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Citizen Candidates and Voting Over Incentive-Compatible Nonlinear Income Tax Schedules

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1. Introduction

The extensive literature on redistributive income taxation that builds on the seminal work of Mirrlees (1971) has primarily been normative. In the Mirrlees model, everybody has the same preferences for consumption and labor supply, but they differ in skill levels (their "types") as measured by their labor productivities. While the distribution of these productivities is common knowledge, the value of any individual's productivity is only known to himself. The government chooses a nonlinear income tax schedule to maximize a social welfare function subject to the constraints that (i) each individual optimally chooses his consumption and labour supply given the tax schedule and (ii) the resulting allocation satisfies the government's budget constraint.

Alternatively, redistributive income tax policy can be studied from a political economy perspective. In the political economy literature, it is often assumed that individuals have single-peaked preferences on the relevant set of alternatives and that majority voting is used to determine what outcome is chosen.[1] See, for example, Austen-Smith and Banks (1999) and Persson and Tabellini (2000). With these assumptions, as shown by Black (1948), any most-preferred alternative of a voter with a median preference peak does as well as any other alternative in a pairwise majority vote; that is, it is a Condorcet winner. However, in general, individual preferences over the set of all income tax schedules that satisfy the incentive and budget constraints in the Mirrlees problem are not single-peaked, so Black's Median Voter Theorem does not apply if majority voting is used to determine redistributive tax policy when only those constraints are imposed. Indeed, Itsumi (1974) and Romer (1975) show that even if the tax schedules are restricted to be linear, quite restrictive assumptions on the preferences and skill distribution are needed to ensure single-peakedness.

Röell (2012) has initiated the study of majority voting over nonlinear income tax schedules when the set of tax schedules being voted on are restricted to those that are selfishly optimal (given the constraints in the Mirrlees problem) for some individual.[2] Specifically, in a finite type version of the Mirrlees model as in Guesnerie and Seade (1982), for each skill type k, Röell first determines the qualitative properties of the tax schedule that maximizes type k's utility subject to the incentive and budget constraints described above and an additional constraint that guarantees each person a minimum utility. This is the tax schedule that someone of type k would choose if he were selfishly dictatorial and respects the minimum-utility constraint. Imposing the minimal-utility

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[1]When we refer to preferences as being single-peaked, we employ the weak definition of single-peakedness of Austen-Smith and Banks (1999, p. 98). In this definition, an individual may have more than one most-preferred alternative—a plateau—and preferences need only be weakly decreasing as one moves farther from this plateau in either direction.

[2]Meltzer and Richard (1981) had previously considered voting rules for which a decisive individual (e.g., a dictator or a median voter) choses his most-preferred feasible linear income tax schedule. Snyder and Kramer (1988) consider majority voting over selfishly optimal nonlinear income tax schedules in a model in which individuals allocate a fixed amount of labor between the taxable and underground sectors.

constraint limits the degree to which low skilled individuals can be exploited in order to further the interests of type k. With the further assumption that preferences are quasilinear in consumption, Röell shows that everybody has single-peaked preferences over these schedules provided that with the schedule chosen by any type k, it is optimal for the adjacent upward incentive constraints to bind for all individuals with lower skill levels than his own, which limits the extent to which minimum-utility constraints bind. Hence, Black's Theorem applies when voting is restricted in this way.

In effect, the model of politics employed by Röell is an application of the citizen-candidate framework of Osborne and Slivinski (1996) and Besley and Coate (1997) in which each candidate proposes his favorite feasible policy.[3] In the Osborne–Slivinksi and Besley–Coate citizen-candidate models, there is a cost for a candidate to contest an election, so not everybody runs for office. In contrast, in Röell's model, there is no cost to proposing a tax schedule for consideration, so everybody is a candidate.[4]

Using a continuum version of Röell's problem, we show that it is possible to provide a complete characterization of the income tax schedule that is utility maximal for a candidate of any skill type k subject to the incentive and government budget constraints when preferences are quasilinear in consumption.[5] This is done by characterizing the income schedule that specifies how much before-tax income each skill type would receive with the selfishly optimal income tax schedule that is proposed by a candidate of type k. With quasilinear-in-consumption preferences, incentive compatibility of an allocation is preserved if everybody's income is changed by a common amount. As a consequence, once candidate k's optimal income schedule has been determined, the corresponding schedule showing how consumption varies with the skill level is easily determined using the government budget constraint (which must bind). The tax an individual pays is the difference between his consumption and before-tax income.

A candidate with the lowest skill type proposes the maxi-min income schedule, whereas one with the highest skill type proposes the maxi-max income schedule. We show that the maxi-max schedule lies everywhere above the maxi-min schedule. For a candidate of any type other than the lowest and highest, we show that he proposes an income schedule that (i) coincides with the maxi-max schedule for the lower part of the skill distribution, (ii) coincides with the maxi-min schedule for the upper part of the skill distribution, and (iii) "bridges" these two segments with a region of bunching that contains the candidate's type.[6] The endpoints of this bunching region are nondecreasing in the type of the

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[3]A preliminary version of Röell's paper was completed in March 1996 prior to the publication of these two articles.

[4]Ledyard (2006, Sec. 3.6) briefly describes a model in which two candidates propose levels of public goods and nonlinear income tax schedules knowing that it is costly to vote so that not everybody votes. Ledyard indicates that his discussion is based on unpublished work with Marcus Berliant.

[5]In Brett and Weymark (2014), we show how this schedule must be modified if the minimum-utility constraint is also taken into account. The addition of this constraint greatly complicates the analysis.

[6]For candidate types sufficiently close to the lowest, it is possible that the solution starts on the bridge, in which case it provides a case of bunching at the bottom similar to that studied by Ebert (1992). If the distribution of types is bounded above, it is possible to have bunching at the top as well.

candidate, from which it follows that everybody has a single-peaked preference over the proposed tax schedules, and so the median voter theorem applies.

In order to obtain the characterization of candidate k's selfishly optimal income schedule, we proceed by first considering a relaxed version of his optimization problem in which we ignore the second-order incentive-compatibility condition that income is nondecreasing in the skill level (Lollivier and Rochet, 1983). This is the first-order approach to candidate k's problem. If the maxi-min and maxi-max income schedules are strictly increasing in the skill level, the solution to this relaxed problem is easy to describe.[7] A candidate of type k wishes to redistribute income from all other types toward his own type. To do this, for types greater than his own, he optimally employs the maxi-min income schedule, whereas for types smaller than his own, he optimally employs the maxi-max income schedule. In the maxi-min case, both candidate k and a maxi-min utility social planner wish to extract as much revenue as incentives allow from the higher types. Because candidate k's optimal before-tax income schedule does not depend on the distribution of consumption, his desire to give that revenue to himself rather than to the least-skilled is of no consequence for the specification of this part of the income schedule. The optimality of using the maxi-max solution for the rest of the skill distribution follows from similar reasoning.

Because the maxi-min schedule lies above the maxi-min schedule, the solution to candidate k's relaxed problem exhibits a downward discontinuity at his own skill level, and so violates the second-order monotonicity condition for incentive compatibility. To obtain the solution when this constraint is taken into account requires "ironing" the schedule described above by introducing a level "bridge" that connects the maxi-max and maxi-min components of the relaxed solution. The standard way of identifying the endpoints of a bunching interval is to use the kind of control-theoretic techniques described in Guesnerie and Laffont (1984). We instead employ the procedure developed by Vincent and Mason (1967, 1968) for smoothing discontinuous control trajectories. Applied to candidate k's problem, solving for the bridge endpoints using the Vincent–Mason approach is a simple unconstrained optimization problem.

In the maxi-min part of candidate k's income schedule, redistribution is constrained by downward incentive-compatibility constraints and gives rise to the familiar positive (or zero for the highest type) marginal income tax rates. Similarly, in the maxi-max part of this schedule, redistribution is constrained by upward incentive-compatibility constraints, which gives rise to negative (or, in some circumstances, zero for the lowest type) marginal income tax rates. As a consequence, there must be a kink in the optimal income tax schedule at the income chosen by candidate k and bunching of some of the types near him.

Single-peakedness of the individual preferences over the set of income tax schedules being voted on is a sufficient condition for the existence of a Condorcet winner; it is

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This corresponds to having the income schedule ending on the bridge.

[7]We present sufficient conditions for these monotonicity conditions to hold and also describe how the analysis needs to be modified when they are not.

not necessary. Provided that individual preferences for consumption and income satisfy the standard single-crossing property introduced by Mirrlees (1971), Gans and Smart (1996) show that if any two of the tax schedules under consideration only cross once, then pairwise majority voting generates a quasitransitive "social" preference on the set of these tax schedules.[8] When there are a finite number of tax schedules being voted on, quasitransitivity is sufficient for the existence of a Condorcet winner.[9] Because there is a continuum of tax schedules in our problem, demonstrating that the tax schedules satisfy the Gans–Smart single-crossing condition would not be sufficient to establish the existence of a Condorcet winner.

Associated with each selfishly optimal income tax schedule is a utility schedule that specifies utility as a function of skill type. Using a continuum version of the Röell model, Bohn and Stuart (2013) show that any pair of these utility curves cross only once, which is a single-crossing property first investigated by Matthews and Moore (1987) in the context of a general adverse selection problem.[10] When certain regularity conditions are satisfied, Bohn and Stuart show that this form of single-crossing is sufficient for the median skill type to be a Condorcet winner. In deriving their results, Bohn and Stuart assume that the minimum-utility constraint is satisfied, but do not require preferences to be quasilinear. As a consequence, their analysis is quite technical.[11] By assuming that individual preferences are quasilinear in consumption and by not considering a minimum-utility constraint, we are not only able to provide a complete characterization of each skill type's selfishly optimal income schedule, we are able to do so using elementary calculus.[12]

The rest of this article is organized as follows. Section 2 introduces the model of the economy. Section 3 contains a detailed account of a citizen candidate's choice of a selfishly optimal income tax schedule. This is followed in Section 4 by an analysis of the voting equilibrium. Section 5 provides concluding remarks. The proofs of our results

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[8]A weak preference relation is quasitransitive if the strict preferences are transitive

[9]The Gans–Smart result generalizes related results in Roberts (1977) for linear income taxes. Using specific functional forms, Austen-Smith and Banks (1999, pp. 113–115) and Persson and Tabellini (2000, pp. 118–121) show why income tax schedules satisfy the single-crossing property when they are linear. As Gans and Smart (1996) note, the single-crossing tax schedule condition is equivalent to the schedules being completely ordered in terms of their progressivity and to the requirement that the individuals choose incomes that are nondecreasing in the skill level regardless of what tax schedule they face, a property that Roberts (1977) calls Hierarchical Adherence. Berliant and Gouveia (2001) have developed sufficient conditions for single-crossing income tax schedules in a model in which the set of skill types is a finite sample from a known distribution, the government's revenue requirement depends on the realized distribution, and voting takes place before the voters know what distribution is realized.

[10]Unlike the single-crossing condition for preferences used in Mirrlees (1971), the Matthews–Moore single-crossing condition does not require there to be only two goods.

[11]The Bohn–Stuart analysis also makes extensive use of an assumption about the curvature properties of an optimized value function. It is not clear what restrictions this assumption imposes on the primitives of the model.

[12]De Donder and Hindricks (2003) use simulations to investigate the existence of a Condorcet winner among the set of selfishly optimal quadratic income tax schedules. Voting over nonlinear income tax schedules when the candidates have some form of vote maximizing objective is considered by Blomquist and Christiansen (1999), Roemer (2012), and Bierbrauer and Boyer (2013).

may be found in the Appendix.

2. The Model

The economy is populated by individuals that differ in labor productivity. Differences in skills are described by a parameter w which is continuously distributed with support $[\underline{w}, \bar{w}]$, density function $f(w) > 0$, and cumulative distribution function $F(w)$. It is assumed that $0 < \underline{w} < \bar{w}$. An individual with skill level w produces w units of a consumption good per unit of labor time in a perfectly competitive labor market and earns a (before-tax) income of

$$y = wl, \tag{1}$$

where l is the amount of labor supplied. Thus, w is this type's wage rate. Income can also be thought of as being labor in efficiency units.

An individual has consumption x, which is also his after-tax income. Preferences over consumption and labor supply are represented by the quasilinear-in-consumption utility function

$$\tilde{u}(l, x) = x - h(l), \tag{2}$$

which is common to all individuals. The function h is increasing, strictly convex, and three-times continuously differentiable. The government can observe an individual's before- and after tax incomes, but not his skill level or labor supply. Using (1), the utility function in terms of observable variables is

$$u(y, x; w) = x - h\left(\frac{y}{w}\right). \tag{3}$$

In terms of consumption and income, the marginal rate of substitution at any bundle (y, x) is decreasing in w when $y > 0$, so the standard Mirrlees (1971) single-crossing condition for preferences is satisfied.

Individuals face an anonymous nonlinear income taxation schedule that specifies the tax paid as a function of income $T(y)$, subject to which individuals choose their most preferred combination of consumption and before-tax income (equivalently, after-tax income and labor supply). Admissible tax schedules are assumed to be piecewise continuously differentiable. By the Taxation Principle (see Hammond, 1979; Guesnerie, 1995), having individuals choose consumption and income subject to an anonymous tax schedule is equivalent to directly specifying these variables as functions of type subject to incentive-compatibility constraints. These schedules, $x(\cdot)$ and $y(\cdot)$, as well as the labor supply schedule $l(\cdot)$ corresponding to $y(\cdot)$, are also piecewise continuously differentiable.[13] Because there are no mass points in the distribution, $T(\cdot)$, $x(\cdot)$, $y(\cdot)$, and $l(\cdot)$ are all continuous (see Hellwig, 2010). The bundle allocated to individuals of type w is $(y(w), x(w))$.

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[13]As shown by Hellwig (2010), it is only necessary to assume that these schedules are integrable. The stronger assumption of piecewise continuous differentiability is typically made to facilitate the use of standard control-theoretic arguments.

The resulting utility level is

$$V(w) = x(w) - h\left(\frac{y(w)}{w}\right), \quad \forall w \in [\underline{w}, \bar{w}]. \tag{4}$$

Incentive compatibility requires that

$$V(w) = \max_{w'} x(w') - h\left(\frac{y(w')}{w}\right), \quad \forall w, w' \in [\underline{w}, \bar{w}]. \tag{5}$$

Because the Mirrlees single-crossing property is satisfied, it follows from Mirrlees (1976) that the first-order (envelope) condition for incentive compatibility is

$$V'(w) = h'\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^2}, \quad \forall w \in [\underline{w}, \bar{w}], \tag{6}$$

and the second-order condition is

$$y'(w) \geq 0, \quad \forall w \in [\underline{w}, \bar{w}].^{14} \tag{7}$$

Consumption must also be nondecreasing in type. Moreover, the single-crossing property and incentive compatibility imply that two types either (i) differ in both income and consumption or (ii) have the same bundle, in which case they are said to be *bunched* (see Laffont and Martimort, 2002, sec. 3.1). Because h is increasing, (6) implies that utility is nondecreasing in w whenever incentive compatibility is satisfied and it is strictly increasing for all w for which $y(w) > 0$.

The income tax schedule must be differentiable almost everywhere. At any income for which it is not differentiable, the marginal tax rate $\tau(w)$ is not well-defined. At incomes for which it is well-defined as the derivative of the tax schedule, $\tau(w)$ is equal to one minus the marginal rate of substitution between consumption and income (i.e., between after-tax and before-tax incomes). As is standard, this expression can be used to define an implicit marginal tax rate for values of y for which $T(y)$ is not differentiable. Thus,

$$\tau(w) = 1 - h'\left(\frac{y(w)}{w}\right)\frac{1}{w}, \quad \forall w \in [\underline{w}, \bar{w}]. \tag{8}$$

Because utility is quasilinear in consumption, marginal tax rates do not depend on consumption.

The only purpose of taxation is to redistribute income, so the government budget constraint is

$$\int_{\underline{w}}^{\bar{w}} [y(w) - x(w)]f(w)\, dw \geq 0. \tag{9}$$

The qualitative features of our analysis are unaffected if the government instead requires a fixed positive amount of revenue.

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[14]The expressions in (6) and (7) are required to hold at all points for which $y(w)$ is differentiable. Because incentive compatibility implies that income is nondecreasing in type, $y(w)$ is differentiable almost everywhere.

3. The Citizen Candidate's Problem

Each individual proposes an income tax schedule and then pairwise majority rule is used to choose which of these schedules is implemented. A tax schedule can only be proposed if the resulting allocation satisfies the incentive-compatibility conditions (6) and (7) and the government budget constraint (9). We call such a schedule an *incentive-compatible income tax schedule*. We suppose that the tax schedule proposed by an individual is his selfishly optimal incentive-compatible tax schedule. That is, it is the incentive-compatible schedule that maximizes his own utility. Individuals of the same type propose the same tax schedule. Hence, we can equivalently think of voting as taking place over types of individuals. In this interpretation of our model, we are viewing individuals as being citizen candidates, as in Osborne and Slivinski (1996) and Besley and Coate (1997). We suppose that there are no costs for a candidate to enter the election (i.e., to propose a tax schedule for consideration) and, so, every type is a candidate. The requirement that a candidate proposes his favorite feasible tax schedule reflects his inability to commit to a tax policy at the time of the election. Voters know this, and this informs their choice of candidates when voting.

Rather than thinking of a candidate as proposing an income tax schedule, it is more convenient to appeal to the Taxation Principle and think of a candidate as choosing an allocation schedule $(y(\cdot), x(\cdot))$ that specifies a bundle $(x(w), y(w))$ for each type $w \in [\underline{w}, \bar{w}]$.[15] Formally, a candidate of type k determines his optimal allocation schedule by solving

$$\max_{x(\cdot), y(\cdot)} V(k) \quad \text{subject to } (4), (6), (7), \text{and } (9). \tag{10}$$

We refer to (10) as *candidate k's problem*.

Two characteristics of candidate k's problem distinguish it from the standard Mirrlees (1971) problem: the form of the objective function and the explicit inclusion of the second-order incentive-compatibility constraint. Mirrlees used a utilitarian objective function, whereas here the utility of a particular type of individual is being maximized When this type is $\underline{w}$, the objective is simply the maxi-min criterion, which has been studied in detail by Boadway and Jacquet (2008). For reasons of tractability, Mirrlees and most subsequent authors only considered the first-order conditions for incentive compatibility, what is known as the *first-order approach*. The second-order incentive-compatibility conditions have been explicitly taken into account by Brito and Oakland (1977) and Ebert (1992). The complete solution to candidate k's problem can be determined from the solution to the relaxed problem in which the second-order monotonicity constraint (7) is ignored, so we begin by considering it. An analysis of the relaxed problem also yields useful insights into the nature of candidate k's optimization problem.

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[15]In this section, the identity of the candidate is fixed, so we do not index the schedules by the candidate's type.

3.1. The First-Order Approach

The relaxed problem in which the monotonicity constraint on before-tax income is suppressed provides a picture of how the form of the objective function in the candidate's problem helps to create a solution to the optimal tax problem that is very different from those found in other optimal nonlinear income tax problems. Formally, this problem is

$$\max_{x(\cdot),y(\cdot)} V(k) \quad \text{subject to } (4), (6), \text{and } (9). \tag{11}$$

We refer to (11) as *candidate k's relaxed problem.*

By modifying the arguments found in Lollivier and Rochet (1983), we show in Proposition 1 that it is possible to formulate an unconstrained optimization problem that provides the before-tax income schedule that solves candidate k's relaxed problem. As is standard in a nonlinear income tax problem, it is optimal for the government budget constraint to bind.[16] Once candidate k's optimal before-tax income schedule has been determined, the corresponding consumption schedule (and, hence, the income tax schedule) can be derived using the incentive-compatibility and binding government budget constraints. With quasilinear-in-consumption utility, the relevant properties of the optimal bundles for each type can be inferred from the before-tax income schedule, so we do not consider the consumption schedule that solves candidate k's relaxed problem explicitly.

Proposition 1. *The optimal schedule of before-tax incomes $y(\cdot)$ for candidate k's relaxed problem is obtained by solving*

$$\max_{y(\cdot)} \int_{\underline{w}}^{k} \left\{ \left[(y(w) - h\left(\frac{y(w)}{w}\right)\right] f(w) + \frac{y(w)}{w^2}h'\left(\frac{y(w)}{w}\right) F(w) \right\} dw$$
$$+ \int_{k}^{\bar{w}} \left\{ \left[(y(w) - h\left(\frac{y(w)}{w}\right)\right] f(w) - \frac{y(w)}{w^2}h'\left(\frac{y(w)}{w}\right) [1 - F(w)] \right\} dw. \tag{12}$$

For ease of exposition, we suppose for now that the solutions to (12) for $k = \underline{w}$ and $k = \bar{w}$ are strictly increasing in w (so there is no bunching) and, hence, that both of these solutions satisfy the monotonicity constraint (7). Later, we shall relax this assumption and also identify sufficient conditions for it be satisfied. Thus, when $k = \underline{w}$, the solution to (12) is the *maxi-min income schedule*, which we denote by $y^R(\cdot)$, and when $k = \bar{w}$, the solution is the *maxi-max income schedule*, which we denote by $y^M(\cdot)$.[17] From (12), we see that the income schedule that solves candidate k's relaxed problem coincides with the maxi-max solution for individuals with skill types smaller than that of the candidate

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[16]If the budget constraint does not bind, because preferences are quasilinear in consumption, each person's consumption can be increased by a common small amount without violating incentive compatibility, thereby increasing the utility of type k individuals.

[17]It is commonplace to call the maxi-min objective "Rawlsian" even though Rawls (1971) used an index of primary goods rather than utility in his criterion. Our notation reflects this common usage.

and coincides with the maxi-min solution for individuals with skill types larger than that of the candidate.

Not only is the optimization problem (12) unconstrained, it can be solved point-wise. Thus, simple differentiation with respect to $y(w)$ provides the first-order conditions for candidate k's relaxed problem, which we write in the implicit form as

$$
\begin{aligned}
\theta^M(w, y(w)) &= 0, \quad \forall w \in [\underline{w}, k), \\
\theta^R(w, y(w)) &= 0, \quad \forall w \in [k, \bar{w}],
\end{aligned}
\tag{13}
$$

where

$$
\theta^M(w, y) = \left[1 - h'\left(\frac{y}{w}\right)\frac{1}{w}\right]f(w) + \left[h''\left(\frac{y}{w}\right)\frac{y}{w^3} + h'\left(\frac{y}{w}\right)\frac{1}{w^2}\right]F(w)
\tag{14}
$$

and

$$
\theta^R(w, y) = \left[1 - h'\left(\frac{y}{w}\right)\frac{1}{w}\right]f(w) - \left[h''\left(\frac{y}{w}\right)\frac{y}{w^3} + h'\left(\frac{y}{w}\right)\frac{1}{w^2}\right][1 - F(w)].^{18}
\tag{15}
$$

Using (8), (13), and (15), the optimal maxi-min marginal tax rates are

$$
\tau^R(y(w)) = \frac{1 - F(w)}{f(w)}\left[h''\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^3} + h'\left(\frac{y(w)}{w}\right)\frac{1}{w}\right], \quad \forall w \in [\underline{w}, \bar{w}].^{19}
\tag{16}
$$

Hence, the marginal tax rate is zero for the highest skilled and positive for all other types with the maxi-min objective. In the absence of the incentive constraints, personalized lump-sum taxes would be used for redistribution. Given our quasilinearity assumption, it then follows that compared to the full-information benchmark, every type except for the highest has his income and labor supply distorted downwards, whereas the highest skilled have the same income and labor supply as in the benchmark case. This pattern of distortions coincides with those found using a utilitarian objective function except that it is optimal in the utilitarian case for the lowest skilled to face a zero marginal tax rate provided that it is not bunched with any other type (see Sadka, 1976; Seade, 1977).

From (8), (13), and (14), the optimal maxi-max marginal tax rates are

$$
\tau^M(y(w)) = -\frac{F(w)}{f(w)}\left[h''\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^3} + h'\left(\frac{y(w)}{w}\right)\frac{1}{w}\right], \quad \forall w \in [\underline{w}, \bar{w}].
\tag{17}
$$

Therefore, with the maxi-max objective, the marginal tax rate is zero for the lowest skilled (if it is not bunched) and negative for all other types. Compared to the full-information benchmark, all types except the lowest skilled (who are not distorted) have their incomes and labor supply distorted upwards.

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[18]We write all first-order conditions for the optimal incomes as equalities, thereby implicitly assuming that the non-negativity constraints on incomes are not binding. The qualitative features of our analysis are unaffected if these constraints are taken into account.

[19]Boadway and Jacquet (2008, eqn. (21), p. 435) state this condition in an implicit form.
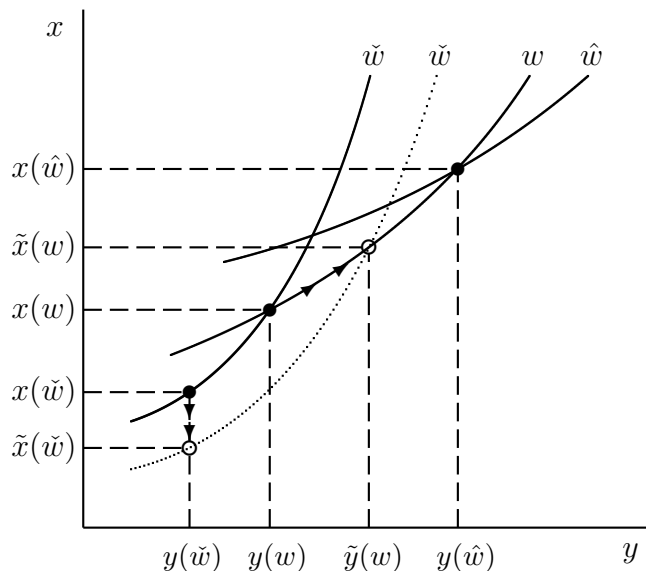
Figure 1: Adjustments to restore incentive compatibility following an increase in $y(w)$ when $y(w) < k$

Using these observations, some intuition can be provided for the first-order conditions (13) for candidate k's relaxed problem. For ease of exposition, it is useful to think of types as being discrete, but with skill levels arbitrarily close to each other. Candidate k wishes to maximize the utility of individuals of his own type. The function $\theta^M(w, y)$ captures the additional consumption (hence, utility) that individuals of this type can gain by increasing $y(w)$ by one unit for some $w < k$. At the solution, this value must be zero. In the first instance, increasing $y(w)$ by one unit makes available $f(w)$ extra units of consumption that can be diverted to the type k individuals. But appropriate adjustments must also be made in order to ensure that incentive compatibility is re-established after this increase. Candidate k wishes to redistribute resources away from lower types towards his own type. Individuals of lower types are distorted upwards, so this type of redistribution is constrained by upward incentive compatibility conditions that prevent individuals of lower types from mimicking types above them. Thus, any increase in $y(w)$ for a $w < k$ must be accompanied by adjustments that ensure that the upward incentive constraints are satisfied. These adjustments are illustrated in Figure 1.

First, each individual of type w can be given $h'(\frac{y(w)}{w})\frac{1}{w}$ additional units of consumption to place him on his initial indifference curve, thereby ensuring that he has no incentive to mimic any other type. This is shown by the adjustment from $(y(w), x(w))$ to $(\tilde{y}(w), \tilde{x}(w))$ in Figure 1. Moreover, this change does not affect the incentives of any types above w.[20] These units of consumption must be subtracted from the $f(w)$ units that can be

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[20]Recall that we are assuming that the maxi-min and maxi-max solutions exhibit no bunching, so

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diverted to type k individuals. This accounts for the second term in the first bracket in (14). Moving individuals of type w upward along their indifference curves in this way slackens the upward incentive constraint for the type $\check{w}$ immediately below w. Because preferences are quasilinear in consumption, by reducing the consumption of everybody whose type is smaller than w by the amount in the final bracket in (14) restores incentive compatibility. This is illustrated by the adjustment from $(y(\check{w}), x(\check{w}))$ to $(y(\check{w}), \tilde{x}(\check{w}))$ for type $\check{w}$ in Figure 1. There are $F(w)$ individuals whose types are smaller than w, so the second term in (14) is the total amount of consumption that type k individuals can re-claim from these types in this way.

Candidate k also wishes to move resources away from types higher than himself towards individuals of his own type. These types are downward distorted, so this kind of redistribution is constrained by downward incentive compatibility constraints. The function $\theta^R(w, y)$ shows the additional consumption that type k individuals can secure for themselves through a one unit increase in $y(w)$ for some $w > k$. The only difference between $\theta^R(w, y)$ and $\theta^M(w, y)$ is in the final term. This difference arises because it is the downward incentive constraints that bind for types above k. Moving individuals of type $w > k$ upward along their indifference curves in the manner described in the preceding paragraph would lead to a violation of the downward incentive constraint for the next highest type. Because preferences are quasilinear in consumption, satisfaction of these constraints can be re-established by giving these individuals and everyone of a higher type more consumption in the amount given in the final bracket in (15). There are $1 - F(w)$ such individuals. Because this consumption must be given to individuals of types different from that of the candidate, these resources are subtracted from the amount available to the type k individuals.

As we have noted, with the maxi-min income schedule, everyone has his income distorted downward compared to the full-information solution except for the highest type who is undistorted, whereas with the maxi-max income schedule, everyone has his income distorted upward compared to the full-information solution except for possibly the lowest type who may be undistorted. Thus, the maxi-max schedule lies everywhere above the maxi-min schedule. As a consequence, for any candidate $k \neq \underline{w}, \bar{w}$, his optimal income schedule has a downward discontinuity at his skill type, as illustrated in Figure 2.[21] We summarize our main findings in Proposition 2.

Proposition 2. *The optimal schedule of before-tax incomes $y(\cdot)$ for candidate k's relaxed problem is given by*

$$y(w) = \begin{cases} y^M(w), & \forall w \in [\underline{w}, k), \\ y^R(w), & \forall w \in [k, \bar{w}]. \end{cases} \tag{18}$$

For $k \neq \underline{w}, \bar{w}$, there is a downward discontinuity in this schedule at $w = k$.

$y(w) \neq y(\hat{w})$, where $\hat{w}$ is the next highest type above w. We are implicitly assuming that units of income are sufficiently small so that $\tilde{y}(w) < y(\hat{w})$.

[21]There is also a downward discontinuity in candidate k's optimal consumption schedule at his type.
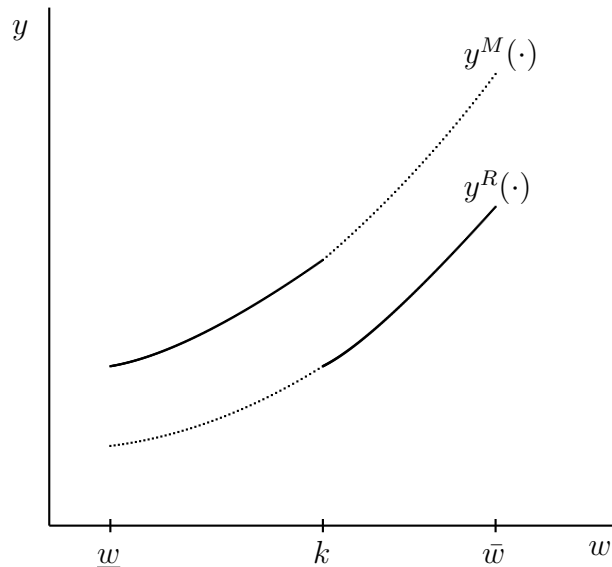
Figure 2: The optimal income schedule for candidate k's relaxed problem

Because the solution to candidate k's relaxed problem for $k \neq \underline{w}, \bar{w}$ features a jump from the maxi-max to the maxi-min tax schedule at this candidate's skill type, there is a discontinuity in the associated marginal tax rates, which are given by

$$
\tau(w) = \begin{cases} \tau^M(w), & \forall w \in [\underline{w}, k), \\ \tau^R(w), & \forall w \in [k, \bar{w}]. \end{cases}
\tag{19}
$$

We thus have a switch from negative marginal tax rates for types just below type k to positive marginal tax rates for types just above this type.

The discontinuities in the income schedule and in the marginal tax rates are intertwined. As we move from types just below type k to types just above it, the upward distortions in incomes switch to downward distortions and the signs of the marginal tax rates change from negative to positive. Because the maxi-max income schedule lies strictly above the maxi-min schedule at $w = k$, it is impossible to reconcile these competing distortions without a downward jump in the income schedule and a change in sign in the marginal tax rates at k.

The downward jump in the solution to candidate k's relaxed problem for the income schedule clearly violates the second-order incentive compatibility conditions. So even if the maxi-min and maxi-max income schedules do satisfy these second-order conditions, we have not found a solution to candidate k's problem in (10). Nevertheless, as we show below, some elements of these solutions feature in the complete solution to his problem. Before we turn to that issue, we first need to consider the circumstances in which the maxi-min and maxi-max income schedules are increasing, as assumed in this subsection.

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3.2. Monotonicity of the Maxi-min and Maxi-max Income Schedules

Of the two components of candidate k's relaxed solution, the part that tracks the maxi-min income schedule $y^R(\cdot)$ is the more familiar (see Boadway and Jacquet, 2008). Increasingness of this schedule is equivalent to the locus of points for which $\theta^R(w, y) = 0$ being increasing. For fixed w, the second-order condition for $y(w)$ to solve the unconstrained optimization problem (12) for this type is that $\theta_y^R(w, y(w)) \leq 0$. Thus, increasingness of $y(\cdot)$ requires that $\theta_y^R(w, y) < 0$ and $\theta_w^R(w, y) > 0$ for all $w \neq \underline{w}, \bar{w}$. From (15), a sufficient condition for $\theta_y^R(w, y) < 0$ is that $h'''(l) \geq 0$. This assumption is satisfied by the commonly-used iso-elastic form. Given that $\theta_y^R(w, y) < 0$, a sufficient condition for $\theta_w^R(w, y) > 0$ is that the labor supply schedule $l(\cdot)$ is upward sloping.[22] Thus, only relatively mild assumptions are needed to ensure that that the maxi-min income schedule is increasing.

Similarly, increasingness of the maxi-max income schedule $y^M(\cdot)$ is equivalent to the locus of points for which $\theta^M(w, y) = 0$ being increasing. However, in the maxi-max case, satisfaction of the second-order condition $\theta_y^M(w, y(w)) \leq 0$ for $y^M(w)$ to solve (12) for this value of w may be problematic. By (12), $y^M(w)$ is found by maximizing

$$G^M(w, y(w)) = \left[y - h\left(\frac{y(w)}{w} \right) \right] f(w) + \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w} \right) F(w) \tag{20}$$

with respect to $y(w)$. Because h is convex, the first term on the right-hand side of (20) is concave. However, when $h'''(l) > 0$, the second term might be convex. If the curvature of the second term dominates that of the first, then G^M is convex in y, and so the second-order condition is violated. Specifically, the second-order condition $\theta_y^M(w, y(w)) \leq 0$ is equivalent to

$$\left[\frac{2F(w) - wf(w)}{w^2} \right] h''\left(\frac{y(w)}{w} \right) + F(w) \frac{y(w)}{w^4} h'''\left(\frac{y(w)}{w} \right) \leq 0. \tag{21}$$

Under a maintained assumption that $h'''(l) > 0$, (21) can be satisfied only when the term in the square bracket on its left-hand side is negative. However, for any unbounded distribution of wages that has a finite expected value, the numerator in this term tends to 2 as w tends to infinity. Thus, if $\bar{w}$ is sufficiently large, it may well be the case that the second-order condition for an optimum of the relaxed version of the maxi-max problem will fail to be satisfied for values at the top end of the type distribution.[23]

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[22]We have $-\theta_w^R(w, y)/\theta_y^R(w, y) = y'(w) = wl'(w) + l(w)$, from which it follows that $\theta_w^R(w, y) > 0$ if $\theta_y^R(w, y) < 0$ and $l'(w) > 0$.

[23]When providing intuition for the first-order conditions (13) for candidate k's relaxed problem we noted that a utility-compensated increase in income and consumption for a type lower than that of candidate k serves to slacken the upward incentive constraints, thereby allowing this candidate to extract additional resources from still lower types. Because the mass of these lower types increases with the candidate's type, the second-order conditions are more likely to violated for candidates of a relatively high type.

For reasons that are closely related to the possible failure of the second-order conditions in the maxi-max case, the function $y^M(\cdot)$ can be rather ill-behaved. A rote application of the implicit function theorem to the top line of (13) yields

$$\frac{dy^M(w)}{dw} = -\frac{\theta_w^M(w, y^M(w))}{\theta_y^M(w, y^M(w))}. \tag{22}$$

It is entirely possible that the left-hand side of (21) is negative for values of w near $\underline{w}$. In fact, this must be the case if $h'''(l) = 0$. Thus, the denominator in (22) may change signs at least once on $[\underline{w}, \bar{w}]$. This, in turn, implies that $y^M(\cdot)$ can have a vertical asymptote in the interior of the type distribution.

Unlike in the case of candidates distinct from $\underline{w}$ and $\bar{w}$, the possible failure of the second-order conditions in the maxi-min and maxi-max cases are not the result of a downward discontinuity in the income schedule. Nevertheless, in order to obtain a complete solution to any candidate k's problem for $k \in (\underline{w}, \bar{w})$ as described in (10), we need to take into account not only the second-order conditions for his problem, but also those for the lowest- and highest-skilled types.

3.3. The Complete Solution

If either the maxi-min or maxi-max income schedule obtained using the first-order approach fails to satisfy the second-order incentive-compatibility condition (7) (i.e., the requirement that the schedule must be nondecreasing), then it is necessary to bunch all types in a decreasing part of the schedule with some types who are in an increasing part, what is known as *ironing*. Any bunching region must be a closed interval. Its endpoints can be determined using the approach described by Guesnerie and Laffont (1984).[24] Because ironing in this kind of situation is well understood and we do not need to know where the endpoints of these bunching regions are for our results, we shall simply suppose that these schedules have been ironed. We let $y^{R*}(\cdot)$ and $y^{M*}(\cdot)$ denote the optimal maxi-min and maxi-max income schedules when the second-order incentive-compatibility constraint has been taken into account.

Once the bunching regions for $y^{R*}(\cdot)$ and $y^{M*}(\cdot)$ have been determined, it is straightforward to modify the objective function (12) in candidate k's relaxed problem for $k = \underline{w}, \bar{w}$ so as to take account of the second-order incentive-compatibility condition (7). Doing so will facilitate the analysis of the other candidate types' problems. Let $\mathcal{B}^M$ and $\mathcal{B}^R$ denote the types that are bunched with some other type in the complete solution to the maxi-max and maxi-min problems, respectively. When w is bunched, we let $[w_-, w_+]$ denote the set of types bunched with w.

In the maxi-max case, only the first integral in (12) applies. Its integrand is replaced

---

[24]The first analysis of ironing in economics appears to have been by Arrow (1968). Arrow was concerned with devising an optimal capital policy with irreversible investment. The irreversability of investment imposes a monotonicity constraint analogous to the one on incomes found here.

by $G^{M*}(w, y(w))$, where

$$
G^{M*}(w, y(w)) = \begin{cases} \left[\left(y(w) - h\left(\frac{y(w)}{w}\right) \right) \right] f(w) + \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) F(w), & \forall w \notin \mathcal{B}^M, \\[2ex] \left[\left(y(w) - h\left(\frac{y(w)}{w}\right) \right) \right] \left[\int_{w_-}^{w_+} f(t)dt \right] + \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) F(w_-), & \forall w \in \mathcal{B}^M. \end{cases}
$$
(23)

Similarly, in the maxi-min case, only the second integral in (12) applies. Its integrand is replaced by $G^{R*}(w, y)(w)$, where

$$
G^{R*}(w, y(w)) = \begin{cases} \left[\left(y(w) - h\left(\frac{y(w)}{w}\right) \right) \right] f(w) + \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) F(w), & \forall w \notin \mathcal{B}^R, \\[2ex] \left[\left(y(w) - h\left(\frac{y(w)}{w}\right) \right) \right] \left[\int_{w_-}^{w_+} f(t)dt \right] - \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) [1 - F(w_+)], & \forall w \in \mathcal{B}^R. \end{cases}
$$
(24)

Ironing does not affect the solution outside a bunching region, so no modifications to the integrands in (12) are needed for types that are not bunched. The intuition for these expressions when there is bunching is similar to that provided above for candidate k's relaxed problem. Now, if an extra unit of consumption is given to type w individuals, it must be given to all individuals who are bunched with them, whose mass is $\int_{w_-}^{w_+} f(t)dt$. For this reason, the $f(w)$ that appears in the first cases of both (23) and (24) is replaced by this integral in the second cases. When w is bunched, in the maxi-max case, some of this extra consumption can be reclaimed from individuals of lower type than those bunched with w, whose mass is $F(w_-)$. The corresponding individuals in the maxi-min case are those individuals of higher type than those bunched with w, whose mass is $[1 - F(w_+)]$. In the second cases of (23) and (24), these expressions are used to replace the $F(w)$ and $[1 - F(w)]$ that appear in the first cases.

Using $y^{R*}(\cdot)$ and $y^{M*}(\cdot)$ instead of $y^R(\cdot)$ and $y^M(\cdot)$ in (18) for $k \neq \underline{w}, \bar{w}$, we obtain the income schedule that must be ironed in order to determine the complete solution to candidate k's problem. Because the only decreasing part of this schedule is the downward discontinuity at his type, only one new bunching region needs to be introduced. In effect, we must build a *bridge* that includes k between the maxi-max and maxi-min parts of this schedule, as illustrated in Figure 3. All types with skill levels in the interval $[w_b, w_B]$ are bunched at a common allocation. The values of the bunching interval endpoints w_b and w_B are determined optimally so as to minimize the loss in candidate k's utility that results from deviating from his relaxed solution.

The endpoints of the bunching interval can be determined using the control-theoretic approach of Guesnerie and Laffont (1984). However, we instead employ a much simpler procedure that was introduced by Vincent and Mason (1967, 1968) to smooth discontinuous control trajectories. Applied to our problem, in this approach, the optimal schedule is first selected for each fixed pair of values of the bridge endpoints w_b and w_B. Then, among these schedules, the one that maximizes candidate k's utility is selected. This is a simple unconstrained optimization problem. In other words, there is no need to
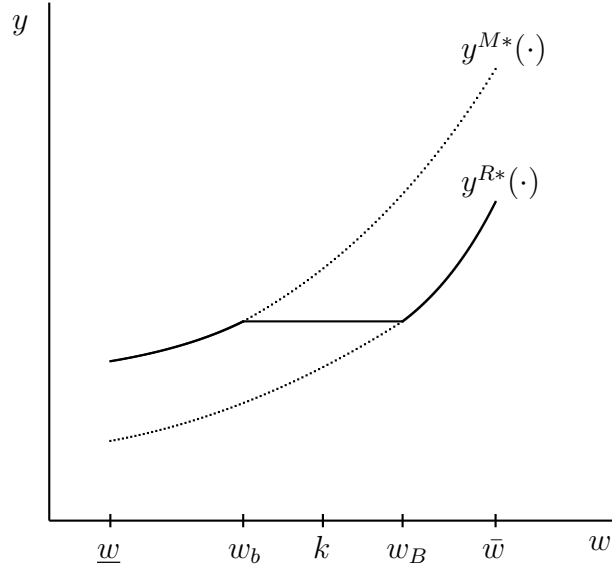
Figure 3: A bridge

use optimal control theory to determine the bridge endpoints. Our choice of technique simplifies the comparative static exercises we perform in Section 4.

The derivatives of $G^{M*}(w,y)$ and $G^{R*}(w,y)$ with respect to income are denoted by $\theta^{M*}(w,y)$ and $\theta^{R*}(w,y)$, respectively. The before-tax income schedule that solves candidate k's problem in (10) is described in Proposition 3.

Proposition 3. *The optimal schedule of before-tax incomes $y^*(\cdot)$ for candidate k's problem is given by*

$$
y^*(w) = \begin{cases}
y^{M*}(w), & \forall w \in [\underline{w}, w_b), \\
y^{M*}(w_b), & \forall w \in [w_b, w_B] \ \text{if} \ w_b > \underline{w}, \\
y^{R*}(w_B), & \forall w \in [w_b, w_B] \ \text{if} \ w_B < \bar{w}, \\
y^{R*}(w), & \forall w \in (w_B, \bar{w}].
\end{cases}
\tag{25}
$$

The optimal values of the bridge endpoints w_b and w_B are determined by the first-order condition

$$
\int_{w_b}^{k} \theta^{M*}(w, y^{M*}(w_b)) dw + \int_{k}^{w_B} \theta^{R*}(w, y^{M*}(w_b)) dw = 0
\tag{26}
$$

if $w_b > \underline{w}$ and by the first-order condition

$$
\int_{w_b}^{k} \theta^{M*}(w, y^{R*}(w_B)) dw + \int_{k}^{w_B} \theta^{R*}(w, y^{R*}(w_B)) dw = 0
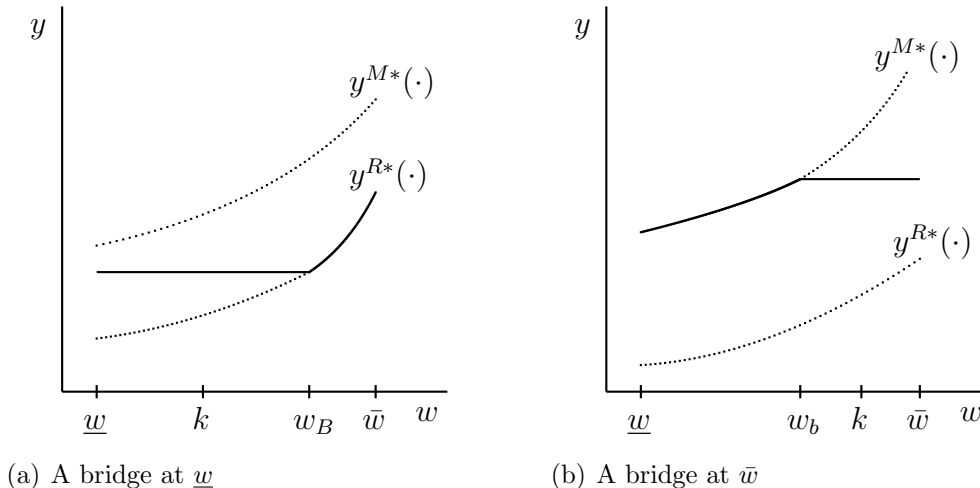\tag{27}
$$

if $w_B < \bar{w}$.

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Figure 4: Bridges at the endpoints of the distribution

The shape of candidate k's optimal income schedule $y^*(\cdot)$ is given by (25). As we have already noted, for types smaller than the lower endpoint of the bridge, candidate k's optimal income schedule coincides with the maxi-max income schedule $y^{M*}(\cdot)$, whereas for types larger than the upper endpoint of the bridge, it coincides with the maxi-min income schedule $y^{R*}(\cdot)$. Provided that $w_B < \bar{w}$, the income for skill type w_B is $y^{R*}(w_B)$. Consequently, all individuals on the bridge receive this income. Analogously, if $w_b > \underline{w}$, then all individuals on the bridge receive $y^{M*}(w_b)$. If both $w_b > \underline{w}$ and $w_B < \bar{w}$, then $y^{M*}(w_b) = y^{R*}(w_B)$. It is possible that $w_b = \underline{w}$, in which case the optimal income schedule starts with the bridge and then tracks the maxi-min solution. It is also possible that $w_B = \bar{w}$, in which case the optimal income schedule first tracks the maxi-max solution and then ends with the bridge.[25] These two possibilities are illustrated in Figure 4.

The first-order optimality conditions for the optimal placement of the bridge endpoints are given in (26) and (27). When both $w_b > \underline{w}$ and $w_B < \bar{w}$ hold, (26) and (27) are equivalent conditions. These two equations are similar to the standard ironing condition found in Guesnerie and Laffont (1984, eqn. (3.16), p. 347).

4. The Political Equilibrium

Majority rule is used to determine the income tax schedule that is implemented. As we observed in the previous section, we can equivalently think of voting as taking place over the types of individuals, with a type k candidate implementing his selfishly optimal incentive-compatible income tax schedule (or, equivalently, his optimal incentive-compatible allocation schedule) if elected. The advantage of this way of formulating the problem is that a type is simply a skill level, and so voting takes place over a one-dimensional issue space.

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[25] It is conceivable that $w_b = \underline{w}$ and $w_B = \bar{w}$, in which case the bridge is the whole income schedule. This possibility is so unlikely that we do not consider it explicitly.

We show that individual preferences over candidates are single-peaked with respect to the skill level. It then follows from Black's Median Voter Theorem (Black, 1948) that there exists a Condorcet winner, that is, a candidate who beats every other candidate in a pairwise majority contest. More precisely, a candidate is a *Condorcet winner* if at least half of the population weakly prefers this candidate to any other candidate.[26] Moreover, any candidate with the median skill level in the population is a Condorcet winner.[27] Thus, the selfishly optimal incentive-compatible tax schedule for an individual with the median skill level (weakly) beats the selfishly optimal incentive-compatible tax schedule for any other type of individual in a pairwise majority vote.

It is now necessary to distinguish allocation schedules by the types that propose them. Let $(x^*(w,k), y^*(w,k))$ denote the optimal allocation assigned to an individual of type w by candidate k's optimal incentive-compatible tax schedule. When $k = \bar{w}$, $y^*(\cdot, k)$ is the maxi-max income schedule $y^{M*}(\cdot)$ and when $k = \underline{w}$, it is the maxi-min income schedule $y^{R*}(\cdot)$. The utility obtained by an individual with skill level w with the schedule proposed by a candidate of type k is

$$V(w,k) = x^*(w,k) - h\left(\frac{y^*(w,k)}{w}\right). \tag{28}$$

The bridge in candidate k's optimal incentive-compatible income schedule is now denoted by $[w_b(k), w_B(k)]$.

While it is only necessary to introduce a bridge between the maxi-max and maxi-min income schedules for types other than $\underline{w}$ and $\bar{w}$, it will simplify the statement of our next result if we define bridges for these two types as well. These bridges are defined to be the set of types that are bunched with the relevant endpoint of the skill distribution. Thus, $w_b(\underline{w}) = \underline{w}$ and $w_B(\bar{w}) = \bar{w}$. In either case, there need not be any bunching, in which case the endpoints of the bridge are the same. In contrast, a bridge must be a nondegenerate interval for any other type.

One determinant of an individual's preferences for the candidates is the amount of before-tax income he will obtain with each candidate's proposal. As a preliminary to identifying how an individual's income varies with the type of candidate, we begin by showing that the endpoints of a bridge are nondecreasing in the candidate's type.

Proposition 4. *The bridge endpoints $w_b(k)$ and $w_B(k)$ are nondecreasing in k for all $k \in [\underline{w}, \bar{w}]$.*

Consider two candidates k_1 and k_2 with $k_1 < k_2$. Because (i) their proposed income schedules coincide with the maxi-max schedule for types below the lower endpoint of the bridge and coincide with the maxi-min schedule for types above the upper endpoint of the bridge and (ii) the endpoints of a bridge are nondecreasing in the type of the candidate, any individual's income is nondecreasing in the type of the candidate.

---

[26]Bohn and Stuart (2013) require any individual who is indifferent between two candidates to either vote for only one of them or to abstain from voting on this pair.

[27]Our assumption that $f(w) > 0$ for all $w \in [\underline{w}, \bar{w}]$ implies that there is a unique median skill level.

Proposition 5. *For all $w, k_1, k_2 \in [\underline{w}, \bar{w}]$ for which $k_1 < k_2$, $y^*(w, k_1) \leq y^*(w, k_2)$.*[28]

Some intuition for the comparative static results described in Propositions 4 and 5 may be obtained by considering the income distortions for different types. For individuals with skill types below the lower endpoint of a bridge, their incomes are determined by the maxi-max schedule. These incomes are distorted upwards relative to the full-information solution. The higher the type of the candidate, the more types there are below him, and so it is beneficial for him to distort the incomes (and labor supplies) of more of these types upwards. In contrast, for individuals with skill types above the upper endpoint of a bridge, their incomes are determined by the maxi-min schedule and, so, are distorted downwards. In this case, the higher the type of the candidate, the fewer the types there are above him, and so it is beneficial for him to distort the incomes (and labor supplies) of fewer of these types downwards.

All candidates face the same government budget and incentive constraints. As a consequence, because each candidate proposes an incentive-compatible income schedule that is best for him, he must weakly prefer what he obtains with his own schedule to what anybody else proposes for him. Formally,

$$V(w, w) \geq V(w, k), \quad \forall w, k \in [\underline{w}, \bar{w}]. \tag{29}$$

Thus, each individual's preferences for the candidates has a peak at his own type. An individual of skill type w has a (weakly) *single-peaked preference* on the candidates' types if

$$V(w, w) \geq V(w, k_1) \geq V(w, k_2) \text{ if } w < k_1 < k_2 \tag{30}$$

and

$$V(w, w) \geq V(w, k_1) \geq V(w, k_2) \text{ if } w > k_1 > k_2. \tag{31}$$

We show in Proposition 6 that everybody's preferences for the candidates are single-peaked.

Proposition 6. *Individual preferences are single-peaked on the types of the candidates.*

Whereas Proposition 5 only considers how an individual's income varies as the type of candidate changes, Proposition 6 compares how his utility varies, and so takes account of both income and consumption changes. Even if two candidates offer an individual the same income, it does not follow that they offer the same consumption as well. For example, suppose that $w < k_1 < k_2$ and that $y^*(w, k_1) = y^*(w, k_2)$ because w is on the maxi-max part of the income schedules proposed by both candidates k_1 and k_2. Further suppose that both endpoints of the bridge are larger with the k_2 schedule than with the k_1 schedule. The latter assumption implies that aggregate before-tax income increases when k_2's schedule is used instead of k_1's and, therefore, there is an additional amount of consumption that can be distributed. Suppose that individual incomes are first adjusted

---

[28]The second inequality is in fact strict for all $w \in (w_b(k_1), w_B(k_2))$ if $w_b(k_1) < w_b(k_2)$ and $w_B(k_1) < w_B(k_2)$.

to the levels stipulated by candidate k_2's income schedule holding consumptions at the levels for candidate k_1's consumption schedule. Because preferences are quasilinear in consumption, if this allocation satisfies the incentive-compatibility constraints, it is then optimal to distribute the surplus consumption to everybody equally as this will preserve incentive compatibility. However, it would then be the case that a type w individual would prefer candidate k_2 to candidate k_1 because he receives more consumption with k_2's proposal than with that of k_1 without any change in income, contradicting Proposition 6. Thus, the first stage of this adjustment procedure violates incentive compatibility. In moving to candidate k_2's schedule, the individuals with skill types in $(w_b(k_1), w_b(k_2))$ have their labor supplies further upward distorted. To restore incentive compatibility, the consumption of everybody of lower skill, including individuals of type w, must have their consumptions reduced even after the additional consumption generated by the increase in the aggregate income has been distributed.[29]

When preferences over incentive-compatible income tax schedules are single-peaked, by Black's Median Voter Theorem (Black, 1948), the tax schedule proposed by the median skill type does at least as well as any other proposed tax schedule in a majority vote. Hence, the following proposition is a direct consequence of Proposition 6 and Black's Theorem.[30]

Proposition 7. *The selfishly optimal incentive-compatible income tax schedule for the median skill type is a Condorcet winner when majority voting is restricted to the incentive-compatible income tax schedules that are selfishly optimal for some skill type.*

Thus, the income tax schedule that is enacted by majority voting maximizes the utility of the median skill type subject to the incentive-compatibility and government budget constraints. In the typical case in which this schedule has three segments, it features marginal wage subsidies for individuals with low skill types except for possibly the least skilled, a bunching region that includes the median type, and positive marginal tax rates for individuals with high skill types except for the most highly skilled. In particular, there must be a kink in the tax schedule.

Given the complexity of the policies under consideration, it is striking that a Condorcet winner exists. Indeed, if voting were over all possible nonlinear income tax schedules, there would be voting cycles. As we have noted in the Introduction, a Condorcet winner exists if only linear income tax schedules that are budget feasible are considered (see Roberts, 1977). The main insight of Röell (2012) is that by restricting attention

---

[29]If $w > k_2 > k_1$, $y^*(w, k_1) = y^*(w, k_2)$ because w is on the maxi-min part of both income schedules, and the bridge endpoints shift as above, then fewer types have their labor supplies downward distorted in moving from candidate k_1's schedule to that of candidate k_2. For this reason, some of the newly generated consumption is allocated to type w individuals to help restore incentive compatibility.

[30]Black (1948) assumes that each voter has a unique most-preferred alternative. The reasoning used to establish his Median Voter Theorem extends to the weakly single-peaked preferences considered here. We conjecture that each skill type has a unique optimal incentive-compatible income tax schedule, but that in some circumstances an interval of candidate types may prefer the same schedule.

to the incentive-compatible tax schedules that are optimal from some individual's perspective, there is a Condorcet winner even when nonlinear income tax schedules are permitted provided that preferences are quasilinear in consumption and there is a finite number of skill types. Proposition 7 confirms that this insight extends to an economy with a continuum of skill types.

5. Conclusion

We have fully characterized the incentive-compatible income schedule that is selfishly optimal for any skill type. The desire of an individual to redistribute resources towards himself both from those who are less skilled and from those who are more skilled has been shown to result in a downward discontinuity in the selfishly optimal income schedule identified using the first-order approach, which is a novel kind of violation of a second-order incentive-compatibility constraint in the optimal income tax literature. Moreover, the upward redistribution from the less skilled results in a non-standard pattern of distortions reminiscent of those found in the screening literature with participation constraints and countervailing incentives, as studied in detail by Jullien (2000). In addition, for sufficiently large skill types, we have observed that there are complications due to potential violations of the second-order incentive-compatibility conditions that do not feature in the existing literature on countervailing incentives. Nevertheless, in spite of all of these complications, we have shown that the selfishly optimal income (and allocation) schedules can be quite simply described.

Our comparative static result describing how the selfishly optimal income schedules vary with the type of candidate is easy to state in terms of bridge endpoints. This simple structure is what gives rise to single-peaked preferences over candidates (and their tax schedules), from which our median voter result follows. Our comparative static result can also be used to analyze the equilibria of other voting rules, such as various forms of plurality rule, or as a building block in understanding the positions that different constituencies might hold and advocate in more institutionally rich models of political decision-making.

The tools we use to identify the endpoints of a bridge are of potential interest in the study of other screening problems in which the first-order approach results in discontinuities in the solution that violate the second-order incentive-compatibility conditions. For example, a policy design problem in which some group is explicitly excluded from the planner's objective but is part of the set of contributors to the policy's finances may well give rise to such a discontinuity.

In Brett and Weymark (2014), we show that our analysis can be modified to take account of the minimum-utility constraint employed by Röell (2012) and Bohn and Stuart (2013) without fundamentally changing the basic structure of a candidate's selfishly optimal income schedule. Interestingly, the addition of such a constraint increases the plausibility of our sufficient conditions for the satisfaction of the second-order conditions in a candidate's optimization problem. The minimum-utility constraint induces a can-

didate to behave as if he is maximizing a weighted average of his own utility and the utility of the least skilled. For types above his own, this modified objective has no effect on this candidate's optimal income schedule because he still wants to redistribute resources downward from these types. For types below this candidate, however, the weighted averaging pulls the income schedule toward the more well-behaved maxi-min schedule. Moreover, the shadow value of the minimum-utility constraint increases with the candidate's type, so that relatively high-skilled candidates (who in our basic model are more prone to violations of the second-order conditions) optimally choose income schedules that are the most shifted away from what they would choose in the absence of the minimum-utility constraint.

The tendency for democratic governments to redistribute from both the poor and the rich toward the middle class is known as Director's Law (see Stigler, 1970). While the income tax schedule implemented by a median voter in our model differs in a number of respects from the schedules that are observed in practice, it is, however, consistent with Director's Law. Exploring whether this feature of income tax policy is preserved when the model is enriched to allow for labor market decisions at the extensive margin or for other policy instruments, such as workfare, is a natural topic for future research.

Acknowledgements

Appendix

Proof of Proposition 1. By (6),

$$V(w) = V(\underline{w}) + \int_{\underline{w}}^{w} \frac{y(t)}{t^2} h'\left(\frac{y(t)}{t}\right) dt. \tag{A.1}$$

Integrating (A.1) over the support of the distribution of types yields

$$\int_{\underline{w}}^{\bar{w}} V(w)f(w)dw = \int_{\underline{w}}^{\bar{w}} V(\underline{w})f(w)dw + \int_{\underline{w}}^{\bar{w}} \int_{\underline{w}}^{w} \frac{y(t)}{t^2} h'\left(\frac{y(t)}{t}\right) f(w)dtdw. \qquad (A.2)$$

Reversing the order of integration in (A.2), we obtain

$$\int_{\underline{w}}^{\bar{w}} V(w)f(w)dw = V(\underline{w}) + \int_{\underline{w}}^{\bar{w}} \frac{y(t)}{t^2} h'\left(\frac{y(t)}{t}\right) \left[\int_{t}^{\bar{w}} f(w)dw\right] dt$$

$$= V(\underline{w}) + \int_{\underline{w}}^{\bar{w}} \frac{y(t)}{t^2} h'\left(\frac{y(t)}{t}\right) [1 - F(t)] \, dt. \qquad (A.3)$$

On the other hand, by (4),

$$\int_{\underline{w}}^{\bar{w}} V(w)f(w)dw = \int_{\underline{w}}^{\bar{w}} x(w)f(w)dw - \int_{\underline{w}}^{\bar{w}} h\left(\frac{y(w)}{w}\right) f(w)dw. \qquad (A.4)$$

As previously noted, it is optimal for the government budget constraint (9) to bind. Substituting the equality form of (9) into (A.4) yields

$$\int_{\underline{w}}^{\bar{w}} V(w)f(w)dw = \int_{\underline{w}}^{\bar{w}} y(w)f(w)dw - \int_{\underline{w}}^{\bar{w}} h\left(\frac{y(w)}{w}\right) f(w)dw. \qquad (A.5)$$

Combining (A.3) and (A.5) implies that

$$V(\underline{w}) = \int_{\underline{w}}^{\bar{w}} y(w)f(w)dw - \int_{\underline{w}}^{\bar{w}} h\left(\frac{y(w)}{w}\right) f(w)dw$$

$$- \int_{\underline{w}}^{\bar{w}} \frac{y(w)}{w^2} h'\left(\frac{y(w)}{w}\right) [1 - F(w)] \, dw. \qquad (A.6)$$

The maximand in (12) is obtained by substituting (A.6) into (A.1) and setting $w = k$. The preceding calculations have accounted for all the constraints in (11), and so the proof is complete. $\square$

Proof of Proposition 3. First, fix the bridge endpoints w_b and w_B and let $y^*(w_b, w_B)$ denote the optimal income on the bridge $[w_b, w_B]$. A bridge cannot begin in the interior of a bunching interval of $y^{M*}(\cdot)$, nor can it end in the interior of a bunching interval of $y^{R*}(\cdot)$, so it is supposed in the rest of this proof that w_b and w_B satisfy these restrictions. The rest of candidate k's optimal income schedule is obtained by solving

$$\max_{y(\cdot)} \left[\int_{\underline{w}}^{w_b} G^{M*}(w, y(w))dw + \int_{w_B}^{\bar{w}} G^{R*}(w, y(w))dw\right] \qquad (A.7)$$

$$\text{subject to } y(w) = y^*(w_b, w_B), \ \forall w \in [w_b, w_B].$$

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This problem can be solved point-wise. Its solution is given implicitly by the first-order conditions

$$\theta^{M*}(w, y(w)) = 0, \quad \forall w \in [\underline{w}, w_b),$$
$$\theta^{R*}(w, y(w)) = 0, \quad \forall w \in [w_B, \bar{w}]. \tag{A.8}$$

Thus,

$$y^*(w) = y^{M*}(w), \quad \forall w \in [\underline{w}, w_b), \tag{A.9}$$

and

$$y^*(w) = y^{R*}(w), \quad \forall w \in w_B, \bar{w}). \tag{A.10}$$

We now need to determine the optimal endpoints of the bridge and the optimal income on the bridge. There are two cases to consider.

Case 1: $w_B < \bar{w}$. Continuity of the solution implies that $y^*(w_B) = y^{R*}(w_B)$. We also have $y^*(w_B) = y^*(w_b)$ because income is a constant on the bridge. If $w_b > \underline{w}$, then by continuity, $y^*(w_b) = y^{M*}(w_b)$. Let

$$\phi(w_B) = \begin{cases} y^{M*^{-1}}(y^{R*}(w_B)), & \text{if } w_b > \underline{w}, \\ w_b, & \text{if } w_b = \underline{w}. \end{cases} \tag{A.11}$$

Because $y^{M*}(\cdot)$ is continuous and piecewise continuously differentiable, so is $\phi(\cdot)$. Candidate k's choice of w_B is therefore the solution to

$$\max_{w_B} B(w_B, k) \equiv \int_{\underline{w}}^{\phi(w_B)} G^{M*}(w, y^{M*}(w))dw + \int_{\phi(w_B)}^{k} G^{M*}(w, y^{R*}(w_B))dw$$
$$+ \int_{k}^{w_B} G^{R*}(w, y^{R*}(w_B))dw + \int_{w_B}^{\bar{w}} G^{R*}(w, y^{R*}(w))dw, \tag{A.12}$$

where use has been made of (A.9) and (A.10) in (A.12). Setting the derivative of $B(w, k)$ with respect to w_B equal to zero, we obtain the first-order condition

$$G^{M*}(w_b, y^{M*}(w_b))\frac{d\phi(w_B)}{dw_B} - G^{M*}(w_b, y^{M*}(w_b))\frac{d\phi(w_B)}{dw_B}$$
$$+ \int_{w_b}^{k} \theta^{M*}(w, y^{R*}(w_B))\frac{dy^{R*}(w_B)}{dw_B}dw + \int_{k}^{w_B} \theta^{R*}(w, y^{R*}(w_B))\frac{dy^{R*}(w_B)}{dw_B}dw \tag{A.13}$$
$$+ G^{R*}(y^{R*}(w_b, y(w_B)) - G^{R*}(y^{R*}(w_b, y(w_B)) = 0.$$

Simplifying (A.13) yields

$$\frac{dy^{R*}(w_B)}{dw_B}\left[\int_{w_b}^{k} \theta^{M*}(w, y^{R*}(w_B))dw + \int_{k}^{w_B} \theta^{R*}(w, y^{R*}(w_B))dw\right] = 0. \tag{A.14}$$

Because a bridge cannot end in the middle of a bunching region of y^{R*}, it must be the case that the derivative in this expression is positive. Hence, (27) holds.

Case 2: $w_b > \underline{w}$. Continuity of the solution now implies that $y^*(w_b) = y^{M*}(w_b)$. Let

$$\psi(w_b) = \begin{cases} y^{R*^{-1}}(y^{M*}(w_b)), & \text{if } w_B < \bar{w}, \\ w_B, & \text{if } w_B = \bar{w}. \end{cases} \tag{A.15}$$

Candidate k's choice of w_b is the solution to

$$\max_{w_b} b(k, w_b) \equiv \int_{\underline{w}}^{w_b} G^{M*}(w, y^{M*}(w))dw + \int_{w_b}^{k} G^{M*}(w, y^{M*}(w_b))dw$$
$$+ \int_{k}^{\psi(w_b)} G^{R*}(w, y^{M*}(w_b))dw + \int_{\psi(w_b)}^{\bar{w}} G^{R*}(w, y^{R*}(w))dw, \tag{A.16}$$

The analogue of (A.14) is

$$\frac{dy^{M*}(w_b)}{dw_b} \left[\int_{w_b}^{k} \theta^{M*}(w, y^{M*}(w_b))dw + \int_{k}^{w_B} \theta^{R*}(w, y^{M*}(w_b))dw \right] = 0, \tag{A.17}$$

from which (27) follows because a bridge cannot begin in the middle of a bunching region of y^{M*}. □

Proof of Proposition 4. We first show that $w_B(k)$ is nondecreasing in k. Consider any value of k for which $w_B(k) < \bar{w}$. Differentiating the left-hand side of (A.14) with respect to k, we obtain

$$\frac{\partial^2 B(w_B, k)}{\partial w_B \partial k} = \frac{dy^{R*}(w_B)}{dw_B} \left[\theta^{M*}(k, y^{R*}(w_B)) - \theta^{R*}(k, y^{R*}(w_B)) \right]$$
$$= \frac{dy^{R*}(w_B)}{dw_B} \left[h'' \left(\frac{y^{R*}(w_B)}{k} \right) \frac{y^{R*}(w_B)}{k^3} + h' \left(\frac{y^{R}(w_B)}{k} \right) \frac{1}{k^2} \right], \tag{A.18}$$

where the second equality follows from (14) and (15). Because $h(\cdot)$ is a strictly convex function, the cross-partial derivative in (A.18) is positive. Hence, by Theorems 10.3 and 10.4 in Sundaram (1996), the function B is supermodular and therefore exhibits increasing differences in (w, k). Thus, by Topkis' Theorem (Topkis, 1978, Theorem 6.1), it follows that $w_B(k)$ is nondecreasing in k for values of k for which $w_B(k) < \bar{w}$.

If $w_B(k) = \bar{w}$, the proof proceeds as above except that $y^*(\bar{w})$ is used instead of $y^{R*}(w_B)$ in (A.11).

The proof that $w_b(k)$ is nondecreasing in k is almost the same as that for the other endpoint of the bridge. In this case, the left-hand side of (A.17) is differentiated with respect to k to obtain

$$\frac{\partial^2 b(w_b, k)}{\partial w_b \partial k} = \frac{dy^{M*}(w_b)}{dw_b} \left[\theta^{M*}(k, y^{M*}(w_b)) - \theta^{R*}(k, y^{M*}(w_b)) \right]$$
$$= \frac{dy^{M*}(w_b)}{dw_b} \left[h'' \left(\frac{y^{M*}(w_b)}{k} \right) \frac{y^{M*}(w_b)}{k^3} + h' \left(\frac{y^{M*}(w_b)}{k} \right) \frac{1}{k^2} \right], \tag{A.19}$$

and $y^*(\underline{w})$ is used instead of $y^{M*}(w_b)$ in (A.15). □

Proof of Proposition 6. We only show (30) as the proof of (31) analogous. Consider three types w, k_1, and k_2 for which $w < k_1 < k_2$. The first inequality in (30) follows immediately from (29).

To show that the second inequality holds as well, first note that by (6),

$$V(w, k_1) = V(k_1, k_1) - \int_w^{k_1} h' \left(\frac{y^*(t, k_1)}{t} \right) \frac{y^*(t, k_1)}{t^2} dt. \tag{A.20}$$

Therefore, using (29),

$$V(w, k_1) \geq V(k_1, k_2) - \int_w^{k_1} h' \left(\frac{y^*(t, k_1)}{t} \right) \frac{y^*(t, k_1)}{t^2} dt. \tag{A.21}$$

It also follows from (6) that

$$V(w, k_2) = V(k_1, k_2) - \int_w^{k_1} h' \left(\frac{y^*(t, k_2)}{t} \right) \frac{y^*(t, k_2)}{t^2} dt. \tag{A.22}$$

Subtracting (A.22) from (A.21) yields

$$V(w, k_1) - V(w, k_2) = \int_w^{k_1} \left[h' \left(\frac{y^*(t, k_2)}{t} \right) \frac{y^*(t, k_2)}{t^2} - h' \left(\frac{y^*(t, k_1)}{t} \right) \frac{y^*(t, k_1)}{t^2} \right] dt. \tag{A.23}$$

By Proposition 5, $y^*(t, k_2) \geq y^*(t, k_1)$ for all t. Therefore, by the convexity of $h(\cdot)$, the expression in (A.23) is nonnegative, as was to be shown. □

References

Arrow, K. J., 1968. Optimal capital policy with irreversible investment. In: Wolfe, J. N. (Ed.), Value, Capital, and Growth: Papers in Honour of Sir John Hicks. Edinburgh University Press, Edinburgh, pp. 1–19.

Austen-Smith, D., Banks, J. S., 1999. Positive Political Theory I: Collective Preference. University of Michigan Press, Ann Arbor.

Berliant, M., Gouveia, M., 2001. On the political economy of income taxation, unpublished manuscript, Department of Economics, Washington University at St. Louis and School of Business and Economics, Universidade Católica Portuguesa.

Besley, T., Coate, S., 1997. An economic model of representative government. Quarterly Journal of Economics 112, 85–114.

Bierbrauer, F., Boyer, P., 2013. Political competition and Mirrleesian income taxation: A first pass. Journal of Public Economics 103, 1–14.

Black, D., 1948. On the rationale of group decision making. Journal of Political Economy 56, 23–34.

Blomquist, S., Christiansen, V., 1999. The political economy of publicly provided private goods. Journal of Public Economics 73, 31–54.

Boadway, R., Jacquet, L., 2008. Optimal marginal and average income taxation under maximin. Journal of Economic Theory 143, 425–441.

Bohn, H., Stuart, C., 2013. Revenue extraction by median voters, unpublished manuscript, Department of Economics, University of California, Santa Barbara.

Brett, C., Weymark, J. A., 2014. Voting over nonlinear income tax schedules with a minimum-utility constraint, in preparation.

Brito, D. L., Oakland, W. H., 1977. Some properties of the optimal income-tax. International Economic Review 18, 407–423.

De Donder, P., Hindricks, J., 2003. The politics of progressive income taxation with incentive effects. Journal of Public Economics 87, 2491–2505.

Ebert, U., 1992. A reexamination of the optimal nonlinear income tax. Journal of Public Economics 49, 47–73.

Gans, J. S., Smart, M., 1996. Majority voting with single-crossing preferences. Journal of Public Economics 59, 219–237.

Guesnerie, R., 1995. A Contribution to the Pure Theory of Taxation. Cambridge University Press, Cambridge.

Guesnerie, R., Laffont, J.-J., 1984. A complete solution to a class of principal-agent problems with an application to the control of aself-managed firm. Journal of Public Economics 25, 329–369.

Guesnerie, R., Seade, J., 1982. Nonlinear pricing in a finite economy. Journal of Public Economics 17, 157–179.

Hammond, P. J., 1979. Straightforward incentive compatibility in large economies. Review of Economic Studies 46, 263–282.

Hellwig, M. F., 2010. Incentive problems with unidimensional characteristics: A unified approach. Econometrica 78, 1201–1237.

Itsumi, Y., 1974. Distributional effects of linear income tax schedules. Review of Economic Studies 41, 371–381.

Jullien, B., 2000. Participation constraints in adverse selection problems. Journal of Economic Theory 93, 1–47.

Laffont, J.-J., Martimort, D., 2002. The Theory of Incentives: The Principle-Agent Problem. Princeton University Press, Princeton, NJ.

Ledyard, J., 2006. Voting and efficient public good mechanisms. In: Weingast, B. R., Wittman, D. A. (Eds.), The Oxford Handbook of Political Economy. Oxford University Press, Oxford, pp. 479–501.

Lollivier, S., Rochet, J.-C., 1983. Bunching and second-order conditions: A note on optimal tax theory. Journal of Economic Theory 31, 392–400.

Matthews, S., Moore, J., 1987. Monopoly provision of quality and warranties: An exploration in thetheory of multi-dimensional screening. Econometrica 55, 441–467.

Meltzer, A. H., Richard, S. F., 1981. A rational theory of the size of government. Journal of Political Economy 89, 914–927.

Mirrlees, J. A., 1971. An exploration in the theory of optimum income taxation. Review of Economic Studies 38, 175–208.

Mirrlees, J. A., 1976. Optimum tax theory: A synthesis. Journal of Public Economics 6, 327–358.

Osborne, M. J., Slivinski, A., 1996. A model of political competition with citizen-candidates. Quarterly Journal of Economics 111, 65–96.

Persson, T., Tabellini, G., 2000. Political Economics: Explaining Economic Policy. MIT Press, Cambridge, MA.

Rawls, J., 1971. A Theory of Justice. Harvard University Press, Cambridge, MA.

Roberts, K. W. S., 1977. Voting over income tax schedules. Journal of Public Economics 8, 329–340.

Röell, A., 2012. Voting over nonlinear income tax schedules, unpublished manuscript, School of International and Public Affairs, Columbia University.

Roemer, J. E., 2012. The political economy of income taxation under asymmetric information: The two-type case. SERIEs: Journal of the Spanish Economic Association 3, 181–199.

Romer, T., 1975. Individual welfare, majority voting, and the properties of a linear income tax. Journal of Public Economics 4, 163–185.

Sadka, E., 1976. On income distribution, incentive effects and optimal income taxation. Review of Economic Studies 43, 261–267.

Seade, J. K., 1977. On the shape of optimal tax schedules. Journal of Public Economics 7, 203–235.

Snyder, J. M., Kramer, G. H., 1988. Fairness, self-interest, and the politics of the progressive income tax. Journal of Public Economics 36, 197–230.

Stigler, G. J., 1970. Director's Law of public income redistribution. Journal of Law and Economics 13, 1–10.

Sundaram, R. K., 1996. A First Course in Optimization Theory. Cambridge University Press, Cambridge.

Topkis, D. M., 1978. Minimizing a submodular function on a lattice. Operations Research 26, 305–321.

Vincent, T. L., Mason, J. D., 1967. A new method for treating optimal trajectories with restricted segments. Contractor Report CR-744, NASA.

Vincent, T. L., Mason, J. D., 1968. Disconnected optimal trajectories. Journal of Optimization Theory and Applications 3, 263–281.