Intra-party decision making and moderation under proportional representation*

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Abstract

We establish coalitional stable party structures of a party formation game under proportional representation. We assume farsighted political players which can commit to form parties, expressive voters and a policy selection process with random proposals subject to intra-party majority rule. Parties may form governments and each government offers a policy lottery. In a stable party structure, a moderating centre party ensures that the median voter either realizes an outcome which is strictly better than the status quo or her ideal point. If agents are risk neutral, party politics is stable against dictatorship.

Keywords: Endogenous political parties, intraparty decision rules, farsighted coalitional stability, political institutions.

JEL codes: C72, D71, D78.

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1 Introduction

1.1 Motivation

In their empirical investigation, Huber and Powell (1994) find that policy outcomes, when measured on a left-right scale, tend to be closer to the preferences of the median voter if the voting system is one of proportional representation (PR) rather than one of majority rule. This is surprising because at least the standard model of two candidate political competition - often seen as an idealized representation of majority voting systems\(^1\) - predicts the outcome preferred by the median voter of the median constituency.\(^2\) Birchfeld and Crepaz (1999) report that PR-systems are characterized by more redistribution relative to systems with MV. They relate their finding to the presence of "collective veto points" in PR-systems. Their argument is that under PR more players have to concede in order to change a policy position. A spatial analysis of the effect of veto points has been provided by Tsebelis (1995). However, in order to make the argument complete one has to show in how far such veto points apply under PR and do not apply in the same way under MV if parties consist of different factions\(^3\). Furthermore, one has to show that political players want to form parties and that those parties agree to form governments in which such veto points become binding. This has to be done in a way, which only takes institutional facts of the democratic decision process such as democratic decision making and the institutional setting of PR as given. In this paper I develop such a model with endogenous parties under PR where a veto is binding in equilibrium. Stable party and government structures of the model support policy outcomes which are close to the ideal point of the median voter.

The main idea driving the results of this paper is that parties can be thought of as democratic organizations which take decisions using majoritarian voting procedures.\(^4\) Once a coalition government is formed it is at least implicitly taken to be the modus operandi that with contentious issues the consent of all the government parties is required. Such differences in decision procedures can only have an effect if the government does not commit itself

\(^1\)See, for example, Schofield (1997).
\(^2\)See Calvert, 1985 for the robustness of the result. For an overview of convergence and non convergence results in models of multi-candidate competition see Myerson, 1999.
\(^3\)On the consequences of a party unanimity rule see Roemer (1999, 2001).
\(^4\)For empirical evidence on the importance of intra-party democracy see Bäck (2008).
to policies by writing them into the coalition treaty or if only such policies can be written into the contract which are renegotiation proof. We argue that parties offer a mechanism for their members to agree on new policy issues as they come along. This can be devised through polls among the party grass-roots on issues such as leadership or policy, formal decision making procedures at party conferences or informal procedures in which legislators coordinate their positions. Although such intra-party decision procedures are no formal part of the parliamentary decision making process, their outcome binds voting behavior of legislators either indirectly (via political pressure) or directly (via party whips).\textsuperscript{5} My approach nevertheless presents a departure from one of the standard assumptions in spatial models where it is typically assumed that parties can freely select the platform under which they run for election\textsuperscript{6} and that policies which are implemented by a coalition government after the election are a convex combination of their party platforms (see, for example Baron, 1993, Baron/Diermeier, 2001, Roemer, 2001).\textsuperscript{7} There is evidence that even in the case where pledges are made and a single party government is in charge, policy outcomes are only partially predetermined.\textsuperscript{8}

1.2 Outline of the model

We assume that socio economic groups can be ordered according to their preferences on a one dimensional policy space and that each group is represented by one policy motivated politician. Politicians may form coalitions - equivalently, parties - or they can drop out of the race. A party structure is a partition of the set of politicians into different parties and abstentions.

\textsuperscript{5}Another possible transmission mechanism runs via the cabinet. As Laver and Shepsle (1994, p. 9-10) report, party affiliation matters for the stance which actors take in the cabinet. Affiliation with factions within the party appear to be less important, which would suggest the effectiveness of some sort of intraparty decision making procedure.

\textsuperscript{6}Schofield/Parks (2000) address the credibility problem involved in selecting a party platform.

\textsuperscript{7}See, however, Persson/Roland/Tabellini (2003) for a non spatial model of policy selection.

\textsuperscript{8}In the United States manifesto pledge fulfillment has been ranging from 50% to 60% over the four electoral cycles starting in 1980 with the exception of the second Reagan term, see Royed/Borrelli (2002). The UK has a better record of pledge fulfillment with over 80% over the same time period (starting in 1979). Furthermore, pledges often give only the direction of policy change. For example, the extent of the subsequent cut in overall taxes is difficult to predict from the British 1979 conservative party manifesto.
Each voter for the party in which the representative of her socio economic group gets organized. Politicians, if they are member of one party, commit themselves to jointly vote in the legislature for the policy which is elected by the party. Each party structure gives rise to a set of (multiparty) governments which have a majority for this structure. For each such government, policies are selected under a random proposal maker process. In order to get approved, a majority in each of the parties which form the government has to get behind a proposal when voted against the status quo policy. A government is selected from the set of undominated majority coalitions of parties if such coalition exists. If no such government exists, the government may be formed from any majority coalition which is part of a cycle. Each agent evaluates a government according to the policy lottery which it generates. Such an assumption is reasonable if the institutional structure determines a distribution of policy outcomes. A stable party structure is a partition where no coalition can deviate in order to raise the expected utility of all its members. We assume that deviating agents are farsighted and behave conservatively (see Chwe, 1994, Diamantoudi/Xue, 2003).

1.3 Results

Under our behavioral assumptions a coalitional stable party structure generally exists. For the case where agents cannot drop out of the political competition, in any stable partition the median voter must be strictly better off with the policy lottery which this partition generates than with the status quo. The only situation in which this does not apply is where the status quo is already her ideal policy. Our result is due the possibility of forming a party close to the center which blocks all extreme policy proposals. This suggests that under proportional representation, policy outcomes should converge towards the median voter position, at least under expectations. If we allow for dropping out of the political competition, voluntary delegation to one agent may be a preferable option. However, if agents are risk neutral, party politics is always stable against delegation. Delegation itself may be challenged by party politics if the status quo is sufficiently close to the median voter’s ideal point.
1.4 Related literature

The model which most closely related to the one presented here is Levy (2004). Levy’s focus is on explaining the existence of political parties while I focus on the role of parties under PR. Whereas she shows that parties matter if the policy space is multidimensional I show that in PR systems parties form which moderate the policy outcome. In Levy’s model, politicians can commit to a policy platform if it is in the Pareto set of party members while in my paper politicians commit to accept the outcome of majoritarian party decisions. In both papers, parties are obtained from stable partitions of the set of politicians but Levy’s stability concept is binding agreements (see Ray and Vohra, 1997). This is an approach where agents are farsighted but coalitional moves are restricted to deviations which result in finer coalition structures. I apply farsighted conservative coaliotional stability (Chwe, 1994, Diamantoudi/Xue, 2003). This stability concept allows for more coalitional moves but assumes that agents behave conservatively when considering possible countermoves by other coalitions. I develop a model with multiple layers of majority voting. Such models have recently been analyzed by Humphreys (2001). Another closely related paper is Kaminski (2006). Kaminski’s focus is on the stability of electoral rules and political parties while my focus is on the relationship between party structure and policy results. In his approach parties are allowed to select platforms, to split or coalesce and a majority is allowed to change the electoral rule. In my paper, coalitional moves are unconstrained and a majority forms the government and selects policies.

Recent approaches to explaining the formation of political parties have been developed by Bandyopadhyay/Oak (2004), Osborne/Tourky (2004) and Jackson/Moselle (2003). Bandyopadhyay/Oak present a citizen candidacy model where the representatives of groups run for election or drop out of the race. The scope of their model are PR systems. Parties in multi party governments negotiate after the government has been formed, i.e. there is no commitment to policies at the government formation stage. Osborne/Tourky present a model where parties are stable coalitions of candidates. Their prediction is the emergence of a two party system under economies of scale irrespective of the electoral system. Jackson/Moselle explain the emergence of parties as an insurance device in a coalition bargaining model.

There are a few contributions which address the effect of voting systems on policy outcomes and party formation: In Austen-Smith (2002) parties represent economic classes where the latter are endogenously formed. Policies
with more redistribution are obtained under PR but policies under MV are closer to the median. Rivière (2003) shows moderating tendencies in PR systems with rational voters in an extension of Myerson/Weber (1993). Morelli (2004) determines effective parties and policies under PR and MV. The setting involves multiple districts with a multitude of candidates running in each district.

I set up the model in section 2. Section 3.1 characterizes outcomes in the case where every representative participates. Section 3.2 considers the case where representatives may drop out of the race. Section 4 discusses the relationship between assumptions and results. Section 5 concludes.

2 The model

I analyse a multi-stage game of party formation and policy making. In the first stage, parties are formed. In the second stage, voters cast their votes for a given party structure. Given the outcome of voting, parties form a government. In the final stage, policies are selected against the status quo policy.

The set of voters consists of \( n \) socio economic groups where each group is characterized by its preference maximum \( \tilde{x}^i \) in policy space \( X = [0, 1] \). Each group has equal size and for each group there is one politician who shares their preferences. The assumption of equal group size is not too restrictive as ideal points of different groups can be arbitrarily close and can be justified as equality in representation.9 The set of politicians is \( N \).

A policy outcome is a realization \( x \in X \). The utility function of a voter/politician belonging to group \( i \) with a preference maximum \( \tilde{x}^i \) is given by a general but symmetric utility representation \( u^i = v(|x - \tilde{x}^i|) \), where \( v \) is a concave and decreasing function.10 On lotteries \( \ell = (\Pr(x), x) \in X \) we define a von Neumann/Morgenstern utility function \( U^i(\ell) = \sum_{x \in X} \Pr(x) u^i(x) \).

9If we were to allow for differences in group size, in equilibrium members of a group may want to randomize their support between two representatives which are organized in different parties. My deterministic approach may be seen as the result of purification of strategies.

10Identical evaluation of distances across agents is restrictive but necessary if we want to relate our results on lotteries to the median voter position in a spatial sense. It is, however, less restrictive than the condition for deriving majority preferences over lotteries which cannot be appreciably extended beyond the case \( u^i = -|x - \tilde{x}^i|^2 \) (see Banks/Duggan, 2006).
2.1 Party formation

At the party formation stage politicians form coalitions between them, or equivalently, parties. In the subsequent stages of the game, players engage in a non-cooperative game which unfolds for the given coalition structure.

Definition 1 A coalition structure $\mathcal{C} = \{S_1, \ldots, S_J\}$ is a partition of $N$ which satisfies $S_i \cap S_j = \emptyset$ and $\bigcup_{i=1}^{J} S_i = N$. A party structure is a partition $\pi(\mathcal{C}) = \{P_1, \ldots, P_K\}$ where for each consecutive subcoalition $S_{ij} \subset S_j \in \mathcal{C}$: $P_k(S_{ij}) = S_{ij}$ if $S_{ij}$ participates and $P_k(S_{ij}) = \emptyset$ if $S_{ij}$ drops out of the race.

Thereby, I focus on equilibria where every party is connected, that is every agent with an ideal point between the ideal points of the most extreme party members is also a party member. For every party structure, the ensuing non-cooperative game in the later stages defines a lottery $\ell(\pi)$ and players order lotteries according to $\succ_i$. An order over party structures is derived from the order over the lotteries they induce.

Definition 2 (Spontaneous formation of coalitions) A coalition $T$ can replace the coalition structure $\mathcal{C} = \{S_1, \ldots, S_J\}$ with coalition structure $\{T, C-T\} = \{S_0^1, \ldots, S_0^J, T\}$ where $S_0^j = S_j \cap (N \setminus T)$.

So a coalition $S$ can replace a partition where this coalition is not formed by a partition where it is formed whilst the other coalitions or their remainders stay in place. If a coalition which is at the centre of a party breaks away, by definition 1 this leaves the two wings of the previous party and the deviating coalition each as a separate party.\footnote{While facilitating the exposition, this assumption can be dropped without changing our results (see the corollary of proposition 2).} It is, however, possible for the new parties to adjust and to merge with neighbouring subcoalitions. In this I assume that agents are farsighted, i.e. they take into account the response of other coalitions once they have deviated. This results in an indirect dominance relation:\footnote{My approach follows Chwe (1994) and Diamantoudi/Xue (2003).}

Definition 3 (Indirect dominance) A deviation from a position $\pi_a$ is attractive for a coalition $S$ of farsighted agents if for some $\pi_b \in \Pi$ the indirect dominance relationship holds $\pi_b \succ \pi_a$ holds: there is a sequence $\pi_k \xrightarrow{S_k} \pi_{k+1}$, for $k = 1, \ldots, K$ with $\pi_1 = \pi_a$ and $\pi_{K+1} = \pi_b$ and $\pi_b \succ S_k \pi_k$ for all $S_k$.}
For each position $\pi$ there are partitions which may be reached once the position has been attained:

**Definition 4** Feasible partitions given $\pi$, $X(\pi)$: $\{\pi\} \cup \{\pi' \in \Pi | \pi' \gg \pi\}$.

We impose a consistency requirement which each feasible position $\pi$ has to fulfill if it is to be considered stable: if a coalition $S$ can move from $\pi$ to another partition $\pi'$ and all stable partitions which can be reached from $\pi'$ are better for all members of $S$ than $\pi$, then $\pi$ is not stable: even conservative agents will necessarily defect from $\pi$. Let $\sigma(\pi) \subset X(\pi)$ denote the set of partitions which are feasible and stable given $\pi$. Following Greenberg (1990) we have:

**Definition 5** A party structure $\pi \in \Pi$ is coalitional farsighted conservative stable, or $\pi \in \sigma$, if there is no $S \subset N$ and $\pi \xrightarrow{S} \pi'$, with $\sigma(\pi') \neq \emptyset$ such that $\sigma(\pi') \succ_S \pi$ for all $\pi'' \in \sigma(\pi')$ and $\sigma$ satisfies external stability: for all $\pi' \in X(\pi) \setminus \sigma(\pi)$, there exists such $S$ such that $\sigma(\pi') \succ_S \pi$ for all $\pi'' \in \sigma(\pi')$.

### 2.2 Voting

Given a party structure $\pi = \{P_1, ..., P_K\}$, voters cast their votes according to $V(\pi)$. We assume that a voter votes for a party if she is represented by it.\(^1\) This means, a voter votes for the party in which the representative which is from the same group as the voter is getting organized. An alternative interpretation, following Osborne/Tourky, is that parties form for a given assembly after elections have taken place.

### 2.3 Government formation

After voters have cast their vote a government is formed. We assume that every government needs an absolute majority of seats in the assembly. This is somewhat restrictive, as we do not allow for minority governments to form.\(^2\) However, our approach accommodates for the possibility of governments with

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\(^1\)This may be motivated as expressive behavior. An implication is that a voter votes for a party which only potentially takes her position. On the empirical relationship between party loyalty and formation of policy preferences, see Hudson (1995).

\(^2\)This may reflect institutional arrangements by which the government has to pass an inaugural vote. See Diermeier/Eraslan/Merlo (2003) for an overview of rules governing the formation of governments.
a supermajority, i.e. with more parties than are necessary to ensure a parliamentary majority.

**Definition 6 (Majority governments given \( \pi \))** Let \( M(\pi) = \bigcup M_i(\pi) \) be the set of party constellations (i.e. multi party governments) \( M_i = \{P_1, \ldots, P_T \mid P \in \pi \} \) which given \( V(\pi) \) command a majority in the assembly.

Because there is uncertainty over which policy is adopted at the policy stage, each government in \( M(\pi) \) induces a lottery. So at the government formation stage, agents have to evaluate prospects with each government. A government \( M_i \) forms when all the parties that are members of \( M_i \) agree on it. A party \( P_k \) agrees on \( M_i \) if a majority of the members of \( P_k \) agree. We say that a government \( M_a \) is stable if, when presented with an alternative government, a majority of members of at least one of the parties which are needed to form the new government objects to the move. If no government is stable, we insist that some government will ultimately form which is potentially stable but we do not know which. We thus define the set of potential governments \( G(\pi) \):

**Definition 7 (Potential governments under PR)** A government \( M_a \) is dominated by a government \( M_b \), in short \( M_a \preceq M_b \), if \( \ell(M_b) \succ_i \ell(M_a) \) for a majority of agents \( i \) of every \( P_k \in M_b \). \( M_a \in M(\pi) \) is stable if \( M_a \preceq M_j \) for no \( M_j \in M(\pi) \). The set of potentially stable governments, \( G(\pi) \), contains all stable governments given \( \pi \). In the case where no government is stable, we define \( S(\pi) = \{M_j \mid M_j \in S(\pi) \} \) if \( M_k \preceq M_j \) for some \( M_k \in S(\pi) \) and \( G(\pi) \) as the union of such sets \( S(\pi) \).

Trivially, \( G(\pi) \) is non empty if \( \pi \neq \emptyset \). \( G(\pi) \) either consists of undominated governments or, if no undominated government exists, it consists of governments which are contained in some cycle in \( M(\pi) \). It is straightforward, that in the latter case \( G(\pi) \) coincides with the admissible set (Kalai/Schmeidler, 1977) of the government formation process. Whenever \( G(\pi) \) contains more than one government we assume that each agent \( i \in N \) evaluates partition \( \pi \) according to the worst government \( M_k \in G(\pi) \) which may form. Pessimism in the case where agents are agnostic about their strategic position in determining outcomes corresponds to convolutionism in a strategic context.15

15 Indeed we may support each \( M_i \in G(\pi) \) as a conservative stable standard of behavior in an appropriately defined government formation situation with farsighted agents (see Xue, 1998). See on the wider implications of pessimism Diamantoudi (2001).
2.4 Policy selection

The parties forming a government select a policy by a proposal maker procedure: A randomly selected politician \( i \) makes a proposal which wins a simple majority in each of the parties in government when voted against the status quo policy \( x^{sq} \). If no proposal is blocked and \( S \) forms the government, \( i \)'s proposal is \( p^i = \tilde{x}^i \) and the lottery associated with this government is

\[
\ell = (\Pr(p^i = \tilde{x}^i), \tilde{x}^i)_{i \in S} \quad \text{with} \quad \Pr(p^i = \tilde{x}^i) = 1/\#S \quad (1)
\]

where \( \#S \) is the cardinality of \( S \). We assume that \( \ell(\emptyset) = (1, x^{sq}) \), i.e. if no party participates, the status quo prevails. In order to deal with blocked proposals we define the (majoritarian) dominance set for each party relative to the status quo:

**Definition 8** Let the dominance set for a party \( P_k \), \( D(P_k, x^{sq}) \): \( \{ x \in X | x \succeq_j x^{sq} \} \) for a majority of all members \( j \) of \( P_k \).

If \( M = \{ P_k \} \) is a single party government, \( i \in P_k \) proposes

\[
p^i = \arg \max [u^i(p^i) \text{ s.t. } p^i \in D(P_k, x^{sq})].
\]

If \( M = \{ P_k \}_{k \leq K} \) is a multiparty government, \( i \in \cup_{k \leq K} P_k \) proposes

\[
p^i = \arg \max [u^i(p^i) \text{ s.t. } p^i \in \cap_{k \leq K} D(P_k, x^{sq})].
\]

For a multiparty government, a proposal must lie in the intersection of the dominance sets of the government parties. For the interpretation of our results it is useful to note that the majoritarian dominance set of a union of two parties strictly contains the intersection of the single party dominance sets in all but trivial cases. This means that if a unitary government party is divided into two and the two parts form a coalition government, the dominance set of the coalition government is strictly contained in the dominance set of the unitary party government.\(^{16}\)

\(^{16}\)Under party unanimity, defining the dominance set accordingly, it is easy to see that the dominance set of a union coincides with the intersection of dominance sets of its parts.
3 Results

Before I go on deriving stable party structures I establish two essential results for the game at hand. The first is a lemma which allows us to relate individual rankings over lotteries if those lotteries are comparable by first order stochastic dominance. The proof of this result only uses the assumption that agents have a common evaluation of distances, \( v \), so the condition under which lemma 1 applies is weaker than the condition for deriving majority preferences over lotteries (see Banks/Duggan, 2006).

**Lemma 1** For any two lotteries \( c_1 \) and \( c_2 \) where \( c_1 \) stochastically dominates \( c_2 \), if \( i \) prefers \( c_1 \) over \( c_2 \) and \( a_i < a_j \), then \( j \) also prefers \( c_1 \) over \( c_2 \).

**Proof.** See appendix

The next result which establishes existence of a coalitional farsighted party structure is due to Chwe (1994). Note that our definitions of the party formation game define \( \Gamma = (N, \Pi, \{\succ_i\}_{i \in N}, \{\longrightarrow\}_{S \subset N, S \neq \emptyset}) \) where \( \Pi \) is finite and \( \{\succ_i\} \) on \( \Pi \) is defined by \( U(\ell(M)) \) for each \( M \in G(\pi) \) and the assignment of potential governments for each \( \pi \) according to \( G(\pi) \) in the definitions of the government formation game.

**Proposition 1** Given \( \Gamma \) there exists \( \sigma(\pi) \subset X(\pi) \), such that \( \sigma(\pi) \neq \emptyset \) for all \( \pi \in \Pi \).

**Proof.** Chwe (1994, proposition 1) shows that for every \( \Gamma \) there exists a largest consistent set (\( LCS(\Gamma) \)). The largest consistent set is non empty (Chwe, 1994, proposition 2) if there are no infinite \( \ll \)-chains. This must hold in the case of finite \( \Pi \). Defining a conservative stable standard of behavior for the Chwe situation (analogous to our definition 5) he establishes that \( \sigma(\pi) = X(\pi) \cap LCS(\Gamma) \) is a conservative stable standard of behavior and that \( \sigma(\pi) \) is non empty for all \( \pi \in \Pi \) (Chwe, 1994, proposition 4).

3.1 The model with no drop outs

Initially, I analyse the case where the representative of each group runs for office. I assume that \( n \) is odd and denominate \( m \) the median voter and \( \hat{m} \) her ideal position. For presentational convenience, I assume that the status quo satisfies \( x^{eq} \geq \hat{m} \) throughout the paper. Let there be at least three groups, \( n \geq 3 \) so we can define an initial partition \( \pi^0 = \{L, m, R\} \). Denominate
\( \hat{m}_P \) the ideal point of the median of \( P \). To deal with the case where \#\( P \) is even, let \( \hat{m}_P \) be the ideal point of the agent who converts a minority into a majority when counted from above and \( \hat{m}_P \) the ideal point of the agent who does the same if the counting starts from below. Note that \( \hat{m}_P \leq \hat{m}_P \) where equality holds if \#\( P \) is odd. \( \hat{x}^1 \) is the ideal point of the left-most agent.

Lemma 2 Say \( \hat{m} \leq x^0 \leq \hat{m}_{\pi^*} \), \( x^0 \succ_m \hat{x}^1 \) and that \( \pi^0 \) has formed. In the case \( \hat{m} < x^0 \), \( \mathcal{L} \) and \( m \) form the government and in the case \( \hat{m} = x^0 \), any \( M_i \in M(\pi^0) \) may be the government.

Proof. \( M(\pi^0) = \{ M_{m_L}, M_{\mathcal{L}m}, M_{LR} \} \) with \( M_{m_L} = \{ \mathcal{L}, m \} \) and \( M_{LR} = \{ \mathcal{L}, R \} \). \( M_{m_L} \) and \( M_{LR} \) implement \( x^0 \) with certainty for \( \hat{m} < x^0 \leq \hat{m}_{\pi^*} \). Any \( k \in M_i \) with \( \hat{x}^k < x^0 \) proposes \( p^k = x^0 \) as a proposal \( p^k < x^0 \) is blocked by \( \hat{m}_{\pi^*} \). Any \( k \in M_i \) with \( \hat{x}^k > x^0 \) proposes \( x^0 \) because a proposal \( p^k > x^0 \) is blocked by the other government party (\( m \) or \( \mathcal{L} \)). Therefore, \( (1, x^0) \sim_m \ell(M_{m_L}) \). Now consider \( M_{m_L} = \{ \mathcal{L}, m \} \) and \( x^0 > \hat{m} \): Any proposal by \( j \in \mathcal{L} \) has to satisfy \( p^j \succeq_m x^0 \) and \( m \)'s own proposal is \( p^m \succ_m x^0 \). Therefore, \( \ell(M_{m_L}) \succ_{\mathcal{L}} \ell(M_{LR}) \) and \( \ell(M_{m_L}) \succ_m \ell(M_{m_L}) \) and \( M_{m_L} \) is the only stable government. Finally, in the case where \( x^0 = \hat{m} \), every \( M_i \in M(\pi^0) \) implements the status quo with certainty and is stable.

Under the condition on the ideal policy of the left-most agent, some policy proposals in \( M_{m_L} \) are blocked. If the proposal by an agent on the left is blocked, this agent proposes the smallest \( x \) which satisfies \( x \succeq_{i^*} x^0 \) for the agent \( i^* \) who is critical in blocking the policy. In forming \( M_{m_L} \), \( m \) becomes critical in blocking proposals. If the multiparty government \( M_{m_L} \) where \( M_{m_L} \) is replaced by a government of a unitary party \( L = m \cup \mathcal{L} \), its median \( m_L \) would be critical in blocking. Under the condition \( x^0 \succeq_m \hat{x}^1 \), \( m \) prefers \( \ell(M_{m_L}) \) over \( \ell(L) \). If \( x^0 \leq \hat{m}_{\pi^*} \), \( M_{m_L} \) offers the lottery \( \ell = (1, x^0) \) because every proposal other than \( x^0 \) is blocked by some party median. If, however, \( x^0 > \hat{m}_{\pi^*} \), \( m_{\pi^*} \) blocks any proposal which is more extreme than \( x^0 \). The following lemma captures those facts:

Lemma 3 Say that (a) \( x^0 > \hat{m}_{\pi^*} \) or (b) \( \hat{x}^1 \succeq_m x^0 \). In case (a), \( \{ m, \mathcal{R} \} \) offers \( m \) a payoff which for \( m \) is better than the payoff with the status quo, in case (b), \( L \) does.

Proof. (a) For \( x^0 > \hat{m}_{\pi^*} \) with \( \{ m, \mathcal{R} \} \) every proposal \( p > x^0 \) is blocked. (b) The worst outcome with \( L = m \cup \mathcal{L} \) is as good as the status quo, so
Because the lotteries are dominance comparable, \( m \) and \( L \) together can guarantee themselves an outcome which is better than the status quo.

Finally consider a centre right party \( R = m \cup R \) for which \( \hat{m}_R \leq \hat{m}_R \). For \( x^{sq} > \hat{m}_R \), the result of lemma 3 similarly applies, i.e. \( R \) offers a lottery which \( m \) prefers to the status quo. For \( x^{sq} < \hat{m}_R \), it offers a lottery which stochastically dominates \( \ell = (1, x^{sq}) \) and for \( \hat{m}_R \leq x^{sq} \leq \hat{m}_R \), it offers the status quo. While \( R \) may form, \( m \) can guarantee that it does not form whenever it offers less to her than the status quo. We, therefore, do not have to particularly treat the case where \( R \) forms separately.

Our next result generalizes the blocking power used in lemma 2:

**Lemma 4** If a government \( K' \) includes as its right-most party some \( P_k \) with \( \hat{m}_{P_k} < x^{sq} \) then \( \ell(M_{mL}) \geq_m (1, x^{sq}) \) for all \( i \leq m \).

**Proof.** Let \( x' \) be the smallest proposal \( p \) for which \( p \geq_i x^{sq} \) for \( i \) with ideal point \( \hat{m}_{P_k} \). Any proposal in \( K' \) satisfies \( x' \leq p \leq x^{sq} \) with the last relationship strict for some \( p \) from which the claim follows.

Subdividing \( L \) into smaller units \( L_1, L_2, ..., L_K \) reproduces the lottery \( \ell(M_{mL}) \) if \( L_1, ..., L_K \) and \( m \) form the government. This is because \( m \) is decisive in blocking a proposal. However, if \( L \) is subdivided, smaller parties on the right of \( m \) could now enter the scene and replace some party \( L_j \) in \( M(\pi) \). If a stable government exists which beats \( \ell(M_{mL}) \), this involves more blocking and is an advantage for \( m \). However, there might be a cycle including this government. We cannot in principle rule out that some agent \( i < m \) joins an agent \( k > m \) in forming a government which \( m \) dislikes.\(^{17}\) \( m \) can, however, enforce a lottery which is better than the status quo:

**Lemma 5** Let \( \Pi_w \) the set of partitions where \( m \) runs as a singleton. Then for every \( \pi \in \Pi_w \), \( m \) realizes a pay off which is strictly better than \( x^{sq} \).

**Proof.** Say in \( \pi \in \Pi_w \), \( \overline{L} \) subdivides into coalitions \( L_1, ..., L_J \) and \( \overline{R} \) into \( R_1, ..., R_K \). If \( m \) forms a party of its own then \( M_0 = \{ L_1, ..., L_J, m \} \) is the set of minimum winning governments. The lotteries \( \ell(M_{mL}) \) and \( \ell(M_0) \) have the same distribution of pay offs, \( \ell(M_0) \sim_m \ell(M_{mL}) \geq_m x^{sq} \) by lemma 2.\(^{17}\) We initially assume that \( x^{sq} \leq \overline{m}_{\overline{L}} \).

\(^{17}\)Banks and Duggan provide an example where a lottery is preferred by an opposing pair of extreme voters.
First, say that a stable government exists. Then either $M_0$ is stable and/or there is a stable government $M_k$ such that $M_k \not\subseteq M_0$ is not true. In the latter case, there must be some $i \leq m$ such that $\ell(M_k) \succeq_i \ell(M_0)$. But because $\ell(M_k)$ stochastically dominates $\ell(M_0)$ (recall that $M_0$ comprises a simple majority of all the agents on the left) it must also be true that $\ell(M_k) \succeq_m \ell(M_0)$ by lemma 1. Finally, $\ell(M_0) \succ_m (1, x^{sq})$ by lemma 3.

Next, say that there is not a stable government, so $M_i \in G(\pi) \Leftrightarrow M_k \subseteq M_i$ for some $M_k \in G(\pi)$. Say $M_0 \notin G(\pi)$ then $M_k \not\subseteq M_0$ for no $M_k \in G(\pi)$. Repeating the argument above, $\ell(M_k) \succeq_m \ell(M_0) \succ_m x^{sq}$ by lemma 1. So consider the case where $M_0 \in G(\pi)$. By lemma 4 all governments where for the righ-most $P_k \in M$, $\hat{m}_P < x^{sq}$, offer a better position than $x^{sq}$. In particular, this is true of $M_0$. Now suppose that a cycle includes a government $M_i$ which offers $(1, x^{sq})$. Because all governments with $\hat{m}_P \geq x^{sq}$ offer $(1, x^{sq})$, for a cycle to exist it must be the case that there is $M_k \in G(\pi)$ and $M_k \subseteq M_i$ where $M_k$ offers a better position than $x^{sq}$ to $m$. If $\ell(M_k) \succ (1, x^{sq})$ we have $\ell(M_k) \succeq_i (1, x^{sq})$ for all $\hat{x}^i \leq m$ because $\ell(M_k)$ is stochastically dominated by $(1, x^{sq})$. But in order to have a majority, there must be at least one party included in $M_i$ where an agent with with $\hat{x}^i \leq \hat{m}$ is decisive: If $M_i$ includes the party where $m$ is a singleton, $m$ herself is decisive. If $M_i$ does not include $m$, there must be a party $L_i \in M_i$ consisting entirely of agents with $\hat{x}^i < \hat{m}$. But then $M_k \subseteq M_i$ is false, a contradiction.

Finally consider the case where $x^{sq} > \hat{m}_P$ and $M_{m\overline{P}}$; by lemma 3, offers a lottery which is stochastically dominated by $(1, x^{sq})$. Say that there is $M_n$ which offers $(1, x^{sq})$. Now $m$ and all agents with $x^i \leq \hat{m}$ prefer $\ell(M_{m\overline{P}})$, so $M_n$ is not in $G(\pi)$. ■

So irrespective of the initial party structure, if $m$ breaks away and runs as a singleton, in any government which may form - and which may or may not include her, $m$ gets a better outcome than $x^{sq}$. This has consequences for the set of stable party structures:

**Proposition 2** A party structure is not stable if $m$ assigns it a pay off of $x^{sq}$ or worse.

**Proof.** Say $\pi' \in \Pi_w$. Then by lemma 5, $\ell(M_i) \succ_m (1, x^{sq})$ for all $M_i \in G(\pi')$. Now consider $\pi'' \notin \Pi_w$ with $\pi'' \gg \pi'$. Obviously, there must be a move $\pi_i \overset{S}{\rightarrow} \pi_{i+1}$ where $m \in S$ and $\pi_i \in \Pi_w$ but $\pi_{i+1} \notin \Pi_w$. But then $\pi'' \succ_S \pi_i$ and in particular, $\pi'' \succ_m \pi_i \succ_m (1, x^{sq})$. Therefore, $\pi'' \in X(\pi')$ implies $\pi'' \succ (1, x^{sq})$. Next define $CDOM(\sigma)(\pi)$: $\{\pi' \in X(\pi): \exists S, \pi_d$ such
that \( \pi' \xrightarrow{S} \pi_d, \sigma(\pi_d) \neq \emptyset \) and \( \pi' <_S \pi_e \forall \pi_e \in \sigma(\pi_d) \), consisting of partitions \( \pi \) which are not stable once \( \pi \) has been reached. Now consider \( \pi \) with an offer \( \ell(M_i) \preceq_m (1, x^{eq}) \) for some \( M_i \in G(\pi) \). But then \( \pi \xrightarrow{m} \pi' \) with \( \pi' \in \Pi_w \). Because \( \sigma(\pi') \subseteq X(\pi'), \sigma(\pi') >_m (1, x^{eq}) \) for all \( \sigma(\pi') \), so \( \pi \in CDOM(\sigma)(\pi) \).

The corollary shows that we can suspend the assumption of definition 1 that only the connected remainders of a party form a new party.

**Corollary 1** Suppose that if \( T \subset S \) leaves \( S \) with party \( P_k = S \), then \( P_k' = S \setminus T \), i.e. even unconnected wings stay together. In that case, proposition 2 also applies.

**Proof.** Problems may occur in the proof of lemma 5 if the remainder of the party which \( m \) leaves behind, \( W \), has two wings, these stay together and have a joint median with \( \hat{m}_W > \hat{m} \). In this situation the following claims may fail to hold: (a) not \( M_k \cup M_0 \) implies that there are some \( i \leq m \) for which \( M_k \preceq_i M_0 \) and (b) there is a decisive agent in \( M_l \) for which \( M_l \succ_i M_k \). Suppose that in any of these cases the lottery \( (1, x^{eq}) \) is offered in \( \hat{\pi}_w \) with \( W \in \hat{\pi}_w \). Show that in this case the left wing of \( W \), \( WL = W \cap \mathcal{L} \) prefers to be on its own and \( \hat{\pi}_w \in CDOM(\hat{\pi}_w, \sigma) \): Note that by proposition 1, \( \sigma(\hat{\pi}_w) \neq \emptyset \). Let \( \Pi^*_w \), the set of partitions with \( m \) and \( WL \) as elements. From the proof of proposition 2, for all \( \pi'' \in \sigma(\pi^*_w), \pi^*_w \in \Pi^*_w \): \( \ell(\pi'') \prec_W (1, x^{eq}) \) therefore, \( \pi'' \succ_W (1, x^{eq}) \) for all \( \pi'' \in \sigma(\hat{\pi}_w) \). Note that \( \ell(\pi^*_w) \) is stochastically dominated by \( (1, x^{eq}) \) and every agent in \( WL \) prefers \( \ell(\pi^*_w) \) because \( m \) does. Now consider \( \pi \) with \( \ell(\pi) = (1, x^{eq}) \) and \( \{m \cup W\} \in \pi \). Then \( \pi \xrightarrow{m} \hat{\pi}_w \) and \( \pi'' \succ_m \pi \) for all \( \pi'' \in \sigma(\hat{\pi}_w) \), so \( \pi \in CDOM(\sigma)(\pi) \).

### 3.2 Model with drop outs

In most of the literature, the decision to run or not to run for office is the focus of a model of party formation (see, for example Bandhyopadhyay/Oak, 2004, or Levy, 2004). Suppose, therefore, that before the party formation game starts, agents can (simultaneously) announce that they drop out of the race and that such an announcement is definite. In accordance with our previous assumption, we maintain that agents are pessimistic and assume that the worst lottery comes up if more than one lottery is compatible with any combination of drop out strategies. Proposition 1 applies to the situation with drop outs as well: If groups are allowed to drop out of the race, they
bring about a set $N' \subset N$ of politicians still in the race. If $N' \neq \emptyset$, we have the "sub" game $\Gamma' = (N, \Pi', \{\succ_i\}_{i \in N}, \{\longrightarrow\}_{S \subset N, S \neq \emptyset})$. A conservative standard of behavior exists for this game and proposition 2 holds with $m$ replaced by $m'$. Because $\ell(\emptyset) = x^{sq}$ and for each $N' \neq \emptyset$ there is $m' \in N'$ which improves over $x^{sq}$, there are always agents who do not want to drop out of the race.

There might now be, however, a case for a "great deviation" in the course of which one agent ends up with sole decision making power. If risk aversion is sufficiently strong and the lottery which agents anticipate is sufficiently dispersed, such a move can never be ruled out. For the case of risk neutral agents, however, we can show that an equilibrium partition without drop outs is stable against this kind of delegation.

**Proposition 3** For $u^i = -|x - \widehat{x}^i|$, every stable partition is stable against delegation.

**Proof.** Say all agents anticipate $\ell(\pi)$ in $N$ and let $E(\ell(\pi)) = \widehat{x}^d + \varepsilon$, $\varepsilon \leq 0$ where $d$ is the delegate. So consider the drop out of $N \setminus d$. There are $k \in N \setminus d$ with $\widehat{x}^k > \widehat{x}^d + \varepsilon$ and/or $\widehat{x}^k < \widehat{x}^d + \varepsilon$ for whom $\ell(\pi)$ is a one-sided lottery (candidates are right and left extremists). For one extreme $k$, $E(\ell(\pi)) \succ_k \ell(1, \widehat{x}^d)$ and, therefore $\widehat{x}^d \succ_{N \setminus d} E(\ell(\pi))$ does not hold. Note that this also holds in the case where $x^{sq} = \widehat{m} = E(\ell(\pi))$: delegation to $m$ would only guarantee $x^{sq}$ and, therefore, not dominate $E(\ell(\pi))$. □

The reason why this result cannot be extended beyond risk neutrality is easily seen if we consider a simple lottery with three outcomes and a delegated outcome which picks the outcome in the middle. Then for every agent there is one lottery outcome which is worse than the outcome under delegation and if we assume a sufficiently high degree of risk aversion, this will result in a preference for delegation. On the other hand, delegation (i.e. a situation where we start with one agent running) is not stable if agents are given the opportunity to announce (simultaneously) that they are running as well and $x^{sq}$ is sufficiently close to the median voter position.

**Proposition 4** If $x^{sq}$ is sufficiently close to $m$, delegation is not an equilibrium.

**Proof.** Say $x^{sq} \longrightarrow \widehat{m}$ from above. In this case it is straightforward that any $\pi \in \Pi_w$ is stable and every stable government offers $\ell(M_0)$ which is
\[(\frac{2}{n}, \hat{m}), (\frac{n-1}{n+1}, p)\] where \(p \rightarrow \hat{m}\) from below. Now say that under delegation the outcome realized is \(\hat{m}\). In that case, every agent with \(\hat{x}^i < \hat{m}\) prefers the lottery over \(\hat{m}\) and enters the race. Recognizing this, every agent with \(\hat{x}^i > \hat{m}\) will also enter the race. Finally, it is straightforward that there is also no outcome different from \(\hat{m}\) which stable under delegation.

So despite the fact that our model generates policy uncertainty, the solution is quite robust even if we open the way to removing the policy uncertainty. In particular, this result becomes even more robust as the status quo approaches the median position in the sense that under "systems competition" proportional representation emerges even if we start with a dictatorship.

## 4 Discussion

In the following section I discuss the necessity of our assumptions for obtaining the results of the party formation model.

### 4.1 Attitudes to risk

Our results on multiparty competition survive even in the case where drop outs are possible if agents are risk neutral. This is because agents are policy oriented and, if risk averse, they might universally prefer delegating policy decisions to one representative rather than living with the ambiguity of having parties. Interestingly, in the model of Moselle/Jackson, parties emerge because they insure against risk. In our model parties generate risk and delegation or proportional representation are different ways of reducing risk. If the status quo is sufficiently close but different from the status quo, proportional representation is the more successful way of dealing with risk.

### 4.2 Non commitment versus commitment

Policy commitments by parties before the election must in some way be compromised if parties enter multiparty governments. Say, therefore, that parties bargain over policy positions after elections. Once we have accepted that bargaining over positions will take place, we are left with two possibilities of influencing post election behavior: The strategic selection of politicians at the preelection stage or the signing up to institutions which govern behavior. The model presented in this paper combines both aspects.
4.3 Majority intra-party decisions

If we replace the assumption of majority intra-party decisions by unanimous intra-party decisions a party will approve of policies within the joint Pareto-set of its party members. Under our assumption that government policies are selected from the set of policies which are jointly acceptable for all parties in the government, we would find that unifying or subdividing parties for a given government does not change policy outcomes. In that case the party structure would have no explanatory power for policies in multiparty governments.

4.4 Proportional representation versus majority voting

We have shown that under PR, the median voter is able to secure an outcome which is strictly better than the status quo. To show that this is non trivial, consider the following highly stylized example of a majoritarian system. Suppose that we have agents A, B, C, D, E ordered along the unit line with their ideal points located at (0, 0.3, 0.6, 0.8, 1). C is the median voter and \( x^q = 0.6 + \varepsilon, 0 < \varepsilon < 0.2 \). Say that any party with a simple majority wins the elections and that if \( \pi' \) results in a draw, \( M(\pi') = \emptyset, \ell(\emptyset) = (1, x^q) \). In this case, \( \pi' = \{(A, B), C, (D, E)\} \) gives \( M(\pi') = \emptyset \) and the realization of \( x^q \). If we say that in the case where two parties in \( \pi' \) tie, a fair coin is cast we can show that even partitions which result in lotteries which are worse for \( m \) than \( (1, x^q) \) can be stable. Suppose that \( x^q = 0.7 \) and that agents are risk neutral. \( \{A, B, (C, D, E)\} \) among others is stable yet it gives \( m \) a pay off of \(-0.233\) which is worse than \( (1, x^q) \).

4.5 Voter behavior

If we replace the assumption of expressive voting by sincere voting, in any partition where some politician gets something worse than the outcome preferred by the median voter she will drop out of the race. The stable partition is trivial: Only the median voter’s representative will run. As far as rational voter behavior is concerned, it results in multiple equilibria even under majority voting (see Myerson/Weber, 1993 and Rivière, 2003).
4.6 Farsighted behavior

The solution concept which we have used is particularly robust in that we know that for any partition $\pi$ there must exist a conservatively coalitional stable standard of behavior which is non empty. Optimistically stable standards of behavior for this game may not exist. Although $m$ is in a fairly strong position in enforcing an outcome, neither can she enforce the outcome on her own (as in Pech, 2006a) nor is the outcome exclusively dependent on the coalition which includes $m$ (as in Pech, 2006b). Solutions according to myopic concepts like strong Nash equilibrium may not exist either. One notable exception is the case where $x^{sq} \rightarrow m$ and where $m$ is a singleton and forms a government with all agents on her left. Any such partition ($\pi \in \Pi_w$) is stable under conservatism or optimism and it is a strong Nash equilibrium.

5 Conclusion

We have shown that $PR$ is moderating if the political game is characterized by non commitment, intra-party majority rule and expressive voter behavior. While the assumptions of our model - particularly non-commitment - diverge from the standard set of assumptions in spatial models of political competition, they are in no way less extreme. Our model explains an empirical regularity which is not easily reconciled with the standard model. This suggests that a complete description of the political process needs to account in some form of the political uncertainty which is generated by the post-election political process.

6 Appendix: Proof of lemma 1

Proof. First we define a differential lottery as follows: For lotteries over a random variable $\xi$ we write $\ell = (\Pr(\xi = x_1), x_1),..., (\Pr(\xi = x_i), x_i),..., (\Pr(\xi = x_n), x_n)$ or equivalently $\ell = (x_i, dF(x_i))_{i=1}^n$ with $dF = \Pr(\xi \leq x_i) - \Pr(\xi \leq x_{i-1})$. Now say the lottery $\ell^b$ stochastically first order dominates $\ell^a$ in the sense that for any $x \in (0, 1)$, $F^b(x) \leq F^a(x)$.

From lottery $\ell^a$ collect $(x_1^a, x_2^a, ..., x_i^a) \ldots x_j^a$ where for each $x_j$ we have $(\Pr(\xi = x_j), x_j) \in \ell^a$ with $\Pr(\xi = x_j) \neq 0$. From lottery $\ell^b$ collect in the same way $(x_1^b, ..., x_i^b, ..., x_n^b)$. For the interval $[x_1^a, x_2^a)$, determine $x_1^a \leq x_1^b, ..., x_i^a \leq x_i^b, ..., x_n^b$ and assign $x_{11}^b := x_1^b, x_{12}^b := x_2^b, ..., x_{1n_i}^b := x_n^b$. If
there does not exist \( x^b_i \in [x^a_1, x^a_2] \) let \( x^a_{i1} := \Delta(x^a_i) \) where \( \Delta(x^a_i) \) is the smallest \( x^b_i > x^a_i \). For the interval \([x^a_2, x^a_3]\), determine \( x^a_3 \leq x^a_{n1+1}, \ldots, x^a_{n} < x^a_3 \) and let \( x_{21} := x^a_{n1+1}, \ldots, x^a_{n1+n2} := x_{2n2} \), otherwise let \( x^b_{21} := \Delta(x^a_i) \). Repeat for all \([x^a_j, x^a_{j+1}]\), \( j > 2 \), \( x^a_{j+1} = 1 \). By stochastic dominance this construction is possible and results in the differential lottery\(^{18}\) \((x_{jk} - x_j, dF^{jk})_{k(j)=nj+1}^{k(j)=n} \) with \( dF^{jk} > 0 \) and \( x_{jk} - x_j \geq 0 \). The utility differential between the lotteries is \( \Delta U^h = \sum_{j=1}^d \sum_{k=1}^{\max(nj-1)} dF^{jk} (u^h(x_{jk}) - u^h(x_j)) \). Now let \( \Delta u^h(\Delta x_{jk}) = u^h(x_{jk}) - u^h(x_j) \) where \( \Delta x_{jk} = x_{jk} - x_j \geq 0 \) and similarly for \( \Delta U^l \). Idealpoints are \( \vec{x}^h > \vec{x}^b \). If \( \Delta u^h(\Delta x_{jk}) > 0 \) (i.e. \( x_j \) is on the left of \( \vec{x}^h \) and \( |x_j - \vec{x}^h| > |x_{jk} - \vec{x}^h| \)) we have \( 0 < \Delta u^h(\Delta x_{jk}) \leq \Delta u^l(\Delta x_{jk}) \). If \( \Delta u^h(\Delta x) < 0 \) (\( x \) is on the right of \( \vec{x}^h \) and \( |x_j - \vec{x}^h| \leq |x_{jk} - \vec{x}^h| \)) we either have \( \Delta u^l(\Delta x_{jk}) > 0 \) or \( \Delta u^l(\Delta x_{jk}) < 0 \) but with \( \Delta u^h(\Delta x_{jk}) \leq \Delta u^l(\Delta x_{jk}) \). Therefore, \( \Delta U^h \leq \Delta U^l \). \( \blacksquare \)

References


\(^{18}\)Here we make the convention \( x^{(0)} \equiv x^i \) and \( x^{nj+1} \equiv x^{i+1} \), so we can conveniently write \( dF^{jk} = \Pr(\xi \leq x_{jk}) - \Pr(\xi \leq x_{j}^{(k-1)}) \). Measures for \( \Pr(\xi \leq x_{jk}) \) are taken under \( F^u \) and measures for \( \Pr(\xi \leq x_{jk}), x_{jk} \neq x^j \) are taken under \( F^b \).


