

Equations (33) and (34) in the Appendix imply that for given t_R, the slope of $K(t_R)$ increases (i.e., a negative slope becomes less steep) as δ increases. Thus, the slope of the dashed K_R curve in Figure 6 (corresponding to higher δ) is less steep than that of the solid K_R curve. The comparative static result for δ now follow immediately from simple geometric observations. In the left panel, the new equilibrium point is below the horizontal line. Given the direction of rotation K_R indicated by the arrow, this corresponds to a lower t_R and hence lower t_D. The cutoff θ_C decreases. θ_L may decrease (as in the graph) or increase, depending on the curvature of K_R.

the position of the conservative cutoff voter, the result follows.

In the right panel, the new equilibrium is above the horizontal line. Again, the rotation direction of K_R implies that at this new equilibrium, taxes are lower. Note that θ_C increases, while θ_L decreases — in the case of $t'_D > 1$ there is no ambiguity about the change of cutoffs.

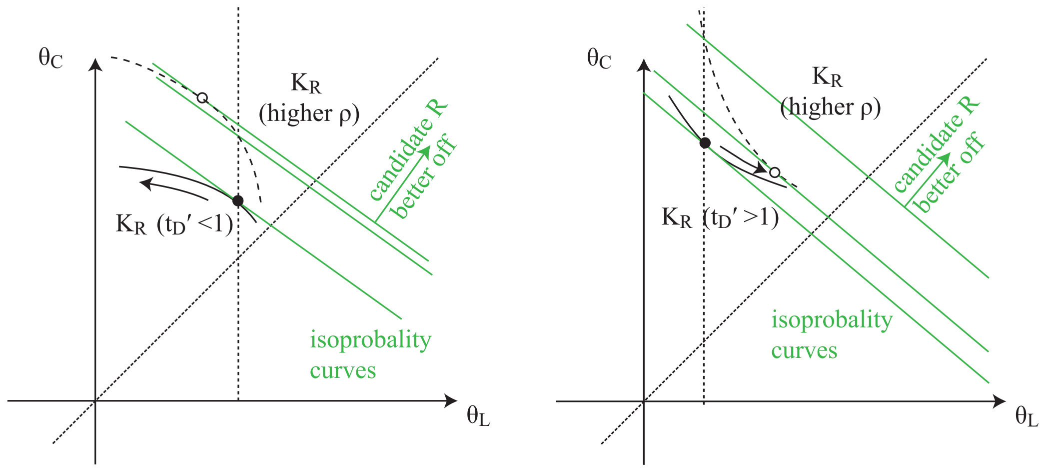

Figure 7: Comparative Statics: Increase of ρ for linear isoprobability curves

Figure 7 shows the effect of increasing ρ for linear isoprobability curves. Increasing ρ moves K_R up and results in a steeper slope along vertical lines. This geometric insight follows again from (5), (6), (33), and (34). Vertical lines connect points on the to K_R curves that correspond to the same tax rate t_R.

In both panels, an increase of ρ results in higher equilibrium tax rates: In the left panel ($t'_D < 1$) the new tangency point is to the left of the vertical line, in the direction of rotation of K_R, while in the right panel, the tangency point is to the right, again in the direction of rotation of K_R. When $t'_D < 1$, more conservative voters but fewer liberal voters support candidate R, i.e., θ_C increases, while θ_L decreases. When $t'_D > 1$, more liberals support candidate R since θ_L increases. However, the effect on θ_C is ambiguous — this mirrors the case depicted in the left panel of Figure (6). In the right panel of Figure (7), θ_C decreases, i.e., candidate R loses the support of some conservatives.

In general, the intuition for the effect of an intensification of cultural preferences is very transparent in our model framework: More intense non-economic preferences among social conservatives, for example, imply that the conservative cutoff type must increase (i.e., has a stronger preference for government spending than before) in order to remain indifferent between Republican and Democrat. As candidates maximize some weighted average of the economic preferences of socially liberal and socially conservative cutoff voters, they now have an incentive to propose higher government spending.

Whether economic differences between candidates become more or less pronounced as t_R increases depends on whether $t'_D > 1$ or $t'_D < 1$, respectively. If $t'_D > 1$, the Democrat increases taxes by more than the Republican. Consequently, the previous socially liberal cutoff voter now strictly prefers the Republican so that θ_L increases. In contrast, there are two counterveiling effects on the conservative cutoff voter (who prefers more spending): On the one hand, this voter's ideological preference has intensified, but on the other hand, the Democrat increases spending by more than the Republican when $t'_D > 1$, which the conservative cutoff voter (who prefers more spending) appreciates. Thus, whether the Republican actually gets more conservative support as their ideological preferences intensify is unclear if $t'_D > 1$.

If, instead, $t'_D < 1$, then the Republican increases government spending by more than the Democrat. Thus, the previous liberal cutoff voter now prefers the Democrat, so that θ_L decreases. In contrast, the previous conservative cutoff now prefers the Republican for both because his ideological preference intensified, and because the Republican's economic platform improved by more than the Democrat's (from the point of view of the conservative cutoff voter). Thus, θ_C increases.

Just like in the discussion of Theorem 3, the effect of a parameter change on the expected economic policy does not only depend on the effect that the parameter change has on equilibrium platforms, but also on who wins the election. As Figures 6 and 7 indicate, the winning probability of candidate D increases as δ increases, and similarly, the winning probability of candidate R rises in response to an increase of ρ.

Consider, for example the case where liberals become more partisan, so that both candidates propose a lower tax rate. However, since the winning probability of candidate D increases, who proposes a higher tax rate than candidate R, the net effect on the expected tax rate is ambiguous.

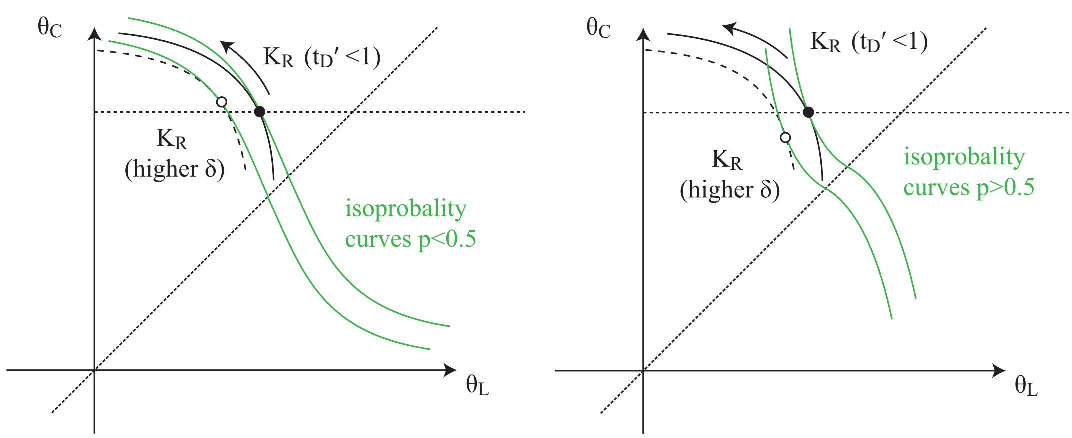

Figure 8: Comparative Statics: Increase of δ for nonlinear isoprobability curves and $t'_D < 1$

When indifference curves are no longer straight lines, the shape of the distribution may also affect the comparative static result. Recall that if $p < 0.5$ then isoprobability curves above the 45 degree line are concave, whereas they are convex for $p > 0.5$. Figure 8 shows comparative statics for the case where $t'_D < 1$. The solid curves depicts the original K_R, while the dashed curve corresponds to a higher level of δ; remember that an increase in δ leads to a flatter K_R curve than before (measured in points that are along the same horizontal line), whenever the slope of K_R is negative.

Remember from the left panel of Figure 6 that, in the case where isoprobability curves are linear and $t'_D < 1$, the new equilibrium cutoff point is necessarily below the horizontal line through old one, and hence corresponds to a lower tax level. The left panel shows that when isoprobability curves are concave ($p < 0.5$, i.e., fewer liberals than conservatives), this need not be the case. The new equilibrium cutoff point may be above the horizontal line through old one, and hence correspond to a higher tax level.[14] In contrast, the right panel depicts the case where $p > 0.5$, i.e., if there are more liberals than conservatives. This has the effect that isoprobability curves are convex above the 45-degree line. In this case, increasing δ results in the same qualitative effect as in the linear case (equilibrium cutoffs below the horizontal line, thus lower taxes), but the effect is magnified compared to the linear case.

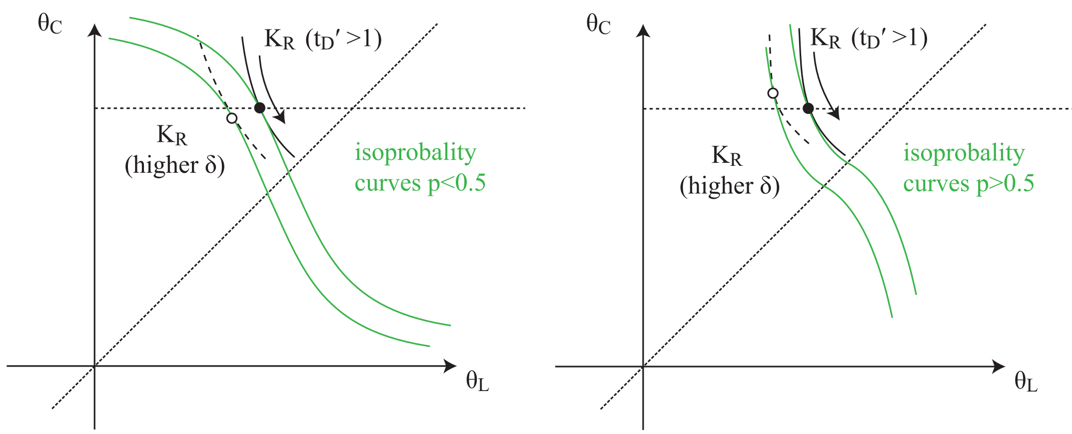

Figure 9: Comparative Statics: Increase of δ for nonlinear isoprobability curves and $t'_D > 1$

Now let $t'_D(t_R) > 1$ for all t_R. The left-panel of Figure 9 shows that the effect of an increase in δ on taxes is again ambiguous when there are fewer liberals than conservatives ($p < 0.5$); taxes may increase while θ_C decreases, i.e., more conservatives vote for the Democrat. The right panel

[14]Note, however, that if p is sufficiently close to 0.5 or if σ is sufficiently large, then isoprobability curves become approximately straight lines, so that the new equilibrium involves a decrease in taxes.

depicts the case where $p > 0.5$. The effect is again magnified compared to the linear case. Proposed tax rates decrease, θ_L decreases, while θ_C increases, i.e., the Democrat gains more liberal votes and loses some conservatives. Again, this second case applies when σ is large or p is close to $1/2$.

Similar comparative statics apply with respect to ρ. If $t'_D > 1$, the case where $p < 0.5$ results in an unambiguous increase in taxes, and t_D increases by more than t_R. This implies that the previous liberal cutoff voter (who prefers lower taxes) now strictly prefers candidate R. Thus, the liberal cutoff θ_L increases. In contrast, it is possible that candidate R loses the support of some conservatives, because the increased ρ may be outweighed by the increased difference in economic policies between the candidates (as the conservative cutoff voter now prefers the economic policy of candidate D even more than before). When $p > 0.5$, taxes may decrease, resulting in a decrease of θ_L and an increase of θ_C. If, instead, $t'_D < 1$ then the case of $p \leq 0.5$ or when σ is large implies an increase of equilibrium tax rates t_D and t_R. Since t_D increases by less than t_R, candidate R loses some liberal support and gains some more conservative support. If $p > 0.5$ we could observe a tax decrease that results in a gain of liberals but a loss of conservatives supporters for candidate R.

We now summarize our results.

Theorem 4.

1. *Suppose that liberal ideology intensity δ increases. Then*

   (a) *If there are more liberals than conservatives ($p \geq 0.5$) or if σ is sufficiently large then both candidates propose a lower tax rate. If $t'_D > 1$, $t_D - t_R$ decreases; θ_L decreases, while θ_C increases, i.e., more liberals but fewer conservatives support candidate D. If $t'_D < 1$, $t_D - t_R$ increases; θ_C decreases, i.e., more conservatives support candidate D, while θ_L may increase or decrease.*

   (b) *If there are fewer liberals than conservatives ($p < 0.5$) and if σ is sufficiently small, then it is possible that both candidates increase their proposed tax rates. If tax rates increase, and if $t'_D < 1$, then θ_L decreases (i.e., more liberals support candidate D) while θ_C increases; if $t'_D > 1$ then θ_C decreases (candidate D gains conservative voters), but θ_L is ambiguous.*

2. *Suppose that conservative ideology intensity ρ increases. Then*

   (a) *If there are fewer liberals than conservatives ($p \leq 0.5$) or if σ is sufficiently large then both candidates increase their proposed tax rates. If $t'_D > 1$, then $t_D - t_R$ increases; θ_L increases (i.e., more liberals support candidate R), while θ_C may increase or decrease. If $t'_D < 1$, then $t_D - t_R$ decreases; θ_L decreases (i.e., fewer liberals support candidate R), while θ_C increases (i.e., more conservatives support candidate R).*

(b) *If there are more liberals than conservatives ($p > 0.5$) and if σ is sufficiently small, then it is possible that both candidates decrease taxes. If tax rates decrease, and if $t'_D > 1$ then candidate R gains liberal voters but may lose conservative voters; if $t'_D < 1$, then candidate R decreases taxes more than candidate D, and θ_C increases while θ_L decreases.*

As a final comparative static experiment we increase both ρ and δ by the same amount, h. That is, ideological intensity increases simultaneously for both liberals and conservatives. This is particularly useful if we want to think about ideology in a spatial framework: Suppose that voter preferences are constant, but that candidate positions on the ideological dimension move away from each other. In this case, voters care more about their ideological favorite winning, i.e., both ρ and δ increase.

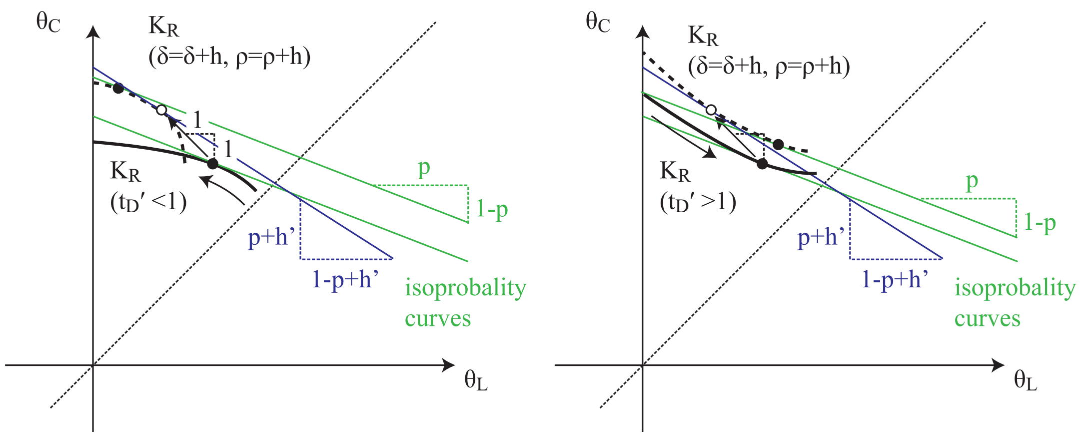

Figure 10: Comparative Statics: Both ρ and δ are increased by the same amount h.

Figure 10 considers a case with $p < 1 - p$, i.e., there are more conservatives than liberals, and linear isoprobability curves. Equations (5) and (6) show that if taxes remain the same, then the new cutoff value moves exactly to the northwest, along a line with slope -1 to the point indicated by the white circle. We refer to this change as the ideology effect.

At the original equilibrium cutoffs, the slope of K_R must be $-p/(1-p)$ because K_R is tangent to the isoprobability curve through (θ_L, θ_C). Equations (33) and (34) imply that after the increase of δ and ρ by amount h, the derivative of K_R in the θ_L direction, which is negative, becomes more negative by some amount h'. The derivative in the θ_C direction, which is positive, increases by the same amount h'. Thus, $p < 1 - p$ implies that the slope must become steeper.[15] The change of the

[15]Note that if $p = 1 - p$ then the slope of K_R would not change, and we would have tangency at the point indicated by the white circle, i.e., taxes would stay the same.

slope of K_R implies that taxes must change, since we do not have tangency at the white circle. We refer to the change in cutoffs because of the tax change as the tax effect.

Specifically, consider the left panel of Figure 10, where $t'_D < 1$. The concavity of K_R immediately implies that the tax effect moves the new tangency point even further to the northwest. Since we move in the direction of the arrow (counterclockwise), taxes increase. Since $t'_D < 1$, the difference between the candidates' tax rates decreases. Thus, candidates differ less on economic policy, and consequently, voters separate more by ideology: The tax effect and the ideology effect reinforce each other.

The right panel of Figure 10 analyzes the case of $t'_D > 1$. Again, if tax rates were kept constant, the cutoff moves to the northwest. However, the convexity of K_R implies that the new tangency point moves toward the southeast, again in the direction of increased taxes. For $t'_D > 1$, the tax effect and the ideology effect have opposite signs. Hence, it is possible that the difference between cutoffs $\theta^*_C - \theta^*_L$ remains almost unchanged, and thus, there is no perceived increase in the extent to which ideology rather than economic interests determine voting behavior, even after ideological polarization has increased. The reason is that when $t'_D > 1$ and taxes increase, the difference in tax rates increases, so that candidates' economic policies differ more. Hence, economic policy becomes more important for voters, too, which can countervail the increased importance of ideology.

The effects are reversed if $p > 1 - p$. In this case, proposed tax rates always decrease. This implies that the ideology effect and the tax effect go in opposite direction when $t'_D < 1$, and they reinforce each other when $t'_D > 1$.

If isoprobability curves are not straight lines, then there is an additional distribution effect. If $p < 1 - p$ as in Figure 10, isoprobability curves are concave above the 45 degree line. Thus, the total effect in the left panel gets even larger, as the new tangency point must be even further to the northwest. In the right panel, the new tangency point would be closer to a 45 degree line compared to the case of linear isoprobability curves.

6 Conclusion

In this paper, we have developed a model in which voters care about both social ideology (which, in our model, is exogenously given for candidates) and the economic positions that candidates take. The interaction between these two dimensions is of first-order importance for our understanding of what determines economic policy: In reality, there are considerable differences in candidates' economic policy platforms, but voter preferences for parties and candidates appear to be influenced by both economic and, probably to an even greater extent, by cultural-ideological positions. A model that explicitly incorporates these non-economic factors provides us with a better understanding of this important interaction, and thus with a better understanding of the determinants of economic policy than a model that abstracts from cultural ideology in order to focus entirely on economic policy issues.

There is an intense and ongoing discussion in both political science and popular discourse as to whether cultural or economic factors become more important as determinants of voter behavior (Green, Palmquist, and Schickler (2002), Frank (2004), McCarty, Poole, and Rosenthal (2006)). Our paper contributes to this literature by providing a unified model framework in which voters care about both cultural-ideologic positions and economic policy. Our main results are as follows: First, any change in equilibrium platforms that is brought about by changes in the voter preference distribution goes in the same direction for both candidates: Either, both candidates propose higher spending, or both propose lower spending than before. Second, an increase in support for the cultural-ideological position of the small-government party (either through an increase in the number of social conservatives, or through an intensification of their cultural preference) leads to an increase in spending. The same holds when both groups become more ideologically polarized, but there are more social conservatives than social liberals. Third, equilibrium policies and voter polarization patterns (as measured by the voting behavior of different ideological groups) are relatively stable with respect to changes in the distribution of economic preferences, at least if the two ideological groups are of similar size. In this respect, our results cast some doubt on claims that an increase in economic inequality in the last decades was the cause of the perceived polarization during the same time period.

7 Appendix

Proof of Lemma 1. For fixed t_D, consider the curve given by $t_R \mapsto (\theta_L^*(t_R), \theta_C^*(t_R))$. Let $S(t_R) = \frac{\partial\theta_C(t_R))}{\partial t_R} / \frac{\partial\theta_L(t_R))}{\partial t_R}$. Thus, (10) and (11) imply that

$$S(t_R) = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}. \tag{23}$$

Let

$$A(t_R) = (w(g_D) - w(g_R)) - (t_D - t_R)a_R w'(g_R) \tag{24}$$

Then

$$\frac{\partial S(t_R)}{\partial t_R} = (\rho + \delta)\frac{w'(g_R)a_R\frac{\partial A(t_R)}{\partial t_R} - A(t)w''(g_R){f_R'}^2(t_R)}{\Big((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)\Big)^2}, \tag{25}$$

where

$$\frac{\partial A(t_R)}{\partial t_R} = -(t_D - t_R){f_R'}^2(t_R)w''(g_R). \tag{26}$$

Next, note that

$$\begin{aligned} f_R'(t_R)\frac{w'(g_R)}{w''(g_R)} - {f_R'}^2(t_R)\frac{A(t_R)}{\frac{\partial A(t_R)}{\partial t_R}} &= a_R\frac{w'(g_R)}{w''(g_R)} + \frac{w(g_D) - w(g_R)}{(t_D - t_R)w''(g_R)} - a_R\frac{w'(g_R)}{w''(g_R)} \\ &= \frac{w(g_D) - w(g_R)}{(t_D - t_R)w''(g_R)}. \end{aligned} \tag{27}$$

Thus,

$$\frac{\partial S(t_R)}{\partial t_R} = -(\rho + \delta)\frac{w''(g_R){f_R'}^2(t_R)(w(g_D) - w(g_R))}{\Big((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)\Big)^2}, \tag{28}$$

The signed curvature of candidate R's response function is given by

$$\kappa_R = \frac{\frac{\partial\theta_L^*}{\partial t_R}\frac{\partial^2\theta_C^*}{\partial t_R^2} - \frac{\partial^2\theta_L^*}{\partial t_R^2}\frac{\partial\theta_C^*}{\partial t_R}}{\left(\left(\frac{\partial\theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial\theta_C^*}{\partial t_R}\right)^2\right)^{3/2}}. \tag{29}$$

Thus,

$$\kappa_R = \frac{\partial S(t_R)}{\partial t_R}\frac{\left(\frac{\partial\theta_L^*(t_R)}{\partial t_R}\right)^2}{\left(\left(\frac{\partial\theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial\theta_C^*}{\partial t_R}\right)^2\right)^{3/2}}. \tag{30}$$

As a consequence (25), (26), (27) and (30) imply (13).

Similarly, it follows that the curvature of $t_D \mapsto (\theta_L(t_D), \theta_C(t_D))$ is given by (14). □

Proof of Lemma 2. In the text. □

Proof of Lemma 4. Applying (10) and (11) it follows that the slope of $(\theta_L^*(t_R), \theta_C^*(t_R)$ is

$$\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}. \tag{31}$$

Similarly, (8) and (9) it follows that the slope of $(\theta_L^*(t_D), \theta_C^*(t_D)$ is

$$\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_D w'(g_D)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_D w'(g_D)}. \tag{32}$$

Thus, (31) and (32) are the same if and only if $a_D w'(g_D) = a_R w'(g_R)$. This proves the first statement.

Substituting $t_D(t_R)$ for t_D in (5) and (6), and taking the derivative with respect to t_R yields

$$\frac{\partial \tilde{\theta}_L}{\partial t_R} = (t'_D - 1)\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)}{(w(g_D) - w(g_R))^2} = (t'_D - 1)\frac{\partial \theta_L^*}{\partial t_R}. \tag{33}$$

$$\frac{\partial \tilde{\theta}_C}{\partial t_R} = (t'_D - 1)\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \rho)a_R w'(g_R)}{(w(g_D) - w(g_R))^2} = (t'_D - 1)\frac{\partial \theta_C^*}{\partial t_R}. \tag{34}$$

This and equations (10), and (11) proves the third statement of the Lemma.

Next, note that (33), (34) and (18) imply that the slope $H(t_R) = S(t_R)$, where $S(t_R)$ is given by (23). Thus, the candidates' reaction functions have the same slope as $(\theta_L(t_R), \theta_C(t_D(t_R))$.

Let

$$B(t_R) = (w(f_D(t_D(t_R))) - w(f(_R(t_R)))) - (t_D - t_R)a_R w'(f_R(t_R)) \tag{35}$$

Then (23) implies that

$$\frac{\partial B(t_R)}{\partial t_R} = -(t_D - t_R){f'_R}^2(t_R)w''(f_R(t_R))). \tag{36}$$

Thus,

$$\frac{\partial H(t_R)}{\partial t_R} = \frac{\partial S(t_R)}{\partial t_R} = (\rho + \delta)\frac{w''(g_R){f'_R}^2(t_R)(w(g_D) - w(g_R))}{\Big((w(g_D) - w(g_R)) - ((t_D - t_R) - \delta)a_R w'(g_R)\Big)^2}, \tag{37}$$

Let $\hat{\kappa}$ be the signed curvature of the curve. Then as in equation (30) it follows that

$$\hat{\kappa} = \frac{\partial H(t_R)}{\partial t_R}\frac{\left(\frac{\partial \tilde{\theta}_L(t_R)}{\partial t_R}\right)^2}{\left(\left(\frac{\partial \tilde{\theta}_L}{\partial t_R}\right)^2 + \left(\frac{\partial \tilde{\theta}_C}{\partial t_R}\right)^2\right)^{3/2}}. \tag{38}$$

Thus, (30), (33), (34), and (38) imply

$$\hat{\kappa} = \frac{\partial S(t_R)}{\partial t_R}\frac{(t'_D - 1)^2\left(\frac{\partial \theta_L^*(t_R)}{\partial t_R}\right)^2}{((t'_D - 1)^2)^{3/2}\left(\left(\frac{\partial \theta_L^*}{\partial t_R}\right)^2 + \left(\frac{\partial \theta_C^*}{\partial t_R}\right)^2\right)^{3/2}} = \frac{\kappa_R}{|t'_D - 1|}.$$

□

Proof of Lemma 3. If $f_R(t_R^*)) < f_D(t_D^*)$ then in equilibrium t_R^* must solve $\max_{t_R} G(k_R(t_R))$, while t_D^* solves $\min_{t_D} G(k_D(t_D))$. The first order conditions are given by $D_{t_R} k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = D_{t_D} k_R(t_D^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0$.

If $f_R(t_R^*)) > f_D(t_D^*)$ then in equilibrium t_R^* must solve $\min_{t_R} G(k_R(t_R))$, while t_D^* solves $\max_{t_D} G(k_D(t_D))$. The first order conditions is therefore $D_{t_R} k_R(t_R^*) \cdot \nabla G(\theta_L^*, \theta_C^*) = 0$. □

Proof of Theorem 1. Lemma 3 immediately implies that (16) must hold. Next, suppose that $f_D(t_D^*) > f_R(t_R^*)$. Then $\theta_C^* > \theta_L^*$. As a consequence, Lemma 1 implies that $\kappa_R, \kappa_D > 0$.

The equation of the isoprobability curve is $(\theta_L, \theta_C(\theta_L))$, where, $\theta_C(\theta_L)$ solves $G(\theta_L, \theta_C(\theta_L)) = G(\theta_L^*, \theta_C^*)$. Thus, the curvature of the isoprobability curve can be computed using equation (12)as

$$\kappa_G(\theta_L^*, \theta_C^*) = \frac{\frac{\partial^2 G(\theta_L^*,\theta_C^*)}{\partial \theta_L^2} \left(\frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_C}\right)^2 - 2\frac{\partial^2 G(\theta_L^*,\theta_C^*)}{\partial \theta_L \partial \theta_C} \frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_L} \frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_C} + \frac{\partial^2 G(\theta_L^*,\theta_C^*)}{\partial \theta_C^2} \left(\frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_L}\right)^2}{\left(\frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_L}\right)^2 + \left(\frac{\partial G(\theta_L^*,\theta_C^*)}{\partial \theta_C}\right)^2}. \tag{39}$$

The isoprobability curve at (θ_L^*, θ_C^*) is concave toward the origin if $\kappa_G < 0$, and convex if $\kappa_G > 0$. Thus, if $\kappa_G < 0$ a necessary condition is that $\kappa_R \geq -\kappa_G$. If, instead, $\kappa_G > 0$ then in a local equilibrium the curvature κ_D cannot be strictly less than κ_G, i.e., $\kappa_D \geq \kappa_G$.

The argument is similar if $f_D(t_D^*) < f_R(t_R^*)$. In this case we must take into account that the curvatures κ_R and κ_D are negative. As a consequence, we get $|\kappa_R| \geq -\kappa_G$ and $|\kappa_D| \geq \kappa_G$.

To prove the reverse implication, note that if (16) holds then Lemma 3 implies that curves k_R and k_D are tangent at t_D^* and t_R^* to the isoprobability curves. If $|\kappa_R| > -\kappa_G$ and $|\kappa_D| > \kappa_G$ then above argument implies that locally the curvature of the isprobability curve is strictly less than of k_D and k_R. As a consequence, we have a local equilibrium.

The final statement of the Theorem requires the curvature condition to hold for all points above the 45 degree line if $f_D(t_D^*) > f_R(t_R^*)$, or for all points below the 45 degree axis, otherwise. Thus, k_R and k_D only touch the isoprobability curve at (θ_L^*, θ_C^*) and we have a semi-global equilibrium. □

Proof of Theorem 2. Note $g_D = g_R$ if candidates at t_D^* and $\hat{t}_R$. Let $t'_R < \hat{t}_R$ such that $t'_R > t_D^* + \rho$. Thus, (6) implies that $\theta_C^*(t_D^*, t'_R) < 0$. Further, $\theta_C^*(t_D^*, 0) > 0$. This. continuity and compactness imply that

$$\bar{\theta}_C = \max_{0 \leq t_R < \hat{t}_R} \theta_C^*(t_D^*, t_R) \tag{40}$$

exists.

Next, note that $\bar{\theta}_C > \theta_C^*$. Clearly, $\bar{\theta}_C \geq \theta_C^*$. Thus, suppose that $\bar{\theta}_C = \theta_C^*$. This, however, means that $\frac{\partial \theta_C^*}{\partial t_R}(t_D^*, t_R^*) = 0$. This and (10) imply $\frac{\partial \theta_L^*}{\partial t_R}(t_D^*, t_R^*) < 0$. However, since $\frac{\partial G}{\partial t_L}(\theta_L^*, \theta_C^*) \neq 0$, this implies that statement 2 of Lemma 3 is violated. Thus, $\bar{\theta}_C > \theta^*_C$.

Equations 5 and 6 imply $(\bar{\theta}_C, \bar{\theta}_C) > (\theta^*_L, \theta^*_C)$. Since the equilibrium is local, any deviation $\tilde{t}_R$ by candidate R with $f_R(\tilde{t}_R) \leq f_D(t^*_D)$ cannot be optimal. Thus, consider a deviation $\tilde{t}_R$ with $f_R(\tilde{t}_R) > f_D(t^*_D)$. Let $\tilde{\theta}_C$, $\tilde{\theta}_L$ be the new cutoff voters. Now candidate R receives the support of all types $\theta_C \geq \tilde{\theta}_C$ and $\theta_L \geq \tilde{\theta}_L$, where $\tilde{\theta}_C < \tilde{\theta}_L$. We now show that $\bar{\theta}_C < \tilde{\theta}_C$.

Suppose by way of contradiction that $\bar{\theta}_C \geq \tilde{\theta}_C$. Let $\bar{t}_R \in \arg\max_{0 \leq t_R < \hat{t}_R} \theta^*_C(t^*_D, t_R)$. Then

$$\bar{t}_R \in \arg\max_{t_R < \hat{t}_R} m(1 - t_R) + \bar{\theta}_C w(f_R(t_R)) + \rho. \tag{41}$$

Else, if (41) is violated then there exists t'_R that gives type $\bar{\theta}_R$ a strictly higher utility. This, however, means that $\bar{\theta}_R$ strictly prefers candidate R to candidate D if candidates choose t^*_D and t'_R, respectively. Thus, by continuity there would exist $\theta'_R > \theta_R$ such that θ'_R strictly prefers R to D. This, however, contradicts 40. Hence, (41) must hold.

Next, note that $m(1 - t_R) + \bar{\theta}_C w(f_R(t_R)) + \rho$ is strictly concave in t_R. Thus, $\bar{t}$ in (41) is the unique maximum, even if we eliminate the constraint that $t_R < \hat{t}_R$.

If $\bar{\theta}_C \geq \tilde{\theta}_C$ then type $\bar{\theta}_C$ is at least as well from candidate R with tax rate $\tilde{t}_R$ than from candidate C with tax rate t^*_D. This, however, means that $\bar{\theta}_C$'s utility is at least as high from candidate R with tax rate $\bar{t}_R$. This, however, contradicts that $\bar{t}_R$ is the unique solution to (41). Thus, $\bar{\theta}_C < \tilde{\theta}_C$. Hence, the winning probability from deviating is $1 - G(\tilde{\theta}_L, \tilde{\theta}_C) \leq 1 - G(\bar{\theta}_C, \bar{\theta}_C)$. Thus, the assumption that $G(\theta^*_L, \theta^*_C) > 1 - G(\bar{\theta}_C, \bar{\theta}_C)$ implies that such a deviation is not optimal.

Next, we consider deviations by candidate D. Since $\hat{t}_D > t^*_R + \delta$, equation (5) implies that $\lim_{t_D \downarrow \hat{t}_D} \theta^*_L(t_D, t^*_R) = \infty$. Thus, we can conclude that

$$\bar{\theta}_L = \min_{\hat{t}_D < t_D \leq 1} \theta^*_L(t_D, t^*_R) \tag{42}$$

exists.

The remainder of the argument is similar to that for $\bar{\theta}_C$. In particular, it follows that $\bar{\theta}_L < \theta^*_L$, and that consequently $(\theta^*_L, \theta^*_C) > (\bar{\theta}_L, \bar{\theta}_L)$. Similar to above it follows that the utility of type $\bar{\theta}_L$ is maximized when candidate D chooses tax rate $\bar{t}_D$ that solves (42). As a consequence, concavity of utility implies that any deviation $t'_D \in [0, 1]$ makes type $\bar{\theta}_L$ worse off. It also follows again that, if $t'_D > \hat{t}_D$ then the new cutoff voter $\tilde{\theta}_L < \hat{\theta}_L$. Candidate D's winning probability after the deviation is $G(\tilde{\theta}_L, \tilde{\theta}_C) < G(\hat{\theta}_L, \tilde{\theta}_C) < G(\hat{\theta}_L, \hat{\theta}_L)$, where the last inequality follows since $\tilde{\theta}_C < \tilde{\theta}_L$ for $t_D > t'_D$. Thus, a sufficient condition for the deviation not to be optimal is $1 - G(\theta^*_L, \theta^*_C) > G(\hat{\theta}_L, \hat{\theta}_L)$, which is equivalent to the condition $1 - G(\bar{\theta}_L, \bar{\theta}_L) > G(\theta^*_L, \theta^*_C)$ in the statement of the theorem. □

Proof of Lemma 5. 1. If$p = 1/2$ and the density function f is symmetric around some point $\bar{\theta}$, then, for a tie to occur in state $\bar{\bar{\omega}}$, $\theta_L - \bar{\bar{\omega}}$ and $\theta_C - \bar{\bar{\omega}}$ must be located symmetrically around $\bar{\theta}$, i.e., the isoprobability curve associated with $\bar{\omega}$ are straight lines of the form $(\bar{\theta}-h+\bar{\bar{\omega}}, \bar{\theta}+h+\bar{\bar{\omega}})$, $h \in \mathbb{R}$, with curvature $\kappa_G \equiv 0$. Thus, if $p = 1/2$, any solution to the first order condition (16) is a local equilibrium and a semi-global equilibrium. Note that this result is independent of the distribution of θ-types, F.

2. If θ_C and θ_L are normally distributed, then Equations (21) and (39) imply that

$$\kappa_G = -\frac{2p(1-p)2e^{-\frac{(\theta_L-\mu-\bar{\omega})^2+(\theta_C-\mu-\bar{\omega})^2}{2\sigma^2}}}{\sigma^2\left(pe^{-\frac{(\theta_L-\mu-\bar{\omega})^2}{2\sigma^2}}+(1-p)e^{-\frac{(\theta_C-\mu-\bar{\omega})^2}{2\sigma^2}}\right)}\left(\frac{\theta_L+\theta_C}{2}-\mu-\bar{\omega}\right). \tag{43}$$

If $p = 0.5$, then (21) implies $\frac{\theta_L+\theta_C}{2} = \mu+\bar{\omega}$, and hence $\kappa_G = 0$, resulting in linear isoprobability curves. If p is close to 0.5 then $\frac{\theta_L+\theta_C}{2} - \mu + \bar{\omega}$ does not differ too much from zero, and hence the curvature κ_G remains close to zero.

On the 45-degree line ($\theta_L = \theta_C$), (21) implies that $\theta_L = \theta_C = \mu+\bar{\omega}$. As a consequence, $\kappa_G = 0$ for any p. Now suppose that $p > 0.5$. For candidates to tie in this case, the Republican needs to attract the support of more conservatives than the Democrat attracts liberals. Since the density f is symmetric, this requirement implies that $-(\theta_L - \mu - \bar{\omega}) < \theta_C - \mu - \bar{\omega}$ for $\theta_L < \theta_C$, and the reverse inequality holds for $\theta_L > \theta_C$. As a consequence $\frac{\theta_L+\theta_C}{2} > \mu + \bar{\omega}$ and hence $\kappa_G < 0$ for $\theta_L < \theta_C$, and $\kappa_G > 0$ for $\theta_L > \theta_C$. Thus, isoprobability curves are convex above the 45 degree line, and concave below the 45 degree line. It follows immediately that this result is reversed for $p < 0.5$, i.e., isoprobability curves are concave above the 45 degree line and convex below.

Next, note that increasing σ will move the curvature closer to zero. In particular, as $\sigma \to \infty$ all exponentials in (43) converge to 1, and hence $\kappa_G \to 0$, i.e., isoprobability curves become straight lines. The slope of the isoprobability curve is given by

$$\theta_C'(\theta_L) = -\frac{pe^{-\frac{(\theta_L-\mu-\bar{\omega})^2}{2\sigma^2}}}{(1-p)e^{-\frac{(\theta_C-\mu-\bar{\omega})^2}{2\sigma^2}}}.$$

which is equivalent to (22). Note that (22) converges to $-p/(1-p)$ as $\sigma \to \infty$.

Let $\theta_L < \theta_C$. Then (21) implies that $(\theta_L + \theta_C)/2 > \mu + \bar{\omega}$ if $p < 0.5$. Thus, the argument of the exponential function in (22) is strictly positive. Increasing σ therefore increases $\theta_C'(\theta_L)$, i.e., the slope becomes more flat. The reverse is true if $p > 0.5$.

Now suppose that the percentage of liberals, p, increases, and consider the isoprobability curve through (θ_L, θ_C). Equation (21) implies that $\bar{\omega}$ must decrease. Thus, the argument of the exponential function in (22) increases. This and the increase of p implies that $\theta_C'(\theta_L)$ decreases, i.e., the slope of the isoprobability curve through (θ_L, θ_C) becomes steeper.

Finally, consider a change in μ. In order for (21) to hold after a change of μ, $\bar{\omega} + \mu$ must remain constant. Thus, a change in μ does not affect $\theta_C'(\theta_L)$.

□

Proof of Theorem 3. See text. □

References

Austen-Smith, D. and M. Wallerstein (2006). Redistribution and affirmative action. *Journal of Public Economics 90*(10-11), 1789–1823.

Besley, T. and S. Coate (1997). An economic model of representative democracy. *Quarterly Journal of Economics 112*, 85–114.

Egan, P. J. (2008). Issue ownership and representation: A theory of legislative responsiveness to constituency opinion. New York University.

Fernández, R. and G. Levy (2008). Diversity and redistribution. *Journal of public economics 92*(5-6), 925–943.

Frank, T. (2004). *What's the matter with Kansas?: How conservatives won the heart of America*. Holt Paperbacks.

Green, D., B. Palmquist, and E. Schickler (2002). *Partisan hearts and minds: Political parties and the social identities of voters*. Yale Univ Pr.

Krasa, S. and M. Polborn (2009). Political competition between differentiated candidates. University of Illinois.

Krasa, S. and M. Polborn (2010a). Competition between Specialized Candidates. *American Political Science Review 104*(4), 745–765.

Krasa, S. and M. K. Polborn (2010b). The binary policy model. *Journal of Economic Theory 145*, 661–688.

Lindqvist, E. and R. Östling (2010). Political Polarization and the Size of Government. *American Political Science Review 104*(3), 543–565.

McCarty, N., K. Poole, and H. Rosenthal (2006). *Polarized America: The dance of ideology and unequal riches*. MIT Press.

Meltzer, A. H. and S. Richard (1981). A rational theory of the size of government. *Journal of Political Economy 89*, 914–927.

Osborne, M. and A. Slivinski (1996). A model of political competition with citizen candidates. *Quarterly Journal of Economics 111*, 65–96.

Stokes, D. (1963). Spatial Models of Party Competition. *American Political Science Review 57*(2), 368–377.