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DEMAND UNCERTAINTY AND EFFICIENCY

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Abstract

I use a flexible price version of the Prescott (1975) "hotels" model to study a dynamic model that allows for storage. The formulation follows the standard competitive analysis tradition with a non-standard definition of markets: The set of markets that open depends on the state of demand. I use three planner's problems to characterize various efficiency concepts that are used in the literature, but focus on the problem of a "weak" planner that faces the same constraints as the sellers in the model. From the point of view of the "weak" planner, the equilibrium outcome is efficient if the probability of becoming active is the same for all buyers. In general, the equilibrium outcome is not efficient from the point of view of a planner that has more information than the sellers in the model except for the case in which the costs of delaying trade are not important. The cost of delay is also relevant for price dispersion: Lower cost of delays may lead to lower price dispersion.

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September 2014

I use a flexible price version of the Prescott (1975) "hotels" model to study a dynamic model that allows for storage. The formulation follows the standard competitive analysis tradition with a non-standard definition of markets: The set of markets that open depends on the state of demand. I use three planner's problems to characterize various efficiency concepts that are used in the literature, but focus on the problem of a "weak" planner that faces the same constraints as the sellers in the model. From the point of view of the "weak" planner, the equilibrium outcome is efficient if the probability of becoming active is the same for all buyers. In general, the equilibrium outcome is not efficient from the point of view of a planner that has more information than the sellers in the model except for the case in which the costs of delaying trade are not important. The cost of delay is also relevant for price dispersion: Lower cost of delays may lead to lower price dispersion.

Keywords: Price dispersion, demand uncertainty, efficiency, sequential trade, inventories, costs of delaying trade, price rigidity.

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1. INTRODUCTION

Uncertainty about demand is an important feature of the environment. Yet the standard competitive model does not offer an explicit description of its resolution. Instead we have a Walrasian auctioneer who resolves the uncertainty and announces the market-clearing price before the beginning of actual trade.

This gap in the standard model is especially felt when capacity is determined prior to the realization of demand. The peak-load-pricing literature initiated by Williamson (1966) follows the Walrasian tradition in assuming that trade occurs only after the resolution of uncertainty. When capacity is not fully utilized the Williamson model predicts that the price will drop to the variable marginal cost. The model does not explain why the price of rooms does not drop in hotels that are not filled to capacity.

To explain the behavior of prices when excess capacity occurs, researchers have assumed price rigidity and/or monopoly power. These issues are important for industrial policy. They are also central for macro-policy. Roughly speaking, price rigidity and monopoly power support new Keynesian models and their policy implications. Moreover, the accumulation of large ("undesired") inventories is often interpreted as the result of prices that do not clear markets and in favor of Keynesian models. On the other hand there is the neo-classical view of market clearing and efficiency.

Prescott (1975) is close to the neo-classical view. He uses a model in which prices are set in advance and show that the resulting allocation is efficient when buyers are homogeneous and demand one unit. In Eden (1990), I proposed an explanation that is in the spirit of Prescott (1975) but does not assume price rigidity. In the model, buyers are homogeneous and each active buyer has a downward sloping demand curve. Active buyers arrive sequentially and sellers must make irreversible selling decisions before they know whether additional buyers will arrive or not. Unlike Williamson's peak load pricing model, here trade occurs before the resolution of the uncertainty about demand and there are many prices. Indeed, sellers know the realization of demand only at the end of the trading process when the price of the unoccupied rooms is no longer relevant (because at this point all active buyers have rooms). I refer to this model as the Uncertain and Sequential Trade (UST) model.[2]

The level of abstraction of the UST model is close to that of the Walrasian model. Nevertheless, the UST model is useful for understanding observations that are inconsistent with the standard formulation of competitive environments. Dana (1998) has shown that a price-taking firm may offer advanced-purchase discounts and consumers with relatively certain demand will take advantage of this offer. Thus, price discrimination may occur in the absence of monopoly power. In Eden (2007) I show that this line of reasoning can explain "dumping": A firm may export at a price that is below the price its sells at home if the demand at home is relatively unstable. Recent studies of prices in the airline industry have tested the "stochastic demand pricing" against the monopoly price discrimination alternative. My reading of this literature is that the behavior of prices in the data is consistent with the predictions of the UST model.[3] The UST model has also been used to address issues of money non-neutrality, price stickiness and price dispersion.[4] In a recent empirical paper (Eden [2014]) I estimate the effect of demand uncertainty on the cross sectional price

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[2] For other versions of the Prescott model, see Dana (1999), Bryant (1980) and Deneckere, Marvel, and Peck (1996). For other type of models that deals with pricing in the presence of demand uncertainty, see Harris and Raviv (1981), Lazear (1986) and Pashigian (1988).

[3] See Escobari and Li (2007), Escobari (2012) and Escobari and Lee (2013) and Cornia, Gerardi and Shapiro (CGS, 2012). Escobari et al. focus on within flight correlation between price dispersion and capacity utilization: Flights that are relatively empty tend to have less price dispersion. CGS find a negative correlation between average capacity utilization and price dispersion: Routes with low average capacity utilization tend to have more price dispersion. In Eden (2013) I argue that both observations are consistent with the UST model.

[4] The observation of prices that do not change often may arise as a direct consequence of the fact that sellers are indifferent about prices in the equilibrium range. See Eden (1994, Figure 4) and Head, Liu, Menzio and Wright (2012).

dispersion of food items sold by supermarkets. According to my estimation, eliminating aggregate demand uncertainty will reduce the cross sectional price dispersion by more than 50%.

To better understand the literature that followed Prescott (1975), I study here the question of efficiency and the role of the cost of delaying trade. I distinguish between two versions of the Prescott model: The rigid price version and the flexible price (UST) version. The assumption about price rigidity does not matter much for the positive implications of the theory but it matters for the policy implications and for the related efficiency issues. Dana (1998) uses the rigid price version of the model and argues that the allocation is not efficient because of the price rigidity assumption. In a recent paper, Deneckere and Peck (DP, 2012) argue that the allocation in a single price version of the model is not efficient. They define the feasible set by the resource constraint with no additional information constraints. Their result therefore implies that a Walrasian auctioneer or a planner that knows the realization of demand can improve matters. In Eden (1990) I argue that the allocation is efficient from the point of view of a planner that faces the same informational constraints as the sellers in the model. Moreover, trade can be described as the execution of Arrow-Debreu contracts that justifies the “competitive label”. The approach in Eden (1990) is in line with the mechanism design literature and the literature on efficiency in search models.[5]

The cost of delaying trade is central to the problem of capacity utilization that is the focus of the Prescott model. The tourist in the Prescott model is not indifferent between having a room today to having it in the next day. Similarly, the tourist that is already unpacked in his room will see a cost for selling his room to a buyer with a higher reservation price or move to a cheaper room that is not occupied. A similar

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[5] See for example, Gale (1987), Mortensen and Wright (2002) and Kircher (2009). The main difference between search models and UST models is in the assumption required to get price dispersion. In the UST model price dispersion arises as a result of uncertainty about aggregate demand while in search models it arises as a result of search frictions.

argument is in Eden and Griliches (1993) who consider the example of restaurants and explicitly assume that one who had lunch will not buy at the same day an additional cheaper lunch. To provide more intuition, I study here the effects of changes in the cost of delaying trade.

I see the main contribution of the paper in putting various results under the same roof and in clarifying the differences between versions of the Prescott model. The new results are obtained in the dynamic versions of the model that allow for storage and in the single period version that allows for heterogeneous buyers.

The dynamic versions of the model analyzed here are related to the models in Bental and Eden (BE, 1993) and Deneckere and Peck (DP, 2012). These models complement each other. The BE model focus on all year around goods with exponential discounting while the DP model focus on seasonal goods with sudden (one-hoss-shay) depreciation. The new results are the general equilibrium and welfare analysis in a model with exponential discounting and the case in which the cost of delaying trade is important in a model with one-hoss-shay depreciation. I use here standard competitive analysis (with non-standard definition of markets). This is different from DP who use game theory.

The main results of the paper are as follows. (a) The efficiency result in Eden (1990) survives heterogeneity about the utility function and the introduction of storage possibilities. But it does not survive heterogeneity about the probability of becoming active. (b) When the costs of delaying trade are important, the equilibrium outcome in models that assume price flexibility may be efficient while the equilibrium outcome in models that assume price rigidity is not efficient, (c) Price rigidity does not impose welfare loss if there is no cost of delays and (d) Price dispersion increases with the cost of delays.

Section 2 discusses some basic issues in a simple framework. Section 3 allows for heterogeneity in the utility function. (Heterogeneity in the probability of becoming active is in Appendix B). Section 4 considers a dynamic version with exponential decay and constant marginal cost. Section 5 allows for increasing marginal cost and productivity shocks. Section 6 discusses production smoothing. Section 7 analyzes an example of the one-hoss-shay depreciation case. Appendix A generalizes this example.

2. A SINGLE PERIOD MODEL WITH HOMOGENEOUS BUYERS

I start with a discrete version of Prescott (1975). This version allows us to address some important issues before attempting a more general analysis.

The economy lasts for one period. There are two goods: X and Y. There are two types of agents: Sellers and buyers. All agents want to consume good Y but only buyers may want to consume good X. The number of buyers that want to consume good X (active buyers) is a random variable $\tilde{N}$ that can take Z possible realizations: $0 < N_1 < N_2 < ... < N_Z$. For notational convenience I use $N_0 = 0$. In state s there are thus N_s active buyers and the remaining $N_Z - N_s$ buyers are not active. The probability of state s is $\pi_s = \Pr ob(\tilde{N} = N_s)$.

Each active buyer wants to consume one unit of good X and is willing to pay a high price for this unit. The number of sellers is known and is normalized to one. Sellers can produce X at the cost of λ units of Y per unit of X.

At the beginning of the period sellers choose capacity and price tags. Sellers can vary price tags across units and sellers take the probability of selling at each price as given. There are Z cutoff prices: $0 < P_1 < P_2 < ... < P_Z$. Sellers expect to sell a unit with a price tag $P_{i-1} < p \le P_i$ if the number of active buyers is greater than N_i and the state is $s \ge i$. Otherwise (if $s < i$) they do not sell the unit. The probability of selling at a price P_i is equal to the probability of selling at the price $P_{i-1} < p \le P_i$ and is given by: $q_i = \sum_{s=i}^{Z} \pi_s$.

Clearly it is not optimal to post $P_{i-1} < p < P_i$ because at this price the probability of making a sale is the same as the probability of making a sale at the price P_i and P_i promises higher revenues. I therefore assume that the sellers choose price tags out of the following Z alternatives: $0 < P_1 < P_2 < ... < P_Z$.

After capacity and price tags are chosen, buyers arrive sequentially in batches. Buyers see all price offers and choose to buy at the cheapest available offer. The first batch of N_1 buyers buys at the cheapest price P_1. Then if $s = 1$ trade ends. Otherwise, if $s > 1$, more buyers arrive. The minimum additional buyers that will arrive if $s > 1$ is $N_2 - N_1$ and this second batch of buyers buys at the price P_2. Then if $s = 2$ trade ends. Otherwise an additional batch of $N_3 - N_2$ buyers will arrive and buy at the price P_3. The trading process continues until the demand of all active buyers is satisfied.

We may describe trade as occurring in a sequence of hypothetical markets. The first batch buys in the first market at the price P_1. If a second batch arrives it buys in the second market at the price P_2 and so on. I use x_i to denote the supply to market i (i.e., the number of units with a price tag P_i) and define equilibrium as follows.

A UST equilibrium is a vector of prices ($P_1, P_2, ..., P_Z$) and a vector of supplies ($x_1, x_2, ..., x_Z$) such that (a) $P_i = \lambda / q_i$ and (b) $x_i = N_i - N_{i-1}$.

Note that in equilibrium expected revenues $q_i P_i$ are the same for all price tags and as a result the seller is indifferent between the Z prices. In equilibrium the expected revenue from supplying a unit to market i is also equal to the cost of producing the unit ($q_i P_i = \lambda$) and the seller's supply to market i is infinitely elastic at the price P_i. The condition $x_i = N_i - N_{i-1}$ is therefore a market clearing condition.

I now turn to discuss the following questions.

1. Can we have equilibrium with a single price?
2. Are prices rigid or flexible?
3. Do the sellers choose prices or quantities?

4. Can a planner improve matters if: (a) he knows the state before the choice of capacity; (b) he knows the state after the choice of capacity but before the arrival of buyers; (c) he has the same information as the sellers in the model.

Price dispersion is necessary

The definition of UST equilibrium requires price dispersion. To motivate this choice I now consider the case in which all sellers post the same price P and argue that sellers can increase expected profits by deviating from the single price strategy if buyers' reservation price, $\bar{P}$, is high. I assume that if there is excess supply at the price P then a seller who posts a lower price can sell with probability one and if there is excess demand, a seller can sell at the reservation price. Under this assumption I show the following claim.

Claim 1: If $\bar{P} > \lambda / \pi_Z$ and all sellers post the single price P, then the individual seller can increase expected profits by posting a different price.

To show this Claim note that at a single price P there are some states in which the market does not clear. We can therefore have one of the following three cases:

(a) In some states there is excess supply.

(b) In some states there is excess demand.

(c) There is excess demand in some states and excess supply in some other states.

In the excess supply case (a), the probability of selling is less than 1. $P > \lambda$ is not an equilibrium price because the individual seller can do better by reducing his price by an arbitrarily small amount and sell with probability 1. $P \leq \lambda$ is not an equilibrium price because the individual seller can do better by not producing.

In the excess demand case (b), the probability of selling is 1. $P > \lambda$ is not consistent with excess demand because at $P > \lambda$ supply is infinite. $P = \lambda$ is not an equilibrium price because the seller can do better by increasing his price to the reservation price. To see this point let σ denote the probability of excess demand. If there is excess demand it must be the case that we have excess demand when demand equal to its highest possible realization and therefore: $\sigma \geq \pi_Z$. The seller can produce a unit and sell it at the price $\bar{P}$ with probability σ. This deviation will increase profits because $\sigma \bar{P} \geq \pi_Z \bar{P} > \lambda$.

In case (c) there is excess supply in some states and excess demand in some other states. The seller can increase his expected profits either by cutting his price by a small amount or by posting the reservation price.

<u>Prices may appear rigid but they are not</u>

Posted prices may appear rigid because they do not respond to the realization of demand (the state). Nevertheless, prices are flexible in the sense that the seller's plan is time consistent and he has no incentive to change prices during trade. To show this claim, note that the probability of state $s \geq i$ given that market i open is:

$$\Pr ob(N = N_s \mid N \geq N_i) = \frac{\Pr ob(N = N_s \cap N \geq N_i)}{\Pr ob(N \geq N_i)} = \frac{\pi_s}{q_i}$$

The expected revenue from supplying to market $j \geq i$ when market i opens is therefore:

$$(1) \qquad P_j \sum_{s=j}^{Z} \frac{\pi_s}{q_i} = P_j \frac{q_j}{q_i} = P_i \quad \text{for all } j \geq i .$$

The second equality in (1) follows from the equilibrium condition: $q_j P_j = q_i P_i$. It implies that after updating of the probabilities all higher index markets have the same expected revenues and therefore the seller has no incentive to change the allocation of the remaining unsold goods across markets. In this sense, prices are perfectly flexible.

It is easier to think of sellers as choosing quantities.

The standard formulation of competitive equilibrium assumes that agents are price-takers. This is not a problem when everyone knows the market-clearing price. What many researchers regard as a gap in the standard formulation is the tatonnement process in which a Walrasian auctioneer learns about the market clearing price in a process that prohibits actual trade.

Formally, the UST approach adopts the price taking assumption because it is easier to think of sellers as choosing quantities rather than prices. But unlike the traditional approach here the set of markets that open depends on the realization of demand and as a result selling itself is a random event. The markets are for the same good and each market is characterized by the probability that it will open and the price in which trade will occur if it opens. We can therefore think of sellers as allocating goods across markets by posting prices. Thus the distinction between choosing prices and choosing quantities is not important in this model.

A planner that learn the state after the choice of capacity cannot improve welfare.

Expected surplus is given by:

$$\bar{P}\sum_{s=1}^{Z}\pi_s N_s - \lambda N_Z = \bar{P}\sum_{i=1}^{Z} q_i (N_i - N_{i-1}) - \lambda \sum_{i=1}^{Z}(N_i - N_{i-1})$$

Note that the amount sold in state s is not equal to the amount produced and the excess capacity in state s is $N_Z - N_s$. A planner who learns about the state before capacity choices are made will improve matters and achieve the expected surplus: $(\bar{P}-\lambda)\sum_{s=1}^{Z}\pi_s N_s$. A planner who learns about the state after capacity choices are made will not be able to improve on the UST outcome.

The above efficiency result depends critically on the assumption that all buyers have the same reservation price. When buyers have different reservation prices, a planner that knows the state after the choice of capacity but before the beginning of trade can improve matters by setting a single price equals to the market-clearing price. At this Walrasian price only the buyers with the relatively high reservation price will buy the good. This is better than the UST equilibrium outcome because in the UST model, capacity is allocated to buyers that arrive early and some of the early arrivals have relatively low reservation prices.

A planner that has information about the state before the beginning of actual trade has more information than the sellers in the model who make irreversible selling decisions before they know the state (they must sell to each batch of buyers before they know whether additional batches will arrive). A "weak" planner that has the same information as the sellers in the model will not be able to improve on the UST outcome.

In what follows I consider the case of heterogeneous buyers and assume that each buyer has a downward sloping demand curve. We may think of a buyer with a downward sloping demand curve as representing many buyers each with a different reservation price. I elaborate on this interpretation later.

3. HETEROGENEOUS BUYERS

As before I assume a single period economy with two goods (X and Y) and Z states. State s occurs with probability π_s. The number of sellers is known and is normalized to 1. Sellers are risk neutral and derive utility from Y only. Sellers can produce X at the per-unit cost of λ units of Y.

Unlike sellers, buyers are heterogeneous. There are J types of buyers. The number of type j (potential) buyers is n_j. All buyers are endowed with a large quantity of Y. In aggregate state s, the utility function that a fraction ϕ_{js} of type j buyers realize is: $u_{js}(x,y) = U_j(x) + y$, where $U_j(x)$ is strictly monotone, strictly concave and differentiable. To simplify, I assume that $U'_j(0) = \infty$. The remaining

$(1-\phi_{js})n_j$ buyers realize the utility function $u_{js}(x,y)=y$ and are not active. The random utility of a type j buyer in aggregate state s is thus:

(2) $u_{js}(x,y)=U_j(x)+y$ with probability ϕ_{js} and $u_{js}(x,y)=y$ otherwise.

An active type j buyer demands $d_j(p)$ units of X at the price p where the individual demand function is defined by:

(3) $$d_j(p)=\arg\max_{x\geq 0} U_j(x)-px.$$

An interior solution to (3) must satisfy the following first order condition:

(4) $$U_j'(x)=p$$

Production (capacity choice) occurs at $t=0$. After production choice is made, buyers realize a utility function and active buyers form a line. I treat all active buyers symmetrically and assume that any segment taken from this line accurately represents the type composition of buyers who want to consume: In state s, $\sum_i \phi_{is}n_i$ buyers want to consume and the fraction of type j buyers in any segment of the line is: $\vartheta_{js}=\dfrac{\phi_{js}n_j}{\sum_i \phi_{is}n_i}$. After the line is formed, active buyers arrive at the market place one by one according to their place in the line and buy at the cheapest available offer. The sequential trade does not take real time (and occurs in meta time).

I start from the following case.

<u>Assumption 1</u>: The probability of becoming active depends only on the aggregate state and not on the buyer's type: $\phi_{js}=\phi_{1s}=\phi_s$ for all j and s.

I choose indices such that demand is increasing in the state: $0=\phi_0<\phi_1<...<\phi_Z=1$. In state s, the number of active buyers is $N_s=\phi_s N$ where $N=\sum_j n_j$ is the number of potential buyers. Under assumption 1, the fraction of type j buyers in any segment of the line, $\vartheta_j=\frac{\phi_s n_j}{\sum_i \phi_s n_i}=\frac{n_j}{N}$, is independent of s.

As in the previous section, trade occurs in a sequence of Walrasian markets described as follows. The minimum number of buyers that will arrive is $\phi_1 N=\min_s\{\phi_s N\}$ and these buyers buy in the first market. The demand in the first market (at the price p) is: $D_1(p)=\phi_1\sum_j n_j d_j(p)$ units. If $s>1$, there are N_s-N_1 buyers who did not buy in the first market. The minimum number of unsatisfied buyers if $s>1$, is $(\phi_2-\phi_1)N=\min_{s>1}\{(\phi_s-\phi_1)N\}$ and this is the number of buyers who will buy in the second market. The demand of this second batch of buyers is: $D_2(p)=(\phi_2-\phi_1)\sum_j n_j d_j(p)$ units. In general, if batch i arrives, its demand at the price p is: $D_i(p)=(\phi_i-\phi_{i-1})\sum_j n_j d_j(p)$ and this is the potential demand in market i. The probability that batch i arrives and market i opens is: $q_i=\sum_{s=i}^{Z}\pi_s$.

The seller is a “conditional price-taker” and behaves as if he can sell any amount at the price P_i if market i opens. The expected revenue from supplying a unit to market i is $q_i P_i$. In equilibrium expected profits are zero and prices satisfy: $q_i P_i=\lambda$. I now modify the definition of equilibrium as follows.

A UST equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i=\lambda/q_i$ and (b) $x_i=D_i(P_i)$.

<u>A “weak” planner</u>

In equilibrium a type j buyer who arrives in batch i consumes $d_j(\lambda/q_i)$ units. To evaluate this outcome I assume a planner that can choose the amount x_{ji} that will be delivered to a type j agent that arrive in batch i. I call this planner “weak” because like the sellers in the model (and unlike the “strong” planner that will be introduce shortly) he must make choices before he knows the realization of demand. The "weak" planner solves the following problem.

(5) $$\max_{x_{ji}} \sum_{i=1}^{Z} q_i(\phi_i - \phi_{i-1}) \sum_{j=1}^{J} n_j U_j(x_{ji}) - \lambda \sum_{i=1}^{Z} \sum_{j=1}^{J} (\phi_i - \phi_{i-1}) n_j x_{ji}$$

The first order conditions to this problem are:

(6) $$q_i U_j'(x_{ji}) = \lambda$$

Since in equilibrium $P_i = \lambda / q_i$, a type j agent that arrive in batch i will choose to consume x_{ji} such that $U_j'(x_{ji}) = P_i = \lambda / q_i$ and therefore the equilibrium outcome satisfies (6). We have thus shown the following claim.

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<u>Claim 2</u>: The UST equilibrium outcome is a solution to the "weak" planner's problem (5).

<u>A "strong" planner</u>

A planner that knows the state before any decision is made (before $t = 0$) will produce exactly the amount that he plans to deliver. In state s , the planner will choose to deliver x_{js} units to type j by solving the following problem.

(7) $$\max_{x_{js}} \sum_j \phi_s n_j U_j(x_{js}) - \lambda \sum_j \phi_s n_j x_{js}$$

The first order conditions for this problem are:

(8) $$U_j'(x_{js}) = \lambda$$

Clearly the UST outcome characterized by (6) is not a solution to the "strong" planner's problem.

A "semi-strong" planner

I now consider the case in which the planner knows the state after the capacity, k, is chosen but before trade occurs. Here I use x_{js} to denote the amount allocated to a type j agent in state s. Under the informational assumption for the "semi-strong" planner, the allocation $(k;x_{11},...,x_{1Z};...;x_{J1},...,x_{JZ})$ is feasible if it satisfies the following condition:

(9) $$\sum_j \phi_s n_j x_{js} = k \quad \text{for all } s$$

This is the definition of feasible allocation in Deneckere and Peck (2012, Definition 1). With this notion of feasible allocations we can write the problem of the "semi-strong" planner as follows.

(10) $$\max_k \sum_s \pi_s V_s(k)$$
$$\text{where } V_s(k) = \max_{x_{js}} \sum_j \phi_s n_j U(x_{js}) - \lambda k \quad \text{s.t. } \sum_j \phi_s n_j x_{js} = k$$

Thus, $V_s(k)$ is the maximum welfare (sum of utilities) that the planner can achieve in state s when capacity is k. The first order conditions for (10) are:

(11) $$U_j'(x_{js}) = U_1'(x_{1s}) \text{ for all } j \text{ and } s$$

(12) $$\sum_s \pi_s U_j'(x_{js}) = \lambda \text{ for all } j.$$

Unlike the weak planner, here the allocation does not depend on the order of arrival (the batch) and unlike the strong planner here only the expected marginal utility is equal to λ (and not the marginal utility in each state). Since the "semi-strong" planner has better information than the sellers in the model he can improve on the UST equilibrium outcome.

The benchmark of the "semi-strong" planner is reasonable if prices are set in advance (at $t = 0$) and sellers observe the state before actual trade occurs. In this case sellers would like to change their prices at the time of trade but cannot do so and a semi-strong planner who does not use rigid prices can improve matters. However, in the UST model, sellers observe only the amount sold at each stage (or the number of the hypothetical markets that were opened) and therefore the weak planner is the appropriate benchmark.

Can the government improve matters? Under Assumption 1, the government can improve matters if it has informational advantage over the sellers in the economy. Once we relax Assumption 1 the government can improve matters even if it has no informational advantage but can discriminate by type. To get the intuition, assume that type 1 is active only in states in which aggregate demand is low. In this case the fact that a type 1 buyer is active is a signal of low demand and therefore the weak planner will assign to a type 1 buyer that arrives in the first batch, more than $d_1(P_1)$ which is the amount he gets under competition. I elaborate in the Appendix. In general, when we relax Assumption 1, the definition of batches is endogenous and so is the probability that market i will open. But prices are still given by $P_i = \lambda / q_i$.

4. A DYNAMIC VERSION

Bental and Eden (BE, 1993) extended the UST model to the case in which the economy lasts forever and storage is possible.[6] To do welfare analysis, I study here a general equilibrium version of the model.

As in the single period case there are $J+1$ types of agents (a seller and J types of buyers). Each agent gets a large endowment of Y each period. The demand of each of the active buyer does not change over time and is given by (3). The probability of becoming active does not depend on the type. The number of active buyers is *iid*. Sellers can store goods but buyers cannot (and in equilibrium they do not have an incentive to do so). The seller uses the discount factor $0 < \beta < 1$ to evaluate future revenues. The discount may also capture storage costs and depreciation.

<u>The "weak" planner's problem:</u>

Each period the "weak" planner chooses the amount x_{ji} that will be delivered to a type j agent that arrives in batch i. Goods that were allocated to batches that did not arrive are not delivered and are carried as inventories to the next period. The planner

[6] The model is different from the standard formulation of competitive equilibrium. In the standard formulation one can always sell the good at the market-clearing price and inventories are held only when agents expect that the price will increase by a sufficient amount to cover depreciation, storage and interest cost. (See for example, Deaton and Laroque [1992]). Since prices do not always increase we should observe periods in which no inventories are held. But in the data such periods are rare. To overcome this problem many researchers have assumed that inventories yield "convenience" which is not unlike the "money in the utility function" approach to the problem of why money is held. Other models assume that inventories enter as an input into the production function (Kydland and Prescott [1982]), generate greater sales at a given price (Kahn and Bils [2000]) and avoid frequent payment of fixed delivery costs (Khan and Thomas [2007]). In Prescott "hotels" type models, the probability of making a sale is typically less than one and sellers may hold inventories simply because their attempt to sell the good failed. This resembles the "undesired inventories" in old Keynesian models but here markets that open are cleared and prices are flexible. In addition to the "failure to sell" reason for holding inventories the BE model allows for purely speculative inventories as in the Deaton-Laroque model and show that this leads to a standard "production smoothing" motive for holding inventories.

can also choose to hold Γ units of purely speculative inventories that will be stored regardless of the state. Thus, in state i, $\Gamma + \sum_{s=i+1}^{Z}(\phi_s - \phi_{s-1})\sum_{j=1}^{J} n_j x_{js}$ units will be carried to the next period as inventories. I use L to denote current production and I to denote the beginning of period inventories.

When $\Gamma = 0$, the amount of inventories in state s is:

$$(13) \qquad I^s = \sum_{i=s+1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j x_{ji}$$

The maximum amount that will be carried as inventories is $I^{\max} = I^1$. The value of inventories is a function, $V(I)$, from the beginning of period inventories $0 \le I \le I^{\max}$ to the real line (R_+) defined by the following Bellman equation:

$$(14)\; V(I) = \max_{L, x_{ji}, \Gamma \ge 0} \sum_{i=1}^{Z} q_i (\phi_i - \phi_{i-1}) \sum_{j=1}^{J} n_j U_j(x_{ji}) - \lambda L + \beta \sum_{i=1}^{Z} \pi_i V\left(\sum_{s=i+1}^{Z} (\phi_s - \phi_{s-1}) \sum_{j=1}^{J} n_j x_{js} + \Gamma \right)$$

$$\text{s.t.} \quad \Gamma + \sum_{i=1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j x_{ji} = L + I .$$

I now show the following Claim.

<u>Claim 3</u>: The solution to the planner's problem (14) is characterized by $L > 0$, $\Gamma = 0$ and the following first order condition:

$$(15) \qquad U_j'(x_{ji}) = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

Proof: Since $U_j'(0) = \infty$, the amounts supplied are strictly positive ($x_{ji} > 0$). Total supply in each period is: $k = \sum_{i=1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j x_{ji} > I^{\max}$ because the supply to the first market $\phi_1 \sum_{j=1}^{J} n_j x_{j1}$ is strictly positive. Since $I \le I^{\max}$, production $L = k - I$ is strictly positive.

Since $L > 0$ we must have $\Gamma = 0$. To see this claim, note that when $\Gamma > 0$ the seller can do better by cutting purely speculative inventories and current production by a unit and increasing production in the next period by a unit.

We can therefore write the first order conditions and the envelope condition as follows:

$$q_i U_j'(x_{ji}) + \beta \sum_{k=1}^{i-1} \pi_k V'\left(\sum_{s=k+1}^{Z} (\phi_s - \phi_{s-1}) \sum_{j=1}^{J} n_j x_{js} \right) = \lambda \tag{16}$$

$$V'(I) = \lambda \tag{17}$$

Substituting (17) in (16) leads to:

$$q_i U_j'(x_{ji}) + (1 - q_i)\beta\lambda = \lambda \tag{18}$$

The first order condition (15) follows from (18). □

Note that a strictly positive amount of production is required to keep total supply at the level k and that inventories are always in the range $[0, I^{\max}]$. Thus (14) is well defined. Furthermore, optimal production fluctuates with inventories: the larger the amount of beginning of period inventories the lower is the amount produced. Here a unit increase in the beginning of period inventories reduces production by a unit.

UST equilibrium

Prices in a typical period are given by $(P_1, ..., P_Z)$. With some abuse of notation, I describe the seller's problem by the following Bellman's equation:

(19) $$V(I) = \max_{x_i,\Gamma,L\geq 0} \sum_{i=1}^{Z} q_i P_i x_i - \lambda L + \beta \sum_{i=1}^{Z} \pi_i V\left(\Gamma + \sum_{s=i+1}^{Z} x_s \right)$$

$$\text{s.t. } \Gamma + \sum_{i=1}^{Z} x_i = L + I$$

Here x_i is the amount the seller allocates to market i and as before the range of $V(I)$ is $0 \leq I \leq I^{\max}$. A solution to (19) with $L, x_i > 0$ must satisfy $\Gamma = 0$ and the following first order and envelope conditions:

(20) $$q_i P_i + \beta(1-q_i) V'\left(\sum_{s=i+1}^{Z} x_s \right) = \lambda$$

(21) $$V'(I) = \lambda$$

Substituting (21) into (20) yields:

(22) $$q_i P_i + (1-q_i)\beta\lambda = \lambda$$

A UST equilibrium is a vector $(P_1,...,P_Z; x_1,...,x_Z)$ that satisfies (22) and the following market clearing conditions:

(23) $$\sum_j (\phi_i - \phi_{i-1}) n_j d_j(P_i) = x_i$$

Equilibrium prices (22) can be written as:

(24) $$P_i = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

Let $x_{ji} = d_j(P_i)$ denotes the amount bought by a type j buyer who arrives in batch i. Then (24) and (4) imply:

(25) $$U_j\,'(x_{ji}) = P_i = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

We can now show the following Claim.

Claim 4: The equilibrium outcome is a solution to the planner's problem (14).

This claim follows from the observation that (25) is the same as (15).

Price dispersion

Prices in (24) are a weighted average between λ and λ/q_i. A higher β reduces the mean and the dispersion measures of the price distribution because all prices get closer to the lowest price λ. When $\beta \to 1$, (24) implies that all prices are approximately equal to λ and price dispersion vanishes. Thus discounting is required to get price dispersion in equilibrium.

In general, price dispersion requires some costs for delaying trade. This cost maybe due to discounting of future profits, storage costs or depreciation.

The "strong" planner's problem

A planner who knows the state will produce exactly the amount that he plans to deliver and will not carry inventories. In state s, the planner will choose to deliver x_{js} units to type j by solving the problem (7). Clearly the UST outcome characterized by (25) is not a solution to the "strong" planner's problem. But when $\beta \to 1$, the "strong" planner cannot improve matters by much. To see this claim note that when $\beta \to 1$, the UST allocation of X is close to the "strong" planner's choice and the benefits from economizing on inventories are small because the maximum amount of inventories held is finite.

5. INCREASING MARGINAL COST AND PRODUCTIVITY SHOCKS

Under the constant returns to scale assumption there is no production-smoothing role for inventories: Inventories are held only because of a failure to make a sale. To get the production-smoothing role and to generalize the efficiency result of the previous section, I now relax the assumption of constant unit cost and allows for productivity shocks.

The utility cost of labor is now a strictly increasing and strictly convex function $v(L)$ where $v'(0)=0$. A unit of labor produces $A=\varepsilon^{-1}$ units of goods where ε is an *iid* random variable with realizations in the range $\varepsilon^{\min}\le\varepsilon\le\varepsilon^{\max}$. The realization of ε is known when production takes place (and before the realization of demand).

I start with the "weak" planner's problem and generalize the Bellman equation (14) as follows.

$$V(I,\varepsilon)=\max_{L,x_{ji},\Gamma\ge0}\sum_{i=1}^{Z}q_i(\phi_i-\phi_{i-1})\sum_{j=1}^{J}n_jU_j(x_{ji})-v(L) \qquad (26)$$

$$+\beta E_{\varepsilon'}\sum_{i=1}^{Z}\pi_iV\left(\sum_{s=i+1}^{Z}(\phi_s-\phi_{s-1})\sum_{j=1}^{J}n_jx_{js}+\Gamma,\varepsilon'\right)$$

$$\text{s.t.}\quad \sum_{i=1}^{Z}(\phi_i-\phi_{i-1})\sum_{j=1}^{J}n_jx_{ji}+\Gamma=AL+I$$

The range of the function $V(I,\varepsilon)$ is $0\le I<I^{\max}<\infty$ and the support of the distribution of ε $(\varepsilon^{\min}\le\varepsilon\le\varepsilon^{\max})$. I assume that $I^{\max}$ is large and finite. Since $I^{\max}$ is large, when $I=I^{\max}$ the marginal utilities are low and production is less than the amount allocated to the first batch and next period's inventories are less than $I^{\max}$. The assumption $v'(0)=0$ insures that optimal production is always positive and the assumption that $U_j^{'}(0)=\infty$ insures a strictly positive amount to each of the active buyers.

I use $\left(x_{ji}(I,\varepsilon),\Gamma(I,\varepsilon),L(I,\varepsilon)\right)$ to denote a solution to (26);

$$I^s(I,\varepsilon)=\sum_{i=s+1}^{Z}(\phi_i-\phi_{i-1})\sum_{j=1}^{J}n_j x_{ji}(I,\varepsilon)+\Gamma(I,\varepsilon)$$

to denote next period's inventories when the current period's state is s and

$$MC(I,\varepsilon)=\varepsilon v'\left(L(I,\varepsilon)\right)$$

to denote the marginal cost. Using this notation I now show the following Claim.

<u>Claim 5</u>: A solution $\left(x_{ji}(I,\varepsilon)>0,\Gamma(I,\varepsilon)\geq 0,L(I,\varepsilon)>0\right)$ to the planner's problem (26) must satisfy the following first order conditions:

(27) $$\beta\sum_{s=1}^{Z}\pi_s E_{\varepsilon'}MC(I^s,\varepsilon')\leq MC(I,\varepsilon) \quad \text{with equality if } \Gamma>0.$$

(28) $$q_i U_j'(x_{ji})+\beta\sum_{s=1}^{i-1}\pi_s E_{\varepsilon'}MC(I^s,\varepsilon') = MC(I,\varepsilon)$$

where $E_{\varepsilon'}$ denotes expectations with respect to next period's productivity shock.

The left hand side of (27) is the expected-discounted-next-period marginal cost. This must be less than the current marginal cost because otherwise, the planner can reduce the expected-present-value of cost by producing more today and storing the additional units. The left hand side of (28) is the expected benefit from allocating a unit to a type j agent in batch i. If the batch arrives (with probability q_i) the planner gets the marginal utility. If it does not arrive the planner gets the expected-discounted-next-period marginal cost. Condition (28) thus says that the expected benefit from producing an additional unit must equal the marginal cost.

Proof: Using $V' = \dfrac{\partial V}{\partial I}$ for the partial derivative, we can write the first order conditions and the envelope condiiton as follows.

$$(29)\quad \beta E_{\varepsilon'} \sum_{s=1}^{Z} \pi_s V'\left(\sum_{k=s+1}^{Z} (\phi_k - \phi_{k-1}) \sum_{j=1}^{J} n_j x_{jk} + \Gamma, \varepsilon' \right) \le \varepsilon v'(L) \quad \text{with equality if } \Gamma > 0$$

$$(30)\qquad q_i U_j'(x_{ji}) + \beta E_{\varepsilon'} \sum_{s=1}^{i-1} \pi_s V'\left(\sum_{k=s+1}^{Z} (\phi_k - \phi_{k-1}) \sum_{j=1}^{J} n_j x_{jk} + \Gamma, \varepsilon' \right) = \varepsilon v'(L)$$

$$(31)\qquad V'(I,\varepsilon) = \varepsilon v'(L)$$

Using the update of (31) and substituting it in (29) and (30) leads to (27) and (28). □

UST equilibrium

The seller's problem is a generalization of (19):

$$(32)\qquad V(I,\varepsilon) = \max_{x_i,\Gamma} \sum_{i=1}^{Z} q_i P_i x_i - v(L) + \beta \sum_{i=1}^{Z} \pi_i E_{\varepsilon'} V\left(\Gamma + \sum_{s=i+1}^{Z} x_s, \varepsilon' \right)$$
$$\text{s.t. } \Gamma + \sum_{i=1}^{Z} x_i = \varepsilon^{-1} L + I$$

Claim 6: A solution $\left(x_i(I,\varepsilon) > 0, \Gamma(I,\varepsilon) \ge 0, L(I,\varepsilon) > 0\right)$ to (32) must satisfy the following first order conditions:

$$(33)\qquad \beta \sum_{i=1}^{Z} \pi_i E_{\varepsilon'} MC(I^i, \varepsilon') \le MC(I,\varepsilon) \text{ with equality if } \Gamma > 0$$

$$(34)\qquad q_i P_i + \beta \sum_{s=1}^{i-1} \pi_s E_{\varepsilon'} MC(I^s, \varepsilon') = MC(I,\varepsilon)$$

Proof: The first order and the envelope conditions are:

(35) $$\beta\sum_{i=1}^{Z}\pi_i E_{\varepsilon'} V'\left(\Gamma+\sum_{s=i+1}^{Z}x_s,\varepsilon'\right)\le \varepsilon v'(L) \quad \text{with equality if } \Gamma>0$$

(36) $$q_i P_i+\beta\sum_{s=1}^{i-1}\pi_s E_{\varepsilon'} V'\left(\Gamma+\sum_{k=i+1}^{Z}x_k,\varepsilon'\right)=\varepsilon v'(L)$$

(37) $$V'(I,\varepsilon)=\varepsilon v'(L)$$

Substituting an updated version of (37) in (35) and (36) leads to (33) and (34). □

A UST equilibrium is a vector of functions $\left(P_1(I,\varepsilon),...,P_Z(I,\varepsilon);x_1(I,\varepsilon),...,x_Z(I,\varepsilon);\Gamma(I,\varepsilon),L(I,\varepsilon)\right)$ that satisfies the first order conditions (33), (34), the resource constraint $\Gamma(I,\varepsilon)+\sum_i x_i(I,\varepsilon)=\varepsilon^{-1}L(I,\varepsilon)+I$ and the market clearing conditions $\sum_j(\phi_i-\phi_{i-1})n_j d_j\left(P_i(I,\varepsilon)\right)=x_i(I,\varepsilon)$ for all (I,ε).

<u>Using the solution to the planner's problem to solve for equilibrium</u>

The first order conditions to the planner's problem (27)-(28) and the first order conditions to the seller's problem (33)-(34) are the same if prices are given by[7]:

(38) $$P_i(I,\varepsilon)=U_j'(x_{ji}) \quad \text{for all } (I,\varepsilon) \text{ and for all } j.$$

Since an interior solution to the consumer's problem (3) must satisfy the first order condition (4), if prices are given by (38), $x_{ji}(I,\varepsilon)=d_j\left(P_i(I,\varepsilon)\right)$ and the equilibrium

[7] Note that (28) implies that the marginal utility of buyers in batch i is the same for all types: $U_j'(x_{ji})=U_1'(x_{1i})$ for all j.

outcome is identical to the planner's choice. Furthermore, since the planner's problem satisfies the constraint in (26), the supplies $x_i(I,\varepsilon) = (\phi_i - \phi_{i-1})\sum_{j=1}^{J} n_j x_{ji}(I,\varepsilon)$ satisfy the market clearing conditions.

It follows that if we have a solution to the planner's problem, we can easily construct a UST equilibrium. This implies the following Claim.

Claim 7: The UST equilibrium outcome is a solution to the "weak" planner's problem (26).

6. PRODUCTION SMOOTHING AND UNDESIRED INVENTORIES

As in the "old" Keynesian description of "undesired inventories", inventories in this model may be held because of a failure to sell. But inventories may also be held to smooth marginal cost which is the motive stressed by the neo-classical approach.

To describe the production-smoothing role of inventories I define,

(39) $$\alpha(I) = E_\varepsilon P_1(I,\varepsilon)$$

The analysis in Bental and Eden (1993) can be used to show that $P_i(I,\varepsilon)$ is decreasing in I and therefore $\alpha(I)$ is a decreasing function. It can also be shown that $P_i(I,\varepsilon)$ is increasing in ε. This leads to the following claim.

Claim 8: $\Gamma(I,\varepsilon)$ is weakly decreasing in ε.

The Claim says that when productivity is low, a larger part of the stock of inventories will be offered for sale. This is the production-smoothing role of inventories.

To show this Claim I use the following arbitrage condition:

(40) $$P_Z(I,\varepsilon) \geq \beta\alpha\left(\Gamma(I,\varepsilon)\right) \text{ with equality if } \Gamma(I,\varepsilon) > 0$$

This arbitrage condition can be derived from the first order conditions under the assumption that the supply to all the Z markets is strictly positive. It must hold in equilibrium because under the assumptions made ($U'(0) = \infty$) the supply to all markets must be strictly positive. The condition must hold because the optimal program of the seller is time consistent. When the last market opens the seller can sell a unit at the price P_Z or he can store it. If he stores it he will get $\beta\alpha(\Gamma)$ because the beginning of next period's inventories are: $I^Z = \Gamma$. Condition (40) says that selling the unit in the last market must yield more revenues in terms of present value than storage.

Figure 1 solves (40) graphically under the assumption: $\varepsilon_1 > \varepsilon_2$. When productivity is high and $\varepsilon = \varepsilon_2$, the price in the last market is low and as a result purely speculative inventories are high.

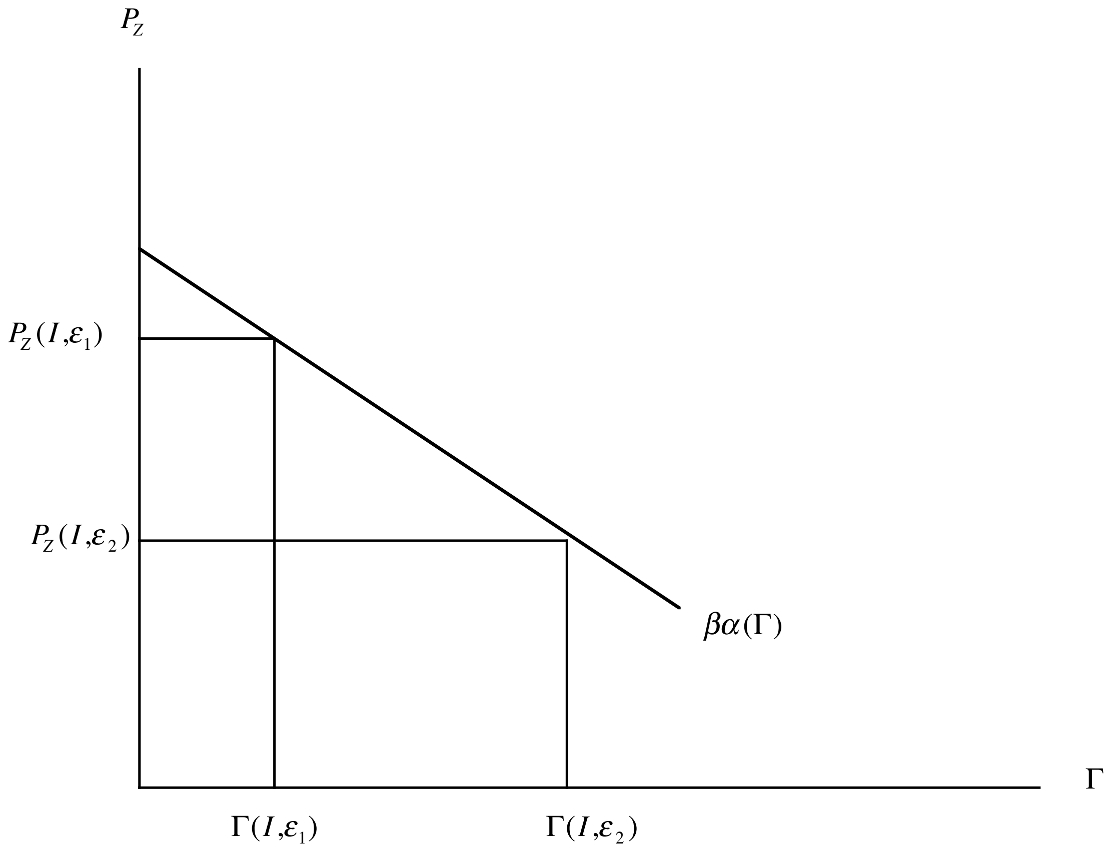

Figure 1: Purely speculative inventories are higher when productivity is higher ($\varepsilon_2 < \varepsilon_1$)

7. ONE-HOSS-SHAY DEPRECIATION

The exponential decay assumption in the previous section (embodied in the parameter β) is convenient for analytical purposes but does not capture important aspects of the goods that are the subject of empirical studies like food items with expiration dates and seasonal goods. Here I relax this assumption and allows for one-hoss-shay depreciation.[8]

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[8] The analysis here has some common elements with the analysis of seasonal goods in Deneckere and Peck (DP, 2012). But there are some important differences. DP use a game theoretic approach while I employ the Walrasian price-taking assumption in non-standard markets. They assume that during the selling period there is no depreciation and that the cost of delay to buyers is small while I allow for delay costs. There are other differences. DP assume that the "sale season" lasts for many periods and allow for new active buyers to enter the market and for information about aggregate demand to be revealed sequentially. I assume that the "sale period" lasts for only 2 periods and all active buyers are

I assume that the economy lasts for 3 periods. Production decisions are made at $t=0$. Trade occurs at $t=1$ and $t=2$. I start with the case in which the number of active buyers $\tilde{N}$ can take 2 possible realizations: $1=N_1<N_2$. As before the probability of state s ($\tilde{N}=N_s$) is π_s.

In the first trading period ($t=1$) there are 2 hypothetical UST markets. Market 1 opens with probability 1 at the price P_1. Market 2 opens with probability π_2 and if it opens trade in this market occurs at the price P_2. If market 2 does not open (with probability $\pi_1=1-\pi_2$) the seller can sell the unsold goods in a Walrasian market that will open in the second trading period $(t=2)$ at the price $p<P_1$.[9]

The utility of an active buyer is given by:

(41) $$U(C+\delta c)+y$$

where C is the amount bought in the first period ($t=1$), c is the amount bought in the second period and $0<\delta\le 1$ is a parameter that reflects the cost of delay. The function U is strictly monotone, strictly concave and differentiable with $U'(0)=\infty$. The delay cost may result from the shortening of the length of time that the buyer uses the good: A consumer who buys a short sleeve shirt at the beginning of the summer gets more use out of it than a consumer who buys it towards the end of the summer. In the case of food items with expiration date it may reflect the tightening of the constraint on the length of period in which the item can be consumed.

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present in the first period. As a result the information about aggregate demand is fully revealed in the first period. DP assume that consumers have unit demand. I assume that they maximize a quazi linear utility function and have a downward sloping demand function. As a result in my model buyers typically buy in both periods while in the DP model they buy only once.

[9] There is no real distinction between the UST and the Walrasian markets. In both cases a market opens only if there is both supply and demand. The UST second market may not open if there is no demand. The second period Walrasian market may not open if there is no supply. And each market that open is cleared. But in the first period there is a sequence of Walrasian markets and price dispersion. In the second period there is at most one market and one price. So I hope this language will help in keeping the two trading periods separated without creating confusion.

A buyer who buys at $t=1$ in the first market will take into account the possibility that a Walrasian market will open at $t=2$ at a cheaper price. He solves the following problem.

$$\max_{C_1} \pi_1 \left(\max_c U(C_1+\delta c) - P_1 C_1 - pc \right) + \pi_2 \left(U(C_1) - P_1 C_1 \right) \tag{42}$$

Here C_1 is the amount he buys at $t=1$ in the first UST market (at the price P_1) and c is the amount he buys at $t=2$ in the Walrasian market (at the price p) if the state of demand is low (state 1).

A buyer who buys at $t=1$ in market 2 knows that the state of demand is high and the Walrasian market will not open in the next period because inventories will not be carried to the next period. He therefore solves the following problem.

$$\max_{C_2} U(C_2) - P_2 C_2 \tag{43}$$

The first order conditions that an interior solution to (42) must satisfy are:

$$\delta U'(C_1+\delta c) = p \tag{44}$$

$$\pi_1 U'(C_1+\delta c) + \pi_2 U'(C_1) = P_1 \tag{45}$$

The first order condition for an interior solution to (43) is:

$$U'(C_2) = P_2 \tag{46}$$

The seller chooses the amount allocated to each of the hypothetical markets at $t = 1$ (x_i) by solving the following problem[10].

(47) $$\max_{x_i} \pi_1(P_1x_1 + px_2) + \pi_2(P_1x_1 + P_2x_2) - \lambda(x_1 + x_2)$$

Note that the revenue per unit allocated to the first market is P_1. The revenue per unit allocated to the second market is random: It is equal to p in the low demand state and P_2 in the high demand state.

The first order conditions for the seller's problem are:

(48) $$P_1 = \lambda$$

(49) $$\pi_2 P_2 + \pi_1 p = \lambda$$

Condition (49) is similar to (22). The revenues are the quoted price in case the market opens and the value of inventories in case it does not open. The left hand side of (49) is therefore the expected revenues that must equal the cost.

Equilibrium is a vector $(C_1, C_2, c, x_1, x_2, P_1, P_2, p)$ that satisfies the buyers' first order conditions (44)-(46), the seller first order conditions (48),(49), the inequalities $p \le P_1 < P_2$, and the following market clearing conditions:

(50) $$C_1 = x_1$$

(51) $$(N_2 - 1)C_2 = x_2$$

(52) $$c = x_2$$

[10] A more general formulation may allow for pure speculations. Let x_3 denote the amount that the seller does not plan to sell in the first period regardless of the number of markets open. Then we can write the seller's problem as:

$$\max_{x_i} \pi_1(P_1x_1 + px_2) + \pi_2\left(P_1x_1 + P_2x_2\right) + px_3 - \lambda(x_1 + x_2 + x_3)$$

It will be shown that in equilibrium $p < P_1$, and therefore the seller chooses $x_3 = 0$.

Note that also here markets that open are cleared.

Solving for the equilibrium vector:

Substituting the market clearing conditions in the buyers' first order conditions (44)-(46) and using $\Delta = N_2 - 1$, leads to:

(53) $$\delta U'(x_1 + \delta x_2) = p$$

(54) $$\pi_1 U'(x_1 + \delta x_2) + \pi_2 U'(x_1) = P_1$$

(55) $$U'(\Delta^{-1} x_2) = P_2$$

Substituting (53) and (55) in (49) leads to:

(56) $$\pi_2 U'(\Delta^{-1} x_2) + \pi_1 \delta U'(x_1 + \delta x_2) = \lambda$$

Substituting (54) in (48) leads to:

(57) $$\pi_1 U'(x_1 + \delta x_2) + \pi_2 U'(x_1) = \lambda$$

We now have 2 equations (56)-(57) with 2 unknowns (x_1, x_2).

Claim 9: There exists a unique equilibrium.

Proof: I start by showing that there exists a unique solution $(\hat{x}_1, \hat{x}_2)$ to (41) and (57). For this purpose note that the slope of the locus of points that solve (56) is:

(58) $$\frac{dx_2}{dx_1} = \frac{\pi_1 \delta U''(x_1 + \delta x_2)}{\pi_2 \Delta^{-1} U''(\Delta^{-1} x_2) + \pi_1 \delta^2 U''(x_1 + \delta x_2)} > -\frac{1}{\delta}$$

The slope of the locus of points that solve (57) is:

(59) $$\frac{dx_2}{dx_1} = \frac{\pi_2 U''(x_1) + \pi_1 U''(x_1 + \delta x_2)}{\pi_1 \delta U''(x_1 + \delta x_2)} < -\frac{1}{\delta}$$

In Figure 2 the locus labeled AA is the solutions to (56) and the locus labeled BB is the solutions to (57). When $x_1 = 0$, the amount x_2 that solves (56) is finite and therefore the locus AA intersects the vertical axis. When $x_2 = 0$, the amount x_1 that solves (57) is finite and therefore BB intersects the horizontal axis. But since $U'(0) = \infty$, the locus BB does not intersect the vertical axis. Therefore there exists a unique solution to (56) and (57) illustrated by Figure 2.

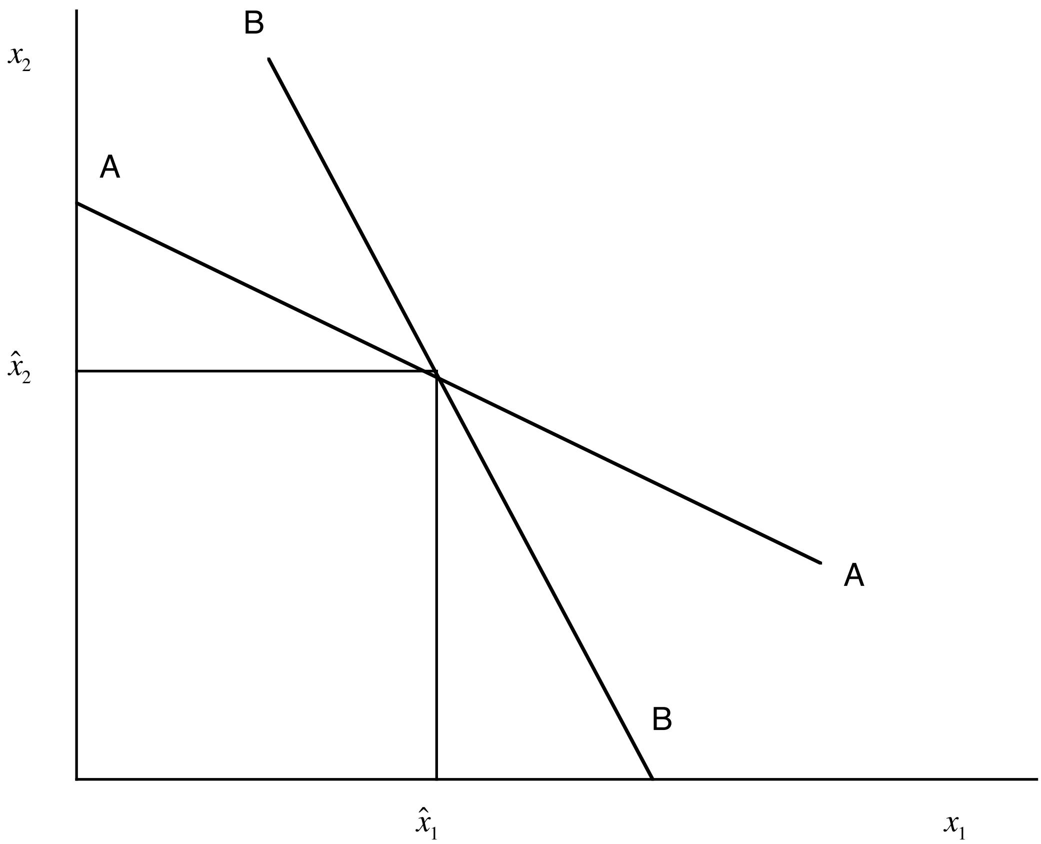

Figure 2: The solution to (56) and (57)

We still need to show that $p \le P_1 < P_2$. To show this note that since $U'(x_1) > U'(x_1 + \delta x_2)$, (57) implies: $U'(x_1) > \lambda$ and $U'(x_1 + \delta x_2) < \lambda$. It follows that $\delta U'(x_1 + \delta x_2) = p < \lambda$. Since $p < \lambda$, (49) implies $P_2 > \lambda = P_1$. □

The case of no cost of delay:

The special case in which $\delta = 1$ provides useful intuition. In this case, (56) and (57) imply:

(60) $$U'(x_1) = U'(\Delta^{-1} x_2)$$

(61) $$\pi_1 U'(x_1 + x_2) + \pi_2 U'(x_1) = \lambda$$

Since (60) implies $x_2 = \Delta x_1$ we can write (61) as:

(62) $$\pi_1 U'\big(x_1(1+\Delta)\big) + \pi_2 U'(x_1) = \lambda$$

Since $U'(0) = \infty$, there exists a unique solution $\hat{x}_1$ to (62). Since $U'\big(x_1(1+\Delta)\big) < U'(x_1)$, (47) implies $U'(\hat{x}_1) > \lambda$. This, (50) and (55) imply that $P_2 = U'(\hat{x}_1) = U'(\Delta^{-1}\hat{x}_2) > \lambda$.

The equilibrium with $\delta = 1$ can be described as follows. The first batch of buyers buys at the price $P_1 = \lambda$ a quantity that is equal to their demand at the higher price P_2. They buy less than their demand at the price P_1 because there is a chance that they will be able to buy more next period at the price $p < \lambda$. In state 2 the second batch arrives and buy the quantity allocated to the second market (x_2) at $t = 1$. In state 1, this quantity is bought by the first batch at $t = 2$.

Comparative statics:

To do comparative statics, I assume that the absolute risk aversion measure is not too high and satisfies the following condition.

(63) $$-\frac{U''(x_1+\delta x_2)}{U'(x_1+\delta x_2)} < \delta^{-2}$$

Claim 10: Under (63) an increase in δ leads to: (a) an increase in x_2 and a decrease in x_1, (b) a decrease in P_2 and (c) an increase in p.

To show (a), note that under (63), an increase in δ will shift the AA curve in Figure 2 up and to the right and the BB curve to the left and down. Part (b) follows from (55) and the increase in x_2. Part (c) follows from (34) and the fact that P_2 went down.

Note that since a reduction in δ increases P_2, an increase in the cost of delay increases price dispersion at $t=1$.

The Weak planner's problem:

The weak planner has to choose production and the allocation to the first batch before he knows whether the second batch will arrive. He therefore solves the following problem.

(64) $$\max_{x_1,x_2} \pi_1 U(x_1+\delta x_2)+\pi_2\left(U(x_1)+\Delta U(\Delta^{-1}x_2)\right)-\lambda(x_1+x_2)$$

The first order conditions for this problem are (56) and (57). Thus,

Claim 11: The equilibrium outcome solves the weak planner's problem (64).

The Semi-strong planner's problem:

The semi-strong planner has to choose capacity (x_1+x_2) before he knows the state but he chooses the amount that he gives to the first batch of buyers at $t=1$ after he knows the state. Since the semi-strong planner will give the entire capacity to the first batch in state 1, we can write his problem as follows.

(65) $$\max_{x_1,x_2} \pi_1 U(x_1+x_2)+\pi_2\left(U(x_1)+\Delta U(\Delta^{-1}x_2)\right)-\lambda(x_1+x_2)$$

This problem is the same as the "weak" planner's problem (64) only if $\delta=1$. Since the semi-strong planner has an informational advantage over the weak planner and the weak planner mimics the equilibrium outcome, this observation leads to the following Claim.

<u>Claim 12</u>: When $\delta<1$, the semi-strong planner can improve on the equilibrium outcome but when $\delta=1$ he cannot.

Claim 12 reiterates the importance of the cost of delay. The intuition is in the value of the informational advantage that the semi-strong planner has over the weak planner (and the sellers in the model). When there is no cost of delay the value of the information about the state is zero because the weak planner can distribute a limited amount at $t=1$ and once he learns about the state, at $t=2$ he can deliver the rest making sure that each buyer gets an equal amount (or more generally an amount that will equate the marginal utility across active buyers) regardless of the order of arrival.

Note that unlike the inventories model in the previous section, here there is price dispersion (at $t=1$) even when $\delta=1$ and there is no cost of delay.

Note also that at $t=1$ buyers in the first batch buy less than their demand at the first market price because they know that there is the possibility of buying at a bargain prices next period. This speculative behavior is similar to the endogenous rationing in DP.[11]

In Appendix A I extend the model to the case of many possible demand realizations and many periods.

8. CONCLUDING REMARKS

This paper studies efficiency, storage, the cost of trade delays and price dispersion in flexible price (UST) versions of the Prescott (1975) model. I use three different benchmarks or planner's problems to judge the equilibrium outcome. The "weak" planner has the same information as the sellers in the model. The "strong" planner knows the state before making any decisions and the "semi-strong" planner knows the state only after capacity decisions are made but before the arrival of buyers. The UST outcome is a solution to the "weak" planner's problem when the probability of becoming active is the same for all buyers. In general, the "semi-strong" and the "strong" planners can improve matters. An exception is the case in which there are no costs for delaying trade. In this case even the "strong" planner cannot improve matters, by much.

The formulation of equilibrium is the same whether we assume rigid or flexible prices. But the formulation of the relevant planner's problem or the definition of feasible allocation is different. The "semi-strong" planner cannot distribute more

[11] To make the connection between the two models let us think of the utility function (26) as describing the preferences of a household that consists of many infinitesimal buyers. The head of the household assigns a reservation price to each member and instructs him to maximize the expected surplus from buying at most one (infinitesimal) unit. The highest reservation price $U'(0)$ is assigned to the member indexed 0 and in general the reservation price $U'(x)$ is assigned to the member indexed x. With this in mind we can get endogenous rationing in the sense described by DP. The members with indices less than $\hat{x}_1$ buy in the first market while those with higher indices are "endogenously rationed".

than the output produced. The “weak” planner faces an additional constraint that he must choose the allocation to each batch of buyers before he knows whether more batches will arrive or not. The “semi-strong” planner is relevant for the rigid price versions in which at the time of trade sellers know the realization of demand, while the “weak” planner is relevant for the flexible price versions in which the realization of demand is fully revealed only at the end of the trading process.

It is not surprising that a “semi-strong” planner can improve matters in rigid price versions of the model. In these models sellers would like to change prices at the time of trade but cannot do so. Therefore, a “semi-strong” planner who has the same information as the sellers but does not use rigid prices can improve matters. It is also not surprising that in the UST versions a planner that has the same information as the sellers cannot improve matters. What maybe surprising are the exceptions to the rules.

An exception to “weak” efficiency occurs when the probability of becoming active is not the same across types. In this case that is discussed in Appendix B, the type composition of buyers is a signal about the state. This does not pose a problem for the rigid price formulation. But it is a problem for the flexible price formulation. To keep the flexible price assumption and the same equilibrium concept I assume that sellers observe only the aggregate amount sold and not the type composition of buyers at each stage of the trading process.

A planner who does not observe the type composition of buyers may be able to improve matters if he can price discriminate in a way that sellers cannot. To see this possibility assume that buyers place orders on the internet and receive the goods by mail. Sellers may not be able to discriminate by zip codes. But a planner (or a policymaker) may be able to do so by varying sales tax (or tariffs in an international setting). For example, if buyers from a specific location want to consume only in high demand states, they should pay a high sale tax because in high demand states there is a higher chance of hitting the capacity constraint.

The efficiency results are extended to the multi-periods case that allows for storage. In the model with exponential decay (ED) inventories are held for two reasons: (a) a failure to make a sale and (b) to smooth the marginal cost. The first resembles the "old" Keynesian description of "undesired" inventories. The second is the neo-classical approach. We get both under the same roof.

The importance of the cost of delay is illustrated by the relatively simple case of constant unit cost. In this case, rigid prices may not reduce welfare if there is no significant discounting. When discounting is not important, sellers will set a single price equal to the marginal cost and will not "regret" this choice even if information about the state becomes available before the beginning of trade. Similarly, when discounting is not important, the optimal policy of a "weak" planner is to keep inventories at some target level and distribute to each active buyer a quantity that does not depend on the state (and equates the marginal utility with the marginal cost of production). Information about the state can be used to eliminate inventories but since discounting is not important the value of doing it is small.

The efficiency results are robust to changes in the assumption about depreciation. I consider the case of one-hoss-shay (OHS) depreciation and found that the resulting allocation is "weakly" efficient when there are costs of delaying trade. It is "strongly" efficient when there are no costs for delaying trade. In this case, a planner will not value information about the state because he can distribute relatively small amounts in the first period, learn about the state and then distribute the rest making sure that the marginal utility of all buyers is the same regardless of the order of arrival. Buyers in the model do what the planner wants them to do. They engage in speculative behavior and buy a relatively small amount in the first period. Then once the state is revealed in the second period they buy the amount that was not sold in the first period at the market-clearing price. It follows that in the absence of delay costs even a "strong" planner cannot improve matters.

On the whole, the results in this paper suggest that observations that have been interpreted as indications of market-failures may actually indicate an efficient way of coping with aggregate demand uncertainty. Among these observations are price discrimination, price dispersion and the accumulation of “undesired” inventories.

APPENDIX A: ONE-HOSS-SHAY DEPRECIATION WITH MANY STATES AND MANY PERIODS

I now extend the analysis of the one-hoss-shay depreciation to the case in which there are many states and many periods. I start with the extension to the many states of demand in a two periods framework.

I assume that the number of active buyers $\tilde{N}$ may take $Z \geq 2$ possible realizations. To simplify notation I assume $N_s - N_{s-1} = 1$ so there is one buyer per batch. As before, the probability that exactly s markets will open in the first period is π_s and the probability that market i will open is $q_i = \sum_{s \geq i} \pi_s$.

The price in the first period hypothetical UST market i is P_i. A single Walrasian market will open in the second period at the price p_s if the number of markets open in the first period is $s < Z$. A buyer who arrives in the first period market i will buy C_i units and will make plans to buy in the next period c_s^i units if $i \leq s < Z$. He will choose these quantities by solving the following problem.

(A1) $$\max_{C_i \geq 0} - P_i C_i + (1/q_i)\pi_Z U(C_i) + (1/q_i)\sum_{s=i}^{Z-1} \pi_s \left(\max_{c_s^i \geq 0} U(C_i + \delta c_s^i) - p_s c_s^i \right)$$

For notational purposes I use $c_Z^i = 0$ and write the first order conditions for (A1) as:

(A2) $$\delta U'(C_i + \delta c_s^i) = p_s \quad \text{for } i \leq s < Z$$

(A3) $$\sum_{s=i}^{Z} \pi_s U'(C_i + \delta c_s^i) = q_i P_i$$

Condition (A2) says that the buyer will choose c_s^i in the second period to equate the marginal utility with the second period price. To interpret (A3) note that $(1/q_i)\pi_s$ is the probability that state s will occur given that $s \geq i$. Since the buyer in market i uses these conditional probabilities, (A3) says that the expected marginal utility for a buyer in market i must equal the price in market i (after dividing both sides of [A3] by q_i).

Sellers will sell in market i, if the price in market i is greater than the value of inventories:

(A4) $$P_i \geq (1/q_i)\sum_{s=i+1}^{Z-1}\pi_s p_s \quad \text{for all } i$$

Under (A4) the seller chooses the supply to market i by solving the following problem.

(A5) $$\max_{x_i} -\lambda x_i + q_i P_i x_i + x_i \sum_{s=1}^{i-1}\pi_s p_s$$

To interpret (A5) note, that the seller pays the cost λ regardless of whether he sells the good or not. With probability q_i he sells the good and gets P_i per unit. If $s < i$, he does not sell the good in the first period but will sell it next period at the price p_s. The first order condition for an interior solution to the seller's problem (A5) is:

(A6) $$q_i P_i + \sum_{s=1}^{i-1}\pi_s p_s = \lambda$$

This condition says that the expected revenue equal the cost. The expected revenue calculations take into account the value of inventories in case market i does not open. In this respect it is similar to (22). But in (22) the value of inventories is the value from reducing production next period. Here there is no production in the second period and it will be shown that the value is less than the cost of production.

To define equilibrium I use $c^i = (c_i^i,...,c_{Z-1}^i)$ to denote the plan (second period's purchases) of a buyer who arrives in market i.

Equilibrium is a vector $(C_1,...,C_Z;c^1,...,c^{Z-1};x_1,...,x_Z;P_1,...,P_Z;p_1,...,p_{Z-1})$ that satisfies (A4), $P_1 < P_2 < ... < P_Z$, the first order conditions (A2), (A3), (A6) and the following market clearing conditions:

(A7) $$C_i = x_i$$

(A8) $$\sum_{j=1}^{s} c_s^j = \sum_{j=s+1}^{Z} x_j$$

Condition (A7) says that the first period market i must clear if it opens. Condition (A8) says that the second period market must clear if it opens (that is if $s < Z$).

The Weak planner's problem

I use x_i to denote the amount that the weak planner distributes to batch i in the first period (if it arrives and carry as inventories if batch i does not arrive) and c_s^i to denote the amount that he will distribute in the second period in state s to buyers that arrive in batch $i \le s$. The weak planner chooses these quantities by solving the following problem.

(A9) $$\max_{x_j,c_s^j} -\lambda\sum_{j=1}^{Z} x_j + \sum_{s=1}^{Z}\pi_s\sum_{j=1}^{s}U(x_j + \delta c_s^j) \quad \text{s.t. (A8).}$$

The first term in (A9) is the cost of production. The second term is the expected sum of the utilities from the consumption of X. The first order conditions for an interior solution to this problem are:

(A10) $$U'(x_j + \delta c_s^j) = U'(x_1 + \delta c_s^1) \quad \text{for all } j \leq s$$

(A11) $$\sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i) + \sum_{s=1}^{i-1} \pi_s \delta U'(x_1 + \delta c_s^1) = \lambda$$

Condition (A10) says that the marginal utility after distributing c_s^j in the second period, must be the same across all active buyers. Condition (A11) says that the expected marginal utility from consuming x_i must equal the cost of production. The first term in the left hand side is the weighted sum of the marginal utilities when batch i arrives. The second term on the left hand side is the weighted sum of the marginal utilities when batch i does not arrive. In this case the marginal utility is $\delta U'(x_1 + \delta c_s^1)$ because the good will be distributed in the second period and will "depreciate" by that time. The first order condition (A11) can also be described as a no arbitrage condition. If the left hand side of (A11) is greater than the right hand side, a planner could do better by increasing production by a unit and allocating the unit to batch i if it arrives and to batch 1 if batch i does not arrive.

We can now show the following Claim.

Claim A: (a) There exists a unique equilibrium, (b) the equilibrium outcome is a solution to the "weak" planner's problem, (c) prices in the second period's Walrasian market are increasing with the state ($p_1 < ... < p_{Z-1}$), (d) the first UST market price is λ (and therefore $\lambda = P_1 < ... < P_Z$) and (e) $p_i < P_i$.

The intuition for (c) is straightforward: In higher states, more stuff is sold and less inventories are carried to the second period. The intuition for (d) is in the arbitrage condition: Market 1 opens with certainty and therefore if $P_1 > \lambda$ the supply will be infinite which is not consistent with market clearing (and if $P_1 < \lambda$, the supply is zero which is also not consistent with market clearing). The intuition for (e) is that when $P_i \geq p_i$ a seller who observes that market i opens will refuse to sell because he can sell it to batch $i+1$ at a higher price if it arrives and sell it in the second period at no loss if batch $i+1$ does not arrive. Since $p_i < P_i$, the price in the second period must be less than the highest transaction price in the first period.

<u>Proof</u>: The first order conditions (A10) and (A11) must hold in equilibrium. To show this claim, note that substituting (A2) and (A3) in (A4) leads to (A11). And (A2) insures that (A10) holds.

I now compute the equilibrium vector from the solution to the planner's problem. Using the planner's allocation to the first batch $(x_1; c_1^1, ..., c_{Z-1}^1)$, we can compute the second period prices:

(A12) $$p_s = \delta U'(x_1 + \delta c_s^1)$$

When more batches arrive there is less to distribute in the second period and therefore the marginal utility $\delta U'(x_1 + \delta c_s^1)$ is increasing in s and therefore: $p_1 < ... < p_{Z-1}$.

We can use the planner's allocation to batch i, $(x_i; c_1^i, ..., c_{Z-1}^i)$, to compute the price in market i:

(A13) $$P_i = (1/q_i) \sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i)$$

To show that $P_i < P_{i+1}$ note that the solution to the planner's problem must satisfy the following condition.

(A14) $$(1/q_i) \sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i) = (1/q_i) \sum_{s=i+1}^{Z} \pi_s U'(x_{i+1} + \delta c_s^{i+1}) + (1/q_i) \pi_i \delta U'(x_i + \delta c_i^i)$$

To interpret (A14) consider the point of view of a planner who observes that batch i arrives but does not know yet if batch $i+1$ will arrive. The left hand side of (A14) is the expected loss from reducing the amount given to batch i in the first period by a unit. The right hand side is the expected gain from supplying a unit to batch $i+1$ if it arrives and supplying the unit to batch i if batch $i+1$ does not arrive. Condition (A14) thus says that at the optimum the expected loss from transferring a unit from x_i to x_{i+1} must equal the expected gain so that a small deviation from the optimal plan does not reduce welfare. From (A14) we get:

(A15) $$(1/q_i)\sum_{s=i+1}^{Z}\pi_s U'(x_i+\delta c_s^i) \le (1/q_i)\sum_{s=i+1}^{Z}\pi_s U'(x_{i+1}+\delta c_s^{i+1})$$

Since $q_{i+1} < q_i$, (A15) implies:

(A16) $$(1/q_i)\sum_{s=i}^{Z}\pi_s U'(x_i+\delta c_s^i) < (1/q_{i+1})\sum_{s=i+1}^{Z}\pi_s U'(x_{i+1}+\delta c_s^{i+1})$$

The inequality $P_i < P_{i+1}$ follows from substituting (A13) in (A16).

Note that (A11) and (A13) imply:

(A17) $$P_1 = \sum_{s=1}^{Z}\pi_s U'(x_1+\delta c_s^1) = \lambda$$

To show that (A4) is satisfied note that (A12) and (A13) imply:

(A18) $$P_i = (1/q_i)\sum_{s=i}^{Z}\pi_s U'(x_i+\delta c_s^i) \ge (1/q_i)\sum_{s=i+1}^{Z-1}\pi_s U'(x_i+\delta c_s^i) = (1/q_i)\sum_{s=i+1}^{Z-1}\pi_s p_s$$

To show (e) note that a seller who observe that market i opens must be indifferent between selling a unit in market i to transferring it to x_{i+1}. This leads to the following arbitrage condition.

(A19) $$P_i = p_i(1/q_i)\pi_i + P_{i+1}(1/q_i)\sum_{s>i}\pi_s$$

Part (e) follows from the observation that P_i is a weighted average of p_i and P_{i+1} and $P_{i+1} > P_i$. This completes the proof.

A unified multi-periods framework

Many goods have both a seasonal and all year round aspects. For example, short sleeve shirt is typically used in the summer that it was bought but can also be stored and used in the next summer. We may capture both aspects by combining the two models: The exponential discounting (ED) model and the one-hoss-shay depreciation (OHS) model.

For this purpose, I assume an economy that lasts for infinitely many periods, and each period is divided into two sub-periods. In the first sub period demand is not known and trade is done in the UST hypothetical markets. The trade in the first sub-period reveals the state and in the second sub-period there is a single Walrasian market with a single price. As in the previous models prices do not change over time and I therefore drop the time index.

As in the OHS model the buyer's problem is described by (A1).

The value of a unit carried as inventories to the next period is $\beta\lambda$ (because the seller can cut next period's production by a unit and save the unit's cost). Therefore, the seller will supply to the second sub-period market only if:

(A20) $$p_s \geq \beta\lambda$$

Assuming that (A20) holds, the seller will choose the amount supplied to the first sub-period UST market i (x_i) by solving the problem (A5). If market i opens he will sell at the price P_i. If market i does not open in the first sub-period, he will sell in the second sub-period $x_i - I_s^i$ units at the price p_s and carry I_s^i units as inventories to the next period. In the second sub-period the seller chooses the amount of inventories by solving the following problem.

(A21) $$\max_{0 \le I_s^i \le x_i} p_s(x_i - I_s^i) + \beta\lambda I_s^i$$

The first order conditions to this problem requires:

(A22) $$I_s^i = 0 \text{ if } p_s > \beta\lambda \text{ and } 0 \le I_s^i \le x_i \text{ if } p_s = \beta\lambda$$

The amount of inventories carried to the next period in state s is $I_s = \sum_{i>s} I_s^i$. Given (A22) the aggregate amount of inventories must satisfy the following condition.

(A23) $$I_s = 0 \text{ if } p_s > \beta\lambda \text{ and } 0 \le I_s^i \le \sum_{i=s+1}^{Z} x_i \text{ if } p_s = \beta\lambda$$

To allow for inventories I modify the equilibrium definition in the previous section as follows.

Equilibrium is a vector $(P_1,...,P_Z; p_1,...,p_{Z-1}; x_1,...,x_Z; C_1,...,C_Z; I_1,...,I_Z)$ that satisfies (a) the incentive to supply conditions (A4), (A20) and $P_1 < P_2 ... < P_Z$; (b) the first order conditions (A2), (A3), (A6), (A23); and (c) the market clearing conditions (A7) and

(A8') $$I_s + \sum_{i=1}^{s} c_s^i = \sum_{i=s+1}^{Z} x_i$$

Note that the exponential discounting (ED) model is a special case that assumes $\delta = 0$, while the OHS is a special case that assumes $\beta = 0$.

The weak planner's problem for the unified model

The planner's problem for the unified model environment is:

(A24) $$\max_{x_j, c_s^j, I_s \geq 0} -\lambda \sum_{j=1}^{Z} x_j + \sum_{s=1}^{Z} \pi_s \left(\beta\lambda I_s + \sum_{j=1}^{s} U(x_j + \delta c_s^j) \right) \quad \text{s.t. (A8')}$$

The first order conditions for this problem are (A11) and

(A25) $$\delta U'(x_j + \delta c_s^j) = \delta U'(x_1 + \delta c_s^1) \geq \beta\lambda \quad \text{with equality if } I_s > 0$$

We can now modify Claim A as follows.

Claim A': (a) There exists a unique equilibrium, (b) the equilibrium outcome is a solution to the "weak" planner's problem, (c) prices in the second period's Walrasian market are increasing with the state $(\beta\lambda \leq p_1 \leq p_2 \leq ... \leq p_{Z-1})$, (d) the first UST market price is λ and (e) $p_i < P_i$.

The proof is similar to the proof of Claim A.

APPENDIX B: RELAXING ASSUMPTION 1

I relax Assumption 1 for the single period case. I use $N_s = \sum_j \phi_{js} n_j$ for the number of active buyers and start with the following special case.

Assumption 2: $U_j(x) = U(x)$, $d_j(p) = d(p)$ for all j and $N_1 < N_2 < ... < N_S$.

Here the type composition of the buyers who arrive in each batch (ϑ_{js}) depends on the state. Because all types have the same demand function, the value of ϑ_{js} is not relevant for computing the demand of each batch and for defining equilibrium. But as we shall see it is relevant for the social planner.

The algorithm for computing the number of buyers in each batch is similar to what we had in the previous case. The minimum number of (active) buyers is: $\Delta_1 = N_1$. The first batch of Δ_1 buyers arrives with certainty. After buyers in this first batch complete trade and go away there are two possibilities. If $s = 1$ trade ends. If $s > 1$, there are $N_s - N_1$ unsatisfied buyers. The minimum number of unsatisfied buyers if $s > 1$ is: $\Delta_2 = \min_s\{N_s - N_1\} = N_2 - N_1$ and this is the number of buyers in batch 2. The probability that $s > 1$ is $q_2 = 1 - \pi_1$ and this is the probability that batch 2 will arrive. Proceeding in this way we define q_s and Δ_s for all $s = 1,...,Z$. As before, it is convenient to think of a sequence of Walrasian markets, where batch i buys in market i and the seller supplies x_i units to market i.

A UST equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \lambda/q_i$ and (b) $x_i = (N_i - N_{i-1})d(P_i) = \Delta_i d(P_i)$.

<u>The "weak" planner's problem</u>:

I assume that the "weak" planner can choose the amount x_{ji} that he will give to a type j buyer who arrives in batch i but does not observe the fraction of type j buyers that arrive in each batch (ϑ_{js}) and like the sellers in the model, must therefore make allocation decisions before he observes the state. The planner's problem is:

$$\text{(B1)} \qquad \max_{x_{ji}} \sum_{j=1}^{J}\sum_{s=1}^{Z}\pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U(x_{ji}) - \lambda \max_s\left(\sum_{j=1}^{J}\sum_{i=1}^{Z}\vartheta_{js}(N_i - N_{i-1})x_{ji}\right)$$

To understand the first term in the objective function, note that $\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U(x_{ji})$ is the total utility that the planner gets from type j buyers in state s. The second term is the production cost of implementing the plan. To simplify, I assume that the maximum amount distributed occurs in state Z:

$Z = \arg\max_s \left(\sum_{j=1}^{J}\sum_{i=1}^{Z}\vartheta_{js}(N_i - N_{i-1})x_{ji} \right)$.

Note that x_{ji} matters only when $s \geq i$ and market i opens. We can therefore find the first order condition to the problem in (A1) by taking the derivative of $(N_i - N_{i-1})U(x_{ji})\sum_{s=i}^{Z}\pi_s\vartheta_{js} - \lambda\vartheta_{jZ}(N_i - N_{i-1})x_{ji}$. The first order condition is:

(B2) $$U'(x_{ji}) = \frac{\lambda\vartheta_{jZ}}{\sum_{s=i}^{Z}\pi_s\vartheta_{js}}$$

This is different from (6), implying that the UST outcome is not a solution to the "weak" planner's problem.

Note that under assumption 1, $\vartheta_{js} = \vartheta_j$ for all j and (B2) is the same as (5). The difference between (B2) and (5) arises when $\vartheta_{jZ} \neq \vartheta_{js}$. In this case, the planner will give type j more relative to the UST outcome, when $\vartheta_{jZ} < \vartheta_{js}$ for all s. In the extreme case when $\vartheta_{jZ} = 0$ he will satiate type j agents because this type arrives only in state in which there is excess capacity.

The general case

I now relax assumption 2. As before buyers arrive in batches but here the size of each batch is endogenous and depends on the prices: $P_1 \leq P_2 \leq ... \leq P_Z$. Roughly speaking, the size of the first batch is the minimum demand at the price P_1. Market 2 opens if there are some buyers who wanted to buy in the first market but could not. In general, market i opens if there is residual demand after transactions in market $i-1$ are complete. The size of batch i is the minimum residual demand.

The definition of equilibrium is essentially a choice of indices: $(y_1,...,y_Z)$. Demand in state s at the price P_1 is: $\sum_j \phi_{js} n_j d_j(P_1)$. I choose indices such that state y_1 is the state of minimum demand at the price P_1: $y_1 = \arg\min_s \left\{ \sum_j \phi_{js} n_j d_j(P_1) \right\}$. State 2 is the minimum demand at the price P_2 out of the states $s \neq y_1$: $y_2 = \arg\min_s \sum_j \phi_{js} n_j d_j(P_i)$ s.t. $s \neq y_1$. And in general: $y_i = \arg\min_s \sum_j \phi_{js} n_j d_j(P_i)$ s.t. $s \neq y_k$ for all $k < i$.

With the above choice of indices, we may describe equilibrium in the following way. The seller puts a price tag of P_i on x_i units and then remains passive. He knows that the lowest priced x_1 units will be sold first with certainty. Then if there is additional demand the x_2 units with the price tag P_2 will be sold and so on. I assume that the seller does not use the type composition of batch i to update the probabilities of the states. We may therefore think of the seller as having many outlets and since trade does not take real time he cannot get aggregate statistics on the type composition during trade.

Since the definition of equilibrium describes this choice of indices it is simpler to write some subscripts in parenthesis. I use $\pi(s)$ instead of π_s for the probability that state s occurs and $\phi_j(s)$ instead of ϕ_{js} for the fraction of type j buyers that are active in state s.

A UST equilibrium is a vector of distinct Z integers $(y_1,...,y_Z)$ and a vector of real numbers $(P_1,...,P_Z;x_1,...,x_Z;\Pi_1,...,\Pi_Z;q_1,...,q_Z;\Phi_{11},...,\Phi_{J1};\Phi_{12},...,\Phi_{J2};...;\Phi_{1Z},...,\Phi_{JZ})$ such that:

(a) $1 \le y_i \le Z$ for all i,

(b) $\sum_j \phi_j(y_i) n_j d_j(P_i) < \sum_j \phi_j(y_k) n_j d_j(P_i)$ for all $i < k \le Z$,

(c) $\Phi_{ji} = \phi_j(y_i)$,

(d) $\Pi_i = \pi(y_i)$,

(e) $q_i = \sum_{s \geq i} \Pi_s$,

(f) $x_i = \sum_j \Phi_{ji} n_j d_j(P_i) - \sum_{s=1}^{i-1} x_s$ and

(g) $P_i = \dfrac{\lambda}{q_i}$.

Part (a) implies that y is a one to one mapping from $(1,...,Z)$ to $(1,...,Z)$.

Part (b) requires that at the price P_i demand in state y_i is less than demand in state y_k for all $k > i$. Suppose for example that $y_1 = 6$ and $y_2 = 3$. Then demand at the price P_1 is lowest in state 6 and demand at the price P_2 is lower in state 3 then in all states $s \neq 6$. The fraction of type j buyers who are active in state 6 is denoted by $\Phi_{j1} = \phi_j(y_1)$ and the fraction of type j buyers who are active in state 3 is denoted by: $\Phi_{j2} = \phi_j(y_2)$. The probability that state 6 occurs is denoted by $\Pi_1 = \pi(y_1)$ and the probability that state 3 occurs is denoted by $\Pi_2 = \pi(y_2)$. Thus Π_1 is the probability that exactly one batch will arrive and Π_2 is the probability that exactly two batches will arrive. The probability $q_i = \sum_{s \geq i} \Pi_s$ is the probability that more than i batches will arrive or the probability that market i opens. Part (f) is a market clearing condition: After transactions in market $i-1$ are complete, the minimum residual demand at the price P_i is $\sum_j \Phi_{ji} n_j d_j(P_i) - \sum_{s=1}^{i-1} x_s$ and this must equal the supply to market i. Part (g) requires that the expected revenue per unit is the same across markets.

<u>The "weak" planner's problem</u>:

I assume that the "weak" planner can observe the aggregate amount distributed and the type of each buyer but not the type composition of the buyers. The "weak" planner chooses Z quantities $(x_1,...,x_Z)$ and Z allocation rules. The first allocation rule is applied to the distribution of the first batch of x_1 units. The second allocation rule is applied to the distribution of the second batch of x_2 units and so on. In detail, the planner distributes x_{j1} units to type j buyers that arrive until the first x_1 units are distributed. He then use the second allocation rule and distributes x_{j2} units to type j buyers that arrive until the second batch of x_2 units are distributed and in general he uses the allocation rule x_{ji} after $\sum_{s=1}^{i-1} x_s$ units were already distributed to distribute the next x_i units.

We may say that buyers who arrive after $\sum_{s=1}^{i-1} x_s$ units were distributed and before $\sum_{s=1}^{i} x_s$ units were distributed, arrive in batch i and x_{ji} is the amount allocated to a type j agent who arrives in batch i.

The choice of x_i and x_{ji} determine the probability that x_i will satisfy the additional demand. If for example x_1 is large and x_{ji} are small, the probability that more buyers will arrive after the distribution of x_1 units is small. Therefore the probabilities of delivery depends on the choice of x_i and x_{ji}.

We may therefore write the "weak" planner's problem in the following way.

Choose Z distinct integers $(y_1,...,y_Z)$ and a vector of real numbers
$(x_1,...,x_Z;\Pi_1,...,\Pi_Z;q_1,...,q_Z;$
$\Phi_{11},...,\Phi_{J1};\Phi_{12},...,\Phi_{J2};...;\Phi_{1Z},...,\Phi_{JZ};x_{11},...,x_{J1};x_{12},...,x_{J2};...;x_{1Z},...,x_{JZ})$

such that:

(a) $1 \le y_i \le Z$ for all i,

(b) $\sum_j \phi_j(y_i) n_j x_{ji} < \sum_j \phi_j(y_k) n_j x_{ji}$ for all $i < k \le Z$

(c) $\Phi_{ji} = \phi_j(y_i)$,

(d) $\Pi_i = \pi(y_i)$,

(e) $x_i = \sum_{j=1}^{J} n_j \Phi_{js} x_{ji} - \sum_{j=1}^{J} n_j \Phi_{ji-1} x_{ji-1} > 0$ And

(f) x_{ji} solve the following problem:

$$\max_{x_{js}} \sum_{j=1}^{J}\sum_{s=1}^{Z}\Pi_s \sum_{i=1}^{s}\Phi_{js} n_j U_j(x_{ji}) - \lambda \max_s \left(\sum_{j=1}^{J}\sum_{i=1}^{Z} n_j \Phi_{js} x_{ji} \right)$$

To solve for the planner's first order condition I assume as before, that the planner wants to distribute the maximum amount in state Z: $Z = \arg\max_s \left(\sum_{j=1}^{J}\sum_{i=1}^{Z} n_j \Phi_{js} x_{ji} \right)$.

Since the planner knows ϑ_{js} he can compute for each state s, the number of buyers served in batch i, $(N_{is} - N_{i-1s})$ and the number of type j buyers served in batch i, $\vartheta_{js}(N_{is} - N_{i-1s})$. In detail, the equations:

$N_{1s}\sum_{j=1}^{J}\vartheta_{js}x_{j1} = x_1$ and $(N_{is} - N_{i-1s})\sum_{j=1}^{J}\vartheta_{js}x_{ji} = x_i$ lead to: $N_{1s} = x_1\left(\sum_{j=1}^{J}\vartheta_{js}x_{j1}\right)^{-1}$ and $N_{is} - N_{i-1s} = x_i\left(\sum_{j=1}^{J}\vartheta_{js}x_{ji}\right)^{-1}$.

The planner will choose the amount allocated to a type j buyers who arrive in batch i by maximizing:

(B3) $$\max_{x_{ji}} U_j(x_{ji})\sum_{s=i}^{Z}\Pi_s\vartheta_{js}(N_{is} - N_{i-1s}) - \lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z})x_{ji}$$

The first order condition for this problem is:

(B4) $$U_j{}'(x_{ji})\Delta_{ji} = \lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z}),$$

where $\Delta_{ji} = \sum_{s=i}^{Z}\Pi_s\vartheta_{js}(N_{is} - N_{i-1s})$ is the expected number of buyers served in market i. The interpretation of this first order condition is as follows. The expected marginal utility from increasing the allocation to type j in market i by one unit: $U_j{}'(x_{ji})\Delta_{ji}$. The cost of doing it is: $\lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z})$, because only in state Z we hit the capacity constraint. Therefore, (B4) says that the marginal benefits equal the marginal cost.

Rearranging (B4) leads to:

(B5) $$U_j'(x_{ji}) = \frac{\lambda \vartheta_{jZ}(N_{iZ} - N_{i-1Z})}{\sum_{s=i}^{Z} \Pi_s \vartheta_{js}(N_{is} - N_{i-1s})}$$

This is different from the UST allocation rule (5) implying that in general, the UST outcome is not a solution to the "weak" planner's problem (and of course not to the "strong" planner's problem).

REFERENCES

Bental, Benjamin and Benjamin Eden (1993). "**Inventories in a Competitive Environment**," Journal of Political Economy, University of Chicago Press, vol. 101(5), pages 863-86, October.

Bryant, J. (1980), "Competitive Equilibrium with Price-Setting Firms and Stochastic Demand," International Economic Review , 21 (1), pp. 519-526.

Cornia Marco & Kristopher S. Gerardi & Adam Hale Shapiro, 2012. "**Price Dispersion Over the Business Cycle: Evidence from the Airline Industry**," Journal of Industrial Economics, Wiley Blackwell, vol. 60(3), pages 347-373, 09.

Dana James D. Jr. "Advance-Purchase Discounts and Price Discrimination in Competitive Markets" Journal of Political Economy, Vol.106, Number 2, April 1998, 395-422.

________ "Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure" The RAND Journal of Economics, Vol. 30, No. 4 (Winter, 1999), pp. 632-660.

Deaton, Angus & Laroque, Guy, 1992. "**On the Behaviour of Commodity Prices**," Review of Economic Studies, Wiley Blackwell, vol. 59(1), pages 1-23, January.

Deneckere Raymond and James Peck., "Dynamic Competition with Random Demand and Costless Search: A Theory of Price Posting". Econometrica, Vol. 80, No. 3 (May, 2012), 1185–1247.

Deneckere, R., H. P. Marvel, and J. Peck (1996), "Demand Uncertainty, Inventories, and Resale Price Maintenance," Quarterly Journal of Economics , August, pp. 885-913.

Eden, Benjamin. "Marginal Cost Pricing When Spot Markets are Complete" Journal of Political Economy, Dec. 1990. Vol. 98, No.6,1293-1306.

________ "**The Adjustment of Prices to Monetary Shocks When Trade Is Uncertain and Sequential**," Journal of Political Economy, University of Chicago Press, vol. 102(3), pages 493-509, June, 1994.

________ "**Inefficient trade patterns: Excessive trade, cross-hauling and dumping**," Journal of International Economics, Elsevier, Elsevier, vol. 73(1), pages 175-188, September, 2007.

_______ "**Efficient barriers to trade: A sequential trade model with heterogeneous agents**," Journal of International Economics, Elsevier, vol. 77(2), pages 234-244, April 2009.

_______ "Price Dispersion and Demand Uncertainty: Evidence from US scanner data" *Vanderbilt University Department of Economics Working Papers*, VUECON-14-00011 (October 2014). For an earlier version, Eden (2013), see WP# 13-00012.

______ and Zvi Griliches, 1993. "**Productivity, Market Power, and Capacity Utilization When Spot Markets Are Complete**," American Economic Review, vol. 83(2), pages 219-23, May.

Escobari, Diego 2012. "**Dynamic Pricing, Advance Sales and Aggregate Demand Learning in Airlines**," Journal of Industrial Economics, Wiley Blackwell, vol. 60(4), pages 697-724, December.

_______ and Li Gan, 2007. "**Price Dispersion under Costly Capacity and Demand Uncertainty**," NBER Working Papers 13075, National Bureau of Economic Research, Inc.

_______ and Lee, Jim, 2013. "**Demand Uncertainty and Capacity Utilization in Airlines**," MPRA Paper 46059, University Library of Munich, Germany.

Gale, Douglas. 1987. "Limit Theorems for Markets with Sequential Bargaining." Journal of Economic Theory 43:20–54.

Harris, M. and A. Raviv (1981), "A Theory of Monopoly Pricing Schemes with Demand Uncertainty," American Economic Review, 71 (3), pp. 347-365.

Head Allen, Lucy Qian Liu, Guido Menzio and Randall Wright, 2012. "**Sticky Prices: A New Monetarist Approach**," Journal of the European Economic

Association, European Economic Association, vol. 10(5), pages 939-973, October.

Kahn James A. & Mark Bils, 2000. "**What Inventory Behavior Tells Us about Business Cycles**," American Economic Review, American Economic Association, vol. 90(3), pages 458-481, June.

Khan Aubhik and Julia K. Thomas, 2007. "**Inventories and the Business Cycle: An Equilibrium Analysis of (S, s) Policies**," American Economic Review, American Economic Association, vol. 97(4), pages 1165-1188, September.

Kircher Philipp, "Efficiency of Simultaneous Search" Journal of Political Economy, Vol. 117, No. 5 (October 2009), pp. 861-913.

Kydland, Finn E & Prescott, Edward C, 1982. "**Time to Build and Aggregate Fluctuations**," Econometrica, Econometric Society, vol. 50(6), pages 1345-70, November.

Lazear, E. (1986), "Retail Pricing and Clearance Sales," The American Economic Review, 76(1), pp. 14-32.

Miron, Jeffrey A & Zeldes, Stephen P, 1988. "**Seasonality, Cost Shocks, and the Production Smoothing Models of Inventories**," Econometrica, Econometric Society, vol. 56(4), pages 877-908, July.

Mortensen, Dale T., and Randall Wright. 2002. "Competitive Pricing and Efficiency in Search Equilibrium." Internat. Econ. Rev. 43:1–20.

Pashigian, B. P. (1988), "Demand Uncertainty and Sales: A Study of Fashion and Markdown Pricing," American Economic Review, 78 (5), pp. 936-953.

Prescott, Edward. C., "Efficiency of the Natural Rate" Journal of Political Economy, 83 (Dec. 1975): 1229-1236.

Williamson, 0. E. "Peak Load Pricing and Optimal Capacity under Indivisibility Constraints." The American Economic Review; Vol. 56, No. 4 (September 1966), pp. 816-827.