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Optimal Income Taxation and Public Provision of Productive Inputs

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Preliminary and Incomplete Version

Abstract

In an economy where individual productivity is unobservable and determined by exogenous ability and endogenous input investment, we characterize the optimal non-linear income tax and the optimal scheme for the public provision of the productive input. Public provision is found to be always welfare improving with respect to pure non-linear taxation. The optimal public provision scheme (whether a pure scheme based on opting-out or topping up mechanisms or a mixed one) depends on the structure of preferences and technology. If exogenous ability is substitute of endogenous input (which happens also when ability and input are technologic complements), and heterogeneity across classes is sufficiently strong, the pure opting-out scheme is optimal. Conversely, when exogenous ability and input are complements, the pure topping up is optimal. Interestingly, in the benchmark two-class case, also the labor supply of rich individuals can be optimally distorted - by a subsidy - at the margin, whenever opting-out scheme is optimal, but its redistributive power is constrained by insufficient individual heterogeneity in input demand.

Keywords: In-kind redistribution; Public provision of private goods; Opting out; Topping up; Non-linear income tax

JEL classification: H42, H21

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1 Introduction

The second-best analysis of the public provision of private goods has challenged the traditional first-best view about redistribution, forging a new argument to justify public social services (Balestrino, 1999, 2000). Whenever household economic condition is imperfectly verified and affects the demand of some goods, in-kind transfers (or quotas on consumption) of these goods Pareto-dominate cash as a redistribution tool (Nichols and Zeckhauser, 1982; Guesnerie and Roberts, 1984). Efficiency gains are driven by higher costs of opportunistic behaviors in taking up subsidies (Blackorby and Donaldson, 1988) and in paying taxes (Guesnerie and Roberts, 1984), which in turn are determined by specific public provision rules.

The conventional wisdom of the considered literature is that social services are consumption goods. [1] However, the economic nature of some publicly provided goods is related more to production than to consumption (Balestrino, 1999, p. 346). Education and healthcare primarily affect household production capacity (i.e., human capital), while their features as consumption goods are relatively less relevant. Also, services such as childcare and elderly-care can be more convincingly modeled as inputs, letting households fully exploit their potential income capacity. Extending the analysis by Greco (2010), we investigate the implications of this production view of social services in terms of redistribution, optimal provision schemes and optimal taxation.

As benchmark we consider a simple second-best framework with endogenous labor supply and nonlinear income taxation. Household income is the product of wage and labor supply. Following Boadway and Marchand (1995, p. 51-55), wage is modeled as an increasing function of two factors: exogenous wealth (or ability) and a productivity-enhancing input (e.g., education, healthcare, childcare or elderly-care) that can be provided by gov-

[1] A remarkable exception is Boadway and Marchand (1995, p. 51-55), who prove that the universal and unconditional public provision of education, affecting the marginal productivity of households, can relax the information constraint on taxation. However, they do not focus on the assessment of the optimal provision mechanism.

ernment and by a market of competitive firms.

Wealth represents a composite asset summing up physical, financial, human and social "exogenous" factors, which determine household potential productivity; input represents any service or intermediate good that improves, in different ways, household potential earnings. Depending on the assumed nature of the productivity-enhancing technology, input can be complementary to or substitute of exogenous wealth. When household capacity to exploit potential earnings is strengthened by input (e.g., childcare or higher education) then it complements wealth. Conversely, input substitutes wealth when it affords households with more of the same factors constituting their production capacity (e.g., social network through schooling and basic living conditions through social housing or health prophylaxis).

Taking pure taxation as a benchmark, the public provision of input reduces the efficiency cost of redistribution in two ways. First, the public provision may alleviate tax distortion on household investment choices by forcing households to use more input (Boadway and Marchand, 1995; Cremer and Gahvari, 1997). Second, if transfers of input can be targeted to the poor more effectively than cash, then they improve the redistribution capacity of public policies by reinforcing self-selection mechanisms (Besley and Coate, 1991; Munro, 1992; Blomquist and Christiansen, 1995).

Government can implement public provision policies in two ways:

1. by supporting private expenditure on social services through conditional transfers (e.g., vouchers, tax allowances); such a policy is also called a *topping-up* scheme given that it amounts to publicly providing a given quantity (or quality) of a social service to everybody allowing for private supplementing;

2. by providing public services (through different institutional arrangements) as an alternative to private services; such policies - also called *opting-out* schemes - leave households free to choose private or public services, though in the latter case private

supplementing is not possible.[2]

These forms of public provision may coexist, giving rise to mixed schemes. For example, households opting for a public school automatically give up tax credits for private schools; hence, savings on tax credits implicitly finance public school expenditure. Also, an increase of tax allowances for households' expenditure on private schools implicitly reduces the additional transfer (with respect to tax allowances) underlying free public schools. Therefore, abstracting from differences in the quality of public and private services, real-world public social programs can generally be represented by a two-pillar provision scheme:

1. the first (topping-up) pillar affords all households with some input that can be privately supplemented;

2. the second (opting-out) pillar provides some additional input as an alternative to private supplementing.

In this framework, we find that public provision is *always* welfare improving. However, optimal provision schemes depend on two structural features of the economy: the complementarity or substitutability of input and wealth in income production and the balance between households' production heterogeneity and government's preference for redistribution. When households' heterogeneity is strong enough (as compared to government's preference for redistribution), the optimal provision scheme is made by a single pillar: a pure opting-out mechanism, when input and wealth are complementary, or a pure topping-up mechanism, when input substitutes wealth. Whenever households' heterogeneity is relatively weak, it is optimal to provide all households with the same amount of input (without private supplementing). Finally, when the balance between households' heterogeneity and preference for redistribution is intermediate, and input is complementary to wealth, a full-fledged two-pillar scheme is optimal. [REVISE THIS PARAGRAPH]

[2]For example, children attending a public school cannot attend, at the same time, a private one. In other cases, private supplementing may be legally forbidden. It is worth remarking that, by means of conditional grants, topping-up mechanisms can also be implemented in sectors where using different services at the same time is not *technologically* feasible (Blomquist and Christiansen, 1995, pp. 564-5).

Our results complement the main findings of the literature on public provision of private goods, providing a somewhat different perspective. We consider social programs characterized by different and potentially coexisting provision rules, generalizing the approach of Blomquist and Christiansen (1998a), who contrast pure topping-up and opting-out schemes. Also, our work highlights that opting-out schemes cannot implement the public provision of goods satisfying households' basic needs.[3]

However, the main difference with the literature regards the effect of households' heterogeneity on the scope for public provision. Consistent to the literature, we find that the degree of heterogeneity affects the optimal structure of social programs. Nevertheless, we show that the public provision of input plays a tax-correction role for any specification of technology, particularly when input demand does not depend on household private wealth. This is at odds with the results highlighting that there is no scope for the public provision of a given commodity whenever households' heterogeneity has no effect on its demand (Balestrino, 1999, 2000).

The reason for this theoretical divergence is that, in our model, government provides an input that directly corrects tax distortion on household production effort. When household production function is separable, input demand is independent of household wealth, and its public provision becomes a perfect tool for correcting tax distortion: in this case, public provision without private supplementing implements the first-best outcome. Conversely, in models with public provision of consumption goods, the government provides such goods to influence the productive effort of individuals (i.e., labor supply) indirectly. Therefore, once this indirect link is broken (i.e., when, by separability of utility function, the demand of the considered commodity does not depend on leisure), there is no more scope for public provision (Blomquist and Christiansen, 1998a, p. 405).

[3] In our model, the input satisfies basic needs when it substitutes wealth. This - under our assumptions - also implies that the former is inferior. Although evidence highlighted that primary healthcare in developing countries seems to be an inferior good (**?**, p. 306), normality of social services is the most common and intuitive case. As discussed in the concluding remarks, our main results can be generalized to models with endogenous labor supply and non-linear taxation, in which the publicly-provided input is a normal good.

The paper is organized as follows: [COMPLETE HERE].

2 The Model

The economy is populated by a large number (a unit measure) of households. The productivity of household i depends on exogenous individual ability θ_i and on investment in input q_i: $w_i = w(\theta_i, q_i)$, where $w(.,.)$ is strictly increasing and twice differentiable in both arguments, and concave in q. In our benchmark case, we assume that $\lambda \in (0,1)$ households have low ability ($\theta_i = \theta_0$), while the others have high-ability ($\theta_i = \theta_1$). Also, we assume constant returns to scale production technology, competitive labor market (hence, gross wage is equal to individual productivity), and unitary price of investment and consumption.

The government maximizes the sum of households utilities, but it can only observe gross income $y_i = w_i \cdot l_i$, conversely gross wage rate (that we assume equal to individual productivity) w_i and household labor supply l_i are not observed. Government can observe after-tax income x_i, but not private consumption c_i and private investment in input q_i^m.

The input is also publicly provided. The government supplies a uniform quantity q^f through a first pillar (topping up scheme) independently of households' consumption and investment choices. Then, a supplementary quantity of input q^s is provided to individuals opting for a second public pillar, and accepting not to privately top-up the public provision. Individuals opting out of the second pillar can privately supplement the first pillar input provision (with q_i^m).

The government budget constraint can be written as $\lambda \cdot (y_1 - x_1) + (1 - \lambda) \cdot (y_2 - x_2) \geq q^f + q^s \cdot I$, where: y_1, x_1, y_2, and x_2 are gross and net incomes of high-ability and low-ability households, respectively; and $I \in [0, 1]$ is the share of population covered by the second-pillar public provision of input.

The utility function of the generic household is $U(c_i, l_i)$, strictly increasing and concave in private consumption, and strictly decreasing and concave in labor supply. Moreover, consumption and leisure are normal goods. By $l_i = \frac{y_i}{w_i}$, $c_i = x_i - q_i^m$ (for households opting out of the second-pillar provision), and $c_i = x_i$ (for households opting for the second-pillar provision), we can write the generic utility of opting-out households as $U(x_i - q_i^m, \frac{y_i}{w(\theta_i, q^f + q_i^m)})$ and the utility of opting-in households as $U(x_i, \frac{y_i}{w(\theta_i, q^f + q^s)})$.

The timing of the model reads as follows. The government determines the non-linear income tax and the first- and second-pillar public provision of input, then households decide whether to reveal or not their ability, and whether to opt out of the second-pillar provision, and in this case the amount of private supplement of the first-pillar provision.

2.1 Household's choices and Policy regimes

In this section we consider the effect of government policies on the household's behavior. Taking as given gross and net incomes, we first analyze input private demand of opting-out households, then household decision to opt out or to accept the second-pillar provision. Finally, we discuss the behavior of households with different abilitys, in particular mimicking behaviors, reacting to different tax schedules.

2.1.1 Private demand of input

Let us assume that a generic household with ability θ decided to opt out. Taking as given gross and net incomes, let $V(x, y, q^f, \theta) \equiv \max_{q^m} U(x - q^m, \frac{y}{w(\theta, q^f + q^m)})$. If $q^m > 0$, by comparative statics of the first order condition with respect to q

$$-(U_c + U_l \cdot \frac{y}{w^2} \cdot w_q) = 0 \tag{1}$$

it is possible to show (see Appendix) that - under the assumed well-behaved preferences and technology - the private demand of input, $q^m(x, y, q^f, \theta)$, is increasing in net income ($\frac{dq^m}{dx} \in (0, 1)$) and in gross income ($\frac{dq^m}{dy} \in (0, \frac{dx}{dy} |_{V(x,y,q^f,\theta)})$), and it is decreasing in first-pillar public provision - though there isn't complete crowding out: $\frac{dq^m}{dq^f} \in (-1, 0)$. The behavior of private input demand with respect to exogenous ability - i.e., whether private input is economic complement of or substitute for ability - depends on the technologic complementarity or substitutability of input and ability:

1. if input and ability are strong technologic complements (say, there is some $\bar{w}_{q\theta} > 0$ such that $w_{q\theta} > \bar{w}_{q\theta}$), then private input is an economic complement of ability ($\frac{dq^m}{d\theta} > 0$);

2. if input and ability are strong technologic substitutes (say, there is some $\underline{w}_{q\theta} < 0$, such that $w_{q\theta} < \underline{w}_{q\theta}$), then private input is an economic strong substitute of ability ($\frac{dq^m}{d\theta} < -\frac{w_\theta}{w_q}$[4]);

3. if ability and input are weak technologic complements or substitutes (say, $w_{q\theta} \in (\underline{w}_{q\theta}, \bar{w}_{q\theta})$), then private input is an economic substitute of ability ($\frac{dq^m}{d\theta} \in (-\frac{w_\theta}{w_q}, 0)$).

It is worth remarking that to observe complementarity between ability and private demand for input, other things equal, we need strong technologic complementarity. In the other cases, the effect of ability on the private demand for input is negative: high-ability households would - other things equal - optimally demand less input. In our analysis, we will assume also another restriction on technology and preferences:

$$w_{q\theta} > w^*_{q\theta} \equiv \frac{w_q \cdot w_\theta}{w} - \beta \cdot (\frac{w_q}{w} - w_{qq}) \qquad (2)$$

In the Appendix, we show that (2) is necessary and sufficient to insure that the Single Crossing Property

$$\frac{d}{d\theta}(\frac{dx}{dy} |_{V(x,y,q^f,\theta)}) = \frac{\partial}{\partial\theta}(\frac{dx}{dy} |_{V(x,y,q^f,\theta)}) + \frac{\partial}{\partial q}(\frac{dx}{dy} |_{V(x,y,q^f,\theta)}) \cdot \frac{dq^m}{d\theta} < 0$$

taking into consideration the effect of θ on private input demand. Also, (2) - hence the Single Crossing Property (SCP) - is never satisfied in the case of strong substitutability

[4]Remark that $-\frac{w_\theta}{w_q}$ is the marginal rate of technologic substitution keeping constant the level of w.

between input and ability (case 2), therefore we will exclude such a case in our analysis.[5]

2.1.2 Opting out choice

We consider now the optimal opting out choice. Such a choice is observed by the government, hence the most general tax schedule could depend also on it.[6] In our analysis, we will consider a restricted tax schedule that does not (explicitly) depend on the choice to join the second-pillar public provision scheme. Thus, the opting-in condition of the generic household can be written as

$$q^s(x, y, q^f, \theta) \equiv \{q' \in \mathcal{R}_+ \mid U(x, \frac{y}{w(\theta, q^f + q')}) = V(x, y, q^f, \theta)\} \tag{3}$$

that is increasing in x and y, and decreasing in q^f (namely, $\frac{dq^s}{dq^f} \in (-1, 0)$) (see Appendix). The effect of θ on q^s depends on the technologic complementarity or substitutability between q and θ:

- if ability and input are strong technologic complements (say, there is a $\tilde{w}_{q\theta} > 0$, such that $w_{q\theta} > \tilde{w}_{q\theta}$), then $\frac{dq^s}{d\theta} > 0$;
- if ability and input are weak technologic complements or substitute (say $w_{q\theta} < \tilde{w}_{q\theta}$), then $\frac{dq^s}{d\theta} < 0$.

[5]Moreover, such in such a case the total effect of exogenous ability on individual productivity is negative and the very concept of high- and low- ability could be questioned.

[6]In this case, the opting-in condition of the generic household can be written as

$$U(x^s, \frac{y^s}{w(\theta, q^f + q^s)}) \geq V(x^m, y^m, q^f, \theta)$$

where y^s, x^s, y^m, and x^m represent gross and net incomes that the considered household would have by opting for second-pillar provision or opting out, respectively. Given all policy variables and the ability level, the minimum second-pillar provision inducing the considered household to opt in can be written as

$$q^s(x^s, y^s, x^m, y^m, q^f, \theta) \equiv \{q' \in \mathcal{R}_+ \mid U(x^s, \frac{y^s}{w(\theta, q^f + q')}) = V(x^m, y^m, q^f, \theta)\}$$

that is decreasing in x^s and y^m and increasing in x^m and y^s (see Appendix), while the sign of the effects of q^f and θ on q^s requires additional assumptions.

In the Appendix, we show that the SCP is always satisfied for opting-in households, namely

$$\frac{d}{d\theta}(\frac{dx}{dy}\mid_{U(x,\frac{y}{w(\theta,q^f,q^s)})}) = \frac{\partial}{\partial\theta}(\frac{dx}{dy}\mid_{U(x,\frac{y}{w(\theta,q^f,q^s)})}) < 0.$$

The SCP for opting-in households always imply that the marginal rate of substitution between net and gross income decreases in θ, while additional restrictions are required to warrant such a property in the case of opting-out households. In particular, we considered restrictions warranting the same sign of the SCP for opting-in and opting-out households. Though such an assumption avoids qualitative changes in the way exogenous ability affects the slope of indifference curves in the space $\{x, y\}$, it is worth to recall that such an effect is stronger (or weaker) for opting-out households with respect to opting-in households if the effect of exogenous ability on private input demand is positive (or negative).

2.1.3 First best benchmark, and mimicking behaviors

In first best, government observes exogenous abilitys of individuals and can implement optimal lump sum taxation. In this case, the traditional result that cash redistribution is superior to in-kind redistribution applies. Thus, in first best there is no role for the public provision of input, and without loss of generality we put $q^f = q^s = 0$. The first best optimization conditions imply non-distorting optimal taxation, namely

$$\frac{dx}{dy}\mid_{V(x,y,0,\theta)} = -\frac{U_l}{U_c \cdot w} = 1$$

for all θ. Moreover, by individual optimization (1) also $\frac{w}{y \cdot w_q} = 1$, for all θ.

The first best allocation may be incentive-incompatible. However, as usual, poor households have no incentive to mimic rich households (see Appendix), while the reverse can happen, namely when redistribution is sufficiently large. In the following, we will assume that this is the case: the first best allocation is incentive-incompatible for the high-ability households, thus the optimal taxation (and public provision) program of the government an incentive constraint has to be introduced.

2.1.4 Policy regimes

As discussed in Greco (2010), in the considered setting we may have four kinds of policy regimes, depending on government policies and structural parameters:

PT in the pure taxation regime, the level of q^f and q^s are such that no individual is constrained by the first pillar and no individual opt for the second pillar public provision, hence $q^s < \min q^s(x_0, y_0, q^f, 0), q^s(x_1, y_1, q^f, 1)$;

INC in the inclusive regime, the level of q^f and q^s are such that all individuals are constrained by the first pillar or opt for the second pillar public provision, hence $q^s > \max q^s(x_0, y_0, q^f, 0), q^s(x_1, y_1, q^f, 1)$;

D we may have two discriminating regimes, depending on the type of households with higher minimum public provision inducing them to opt in the second pillar scheme:

- **DLS** when $q^s(x_0, y_0, q^f, 0) < q^s(x_1, y_1, q^f, 1)$, the low-ability households opt for the second-pillar provision while the high-ability households opt out;
- **DHS** when $q^s(x_0, y_0, q^f, 0) > q^s(x_1, y_1, q^f, 1)$, the high-ability households opt for the second-pillar provision while the low-ability households opt out.

3 Optimal Taxation and Public Provision

The government may choose the relevant policy regime, by setting appropriate tax schedule and public provision levels for the first and second pillar. However, the nature of the prevailing discriminating regime, depends on structural features such as individual preferences and technology. Passing from one policy regime to the other introduces, in the two-class setting, discontinuities in the structure of government objective and constraints, therefore we will first find optimal solution constrained within each policy regime, then analyze optimal global solutions.

In the pure taxation regime, the optimal tax problem is the classical one, low ability individuals receive a lump sum transfer and their labor income is taxed with distorting taxation. The taxation problem leads to the same conclusions also in the inclusive regime regime. The only difference is that in this case the government also controls the (uniform) public provision of input. [HERE DISCUSSION ON UNIFORM PUBLIC PROVISION OF INPUT].

3.1 Discriminating Regimes

We assume that preferences and technology are such that low-ability households opt-in for lower level of q^s than high-ability (DLS). In this discriminating regimes, the low-ability opt in, the high-ability opt out, and the mimicker is forced to opt in the second pillar and take q^s. Therefore the maximization problem of the government is

$$\begin{aligned}
\max_{\{x_0,y_0,x_1,y_1,q^f,q^s\}} \lambda \cdot U(x_0, \frac{y_0}{w(0,q^f+q^s)}) + (1-\lambda)\cdot V(x_1,y_1,q^f,1) \\
s.t.: \\
\lambda\cdot(y_0-x_0)+(1-\lambda)\cdot(y_1-x_1)-q^f-\lambda\cdot q^s \geq 0 \\
V(x_1,y_1,q^f,1) \geq U(x_0, \frac{y_0}{w(1,q^f+q^s)}) \\
q^s(x_1,y_1,q^f,1) \geq q^s \\
q^s \geq q^s(x_0,y_0,q^f,0) \\
q^f \geq 0 \\
q^s(x_1,y_1,q^f,1) \geq 0
\end{aligned} \tag{4}$$

By the optimization conditions of the program (4) we have

Proposition 1 *If low-ability demand less private input, the optimal policy mix in the* *discriminating regime,* $\{x_0^*, y_0^*, x_1^*, y_1^*, q^{f*}, q^{s*}\}$*, has the following features:*

- *if exogenous ability and input are economic substitutes,*
  - *and there is enough scope for the second-pillar provision, hence* $q^s(x_1, y_1, q^f, 1) > q^{s*}$*, then: the optimal taxation schedule never distorts high-ability labor supply and the optimal tax distortion of low-ability labor supply has the usual shape; moreover the optimal public provision scheme is pure opting-out:* $q^{f*} = 0$*, and* q^{s*} *implies over-investment for the low-ability households;*
  - *if there is not enough scope for the second-pillar provision, hence* $q^s(x_1, y_1, q^f, 1) = q^{s*}$*, then: the optimal taxation schedule distorts also the high-ability labor supply with a marginal subsidy; moreover, the optimal public provision scheme mixes topping-up and opting-out;*
- *if exogenous ability and input are economic complements, then: the optimal taxation schedule never distorts high-ability labor supply, and the low-ability labor supply is overtaxed (with respect to the case without public provision); moreover the public provision scheme is pure topping-up.*

We now consider the other discriminating case, where high-ability households opt-in for lower level of q^s than low-ability (DHS). The maximization problem of the government is

$$\max_{\{x_0,y_0,x_1,y_1,q^f,q^s\}} \lambda \cdot V(x_0, y_0, q^f, 0) + (1-\lambda) \cdot U(x_1, \frac{y_1}{w(1, q^f + q^s)})$$

$$s.t.: \tag{5}$$

$$\lambda \cdot (y_0 - x_0) + (1-\lambda) \cdot (y_1 - x_1) - q^f - (1-\lambda) \cdot q^s \geq 0$$

$$U(x_1, \frac{y_1}{w(1, q^f + q^s)}) \geq V(x_0, y_0, q^f, 1)$$

$$q^s(x_0, y_0, q^f, 0) \geq q^s$$

$$q^s \geq q^s(x_1, y_1, q^f, 1)$$

$$q^f \geq 0$$

$$q^s(x_0, y_0, q^f, 0) \geq 0$$

Proposition 2 *[HERE PROPOSITION FOR THE $D_H SCASE$]*

3.2 Welfare Analysis and Numerical Characterization of Regimes

Which policy (among the regime-specific optima) is globally optimal? In this section, we first provide a qualitative answer to this question, then we characterize the optimality of alternative regime by means of numerical specifications of our general setting.

3.2.1 Numerical Characterization of Regimes

This section shows the results of our propositions and how parameters affect these results. In particular, we study the role played by some important parameters in determining different optimum tax schedules.

Following Heatcote *et al.* (2009), we use a separable utility function:

$$U(c, l) = \frac{c^{1-\rho} - 1}{1-\rho} - \frac{l^{1+k}}{1+k}$$

The parameter ρ represents the coefficient of relative risk-aversion (as baseline case, we take $\rho = 3$) and $\frac{1}{k}$ is the Frisch elasticity of labor supply. In particular, we set $k = 2$ in order to account for the standard compensated elasticity of labor estimated in literature (0.33). Therefore, we can rewrite the utility function as follows:

$$U(c,h) = \frac{(x - q^m)^{1-\rho} - 1}{1-\rho} - \frac{\left(\frac{y}{w(\theta, q^f + q^m)}\right)^{1+k}}{1+k}$$

Since this functional form is quite common in the microeconomic literature that estimates consumption and labor elasticity, the main advantage of this specification is the facility to calibrate it.

Consistently with the literature on Mincer's earning functions, we use the following Cobb-Douglas specification

$$w(\theta, q^f + q^m) = \theta^\alpha (q^f + q^m)^\beta$$

Using a log transformation, it is easy to see that α and β represent the estimated coefficients of a Mincer's regression with regressors θ and q, respectively. However, we also consider a linear specification for the earning function in order to show the validity of our propositions, when θ and q are substitutes.

Starting from a benchmark parametrization, we want to show how our parameters affect the optimal policy.

Table 1 reports, the value we used as initial case.

Table 1: Benchmark Case

Parameter	Value
θ_0	1
θ_1	3
α	0.5
β	0.5
α	0.2
k	2
ρ	3

Here, we assume that θ and q have the same weight. This assumption is in line with Heckman *et al.* (2006), where the authors show that inherited abilities and family background as much important as schooling in the wage determination processes. However, in the second part of this section, we will use country-specific coefficients to study different, real economic systems. Starting from Table 1, the optimal policy refers to case 1 of our discriminatory regime. We summarize the optimal policy in Table 2

Table 2: Optimal Policies (Discriminatory Case 1)

Instruments	Benchmark	$\alpha = 0.4$, $\beta = 0.6$	$k = 6$	$\theta_1 = 5$	$\lambda = 0.55$
y_0	0.380	0.229	0.562	0.099	1.176
x_0	0.614	0.571	0.605	0.724	1.481
y_1	2.898	3.810	2.270	3.872	1.823
x_1	2.320	3.281	1.801	3.168	0.701
q^f	0.155	0.050	0.145	0.030	0.235
q^s	0.035	0.087	0.136	0.020	0.014
Welfare	-0.960	-1.124	-0.975	-0.470	-3.833

Notice that, when policy is optimal, both the public provision and taxation, are used as redistribution devices.

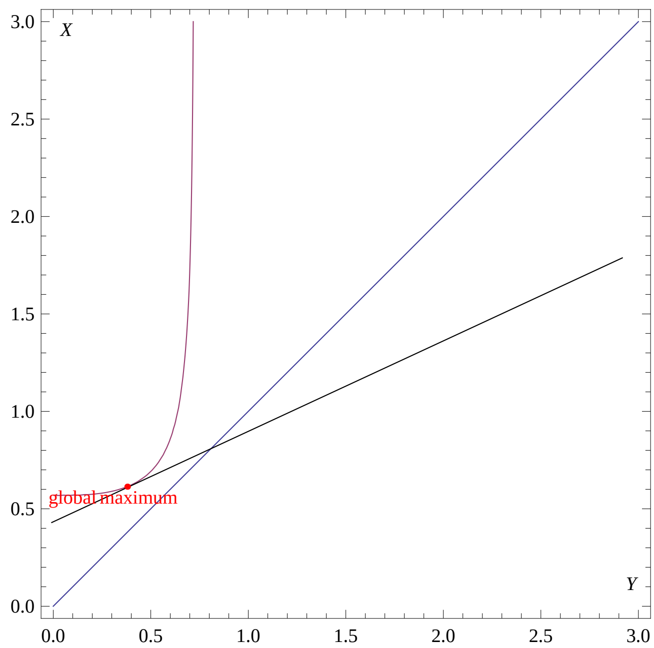

Figure 1: Low-ability type

In the previous analysis, θ and q are economic complements and then we want to show that the optimal schedule does not distort the high-ability type. Figures 1 and 2 show that the optimal policy implies a marginal taxation for the low-ability workers and no distortion for the high-ability one, respectively. Indeed, the slope of the high-ability individual's indifference curve at the global maximum point is 1, where the 45 degree lines are in blue.

The first part of Proposition 1 implies that, in case of substitutability between θ and q, we may have $q^f = 0$, when $q^s(x_1, y_1, q^f, \theta_1) > q^{s*}$. To replicate this result, we use the following wage function: $w(\theta, q^f + q^m) = \alpha\theta + \beta(q^f + q^m)$. Using the baseline setting proposed in Table 1, we obtain the following result $\{y_0, x_0, y_1, x_1, q^f, q^s\} = \{0.364, 0.791, 1.721, 1.294, 0, 0.0002\}$ with a welfare level equal to -0.365. The low level of q^s is due to the low level of $q^s(x_1, y_1, q^f, \theta_1) = 0.0006$.

The case in which θ and q are economic substitutes and the optimal policy implies the use of both pillars can be found assuming $\theta_1 = 2$. The optimal policy is described by

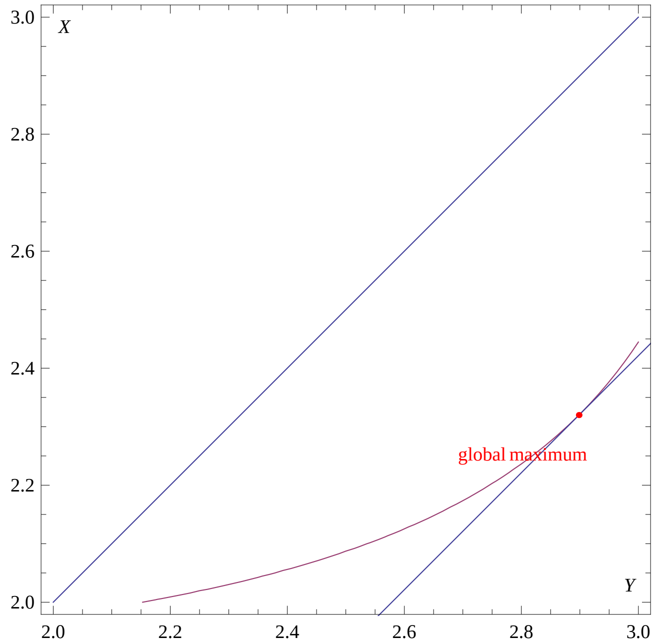

Figure 2: High-ability type

the following vector $\{y_0, x_0, y_1, x_1, q^f, q^s\} = \{0.512, 0.699, 1.144, 0.917, 0.001, 0.039\}$ with a welfare level equal to -0.703. Figures 3 and 4 provide a graphical representation of this optimal policy in the space (y, x), for the low-ability type and for the high-ability type, respectively. With respect to Figure 2, now, Figure 4 shows that the optimal taxation schedule distorts the high-ability supply with a marginal subsidy.

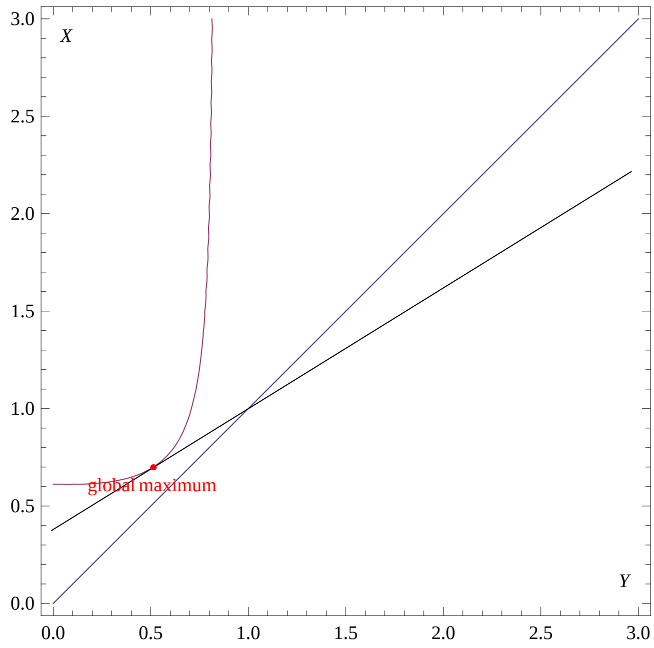

Figure 3: Low-ability type

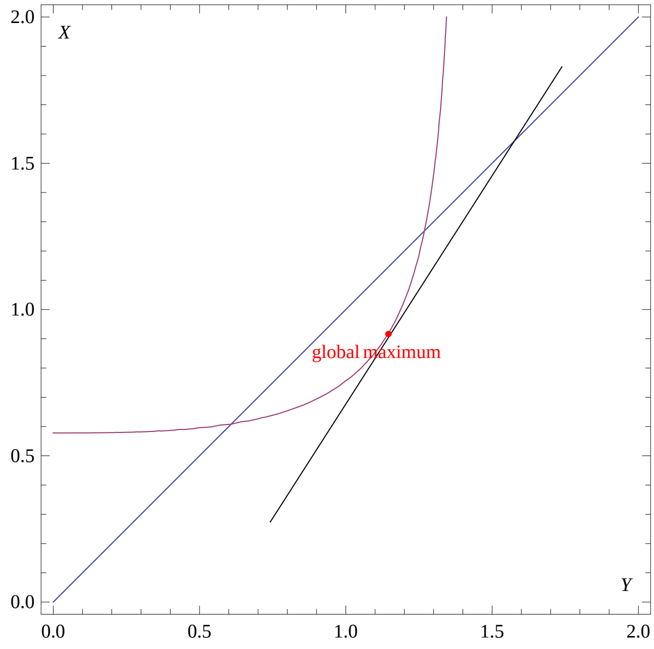

Figure 4: High-ability type

[HERE NUMERICAL ANALYSIS: WELFARE COMPARISON OF POLICY REGIMES]

4 Extension: Multi-class Economy

[HERE ANALYSIS OF THE CASE WHERE THE EXOGENOUS ability IS A CONTINUOUS VARIABLE ON A FINITE SUPPORT]

5 Conclusions

[INCOMPLETE]

1 Technical Appendix

1.1 Household's input demand

Let us consider the behavior of households opting out of the second pillar. (To save space we omit household's index, i). Taking as given the government's tax policy and the first-pillar public provision of investment (q^f) as well as the generic household's choice about labor supply (hence, the net - x - and gross - y - incomes), the optimal private investment of the household is determined by the program

$$\max_{q^m} U(x - q^m, \frac{y}{w(\theta, q^f + q^m)}) \quad s.t. \quad q^m \geq 0$$

The first order condition is

$$\frac{dU}{dq^m} = \phi = -U_c - U_l \cdot \frac{y}{w^2} \cdot w_q = 0 \tag{6}$$

By concavity of $U(.,.)$ (hence, $U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 < 0$) and $w(.,.)$ (hence, $w_{qq} \leq 0$), the second order condition with respect to q^m is satisfied

$$\begin{aligned} \frac{d^2U}{dq^{m2}} = \phi_{q^m} = U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 + \\ + U_l \cdot (\frac{2 \cdot y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}) < 0 \end{aligned} \tag{7}$$

1.1.1 Comparative statics

We now characterize the shape of $q^m(x, y, q^f, \theta)$. Applying the implicit function theorem to the first order condition with respect to q^m, we know that

$$\frac{dq^m}{dz} = -\frac{\phi_z}{\phi_{q^m}}$$

where $z \in \{x, y, q^f, \theta\}$. By weak concavity of $w(.,.)$ in q, we know that $\phi_{q^m} < 0$, thus the effect of z on $q^m(x, y, q^f, \theta)$ depends on the differential of the first order condition, ϕ_z. We summarize all results in the following Lemmas.

Lemma 3 *Private input demand increases in net income less than one-to-one:* $\frac{dq^m}{dx} \in (0, 1)$.

Proof. $\frac{dq^m}{dx} > 0$ if and only if $\phi_x = -U_{cc} - U_{lc} \cdot \frac{y}{w^2} \cdot w_q > 0$. By (6), $-\frac{U_c}{U_l} = \frac{y}{w^2} \cdot w_q$, thus $\phi_x = -U_{cc} + \frac{U_c}{U_l} \cdot U_{lc} > 0$ if and only if leisure is a normal good. Moreover, $\frac{dq^m}{dx} < 1$ if $\phi_x < -\phi_{q^m}$ or $(-U_{cl} + \frac{U_c}{U_l} \cdot U_{ll}) \cdot \frac{y}{w^2} \cdot w_q - U_l \cdot (2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}) > 0$, that is satisfied if consumption is a normal good (hence, $-U_{cl} + \frac{U_c}{U_l} \cdot U_{ll} > 0$). ■

Lemma 4 *Private input demand increases in gross income less than the marginal rate of substitution between net and gross income:* $\frac{dq^m}{dy} \in (0, \frac{dx}{dy} |_{V(x,y,q^f,\theta)})$.

Proof. $\frac{dq^m}{dy} > 0$ if $\phi_y = -U_{cl} \cdot \frac{1}{w} - U_{ll} \cdot \frac{y}{w^3} \cdot w_q - U_l \cdot \frac{w_q}{w^2} > 0$. By (6), $-\frac{U_c}{U_l} = \frac{y}{w^2} \cdot w_q$, thus $\phi_y = \frac{1}{w} \cdot \left(-U_{cl} + \frac{U_c}{U_l} \cdot U_{ll} - U_l \cdot \frac{w_q}{w}\right) > 0$ that is true if consumption is a normal good (hence, $-U_{cl} + \frac{U_c}{U_l} \cdot U_{ll} > 0$). Moreover, $\frac{dq^m}{dy} = \frac{w}{y \cdot w_q} \cdot \alpha_y$ or - by the first order conditions for q^m and consumption-labor choices - $\frac{dq^m}{dy} = \frac{dx}{dy} |_{V(x,y,q^f,\theta)} \cdot \alpha_y$, where

$$\alpha_y = \frac{U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 + U_l \cdot \frac{y}{w^3} \cdot w_q^2}{U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq})} < 1$$

if $U_{cc} + U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_l \cdot (\frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}) < 0$, that - by the first order condition - is satisfied if leisure is a normal good (hence, $U_{cc} - \frac{U_c}{U_l} \cdot U_{cl} < 0$) and $w(.,.)$ is concave in q. ■

Lemma 5 *The first-pillar public provision crowds partially out private input demand:* $\frac{dq^m}{dq^f} \in (-1, 0)$.

Proof. $\frac{dq^m}{dq^f} < 0$ if $\phi_{q^f} = \frac{y}{w^2} \cdot w_q \cdot (U_{cl} + \frac{y}{w^2} \cdot w_q \cdot U_{ll}) + U_l \cdot \left(2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}\right) < 0$. By (6), $\phi_{q^f} = \frac{y}{w^2} \cdot w_q \cdot (U_{cl} - \frac{U_c}{U_l} \cdot U_{ll}) + U_l \cdot \left(2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}\right) < 0$, that is true if consumption is a normal good (hence, $U_{cl} - \frac{U_c}{U_l} \cdot U_{ll} < 0$). Moreover, $\frac{dq^m}{dq^f} > -1$ if $\phi_{q^f} > \phi_{q^m}$ or $U_{cc} + \frac{y}{w^2} \cdot w_q \cdot U_{cl} < 0$, that is satisfied if leisure is a normal good (hence, $U_{cc} - \frac{U_c}{U_l} \cdot U_{cl} < 0$). ■

Let us remark that, for any given level of household's wage (e.g., w'), the marginal rate of technical substitution between input and ability that keeps constant wage is given by $\frac{dq}{d\theta} = -\frac{w_\theta}{w_q} |_{w(\theta,q)=w'} < 0$.

Lemma 6 *Private input is an economic*

- *complement of ability (i.e., $\frac{dq^m}{d\theta} > 0$), if input and ability are strong technologic complements (i.e., $w_{q\theta} > \bar{w}_{q\theta} > 0$);*
- *strong substitute for ability (i.e., $\frac{dq^m}{d\theta} < -\frac{w_\theta}{w_q}$), if input and ability are strong technologic substitutes (i.e., $w_{q\theta} < \underline{w}_{q\theta} < 0$);*
- *substitute for ability (i.e., $\frac{dq^m}{d\theta} \in (-\frac{w_\theta}{w_q}, 0)$), if ability and input are weak technologic complements or substitutes (i.e., $w_{q\theta} \in (\underline{w}_{q\theta}, \bar{w}_{q\theta})$).*

Proof.

$$\phi_\theta = \frac{y}{w^2} \cdot w_\theta \cdot (U_{cl} + \frac{y}{w^2} \cdot w_q \cdot U_{ll}) + U_l \cdot \left(2 \cdot \frac{y}{w^3} \cdot w_q \cdot w_\theta - \frac{y}{w^2} \cdot w_{q\theta}\right) \tag{8}$$

By (6), $U_{cl} + \frac{y}{w^2} \cdot w_q \cdot U_{ll} = U_{cl} - \frac{U_c}{U_l} \cdot U_{ll} < 0$ if consumption is normal. (8) is negative (or positive) if and only if $w_{q\theta} < \bar{w}_{q\theta}$ (or $w_{q\theta} > \bar{w}_{q\theta}$), where

$$\bar{w}_{q\theta} \equiv 2 \cdot \frac{w_q \cdot w_\theta}{w} + \frac{U_{cl} - \frac{U_c}{U_l} \cdot U_{ll}}{U_l} > 0. \tag{9}$$

Moreover, $\frac{dq^m}{d\theta} = -\frac{w_\theta}{w_q} \cdot \alpha_\theta$, where

$$\alpha_\theta \equiv \frac{U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot \frac{w_q}{w_\theta} \cdot w_{q\theta})}{U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq})} < 1$$

if $U_{cc} + U_l \cdot \frac{y}{w^2} \cdot (\frac{w_q}{w_\theta} \cdot w_{q\theta} - w_{qq}) < 0$ or $w_{q\theta} > \underline{w}_{q\theta}$ where

$$\underline{w}_{q\theta} = \frac{w_\theta}{w_q} \cdot (-\frac{w^2}{y} \cdot \frac{U_{cc}}{U_l} + w_{qq}) < 0. \tag{10}$$

■

1.1.2 Single Crossing Property

The effect of θ on the marginal rate of substitution between net and gross income depends, in this setting, also on the reaction of q^m to such a change (Boadway and Marchand, 1995).

Lemma 7 *The single crossing property*

$$\frac{d\frac{dx}{dy}\mid_{V(x,y,q^f,\theta)}}{d\theta} = \frac{\partial\frac{dx}{dy}\mid_{V(x,y,q^f,\theta)}}{\partial\theta} + \frac{\partial\frac{dx}{dy}\mid_{V(x,y,q^f,\theta)}}{\partial q} \cdot \frac{dq}{d\theta} < 0 \tag{11}$$

is satisfied if and only if $w_{q\theta} > w^*_{q\theta}$. *Moreover, the single crossing property is violated whenever* $w_{q\theta} < \underline{w}_{q\theta}$.

Proof. Remark that

$$\frac{\partial\frac{dx}{dy}\mid_{V(\theta)}}{\partial\theta} = -\frac{w_\theta}{w_q} \cdot \frac{U_l}{U_c^2 \cdot w} \cdot (-\frac{U_c}{U_l} \cdot U_{cl} + \frac{U_c^2}{U_l^2} \cdot U_{ll} - U_c \cdot \frac{w_q}{w}) < 0$$

and

$$\frac{\partial\frac{dx}{dy}\mid_{V(\theta)}}{\partial q} = -\frac{U_l}{U_c^2 \cdot w} \cdot (U_{cc} - 2 \cdot \frac{U_c}{U_l} \cdot U_{cl} + \frac{U_c^2}{U_l^2} \cdot U_{ll} - U_c \cdot \frac{w_q}{w}) < 0$$

then, (11) is satisfied if and only if

$$\frac{dq}{d\theta} > -\frac{\frac{\partial\frac{dx}{dy}\mid_{V(\theta)}}{\partial\theta}}{\frac{\partial\frac{dx}{dy}\mid_{V(\theta)}}{\partial q}}$$

that is equivalently written as $\beta > \alpha_\theta$, and boils down to

$$w_{q\theta} > w^*_{q\theta}$$

where

$$w^*_{q\theta} \equiv \frac{w_q \cdot w_\theta}{w} - \beta \cdot (\frac{w_q}{w} - w_{qq})$$

and

$$\beta \equiv \frac{-\frac{U_c}{U_l} \cdot U_{cl} + \frac{U_c^2}{U_l^2} \cdot U_{ll} - U_c \cdot \frac{w_q}{w}}{U_{cc} - 2 \cdot \frac{U_c}{U_l} \cdot U_{cl} + \frac{U_c^2}{U_l^2} \cdot U_{ll} - U_c \cdot \frac{w_q}{w}} \in (0,1)$$

under the assumption that c is normal and $U(.,.)$ is strictly concave. Remark that when $w_{q\theta} < \underline{w}_{q\theta}$, then $\alpha_\theta \geq 1$, hence $\beta > \alpha_\theta$ - and (11) - is violated. ■

1.2 Minimum second-pillar provision

We analyze the structure of the minimum second-pillar provision inducing the household with ability θ to opt in. We first consider the case of a general tax schedule (i.e., depending on opting-in/out choice), then the case of a restricted tax schedule.

1.2.1 General Tax Schedule

Given all policy variables and the ability level, the minimum second-pillar provision inducing the considered household to opt in can be written as

$$q^s(x^s, y^s, x^m, y^m, q^f, \theta) \equiv \{q' \in \mathcal{R}_+ \mid U(x^s, \frac{y^s}{w(\theta, q^f + q')}) = V(x^m, y^m, q^f, \theta)\}$$

We can characterize the shape of $q^s(x^s, y^s, x^m, y^m, q^f, \theta)$ applying the implicit function theorem. In particular,

$$\frac{dq^s}{dz} = -\frac{\frac{dU(x^s, \frac{y^s}{w^s})}{dz} - \frac{dV(x^m, y^m, q^f, \theta)}{dz}}{\frac{dU(x^s, \frac{y^s}{w(\theta, q^f + q')})}{dq^s}}$$

for any $z \in \{x^s, y^s, x^m, y^m, q^f, \theta\}$. Therefore,

$$\frac{dq^s}{dx^s} = \frac{U_c^s}{U_l^s \cdot \frac{y^s}{w^{s2} \cdot w_q^s}} = -\frac{\frac{w^s}{y^s \cdot w_q^s}}{\frac{dx}{dy} \mid_{U(x^s, \frac{y^s}{w^s})}} < 0$$

where $U_c^s \equiv \frac{\partial U(x^s, \frac{y^s}{w^s})}{\partial c}$, $U_l^s \equiv \frac{\partial U(x^s, \frac{y^s}{w^s})}{\partial l}$, and $\frac{dx}{dy} \mid_{U(x^s, \frac{y^s}{w^s})} \equiv -\frac{U_l^s}{U_c^s \cdot w^s}$.

$$\frac{dq^s}{dy^s} = \frac{w^s}{y^s \cdot w_q^s} > 0$$

$$\frac{dq^s}{dx^m} = -\frac{U_c^m}{U_l^s \cdot \frac{y^s}{w^{s2} \cdot w_q^s}} = \frac{U_c^m}{U_c^s} \cdot \frac{\frac{w^s}{y^s \cdot w_q^s}}{\frac{dx}{dy} \mid_{U(x^s, \frac{y^s}{w^s})}} > 0$$

where $U_c^m \equiv V_{x^m}$.

$$\frac{dq^s}{dy^m} = -\frac{\frac{U_l^m}{w^m}}{U_l^s \cdot \frac{y^s}{w^{s2} \cdot w_q^s}} = -\frac{\frac{U_l^m}{w^m}}{\frac{U_l^s}{w^s}} \cdot \frac{w^s}{y^s \cdot w_q^s} < 0$$

where $\frac{U_l^m}{w^m} \equiv V_{y^m}$. The effect of q^f and θ on q^s is ambiguous and depends on the structure of the tax schedule for individuals opting in or out of the second-pillar scheme.

$$\frac{dq^s}{dq^f} = -(1 - \eta \cdot \frac{\frac{w_q^m}{w^m}}{\frac{w_q^s}{w^s}})$$

where $\eta \equiv \frac{U_l^m \cdot \frac{y^m}{w^m}}{U_l^s \cdot \frac{y^s}{w^s}}$;

$$\frac{dq^s}{d\theta} = -\frac{w_\theta^s}{w_q^s} \cdot (1 - \eta \cdot \frac{\frac{w_\theta^m}{w^m}}{\frac{w_\theta^s}{w^s}})$$

1.2.2 Restricted Tax Schedule

In this case, the minimum second-pillar provision becomes

$$q^s(x, y, q^f, \theta) \equiv \{q' \in \mathcal{R}_+ \mid U(x, \frac{y}{w(\theta, q^f + q')}) = V(x, y, q^f, \theta)\} \tag{12}$$

Let us remark that, in this particular case, as far as $q^m(x,y,q^f,\theta) > 0$, $U_c^s < U_c^m$, $|U_l^s| > |U_l^m|$, and $q^m(x,y,q^f,\theta) > q^s(x,y,q^f,\theta) > 0$ (hence, $w^m > w^s$. Moreover, in this case, changes in net and gross incomes affect both sides of (12). Thus,

$$\frac{dq^s}{dx} = (\frac{U_c^m}{U_c^s} - 1) \cdot \frac{\frac{w^s}{y \cdot w_q^s}}{\frac{dx}{dy}\mid_{U(x^s,\frac{y}{w^s})}} > 0$$

and

$$\frac{dq^s}{dy} = \frac{w^s}{y \cdot w_q^s} \cdot (1 - -\frac{\frac{U_l^m}{w^m}}{\frac{U_l^s}{w^s}}) > 0$$

In this case, $\eta < 1$, thus

$$\frac{dq^s}{dq^f} = -(1 - \eta \cdot \frac{\frac{w_q^m}{w^m}}{\frac{w_q^s}{w^s}}) \in (-1,0)$$

Given η, the sign of

$$\frac{dq^s}{d\theta} = -\frac{w_\theta^s}{w_q^s} \cdot (1 - \eta \cdot \frac{\frac{w_\theta^m}{w^m}}{\frac{w_\theta^s}{w^s}})$$

depends on the technical complementarity/substitutability between q and θ, namely $\frac{dq^s}{d\theta} < 0$ if and only if $1 - \eta \cdot \frac{\frac{w_\theta^m}{w^m}}{\frac{w_\theta^s}{w^s}} > 0$ or

$$\frac{w_\theta^s}{w^s} - \eta \cdot \frac{w_\theta^m}{w^m} = \frac{w_\theta^m}{w^m} \cdot (1-\eta) - \int_{q^s}^{q^m} (\frac{w_{q\theta}}{w} - \frac{w_q \cdot w_\theta}{w^2}) \cdot dq > 0$$

Thus, given η, a sufficient condition for $\frac{dq^s}{d\theta} < 0$ (or $\frac{dq^s}{d\theta} > 0$) is that $w_{q\theta} < \tilde{w}_{q\theta}$ (or $w_{q\theta} > \tilde{w}_{q\theta}$), where

$$\tilde{w}_{q\theta} \equiv \frac{w_q \cdot w_\theta}{w} + \frac{w_\theta^m}{w^m} \cdot \frac{1-\eta}{q^m - q^s} \cdot w > 0.$$

1.2.3 Single Crossing Property for opting-in households

Opting-in households do not privately demand any input, thus only the direct effect of θ on the marginal rate of substitution between net and gross income is relevant; hence

$$\frac{d\frac{dx}{dy}\mid_{U(x,\frac{y}{w(\theta,q^f,q^s)})}}{d\theta} = \frac{\partial\frac{dx}{dy}\mid_{U(x,\frac{y}{w(\theta,q^f,q^s)})}}{\partial\theta} < 0$$

as shown in Lemma 7.

1.3 Incentive Compatibility of First Best Allocations

Given individual optimization of private input investment, first best allocations can be characterized by the maximization of a social welfare function under the feasibility constraint (amounting to government's budget constraint):

$$\max_{x(0),y(0),x(1),y(1)} \quad \lambda V(x(0),y(0),0) + (1-\lambda)\cdot V(x(1),y(1),1)$$
$$s.t. \quad \lambda\cdot(y(0)-x(0)) + (1-\lambda)\cdot(y(1)-x(1)) \geq 0 \quad (\mu)$$

by the first order conditions, the following optimization conditions arise

$$\frac{dx}{dy}\mid_{V(x,y,0,\theta)}=1$$

(and, by individual optimization, also $\frac{w}{y\cdot w_q}=1$), for all $\theta\in\{0,1\}$.

Lemma 8 *The first best allocation is incentive-compatible for low-ability households.*

Proof. Let T be the first best income transfer received by low-ability households, thus by government budget constraint, $x_0 = y_0 + T$ and $x_1 = y_1 - \frac{\lambda}{1-\lambda}\cdot T$. The first best allocation is incentive compatible for low-ability individuals if

$$U(y_0+T-q(y_0+T,y_0,0,0),\frac{y_0}{w(0,q(y_0+T,y_0,0,0))}) \geq \tag{13}$$
$$U(y_1-\frac{\lambda}{1-\lambda}\cdot T-q(y_1-\frac{\lambda}{1-\lambda}\cdot T,y_1,0,0),\frac{y_1}{w(0,q(y_1-\frac{\lambda}{1-\lambda}\cdot T,y_1,0,0))})$$

Given that $q(x,y,0,0)$ is the optimal quantity of input demanded by low-ability households under $\{x,y\}$ - hence, the first order condition (1) - is always satisfied - we apply the envelope theorem and write (13) as

$$\int_{y_0}^{y_1} \hat{U}_c(T) \cdot (1 + \frac{\hat{U}_l(T)}{\hat{U}_c(T) \cdot \hat{w}(T)}) \cdot dy - \int_{-\frac{\lambda}{1-\lambda} \cdot T}^{T} \hat{U}_c(y_1) \cdot d\tau \leq 0 \tag{14}$$

where

$$\begin{aligned}
\hat{U}_c(T) &\equiv \frac{\partial}{\partial c} U(y + T - q(y + T, y, 0, 0), \frac{y}{w(0, q(y + T, y, 0, 0))}) \\
\hat{U}_l(T) &\equiv \frac{\partial}{\partial l} U(y + T - q(y + T, y, 0, 0), \frac{y}{w(0, q(y + T, y, 0, 0))}) \\
\hat{w}(T) &\equiv w(0, q(y + T, y, 0, 0)) \\
\hat{U}_c(y_1) &\equiv \frac{\partial}{\partial c} U(y_1 + \tau - q(y_1 + \tau, y, 0, 0), \frac{y_1}{w(0, q(y_1 + \tau, y_1, 0, 0))})
\end{aligned}$$

for all $y \in [y_0, y_1]$ and all $\tau \in [-\frac{\lambda}{1-\lambda} \cdot T, T]$. By the SCP, if $y_0 < y_1$ (or $y_0 > y_1$) $1 + \frac{\hat{U}_l(T)}{\hat{U}_c(T) \cdot \hat{w}(T)} < 0$ (or $1 + \frac{\hat{U}_l(T)}{\hat{U}_c(T) \cdot \hat{w}(T)} > 0$), thus (14) is always satisfied with strict inequality. ■

As regards the first best tax schedule for households with high exogenous ability, we cannot conclude anything, given that - by the same kind of arguments used in Lemma 8:

$$\begin{aligned}
U(y_1 - \frac{\lambda}{1-\lambda} \cdot T - q(y_1 - \frac{\lambda}{1-\lambda} \cdot T, y_1, 0, 1), \frac{y_1}{w(1, q(y_1 - \frac{\lambda}{1-\lambda} \cdot T, y_1, 0, 1))}) + \\
-U(y_0 + T - q(y_0 + T, y_0, 0, 1), \frac{y_0}{w(1, q(y_0 + T, y_0, 0, 1))}) = \\
\underbrace{\int_{y_0}^{y_1} \hat{U}_c(-T) \cdot (1 + \frac{\hat{U}_l(-T)}{\hat{U}_c(-T) \cdot \hat{w}(-T)}) \cdot dy}_{>0} - \underbrace{\int_{-\frac{\lambda}{1-\lambda} \cdot T}^{T} \hat{U}_c(y_1) \cdot d\tau}_{>0}
\end{aligned}$$

In our analysis, we assume that the tax schedule for high-ability households is incentive incompatible. The same arguments hold also when we consider the public provision of private input. In the inclusive case, the SCP holds and all the above arguments apply, given that individuals do not control q any more. In the discriminating case, the low-ability households opting for the second-pillar provision receive at least the opt-out utility, thus the incentive constraint is satisfied. Also high-ability households opting for the second pillar provision receive at least their opting-out utility, which may reduce the incentive problem characterizing second best redistribution.

1.4 Optimal Tax and Public Provision

In the following the optimization problems under different policy regimes are considered.

1.4.1 Pure Taxation Regime

The government's program is

$$\max_{\{x_0, y_0, x_1, y_1\}} \lambda \cdot V(x_0, y_0, q^f, 0) + (1-\lambda) \cdot V(x_1, y_1, q^f, 1)$$
$$s.t.$$
$$\lambda \cdot (y_0 - x_0) + (1-\lambda) \cdot (y_1 - x_1) \geq 0$$
$$V(x_1, y_1, q^f, 1) \geq V(x_0, y_0, q^f, 1)$$

Thus, the Lagrangian is

$$\mathcal{L} = \lambda \cdot V(x_0, y_0, q^f, 0) + (1-\lambda) \cdot V(x_1, y_1, q^f, 1) + \mu_1 \cdot [\lambda \cdot (y_0 - x_0) + (1-\lambda) \cdot (y_1 - x_1)] + \\ + \mu_2 \cdot (V(x_1, y_1, q^f, 1) - V(x_0, y_0, q^f, 1))$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_0} = \lambda \cdot V_{x_0} - \lambda \cdot \mu_1 - \mu_2 \cdot \widehat{V}_{x_0} = 0 \tag{15}$$
$$\frac{\partial \mathcal{L}}{\partial y_0} = \lambda \cdot V_{y_0} + \lambda \cdot \mu_1 - \mu_2 \cdot \widehat{V}_{y_0} = 0 \tag{16}$$
$$\frac{\partial \mathcal{L}}{\partial x_1} = (1-\lambda) \cdot V_{x_1} - (1-\lambda) \cdot \mu_1 + \mu_2 \cdot V_{x_1} = 0 \tag{17}$$
$$\frac{\partial \mathcal{L}}{\partial y_1} = (1-\lambda) \cdot V_{y_1} + (1-\lambda) \cdot \mu_1 + \mu_2 \cdot V_{y_1} = 0 \tag{18}$$

By (17) and (18), $-\frac{V_{y_1}}{V_{x_1}} = \frac{dx}{dy} \mid_{V(x_1,y_1,0,1)} = 1$. By (15) and (16), after some algebra we obtain

$$\frac{dx}{dy} \mid_{V(x_0,y_0,0,0)} = 1 - \frac{\frac{\mu_2}{\lambda} \cdot \frac{\widehat{V}_{x_0}}{V_{x_0}}}{1 - \frac{\mu_2}{\lambda} \cdot \frac{\widehat{V}_{x_0}}{V_{x_0}}} \cdot \left(\frac{dx}{dy} \mid_{V(x_0,y_0,0,0)} - \frac{dx}{dy} \mid_{\widehat{V}(x_0,y_0,0,1)}\right)$$

where $\frac{dx}{dy} \mid_{V(x_0,y_0,0,0)} \equiv -\frac{V_{y_0}}{V_{x_0}}$ and $\frac{dx}{dy} \mid_{V(x_0,y_0,0,1)} \equiv -\frac{\widehat{V}_{y_0}}{\widehat{V}_{x_0}}$. By (15), $1 - \frac{\mu_2}{\lambda} \cdot \frac{\widehat{V}_{x_0}}{V_{x_0}} > 0$, and by the SCP, $\frac{dx}{dy} \mid_{V(x_0,y_0,0,0)} - \frac{dx}{dy} \mid_{\widehat{V}(x_0,y_0,0,1)} > 0$, hence $\frac{dx}{dy} \mid_{V(x_0,y_0,0,0)} < 1$ (the optimal distortion implies a positive marginal tax on low-ability labor supply).

1.4.2 Inclusive Regime

Let $q = q^f + q^s$ be the total provision of public good. Under this regime the optimization problem is

$$\max_{\{x_0,y_0,x_1,y_1,q\}} \lambda \cdot U(x_0, \frac{y_0}{w(0,q)}) + (1-\lambda) \cdot U(x_1, \frac{y_1}{w(1,q)})$$
$$s.t.$$
$$\lambda \cdot (y_0 - x_0) + (1-\lambda) \cdot (y_1 - x_1) - q \geq 0$$
$$U(x_1, \frac{y_1}{w(1,q)}) \geq U(x_0, \frac{y_0}{w(1,q)})$$
$$q \geq q_{\max}$$

where $q_{\max} \equiv \{q^f + q^s \mid \max\{q^s(x_0, y_0, q^f, 0), q^s(x_1, y_1, q^f, 1)\} \leq q^s\}$.

The Lagrangian is

$$\begin{aligned} \mathcal{L} = \lambda \cdot U(x_0, \frac{y_0}{w(0,q)}) + (1-\lambda) \cdot U(x_1, \frac{y_1}{w(1,q)}) + \\ +\mu_1 \cdot [\lambda \cdot (y_0 - x_0) + (1-\lambda) \cdot (y_1 - x_1) - q] \\ +\mu_2 \cdot \left(U(x_1, \frac{y_1}{w(0,q)}) - U(x_0, \frac{y_0}{w(1,q)})\right) + \eta_0 \cdot (q - q_{\max}) \end{aligned}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_0} = \lambda \cdot U_{c_0} - \lambda \cdot \mu_1 - \mu_2 \cdot \widehat{U}_{c_0} = 0 \quad (19)$$

$$\frac{\partial \mathcal{L}}{\partial y_0} = \lambda \cdot \frac{U_{l_0}}{w_0} + \lambda \cdot \mu_1 - \mu_2 \cdot \frac{\widehat{U}_{l_0}}{\widehat{w}} = 0 \quad (20)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = (1 - \lambda + \mu_2) \cdot U_{x_1} - (1 - \lambda) \cdot \mu_1 = 0 \quad (21)$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = (1 - \lambda + \mu_2) \cdot \frac{U_{l_1}}{w_1} + (1 - \lambda) \cdot \mu_1 = 0 \quad (22)$$

$$\frac{\partial \mathcal{L}}{\partial q} = -\lambda \cdot U_{l_0} \cdot \frac{y_0}{w_0^2} \cdot w_{0q} - (1 - \lambda + \mu_2) \cdot U_{l_1} \cdot \frac{y_1}{w_1^2} \cdot w_{1q} - \mu_1 + \mu_2 \cdot \widehat{U}_{l_0} \cdot \frac{y_0}{\widehat{w}^2} \cdot \widehat{w}_q + \eta_0 \quad (23)$$

By (21) and (22), $\frac{dx}{dy} \mid_{U(x_1, \frac{y_1}{w_1})} \equiv -\frac{U_{l_1}}{U_{x_1} \cdot w_1} = 1$. By (19) and (20), after some algebra we obtain that $\frac{dx}{dy} \mid_{U(x_0, \frac{y_0}{w_0})} \equiv -\frac{U_{l_0}}{U_{x_0} \cdot w_0} < 1$. Moreover, [HERE A LEMMA ON THE OPTIMAL UNIFORM PROVISION OF INPUT].

1.4.3 Discriminating Regimes

We consider first the case of low-ability in the second pillar and high-ability out. The Lagrangian corresponding to the program (4) is

$$\begin{aligned}
\mathcal{L} = \lambda \cdot U(x_0, \frac{y_0}{w(0, q^f + q^s)}) + (1 - \lambda) \cdot V(x_1, y_1, q^f, 1) \\
+\mu_1 \cdot \left[\lambda \cdot (y_0 - x_0) + (1 - \lambda) \cdot (y_1 - x_1) - q^f - \lambda \cdot q^s\right] \\
+\mu_2 \cdot \left(V(x_1, y_1, q^f, 1) - U(x_0, \frac{y_0}{w(1, q^f + q^s)})\right) \\
+\eta_1 \cdot (q^s(x_1, y_1, q^f, 1) - q^s) + \eta_0 \cdot (q^s - q^s(x_0, y_0, q^f, 0)) \\
+\varphi_0 \cdot q^f + \varphi_1 \cdot q^s(x_1, y_1, q^f, 1)
\end{aligned}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x_0} = \lambda \cdot U_{c_0} - \lambda \cdot \mu_1 - \mu_2 \cdot \widehat{U}_{c_0} - \eta_0 \cdot q^s_{x_0} = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial y_0} = \lambda \cdot \frac{U_{l_0}}{w_0} + \lambda \cdot \mu_1 - \mu_2 \cdot \frac{\widehat{U}_{l_0}}{\widehat{w}} - \eta_0 \cdot q^s_{y_0} = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = (1 - \lambda + \mu_2) \cdot V_{x_1} - (1 - \lambda) \cdot \mu_1 + (\eta_1 + \varphi_1) \cdot q^s_{x_1} = 0 \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = (1 - \lambda + \mu_2) \cdot V_{y_1} + (1 - \lambda) \cdot \mu_1 \cdot (\eta_1 + \varphi_1) \cdot q^s_{y_1} = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial q^f} = (1 - \lambda + \mu_2) \cdot V_{q^f} + \mu_2 \cdot \widehat{U}_{l_0} \cdot \frac{y_0}{\widehat{w}^2} \cdot \widehat{w}_q - \lambda \cdot U_{l_0} \frac{y_0}{w_0^2} \cdot w_{0q} - \mu_1 + \\ +(\eta_1 + \varphi_1) \cdot q^s_{1q^f} - \eta_0 \cdot q^s_{0q^f} + \varphi_0 = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial q^s} = -\lambda \cdot U_{l_0} \cdot \frac{y_0}{w_0^2} \cdot w_{0q} - \mu_1 \cdot \lambda + \mu_2 \cdot \widehat{U}_{l_0} \cdot \frac{y_0}{\widehat{w}^2} \cdot \widehat{w}_q - \eta_1 + \eta_0 = 0 \tag{29}$$

By (26) and (27), we get

$$\frac{dx}{dy} \mid_{V(x_1, y_1, q^f, 1)} = 1 + \frac{\eta_1 + \varphi_1}{1 - \lambda + \mu_2} \cdot \frac{q^s_{x_1} + q^s_{y_1}}{V_{x_1}} \tag{30}$$

where $\frac{dx}{dy} \mid_{V(x_1,y_1,q^f,1)} \equiv -\frac{V_{y_1}}{V_{x_1}}$; thus, high-ability labor income is not distorted at the margin only if $\eta_1 = \varphi_1 = 0$, hence only if $q^s(x_1, y_1, q^f, 1) > q^s \geq 0$; otherwise - when the upper constraint to the second-pillar public provision is binding or the first-pillar public provision is sufficiently high that any second-pillar provision induces households to opt in, the high-ability labor income can be optimally distorted with a marginal subsidy.

By (24) and (25), we get

$$\frac{dx}{dy} \mid_{U(x_0, \frac{y_0}{w_0})} = 1 - \frac{\frac{\mu_2}{\lambda} \cdot \frac{\widehat{U}_{c_0}}{U_{c_0}} \cdot \left(\frac{dx}{dy} \mid_{U(x_0, \frac{y_0}{w_0})} - \frac{dx}{dy} \mid_{\widehat{U}(x_0, \frac{y_0}{\widehat{w}})} \right) + \frac{\eta_0}{\lambda \cdot U_{c_0}} \cdot (q^s_{x_0} + q^s_{y_0})}{1 - \frac{\mu_2}{\lambda} \cdot \frac{\widehat{U}_{c_0}}{U_{c_0}}} \tag{31}$$

where $\frac{dx}{dy} \mid_{U(x_0, \frac{y_0}{w_0})} \equiv -\frac{U_{l_0}}{U_{c_0} \cdot w_0}$ and $\frac{dx}{dy} \mid_{\widehat{U}(x_0, \frac{y_0}{\widehat{w}})} \equiv -\frac{\widehat{U}_{l_0}}{\widehat{U}_{c_0} \cdot \widehat{w}}$; in this case, the low-ability labor income can be taxed with an heavier marginal tax rate if the lower constraint to the second-pillar public provision is binding (when $\eta_0 > 0$, necessarily $q^s = q^s(x_0, y_0, 0, q^f)$).

Vice versa, when $\eta_0 = 0$, we have $q^s > q$

Substituting (26) and (28) in (29), and observing that $V_{x_1} - V_{q^f} = 0$, we get

$$\eta_1 - \eta_0 \cdot (1 + q^s_{0q^f}) + \varphi_0 = (\eta_1 + \varphi_1) \cdot (q^s_{x_1} - q^s_{q^1 f}) \geq 0 \tag{32}$$

if $q^f > 0$, then $\varphi_0 = 0$, consequently $\eta_1 > 0$ and $\eta_0 = 0$. Therefore, when the first-pillar is active ($q^f > 0$, the high-ability households is subsidized at the margin.

Lemma 9 $\varphi_0 > 0$ *(hence,* $\varphi_1 = 0$*) and* $\eta_0 > 0$ *(hence,* $\eta_1 = 0$*) may happen only if* θ *and* q *are strong technologic complements (*$w_{q\theta} > \overline{w}_{q\theta}$*.*

Proof. From (29) we have

$$\mu_1 = U_{c_0} \left(1 - \frac{\mu_2}{\lambda} \cdot \frac{\widehat{U}_{c_0}}{U_{c_0}} \right) - \frac{\eta_0}{\lambda} \cdot q^s_{x_0} > 0$$

hence, necessarily $\alpha \equiv \frac{\mu_2}{\lambda} \cdot \frac{\widehat{U}_{c_0}}{U_{c_0}} \in (0, 1]$. By (28),

$$-\left(1 + \frac{U_{l_0}}{U_{c_0} w_0} \frac{y_0}{w_0} w_q(0) \right) = -\alpha \left(1 + \frac{\widehat{U}_{l_0}}{\widehat{U}_{c_0} \widehat{w}} \frac{y_0}{\widehat{w}} \widehat{w}_q \right) - \frac{\eta_0}{\lambda U_{c_0}} (1 + q^s_{x_0})$$

If θ and q are substitutes, we have $-\left(1 + \frac{U_{l_0}}{U_{c_0} w_0} \frac{y_0}{w_0} w_q(0)\right) > -\left(1 + \frac{\widehat{U}_{l_0}}{\widehat{U}_{c_0} \widehat{w}} \frac{y_0}{\widehat{w}} \widehat{w}_q\right)$. Together with the fact that $\alpha \in [0, 1)$, we can conclude that

$$-\frac{\eta_0}{\lambda U_{c_0}} (1 + q^s_{x_0}) > 0$$

Then, $\eta_0 > 0$ never happens. ($\eta_0 > 0$ is possible only in the case of strong complementarity between θ and q) [CHECK AND COMPLETE] ■

[HERE ANALYSIS OF DISCRIMINATING REGIME D$_H S$]

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