

Voting power and social interaction

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Summary. In a simple yet illustrative model basic notions of classical voting theory are extended to serve the goal of identifying the most influential actors in social influence dynamics. Feature of the model is that now the indirect power of those actors in democratic processes without institutional power (like insurgency and lobbyists) can be measured for a given social network structure. The concept is sensitive to the parameters defining the social dynamics, as is illustrated by the discussion of a couple of simple learning models.

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1 Introduction

Power in democracies is transferred and institutionalized through voting procedures. The voters have direct power in the decision-making process, whereas others have indirect power through influence relations with the voters. This paper contributes to the theoretical analysis of democratic processes by distinguishing and quantifying the role of all actors through their social relations. This paper advances on the traditional power analysis in two ways. Firstly, opposed to *a priori* analysis of voting bodies with independent voters, we explicitly allow for correlated voting behavior through an endogenous process of opinion formation. Secondly, an important feature of the model is the quantitative analysis of power of non-voters as participants in this process. For instance, despite the fact that big industries are not credited with direct voting power, strong activity of lobbyist provides them with indirect political power. Another example is insurgency, which is commonly understood as an unlawful movement, excluded from direct representative power. However, insurgency movements may be united with political parties in common goals – like in Ireland the IRA can be associated with the political Sinn Féin.

We focus on measures of what has been called *I-power*, which is supposed to measure the influence of a voter over the outcome of a vote (in contrast to the idea of power as division of spoils). The classical measure of I-power proposed by Penrose(1946, 1952)⁴ is based on a random voting model in which each member votes for or against with equal probability independently of all other members. This is not a behavioral assumption but is a method of *a priori* analysis. The latter models the voting system as an abstract shell, without taking into consideration voters preferences, the range of issues over which a decision is taken or the degree of affinity between the voters. This abstraction seems to be necessary to evaluate the decision rule itself (for a more elaborated discussion on a priori voting power see e.g. Felsenthal and Machover 1998).

A common criticism of the widely used Penrose measure is that it fails to take

⁴also known as the *absolute Banzhaf index*.

account of actual behaviour of voters in the particular voting bodies under consideration. The classical treatment of *a priori* voting power encompasses independent voting behaviour of the actors. Therefore it does not seem fit to make judgements about actual *a posteriori* voting power, where interdependencies may exist. For instance, under qualitative majority voting in the 1958 EU Council of Ministers, Luxembourg was a dummy voter and therefore credited with voting power 0. Its voting weight was too low so that it could never bring out a decisive vote. Nevertheless one may question whether Luxembourg was indeed powerless. Since, as a member of the Benelux, it was well able to influence the voting behavior of Belgium and the Netherlands and in turn the outcomes of the voting process in the EU Council of Ministers.

Even more extreme examples include the discussion of power of lobbyists. Wright (1990) studies the influence of lobbyists on policy decisions in the US House of Representatives. Information about which groups worked together on two controversial issues and which representatives they lobbied was obtained through personal interviews and a mail survey of professional lobbyists. He argues that the influence on representatives' policy decisions is best explained by and depending on the number of lobbying contacts from interest groups on each side of an issue. In particular, this study points out that campaign contributors proved somewhat useful for explaining groups' lobbying patterns; but it appears to be lobbying, not money, that shapes and reinforces representatives' policy decisions. Wright's paper takes a different approach than the mainstream in the literature which focuses on the role of money. In his work, monetary investments may be represented by certain positions in the social network. We see our work on a posteriori power as complimentary to the work of Wright as it provides a measurement of power as influence of agents who are not necessarily part of the voting body, as in the case of lobbyists. Although lobbyists have no constitutional power, their very existence shows the capability to exert power by influencing the members in the assembly. Again, classical *a priori* voting theory cannot be applied to judge their power.

We suggest a voting model in which the social structure between the voters is

made explicit. In particular we suggest to include learning dynamics as protocol by which the voters update their *a priori* approval probabilities.⁵ In their 1979, Dubey and Shapley generalize the Penrose measure ψ to the case where all voters vote independently. Let x_k denote the probability of the event that individual k votes 'yes'. Under the assumption that all x_k 's are arbitrary and independent for all $k \in N$ of the assembly N their approach coincides with

$$\psi_k = \frac{\partial A}{\partial x_k}, \quad (1)$$

where A denotes the probability of the event that the outcome of a vote is positive (which Coleman (1971) called the ability of the collectivity to act). Thus ψ_k is the marginal contribution of the propensity of k to vote for a bill to the probability of the bills passing. In this paper we seek to construct an empirically relevant power measure by relaxing the idea of independence and show how to replace it by the use of information about real or assumed voting patterns.

We suggest a measure of voting power taking into account exercise of *influence on* voting behaviour of others. In particular, power is exerted through an influence on voters entitled with constitutional voting power. We formalize this influence by stating that the initial *a priori* affirmative probabilities $p = (p_1, \dots, p_n)$ are transformed to *a posteriori* probabilities $x(p) = (x_1(p), \dots, x_n(p))$. Here x is an appropriate model of social interaction. Inspired by (1) we measure power in presence of interdependence as follows:

$$\psi_k^x = \left. \frac{\partial (A(x(p)))}{\partial p_k} \right|_{p=(\frac{1}{2}, \dots, \frac{1}{2})} \quad (2)$$

Or, shorter, by application of the chain rule

$$\psi_k^x = \frac{\partial A}{\partial x_1} \cdot \frac{\partial x_1}{\partial p_k} + \dots + \frac{\partial A}{\partial x_n} \cdot \frac{\partial x_n}{\partial p_k}. \quad (3)$$

⁵Different from other literature we do not need notions like coalition or coalition structure, although these may arise by correlation of approval probabilities. In this sense our theory is more general.

Hence power of k is the ability to influence a voter j who in turn exerts power. Note that this definition doesn't require k to be essential part of the voting body. Here, the term $\partial x_j / \partial p_k$ measures the sensitivity with which j reacts to changes in k 's opinion.

In this paper we will discuss examples derived from social network theory, where $\partial x_j / \partial p_k$ is determined by the architecture of the network. Social network analysis views social relationships in terms of nodes and ties. In this, we follow the seminal work *The Strength of Weak Ties* of Granovetter (1973). The notion *strength of a tie* is used as an assessment of the intensity of a bond between two actors⁶ and – in our context – it shapes the influence dynamics. In particular, in our model $\partial x_j / \partial p_k$ is the strength of a tie.

Note that our approach is different from centrality which is one of the most studied concepts in social network analysis. Numerous measures have been developed, including degree centrality, closeness, betweenness, eigenvector centrality, flow betweenness, the rush index, the influence measures of Katz (1953), Hubbell (1965), and Hoede (1978), Taylor's (1969) measure, etc. See Freeman (1979) for an overview. However, in the context of social choice, centrality fails to take into account the constitutional voting power of the actors. Hence, our measure is given by two major ingredients: (1) the architecture of the network and (2) the voting game, i.e. the constitutional voting power. The vast majority of the literature, e.g., Aumann and Drèze (1974), vd Brink and vd Laan (2005) on voting with interdependent voters assumes the existence of a coalition structure. Whereas our approach does not exclude those cases where dynamic interaction leads to such structured voting assembly, we do not take this as given.

⁶Moreover, graph theory (or the mathematical study of abstract representations of networks), can be extended to include negative ties such as animosity among persons. In the present account, however, we shall abstract from negative ties.

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