Flat rate taxes and relative poverty

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Abstract

In this paper, we consider the effects of both a flat rate tax and a negative income tax on the level of relative poverty. Specifically, we analyze how revenue-neutral parameter changes for these tax systems affect the level of relative poverty. Using micro-level data for Canada, we find that, poverty could be reduced from its current level with quite reasonable marginal tax rates under either of these systems.

Key words poverty · flat rate tax · negative income tax
JEL Classification H24 · I32 · I38

1 Introduction

Flat rate income taxes have garnered much attention in both the political and academic arenas for quite some time now. This attention has grown in recent years, with several Eastern European economies (e.g., Russia) adopting flat rate tax (FRT) systems. While there is some agreement about the reduced

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complexity and improved efficiency that such tax systems offer, there have been concerns about possibly undesirable distributional impacts.

Several studies, notably Davies and Hoy (2002) and Chiu (2006), consider how the adoption of an FRT, or its closely related alternative, a negative income tax (NIT), would affect income inequality. These studies find that an FRT (or NIT) can reduce inequality with very reasonable marginal tax rates. In this paper, we consider the implications of both an FRT and NIT on relative poverty. We find that, similarly, either of these alternative tax systems can reduce relative poverty from its current level with very reasonable marginal tax rates.

2 Poverty measurement

Throughout this paper we employ the following notation. Suppose that a population of $n$ households has pre-tax income vector $y = (y_1, \ldots, y_n)$, where $y_i \geq 0$ for all $i$. The equivalized pre-tax income vector of this population is $y^e = y/e$, where $e = (e_1, \ldots, e_n)$ is a vector of equivalence weightings, with $e_i \geq 1$ for all $i$. Without any loss of generality, we assume that the elements of $y^e$ are sorted in non-decreasing order, i.e., $y^e_1 \leq y^e_2 \leq \ldots \leq y^e_n$.

Taxes paid by the $i$th household are denoted by $T_i$, which may be positive or negative. The equivalized post-tax income of the $i$th household is therefore $(y_i - T_i)/e_i \geq 0$, which will always be non-negative.

A population will be said to contain $q \in [0, n]$ poor households, for which equivalized post-tax incomes are less than or equal to a poverty line $z > 0$. Here, we take a relative approach to identifying poverty, setting the poverty line at some fraction, $\beta \in (0, 1)$, of the median level of equivalized post-tax incomes.

A poverty measure is a real valued function $P : \mathbb{R}^q \to \mathbb{R}$, applied here to the $q$-vector of equivalized post-tax incomes of the poor. That is, it is implicitly
assumed that poverty measures are independent of the incomes of the non-poor.\footnote{This is known as the focus axiom in the poverty measurement literature. This axiom does not preclude the poverty line from being dependent on the incomes of the non-poor.} Specifically, we focus our attention here on poverty measures in the class introduced by Foster et al. (1984). Applied to equivalized post-tax incomes, measures in this class have the form

$$P_\gamma = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{z - (y_i - T_i)/e_i}{z} \right)^\gamma, \quad \gamma \geq 0. \quad (1)$$

The parameter $\gamma$ can be viewed as an indicator of “poverty aversion”. Higher values of $\gamma$ put a greater weight on larger relative poverty gaps (the relative differences between the poverty line and the equivalized post-tax incomes of poor households). Note that $P_0 = q/n$ is the so-called headcount ratio (i.e., the proportion of households in a population that are poor). For the empirical illustrations presented below, we consider values of 0, 1, and 2 for $\gamma$.

3 Flat rate tax

Under a flat rate tax (FRT), the tax paid by the $i$th household is

$$T_{F,i} = t_F \max(y_i - e_iA_F, 0), \quad i = 1, \ldots, n,$$

where $t_F \in (0, 1)$ is the marginal tax rate and $A_F \geq 0$ is the allowance level. The equivalized post-tax income of the $i$th household is therefore

$$\frac{y_i - T_{F,i}}{e_i} = \frac{y_i - t_F \max(y_i - e_iA_F, 0)}{e_i} = \frac{y_i^e - t_F \max(y_i^e - A_F, 0)}{e_i}.$$
This should make it clear that the ordering of pre- and post-tax equivalized incomes is identical, i.e.,

$$y^e_i - t_F \max(y^e_i - A_F, 0) > y^e_j - t_F \max(y^e_j - A_F, 0),$$

for all $y^e_i > y^e_j$. Thus, for example, a household with equivalized pre-tax income equal to the median level will have equivalized post-tax income equal to the median level.

Here, we are interested in analyzing how changes to an FRT (specifically, changes to the allowance level, $A_F$) can affect the level of poverty. In considering any change in tax policy, we assume that the amount of tax revenue, which we will denote by $R$, is exogenous to the choice of the tax system and its parameters. As a result, any change in tax policy must be revenue-neutral.

Under an FRT, tax revenue is

$$R = \sum_{i=1}^{n} T_{F,i} = t_F \sum_{i=1}^{n} \max(y_i - e_i A_F, 0).$$

Thus, if the allowance level under an FRT is $A_F$, a tax rate of

$$t_F(A_F) = \frac{R}{\sum_{i=1}^{n} \max(y_i - e_i A_F, 0)}.$$  \hspace{1cm} (2)

will be required to generate tax revenue $R$. Note that, as long as the pre-tax equivalized income distribution is dense on the appropriate interval, an increase in the allowance level will require an increase in the tax rate if total tax revenue is to be held constant. That is, $t_F(A_F)$ is increasing in $A_F$.

To see how changes in the allowance level will affect poverty, start by considering an allowance level of $A_F^n = 0$. Here, all of the income of every household in
the population is taxed at a rate of $t_F^a = t_F(0)$. Thus, the vector of equivalized post-tax incomes is $y(1 - t_F^a)/e = y^r(1 - t_F^a)$. Since this a linear map of the vector $y^r$, the poverty line here is simply

$$z^a = \beta \bar{y}^r(1 - t_F^a),$$

where $\bar{y}^r$ is the median level of equivalized pre-tax incomes. Therefore, any household with equivalized pre-tax income of $\beta \bar{y}^r$ or less will be considered poor.

Next, consider an allowance level of $A_F^b = \bar{y}^r$. Here, every household with equivalized pre-tax income less than or equal to the median level will pay no tax. Thus, the median level of equivalized post-tax incomes will be equal to the pre-tax level. As a result, the poverty line is just $\beta$ times this quantity, i.e., $z^b = \beta \bar{y}^r$. Alternatively, we may write $z^b = z^a/(1 - t_F^a)$, which should make it clear that $z^b > z^a$, since $t_F^a \in (0, 1)$.

Note that, with an allowance level of $A_F^b$, households with an equivalized pre-tax income of $\beta \bar{y}$ or less will have an equivalized post-tax income equal to $z^b$ or less, and would thus be considered poor. As pointed out above, this would also be also the case with an allowance level of $A_F^a = 0$. As a result, the number of poor households, $q$, is the same in either case.

While this should make it clear that the headcount ratio is the same in either of the cases considered above, it is interesting to note that the same is true of any poverty measure in the class of Foster et al. (1984). To see this, let $P_{\gamma}^a$ denote the level of poverty when the allowance level is $A_F^a = 0$. Similarly, let $P_{\gamma}^b$ denote the level of poverty when the allowance level is $A_F^b = \beta \bar{y}$. Substituting the appropriate tax parameters into equation (1), we see that

$$P_{\gamma}^a = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{z^a - y^r_e(1 - t_F^a)}{z^a} \right)^\gamma$$

5
\[
= \frac{1}{n} \sum_{i=1}^{g} \left( \frac{z^b(1 - t^a_F) - y^e_i(1 - t^a_F)}{z^b(1 - t^a_F)} \right)^\gamma \\
= \frac{1}{n} \sum_{i=1}^{g} \left( \frac{z^b - y^e_i}{z^b} \right)^\gamma \\
= P^b_{\gamma}.
\]

Intuitively, this can be explained as follows. Although the the equivalized post-tax income of each poor household is higher when the allowance level is \(A^b_F\), the poverty line is higher by exactly the same amount. Thus, the relative poverty gap of every poor household is unchanged.

Note that for all allowance levels greater than \(A^b_F = \tilde{y}^e\), the median level of equivalized post-tax incomes, and thus the poverty line, will be unchanged. Since the post-tax equivalized incomes of all poor households will also be unchanged, this implies that the level of poverty will be constant.

Moving in the opposite direction, consider an allowance level of \(A^c_F = \beta\tilde{y}^e\), with associated tax rate \(t^c_F = t_F(\beta\tilde{y}^e)\). A household with equivalized pre-tax income equal to the median level will have equivalized post-tax income of

\[
\tilde{y}^e - t^c_F(\tilde{y}^e - \beta\tilde{y}^e) = \tilde{y}^e(1 - t^c_F(1 - \beta)),
\]

which implies that the poverty line will be

\[
z^c = \beta\tilde{y}^e(1 - t^c_F(1 - \beta)).
\]

Of course, this is less than \(z^b = \beta\tilde{y}^e\), since \(1 - t^c_F(1 - \beta) < 1\).

Note that, a household with equivalized pre-tax income of \(\beta\tilde{y}^e\) will be considered poor (in a weak sense) when the allowance level is \(A^b_F = \tilde{y}^e\), but not be considered poor when the allowance level is \(A^c_F = \beta\tilde{y}^e\). In fact, the same can be said for any household with equivalized pre-tax income on the interval \([\beta\tilde{y}^e, \tilde{y}^e)\).
Accordingly, there will be less poor households when the allowance level is \( A^c_F \) than when the allowance level is \( A^b_F \), implying a lower headcount ratio. Moreover, the relative poverty gap of every household with pre-tax income less than \( \beta \bar{y}^e \) will be lower. Thus, poverty will be lower.

Finally, note that as the allowance level is increased from \( A^c_F = \beta \bar{y}^e \) to \( A^b_F = \bar{y}^e \), the poverty line will increase, since the tax paid by a household with equivalized pre-tax income of \( \bar{y}^e \) will decrease. At the same time, the equivalized post-tax incomes of all households with equivalized pre-tax income less than or equal to \( \beta \bar{y}^e \) will be the same as when the allowance level is \( A^b_F \). Thus, the relative poverty gaps of these households will be increasing. Moreover, there will be households with equivalized pre-tax incomes on the interval \( (\beta \bar{y}, \bar{y}) \) who will become poor (even though their post-tax incomes are rising). In other words, the number of poor households will be increasing. Therefore, poverty will be monotonically increasing as the allowance level is increased from \( A^c_F \) to \( A^b_F \).

The effect of a change in the allowance level on poverty when the allowance level is less than \( A^c_F = \beta \bar{y}^e \) is not so obvious. While poverty will be higher with an allowance level \( A^a_F = 0 \) than with an allowance level of \( A^c_F \), it is not clear that poverty will decrease monotonically between these two levels. While the poverty line will be lower with an allowance level below \( A^c_F \), some poor households will have paid taxes, and thus have lower equivalized post-tax incomes. The effect on both the number of poor households and the poverty gap of these households is thus ambiguous.

The above results are summarized in the following proposition.

**Proposition 3.1.** If the poverty line is some fraction, \( \beta \in (0, 1) \), of the median level of equivalized post-tax incomes, poverty under any revenue-neutral FRT is
minimized at some allowance level of on the interval \((0, \beta \bar{y}^*]\), where \(\bar{y}^*\) is the median level of equivalized pre-tax incomes.

We illustrate this finding using data from Statistics Canada’s Survey of Labour and Income Dynamics for 2004. This survey contains pre- and post-tax income data for 28,936 households. Total tax revenue, \(R\), is set equal to the sum of all federal and provincial income tax paid by these households (i.e., the sum of the differences between post- and pre-tax household incomes). The equivalence scale utilized here is equal to the square-root of household size. That is, if household \(i\) has \(s_i\) members, then \(e_i = \sqrt{s_i}\).

Figure 1 shows how the level of poverty, as measured by the Foster et al. (1984) class of measures, \(P_\gamma\), with \(\gamma = 0, 1,\) and 2, is affected by changes in the allowance level associated with a revenue-neutral FRT. Note that, when the allowance level is zero, the level of poverty, whether measured by \(P_0\), \(P_1\), or \(P_2\), is the same as when the allowance level is at or above the median level of pre-tax equivalized incomes ($32,850). The poverty level is lower when the allowance level is equal to 50% of the median level of pre-tax equivalized incomes ($16,425), but hits a minimum at some point lower than this. Specifically, the poverty-minimizing allowance levels are $14,172, $13,453, and $11,947, for \(P_0\), \(P_1\), and \(P_2\), respectively. These allowance levels correspond to tax rates of 25.3%, and 24.7%, and 23.5%, respectively.

Interestingly, these minimum poverty levels (0.1249, 0.0422, and 0.0224, respectively), are not substantially lower than the levels under the current GRT system (0.1350, 0.0450, and 0.0245, respectively). In other words, ignoring behavioural impacts, it is not possible to achieve a substantial reduction in the current level of poverty under any FRT. However, it should also be pointed out that these poverty-minimizing tax rates, which represent combined federal and

\(^2\)From this sample, we remove 46 households with negative levels of pre-tax income.
provincial rates, are quite low as compared with the highest combined federal and provincial marginal tax rates under the status quo, which range from 39% in Alberta to 47% in Newfoundland.

Before moving on, it is useful to compare these effects on poverty with the effects on inequality. Davies and Hoy (2002, Proposition 2.1) show that any revenue-neutral increase in the allowance level results in a post-tax income distribution which Lorenz dominates the original one. In other words, the level of inequality, as measured by any reasonable inequality measure, will monotonically be decreasing as the allowance level is increased. Comparing this result to Proposition 2.1, we see that, increases in the allowance level beyond the poverty-minimizing level (or, at least beyond a level of $\beta y^*$), will simultaneously lead to an increase in poverty and a decrease in inequality.

4 Negative income tax

We now consider the case of a negative income tax (NIT), which is a slight variation on an FRT. An NIT replaces the allowance level with a fully refundable credit or “demogrant”. Under an NIT, the tax paid by the $i$th household is

$$T_{N,i} = -e_iD + t_Ny_i, \quad i = 1, \ldots, n,$$

where $D \geq 0$ is the demogrant, and $t_N \in (0, 1)$ is the proportional tax rate. The post-tax equivalized income of the $i$th household is thus

$$\frac{y_i - T_{N,i}}{e_i} = \frac{e_iD + y_i(1 - t_N)}{e_i} = D + y_i^e(1 - t_N).$$

As with an FRT, the pre- and post-tax orderings of equivalized household
incomes are identical, since

\[ D + y_i^c (1 - t_N) > D + y_j^c (1 - t_N), \]

for \( y_i^c > y_j^c \). This again implies that a household with equivalized pre-tax income equal to the median level will have equivalized post-tax income equal to the median level.

Tax revenue under an NIT is

\[
R = \sum_{i=1}^{n} T_{N,i} = -D \sum_{i=1}^{n} e_i + t_N n \bar{y},
\]

where \( \bar{y} \) is the mean level of pre-tax (non-equivalized) incomes. Therefore, in order to generate tax revenue \( R \), an NIT with demogrant \( D \) will require a tax rate of

\[
t_N(D) = \frac{R + D \sum_{i=1}^{n} e_i}{n \bar{y}}.
\]

This should make it easy to see that \( \partial t_N(D) / \partial D > 0 \). That is, increasing the demogrant will require an increase in the tax rate if tax revenue is to be held constant.

To consider the effect of changes to the demogrant on the level of poverty, it is useful to consider what would happen to a household with equivalized pre-tax income equal to the median level, \( \tilde{y}^c \). The equivalized post-tax income of such a household is

\[
D + \tilde{y}^c (1 - t_N) = D + \left( \frac{R + D \sum_{i=1}^{n} e_i}{n \bar{y}} \right) \tilde{y}^c,
\]
which implies that the poverty line is

\[
z(D) = \beta D + \beta \left( 1 - \frac{R + D \sum_{i=1}^{n} e_i}{n \bar{y}} \right) \bar{y}^c.
\]  

(3)

It is interesting to note that,

\[
\frac{\partial z(D)}{\partial D} = \beta \left( 1 - \frac{\bar{y}^c}{\bar{y}} \right),
\]  

(4)

which is positive if we assume that \( \bar{y}^c < \bar{y} \). That is, the poverty line will be increasing with the demogrant level if we assume that the median level of equivalized pre-tax incomes is less than the mean level of non-equivalized pre-tax incomes. Since any conceivable pre-tax income real-world would be heavily skewed to the right, it is very reasonable to assume that the median level of pre-tax incomes would be less than the mean level. Moreover, since equivalized pre-tax incomes will always be less than (or equal to, in the case of an individual) non-equivalized pre-tax incomes, this assumption can be seen to be very mild indeed.

While an increase in the demogrant will increase the poverty line, it will also increase the post-tax equivalized incomes of some households. To see this, note that the post-tax equivalized income of the \( i \)-th household is

\[
D + \tilde{y}_i(1 - t_N) = D + \left( R + D \sum_{i=1}^{n} e_i \right) \frac{\tilde{y}_i}{\bar{y}}.
\]

Differentiating this with respect to \( D \) yields

\[
\frac{\partial (D + \tilde{y}_i(1 - t_N))}{\partial D} = \left( 1 - \frac{\tilde{y}_i}{\bar{y}} \right),
\]  

(5)

Thus, the post-tax equivalized income of every household with pre-tax equivalized income less than the mean level of non-equivalized pre-tax incomes will
increase with an increase in the demogrant. As above, if it is assumed that \( \bar{y} < \bar{y} \), then the post-tax equivalized income of every household with pre-tax equivalized income below the median level will be increasing with an increase in the demogrant. Since the poverty line is a fraction of the median level of post-tax equivalized incomes, this implies that the post-tax equivalized income of all poor households will be increasing with an increase in the demogrant.

Comparing equations (4) and (5), it is evident that, when the demogrant is increased, the poverty line will be increasing at a slower rate than the post-tax equivalized incomes of all (potentially) poor households, since \( \beta \in (0,1) \). This implies that, when the demogrant is increased, some households may have their post-tax incomes pushed above poverty line (potentially decreasing the headcount ratio), while those remaining poor will have smaller relative poverty gaps. Thus, poverty will be monotonically increasing with the demogrant level.

However, this can only continue up to the point where the demogrant \( D^a = z(D) \). Beyond this point even a household with no pre-tax income will have post-tax income greater than the poverty line. Hence, there will be no poverty. Setting the right-hand-side of equation (3) equal to \( D \), and solving yields

\[
D^a = \frac{\beta \bar{y}^e (n\bar{y} - R)}{(1 - \beta)n\bar{y} + \beta \bar{y}^e \sum_{i=1}^{n} e_i}.
\]

Note that it will always be the case that point is attainable in the sense that \( t_N(D^a) < 1 \).

The above results are summarized in the following proposition.

**Proposition 4.1.** If the poverty line is a fraction of the median level of equivalized post-tax incomes, poverty under any revenue-neutral NIT will be monotonically decreasing as the demogrant is increased up to \( D^a \) (see above). At demogrant levels greater than or equal to \( D^a \), there will be no poverty.
This finding is illustrated using the Canadian household survey data (see Section 3) in Figure 2. Poverty, as measured by $P_0$, $P_1$, or $P_2$, is zero for demogrant levels at or above $15,061$, the figure calculated for $D^a$. This corresponds to a tax rate of 54.2%. Compared to the highest combined federal and provincial marginal tax rates under the status quo (which range from 39% in Alberta to 47% in Newfoundland), this is fairly high. However, such a demogrant ($15,061$) leads to the complete elimination of poverty. Clearly, it would be possible substantially reduce poverty from its current level at some lower demogrant. Demogants of $2824.48$, $1591.36$, and $894.69$ (with associated tax rates of 23.7%, 20.6%, and 18.9%), would generate the same level of poverty, as measured by $P_0$, $P_1$, and $P_2$, respectively, as under the existing GRT. To cut existing poverty levels in half, demogants of $7818.18$, $5130.51$, and $3636.36$ (with associated tax rates of 36.1%, 29.4%, and 25.7%), respectively, would be required. As all of these tax rates are lower than the highest combined federal and provincial marginal tax rates under the status quo, this seems to be a realistic achievement.

5 Conclusion

In this paper, we have considered the implications of both an FRT and NIT on the level of relative poverty. We provide a systematic analysis of how revenue-neutral changes to the parameters in these tax systems would affect the level of poverty. It is found that, either of these alternative tax systems can reduce relative poverty from its current level with very reasonable marginal tax rates.
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References


Figure 1: The level of poverty as a function of the allowance level for an FRT. The solid, dashed, and dot-dashed lines represent the levels of $P_0$, $P_1$, and $P_2$, respectively.
Figure 2: The level of poverty as a function of the demogrant level for an NIT. The solid, dashed, and dot-dashed lines represent the levels of $P_0$, $P_1$, and $P_2$, respectively.