Constrained School Choice: An Experimental Study*

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Abstract: This paper reports about a series of experiment on school choice, based on a previous experiment made by Chen and Snmez (*J. Econ. Th.*, 2006). Our experiment differs from Chen and Snmez's one in that subjects were not allowed to submit a preference list containing all schools. That is, subjects choices was constrained, a feature often present in real-life school choice procedure. Our experimental study shows that due to this constraint (i) subjects's submitted list respect more the true ordering than in the unconstrained case, (ii) there is a strong "safety school" effect, (iii) contrary to the unconstrained case the Gale-Shapley and TTC mechanisms outperform the Boston mechanism in terms of efficiency.

JEL classification: C72, C78, D78, I20

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1 Introduction

A recent paper by Abdulkadiroğlu and Sönmez (2003) has lead to an upsurge of enthusiasm in the use of matching theory for the design and study of school choice mechanisms.[1] Abdulkadiroğlu and Sönmez (2003) discuss critical flaws of some US school districts' procedures to assign children to public schools. They point out that the widely used Boston mechanism has the serious shortcoming that it is not in the parents' best interest to reveal their true preferences. This

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[1] Recent papers include Abdulkadiroğlu (2005), Abdulkadiroğlu, Pathak, and Roth (2005), Abdulkadiroğlu, Pathak, Roth, and Sönmez (2005), Chen and Sönmez (2006), Erdil and Ergin (2007), Ergin and Sönmez (2006), Haeringer and Klijn (2007), Kesten (2005), Kojima (2007), and Pathak and Sönmez (2006).

implies that parents are forced to play a complicated strategic game with high uncertainty and important consequences. Risk aversion may lead them to reverse the orders of the schools in order to guarantee access to a decent school. This will lead to inefficiencies.[2] Using a mechanism design approach, they propose and analyze two alternative student assignment mechanisms that do not have this shortcoming: the Gale-Shapley mechanism and the Top Trading Cycles mechanism. The only experimental study that has been carried out to evaluate the theoretical predictions and to assess the performance of the mechanisms is Chen and Sönmez (2006), henceforth denoted by CS06. Consistent with theory they find a high preference manipulation under the Boston mechanism. Moreover, they find that efficiency under the Boston mechanism is significantly lower than under the other two mechanisms. This gives additional weight to Abdulkadiroğlu and Sönmez's (2003) recommendation to replace the Boston mechanism by either of the other two mechanisms.

There is a seemingly unimportant aspect of the actual mechanisms implemented that we will show to have strong effects on their performance: parents are only asked to reveal their preferences up to a limited number of schools. For instance, in the school district of New York City each year more than 90,000 students are assigned to about 500 school programs through a variant of the Gale-Shapley mechanism (Abdulkadiroğlu, Pathak and Roth, 2005). Parents are asked to elicit a preference list containing a maximum number of schools (currently up to 12).[3] This apparently innocuous restriction is reason for concern.[4] When individuals' choice is constrained by the number of schools they can include in their choice list, one could think that they will only report their most preferred options (until the list is exhausted). Such strategies are known in the two-sided matching literature as *truncation strategies* (or simply, *truncations*) and have been shown to be optimal in certain environments —see Roth and Rothblum (1999) and Ehlers (2004). The risk of using a truncation strategy is that the mechanisms may exhaust the options listed, in which case the individual's possible further preferences will not be considered. We would therefore expect a student to employ a truncation strategy only if he is comfortable and confident enough about which school he will be assigned to. In particular, if a participant fears rejection by his most preferred programs, it can be advantageous not to apply to these programs and use instead its allowed application slots for less preferred programs. Also, it will be a "safe" strategy to include a school that ensures a decent school seat. This is what we will call the "safety school" effect. Therefore, imposing a curb on the length of the submitted lists, though having the merit of "simplifying matters," has the perverse effect of forcing participants not to be truthful, and eventually compel them to adopt strategic behavior. In other words, we are back to the situation of the Boston mechanism where participants were forced to play a complicated admissions game. Participants may adopt strategic behavior because the "quantitative" effect (i.e., participants cannot reveal their complete preference lists) is likely to have a "qualitative"

[2] Moreover, since parents typically do not reveal their true preferences, public administration is hindered to evaluate society's needs and demand for different types of schools in an appropriate way.

[3] In fact, students that are not assigned a seat are asked to elicit a second list containing only schools with *vacant* seats. It is easy to see that also this variant of the Gale-Shapley mechanism distorts the strategy-proofness of the "pure" mechanism.

[4] Obviously, in case the number of acceptable schools for an individual is smaller than the number of slots in the list there would be no problem. However, in case the number of acceptable schools is larger than the number of slots in the list the individuals will have to think out which schools to include in the list. Abdulkadiroğlu, Pathak and Roth (2005) report that about 25% of the students exhaust their list, which suggests that the constraint indeed may be binding for a substantial proportion of students.

effect (i.e., participants may self-select by not declaring their most preferred options).

The goal of our experimental study is to explore the extent of the effects described above when imposing a maximal length of students' submittable preference lists. In particular, we are interested in the rate of different types of preference manipulation and the efficiency loss. To this end, we experimentally study the school choice problem (Abdulkadiroğlu and Sönmez, 2003). In a school choice problem a number of students has to be assigned to a number of schools, each of which has a limited seat capacity. Students have preferences over schools and remaining unassigned and schools have exogenously given priority rankings over students.[5] The school choice model is closely related to the well-known college admissions model of the two-sided matching literature. The key difference between the two models is that, in school choice, school seats are mere "objects," whereas in college admissions, colleges have preferences over students. In college admissions, both sides of the market are strategic agents who can misrepresent their preferences, whereas in school choice, only students are strategic agents and school priorities are exogenously determined.

In the unconstrained setting, the three mechanisms studied in Abdulkadiroğlu and Sönmez (2003) have different theoretical properties. As has been mentioned, the Gale-Shapley mechanism and the Top Trading Cycles mechanism are strategy-proof, but the Boston mechanism is not. The Gale-Shapley mechanism is also stable, i.e., no student ever envies some lower priority student. In fact, it always generates the best stable assignment for the students. However, it is not Pareto efficient. In contrast, the Top Trading Cycles mechanism is Pareto efficient but not stable. The Boston mechanism is Pareto efficient but not strategy-proof (nor stable). As a consequence, the Boston Mechanism is (typically) not Pareto efficient with respect to the true preferences. Haeringer and Klijn (2007) study the three mechanisms in the constrained setting. Obviously, in this setting stategy-proofness is not well-defined since it is typically not possible for parents to submit their full list of preferences. In fact, in general there is no weakly dominant strategy. Therefore, they explore the preference revelation game where students can only declare up to a fixed number of schools to be acceptable. They study the Nash equilibrium outcomes and find rather stringent necessary and sufficient conditions on the priorities to guarantee stability under the Gale-Shapley and Top Trading Cycles mechanisms. This stands in sharp contrast with the Boston mechanism which yields stable Nash equilibrium outcomes, independently of the constraint. Their study assumes a complete information environment. Ergin and Sönmez (2006, Example 4) show that even in the unconstrained environment the results for the Boston mechanism do not carry over to incomplete information environments. It is well-known that incomplete information is a difficult setting for theoretical analysis. Therefore, it seems convenient to complement theoretical studies of complete information environments with experimental studies with incomplete information environments to explore to what extent the results carry over to the incomplete information environment. In our experimental study we recur to CS06's designed and randomized school choice problems. In other words, the only difference between our set-up and CS06's is the constraint on the length of submittable preferences. Our aim is to isolate as much as possible the effect of the restriction on preference revelation.

[5] Very often local or state laws determine the priority rankings. Typically, students who live closer to a school or have siblings attending a school have higher priority to be admitted at the school. In other situations, priority rankings may be determined by one or several entrance exams. Then students who achieve higher test scores in the entrance exam of a school have higher priority for admission at the school than students with lower test scores.

We are interested in testing how the constraint on the submittable preference lists affects

- the rationality of preference revelation under the Gale-Shapley and Top Trading Cycles mechanisms;
- the rate of each of the different types of preference manipulation, as identified by CS06, for each of the three mechanisms;
- the resulting efficiency loss for each of the three mechanisms.

The experimental evidence confirms that individuals do not submit truncations and behave rationally and strategically, most of them exhibiting a "safety school" effect. As a result the performance of the Gale-Shapley and Top Trading Cycles mechanisms is no longer substantially better than the Boston mechanism.

The remainder of the paper is organized as follows. In Section 2, we recall the model of school choice. In Section 3, we describe the three mechanisms. In Section 4, we informally describe the hypotheses of what happens if a constraint is imposed on the length of submittable preference lists. In Section 5, we describe the experimental design. In Section 6, we present the main findings of our experiment and contrast them with CS06's. Finally, Section 7 concludes and comments on possible policy implications of our results.

2 School Choice

A school choice problem (Abdulkadiroğlu and Sönmez, 2003) is defined by a set of schools and a set of students, each of which has to be assigned a seat at not more than one of the schools. Each student is assumed to have strict preferences over the schools and the option of remaining unassigned. Each school is endowed with a strict priority ordering over the students and a fixed capacity of seats. If a student prefers the option of remaining unassigned to being assigned a seat at school, then this school is called *unacceptable* for the student. Otherwise, the school is *acceptable*.

An outcome of a school choice problem is a *matching*, i.e., an assignment of school seats to students such that each student is assigned at most one seat and each school receives no more students than its capacity. A student is *unassigned*, or *assigned to himself*, at a matching if he is not assigned any seat. Since in the context of school choice only the welfare of the students matters, a matching is *Pareto efficient* if there is no matching that assigns each student a weakly better school and at least one student a strictly better school. (With a slight abuse of terminology, here a "school" can also mean remaining unassigned.) A matching is *stable*

- it is *individually rational*, i.e., each student is unassigned or assigned a seat at some acceptable school;
- it is *non wasteful* (Balinski and Sönmez, 1999), i.e., no student prefers a vacant seat to his assigned seat (this latter possibly being unassigned); and
- there is no *justified envy*, i.e., there is no student-school pair (i, s) such that
  (a) i prefers s to his assigned seat (this latter possibly being unassigned) and
  (b) i has higher priority at s than some other student who is assigned a seat at s.

A (student assignment) *mechanism* systematically selects a matching for each school choice problem. A mechanism is stable if it always selects a stable matching. Similarly, one can speak of a Pareto-efficient mechanism. Finally, a mechanism is *strategy-proof* if no student can ever benefit by unilaterally misrepresenting his preferences.

3 Three Competing Mechanisms

In this section we describe the mechanisms that we study in the context of constrained school choice: the Boston, Gale-Shapley mechanism and Top Trading Cycles mechanisms. In our description we distinguish between the students' and the schools' point of view. The reason is that in the eventual computations the three mechanisms are the "same" from the students' point of view. In fact, the three mechanisms only differ in the way a student is "rejected" by a school.

The Boston (BOS), Gale-Shapley (GS) and Top Trading Cycles (TTC) mechanisms:

Step 1: For each school, a priority ordering of students is determined (based on state and local laws/policies, etc.).

Step 2: Each student submits a preference ranking of the schools.

Step 3: The assignment of seats is obtained in several rounds.

ROUND k, $k \geq 1$ [STUDENTS]: Each student that has not been removed yet but is rejected in the previous round[6] points to the next highest ranked school in his submitted list that has not been removed yet (if there is no such school then the student points to himself).

ROUND k, $k \geq 1$ [SCHOOLS, **Boston**]: Each school assigns seats one at a time to the students that point to it following its priority order. If the school capacity is or was attained, the school rejects any remaining students that point to it. If a student points to himself, he is assigned to himself. Any student that is assigned is removed.
The Boston mechanism terminates when all students have been removed.

ROUND k, $k \geq 1$ [SCHOOLS, **Gale-Shapley**]: Each school tentatively assigns seats one at a time to the students that point to it following its priority order. When the school capacity is attained the school rejects any remaining students that point to it. If a student points to himself, he is tentatively assigned to himself.
The Gale-Shapley mechanism terminates when no student is rejected. The tentative matching becomes final.

ROUND k, $k \geq 1$ [SCHOOLS, **Top Trading Cycles**]: Each school that has not been removed yet points to the student with highest priority among the students that have not been removed yet. There is at least one cycle. If a student is in a cycle he is assigned a seat at the school he points to (or to himself if he is in a self-cycle). Students that are assigned a seat are removed. If a school is in a cycle then its number of vacant seats is decreased by 1. If a school has no longer vacant seats then it is removed.
The Top Trading Cycles mechanism terminates when all students or schools have been removed.

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[6] In order to incorporate the initial step correctly, we use the convention that at "Round 0" all students are rejected.

In the unconstrained setting, i.e., when students can submit any preference list (in Step 2), the Gale-Shapley and Top Trading Cycles mechanisms are strategy-proof, but the Boston mechanism is not. The Gale-Shapley mechanism is also stable. In fact, it always generates the best stable matching for the students. However, it is not Pareto efficient. In contrast, the Top Trading Cycles mechanism is Pareto efficient but not stable. The Boston mechanism is Pareto efficient, but since it is not strategy-proof (nor stable), Pareto efficiency is likely to be distorted in practice. Obviously, by imposing a maximal length on submittable preference lists, the Gale-Shapley and Top Trading Cycles mechanisms are no longer strategy-proof. Our goal is to understand its consequent impact on the superiority of the latter two mechanisms.

4 Constrained School Choice: Hypotheses

The constraint on the length of submittable preference lists will have several effects on individuals' strategies. The individuals will not only have to think how to order schools, but also which schools to include in their list. Choosing "inappropriate" schools may lead a mechanism to completely exhaust an individual's listed schools, leaving the individual unassigned. This very apparent risk will force individuals to think harder. As pointed out by Haeringer and Klijn (2007), a minimal rationality requirement for the Gale-Shapley and Top Trading Cycles mechanisms is the preservation of the true order. In other words, any two schools in the submitted preference list are ranked in the same relative order as in the true preference list. Therefore our first hypothesis is the following.

Hypothesis 1 *Under GS and TTC, the constraint increases the proportion of individuals that exhibit rational behavior.*

Again, the fact that an individual's possible choices will not be considered once his list is exhausted imposes a high risk on the individual. This would make individuals search for "safe" strategies which reduce the risk of remaining unassigned. We will refer to "safety school" as the school in which an individual is most likely to be accepted, i.e., one for which he has high priority.

Hypothesis 2 *Under all mechanisms, individuals will exhibit a "safety school" effect, i.e., they will list the school with highest entrance probability.*

Related to the idea of safety school, we define "truncated truthtelling" as replicating the true preferences up to the "safety school." In fact, for the Boston mechanism we require the individual to exactly replicate the true preferences (until he exhausts his list), unless his "safety school" is ranked first, in which case we only require that he lists that school in the first position. We expect that under all three mechanisms the rate of "truncated truthtelling" in the constrained setting is smaller than that in the unconstrained setting. The difference under the Boston mechanism, however, is likely to be small. The reason is that already in the unconstrained setting the Boston mechanism gives incentives to manipulate. Therefore, in the constrained setting the risk of the Boston mechanism reaching the end of an individual's submitted list is likely to have only a minor additional impact on the incentives to manipulate.

Treatment	Mechanism	Environment	# of subjects per session	Total # of subjects
BOS_d	Boston	Designed	36	72
GS_d	Gale-Shapley	Designed	36	72
TTC_d	Top Trading Cycle	Designed	36	72
BOS_r	Boston	Random	36	72
GS_r	Gale-Shapley	Random	36	72
TTC_r	Top Trading Cycle	Random	36	72

Table 1: Features of experimental sessions

Hypothesis 3 *Under GS and TTC, the rate of "truncated truthtelling" in the constrained setting is smaller than that in the unconstrained setting. The difference under BOS will be small.*

The lack of information that individuals have when deciding and the fact that there is no weakly dominant strategy implies that the optimality of individuals' strategies *ex post* will vary. We therefore expect that the constraint will bring along high efficiency costs. Compared to the unconstrained setting, we expect the Gale-Shapley and Top Trading Cycle mechanisms to perform much worse, and the Boston mechanism to perform again only slightly worse.

Hypothesis 4 *Under all three mechanisms, the constraint produces an efficiency loss.*

The absence of a weakly dominant strategy and the uncertainty involved when individuals decide will most probably break down the nice efficiency properties that the Gale-Shapley and Top Trading Cycles mechanisms have in the unconstrained setting.

Hypothesis 5 *The efficiency gap between BOS on the one hand and GS and TTC on the other will be small.*

5 Experimental Design

In this section we describe the experimental design which is almost identical to CS06's. *The only but key difference is that in all our sessions the participants were only allowed to submit a preference list of up to 3 schools, thereby mimicking a constraint in many real-life situations of school choice.* Our goal is obtain a clear-cut and self-contained comparison between the two experimental studies that facilitates identifying the consequences of the restriction on strategic behavior.

We use a 3×2 design: each of the three mechanisms is examined in a designed environment as well as a random environment. For each of the six treatments, two independent sessions were carried out. Table 1 summarizes features of experimental sessions. For each mechanism and for each environment, we conducted by hand two independent sessions between May and November 2006, one at the Universitat Autònoma de Barcelona and one at the Universitat Pompeu Fabra.

In each session, there are 36 students and 36 school seats across seven schools. There are three seats each at schools A and B, and six seats each at schools C, D, E, F and G. Students 1–3 live within the school district of school A, students 4–6 live within the school district of school B, students 7–12 live within the school district of school C, students 13–18 live within the school district of school D, students 19–24 live within the school district of school E, students 25–30 live within the school district of school F and students 31–36 live within the school district of school G.

The designed environment builds on three important real-life factors: a school's quality, its proximity, and a random factor. (We refer to CS06 for details on the design of the rankings and monetary payoffs.) Based on the induced rankings, one obtains the monetary payoffs for each participant as a result of the school he is assigned to at the end of the experiment. Payoffs are presented on the left-hand side of Table 2. The random environment is used to check the robustness of the results with respect to changes in the environment. Payoffs for the random environment are presented on the right-hand side of Table 2. Boldfaced numbers in the table indicate that the participant lives within the school district of that school.

All three mechanisms were implemented as one-shot games of incomplete information. Each subject knows his own payoff table, but not the other participants' payoff tables. He also knows that different participants might have different payoff tables. Each session consists of one round only. The sessions last approximately 45 minutes, with the first 20–25 minutes being used for instructions. The conversion rate is €1.[7] In addition to the earnings from the experiment, each subject also receives a participation fee of €3.

In the experiment, each participant is randomly assigned an ID number and is seated in a chair in a classroom. The experimenter reads the instructions aloud. Subjects are allowed to ask questions, which are answered in public. Subjects are then given 15 minutes to read the instructions again at their own pace and to make their decisions. Next, the experimenter collects the decisions and asks a volunteer to draw ping-pong balls out of an urn, which generates the lottery. The experimental session ends after the lottery is generated and publicly announced. The experimenter then introduces the subject decisions and the lottery in a computer program with the appropriate algorithm to compute the assignment, announces the results, proceeds to the corresponding payments to the subjects.

6 Results

As mentioned earlier, the subjects in our experimental study cannot (fully) reveal their true preferences because the form they have to fill out does not allow them to list all schools. This implies that once a mechanism exhausts a list it will no longer obey the individual's preferences. Therefore, the individuals face the additional risk of remaining unassigned. This will force individuals to think harder about their choices and a likely consequence is that they make fewer "mistakes." Haeringer and Klijn (2007) pointed out that under the Gale-Shapley and Top Trading Cycles mechanisms, an individual behaving rationally should never reverse the ranking of schools in the list with respect to the true preferences. This is also true for the unconstrained setting. Our first finding is that in the constrained setting very few individuals reverse the order, whereas for the unconstrained setting a significantly larger amount of individuals do. Therefore we can confirm Hypothesis 1. To obtain a genuine comparison we consider the number of individuals that reversed the ranking of their first three options in the list (for both the

[7]We used the exchange rate 1:1 of dollar:euro to convert the monetary payoffs from CS06 into euros.

Student	Designed, School							Randomized, School						
ID	A	B	C	D	E	F	G	A	B	C	D	E	F	G
1	**13**	16	9	2	5	11	7	**14**	8	10	13	16	12	3
2	**16**	13	11	7	2	5	9	**7**	15	1	6	14	3	4
3	**11**	13	7	16	2	9	5	**4**	10	5	1	12	8	15
4	16	**13**	11	5	2	7	9	8	**7**	14	9	4	11	1
5	11	**16**	2	5	13	7	9	11	**9**	14	7	12	5	4
6	16	**13**	7	9	11	2	5	4	**11**	5	9	3	16	7
7	13	16	**9**	5	11	7	2	14	10	**8**	15	11	6	5
8	16	9	**11**	2	13	7	5	6	9	**12**	5	14	10	8
9	16	13	**2**	5	9	7	11	12	10	**13**	16	9	15	3
10	16	7	**9**	5	2	11	13	16	5	**13**	12	3	1	4
11	7	16	**11**	9	5	2	13	5	11	**8**	2	16	10	7
12	13	16	**9**	11	2	7	5	9	6	**7**	4	10	13	11
13	9	16	2	**13**	11	5	7	4	7	13	**11**	8	10	1
14	16	5	2	**9**	7	13	11	10	1	7	**5**	14	13	16
15	13	16	9	**11**	2	7	5	16	13	5	**9**	8	3	6
16	16	13	11	**5**	9	7	2	7	12	5	**8**	15	9	4
17	13	16	5	**7**	2	9	11	14	11	3	**4**	10	6	8
18	16	13	5	**9**	7	11	2	10	1	6	**11**	15	2	8
19	11	16	7	5	**13**	9	2	8	12	16	5	**14**	4	13
20	16	13	7	9	**5**	2	11	15	4	1	2	**11**	14	3
21	13	16	2	7	**9**	11	5	3	16	8	6	**7**	10	2
22	16	11	7	2	**9**	5	13	1	8	14	15	**5**	3	4
23	16	13	7	2	**5**	11	9	11	16	12	1	**3**	7	15
24	16	13	11	5	**9**	2	7	12	11	6	3	**9**	4	14
25	13	16	2	5	11	**9**	7	8	14	7	15	1	**5**	12
26	16	13	5	9	7	**2**	11	11	16	9	7	4	**12**	1
27	7	11	5	2	13	**9**	16	7	13	15	14	3	**10**	12
28	16	13	7	2	11	**5**	9	4	15	10	11	9	**6**	7
29	7	11	16	13	2	**9**	5	11	12	7	14	6	**10**	9
30	16	9	7	2	5	**11**	13	13	8	3	12	16	**2**	11
31	11	16	7	2	5	9	**13**	6	3	10	14	11	16	**1**
32	13	9	16	2	5	7	**11**	13	12	7	11	2	16	**14**
33	13	16	11	9	7	5	**2**	2	5	7	8	15	10	**6**
34	16	11	2	7	5	13	**9**	8	10	3	14	16	1	**12**
35	7	16	2	5	11	13	**9**	10	12	2	7	3	14	**8**
36	16	13	5	7	9	2	**11**	16	3	14	13	8	10	**11**

Table 2: Payoff table in the designed and the randomized environments

Treatment	GS_d	GS_r	GS_j	TTC_d	TTC_r	TTC_j
Unconstrained (%)	96	92	94	93	90	92
Constrained (%)	81	75	77	79	68	74
p-value	0.0048	0.006	0.00	0.016	0.0012	0.0000

Table 3: Rational behavior

constrained and the unconstrained setting).

Result 6.1 (Higher rationality under constrained GS and TTC) *Under GS, 94% of the individuals in the constrained setting and 77% in the unconstrained setting behave "rationally," i.e., they do not reverse their ranking of schools in the list with respect to their true preferences. Under TTC we find 92% and 74% for the constrained and unconstrained setting, respectively.*

Support: As shown in Table 3, under both mechanisms and for all treatments (designed, random, and joint) the percentage of rational students is significantly higher in the unconstrained setting than in the constrained setting. The last row presents the p-values when performing a z-test, the null hypothesis being that percentages are equal. ■

Note, on the other hand, that the proportion of rational behavior is not significantly different between the Gale-Shapley and Top Trading Cycles mechanisms.

Full truthtelling is not a valid option in the constrained setting. A simple strategy for this setting with a spirit of truthtelling is *truncated truthtelling*. In the context of the Gale-Shapley and Top Trading Cycles mechanisms, we define truncated truthtelling as replicating the true preferences up to the district school. More precisely, if the district school is ranked third or higher it requires that the elements of the list up to the district school coincide with the true preferences. If not, it requires that the list coincides with the true preferences, up to the third school. For the Boston mechanism we require that the subject exactly replicates his true preferences up to the third school, unless his most preferred school is his district school, in which case we only require that this school his ranked first. Though possibly an intuitive strategy to recur to in an incomplete information environment, truncated truthtelling is not a weakly dominant strategy for any of the three mechanisms. The next result shows that most individuals are aware of this and do not reveal their (truncated) true preferences. Since the Boston mechanism is not even strategy-proof in the unconstrained setting, here the difference between both settings is almost not notable. Again, to obtain a genuine comparison we consider only the first three options in the list for the unconstrained setting.

Result 6.2 (Less truncated truthtelling under constrained choice) *Under GS, truncated truthtelling is reduced from 65% to 21%. Also, under TTC truncated truthtelling has gone down from 47% to 21%. Under BOS the reduction is smaller (but sometimes still significant).*

Treatment	GS_d	GS_r	GS_j	TTC_d	TTC_r	TTC_j	BOS_d	BOS_r	BOS_j
Unconstrained (%)	72	57	65	50	43	47	14	27	23
Constrained (%)	25	18	21	22	19	21	18	8	13
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.51 [8]	0.00	0.03

Table 4: Truncated truthtelling

Support: As shown in Table 4 truncated truthtelling goes down significantly. The last row depicts the p-values when performing a z-test where the null hypothesis is that the percentage of truncated truthtelling is the same across the constrained and unconstrained settings. ■

Result 6.2 confirms Hypothesis 3. Also, as an important consequence of Result 6.2, the nice properties of the Gale-Shapley and Top Trading Cycles mechanisms that can be expected in the unconstrained setting due to the strategy-proofness of the mechanisms, are likely to be distorted. The reason is that if individuals do not even use truncated truthtelling then the underlying computations in the mechanisms diverge rapidly from those under the true preferences.

The next result shows that in terms of (truncated) truthtelling the three mechanisms perform equally bad.

Result 6.3 (BOS, GS and TTC are equally bad in terms of truncated truthtelling under constrained choice) *In the constrained setting, the level of truncated truthtelling does not significantly vary across GS, TTC and BOS.*

Support: The z-statistic when comparing BOS vs. GS is $z = 0.43$ (p-value=0.67) for the designed treatment and $z = -0.15$ (p-value=0.88) for the random treatment. For BOS vs. TTC, $z = -0.31$ (p-value=0.67) for the designed treatment and $z = 1.93$ (p-value=0.054) for the random treatment. Finally, for GS vs. TTC, $z = 0.425$ (p-value=0.67) for the designed treatment and $z = 0.155$ (p-value= 0.88) for the random treatment. ■

The previous result stands in sharp contrast with CS06's finding that in the unconstrained setting the Boston mechanism performs much worse in terms of (pure) truthtelling:

Result 1 (CS06 – Truthful preference revelation) In both the designed and random environments, the proportion of truthful preference revelation under BOS is significantly lower than that under either GS or TTC. The proportion of truthful preference revelation under TTC is significantly lower than that of GS in the designed environment, and is weakly lower in the random environment.

CS06 introduced the following three categories of preference misrepresentations. Note that all three preference misrepresentations can also be employed in the constrained setting.

- *District School Bias* (DSB): A participant puts his district school into a higher position than that in the true preference order.

[8] Note that in this case the difference is negative but not significant.

Treatment	DSB	SSB	SPB
GS_d	8.5 (0.00)	5.181 (0.00)	1.196 (0.23)
TTC_d	4.68 (0.00)	2.4 (0.02)	-0.87 (0.38)
BOS_d	0.57 (0.57)	1.37 (0.17)	1.56 (0.11)
GS_r	7.7 (0.00)	6.74 (0.00)	1.39 (0.16)
TTC_r	4.92 (0.00)	6.37 (0.00)	-1.1 (0.26)
BOS_r	8.54 (0.00)	6.5 (0.00)	1.08 (0.27)

Table 5: z-statistics (p-values) for the test of proportions of misrepresentations between the constrained and the unconstrained setting

- *Small School Bias* (SSB): A participant puts school A or B (or both) into lower positions than those in the true preference ordering.
- *Similar Preference Bias* (SPB): A participant puts schools with the highest payoffs in lower positions.

Result 6.4 (Increased misrepresentations under constrained GS and TTC due to DSB and SSB) *Under GS and TTC, the increase in misrepresentation is mainly due to DSB and SSB.*

Support: Table 5 contains the z-statistics and the p-values obtained from a test of proportions comparing the percentage of types of misrepresentations between the constrained and the unconstrained setting. As shown in Table 6, the increase in misrepresentation can be characterized by a greater DSB and SSB. The differences between the constrained and unconstrained setting are highly significant. The differences in SPB are not significant. ■

Result 6.5 (Equal DSB and SSB under constrained choice) *In the constrained setting, the three mechanisms have similar levels of DSB and SSB. On the other hand, BOS exhibits a higher level of SPB than GS and TTC.*

Support: Table 7 contains the z-statistics and the p-values obtained from a test of proportions comparing the percentage of types of misrepresentations between the different mechanisms in the constrained setting. We find that only SPB is higher under the Boston mechanisms relative to GS and TTC, for both the designed and the random treatment. The differences in DSB and SSB between any pair of mechanism is not significant for neither the designed nor the random treatment. ■

In each treatment, almost 60% of the individuals had their district school ranked 4th or below in their true preferences. These are the individuals that under GS and TTC would have a clear incentive to misrepresent their preferences by including their district school in the list. The reason is that under GS and TTC the district school is a "safety school," i.e., including the district school guarantees being assigned a seat. Therefore we define *Safety School Bias* as the preference manipulation of including the district school in the list given that this school is

Treatment	DSB	SSB	SPB	other
GS_d - C	70	67	33	0
TTC_d - C	68	66	33	1
BOS_d - C	79	79	60	0
GS_r - C	78	82	39	0
TTC_r - C	73	79	34	0
BOS_r - C	76	90	58	0
GS_d - U	8	24	24	1
TTC_d - U	29	46	40	3
BOS_d - U	75	69	47	0
GS_r - U	14	26	28	4
TTC_r - U	32	26	43	3
BOS_r - U	60	38	49	3

Table 6: Percentage of submitted lists with the specified misrepresentation, for the constrained (C) and unconstrained (U) setting

Mechanism	DSB	SSB	SPB
GS-TTC_d	0.25 (0.79)	0.127 (0.9)	0 (1)
GS-TTC_r	0.7 (0.49)	0.46 (0.65)	0.62 (0.53)
GS-BOS_d	-1.24 (0.22)	-1.6 (0.1)	3.25 (0.001)
GS-BOS_r	0.29 (0.78)	-1.38 (0.17)	2.281 (0.02)
BOS-TTC_d	-1.495 (0.13)	-1.75 (0.08)	-3.25 (0.001)
BOS-TTC_r	-0.413 (0.67)	-1.824 (0.07)	-2.9 (0.003)

Table 7: z-statistics (p-values) for the test of proportions of misrepresentations between mechanisms in the constrained setting

Mechanism	Constrained (%)	Unconstrained (%)	z-stat (p-value)
GS_d	90	7	13.66 (0.00)
TTC_d	85	26	6.78 (0.00)
BOS_d	50	35	1.74 (0.08)
GS_r	88	11	11.28 (0.00)
TTC_r	89	23	8.22 (0.00)
BOS_r	40	29	1.4 (0.16)

Table 8: "Safety school" misrepresentations

ranked 4th or below in the true preferences. Clearly, Safety School Bias is a particular case of District School Bias. For the Boston mechanism the "safety school" effect would involve making the district school the first choice in the list.

Result 6.6 ("Safety school" effect under constrained GS and TTC) *Under GS (TTC), 88% (87%) of the individuals whose district school is listed 4th or below in the true preferences nevertheless includes this school in their list. The difference with respect to the unconstrained setting is very large and significant.*

Support: Table 8 shows the proportion of students who did not have their district school as one of their three most preferred schools and still included it in the list. The z-statistics and the p-values obtained from a test of proportions show that this bias is significantly larger in the constrained setting. ■

The following result shows that this effect is relatively smaller and not always significantly different from that in the unconstrained setting. This is due to the fact that in the unconstrained setting there is already an incentive to be cautious and follow a "safe strategy."

Result 6.7 (Smaller "safety school" effect under constrained BOS) *Under BOS, 45% of the individuals whose district school is listed 4th or below in the true preferences nevertheless includes this school in their list. Compared to the unconstrained setting, the effect is larger but not always significant.*

Support: Table 8 shows the proportion of students who did not have their district school as one of their three most preferred schools and still included it as their first choice. The z-statistics and the p-values obtained from a test of proportions show that this bias is only significantly larger at a 10% significance level in the designed treatment. ■

In view of the last two results, we can confirm Hypothesis 2, i.e., there exists a "safety school" effect for all three mechanisms.

Next, we turn to comparisons of efficiency losses. For our results on the efficiency of the mechanisms, as in CS06 we use the recombinant estimator of Mullin and Reiley (2006) and compare the per capita payoffs for each mechanism. Taking advantage of the fact that the experiment was a one-shot simultaneous game and that we have two identical sessions for each

Mechanism	Mean	Variance	Covariance	Asymmmetric Var
GS_d	10.89	0.148	0.00132	0.024
TTC_d	11.221	0.117	0.0022	0.0397
BOS_d	10.39	0.1395	0.0023	0.0414
GS_r	12.11	0.4	0.0048	0.087
TTC_r	12.72	0.298	0.0052	0.0928
BOS_r	11.464	0.22	0.00354	0.0637

Table 9: Summary statistics of the 30.000 recombinations' mean payoffs

treatment, we recombine the 72 students into two groups of 36 (each with a representation of the 36 types of players). We do this 30.000 times and use the resulting mean payoffs in the recombinations to asess the efficiency of the mechanisms.[9] Table 9 shows the summary statistics on the mean payoff of the 30.000 recombinations. Let x > y denote that a measure under mechanism x is greater than the corresponding measure under mechanism y at the 5% significance level or less. Let x $\sim$ y denote that a measure under mechanism x is not significantly different from the corresponding measure under mechanism y at the 5% significance level.

Result 6.8 *In the designed environment, the efficiency ranking is the following:* $TTC_d > GS_d > BOS_d$. *In the random environment* $TTC_r \sim GS_r \sim BOS_r$, *but* $TTC_r > BOS_r$.

Support: The t-stat and their corresponding p-values for the different hypothesis are, 1.94 (0.026) for $GS_d > BOS_d$, 2.916 (0.0018) for $TTC_d > BOS_d$ and 1.33 (0.0921) for the designed environment. For the random environment, 1.678 (0.0468) for $GS_r > BOS_r$, 3.173 (0.0008) for $TTC_r > BOS_r$ and 1.4252 (0.077) for $TTC_r > GS_r$. ■

Result 6.9 *For both environments, designed and random, BOS and GS are significantly less efficient in the constrained case then in the unconstrained case. For TTC the difference is not significant.*

Support: The third column in Table 10 shows the t-statistics and their corresponding p-value to test for the mean in the constrained case being smaller than the mean in the unconstrained case. Only TTC's mean differences are not significant. ■

There are two effects driving the relative efficiency of the mechanisms in the constrained versus the unconstrained case. First, individuals not behaving irrationally, i.e., no reversing the order of their true ranking in the submitted list, increases efficiency. Secondly, truthful representation of preferences increases the efficiency of the mechanism if individuals are assigned in one of their top choices. In the constrained case we observe less irrational, which favors efficiency, but an increased manipulation, which decreases efficiency. The relative size of each effect explains the final results presented.

[9]Chen and Sönmez run 200 recombinations, but the results in that case are not robust, so we increased substantially the number of replications to insure robustness of the results.

Mechanism	Constrained	Unconstrained	t-stat (p-value)
GS_d	11.71	10.89	3.88 (0.0001)
TTC_d	11.41	11.22	0.72 (0.24)
BOS_d	11.15	10.38	3.12 (0.0009)
GS_r	12.78	12.12	1.87 (0.03)
TTC_r	12.34	12.71	0.88 (0.19)
BOS_r	12.83	11.46	4.23 (0.0000)

Table 10: Efficiency comparisions between the Constrained and the Unconstrained environment

7 Conclusion

Recent work by Abdulkadiroğlu and Sönmez (2003) has lead district schools in the US and other countries to question the mechanisms in use to assign children to schools. A series of studies have analyzed the properties of the vastly used Boston mechanism revealing large manipulation rates. Two other mechanisms, the Gale-Shapley and the Top Trading Cycles mechanisms have been proposed and implemented. The argument to move from the Boston mechanism to the Gale-Shapley and the Top Trading Cycles mechanisms is the strategy-proofness of the latter two mechanisms in combination with their other desirable properties (stability and efficiency). But this argument ignores a commonly used "simplification" when parents are asked to fill out the preference lists: the number of slots is smaller than the number of schools available. Our experimental study makes clear that the superiority of the Gale-Shapley and the Top Trading Cycles mechanisms over the Boston mechanism does not necessarily carry over to the constrained setting. Therefore, replacing the Boston mechanism without removing the constraint on the number of schools that parents can list will not necessarily lead to an improvement in the performance of the school choice procedure.

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A Appendix: Experimental Instructions

Instructions - Mechanism B

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you.

Procedure

- There are 36 participants in this experiment. You are participant #1.
- In this simulation, 36 school slots are available across seven schools. These schools differ in size, geographic location, specialty, and quality of instruction in each specialty. Each school slot is allocated to one participant. There are three slots each at schools A and B, and six slots each at schools C, D, E, F and G.
- **Your payoff** amount depends on the school slot you hold at the end of the experiment + €3 for participating. Payoff amounts are outlined in the following table. These amounts reflect the desirability of the school in terms of location, specialty and quality of instruction.

Slot received at School:	A	B	C	D	E	F	G
Payoff to Participant #1 (in euros)	13	16	9	2	5	11	7

  The table is explained as follows:

  - You will be paid €13 if you hold a slot at school A at the end of the experiment.
  - You will be paid €16 if you hold a slot at school B at the end of the experiment.
  - You will be paid €9 if you hold a slot at school C at the end of the experiment.
  - You will be paid €2 if you hold a slot at school D at the end of the experiment.
  - You will be paid €5 if you hold a slot at school E at the end of the experiment.
  - You will be paid €11 if you hold a slot at school F at the end of the experiment.
  - You will be paid €7 if you hold a slot at school G at the end of the experiment.

  ***NOTE* different participants might have different payoff tables.** That is, payoff by school might be different for different participants.

- During the experiment, each participant first completes the Decision Sheet by indicating school preferences. The Decision Sheet is the last page of this packet. Note that you need to rank at most three schools in order to indicate your preferences.
- After all participants have completed their Decision Sheets, the experimenter collects the Sheets and starts the allocation process.
- Once the allocations are determined, the experimenter informs each participant whether he/she has been assigned a slot, and if so, which slot and payoff.

Allocation Method

- In this experiment, participants are defined as belonging to the following school districts.
  - Participants #1 – #3 live within the school district of school A,
  - Participants #4 – #6 live within the school district of school B,
  - Participants #7 – #12 live within the school district of school C,
  - Participants #13 – #18 live within the school district of school D,
  - Participants #19 – #24 live within the school district of school E,
  - Participants #25 – #30 live within the school district of school F,
  - Participants #31 – #36 live within the school district of school G.
- In addition, for each school, a separate **priority order** of the students is determined as follows:
  - **Highest Priority Level:** Participants who rank the school as their first choice AND who also live within the school district.
  - **2nd Priority Level:** Participants who rank the school as their first choice BUT who do not live within the school district.
  - **3rd Priority Level:** Participants who rank the school as their second choice AND who also live within the school district.
  - **4th Priority Level:** Participants who rank the school as their second choice BUT who do not live within the school district.
  - **5th Priority Level:** Participants who rank the school as their third choice AND who also live within the school district.
  - **6th Priority Level:** Participants who rank the school as their third choice BUT who do not live within the school district.
- The ties between participants at the same priority level are broken using a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, ..., as well as the last in the line. To determine this fair lottery, a participant will be asked to draw 36 ping pong balls from an urn, one at a time. Each ball has a number on it, corresponding to a participant ID number. The sequence of the draw determines the order in the lottery.
- Therefore, to determine the priority order of a student for a school:
  - The first consideration is how highly the participant ranks the school in his/her Decision Sheet,
  - The second consideration is whether the participant lives within the school district or not, and
  - The last consideration is the order in the fair lottery.

- Once the priorities are determined, slots are allocated in four rounds.

Round 1. a. An application to the first ranked school in the Decision Sheet is sent for each participant.

b. Each school accepts the students with higher priority order until all slots are filled. These students and their assignments are removed from the system. The remaining applications for each respective school are rejected.

Round 2. a. The rejected applications are sent to his/her second ranked school in the Decision Sheet.

b. If a school still has available slots remaining from Round 1, then it accepts the students with higher priority order until all slots are filled. The remaining applications are rejected.

Round 3. a. The rejected applications are sent to his/her third ranked school in the Decision Sheet.

b. If a school still has available slots remaining from Round 2, then it accepts the students with higher priority order until all slots are filled. The remaining applications are rejected.

Round 4. A participant whose application was rejected in Round 3 remains unassigned.

- If a participant has been rejected by all schools listed in his/her Decision Sheet, then he/she remains unassigned. The final payoff to a participant will be €3 + the payoff for the obtained slot (in case he/she is assigned a slot).

An Example:

We will go through a simple example to illustrate how the allocation method works.

Students and Schools: In this example, there are six students, 1–6, and four schools, Clair, Erie, Huron and Ontario.

Student ID Number: $1, 2, 3, 4, 5, 6$	Schools: Clair, Erie, Huron, Ontario

Slots and Residents: There are two slots each at Clair and Erie, and one slot each at Huron and Ontario. Residents of districts are indicated in the table below.

School	Slot 1	Slot 2	District Residents
Clair	☐	☐	1 2
Erie	☐	☐	3 4
Huron	☐		5
Ontario	☐		6

Lottery: The lottery produces the following order.

$\boxed{1 - 2 - 3 - 4 - 5 - 6}$

Submitted School Rankings: The students submit the following school rankings:

	1st Choice	2nd Choice	3rd Choice
Student 1	Huron	Clair	
Student 2	Huron	Ontario	Clair
Student 3	Ontario	Clair	Erie
Student 4	Huron	Clair	Ontario
Student 5	Ontario	Huron	Clair
Student 6	Clair	Erie	Ontario

Priority: School priorities depend on: (1) how highly the student ranks the school, (2) whether the school is a district school, and (3) the lottery order.

Clair : Student 6 ranks Clair first. Students 1, 3 and 4 rank Clair second; among them, student 1 lives within the Clair school district. Students 2 and 5 rank Clair third. Using the lottery order to break ties, the priority for Clair is 6-1-3-4-2-5.

$$\overbrace{6}^{\text{1st Choice}}\ \overbrace{\underbrace{1}_{\text{Resident}}\ \underbrace{3\quad 4}_{\text{Non-Resident}}}^{\text{2nd Choice}}\ \overbrace{\underbrace{2\quad 5}_{\text{Non-Resident}}}^{\text{3rd Choice}}$$

Erie : No student ranks Erie first. Student 6 ranks Erie second. Student 3 ranks Erie third. So, the priority for Erie is 6-3.

$$\overbrace{\text{None}}^{\text{1st Choice}}\ \overbrace{6}^{\text{2nd Choice}}\ \overbrace{3}^{\text{3rd Choice}}$$

Huron : Students 1, 2 and 4 rank Huron first. Student 5 ranks Huron second. Using the lottery order to break ties, the priority for Huron is 1-2-4-5.

$$\overbrace{\underbrace{1\quad 2\quad 4}_{\text{Non-Residents}}}^{\text{1st Choice}}\ \overbrace{5}^{\text{2nd Choice}}\ \overbrace{\text{None}}^{\text{3rd Choice}}$$

Ontario : Students 3 and 5 rank Ontario first. Student 2 ranks Ontario second. Students 4 and 6 rank Ontario third; among them student 6 lives within the Ontario school district. Using the lottery order to break ties, the priority for Ontario is 3-5-2-6-4.

$$\overbrace{\underbrace{3\quad 5}_{\text{Non-Residents}}}^{\text{1st Choice}}\ \overbrace{2}^{\text{2nd Choice}}\ \overbrace{\underbrace{6}_{\text{Resident}}\ \underbrace{4}_{\text{Non-Resident}}}^{\text{3rd Choice}}$$

Allocation: This allocation method consists of the following rounds.

Round 1 : Each student applies to his/her first choice: Students 1, 2 and 4 apply to Huron, students 3 and 5 apply to Ontario and student 6 applies to Clair.

- School Clair accepts Student 6.
- School Huron accepts Student 1 and rejects Students 2,4.
- School Ontario accepts Student 3 and rejects Student 5.

Applicants		School		Accept	Reject		Slot 1	Slot 2
6	⟶	Clair	⟶	6		⟶	$\boxed{6}$	$\boxed{}$
	⟶	Erie	⟶			⟶	$\boxed{}$	$\boxed{}$
1, 2, 4	⟶	Huron	⟶	1	2, 4	⟶	$\boxed{1}$	—
3, 5	⟶	Ontario	⟶	3	5	⟶	$\boxed{3}$	—

Accepted students are removed from the subsequent process.

Round 2 : Each student who is rejected in Round 1 then applies to his/her second choice: Student 2 applies to Ontario, student 4 applies to Clair, and student 5 applies to Huron.

- No slot is left at Ontario, so it rejects student 2.
- Clair accepts student 4 for its last slot.
- No slot is left at Huron, so it rejects student 5.

Applicants		School		Accept	Reject		Slot 1	Slot 2
4	⟶	Clair	⟶	4		⟶	$\boxed{6}$	$\boxed{4}$
	⟶	Erie	⟶			⟶	$\boxed{}$	$\boxed{}$
5	⟶	Huron	⟶		5	⟶	$\boxed{1}$	—
2	⟶	Ontario	⟶		2	⟶	$\boxed{3}$	—

Round 3 : Each student who is rejected in Rounds 1-2 applies to his/her third choice: Students 2 and 5 apply to Clair.

- No slot is left at Clair, so it rejects students 2 and 5.

Applicants		School		Accept	Reject		Slot 1	Slot 2
2, 5	⟶	Clair	⟶		2, 5	⟶	$\boxed{6}$	$\boxed{4}$
	⟶	Erie	⟶			⟶	$\boxed{}$	$\boxed{}$
	⟶	Huron	⟶			⟶	$\boxed{1}$	—
	⟶	Ontario	⟶			⟶	$\boxed{3}$	—

Round 4 : Students 2 and 5 were rejected by Clair. Since the School Rankings of students 2 and 5 have been exhausted these students remain unassigned.

Based on this method, the final allocations are:

Student	1	2	3	4	5	6
School	Huron	–	Ontario	Clair	–	Clair

You will have 15 minutes to go over the instructions at your own pace, and make your decisions. Feel free to earn as much cash as you can. Are there any questions?

Decision Sheet - Mechanism B

- Recall: You are participant #1 and you live within the school district of School A.
- Recall: **Your payoff** amount depends on the school slot you hold at the end of the experiment. Payoff amounts are outlined in the following table.

School:	A	B	C	D	E	F	G
Payoff in euros	13	16	9	2	5	11	7

You will be paid €13 if you hold a slot of School A at the end of the experiment.
You will be paid €16 if you hold a slot of School B at the end of the experiment.
You will be paid €9 if you hold a slot of School C at the end of the experiment.
You will be paid €2 if you hold a slot of School D at the end of the experiment.
You will be paid €5 if you hold a slot of School E at the end of the experiment.
You will be paid €11 if you hold a slot of School F at the end of the experiment.
You will be paid €7 if you hold a slot of School G at the end of the experiment.

Please write down a ranking of up to 3 schools.

1st choice	2nd choice	3rd choice

Your I.D : #1 **Your Name** (print): ____________________

This is the end of the experiment for you. Please remain seated until the experimenter collects your Decision Sheet.

After the experimenter collects all Decision Sheets, a participant will be asked to draw ping pong balls from an urn to generate a fair lottery. The lottery, as well as all participants' rankings will be entered into a computer after the experiment. The experimenter will inform each participants of his/her allocation slot and respective payoff once it is computed.

Session Number : 1 Mechanism 1 Payoff Matrix 1

Instructions - Mechanism G

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Allocation Method

- In this experiment, participants are defined as belonging to the following school districts.
  - Participants #1 – #3 live within the school district of school A,
  - Participants #4 – #6 live within the school district of school B,
  - Participants #7 – #12 live within the school district of school C,
  - Participants #13 – #18 live within the school district of school D,
  - Participants #19 – #24 live within the school district of school E,
  - Participants #25 – #30 live within the school district of school F,
  - Participants #31 – #36 live within the school district of school G.
- A priority order is determined for each school. Each participant is assigned a slot at the **best possible** school reported in his/her Decision Sheet that is consistent with the priority order below.
- The priority order for each school is separately determined as follows:
  - **High Priority Level:** Participants who live within the school district.

    Since the number of High priority participants at each school is equal to the school capacity, each High priority participant is guaranteed an assignment that is at least as good as his/her district school based on the ranking indicated in his/her Decision Sheet.
  - **Low Priority Level:** Participants who do not live within the school district.

    The priority among the Low priority students is based on their respective order in a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, . . ., as well as the last in the line. To determine this fair lottery, a participant will be asked to draw 36 ping pong balls from an urn, one at a time. Each ball has a number on it, corresponding to a participant ID number. The sequence of the draw determines the order in the lottery.
- Once the priorities are determined, the allocation of school slots is obtained as follows:
  - An application to the first ranked school in the Decision Sheet is sent for each participant.
  - Throughout the allocation process, a school can hold no more applications than its number of slots.

    If a school receives more applications than its capacity, then it rejects the students with lowest priority orders. The remaining applications are retained.
  - Whenever an applicant is rejected at a school, his application is sent to the next highest school on his Decision Sheet.

  - Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the number of the slots are rejected, while remaining applications are retained.
  - The allocation is finalized when no more applications can be rejected.

    Each participant is assigned a slot at the school that holds his/her application at the end of the process.

- If a participant has been rejected by all schools listed in his/her Decision Sheet, then he/she remains unassigned. The final payoff to a participant will be €3 + the payoff for the obtained slot (in case he/she is assigned a slot).

An Example:

We will go through a simple example to illustrate how the allocation method works.

Students and Schools: In this example, there are six students, 1-6, and four schools, Clair, Erie, Huron and Ontario.

Student ID Number: $1, 2, 3, 4, 5, 6$ Schools: Clair, Erie, Huron, Ontario

Slots and Residents: There are two slots each at Clair and Erie, and one slot each at Huron and Ontario. Residents of districts are indicated in the table below.

School	Slot 1	Slot 2	District Residents
Clair	☐	☐	1 2
Erie	☐	☐	3 4
Huron	☐		5
Ontario	☐		6

Lottery: The lottery produces the following order.

1 – 2 – 3 – 4 – 5 – 6

Submitted School Rankings: The students submit the following school rankings:

	1st Choice	2nd Choice	3rd Choice
Student 1	Huron	Clair	
Student 2	Huron	Ontario	Clair
Student 3	Ontario	Clair	Erie
Student 4	Huron	Clair	Ontario
Student 5	Ontario	Huron	Clair
Student 6	Clair	Erie	Ontario

Priority : School priorities first depend on whether the school is a district school, and next on the lottery order:

	Resident	Non-Resident
Priority order at Clair:	**1, 2**	– 3 – 4 – 5 – 6
Priority order at Erie:	**3, 4**	– 1 – 2 – 5 – 6
Priority order at Huron:	**5**	– 1 – 2 – 3 – 4 – 6
Priority order at Ontario:	**6**	– 1 – 2 – 3 – 4 – 5

The allocation method consists of the following steps:

Step 1 : Each student applies to his/her first choice: Students 1, 2 and 4 apply to Huron, students 3 and 5 apply to Ontario, and student 6 applies to Clair.

- Clair holds the application of student 6.
- Huron holds the application of student 1 and rejects students 2 and 4.
- Ontario holds the application of student 3 and rejects student 5.

Applicants		School		Hold	Reject
6	⟶	Clair	⟶	$\boxed{6}\,\boxed{}$	
	⟶	Erie	⟶	$\boxed{}\,\boxed{}$	
1, 2, 4	⟶	Huron	⟶	$\boxed{1}$ –	2, 4
3, 5	⟶	Ontario	⟶	$\boxed{3}$ –	5

Step 2 : Each student rejected in Step 1 applies to his/her next choice: Student 2 applies to Ontario, student 4 applies to Clair, and student 5 applies to Huron.

- Clair considers the application of student 4 together with the application of student 6, which was on hold. It holds both applications.
- Huron considers the application of student 5 together with the application of student 1, which was on hold. It holds the application of student 5 and rejects student 1.
- Ontario considers the application of student 2 together with the application of student 3, which was on hold. It holds the application of student 2 and rejects student 3.

Hold	New applicants		School		Hold	Reject
$\boxed{6}\,\boxed{}$	4	⟶	Clair	⟶	$\boxed{6}\,\boxed{4}$	
$\boxed{}\,\boxed{}$		⟶	Erie	⟶	$\boxed{}\,\boxed{}$	
$\boxed{1}$ –	5	⟶	Huron	⟶	$\boxed{5}$ –	1
$\boxed{3}$ –	2	⟶	Ontario	⟶	$\boxed{2}$ –	3

Step 3 : Each student rejected in Step 2 applies to his/her next choice: Students 1 and 3 apply to Clair.

- Clair considers the applications of students 1 and 3 together with the applications of students 4 and 6, which were on hold. It holds the applications of students 1 and 3 and rejects students 4 and 6.

Hold	New applicants		School		Hold	Reject
$\boxed{6}\boxed{4}$	1, 3	$\longrightarrow$	Clair	$\longrightarrow$	$\boxed{1}\boxed{3}$	4, 6
$\boxed{}\boxed{}$		$\longrightarrow$	Erie	$\longrightarrow$	$\boxed{}\boxed{}$	
$\boxed{5}$ –		$\longrightarrow$	Huron	$\longrightarrow$	$\boxed{5}$ –	
$\boxed{2}$ –		$\longrightarrow$	Ontario	$\longrightarrow$	$\boxed{2}$ –	

Step 4 : Each student rejected in Step 3 applies to his/her next choice: Student 4 applies to Ontario and student 6 applies to Erie.

- Ontario considers the application of student 4 together with the application of student 2, which was on hold. It holds the application of student 2 and rejects student 4.
- Erie holds the application of student 6.

Hold	New applicants		School		Hold	Reject
$\boxed{1}\boxed{3}$		$\longrightarrow$	Clair	$\longrightarrow$	$\boxed{1}\boxed{3}$	
$\boxed{}\boxed{}$	6	$\longrightarrow$	Erie	$\longrightarrow$	$\boxed{6}\boxed{}$	
$\boxed{5}$ –		$\longrightarrow$	Huron	$\longrightarrow$	$\boxed{5}$ –	
$\boxed{2}$ –	4	$\longrightarrow$	Ontario	$\longrightarrow$	$\boxed{2}$ –	4

Student 4 was rejected by Ontario. Since the School Ranking of student 4 has been exhausted this student remains unassigned. Based on this method, the final allocations are:

Student	1	2	3	4	5	6
School	Clair	Ontario	Clair	–	Huron	Erie

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Instructions - Mechanism T

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Allocation Method

- In this experiment, participants are defined as belonging to the following school districts.
  - Participants #1 – #3 live within the school district of school A,
  - Participants #4 – #6 live within the school district of school B,
  - Participants #7 – #12 live within the school district of school C,
  - Participants #13 – #18 live within the school district of school D,
  - Participants #19 – #24 live within the school district of school E,
  - Participants #25 – #30 live within the school district of school F,
  - Participants #31 – #36 live within the school district of school G.
- Each participant is first tentatively assigned to the school within his/her respective district. Next, Decision Sheet rankings are used to determine mutually beneficial exchanges between two or more participants. The order in which these exchanges are considered is determined by a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, ..., as well as the last in the line. To determine this fair lottery, a participant will be asked to draw 36 ping pong balls from an urn, one at a time. Each ball has a number on it, corresponding to a participant ID number. The sequence of the draw determines the order in the lottery.
- The specific allocation process is explained below.
  - Initially all slots are available for allocation.
  - All participants are ordered in a queue based on the order in the lottery.
  - Next, an application to the highest ranked school in the Decision Sheet is submitted for the participant at the top of the queue.
    * If the application is submitted to his district school, then his/her tentative assignment is finalized (thus he/she is assigned a slot at his district school). The participant and his/her assignment are removed from subsequent allocations. The process continues with the next participant in line.
    * If the application is submitted to another school, say school S, then the first participant in the queue who tentatively holds a slot at School S is moved to the top of the queue directly in front of the requester.
  - Whenever the queue is modified, the process continues similarly: An application is submitted to the highest ranked school with available slots for the participant at the top of the queue.
    * If the application is submitted to his district school, then his/her tentative assignment is finalized. The process continues with the next participant in line.

    * If the application is submitted to another school, say school S, then the first participant in the queue who tentatively holds a slot at school S is moved to the top of the queue directly in front of the requester. This way, each participant is guaranteed an assignment which is at least as good as his/her district school based on the preferences indicated in his/her Decision Sheet.
  - A mutually-beneficial exchange is obtained when a cycle of applications are made in sequence, which benefits all affected participants, e.g., I apply to John's district school, John applies to your district school, and you apply to my district school. In this case, the exchange is completed and the participants as well as their assignments are removed from subsequent allocations.
  - The process continues until there is no longer a mutually-beneficial exchange or a student submitting an application.

- If a participant has been rejected by all schools listed in his/her Decision Sheet, then he/she remains unassigned. The final payoff to a participant will be €3 + the payoff for the obtained slot (in case he/she is assigned a slot).

An Example:

We will go through a simple example to illustrate how the allocation method works.

Students and Schools: In this example, there are six students, 1–6, and four schools, Clair, Erie, Huron and Ontario.

Student ID Number: $1, 2, 3, 4, 5, 6$	Schools: Clair, Erie, Huron, Ontario

Slots and Residents: There are two slots each at Clair and Erie, and one slot each at Huron and Ontario. Residents of districts are indicated in the table below.

School	Slot 1	Slot 2	District Residents
Clair	☐	☐	1 2
Erie	☐	☐	3 4
Huron	☐		5
Ontario	☐		6

Tentative assignments: Students are tentatively assigned slots at their district schools.

School	Slot 1	Slot 2	
Clair	$\boxed{1}$	$\boxed{2}$	Students 1 and 2 are **tentatively assigned** a slot at Clair;
Erie	$\boxed{3}$	$\boxed{4}$	Students 3 and 4 are **tentatively assigned** a slot at Erie;
Huron	$\boxed{5}$	–	Student 5 is **tentatively assigned** a slot at Huron;
Ontario	$\boxed{6}$	–	Students 6 is **tentatively assigned** a slot at Ontario.

Lottery: The lottery produces the following order.

$$\boxed{1 - 2 - 3 - 4 - 5 - 6}$$

Submitted School Rankings: The students submit the following school rankings:

	1st Choice	2nd Choice	3rd Choice
Student 1	Huron	Clair	
Student 2	Huron	Ontario	Clair
Student 3	Ontario	Clair	Erie
Student 4	Huron	Clair	Ontario
Student 5	Ontario	Huron	Clair
Student 6	Clair	Erie	Ontario

This allocation method consists of the following steps:

Step 1 : A fair lottery determines the following student order: 1-2-3-4-5-6. Student 1 has ranked Huron as his top choice. However, the only slot at Huron is tentatively held by student 5. So student 5 is moved to the top of the queue.

Step 2 : The modified queue is now 5-1-2-3-4-6. Student 5 has ranked Ontario as his top choice. However, the only slot at Ontario is tentatively held by student 6. So student 6 is moved to the top of the queue.

Step 3 : The modified queue is now 6-5-1-2-3-4. Student 6 has ranked Clair as her top choice. The two slots at Clair are tentatively held by students 1 and 2. Between the two, student 1 is ahead in the queue. So student 1 is moved to the top of the queue.

Step 4 : The modified queue is now 1-6-5-2-3-4. Remember that student 1 has ranked Huron as his top choice. A cycle of applications is now made in sequence in the last three steps: student 1 applied to the tentative assignment of student 5, student 5 applied to the tentative assignment of student 6, and student 6 applied to the tentative assignment of student 1. These mutually beneficial exchanges are carried out: student 1 is assigned a slot at Huron, student 5 is assigned a slot at Ontario, and student 6 is assigned a slot at Clair. These students as well as their assignments are removed from the system.

Step 5 : The modified queue is now 2-3-4. There is one slot left at Clair and two slots left at Erie. Student 2 applies to Clair, which is her top choice between the two schools with remaining slots. Since student 2 tentatively holds a slot at Clair, her tentative assignment is finalized. Student 2 and her assignment are removed from the system.

Step 6 : The modified queue is now 3-4. There are two slots left at Erie. Student 3 applies to Erie, which is the only school with available slots. Since Student 3 tentatively holds a slot at Erie, her tentative assignment is finalized. Student 3 and her assignment are removed from the system.

Step 7 : The only remaining student is student 4. There is one slot left at Erie, but since Erie is not listed in student 4's Decision Sheet, she remains unassigned.

Final assignment Based on this method, the final allocations are:

Student	1	2	3	4	5	6
School	Huron	Clair	Erie	–	Ontario	Clair

Illustration

	Queue	Available Slots	The top student in the queue applies to a school.	At the end of the step
Step 1	1-2-3-4-5-6	Clair Clair Erie Erie Huron Ontario	1 applies to his 1st choice Huron, which is tentatively assigned to 5.	5 comes to the top. 1-2-3-4-**5**-6
Step 2	5-1-2-3-4-6	Clair Clair Erie Erie Huron Ontario	5 applies to his 1st choice Ontario which is tentatively assigned to 6.	6 comes to the top. 5-1-2-3-4-**6**
Step 3	6-5-1-2-3-4	Clair Clair Erie Erie Huron Ontario	6 applies to her 1st choice Clair, which is tentatively assigned to 1 and 2.	1 comes to the top. 6-5-**1**-2-3-4
Step 4	1-6-5-2-3-4	Clair Clair Erie Erie Huron Ontario	A cycle happens in the last 3 steps.	1 gets a slot at Huron. 5 gets a slot at Ontario. 6 gets a slot at Clair.
Step 5	2-3-4	Clair Erie Erie	2 applies to her 3rd choice Clair, because her 1st and 2nd choices (Huron and Ontario) are no longer available.	2 gets a slot at Clair, because she is a resident in Clair.
Step 6	3-4	Erie Erie	3 applies to Erie which is still available.	3 gets a slot at Erie, because she is a resident in Erie.
Step 7	4	Erie	4 does not apply to any other school since none of the schools in her list has a vacant slot.	4 remains without a slot, and Erie remains with a vacant seat.

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