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Conundrums for Nonconsequentialists*

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Keywords: consequentialism, welfarism, social choice, welfare economics.

JEL classification numbers: D63, D71.

1 Introduction

Consequentialism is the approach to normative evaluation in which only the consequences that are obtained from an alternative are of ethical significance. Here, I consider the more specific form of consequentialism used in social choice theory and welfare economics when the objective is to provide a normative ranking of a set of alternatives. This ranking is consequentialist if it only depends on a ranking of the associated consequences. In many applications, the consequences are utilities for a group of individuals. *Utility consequentialism* requires all non-utility features of the alternatives to be ignored when ranking them. This approach to normative evaluation is also known as *welfarism*.

* This article is an expanded version of the first part of my Presidential Address to the Society for Social Choice and Welfare that I delivered on June 19, 2008 at Concordia University in Montreal. I have benefited from discussing the subject of my lecture with Marc Fleurbaey.

Consequentialism precludes taking account of many important values, such as liberal rights and the fairness of social allocations. Because of this limitation, there has been a great deal of interest in non-welfarist or, more generally, nonconsequentialist approaches to normative evaluation in recent decades. However, nonconsequentialist principles often conflict with other cherished values. The most well-known example of such a conflict is provided by Amartya Sen's liberal paradox (Sen, 1970b). This paradox shows that if individuals are given the right to determine the social ranking on some pairs of alternatives, then there are configurations of individual preferences for which it is not possible to also satisfy the Weak Pareto Principle (which requires the social ranking to respect unanimous strict preferences) provided that the social ranking is acyclic.

Kaplow and Shavell (2001) contend that the conflict between nonconsequentialism and the Weak Pareto Principle is more widespread than the one identified by Sen. They argue that *any* non-welfarist approach to social evaluation, not just a respect for liberal rights, must violate this form of the Pareto Principle given some weak regularity conditions. Their proof of this claim utilizes the single-profile framework employed in traditional welfare economics. As we shall see, their theorem is a consequence of the equivalence of welfarism and Pareto Indifference (which requires universal indifference to be respected) in a single-profile setting when the social ranking of the alternatives is a quasiordering (Blackorby, Donaldson, and Weymark, 1990).

When consequences are multidimensional, appeal is often made to some form of a *dominance principle* that requires one alternative to be preferred to a second if the former vector dominates the latter in the space of consequences. The Weak Pareto Principle is a dominance principle applied to vectors of individual utilities. There are now a number of single-profile impossibility theorems that demonstrate the incompatibility of dominance criteria with various nonconsequentialist principles given some rationality restrictions on the rankings being considered. Each of these impossibility theorems provides nonconsequentialists with a difficult conundrum: How should the fundamental incompatibilities between the desiderata of the theorem be resolved?

In this article, I consider some of these single-profile impossibility theorems and examine what they have in common and how they differ. In particular, I identify groups of results that have similar formal structures and are established using similar proof strategies. In order to highlight the underlying structural similarities and dissimilarities of the theorems that I discuss, I sometimes present variants of the theorems that were established in the literature, rather than the original results themselves. Furthermore, for the same reason, I sometimes use stronger axioms than are necessary, rather than provide the most general form of an impossibility theorem. The theorems that I have chosen to consider in some detail illustrate the main kinds of impossibility results that have appeared in the literature. Many other nonconsequentialist impossibility theorems have been established, some of which are mentioned in subsequent sections.[1]

I begin in Section 2 with some notation and definitions. In Section 3, I present the single-profile characterization of welfarism in terms of Pareto Indifference. The Kaplow–Shavell Theorem is considered in Section 4. In Section 5, I discuss the conflicts identified by Sen (1970a) and Brun and Tungodden (2004) between the Pareto Principle and dominance principles that involve permuting the positions of individuals. Pareto conflicts with inequality aversion principles identified by Gibbard (1979) and Fleurbaey and Trannoy (2003) are considered in Section 6. The consequences in Sections 3–6 are all utilities. In Section 7, I consider non-welfarist forms of nonconsequentialism. Specifically, I discuss impossibility theorems about the measurement of standards of living due to Pattanaik and Xu (2007) and an abstract theorem due to Hare (2007) that he has applied to the problem of whether taking account of proximity is morally justified when deciding whether to aid the needy. Finally, in Section 8, I offer some concluding remarks about how one might resolve a conflict between nonconsequentialist and dominance principles without abandoning social rationality.

2 Preliminaries

Let X be the set of *alternatives*. Depending on the application, members of X could be, for example, social alternatives, states of the world, or actions. The set of individuals is $N = \{1, \ldots, n\}$, where $n \geq 2$. Alternatives in X are mapped into a set of *consequences* C. What these consequences are and how X is mapped into C differs in each of the problems considered here. However, in each case, C is a subset of an m-dimensional Euclidean space $\mathbb{R}^m$.[2]

Let A be a set which, depending on the context, shall be either X or C. A *weak preference* relation is a binary relation R on A that is interpreted as meaning "is weakly preferred to". The corresponding *strict preference* relation P ("is strictly preferred to") and *indifference* relation I ("is indifferent to") are defined by setting, for all $a, b \in A$, (i) aPb if and only if aRb and $\neg(bRa)$ and (ii) aIb if and only if aRb and bRa.

[1] After my presentation in Montreal, I learned that Prasanta Pattanaik and Yongsheng Xu were exploring the structural unity of examples that exhibit a conflict between dominance, context-dependence (a form of nonconsequentialism), and a continuity (and possibly a rationality) condition imposed on the ranking being considered. I am grateful to them for providing me with a preliminary version of Pattanaik and Xu (2012).

[2] The following notation is used for vector equalities and inequalities. For all $x, y \in \mathbb{R}^m$, (i) $x \geq y$ if and only if $x_i \geq y_i$ for all i, (ii) $x > y$ if and only if $x \geq y$ and $x \neq y$, (iii) $x \gg y$ if and only if $x_i > y_i$ for all i, and (iv) $x = y$ if and only if $x_i = y_i$ for all i. The origin in $\mathbb{R}^k$ is denoted by $\mathbf{0}_k$.

The relation R is *reflexive* if aRa for all $a \in A$, *complete* if aRb or bRa for all distinct $a, b \in A$, *transitive* if aRb and bRc imply aRc for all $a, b, c \in A$, and *acyclic* if $a_1 P a_2, \dots, a_{s-1} P a_s$ imply $\neg(a_s R a_1)$ for all $a_1, \dots a_s \in A$. The relation R is a *quasiordering* if it is reflexive and transitive and it is an *ordering* if it is a complete quasiordering.

3 Single-Profile Welfarism

In this section, I consider a single-profile version of utility consequentialism. Each individual $i \in N$ has a utility function $U_i \colon X \to \mathbb{R}$, interpreted as being a comprehensive measure of well-being.[3] There is a fixed *profile* of utility functions $U = (U_1, \dots, U_n)$. For each $x \in X$, U determines a vector of individual utilities $U(x) = (U_1(x), \dots, U_n(x))$. The set of utility vectors that are achievable with the profile U and set of alternatives X is $U(X) = \{U(x) \mid x \in X\}$. Here, the set of consequences C is $U(X)$ and, hence, $m = n$.

Let R_U denote a weak social preference relation on X. The use of the subscript indicates that this preference is conditional on the profile of utility functions U being considered. The corresponding strict preference and indifference relations are P_U and I_U, respectively. It is supposed that R_U is either an ordering or a quasiordering. A weak social preference relation R^*_U on the set of consequences $U(X)$ is called a *social welfare ranking*. The corresponding strict preference and indifference relations are P^*_U and I^*_U, respectively.

In the single-profile setting being considered here, utility consequentialism requires that the social ranking of the alternatives in X be determined by a social welfare ranking of the achievable utility vectors $U(X)$, a property known as *Single-Profile Welfarism*.

Single-Profile Welfarism. There exists a a social welfare ranking R^*_U on $U(X)$ such that for all $x, y \in X$,

$$x R_U y \leftrightarrow U(x) R^*_U U(y). \tag{1}$$

Pareto Indifference requires two alternatives to be socially indifferent if everybody is indifferent between them.

Pareto Indifference. For all $x, y \in X$, if $U(x) = U(y)$, then $x I_U y$.

In general, these two conditions are not equivalent. For example, suppose that $n = 2$, $X = \{x, y, z\}$, $U(x) = U(y)$, $U_1(z) > U_1(x)$, and $U_2(z) < U_2(x)$. Define R_U by setting $x I_U y$, $x P_U z$, and $z P_U y$. Pareto Indifference is satisfied, but there is no binary relation R^*_U on $U(X)$ for which (1) holds, so Single-Profile Welfarism is not.

[3] For discussions of the importance for normative social evaluation of interpreting utility functions in this way, see Sen (1990) and Blackorby, Bossert, and Donaldson (2005).

In the preceding example, R_U is not transitive. Theorem 1 demonstrates that Pareto Indifference is a necessary and sufficient condition for Single-Profile Welfarism provided that R_U is a quasiordering. Moreover, in this case, R_U^* is also a quasiordering. Furthermore, if R_U is complete, then so is R_U^*.[4]

Theorem 1. *For the triple $\langle X, U, R_U \rangle$, if $|X| \geq 3$ and R_U is a quasiordering (resp. ordering) of X, then Pareto Indifference is satisfied if and only Single-Profile Welfarism is satisfied with R_U^* a quasiordering (resp. ordering) of $U(X)$.*

Proof. (i) Suppose that R_U is a quasiordering and that Pareto Indifference is satisfied. I first show that if xR_Uy for some $x, y \in X$ and there exist $w, z \in X$ such that $U(w) = U(x)$ and $U(z) = U(y)$, then wR_Uz.[5] By Pareto Indifference, wI_Ux and yI_Uz. Together with the assumption that xR_Uy, transitivity of R_U implies that wR_Uz.

Define R_U^* on $U(X)$ as follows. Consider any $u, v \in U(X)$. By the definition of $U(X)$, there exist $x, y \in X$ such that $U(x) = u$ and $U(y) = v$. Let uR_U^*v if and only if xR_Uy and vR_U^*u if and only if yR_Ux. By the preceding argument, the ranking of u and v defined in this way is independent of the alternatives in X used to generate u and v. Thus, Single-Profile Welfarism is satisfied.

If $u = v$, then y can be chosen to be x. It then follows from the reflexivity of R_U that R_U^* is also reflexive. Consider any $t, u, v \in U(X)$ for which tR_U^*u and uR_U^*v. By construction, there exist $x, y, z \in X$ with $U(x) = t$, $U(y) = u$, and $U(z) = v$ such that xR_Uy and yR_Uz. Transitivity of R_U implies that xR_Uz and, hence, by the definition of R_U^*, that tR_U^*v. Thus, R_U^* is transitive. If R_U^* is complete, then by the definition of R_U^*, so is R_U^*.

(ii) If Single-Profile Welfarism is satisfied and R_U^* is reflexive, it follows immediately from (1) that Pareto Indifference is satisfied. □

As the proof of Theorem 1 shows, only reflexivity of R_U^* is needed to conclude that Single-Profile Welfarism implies Pareto Indifference. However, the reverse implication utilizes the full force of the transitivity of R_U.

Theorem 1 demonstrates that a commitment to Single-Profile Welfarism amounts to endorsing Pareto Indifference. Thus, a utility nonconsequentialist in the single-profile context being considered here must reject Pareto Indifference, at least if R_U is a quasiordering. Blackorby, Donaldson, and Weymark (1990) have argued that there may be good reasons for doing so. For example, Pareto Indifference is incompatible with granting an individual the right to choose between alternatives that only differ in some feature that falls within his protected private sphere (e.g., the colour of his bedroom walls) independent of the utility consequences. Furthermore, Pareto Indifference does not permit the motivations an individual has for assigning utilities to alternatives

[4] When R_U is an ordering, Theorem 1 follows from combining Propositions 1 and 2 in Blackorby, Donaldson, and Weymark (1990).

[5] This property is known as *Profile-Dependent Neutrality.*

(e.g., the pleasure a sadist obtains from torturing someone) to play any role in determining the social ranking.[6]

4 The Kaplow–Shavell Theorem

Kaplow and Shavell (2001) have argued that any non-welfarist procedure for evaluating social policies, such as one that incorporates considerations of fairness, must violate the Weak Pareto Principle. That is, in some circumstances, such a procedure must not respect a unanimous strict ranking of the alternatives by the individuals. This conclusion makes use of some auxiliary assumptions, notably a continuity assumption. The implications of this view are developed at great length in Kaplow and Shavell (2002). For reasons discussed in Section 8, Fleurbaey, Tungodden, and Chang (2003) believe that what Kaplow and Shavell have shown in their formal theorem establishes a less far-reaching result than what their informal statement suggests.

In this section, I present a version of the Kaplow–Shavell Theorem for a social welfare ordering R_U on the set of alternatives X and its corresponding social welfare ranking R_U^* of $U(X)$.[7] I also discuss how their theorem relates to Theorem 1. The role that the continuity assumption plays in their analysis is given particular attention because it is different from the role that continuity assumptions play in some of the theorems discussed in subsequent sections.

Kaplow and Shavell assume that there is at least one divisible private good, but do not require that any of the other features of an alternative exhibit any special structure. Accordingly, in this section, it is assumed that $X = X^1 \times X^2$, where $X^1 = \mathbb{R}^n_+$. A vector $x^1 = (x^1_1, \ldots, x^1_n)$ in X^1 specifies, for each person i, the amount x^1_i of some divisible private good that i is allocated. As in Section 3, there is a single profile of utility functions U and the set of consequences C is $U(X)$. Furthermore, the social preference relation R_U is assumed to be an ordering.

Weak Pareto regards any change that makes everybody better off as being a social improvement.

Weak Pareto. For all $x, y \in X$, if $U(x) \gg U(y)$, then xP_Uy.

Kaplow and Shavell impose a relatively weak monotonicity condition on the individual preferences, what I call *Common Monotonicity*. Common Monotonicity says that if everybody's allocation of the divisible private good

[6] These objections to Pareto Indifference were adapted from objections advanced by Sen (1977, 1979) to the strong neutrality principle of social choice theory. For further discussions of this issue, see Sen (1990) and Bossert and Weymark (2004).

[7] Kaplow and Shavell instead state their theorem in terms of real-valued representations of R_U and R_U^*. However, their arguments do not require either R_U or R_U^* to be representable.

is increased by a common amount holding the other features of the alternatives fixed, then everybody is made better off. This assumption ensures that Weak Pareto is not vacuous.

Common Monotonicity. For all $x, y \in X$ with $x^2 = y^2$, if $x_i^1 = y_i^1 + \delta$ for all $i \in N$ for some $\delta > 0$, then $U(x) \gg U(y)$.

Kaplow and Shavell also impose a continuity condition on the social ordering R_U. It requires R_U to be continuous on X^1 for any fixed x^2 in X^2, a property I call *Continuity on* X^1.

Continuity on X^1. For all $x \in X$, the sets $\{z^1 \in \mathbb{R}_+^n \mid (z^1, x^2) R_U x\}$ and $\{z^1 \in \mathbb{R}_+^n \mid x R_U (z^1, x^2)\}$ are closed.

The Kaplow-Shavell Theorem shows that if R_U is an ordering and both Common Monotonicity and Continuity on X^1 are satisfied, then it is not possible to take account of non-welfare information in this single-profile setting without violating the Weak Pareto Principle.

Theorem 2. *For the triple $\langle X, U, R_U \rangle$, if $X = X^1 \times X^2$ with $X^1 = \mathbb{R}_+^n$, R_U is an ordering, and both Common Monotonicity and Continuity on X^1 are satisfied but Single-Profile Welfarism is not, then Weak Pareto is violated.*

Proof. The theorem is established by showing that if Common Monotonicity, Continuity on X^1, and Weak Pareto are satisfied, then so is Single-Profile Welfarism when R_U is an ordering. I first show that Pareto Indifference is satisfied. On the contrary, suppose that it is not. Thus, there exist x, y with $U(x) = U(y)$ for which $x P_U y$.[8] Let z be such that $z^2 = y^2$ and $z_i^1 = y_i^1 + \delta$ for all $i \in N$ for some $\delta > 0$. By choosing δ sufficiently small, Continuity on X^1 implies that $x P_U z$. However, by Common Monotonicity, $U(z) \gg U(y)$ and, hence, $U(z) \gg U(x)$. Therefore, Weak Pareto is violated, a contradiction. Hence, Pareto Indifference is satisfied and, by Theorem 1, so is Single-Profile Welfarism. □

As my proof of Theorem 2 demonstrates, what Kaplow and Shavell have done is to identify restrictions for which Weak Pareto implies Pareto Indifference. Specifically, provided that R_U is an ordering, Weak Pareto implies Pareto Indifference if Common Monotonicity and Continuity on X^1 are satisfied. Using somewhat different continuity and monotonicity assumptions about the profile U and the social preference R_U, Suzumura (2001) has shown that if it is always possible to reverse a social preference $x P_U y$ by increasing any individual's consumption of the divisible private good in y sufficiently, then the standard versions of the Pareto Principle, including Weak Pareto and Pareto Indifference, are all mutually equivalent. Thus, in the single-profile setting considered here, the conflict between permitting non-welfare information to

[8] Note that the completeness of R_U is used at this point in the argument.

play a role in the social evaluation and the Weak Pareto Principle identified by Kaplow and Shavell is simply an implication of the fact established in Theorem 1 that Pareto Indifference and Single-Profile Welfarism are equivalent conditions when R_U is an ordering.[9]

Requiring R_U to be an ordering and the profile U to satisfy Common Monotonicity are relatively uncontroversial assumptions. Given these two assumptions, the only role of the continuity assumption is to show that by adopting Weak Pareto, one is also committed to Pareto Indifference, which in turn commits one to Single-Profile Welfarism. Chang (2000, Section IV.A) argues that Kaplow and Shavell's continuity condition is not compelling. He provides examples that violate this condition in which the ordering R_U is obtained by serially applying different criteria, including the Weak Pareto Principle. If one rejects Continuity on X^1, then it is is possible to be a non-welfarist and satisfy Weak Pareto when only a single profile is considered. However, in view of Theorem 1, one must nevertheless abandon Pareto Indifference.

5 Pareto Conflicts with Permutation Dominance Principles

Sen (1970a, Theorem 9*2) has shown that the Suppes (1966) Grading Principle is inconsistent with Weak Pareto for some possible profiles of preferences. The Suppes Grading Principle is a non-welfaristic principle for socially ranking alternatives based on utility dominance after possibly permuting the positions of the individuals in one of the alternatives. For a fixed profile, Brun and Tungodden (2004) consider a more structured economic environment in which alternatives consist of a commodity bundle for each individual, individuals only care about what they receive, and the individual preferences for own consumption satisfy the standard assumptions of microeconomic theory. In this framework, the Suppes Grading Principle compares alternatives by applying a utility dominance criterion after the commodity bundles in one of the alternatives have been permuted. Brun and Tungodden consider a related *permutation dominance principle* in which dominance is applied to commodity bundles, not utilities. In their Observation, they show that their principle is incompatible with the Strong Pareto Principle provided that the preferences for own consumption are not all the same.

In this section, I use the framework and proof strategy employed by Brun and Tungodden to show that Strong Pareto violates both a slight strengthening of their dominance principle and the Suppes Grading Principle. This

[9] Campbell and Kelly (2002, pp. 80–81) claim that Kaplow and Shavell's proof of their theorem does not make use of the transitivity of R_U. This is not correct because transitivity is implicitly used in making their "Observation" (which is a restatement of Theorem 1 in terms of real-valued representations). Campbell and Kelly note that a somewhat weaker continuity condition suffices to esablish Theorem 2.

result illustrates the basic conflict identified by Sen and by Brun and Tungodden. I also show that similar reasoning can be used to establish a Pareto Indifference version of this result. This permits me to relate the incompatibilities between any single-profile non-welfarist principle and various versions of the Pareto Principle discussed in the preceding two sections with the more specific conflicts considered in this section.

I now suppose that $X = \prod_{i=1}^{n} X^i$, where $X^i = \mathbb{R}^k_+$ with $k \geq 2$. An alternative has the form $x = (x^1, \ldots, x^n)$, where for each $i \in N$, x^i is the *commodity bundle* that specifies the quantities of k divisible private goods for person i. Brun and Tungodden (2004) have interpreted these goods as being either functionings as in Sen (1985) or primary goods as in Rawls (1971). Functionings are achievements, what an individual does or becomes. Primary goods are goods that facilitate the achievement of a good life whatever one's conception of a good life turns out to be. The vector x is sometimes written as (x^i, x^{-i}), where x^{-i} is the vector of commodity bundles of everyone but i. As in the preceding sections, there is a single profile of utility functions U and the set of consequences C is $U(X)$.[10]

The utility function U^i is *self-regarding* if $U^i(x^i, x^{-i}) = U^i(x^i, y^{-i})$ for all $x, y \in X$. If U^i is self-regarding, i's utility only depends on his own consumption and, hence, U^i can be equivalently expressed using a *utility function for own consumption* $\widetilde{U}^i$ defined on X^i. U^i is a *classical private goods utility function* if it is self-regarding and $\widetilde{U}^i$ is continuous, increasing in each of its arguments, and strictly quasiconcave. It is assumed that each person's utility function satisfies these restrictions.

Classical Private Goods Profile. For each $i \in N$, U^i is a classical private goods utility function.

The Pareto conflicts considered in this section presuppose that not everybody has the same preferences for own consumption, what I call *Nonidentical Preferences*. Individual i has the *same preferences for own consumption* as individual j if $\widetilde{U}^i$ is an increasing transform of $\widetilde{U}^j$.

Nonidentical Preferences. There exist $i, j \in N$ who do not have the same preferences for own consumption.

Strong Pareto regards any change that makes at least one person better off without harming anyone else as being a social improvement.

Strong Pareto. For all $x, y \in X$, if $U(x) > U(y)$, then $xP_U y$.

Note that this definition of Strong Pareto differs from the standard one which also stipulates that Pareto Indifference is satisfied.

[10] The results in this and the following section can be restated in terms of a single profile of individual preferences on X, but in order to relate these results to welfarism, it is more convenient to express them in terms of individual utilities.

I now consider a dominance condition that combines anonymity and dominance properties for commodity bundles. *Permutation Dominance* regards alternative x to be socially preferred to y if there is a permutation of the commodity bundles in x that provides someone with more of every good than in y and no less of any good for everybody else.

Permutation Dominance. For all $x, y \in X$, if there exists a permutation $\pi\colon N \to N$ such that $x^{\pi(i)} \geq y^i$ for all $i \in N$ with $x^{\pi(j)} \gg y^j$ for some $j \in N$, then xP_Uy.

Permutation Dominance is a strengthening of the *Strong Dominance* condition considered by Brun and Tungodden (2004). Strong Dominance modifies the antecedent in Permutation Dominance by requiring that $x^{\pi(i)} \gg y^i$ for all $i \in N$.

Permutation Dominance is closely related to the *Suppes Grading Principle* (Suppes, 1966). While, in general, the latter principle does not presuppose that the individual utility functions are self-regarding, in order to compare it with Permutation Dominance, I shall suppose that they are. With this proviso, the Suppes Grading Principle says that if it is possible to permute the individual commodity bundles in x in such a way that the permuted alternative Pareto dominates y, then x is socially preferred to y.

Suppes Grading Principle. For all $x, y \in X$, if there exists a permutation $\pi\colon N \to N$ such that $\widetilde{U}^i(x^{\pi(i)}) \geq \widetilde{U}^i(y^i)$ for all $i \in N$ with a strict inequality for some $j \in N$, then xP_Uy.

If U is a classical private goods profile, then the Suppes Grading Principle is a more stringent requirement than Permutation Dominance because the antecedent in the Suppes Grading Principle is implied by the antecedent in Permutation Dominance.

Theorem 3 illustrates the conflict identified by Sen and by Brun and Tungodden between the Pareto Principle and their dominance principles using the economic stucture employed by Brun and Tungodden, but with Permutation Dominance used instead of Strong Dominance.

Theorem 3. *For the triple $\langle X, U, R_U\rangle$, if $X = \prod_{i=1}^n X^i$, where $X^i = \mathbb{R}^k_+$ and $k \geq 2$, and U is a classical private goods profile with nonidentical preferences, then Strong Pareto is incompatible with Permutation Dominance and with the Suppes Grading Principle.*

Proof. Suppose that both Strong Pareto and Permutation Dominance are satisfied. Because U is a classical private goods profile with nonidentical preferences, there exist $i, j \in N$ for which two indifference curves for own consumption cross. Hence, it is possible to choose alternatives x and y such that (i) $\widetilde{U}^i(x^i) > \widetilde{U}^i(y^i)$, (ii) $\widetilde{U}^j(x^j) > \widetilde{U}^j(y^j)$, (iii) $y^i \gg x^j$, (iv) $y^j \gg x^i$, and (v) $x^h = y^h$ for all $h \neq i, j$. The commodity bundles for i and j are illustrated in Figure 1 for the case in which there are two goods. By Strong Pareto, xP_Uy.

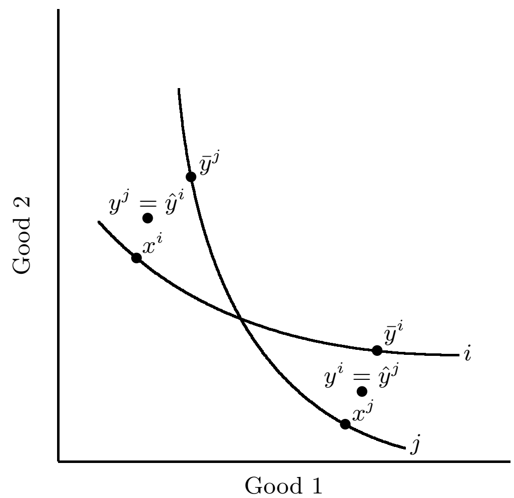

Fig. 1. Illustration of the proofs of Theorems 3 and 4

Let $\hat{y}$ denote the alternative that is obtained by permuting the commodity bundles of i and j in y. The constructions for i and j are illustrated in Figure 1 for the two-good case. Because $\hat{y}^i \gg x^i$, $\hat{y}^j \gg x^j$, and $\hat{y}^h = x^h$ for all $h \neq i, j$, Permutation Dominance implies that $yP_U x$, a contradiction.

The same argument shows that Strong Pareto and the Suppes Grading Principle are inconsistent because $\widetilde{U}^i(\hat{y}^i) > \widetilde{U}^i(x^i)$, $\widetilde{U}^j(\hat{y}^j) > \widetilde{U}^j(x^j)$, and $\widetilde{U}^h(\hat{y}^h) = \widetilde{U}^h(x^h)$ for all $h \neq i, j$.[11] □

A notable feature of Theorem 3 is that R_U is not assumed to satisfy any rationality condition. The proof merely exploits the fact that it is logically impossible for one alternative to be both socially preferred to and socially worse than a second.

A simple modification of the argument used to prove Theorem 3 shows that Pareto Indifference is inconsistent with both Permutation Dominance and the Suppes Grading Principle if U is a classical private goods profile with nonidentical preferences.

Theorem 4. *For the triple $\langle X, U, R_U \rangle$, if $X = \prod_{i=1}^n X^i$, where $X^i = \mathbb{R}^k_+$ and $k \geq 2$, and U is a classical private goods profile with nonidentical preferences, then Pareto Indifference is incompatible with Permutation Dominance and with the Suppes Grading Principle.*

[11] By choosing y^h so that $y^h \gg x^h$ for all $h \neq i, j$, the same proof strategy shows that Weak Pareto is inconsistent with Strong Dominance, Permutation Dominance, and the Suppes Grading Principle when U is a classical private goods profile with nonidentical preferences.

Proof. As in the proof of Theorem 3, suppose that i and j have different preferences. The assumptions on U imply that it is possible to choose alternatives x and $\bar{y}$ such that (i) $U^i(x^i) = U^i(\bar{y}^i)$, (ii) $U^j(x^j) = U^i(\bar{y}^j)$, (iii) $\bar{y}^i \gg x^j$, (iv) $\bar{y}^j \gg x^i$, and (v) $x^h = \bar{y}^h$ for all $h \neq i, j$. For i and j, see Figure 1 for the two-good case. By Pareto Indifference, $xI_U\bar{y}$. By either Permutation Dominance or the Suppes Grading Principle, $\bar{y}P_Ux$, a contradiction. □

In the definitions of Permutation Dominance and the Suppes Grading Principle, if $\pi(i) \neq i$, then $x^{\pi(i)}$ is not i's consumption bundle in x unless $\pi(i)$ and i happen to have the same bundle. As a consequence, both of these conditions are non-welfarist criteria for making social evaluations. Theorem 1 applies to any non-welfarist principle and to any set of alternatives, not just to the specific non-welfarist criteria and structured set of alternatives considered in this section. Nevertheless, Theorem 4 does not follow from the conclusion of Theorem 1 that Single-Profile Welfarism is inconsistent with Pareto Indifference because, unlike Theorem 1, Theorem 4 does not presuppose that R_U is a quasiordering. What Theorem 4 demonstrates is that with a more structured set of alternatives, there can be a conflict between a more specific non-welfarist criterion and Pareto Indifference without any social rationality restriction whatsoever.

It is also noteworthy that Theorems 3 and 4 make no use of any continuity assumption for R_U. In contrast, the Kaplow–Shavell Theorem makes essential use of such an assumption.

6 Pareto Conflicts with Inequality Aversion Principles

I now turn to conflicts between the Pareto Principle and two inequality aversion principles. The first is a version of the Rawlsian Difference Principle and the second is a multidimensional generalization of the Pigou–Dalton Transfer Principle. I present Pareto Indifference versions of the Weak Pareto impossibility results established by Gibbard (1979) for the former principle and by Fleurbaey and Trannoy (2003) for the latter. I also consider how these impossibility theorems are related to the results discussed in the preceding sections.

6.1 The Minimal Difference Principle

The Rawlsian *Difference Principle* (Rawls, 1971) advocates designing social institutions so as to make the least advantaged as well off as possible, where advantage is determined by an index of primary goods. Rawls identified a number of primary goods (such as rights and liberties, power and opportunities, self-respect, and income and wealth), but did not specify how these goods are to be aggregated into an index. The Difference Principle provides a non-welfarist criterion for ranking social alternatives. However, even if everybody has the same amount of all primary goods except for income, in order to make this principle operational, there remains the difficulty of identifying who is the least advantaged because individuals have different preferences for commodities with the consequence that how well income advances their interests depends on commodity prices. This problem does not arise if prices are held fixed because then the least advantaged is simply the person with the smallest income. Gibbard (1979) has shown that even if the Difference Principle is restricted to such fixed-price comparisons, there is a conflict with the Weak Pareto Principle provided that all preferences are not identical.

For Gibbard, an alternative is a price vector $p \in \mathbb{R}^k_{++}$ and a vector of incomes $\mu = (\mu^1, \ldots, \mu^n) \in \mathbb{R}^n_{++}$ for the n individuals. These are dual variables to the commodity bundles considered in Section 5. To facilitate the comparison of the impossibility result in this section with the other impossibility results considered in this article, I employ a primal approach. Specifically, I assume that the set of alternatives X is the same as in Section 5. Furthermore, it is assumed that U is a classical private goods profile with nonidentical preferences. The set of consequences C is again $U(X)$.

The assumptions employed here imply that for each price-income pair $(p, \mu^i) \in \mathbb{R}^{k+1}_{++}$, person i has a unique *demand vector* $d^i(p, \mu^i)$. This is the commodity bundle that maximizes i's utility function subject to his budget constraint. The primal form of Gibbard's *Minimal Difference Principle* says that alternative x is socially preferred to y if x and y are vectors of demands for two price-income situations in which the prices are the same in both situations and the smallest income in the first situation is larger than the smallest income in the second situation.

Minimal Difference Principle. For all $x, y \in X$, if there exist $(p, \mu), (p, \bar{\mu}) \in \mathbb{R}^{k+n}_{++}$ such that (i) $x^i = d^i(p, \mu^i)$ and $y^i = d^i(p, \bar{\mu}^i)$ for all $i \in N$ and (ii) $\min_i \mu^i > \min_i \bar{\mu}^i$, then $xP_U y$.

Theorem 5 demonstates that this principle is inconsistent with Pareto Indifference provided that U is a classical private goods profile with nonidentical preferences and the social ranking R_U is a quasiordering.

Theorem 5. *For the triple $\langle X, U, R_U \rangle$, if $X = \prod_{i=1}^n X^i$, where $X^i = \mathbb{R}^k_+$ and $k \geq 2$, U is a classical private goods profile with nonidentical preferences, and R_U is a quasiordering, then Pareto Indifference is incompatible with the Minimal Difference Principle.*

Proof. Suppose that both Pareto Indifference and the Minimal Difference Principle are satisfied. Because U is a classical private goods profile with nonidentical preferences, there exist $i, j \in N$ with nonidentical indifference curves for own consumption. Without loss of generality, suppose that for fixed values of goods 3 through k, there exists an indifference curve for i that intersects an indifference curve for j from above. It is then possible to choose alternatives x,

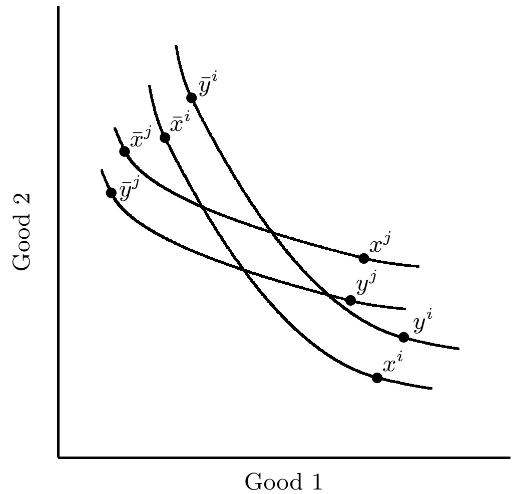

Fig. 2. Illustration of the proof of Theorem 5

y, $\bar{x}$, and $\bar{y}$ and price-income situations (p, μ_1), (p, μ_2), (q, μ_3), and (q, μ_4) with $p_1/p_2 < q_1/q_2$ such that (i) $x^h = d^h(p, \mu_1^h)$, $y^h = d^h(p, \mu_2^h)$, $\bar{x}^h = d^h(q, \mu_3^h)$, and $\bar{y}^h = d^h(q, \mu_4^h)$ for all $h \in N$, (ii) $U^h(x^h) = U^h(\bar{x}^h)$ and $U^h(y^h) = U^h(\bar{y}^h)$ for all $h \in N$, (iii) $\mu_1^h > \mu_1^i$ and $\mu_2^h > \mu_2^i$ for all $h \neq i$, (iv) $\mu_3^h > \mu_3^j$ and $\mu_4^h > \mu_4^j$ for all $h \neq j$, (v) $\mu_2^i > \mu_1^i$, and (vi) $\mu_3^j > \mu_4^j$. This construction is illustrated in Figure 2 when there are two goods and two individuals.[12]

By the Minimal Difference Principle, yP_Ux and $\bar{x}P_U\bar{y}$. By Pareto Indifference, $xI_U\bar{x}$ and $\bar{y}I_Uy$. Because yP_Ux, $xI_U\bar{x}$, $\bar{x}P_U\bar{y}$, and $\bar{y}I_Uy$, the transitivity of R_U implies that yP_Uy, which contradicts the reflexivity of R_U. □

The strategy used to prove Theorem 5 is more transparent when there are only two goods and two individuals. In Figure 2, person i has a relative taste for good 1, whereas person j has a relative taste for good 2. The commodity bundles in x and y are chosen when good 1 is relatively cheap; that is, when the price vector is p. For these bundles to be demand vectors, i must have the smallest income in both of these cases and he must have more income when he chooses y^i than when he chooses x^i. Hence, by the Minimal Difference Principle, yP_Ux. For $\bar{x}$ and $\bar{y}$, similar reasoning using the price vector q shows that j is the least advantaged in both cases and that $\bar{x}P_U\bar{y}$. Pareto Indifference implies that $xI_U\bar{x}$ and $\bar{y}I_Uy$. These four rankings are inconsistent with R_U being a quasiordering.

[12] In order to avoid visual clutter, the budget lines are not shown in the diagram. The marginal rate of substitution is the same at x^i, x^j, y^i, and y^j. They are also equal at $\bar{x}^i$, $\bar{x}^j$, $\bar{y}^i$, and $\bar{y}^j$.

6.2 The Multidimensional Transfer Principle

For unidimensional distributions of income, the *Pigou–Dalton Transfer Principle* expresses an aversion to inequality. This principle regards a transfer of income from a richer to a poorer person that does not reverse their ranking in the income distribution as being a social improvement. When there is more than one good, this principle needs to be reformulated so as to take account of differences in the individual preferences. Fleurbaey and Trannoy (2003) have introduced a natural multidimensional version of this transfer principle and shown that it conflicts with the Weak Pareto Principle when all preferences are not identical.[13] I present a Pareto Indifference version of their result.

As above, U is a classical private goods profile with nonidentical preferences for the set of alternatives X used in Section 5 and in Theorem 5 and the set of consequences C is $U(X)$. Consider implementing a Pigou–Dalton transfer between individuals i and j for each good separately. If i has more of one good that j initially, but the reverse is true for some other good, it is unclear if inequality has been reduced by such a transfer. However, if one of these individuals has at least as much of every good as the other and strictly more of at least one of the goods, then this multidimensional transfer is unambiguously inequality reducing. Fleurbaey and Trannoy's *Multidimensional Transfer Principle* regards such a transfer as being a social improvement.

Multidimensional Transfer Principle. For all $x, y \in X$, if there exist $i, j \in N$ and $\delta > \mathbf{0}_k$ such that $x^i + \delta = y^i \leq y^j = x^j - \delta$ and $x^h = y^h$ for all $h \neq i, j$, then yP_Ux.

Theorem 6 shows that the impossibility result established in Theorem 5 is also valid if the Multdimensional Transfer Principle is substituted for the Minimal Difference Principle.

Theorem 6. *For the triple $\langle X, U, R_U\rangle$, if $X = \prod_{i=1}^n X^i$, where $X^i = \mathbb{R}^k_+$ and $k \geq 2$, U is a classical private goods profile with nonidentical preferences, and R_U is a quasiordering, then Pareto Indifference is incompatible with the Multdimensional Transfer Principle.*

Proof. Suppose that both Pareto Indifference and the Multidimensional Transfer Principle are satisfied. Because U is a classical private goods profile with nonidentical preferences, there exist $i, j \in N$ with nonidentical indifference curves for own consumption. Hence, it is possible to choose alternatives x, y, $\bar{x}$, and $\bar{y}$ and vectors $\delta > \mathbf{0}_k$ and $\bar{\delta} > \mathbf{0}_k$ such that (i) $x^i + \delta = y^i \leq y^j = x^j - \delta$, (ii) $x^h = y^h$ for all $h \neq i, j$, (iii) $\bar{y}^j + \bar{\delta} = \bar{x}^j \leq \bar{x}^i = \bar{y}^i - \bar{\delta}$, (iv) $\bar{x}^h = \bar{y}^h$ for all $h \neq i, j$, and (v) $U^h(x^h) = U^h(\bar{x}^h)$ and $U^h(y^h) = U^h(\bar{y}^h)$ for $h = i, j$.[14]

[13] For a compact presentation of the Fleurbaey–Trannoy Theorem, see Fleurbaey (2006, Section 9.5).

[14] For a formal proof of this claim, see Fleurbaey and Trannoy (2003, Lemma 1).

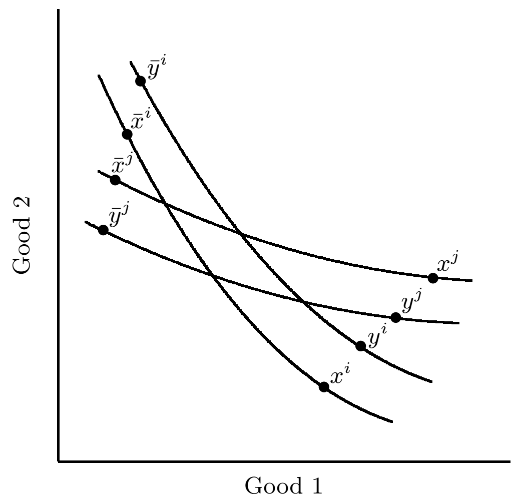

Fig. 3. Illustration of the proof of Theorem 6

For individuals i and j, this construction as illustrated in Figure 3 for the two-good case.

By the Multidimensional Transfer Principle, yP_Ux and $\bar{x}P_U\bar{y}$. By Pareto Indifference, $xI_U\bar{x}$ and $\bar{y}I_Uy$. Because yP_Ux, $xI_U\bar{x}$, $\bar{x}P_U\bar{y}$, and $\bar{y}I_Uy$, the transitivity of R_U implies that yP_Uy, which contradicts the reflexivity of R_U. □

The proofs of Theorems 5 and 6 are remarkably similar. For each of the focal individuals i and j, their four consumption bundles are chosen from two pairs of intersecting indifference curves and the relative positions of these bundles are similar in the two cases. The precise locations of the bundles differs in the two proofs, with vector dominance and commonality of marginal rates of substitution used in the proof of Theorem 5 to pin these locations down so as to appeal to the Minimal Dominance Principle, whereas vector dominance and commonality of the size of transfers is used in the proof of Theorem 6 so as to appeal to the Multidimensional Transfer Principle. The same preference cycle is generated in both cases. Unlike with the Kaplow–Shavell Theorem, no continuity assumption for R_U is required.

The Minimal Dominance and Multidimensional Transfer Principles are non-welfarist. Because the social ranking R_U is assumed to be a quasiordering in both Theorems 5 and 6, these two theorems are in fact corollaries to the single-profile welfarism theorem (Theorem 1). In contrast, the Weak Pareto versions of Theorems 5 and 6 established by Gibbard (1979) and Fleurbaey and Trannoy (2003) are not corollaries to Theorem 1 because they do not utilize Pareto Indifference. The Weak Pareto variants can be established using proofs that are very similar to the ones used here to prove their Pareto Indifference counterparts. Thus, while it is possible to use Theorem 1 to provide an indirect proof of Theorems 5 and 6, the direct proofs provide a link with the Weak Pareto versions of these results.

Moreover, the role that having noindentical prefernces plays in the direct proofs of Theorems 5 and 6 is similar to the role that it plays in the proof of Theorem 4. However, the latter result differs in some fundamental respects from the other two theorems. First, its proof only requires considering two alternatives, whereas the direct proofs of Theorems 5 and 6 require considering four. Second, even though the Pareto Principle in Theorem 4 is Pareto Indifference, it is not a corollary to Theorem 1 because it does not impose any rationality restriction on R_U.

7 Non-Welfarist Impossibility Theorems

In the preceding sections, the consequences have been utilities. I now turn to two results in which the consequences need not have this interpretation. The first is a theorem due to Pattanaik and Xu (2007) about standard of living measurement that I call the Pattanaik–Xu Theorem. The second is an abstract theorem due to Hare (2007) that I call the Hare Theorem. Hare illustrates his theorem with a number of concrete applications. Here, I only consider its interpretation as a theorem about aiding the nearby or distant needy. In both theorems, the ranking of the consequences depends on some conditioning variables, what Pattanaik and Xu (2012) call *context dependence*. They distinguish between two types of context dependence, of which the kind considered in this section is *type 1 dependence*. Pattanaik and Xu (2012) have established a quite general theorem about type 1 dependence from which slight variants of the theorems considered in this section follow as special cases.[15]

7.1 The Pattanaik–Xu Theorem

Now assume that $X = \mathbb{R}^m_+$, with $m \geq 2$. Pattanaik and Xu (2007) interpret a vector $x \in X$ as being the quantities of m divisible functionings for some individual, but it can also be interpreted as being a commodity bundle listing this individual's quantities of m divisible goods. I shall use the latter interpretation. Pattanaik and Xu permit the quantity of each good to have a finite upper bound; for simplicity, here it is supposed that X is unbounded from above. As in previous sections, $N = \{1, \ldots, n\}$ is the set of individuals, with $n \geq 2$.

[15] Type 2 dependence requires there to exist two situations with the same consequences that are not indifferent. When the consequences are utility vectors, type 2 dependence is equivalent to requiring Single-Profile Welfarism to be violated. Pattanaik and Xu (2012) have esablished a version of the Kaplow–Shavell Theorem in this more abstract setting and shown how their impossibility result can be overturned by weakening the continuity condition.

The objective is to compare commodity bundles in terms of their standards of living both intrapersonally and interpersonally. This is done by means of a *standard of living relation* $\succeq$ on $N \times X$, with corresponding asymmetric factor $\succ$ and symmetric factor $\sim$, respectively. For all $i, j \in N$ and all $x, y \in X$, $(i, x) \succeq (j, y)$ is interpreted as meaning that i has a standard of living with the commodity bundle x at least as high as j's standard of living with the commodity bundle y. For each $i \in N$, $\succeq$ defines a conditional ranking $\succeq_i$ on X that can be thought of as being i's preference.

Pattanaik and Xu suppose that intrapersonal standard of living comparisons should not be invariant across individuals, but should instead respect differences in individual values and the cultural norms in the societies in which they live. A very minimal version of this requirement is provided by their *Minimal Relativism* condition, which requires that there exist at least two individuals and two commodity bundles for which the standard of living comparison differs.

Minimal Relativism. There exist $i, j \in N$ and $x, y \in X$ such that $(i, x) \succ (i, y)$ and $(j, y) \succ (j, x)$.

Thus, the conditional rankings $\succeq_i$ on X for $i \in N$ cannot be the same for all individuals. If X is thought of as being the set of consequences C, Minimal Relativism is a nonconsequentialist principle because the intrapersonal standard of living comparisons are permitted to depend on the nonconsequentialist information in N. In other words, the ranking of X depends on the context, here provided by the identity of the individual being considered. A consequentialist would require these conditional rankings to all be the same. Minimal Relativism is an analogue of the Nonidentical Preferences condition considered in the preceding two sections.

PX Dominance. For all $i, j \in N$ and all $x, y \in X$ for which $x \gg y$, $(i, x) \succ (j, y)$.

PX Dominance uses vector dominance in the space of consequences to make standard of living comparisons for individual-commodity bundle pairs.[16] Specifically, if one commodity bundle x strictly dominates a second commodity bundle y, then no matter who has x and y, the person with x has a higher standard of living than the person with y. This condition is a slight weakening of the Dominance axiom of Pattanaik and Xu (2007); their antecedent that $x > y$ is replaced here with $x \gg y$.

Conditional Continuity. For all $i \in N$ and all $x \in X$, the sets $\{z \in X \mid (i, z) \succeq (i, x)\}$ and $\{z \in X \mid (i, x) \succeq (i, z)\}$ are closed.

[16] The terminology has been chosen to distinguish this dominance principle from the one considered in the next subsection.

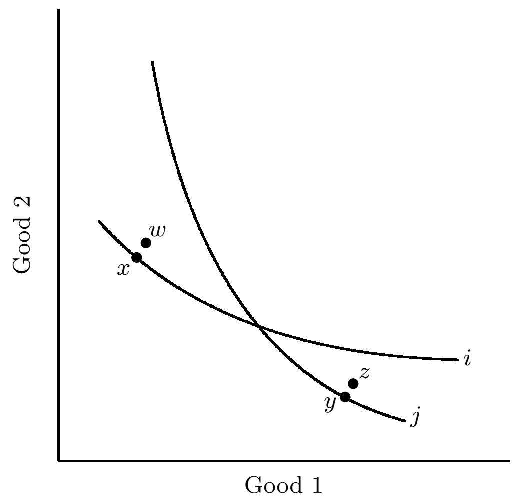

Fig. 4. Illustration of the proof of Theorem 7

Conditional Continuity requires each of the conditional rankings $\succeq_i$ to be continuous. This continuity assumption only applies to intrapersonal comparisons of consequences, unlike in the Kaplow–Shavell Theorem where continuity is used interpersonally.

Theorem 7 is the *Pattanaik–Xu Theorem*. It is a variant of Proposition 1 in Pattanaik and Xu (2007).[17]

Theorem 7. *For the triple $\langle X, U, \succeq \rangle$, if $X = \mathbb{R}^m_+$ with $m \geq 2$ and $\succeq$ is acyclic, then Minimal Relativism, PX Dominance, and Conditional Continuity are incompatible.*

Proof. By Minimal Relativism and PX Dominance, there exist $i, j \in N$ and $x, y \in X$ with $\neg(x \gg y)$ and $\neg(y \gg x)$ such that $(i, x) \succ (i, y)$ and $(j, y) \succ (j, x)$. By Conditional Continuity, (a) $(i, x) \succ (i, z)$ for any z arbitrarily close to y for which $z \gg y$ and (b) $(j, y) \succ (j, w)$ for any w arbitrarily close to x for which $w \gg x$. By PX Dominance, $(j, w) \succ (i, x)$ and $(i, z) \succ (j, y)$. However, $(j, y) \succ (j, w)$, $(j, w) \succ (i, x)$, $(i, x) \succ (i, z)$, and $(i, z) \succ (j, y)$ contradict acyclicity. $\square$

The proof of Theorem 7 is illustrated in Figure 4 for the two-good case. The relative positions of w, x, y, and z with respect to each other and with respect to the i and j indifference curves in Figure 4 are the same as the relative

[17] Theorem 7 is also closely related to Theorem 5 in Fleurbaey (2007). Fleurbaey uses the individuals' utility functions as the conditioning variables rather than their identities.

positions of y^i, x^i, x^j, and y^i in Figure 1. The similarity between these two figures might lead one to believe that Theorems 3 and 7 are fundamentally the same, but this is not the case. While both proofs exploit some of the same ideas, in the former case, the comparisons are between allocations of commodity bundles to every individual, whereas in the latter case, the comparisons are between commodity bundles for particular individuals. Furthermore, in the proof of Theorem 7, because PX Dominance only takes account of two individuals' commodity bundles, it does not matter what commodity bundle anybody else has. In contrast, in the proof of Theorem 3, in order to apply Strong Pareto, it matters what everybody consumes. Moreover, Theorem 7 makes use of a social rationality condition on $\succeq$ (i.e., acyclicity), whereas Theorem 3 does not.

Proposition 1 in Pattanaik and Xu (2007) employs a slightly different dominance condition, what they call *Weak Dominance*, in which $x \gg y$ and $(i, x) \succ (j, y)$ are replaced by $x > y$ and $(i, x) \succeq (j, y)$, respectively, in the statement of PX Dominance. With this alternative dominance principle, it is necessary to strengthen acyclicity to transitivity in order to obtain a contradiction between Minimal Relativism, Weak Dominance, and Conditional Continuity. Both the acyclic and transitive cases are covered by Proposition 1 in Pattanaik and Xu (2012).

7.2 The Hare Theorem

In Hare's Theorem, it is assumed that X is the union of two disjoint sets X^1 and X^2. No other structure is imposed on X. Alternatives in X can be given many different interpretations; for concreteness, following Hare, I call them *entities*. The set of consequences C is an m-dimensional subset of $\mathbb{R}^m$ for some $m \geq 2$. For now, no interpretation of the alternatives in C is needed. Each entity in X uniquely determines a consequence in C as described by the function $f\colon X \to C$. An evaluator has a preference binary relation R on X with corresponding asymmetric and symmetric factors P and I, respectively. Note that this preference is on the set of entities X, not the set of consequences C.

The set of consequences that can be obtained by entities in X^1 need not be the same as the set of consequences that can be obtained by entities in X^2. Nevertheless, these two sets are required to have two full-dimensional sets of consequences that do not vector dominate each other in common. I call this assumption *Conditional Consequence Richness*.

Conditional Consequence Richness. There exist open (relative to $\mathbb{R}^m$) subsets A and B of C for which $\neg(u \gg v)$ and $\neg(v \gg u)$ for all $u \in A$ and all $v \in B$ such that $(A \cup B) \subseteq f(X^1)$ and $(A \cup B) \subseteq f(X^2)$.

The openness part of this definition ensures that for any entity x in X^j, $j = 1, 2$, whose consequence $f(x)$ is in either A or B, it is possible to find a different entity in the same set whose consequence is arbitrarily close to $f(x)$ that vector dominates $f(x)$. The requirement that no consequence in A vector dominates any consequence in B and vice versa ensures that vector dominance in the space of consequences does not completely determine the preference $\succeq$.[18]

H Dominance. For all $x, y \in X$, if $f(x) \gg f(y)$, then xPy.

As is the case with PX Dominance, vector dominance in the space of consequences is used to make inferences about the the binary relation of interest (the evaluator's preference R on the set of entities in the case of H Dominance and the standard of living relation $\succeq$ in the case of PX Dominance). H Dominance requires an evaluator to prefer one entity to another if the former results in consequences that vector dominate the consequences obtained with the latter.

Variable Trade-Offs. For all $x, y \in X^1$, if $f(x) \in A$ and $f(y) \in B$, then xPy, whereas for all $x, y \in X^2$, if $f(x) \in A$ and $f(y) \in B$, then yPx.

As with Minimal Relativism, Variable Trade-Offs requires the ranking of the entities to be sensitive to nonconsequentialist information. Specifically, the ranking of consequences in A relative to those in B is reversed if these consequences are obtained from entities in X^1 rather than from entities in X^2.

Theorem 8 is *Hare's Theorem.* It is a formal statement of a result stated somewhat informally in Hare (2007) with his transitivity assumption weakened to acyclicity.

Theorem 8. *For the triple $\langle X, U, R\rangle$, if $X = X^1 \cup X^2$ with $X^1 \cap X^2 = \varnothing$, $C \subseteq \mathbb{R}^m$ with $m \geq 2$, and R is acyclic, then Conditional Consequence Richness, H Dominance, and Variable Trade-Offs are incompatible.*

Proof. By Conditional Consequence Richness, there exist (i) $w, x \in X^1$ such that $f(w) = u \in A$ and $f(x) = v \in B$ and (ii) $y, z \in X^2$ such that $f(y) = \bar{u} \in A$ and $f(z) = \bar{v} \in B$, where $\bar{u} \gg u$ and $v \gg \bar{v}$. By Variable Trade-Offs, wPx and zPy. By H Dominance, yPw and xPz. Thus, R is cyclic. $\square$

Although the evaluator's preference R is on the set of entities X, it is nevertheless possible to illustrate the proof of Theorem 8 in the space of consequences C as in Figure 5, thereby facilitating the comparison of Theorems 7 and 8.[19] The two disks are the consequent sets A and B described in Conditional Consequence Richness. Strictly speaking, there need not be well-defined

[18] Hare does not state Conditional Consequence Richness as a separate axiom. Part of it is included in his version of the Variable Trade-Offs axiom introduced below and part is implicit.

[19] Because f assigns a unique consequence vector to each $x \in X$, the binary relation R used in the Hare Theorem can be equivalently defined on $C \times X$, just as $\succeq$

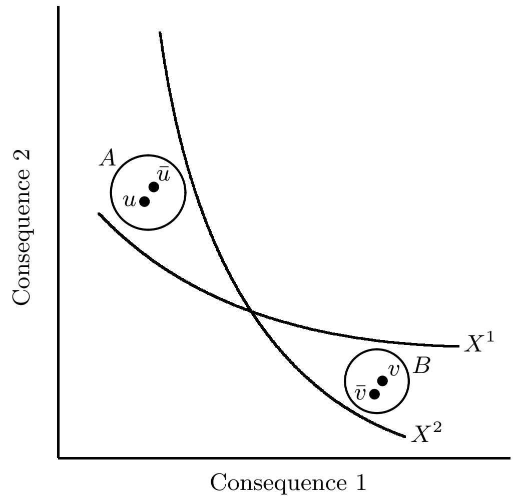

Fig. 5. Illustration of the proof of Theorem 8

indifference curves in C corresponding to either X^1 or X^2. The two curves shown in Figure 5 are merely meant to indicate (i) that any entity in X^1 that generates a consequence in A such as u is preferred to any entity in X^1 that generates a consequence in B such as v and (ii) that any entity in X^2 that generates a consequence in B such as $\bar{v}$ is preferred to any entity in X^2 that generates a consequence in A such as $\bar{u}$. When combined with the dominance principle that requires any entity in X that generates $\bar{u}$ (resp. v) to be preferred to any entity that generates u (resp. $\bar{v}$), a strict preference cycle for R is obtained. Even though the proof of Theorem 8 is somewhat more indirect than that of Theorem 7 because of the need to consider the entities underlying consequences, the logic underlying them is essentially the same.

Hare (2007) applies Theorem 8 to the ethical issue of whether it is morally legitimate for a prosperous individual to condition assistance given to someone in great need based on his proximity (either in terms of distance or kinship) when the sacrifice required is moderate. In this application, the set of individuals is $N = \{1, 2\}$, where individual 1 is the person in the position of offering assistance and individual 2 is the person needing it. Let $\omega > 0$ be a small number that is much less than individual 1's wealth. He contemplates sacrificing any amount of money in $[0, \omega]$ to aid individual 2. Let α denote that the second individual is nearby and β that he is distant. The set of entities

is defined on $N \times X$ in the Pattanaik–Xu Theorem. Pattanaik and Xu (2012) exploit this observation to draw out some of the connections between these two results. Note that in the Hare Theorem, it is X that provides the conditioning (i.e., contextual) variables, whereas in the Pattanaik–Xu Theorem, it is N.

is $X = X^1 \cup X^2$, where $X^1 = [0, \omega] \times \{\alpha\}$ and $X^2 = [0, \omega] \times \{\beta\}$. In this application, consequences are the individuals' utilities, so $m = 2$. For simplicity, let the set of consequences C be $\mathbb{R}^2_+$ and identify f with a profile of utility functions $U\colon X \to \mathbb{R}^2_+$. The preference relation R on X is now interpreted as being the moral preferences of the first individual. Hare also supposes that it is possible to make intrapersonal comparisons of utility differences and that the utilities of individual 2 are much greater in region A of the consequence space than in region B, wheres the utilities of individual 1 in region A are only slightly smaller than in region B.

With these interpetations of Hare's abstract framework, Variable Trade-Offs is a non-welfarist principle that he regards as capturing the "morally undemanding" view that the moderately prosperous person is obligated to make small sacrifices for the nearby needy, but not for the distant needy, even if the benefits from doing so for the beneficiary are substantial. Hare does not explictly consider the Conditional Full Dimensionality axiom, so he regards his theorem as saying that if a potential donor wants to be rational (here, requiring R to be acyclic), then he must reject either H Dominance or Variable Trade-Offs, both of which Hare regards as being the behaviour of an ogre.

8 Concluding Remarks

The impossibility theorems discussed in the preceding sections present a nonconsequentialist with the conundrum of how to resolve the fundamental incompatibilty of the principles that he would like satisfied. In these concluding remarks, I describe three kinds of solutions to this dilemma that have been considered in the literature. The first shifts the focus to a multi-profile setting and only allows nonconsequentialist principles to play a role in inter-profile comparisons. The other two solutions retain the single-profile setting but either reject one or more of the principles or restrict their scope.

Fleurbaey, Tungodden, and Chang (2003) take issue with the claim made by Kaplow and Shavell (2001) that Theorem 2 establishes that any nonwelfarist method of policy assessment must violate the Weak Pareto Principle. They argue that welfarism is generally understood to be the claim that a single social welfare ordering of utility vectors is used to determine the social ordering of the alternatives for each possible profile of utility functions. In this multi-profile setting, in addition to Pareto Indifference, welfarism requires the social ranking of a pair of alternatives to be the same for both profiles if these profiles agree on the individual utilities assigned to them.[20] Even if one is

[20] For a formal statement of the multi-profile welfarism theorem, see Bossert and Weymark (2004, Theorem 2.2). Blackorby, Bossert, and Donaldson (2005) have considered a richer multi-profile framework in which the consequence space includes non-welfare information in addition to the individual utilities. They have

willing to accept some form of the Pareto Principle, this independence condition is controversial and, therefore, one may well be a non-welfarist in the multi-profile sense without running afoul of any Pareto principle. Fleurbaey, Tungodden, and Chang (2003) provide an example of a Paretian social welfare functional based on fairness principles that is non-welfarist because it violates the independence described above (see also Chang, 2000, Section IV.B). Their example is, however, welfarist for each profile considered separately.[21]

In the single-profile setting that has been the focus of this article, one possible way of dealing with incompatible principles is to reject one or more of them. Hare (2007), for example, rejects Variable Trade-Offs, which is his context-dependence axiom. He argues that proximity is not a relevant consideration when deciding whether to aid the nearby or distant needy. Pattanaik and Xu (2012) also suggest dropping one of the axioms, but argue that which one this should be depends on the application. For the applications that they consider, they argue that the dominance principle is the one that should be rejected. For example, with PX Dominance, they note that if individual i has more of every good with the commodity bundle x than individual j has with the commodity bundle y, then it cannot be inferred that i is better off than j; other factors may be relevant when making interpersonal welfare comparisons even when there is commodity bundle dominance. Further grounds for rejecting dominance conditions when the comparisions are between vectors of individual utilities have already been provided in the discussion of Pareto principles in Section 3.

Rather than reject a desiderata outright, its scope can be restricted. For example, Fleurbaey and Trannoy (2003) and Fleurbaey (2006) suggest restricting the Multidimensional Transfer Principle considered in Section 6.2. One way of doing so is to only apply this principle when the transfers are between individuals with the same preferences. Alternatively, this principle could be applied only when all of the commodity bundles are proportional to each other. With either of these restrictions, not only is there no conflict with standard Pareto principles and an appropriate social rationality condition, it is also possible to satisfy other desirable criteria, such as ones that incorporate equity considerations. Similarly, the set of alternatives to which a dominance principle applies may be restricted, as in Fleurbaey (2007, 2011) and Fleurbaey, Schokkaert, and Decancq (2009), who restrict vector dominance comparisons to a single monotonic path in the set of alternatives.[22]

identified conditions, including a strong unrestricted domain condition, that precludes the non-welfare information from playing any role in the social ranking of the alternatives.

[21] Suzumura (2011, p. 675) describes the differing interpretations of Kaplow and Shavell's claim as reflecting the conflict between those who support a single-profile approach to normative social evaluation and those who support a multi-profile approach.

[22] These articles employ what Fleurbaey (2011) calls the *equivalence approach*. In this approach, two alternatives are ranked in the same way as the alternatives that

None of these resolutions to the nonconsequentialist's condundrums is completely satisfactory. But they may be the best we can do given the fundamental incompatibility of nonconsequentialism, dominance, and social rationality in the applications considered here.

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they are indifferent to in some reference set. When the reference set is a monotonic path, it follows that vector dominance is only used to rank alternatives that lie on this path. The most fully developed exploration of the equivalence approach is provided by Fleurbaey and Maniquet (2011). Some complications that arise when preferences are incomplete are considered in Fleurbaey and Schokkaert (2013).

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