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### Costly intermediation and the friedman rule

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#### Abstract

I examine the implementation of the Friedman rule under the assumption that age dependent lump sum transfers are possible and private intermediation is costly. This is done both in an infinitely lived agents model and in an overlapping generations model. I argue that in addition to a zero nominal-interest-rate policy (the so called Friedman rule) a transfer to young agents, or a government loan program is required for satiating agents with real balances. The paper also contributes to the understanding of Friedman's original article and discusses related questions about the size of the financial sector. It is shown that the adoption of the (modified) Friedman rule will crowd out private lending and borrowing. I also look at the social value of a market for contingent claims and argue that resources spent on operating a market for accidental nominal bequests are a waste from the social point of view in spite of the fact that individuals have an incentive to trade in such markets.

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## COSTLY INTERMEDIATION AND THE FRIEDMAN RULE

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I examine the implementation of the Friedman rule under the assumption that age dependent lump sum transfers are possible and private intermediation is costly. This is done both in an infinitely lived agents model and in an overlapping generations model. I argue that in addition to a zero nominal-interest-rate policy (the so called Friedman rule) a transfer to young agents, or a government loan program is required for satiating agents with real balances. The paper also contributes to the understanding of Friedman's original article and discusses related questions about the size of the financial sector. It is shown that the adoption of the (modified) Friedman rule will crowd out private lending and borrowing. I also look at the social value of a market for contingent claims and argue that resources spent on operating a market for accidental nominal bequests are a waste from the social point of view in spite of the fact that individuals have an incentive to trade in such markets.

Key words: The Friedman Rule, Accidental bequests, The optimal size of the financial sector, Government loans.

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## 1. INTRODUCTION

The recent financial crisis has led to a renewed interest in monetary economics and old questions that were never fully answered are now at the top of the research agenda. While many of the same issues are relevant, the focus (or perhaps the terminology) has shifted from “money” to “liquidity”. Like money, liquidity is not easy to define. Roughly speaking, I use liquidity for instruments and policies that help to mitigate the effects of trade frictions.

Does the government have an advantage over the private sector in supplying liquidity? In one area, namely money creation there is almost a consensus that the government does have an advantage. The government has also an undisputed advantage in collecting tax payments. Can this advantage be used to increase liquidity? Similarly, many believe that the government has an advantage in enforcing uncollateralized loan contracts.<sup>1</sup> Should it play a role in the credit market?

Here I look at these questions through the lenses of Friedman (1969) original optimum quantity of money article. In his seminal article, Friedman argues that the social cost of producing real balances is zero and therefore a policy aimed at satiating agents with real balances is optimal. I argue here that satiating agents with real balances requires perfect credit markets or a particular type of a tax/transfer policy. This suggests that in order to realize the full gains from the government’s advantage in money creation, it must also realize its advantage in other areas such as collecting tax and uncollateralized loan payments.

I follow Friedman in using the “money in the utility function approach” and discuss problems that arise as a result of heterogeneity and market incompleteness. This is done in a version of Friedman’s original model with infinitely lived agents (the *IL*

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<sup>1</sup> There is little difference between collecting taxes and collecting uncollateralized loan payments. Therefore, the advantage in collecting taxes may imply an advantage in collecting loan payments.

model) and in a Blanchard (1985) type of overlapping generations (*OG*) model. In the *IL* model, the cost of intermediation matters for the transition to the optimal steady state. In the *OG* model, it matters also in the steady state because of the built in age heterogeneity.

I start with the case in which the cost of private intermediation is prohibitive. To see the need for a transition period in the *IL* model that will arise in the absence of intermediation, let us assume that the price level drops after the adoption of the Friedman rule, to a level that *on average* satiate agents with real balances. After the drop, some agents will have more money than they want to hold and some will have less money than they want to hold. Those with “too much” money will decumulate real balances by consuming more than their income while those who do not have enough money will accumulate money by consuming less than their income. This process will continue until they reach the efficient steady state and during the transition period agents do not hold the optimal quantity of money.

In the *OG* model agents may hold suboptimal levels of real balances even in a steady state with zero nominal interest rate because young agents may start with suboptimal levels of money.

The government may restore efficiency by realizing its advantage in collecting tax and uncollateralized loan payments. It can make a transfer to the young and collect taxes to cover the “interest payments” on the transfer or it can institute a loan program. The equivalence between the transfer payment solution and the loan solution can be explained in terms of a Ricardian equivalence type of reasoning. The tax implications of the “transfer solution” can be viewed as interest payments and the initial transfer can be viewed as a loan that agents are forced to accept. The use of “force” is however not consequential because the optimal policy allows agents to save at the same interest rate that they pay on the “forced loan”. Therefore an agent can simply deposit the unwanted

fraction of the “forced loan” in government bonds and use the interest payment to cover the tax obligations.

I also attempt the broader question of the optimal size of the financial sector. I show that the optimal policy (that includes government loans or a transfer to the young) will crowd out costly private lending and borrowing. In this sense the financial sector that exists when the optimal policy is not implemented is “too big”. I also show that resources spent on contingent contracts on accidental nominal bequests are a waste from the social point of view. Trade in claims on accidental bequests is likely to emerge and may be important if the transfer solution is used to satiate agents with real balances. The problem is less severe under the loan program because in this case agents do not own most of the money they hold and the accidental bequest is small.

## 2. A VERSION OF THE FRIEDMAN MODEL

A short review of the large literature that followed Friedman’s original article may be useful. Friedman developed his main argument in the context of “a hypothetical simple society” in which money is the only asset (no bonds and no physical capital). In this case the optimal real rate of return on money (which is approximately equal to the deflation rate) is the subjective interest rate. Friedman then introduced default-free bonds and riskless physical capital and argued that the policy maker can achieve his objective by choosing a rate of deflation that will make the nominal interest rate equal to zero. This has become known as the Friedman rule. See Eden (2005, ch. 2) for an exposition of the Friedman model.

Friedman made his argument in terms of first order (no arbitrage) conditions that must hold in a steady-state equilibrium, but did not specify the agents’ problems. Brock (1974, 1975) used the representative agent assumption to formalize Friedman’s analysis. Woodford (1990) summarized the earlier literature and showed that the Friedman rule is

optimal in infinitely lived representative agent models that allow lump sum taxes and transfers. Cole and Kocherlakota (1998) showed that the Friedman rule is both sufficient and necessary for achieving the optimal allocation in a cash-in-advance economy under the representative agent assumption.

Phelps (1973) challenged Friedman's conclusions on the grounds that when only distortive taxes are possible, money should be taxed like any other good. Subsequent literature examined this argument and found that the Friedman rule is rather robust. See, for example, Kimbrough (1986), Chari, Christiano and Kehoe (1996), Correia and Pedro (1996) and da Costa and Werning (2008).

The literature that allows for heterogeneity with incomplete markets poses another challenge for the Friedman rule. Bhattacharya, Haslag and Martin (BHM, 2005) review this literature and argue that the adoption of the Friedman rule may not lead to a Pareto improvement. I will revisit their argument shortly.

It may be worth noting that Friedman (1969) provides an extensive discussion of the income distribution aspects of his proposal and explicitly abstracts from "distributional effects" (page 14) when discussing welfare. He thus focuses on efficiency rather than Pareto ranking.

Since private intermediation is costly, achieving efficiency and satiation with real balances is not trivial. I argue here that since the government has an advantage in collecting tax and loan payments it can reduce the need for a transition period in which agents hold suboptimal balances.

Another problem for implementing the Friedman rule is the presence of menu type costs for changing prices. I start with the case of a technological advanced society in which cash is relatively unimportant. As noted by Friedman (1969, page 38), in this case it is possible to achieve optimal liquidity by paying explicit interest on money and maintaining price stability.

## 2.1 The Environment and a Planner's Problem

I assume  $n$  agents (indexed  $h$ ) who live forever and consume a non-storable consumption good (corn) and liquidity services. (See the Appendix for the derivation of the indirect utility function in a many goods economy). Agent  $h$ 's period utility depends on "total consumption" which aggregates corn consumption and liquidity services. Liquidity services  $f^h(m_{tb}^h, Y_t^h)$  depend on the amount of real balances held at the beginning of the period ( $m_{tb}^h$ ) and on corn consumption ( $Y_t^h$ ). Total consumption is  $C_t^h = Y_t^h + f^h(m_{tb}^h, Y_t^h)$  and agent  $h$ 's utility is  $\sum_{t=1}^{\infty} (\beta^h)^t U^h(C_t^h)$  where  $U^h$  is strictly concave and monotone and  $0 < \beta^h < 1$  is agent  $h$ 's discount factor. The liquidity services function  $f^h(m_{tb}^h, Y_t^h)$  is strictly concave and for any given  $Y_t^h$  it reaches a unique maximum at the point when the partial derivative is zero:  $f_1^h(m_{tb}^h, Y_t^h) = 0$ .<sup>2</sup> Agents get an endowment of corn at the beginning of each period. Each agent gets an endowment of  $\bar{Y}$  per period and the aggregate endowment is:  $\Sigma = n\bar{Y}$  per period. (I will use cap sigma both for summation and for the aggregate endowment).

I start with the assumption that the economy is fully controlled by a central planner who gets the endowment of corn. In addition, the planner has an unlimited amount of another good called money (or real balances). At the beginning of each period the planner gives each agent a certain amount of money as an interest free loan to be returned at the end of the period.

The planner maximizes the weighted sum of the utilities of the agents in the economy, and solve the following problem:

$$(1) \quad \max_{Y_t^h, m_{tb}^h} \sum_{h=1}^n \omega^h \sum_{t=1}^{\infty} (\beta^h)^t U^h \left( Y_t^h + f^h(m_{tb}^h, Y_t^h) \right) \quad \text{s.t.} \quad \sum_{h=1}^n Y_t^h = \Sigma \quad \text{for all } t \geq 1,$$

where  $\omega^h$  is the weight that he assigns to agent  $h$ .

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<sup>2</sup> Friedman (1969, page 17) argue that the marginal product of money in producing liquidity services will be negative after some point because of the need to hire guards to protect the cash hoard. This may also apply for a cashless society because a stolen debit card can be used to withdraw money from the checking account but not from the savings account. The amount of resources devoted to prevent theft from the checking account is related to the amount held in this account.

The first order conditions for this problem require:

$$(2) \quad f_1^h(m_{ib}^h, Y_t^h) = 0 \quad \text{for all } h \text{ and } t .$$

Thus since real balances are a “free good” efficiency requires that all agents are satiated with real balances at all times.

Note that our planner can change the allocation of real balances without changing the allocation of corn. When  $f_1^h(m_{ib}^h, Y_t^h) > 0$ , increasing real balances (without changing the allocation of corn) improves the welfare of all individuals.

I now turn to the description of the actual economy in which agents get the endowment of corn and decide how much money to hold.

## 2.2 The Loan Implementation When Paying Interest on Money is Possible

I start with the relatively simple case in which paying interest on money is possible, prices are stable and the only source of heterogeneity is in the initial money holdings. I show that the government can increase equilibrium liquidity by instituting a loan program and given a sufficiently high borrowing limit, we can Pareto rank alternative policies.

There are two assets: money and government bonds. The price of corn is one dollar per unit and all magnitudes are therefore in units of corn (real terms).

It may help to think of the two assets as two types of deposit accounts in a government run bank: a checkable account and a savings account. The checkable account yields liquidity services and pays the gross real interest rate  $R_m = 1 + r_m$ . The savings account pays the gross real interest  $R = 1 + r \geq R_m$  and does not yield liquidity services. The end of period balances are  $m_t^h \geq 0$  in the checking account and  $b_t^h$  in the savings account. At the beginning of each period  $t \geq 1$ , the agent gets interest payment and a money transfer from the government of  $g$  units that is deposited to his checking account. The beginning-of-period real balances are:  $m_{ib}^h = R_m m_{t-1}^h + g$ . The amount in the savings

account can be negative but there is a limit to the amount of borrowing allowed:

$$b_t^h \geq B > -\infty.$$

Agent  $h$  starts period  $t = 0$  with  $m_{-1}^h$  units of real balances that are different across agents. At  $t = 0$ , the government gives each agent a transfer of  $\tau$  units of real balances (that may be different from  $g$  - the transfer that he will get in all future periods  $t \geq 1$ ). After the initial transfer, the beginning-of-period  $t = 0$  real balances are:

$$z^h = m_{-1}^h + \tau.$$

The average amount of initial money holdings after the initial transfer is:

$m = (1/n) \sum_{h=1}^n z^h$ . The end-of-period real balances are:  $m_0^h = z^h - b_0^h$ , where  $-b_0^h$  is the amount he borrows from the government ( $b_0^h$  is the amount in his savings account that may be negative).

Starting from period  $t = 1$  each agent gets an endowment of  $\bar{Y}$  units of corn per period. I simplify and assume that liquidity services can be written only as a function of the beginning of period real balances (there is no interaction with corn consumption) and this function is the same for all agents. Total consumption is given by  $C_t^h = Y_t^h + f(m_{ib}^h)$  where  $Y_t^h$  is the consumption of corn and  $f(m_{ib}^h)$  is the liquidity service in units of corn. The function  $f(m_{ib}^h)$  is strictly concave and there is a satiation level  $\bar{m}$  such that:

$$f'(m) > 0 \text{ when } m < \bar{m}, f'(\bar{m}) = 0 \text{ and } f'(m) < 0 \text{ when } m > \bar{m}.$$

At  $t = 0$  the government announces a policy  $(R, R_m \leq R, -\infty < B \leq 0, g, \tau)$ . The policy variables do not change over time and therefore they are not indexed by  $t$ . Agents take the policy as given and solve the following problem.

9

$$\begin{aligned}
 (3) \quad & \max_{Y_t^h, m_t^h, b_t^h} \sum_{t=1}^{\infty} \beta^t U(C_t^h) \\
 & \text{s.t.} \\
 & C_t^h = Y_t^h + f(m_{tb}^h) \\
 & m_{tb}^h = R_m m_{t-1}^h + g \\
 & m_0^h + b_0^h = m_{-1}^h + \tau = z^h \\
 & b_t^h + m_t^h + Y_t^h = \bar{Y} + R_m m_{t-1}^h + R b_{t-1}^h + g \\
 & z^h \geq 0 \text{ is given, } m_t^h \geq 0, b_t^h \geq B.
 \end{aligned}$$

Using dynamic programming we can write (3) as:

$$\begin{aligned}
 (4) \quad & V(z) = \max_{m_0 \geq 0, b_0 \geq B} v(m_0, b_0) \text{ s.t. } m_0 + b_0 = z \text{ and} \\
 & v(m, b) = \max_{m' \geq 0, b' \geq B} U(\bar{Y} + R_m m + R b - m' - b' + g + f(R_m m + g)) + \beta v(m', b')
 \end{aligned}$$

The function  $v(m, b)$  defined by the Bellman equation, is the maximum utility that one can get if he starts period  $t \geq 1$  with  $m$  units of real balances and  $b$  units of bonds. Later, I use the symbol  $v$  to denote different (but similar) value functions. The function  $V(z)$  is the maximum utility that the consumer can achieve if his post initial transfer balances are  $z$  units.

The first order conditions that an interior solution to (3) must satisfy are:

$$(5) \quad \frac{U'(C_t)}{\beta U'(C_{t+1})} = R = R_m (1 + f'(m_{t+1b}))$$

The first equality is the standard Euler equation. It equates the marginal rate of substitution to the real interest rate on bonds. The second equality says that the total real rate of return on money (the sum of  $R_m$  and the marginal product  $R_m f'$ ) must equal the real rate of return on bonds. Note that the relevant marginal product is at  $t + 1$  because liquidity services depend on the beginning of period real balances.

Equilibrium is a policy announcement  $(R, R_m \leq R, -\infty < B \leq 0, g, \tau)$  an initial level of real balances  $(z^1 = m_{-1}^1 + \tau, \dots, z^n = m_{-1}^n + \tau)$ , the amounts of funds in the two accounts  $\{b_t^h, m_t^h\}_{t=0}^\infty$  and the amount of corn consumption  $\{Y_t^h\}_{t=1}^\infty$  such that: (a) given the policy announcement and the initial level of real balances, the choices of the consumers are optimal (solve [3]) and (b) the market for corn is cleared,  $\sum_{h=1}^n Y_t^h = n\bar{Y}$  for all  $t \geq 1$ .

A steady state equilibrium characterized by the level of real balances  $m$  is an equilibrium which satisfies the added condition:  $m_t^h = m = (\frac{1}{n})\sum_{h=1}^n z^h$  for all  $h$  and  $t$ .

Thus in the steady state, real balances do not change over time. Since all agents use the same  $f$  function they hold the same level of real balances. Corn consumption and bond holdings may still be different across agents. When  $|B|$  is sufficiently large, agents with initial holdings of real balances that is less than the average  $m$  take a loan from the government and those whose initial holdings is more than  $m$  make a loan to the government. This is the only time agents adjust their portfolios. After this adjustment agents consume their permanent income. This is stated in the following Claim.

Claim 1: A policy announcement  $(R = 1 + \rho, R_m, B, g = -r_m m, \tau = m - (\frac{1}{n})\sum_{h=1}^n m_{-1}^h)$  with a large enough limit on borrowing  $|B|$ , and the following choice variables

$$m_{t+1}^h = R_m m + g = m$$

$$b^h = z^h - m = m_{-1}^h + \tau - m = m_{-1}^h - (\frac{1}{n})\sum_{i=1}^n m_{-1}^i \text{ for all } t \geq 0,$$

$$Y_t^h = \bar{Y} + r b^h + r_m m + g = \bar{Y} + r b^h, C_t^h = C^h = \bar{Y} + r b^h + f(m) \text{ for all } t \geq 1,$$

is a steady state equilibrium if  $m$  satisfies the first order conditions (5), which can now be written as:

$$(6) \quad R = 1 + \rho = R_m (1 + f'(m)).$$

Assuming  $m \leq \bar{m}$ , there is a unique solution to (6) for any given policy choice  $R_m$  and we can therefore introduce the following Claim.

Claim 2: The steady state welfare of all agents is increasing in  $R_m$  ( $R_m \leq R = 1 + \rho$ ).

The proof of the Claim uses (6) to show that an increase in  $R_m$  leads to an increase in  $m$  and in the liquidity services  $f(m)$  for all agents. Since  $b^h$  is the same across steady states, total consumption  $C^h = \bar{Y} + rb^h + f(m)$  is increasing in  $m \leq \bar{m}$  for all  $h$ . To get the first best, the government must choose:  $R = R_m = 1 + \rho$ .

To get the intuition, we may think of the transition to a steady state as occurring in two stages. In the first stage agents lend or borrow to achieve a common amount of real balances. Then agents get an initial transfer of  $\tau$ . Since achieving a steady state with more real balances requires a larger  $\tau$  and since all agents end up holding this initial transfer and derive liquidity services from it, all agents are better off in a steady state with more real balances.

The assumption of rigid prices allows for Pareto ranking in spite of the heterogeneity in the initial distribution of money. Unlike Bhattacharya, Haslag and Martin (BHM, 2005), here achieving a steady state with higher real balances does not change the value of the initial holding  $m_{-1}^h$  and therefore there is no redistribution effect that may reduce the welfare of some individuals.

### 2.3 An Initial Transfer Implementation

An alternative way of achieving the first best is to replace the loan by a large transfer of money at  $t = 0$  and adopt the policy:  $R = R_m = 1 + \rho$ ,  $B = 0$ ,  $g = -r \left( \bar{m} + \frac{1}{n} \sum_{h=1}^n m_{-1}^h \right)$  and  $\tau = \bar{m}$ .

Note that after the initial transfer, agent  $h$  has  $z^h - \bar{m} = m_{-1}^h$  unproductive real balances. He deposits the unproductive balances in the savings account (by choosing

$b^h = m_{-1}^h$ ) and consumes his permanent income:

$$Y_t^h = \bar{Y} + r\bar{m} + rb^h + g = \bar{Y} + r\left(m_{-1}^h - \left(\frac{1}{n}\right)\sum_{i=1}^n m_{-1}^i\right).$$

In general, there is no difference here between the transfer solution and the loan solution.

We can consider alternative policies:

$$\left[ R = 1 + \rho, R_m, B = 0, g = -r_m \left( m + \left(\frac{1}{n}\right)\sum_{h=1}^n m_{-1}^h \right) \tau = m \right]$$

and show that total consumption in the steady state is the same as under the loan program and Claims 1 and 2 hold for this case as well. This is because of the Ricardo type equivalence between the loan and the transfer that was mentioned in the introduction: The transfer may be viewed as a loan and the increase in future tax obligations as interest payments. The only difference is that here the agent does not choose the size of the loan but this is not important because he can always deposit the unproductive balances in the savings account.

#### 2.4 Flexible Prices

The “baseline” model considered by Friedman assumes perfectly flexible prices. In this case, paying explicit interest on money is not necessary and the optimal steady state can be achieved by choosing an inflation rate  $\pi$  such that the gross real rate of return on money is:  $\frac{1}{1+\pi} = 1 + \rho$  and  $\pi \approx -\rho$ .

We can reach the steady state immediately, without a transition period, and without having a government run bank, provided that we allow for large jumps in the price level at the time of the announcement of the optimal policy. I focus here on two cases.

Initial hyperinflation: The government can achieve any steady state (without a transition period and without a government run bank) in the following way. It first announces the interest rates  $(R = 1 + \rho, R_m \leq R)$ . After the announcement, the steady state level of real balances  $m$  is determined by (6). The government then makes a large transfer of  $T$  dollars per agent. After the transfer the price level jumps to  $P_0$  and agent  $h$  holding of

real balances is:  $z^h = \frac{M_{-1}^h + T}{P_0}$ , where  $M_{-1}^h$  is the initial money endowment in nominal terms. When  $T$  is sufficiently large, the real value of the initial endowment of money holding is small and  $z^h \approx m$  for all agents. Thus a large transfer eliminates the difference in the initial money holding and the need for a transition period to the steady state.

Thus, when  $T$  is sufficiently large, we can Pareto rank alternative steady states and the optimal steady state is achieved when choosing  $R_m = R = 1 + \rho$ .<sup>3</sup>

I now turn to show that the introduction of government bonds is enough to eliminate the transition period.

The role of government bonds in eliminating the transition period: The government can eliminate the transition period without an initial transfer ( $T = 0$ ) and without lending, provided that agents are allowed to save in government bonds and all agents hold a strictly positive amount of money. In this case, after the government announces the interest rates ( $R = 1 + \rho, R_m \leq R$ ) the price level is determined so that the real balances of the individual(s) with the lowest money holding is the steady state level  $m$ . All other agents will have more money than they want to hold and will deposit the unproductive balances in government bonds.

To see how this works, I assume  $B = 0$ , so that agents cannot borrow from the government but can still lend to it. I use  $\underline{M}$  to denote the lowest amount of nominal balances held at  $t = 0$ :  $M_0^h \geq \underline{M}$  for all  $h$  with strict equality for some  $h$ .

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<sup>3</sup> Without the initial transfer and without lending and borrowing, we will have a transition period during which agents with initial holdings of money that are less than average will accumulate real balances and agents with initial holdings that are higher than average will decumulate real balances. This will continue until all agents hold the steady state level of real balances. As was noted by BHM different announcements of  $R_m \leq R = 1 + \rho$  imply different distribution of initial wealth. A higher  $R_m$  will in general benefit agents with relatively high amount of initial money holdings and may make the Pareto ranking of alternative (non steady state) equilibria impossible.

After the announcement of the policy,  $m$  is determined by (6) and the price level is determined by:  $m = \frac{M}{P_0}$ . Agents' optimal lending is:  $b_0^h = \frac{M_0^h - M}{P_0} = \frac{M_0^h}{P_0} - m$ . Let  $b = (\frac{1}{n}) \sum_{h=1}^n b_0^h$  denote the average holding of bonds. The budget constraint of the government is now:  $g = -rb - r_m m$ . The consumption of corn is equal to permanent income:  $Y_t^h = \bar{Y} + rb_0^h + rm = \bar{Y} + r(b_0^h - b)$ .

Note that an individual with an average holding of money will hold an average amount of government bonds. The average individual will use the interest payments on his assets (bonds and money) holding to cover the lump sum tax and will consume  $\bar{Y}$  units of corn per period. The average individual will definitely prefer steady states with high  $m$  and high level of average total consumption.

The "poorest" individuals will not hold government bonds and will consume  $\bar{Y} - rb$  units of corn per period. Since the average bond holding  $b$  is decreasing in  $P_0$ , it is possible that the welfare of the "poorest" individuals is relatively high in steady state with high  $P_0$  and low  $m$ . Therefore, Pareto ranking of alternative steady states may not be possible.

We can say however that efficiency requires the Friedman rule because only when the government announces  $R_m = R = 1 + \rho$ , agents will be satiated with real balances.

## 2.5 Differences in the Demand for Money

Pareto ranking of alternative choice of  $R_m$  is not possible when agents have different  $f$  functions. In this case the demand for money in the steady state is the solution  $m^h$  to  $R = R_m (1 + f^h(m))$ , where  $f^h(m)$  is the production of liquidity services technology available to agent  $h$ . As before, the lump sum transfer for  $t \geq 1$  is given by:  $g = -r_m m$ , where  $m = (\frac{1}{n}) \sum_{h=1}^n m^h$  is average real balances in the steady state. When we increase  $R_m$  (holding  $R = 1 + \rho$  constant) average money holding increases and the transfer decreases. For agents who derive large amount of services from holding real

balances, the increase in real balances will more than compensate for the decrease in the transfer. But for agents who derive little or no liquidity services, the reduction in the transfer will dominate and their welfare will go down.

To illustrate, I assume that agents' initial endowment of real balances is zero ( $m_{-1}^h = 0$  for all  $h$ ) and the initial transfer of money is equal to the average amount of real balances in the steady state ( $\tau = m$ ). The amount that agent  $h$  will deposit in his savings account is:  $b^h = \tau - m^h = m - m^h$ . The transfer from the government in the steady state is:  $g = -r_m m$ . And the consumption of corn in the steady state is:

$Y_t^h = \bar{Y} + r b^h + r_m m^h + g = \bar{Y} + (m - m^h)(r - r_m)$ . When  $r > r_m$  the corn consumption of agents with higher than average demand for money ( $m^h > m$ ) is less than their endowment ( $Y^h < \bar{Y}$ ) and agents with lower average demand for money is higher than their endowment. In this sense, agents with higher demand for money transfer resources to agents with lower demand for money. When we reduce the tax on real balances ( $r - r_m$ ) tax revenues typically go down and the transfer from those with above average demand to those with below average demand goes down. The welfare of some agents may actually decline in spite of the fact that the distortion created by the tax has been reduced. For example, the consumption of corn of an agent that does not use real balances ( $m^h = 0$ ) is:  $\bar{Y} + (r - r_m)m$ . This agent's corn consumption is above his endowment by the average tax revenues and his welfare will go down if we adopt the Friedman rule and eliminate the tax on real balances.

This argument was made by BHM (2005, Proposition 1) who use the Lagos-Wright search model. In their model agents that have less than average demand for the goods produced in the decentralized market have less than average demand for money.

The argument is not special for money. If we impose a tax on yellow cheese and distribute the tax revenues equally, it may benefit agents who do not consume yellow cheese. But this is not usually used to argue for a tax on yellow cheese. I therefore think

that the bar of adopting a policy only if it leads to a Pareto improvement is too high and I adopt here the lower bar of efficiency.

### 3. SATIATING AGENTS WITH REAL BALANCES IN AN OVERLAPPING GENERATIONS MODEL

There are some important differences between the *OG* model presented here and the *IL* model of the previous section. As a result of age heterogeneity the optimal real interest rate is different. The newly born may receive different bequests and as a result the problem of heterogeneity is present in the steady state and not only in the transition period. And there may be a difference between the transfer solution to the heterogeneity problem and the loan solution.<sup>4</sup>

The optimal real interest rate in the *IL* model is  $R = 1/\beta$  and the policy maker has no choice in this matter. In the *OG* model there is more than one real interest rate that is consistent with a steady-state equilibrium. A policy-maker who runs a government loans program will therefore face a non-trivial choice.<sup>5</sup>

I focus here on the steady state and on the loan implementation under constant prices of 1 dollar per unit of corn.

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<sup>4</sup> Friedman (1969) assumes that the members of his hypothetical simple society are “immortal and unchangeable”. In footnote 1 he says: “this is equivalent to regarding the community as having a constant distribution of persons by age, sex, etc. Each of our infinitely long-lived individuals stands, as it were, for a family line in the alternative population of changing individuals but unchanging aggregates”. I do not think that it is easy to defend this footnote because of the age heterogeneity that is present in the *OG* model but not in the *IL* model.

<sup>5</sup> The difference between the two models’ implications about the real interest rate has created a controversy that to my mind has not been fully resolved. Cass and Yaari (1966) question the interpretation of the interest rate in Samuelson’s (1958) model and argue that it is not a price used in actual transactions and not even an implicit price. (page 354). Here the government set interest rate is a price used in transactions between the government and private agents.

### 3.1 The Environment and a Planner's Problem

As in Blanchard (1985), agents face uncertainty about life and the probability that an agent will die in the current period ( $\delta$ ) does not depend on his age (indexed by  $t$ ; note the difference between the use of this index to describe calendar time in the previous section and the use of it here). The survival probability is denoted by  $\sigma = 1 - \delta$ . Death and birth occur at the beginning of the period and live agents age  $t \geq 1$  get an endowment of  $\bar{Y}$  units of corn (the newly born age  $t = 0$  do not consume and do not get the endowment). Agents are distributed over  $J$  families or types (indexed  $j$ ). The number of agents who are born from each type is the same as the number of agents who die from that type and is equal to  $n$ . At each point in time there are thus  $\sigma^t n$  age  $t$  type  $j$  agents. The planner chooses the amount of corn consumption  $Y_t^j$  and the amount of real balances,  $m_t^j$  that he will give to agent age  $t$  type  $j$ . The allocation of corn to age  $t$  agents is  $n \sum_{j=1}^J \sigma^t Y_t^j$ . Summing over types yields the aggregate allocation of corn:

$n \sum_{t=1}^{\infty} \sum_{j=1}^J \sigma^t Y_t^j$ . Aggregate supply is:  $nJ\bar{Y} \sum_{t=1}^{\infty} \sigma^t = \frac{nJ\bar{Y}\sigma}{1-\sigma}$ . Using  $\Sigma = \frac{J\bar{Y}\sigma}{1-\sigma}$  we can

write the resource constraint as:

$$(7) \quad \sum_{t=1}^{\infty} \sum_{j=1}^J \sigma^t Y_t^j = \Sigma$$

Money is a free good. The planner maximizes welfare in the steady state and solves the following problem:

$$(8) \quad \max_{Y_t^j \geq 0, m_t^j \geq 0} \sum_{j=1}^J \omega^j \sum_{t=1}^{\infty} (\beta\sigma)^t U(Y_t^j + f(m_t^j)) \text{ s.t. (7),}$$

where  $\omega^j$  is the weight of type  $j$ .

The first order conditions for this problem require (for all  $t$  and  $j$ ):

$$(9) \quad f'(m_t^j) = 0$$

$$(10) \quad \frac{U'(Y_{t-1}^j + f(m_{t-1}^j))}{\beta U'(Y_t^j + f(m_t^j))} = 1$$

$$(11) \quad \omega^j U'(Y_t^j + f(m_t^j)) = \omega^1 U'(Y_t^1 + f(m_t^1))$$

### 3.2 Steady-State Equilibrium

Agents leave their assets holdings as accidental bequest when they die. The heirs are responsible for paying any outstanding loans left by their parents so that the value of the bequest may be negative. The accidental bequests of type  $j$  agents are distributed equally among the newly born type  $j$  agents. In the steady state, bond holdings are different across types and therefore accidental bequests are different across types: A newly born type  $j$  gets a bequest of  $z^j$  dollars. Types are identical in all other respects.

As before there is a government run bank that offers two accounts: checking and savings. I start with the case in which there is no private lending and borrowing and no markets for contingent claims.

The problem of a newly born agent with initial money holdings of  $z$  units is:

$$(12) \quad V(z) = \max_{m_0 \geq 0, b_0 \geq B} v(m_0, b_0) \text{ s.t. } m_0 + b_0 = z \text{ and}$$

$$(13) \quad v(m, b) = \max_{m' \geq 0, b' \geq B} U(\bar{Y} + R_m m + Rb - m' - b' + g + f(R_m m + g)) + \beta \sigma v(m', b')$$

This problem is similar to (4). The only difference is that here the future is “discounted” by  $\beta \sigma$  instead of just  $\beta$ . The first order conditions that an interior solution to (12) and (13) must satisfy are:

$$(14) \quad \frac{U'(C)}{\beta \sigma U'(C')} = R$$

$$(15) \quad R_m (1 + f'(R_m m' + g)) = R$$

Condition (14) is a standard Euler condition. Condition (15) determines the level of real balances and has a unique solution:  $m = m'$ . Thus the beginning-of-period real balances do not change with age.

A steady-state equilibrium is a vector of accidental bequests received by the newly born,  $(z^1, \dots, z^J)$ , a level of real balances  $m = (\frac{1}{J}) \sum_{j=1}^J z^j$ , a policy choice  $(R, R_m \leq R, -\infty < B \leq 0, g)$  and a sequence of endogenous variables  $\{m_t^j = m, b_t^j, Y_t^j\}$  such that: (a) given the policy choice, the endogenous variables are a solution to (12) and (13), (b) the market clearing condition (7) is satisfied, (c) the government transfer to agents age  $t \geq 1$  is:  $g = -r_m m$  and (d)  $z^j = \delta \sum_{t=0}^{\infty} \sigma^t (Rb_t^j + m)$ .

The definition is standard except for (d) that requires equality between the accidental bequest received by the newly born and the value of the portfolio of those who died in the current period. Here, I assume that interest payment and transfers occur before a fraction  $\delta$  of the adult population dies. The value of the portfolios held by type  $j$  agents at the beginning of the period (before some agents die) is  $n \sum_{t=0}^{\infty} \sigma^t (Rb_t^j + m)$  and the newly born get a fraction  $\delta$  of this value.

The balanced budget requirement  $g = -r_m m$  leads to  $m_{tb} = R_m m + g = m$  and we can use (15) to solve for the steady state real balances:

$$(16) \quad f'(m) = \frac{R}{R_m} - 1$$

When  $R$  is held constant, we can write the solution to (16) as  $m(R_m)$ .

A change in  $R_m$  will lead to a transition to a new steady state. In the case of flexible prices, the change will result in a price level change. Here I assume constant prices and therefore the change requires special transfers. When the government changes the real rate of return on money from  $R_m$  to  $\hat{R}_m$ , it must transfer  $m(\hat{R}_m) - m(R_m)$  dollars (here also units of real balances) to all the agents in the economy.

We can rank welfare across steady states with the same  $R$  and with a large enough borrowing limit  $|B|$ .

Claim 3: The steady state welfare of all agents increase with  $R_m \leq R$ .

To show this Claim note that when the policy maker increases  $R_m$  he makes a transfer to all agents and they hold it and derive utility services from it. The government can satiate agents with real balances by setting  $R_m = R$ .

I now turn to the choice of the real interest rate  $R$ .

Claim 4: There exist at least two steady states: An efficient steady state with  $R = 1/\sigma$  and an inefficient steady state with  $R = 1/\beta\sigma$ .

The proof is in the Appendix.

To better understand the reason for setting  $R = 1/\sigma$ , note that from the planner's point of view, the price of consumption at age  $t$  in terms of consumption at age  $t+1$  is  $1/\sigma$ : The planner can take a unit from age  $t$  type  $j$  agents and give more than a unit (i.e.,  $1/\sigma$ ) to age  $t+1$  type  $j$  agents without changing the amount he gives to other agents. He can do it because there are less age  $t+1$  agents. Therefore the choice of  $R = 1/\sigma$  that reflects the true social cost of current consumption, leads to an efficient allocation.

Note that (14) and  $\beta < 1$  imply that when  $R = 1/\sigma$ , corn consumption decreases with age. In this case, young agents will consume above their endowment. Agents who live for a long time will consume below their endowment and their consumption will approach zero as their age goes to infinity. Note also that the optimal interest rate does not depend on the time preference parameter  $\beta$ . This is an important difference between the infinitely lived agents ( $IL$ ) model and the overlapping generations model in Samuelson (1958). It may be argued (and indeed Friedman [1969] does make this point) that the only reason for discounting the future is uncertainty about life and therefore  $\beta = 1$ . In this case, the two models yield the same prescription about the interest rate.

#### 4. THE OPTIMAL SIZE OF THE FINANCIAL SECTOR

The financial sector may be too big because of the incentives to create substitutes for money. Counterfeiting money is an extreme example. Clearly, resources spent by the private sector on counterfeiting money are a waste from the social point of view but from the individual point of view there are incentives to do it. Note that agents have an incentive to counterfeit also when they are satiated with liquidity services because they can always use additional money to buy corn. (So, literally speaking, they are never satiated with money, they are only satiated with liquidity services).

The private sector creates assets that are only in part substitutes for money. In this case, it is not trivial to distinguish between assets that serve a useful social function and assets that are mostly counterfeits for money. See Maya Eden (2012) for a discussion of the social value of intermediation aimed at the allocation of physical capital.

More generally, the problem is to map the areas in which the government has an advantage over the private sector in supplying liquidity. Here I use the above *OG* model to provide partial answers under the assumption that the government has an advantage in creating money and in collecting uncollateralized loan and tax payments.

##### 4.1 Private Intermediation

I relax the assumption that the cost of private intermediation is prohibitive, and assume instead a private intermediation sector that accepts one period deposits and makes one period loans. I follow Mehra, Piguillem and Prescott (2011) in assuming that the cost of intermediation is proportional to the amount intermediated. They also assume that the government debt is intermediated at zero cost. I take it to mean that the government can borrow and lend money without using any resources.

I use  $d_t^j$  to denote the amount of private bonds owned by a type  $j$  agent at age  $t$  (negative amounts are loans). I use  $d = (d_0^1, \dots, d_0^J; d_1^1, \dots, d_1^J; d_2^1, \dots, d_2^J; \dots)$  to describe the

holding of private bonds in the steady state and the index function  $I_t^j = 1$  if  $d_t^j < 0$  and  $I_t^j = 0$  otherwise. I assume that individuals face a limit  $D$  on private borrowing.

A vector of private bond holdings is feasible if:

$$(17) \quad \sum_{t=0}^{\infty} \sigma^t \sum_j d_t^j = 0 \text{ and } d_t^j \geq D \text{ for all } t, j.$$

The aggregate cost of private intermediation is:

$$(18) \quad TC(d) = \phi n \sum_{t=0}^{\infty} \sigma^t \sum_j I_t^j d_t^j \text{ if (17) is satisfied and } TC(d) = \infty \text{ otherwise.}$$

Here  $0 < \phi < 1$  is a cost parameter.

Intermediation is done by a competitive banking sector that pays (charges) the interest  $R_d$  on deposits (loans) and an additional “service charge” of  $\phi Id$  for loans.

The problem of the representative type  $j$  agent is now:

$$(19) \quad \max_{Y_t^j, m_t^j, b_t^j, d_t^j} \sum_{t=1}^{\infty} (\beta \sigma)^t U(C_t^j)$$

s.t.

$$C_t^j = Y_t^j + f(m_{tb}^j); \quad m_{tb}^j = R_m m_{t-1}^j + g$$

$$m_0^j + b_0^j + (1 - \phi I_0^j) d_0^j = z^j$$

$$m_t^j + b_t^j + (1 - \phi I_t^j) d_t^j + Y_t^j = \bar{Y} + R_m m_{t-1}^j + R b_{t-1}^j + R_d d_{t-1}^j + g$$

$$z^j \geq 0 \text{ is given, } m_t^j \geq 0, \quad b_t^j \geq B, \quad d_t^j \geq D$$

This problem can be written as:

$$(20) \quad \max_{m_0 \geq 0, b_0 \geq B, d_0 \geq D} v(m_0, b_0, d_0) \text{ s.t. } m_0 + b_0 + (1 - \phi I) d_0 = z; \text{ and}$$

$$(21) \quad v(m, b, d) = \max_{m' \geq 0, b' \geq B, d' \geq D} U(\bar{Y} + R_m m + R b + R_d d + g - m' - b' - (1 - \phi I) d' + f(R_m m + g)) \\ + \beta \sigma v(m', b', d')$$

The first order conditions that an interior solution to (19) must satisfy are:

$$(22) \quad \frac{U'(C_{t-1}^j)}{\beta \sigma U'(C_t^j)} = R = \frac{R_d}{1 - \phi I_t^j} = R_m (1 + f'(R_m m_{t-1}^j + g))$$

Claim 5: (22) and (17) can be satisfied only if  $d_t^j = 0$  for all  $t, j$ .

To show the Claim, note that the effective interest for borrowers who use private intermediation ( $= R_D / (1-\phi)$ ) is higher than the interest for lenders ( $= R_D$ ). Thus, (22) cannot hold for both lenders and borrowers.

The implication of Claim 5 is that costless government intermediation will crowd out costly private intermediation. This is not surprising. What may be more surprising is the realization that satiating agents with real balances requires the crowding out of private intermediation. In the absence of government intermediation, the choice  $R_m = R_D$  will satiate lenders with money, but since in this case  $R_m < R_D / (1-\phi)$ , borrowers will hold suboptimal level of real balances. This is different from the description in Friedman (1969) who assumed that private bonds are held in the optimal steady state. I will discuss this difference later.

There is of course another reason why any equilibrium outcome with private intermediation cannot coincide with the solution to the planner's problem (8). The clearing of the market for corn requires:

$$(23) \quad \sum_{t=1}^{\infty} \sum_{j=1}^J \sigma^j Y_t^j = \Sigma - \phi \sum_{t=0}^{\infty} \sum_{j=1}^J I_t^j d_t^j \geq \Sigma$$

This will coincide with the resource constraint (7) only when  $d_t^j = 0$  for all  $t, j$  and the market for private intermediation is not active. Otherwise, the inequality in (23) is strict and the aggregate consumption of corn is less than the aggregate endowment.

Note that the Ricardian type equivalence between transfers and loans holds also here. The government can shut the loan program by imposing  $B = 0$  and give each newly born agent  $\bar{m}$  units of real balances as a transfer. As in section 2.3, agents use the unproductive balances to buy government bonds and the interest on money and bond holdings is used to pay the lump sum tax. This equivalence may not hold once we allow for costly markets for contingent claims.

#### 4.2 A Market for Contingent Claims on Nominal Bequests

From the individual point of view there is an incentive to sell bequests (contingent contracts that promise assets in the case of death) even when operating this market requires resources. But from the social point of view, the resources spent on operating a market for accidental bequests may be a waste. This may be important when agents are satiated with real balances and accidental bequests are large.

To illustrate, I assume that all agents are identical and start with the case in which the government gives the newly born representative agent  $z$  units of real balances. There are no bonds (private or government) but there is a costly market for accidental bequests. The price of a promise to deliver a unit of accidental bequest with probability  $\pi$  is proportional to the probability of delivery and is given by  $q\pi$ . An agent age  $t$  that holds  $m_t$  units of real balances can therefore sell a promise to “deliver”  $R_m m_t + g$  units at time  $t+1$  if he does not survive (with probability  $\delta = 1 - \sigma$ ). The representative agent can buy a portfolio of promises on accidental bequests that will deliver a unit with certainty. This is possible because the fraction of agents that die is known and prices are actuarially fair. I use  $A_t$  to denote the amount that the agent will get (with certainty; in units of corn) at age  $t+1$  from the portfolio of bequests bought at age  $t$ .

The seller pays a fraction  $0 \leq \chi \leq 1$  of the amount of claims that he sells to cover real transaction costs. An agent who dies at time  $t+1$  will have  $R_m m_t + g$  dollars in his account and can sell claims on this amount at time  $t$  for  $q\delta(1 - \chi)(R_m m_t + g)$  dollars.

The budget constraint of the representative agent is thus:

$$(24) \quad Y_t + qA_t + m_t = \bar{Y} + A_{t-1} + q\delta(1 - \chi)(R_m m_t + g) + R_m m_{t-1} + g$$

The market clearing conditions are:

$$(25) \quad \sum_{t=0}^{\infty} \sigma^t A_t = q\delta \sum_{t=0}^{\infty} \sigma^t (R_m m_t + g)$$

$$(26) \quad \sum_{t=1}^{\infty} \sigma^t Y_t = \Sigma - \chi \sum_{t=0}^{\infty} \sigma^t A_t$$

When  $A_t > 0$ , the amount of corn that can be distributed according to (26) is less than the amount that can be distributed according to the resource constraint (7).

Therefore, the equilibrium outcome will coincide with the planner's solution to (8) only if  $A_t = 0$  for all  $t$ .

To achieve efficiency the policy-maker may impose a 100% inheritance tax. Alternatively, the government may give the newly born  $z$  dollars as a loan rather than a gift. This will be only a partial solution in the case of heterogeneous agents because in this case some will die with strictly positive net worth and a market for accidental bequest may emerge.

The case of nominal bequest is different from the case in which the accidental bequest is physical capital or corn. A market for "real accidental bequest" may actually improve matters because agents fail to take into account the positive effect of leaving physical capital as accidental bequest. An external effect of this type is absent in the case of money because the amount of real balances held is determined by the demand for it (which depends on  $R_m$ ) and is independent of the amount of accidental bequest. To make this point, I now add the possibility of investing in physical capital.

#### 4.3 Real Investment Opportunity and a Market for Real Bequests

I add the possibility of sowing corn: An individual that sows  $k$  units of corn at age  $t$  will harvest at age  $t+1$ ,  $F(k)$  units of corn. I assume that  $F$  is differentiable, strictly monotone and strictly concave. Note that physical capital (corn in the soil) fully depreciates after one period. As in the previous section, all agents are identical.

Agents who die do not get the fruits of their investment: Their heirs do. The aggregate harvest is  $n \sum_{t=1}^{\infty} \sigma^{t-1} F(k_{t-1})$  and the resource constraint in the steady state is:

$$(27) \quad \sum_{t=1}^{\infty} \sigma^t (Y_t + k_t) = \Sigma + \sum_{t=1}^{\infty} \sigma^{t-1} F(k_{t-1})$$

The planner's problem is:

$$(28) \quad \max_{Y_t \geq 0, m_t \geq 0, k_t \geq 0} \sum_{t=1}^{\infty} \beta^t U(Y_t + f(m_t)) \quad \text{s.t. (27)}.$$

The first order conditions for this problem require:

$$(29) \quad \frac{U'(C_{t-1})}{\beta U'(C_t)} = 1 = F'(k_t)$$

Steady State Equilibrium: I start with the case in which there is no market for bequests and no private intermediation. The budget constraint of the representative agent is now:  $Y + m' + b' + k' = \bar{Y} + R_m m + Rb + F(k) + g$ , and the newly born solves the following problem:

$$(30) \quad \max_{m_0 \geq 0, b_0 \geq 0, k_0 \geq 0} v(m_0, b_0, k_0) \quad \text{s.t.} \quad m_0 + b_0 + k_0 = z \quad \text{and}$$

$$v(m, b, k) = \max_{m \geq 0, b \geq 0, k \geq 0} U(\bar{Y} + F(k) + R_m m + Rb + g + f(R_m m + g) - m' - b' - k') \\ + \beta \sigma v(m', b', k')$$

The first order conditions for an interior solution to this problem are:

$$(31) \quad \frac{U'(C)}{\beta \sigma U'(C')} = R = F'(k') = R_m (1 + f'(R_m m' + g))$$

Since any steady state allocation must satisfy (31) there is a difficulty in achieving the first best. The problem is in the choice of the real interest rate  $R$ . If the policy-maker chooses  $R = 1/\sigma$ , the representative agent's choice (that satisfies [31]) will satisfy the first equality in the planner's first order conditions (29) but not the second. If the policy-maker chooses  $R = 1$ , the representative agent's choice will satisfy the second equality in (29) but not the first.

The reason for the difficulty is in the external effect problem discussed above: Agents fail to take the positive effect that their accidental bequest have on their heirs into account. Paying a subsidy of  $(1/\sigma) - 1$  per unit of harvest will solve the problem. A market for accidental bequest may also internalize the externality if its operation does not take too much resources.

## 5. CONCLUDING REMARKS

I have examined the implementation of the Friedman rule under the assumption that age specific lump sum transfers are possible. This was done both for an infinitely lived agents (*IL*) model and for an overlapping generations (*OG*) model.

Heterogeneity with imperfect capital markets pose problems for satiating agents with money. A lump sum transfer or a government loan program can usually solve these problems.

The crowding out of private lending and borrowing is an important by product of the optimal policy. Lucas (2000) estimates the welfare gain from reducing the nominal interest rate from 4% to zero at around 1% of GDP. Mehra, Piguillem and Prescott (2011) estimate a cost of private intermediation that is as at least 3.4% of GDP. It thus seems that the government transfer/loan component of the optimal liquidity policy is more important than the zero nominal interest rate component. The relative importance of the zero nominal interest will decline as cash becomes less important.

The crowding out result is different from Friedman (1969) who envisions an active market for default free bonds even in the optimal steady state. My first inclination was to look for ideological reasons to explain this difference. Being a champion of free markets, I thought that Friedman could not entertain the idea of a government run loan program. This is not the case. In his celebrated book, *Capitalism and Freedom*, Friedman argues that the government has an advantage in enforcing uncollateralized loan contracts and advocates a government loan program for financing education. It is not clear to me why Friedman insists on the use of government loans to finance education and not for smoothing consumption.

The reason for the difference between the model here and Friedman's (1969) model is in the assumption about the cost of intermediation. Friedman implicitly assumes no cost of private intermediation and no cost for creating private riskless assets. The

assumption that money yields liquidity services in what seems to be a frictionless world is problematic. The Appendix elaborates.

I also attempted the broader question of the role of government in the financial sector. I argue that operating a market for accidental nominal bequests is a waste of resources from the social point of view because trading in contingent claims on money does not change the equilibrium amount of real balances.

Markets for contingent claims on money may be important when the nominal interest rate is zero and agents hold large amounts of money. The problem may be more severe when implementing the Friedman rule by an initial transfer rather than a loan. In the initial transfer scheme young agents get a large amount from the government and they pay taxes that cover the interest payments (explicit or implicit) on the initial transfers. But they have full ownership of the transfer and can therefore sell it in the accidental bequests market. In the loan implementation, agents pay the interest on money during their lifetime but they cannot sell the principle that is owned by the government. This speaks in favor of the loan implementation.

Appendix A: The money in the utility function approach and the crowding out result

To better understand the crowding out result, I now turn to a discussion of the micro foundations of the money (and bonds) in the utility function approach. Baumol (1952) and Tobin (1956) were an early important contribution to the micro foundations literature. They stressed the fixed cost of trading in bonds.

The fixed cost is an important component in the cost of intermediation. But since fixed costs complicate the model many authors abstracted from it. To capture the difference between money and bonds it became customary to divide each period into sub-periods assuming that the amount of money holdings can be costlessly changed from one sub-period to another but the amount of bond holding can be changed only at the end of the period. The early attempt by Patinkin (1965, chapter 5.2) may serve as an example.

In the Patinkin model, contracts are signed at the beginning of the period (Monday) and are executed during the period (week). Patinkin divides the week into many sub-periods (hours). Contracts are drawn randomly at the beginning of each sub-period and individuals are called upon to make and receive payments. Money is held to avoid the inconvenience associated with not being able to make a payment. Bond payments are not random and occur at the end of the period.

I now turn to a partial model that builds on the Patinkin model and uses elements from shopping time models and search models.<sup>6</sup> As a byproduct the discussion here provides an explanation for the relationship between the beginning of period real balances and “liquidity services” (the function  $f$ ).

I consider the problem of an agent who gets an endowment of  $G > 1$  goods:  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_G)$ . The agent starts the period with  $M_b$  dollars in his checking account and  $B_b$  dollars in his savings account (after interest payments were made). He plans to carry

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<sup>6</sup> See for example, Brock (1974), Jones (1976), McCallum and Goodfriend (1987) and Kiyotaki and Wright (1989).

$M$  dollars and  $B$  dollars worth of bonds to the next period. He thus plans to spend a total of  $I = \sum_i P_i \bar{x}_i + M_b - M + B_b - B$  dollars, where  $P = (P_1, \dots, P_G)$  are dollar prices.

As in the Patinkin model the consumer chooses the consumption vector,  $x = (x_1, \dots, x_G)$ , at the beginning of the period and then execute transactions during the period. There are many subperiods or rounds of trade. In each round pairs are formed in a random manner and each pair can trade either goods for goods or goods for money. Exchanging goods for bonds directly is not allowed. I start with the assumption that the cost of a trip to the bank during the period is prohibitive.

In this environment having money at the beginning of the period may cut on the number of rounds that are required to execute trades that were chosen at the beginning of the period. This is because having “enough” money at the beginning of the period allows the agent to buy goods before he sells goods.

To illustrate, I consider the case in which there are three types of agents and three goods. The prices of all goods are the same. Agents type 1 want to buy a unit of good 1 and sell a unit of good 2. Agents type 2 want to buy a unit of good 2 and sell a unit of good 3. Agents type 3 want to buy a unit of good 3 and sell a unit of good 1. Suppose now that an agent type 1 meets an agent type 3. If the agent type 1 has enough money, he may buy a unit of good 1. Otherwise, he will have to pass this trading opportunity and wait until he sell good 2 before he can buy good 1.

In general, the time (measured by the number of rounds) that the agents will spend in the market depends on the state (history of the meetings in the rounds of trade), the shopping list (or the excess demand vector),  $x - \bar{x} = (x_1 - \bar{x}_1, \dots, x_G - \bar{x}_G)$ , nominal prices and the beginning of period nominal balances,  $M_b$ . It also depends on the beginning of period money balances held by others. For example, it is possible that the individual agent will meet someone who wants to buy what he has for sale but does not have enough money.

Taking the amount of money held by others as given, I assume that from the individual agent's point of view the time spent in the market in state  $s$  is:

$l_s(x - \bar{x}, P, M_b)$ . The function  $l_s$  is weakly decreasing in  $M_b$ : The more money you have at the beginning of the period the less time you have to spend in the market because having "enough" money allows buying before selling.

The number of rounds required to complete trade will not change if we double the beginning of period money and prices. I therefore assume:

$$(A1) \quad l_s(x - \bar{x}, P, M_b) = l_s(x - \bar{x}, \lambda P, \lambda M_b) \text{ for all } \lambda > 0.$$

I now choose  $\lambda = 1/p_1$  and write (1) as:

$$(A2) \quad l_s(x - \bar{x}; 1, p_2, \dots, p_G; m_b),$$

where  $p_i = p_i/p_1$  is the price of good  $i$  in terms of good 1 and  $m_b = M_b/p_1$  is the value of the beginning of period money in terms of good 1.

The agent's single period utility function,  $U(x, l)$ , is increasing in  $x$  and decreasing in  $l$ . Assuming expected utility and using  $\Pi_s$  to denote the probability of state  $s$ , we can now write the indirect utility:

$$(A3) \quad V(I, \bar{x}, P, m_b) = \max_x \sum_s \Pi_s U(x, l_s(x - \bar{x}, p, m_b)) \text{ s.t. } \sum_i P_i x_i = I$$

Since any vector  $x$  that satisfies  $\sum_i P_i x_i = I$  also satisfies  $\sum_i \lambda P_i x_i = \lambda I$ , the indirect utility function is also homogeneous:

$$(A4) \quad V(I, \bar{x}, P, m_b) = V(\lambda I, \bar{x}, \lambda P, m_b) \text{ for all } \lambda > 0.$$

Again, I choose  $\lambda = 1/p_1$ , and write:

$$(A5) \quad V(Y, \bar{x}, p, m_b) = V(I, \bar{x}, P, m_b),$$

where  $Y = I/p_1$  is spending in terms of good 1 and  $p = (1, p_2, \dots, p_G)$  are relative prices.

Assuming that the endowment and relative prices do not change over time, we can write (A5) with some abuse of notation as:

$$(A6) \quad V(Y, m_b)$$

In previous sections I simplified by assuming a pair of functions  $U$  and  $f$  such that<sup>7</sup>

$$(A7) \quad U(Y + f(m_b)) = V(Y, m_b).$$

As was shown above this simplification yields a demand for money that depends only on the rate of return on money and not on income. But since income was held constant this simplification may be justified.<sup>8</sup>

Allowing for a trade between bonds and money during the period: I now allow

withdrawals from the savings account during the period but getting a loan can be done only at the end of the period at some cost. To withdraw funds from the savings account requires a trip to the bank and the time spent on making the trip cannot be used to execute transactions. But nevertheless an agent who is short of cash may choose to make the trip to the bank because the expected time he saves by having more money is greater than the time it takes to make the trip. I therefore assume that the time spent in the market in state  $s$  is given by:  $l_s(x - \bar{x}, p, m_b, b_b)$ , where:

$$(A8) \quad \frac{\partial l_s(x - \bar{x}, p, m_b, b_b)}{\partial m_b} \geq \frac{\partial l_s(x - \bar{x}, p, m_b, b_b)}{\partial b_b} \geq 0$$

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<sup>7</sup> The above description leads to a function  $f$  that is increasing up to the satiation level and then it becomes flat. That is, there exists  $\bar{m}$  such that  $f'(m) > 0$  when  $m < \bar{m}$  and  $f'(m) = 0$  when  $m \geq \bar{m}$ . Friedman assumes that holding more money than the satiation level reduces utility because of the need to employ "body guards" to protect large amount of cash. This argument may also apply to debit cards that may be stolen. In any case, most of Friedman's argument does not require the assumption of a unique satiation level. I have also neglected the amount of money held by others. If the agent meets someone who wants to buy what he wants to sell but does not have enough money the agent will have to give up this trading opportunity. This market externality was discussed in Diamond (xx) and others. But this externality will not change the main result. Welfare increases when we move to a steady state in which everyone has more money.

<sup>8</sup> When income changes over time, the specification  $C_t = Y_t + \bar{Y}_t f\left(\frac{m_b}{\bar{Y}_t}\right)$  will yield a demand for money that is proportional to income in the steady state. The velocity will depend on the real rate of return on money in the steady state.

Abstracting from negative marginal products (non-pecuniary returns), this assumption is perfectly consistent with the assumptions made by Friedman. It implies that if  $\frac{\partial l_s(x - \bar{x}, p, m_b, b_b)}{\partial m_b} = 0$  then  $\frac{\partial l_s(x - \bar{x}, p, m_b, b_b)}{\partial b_b} = 0$ .

I now define the indirect utility function:

$$(A9) \quad V(Y, \bar{x}, p, m_b, b_b) = \max_x \sum_s \Pi_s U(x, l_s(x - \bar{x}, p, m_b, b_b)) \quad \text{s.t.} \quad \sum_i p_i x_i = Y$$

Since when agents are satiated with money they will never make a trip to the bank during the period, it follows that:

$$(A10) \quad V(Y, \bar{x}, p, \bar{m}, b_b) = V(Y, \bar{x}, p, \bar{m}, 0) \quad \text{for all } b_b$$

I assume that making a loan (creating a private bond) is costly. Since at the optimal steady state bonds do not yield any services no one will pay the cost for creating a private bond and there will be no trade in private bonds.

#### Appendix B: Proof of Claim 4

To show that there exists a steady state with  $R = 1/\sigma$ , note that in the steady state real balances do not change over time. Corn consumption must therefore satisfy the budget constraint:  $\sum_{t=1}^{\infty} R^{-t} Y_t^j = b_0^j + \bar{Y} \sum_{t=1}^{\infty} R^{-t}$ . Substituting  $R = 1/\sigma$  leads to:

$$\sum_{t=1}^{\infty} \sigma^t Y_t^j = b_0^j + \bar{Y} \sum_{t=1}^{\infty} \sigma^t. \quad \text{Since } b_0^j = z^j - m = z^j - (\gamma_j) \sum_i z^i, \text{ summing over } j \text{ leads to } \sum_j b_0^j = 0 \text{ and to (7). Thus, the choice } R = 1/\sigma \text{ is feasible and leads to market clearing.}$$

The accidental bequest received by the newly born is:

$$\sum_j z^j = \delta \sum_{t=0}^{\infty} \sum_j \sigma^t (R b_t^j + m) = mJ + \delta R \sum_{t=0}^{\infty} \sum_j \sigma^t b_t^j. \quad \text{Since the present value of the}$$

balances held in the savings account is zero (if it is positive the agent can increase his consumption; if it is negative the constraint on the size of the loan is violated) we have:

$$\sum_{t=0}^{\infty} R^{-t} b_t^j = \sum_{t=0}^{\infty} \sigma^t b_t^j = 0 \quad \text{and} \quad \sum_j z^j = mJ. \quad \text{Thus the average accidental bequest is}$$

equal to the amount of money held in the steady state.

To show that there exists a steady state with  $R = \frac{1}{\beta\sigma}$ , note that in this case agents will choose smooth consumption paths:  $Y_t^j = \bar{Y} + rb_0^j$ . Summing over  $j$  leads to  $\sum_j Y_t^j = J\bar{Y}$  and Summing over  $t$  leads to (7).

The choice  $R = \frac{1}{\beta\sigma}$  leads to smooth consumption paths. But this choice is not optimal. Only the choice  $R = \frac{1}{\sigma}$  leads to an outcome that satisfies the first order condition (10) for the planner's problem.  $\square$

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