Vanderbilt University Department of Economics Working Paper Series
Nashville, TN 37235-1819

Vanderbilt University Department of Economics Working Papers 16-00018

Impossibilities for strategy-proof committee selection mechanisms with vetoes

Martin Van der linden
Department of Economics, Vanderbilt University

Abstract

Many mechanisms used to select a committee of k members out of a candidates endow voters with some veto power over candidates. Impossibility results are provided showing that, in most cases, even limited veto power implies that the mechanism is not strategy-proof. These impossibilities hold on a large set of domains including the domain of additive preferences and even when probabilistic mechanisms are allowed.

I am grateful to Eun Jeong Heo and John Weymark for helpful discussions and comments and to Benoit Decerf, Paul Edelman, Greg Leo, and Myrna Wooders for useful remarks. I thank CEREC and CORE for their hospitality during visits. Support from the Kirk Dornbush summer research grant and the National Science Foundation grant IIS-1526860 is gratefully acknowledged.
Citation: Martin Van der linden, (2016) "Impossibilities for strategy-proof committee selection mechanisms with vetoes", *Vanderbilt University Department of Economics Working Papers*, VUECON-16-00018.
Contact: Martin Van der linden - martin.van.der.linden@vanderbilt.edu.
Submitted: September 19, 2016. **Published:** September 21, 2016.
URL:http://www.accessecon.com/Pubs/VUECON/VUECON-16-00018.pdf

1 Introduction

The literature on mechanism design often regards it to be desirable for no voter to have veto power over any alternatives. Such a *no-veto power* requirements are, for example, prominent in implementation theory where, combined with monotonicity requirements, they provide sufficient conditions for Nash implementability (Maskin, 1999).

In practice, however, it is common to endow voters with veto power. Examples include jury selection (see Flanagan (2015) for a recent review), other judicial procedures such as the selection of arbitrators (de Clippel et al., 2014) and Special Masters (see, e.g., Valdivia v. Schwarzenegger[1]), and the selection of a candidate for the papal throne.[2] The importance of endowing voters with some veto power is also acknowledged in the social choice literature on rights (Sen, 1970).[3]

In many procedures with veto power, the players' actions are in fact limited to vetoes. For example, in the *strike and replace* procedure for jury selection, the defense and the plaintiff can veto each potential juror one at a time when he or she is drawn from the pool (provided they have not yet exhausted all of their vetoes). Mueller (1978) and Moulin (1981) have shown that similar procedures in which voters take turns vetoing alternatives can be used to implement desirable social choice functions using backward induction.[4]

One issue with veto procedures is that, despite having interesting equilibria, they are often manipulable. Examples of manipulable veto procedures include the procedures studied in Mueller (1978), Moulin (1981), de Clippel et al. (2014) and Flanagan (2015). Moreover, evidence from experimental and field data show that voters do attempt to manipulate procedures involving veto power and sometimes fail to reach an equilibrium (Yuval, 2002; de Clippel et al., 2014). Because veto power is so pervasive in practice, it is therefore legitimate to ask whether interesting veto procedures exist that leave no room for manipulation.

In this paper, I answer this question negatively for the standard problem of selecting a committee of k members out of a candidates. I show that

[1] Stipulation and Amended Order Re Special Master Order of Reference, Valdivia v. Schwarzenegger, No. CIV-S-94-0671 (E.D. Cal. filed Aug. 19, 2005)

[2] The right to veto *one* candidate to the papal throne was exercised by France, Austria and Spain in various shapes and forms from the late 16th until the beginning of the 20th Century (O'Malley, 2015, p.41).

[3] In the literature on rights, voters are allowed to veto a *social preference* for some alternative x over another alternative y.

[4] Among other properties, the social choice function in Mueller (1978) and Moulin (1981) are Pareto efficient and never select a voters' worst alternative.

endowing as few as two agents with the power to veto a *single* candidate makes most mechanisms manipulable. This is true for small sub-domains of the set of additive preferences and when probabilistic mechanisms are allowed.

On these subdomains, I show that strategy-proof mechanisms with veto power must have ranges that (i) do not contain degenerate lotteries in which a committee is chosen for sure (Theorem 1) and (ii) have as a limit point a lottery in which the probability to select a particular candidate is zero (Theorem 2). Condition (i) implies that every *deterministic* mechanism with veto power violates strategy-proofness (Corollary 1). Condition (i) also restricts the efficiency of strategy-proof mechanisms with veto power. For example, a strategy-proof mechanism with veto power cannot always select committees that voters unanimously prefer (Corollary 2). Condition (ii) implies that a wide class of selection mechanisms constructed from extensive game forms violate strategy-proofness (Corollary 3).

Related Literature

For deterministic mechanisms with an unrestricted domain, the Gibbard-Satterthwaite Theorem implies that every strategy-proof mechanism for selecting fixed-size committees with more than three committees in its range is dictatorial. At least two approaches have been used to overcome this negative result.

The first weakens the unrestricted domain assumption. This typically involves assuming that voters' preferences satisfy some separability condition. Unfortunately, Barberà et al. (2005) show that, even when preferences are separable, only a restricted set of non-dictatorial selection mechanisms are strategy-proof.[5]

Another approach allows for probabilistic selection mechanisms that select *lotteries* over committees rather than sure committees. In many problems, however, strategy-proofness cannot be combined with other desirable properties even using a probabilistic mechanism. In voting models with a finite set of alternatives, strategy-proofness is, for example, incompatible with unanimity except for (possibly random) dictatorial mechanisms (Hylland, 1980; Schummer, 1999; Benoît, 2002; Dutta et al., 2006; Nandeibam, 2012; Chatterji et al., 2012).

[5] The characterization in Barberà et al. (2005) is more permissive for *additive* preferences. But even on this smaller domain, the class of strategy-proof selection mechanisms remains a small subclass of the rules known as *voting by committees* (the "committees" in *voting by committees* are committees of *voters* and should not be confused with the selected committee of k members). See also Barberà et al. (1991) for the problem of selecting a committee without size constraints.

In this paper, I combine both approaches by considering probabilistic mechanisms on a domain smaller than the domain of separable preferences. I show that strategy-proofness is, in general, incompatible with giving voters a minimal veto power over candidates. This finding contrasts with Ju (2003), who studies domain restrictions for which strategy-proof mechanisms exist that do *not* give voters veto power over candidates. My results provide new evidence of the difficulty of combining strategy-proofness with other desirable requirements and of the limited freedom one gains by imposing domain restrictions and allowing for probabilistic mechanisms.

2 Model and definitions

The set of **voters** is $N \coloneqq \{1, \ldots, n\}$ with $n \geq 2$. The set of **candidates** is $A \coloneqq \{1, \ldots, a\}$ with $a \geq 2$. The set of possible **committees** $\mathcal{A}_k$ is the set of subsets of A with k elements ($k < a$). Let $\Delta\mathcal{A}_k$ be the set of lotteries on $\mathcal{A}_k$. Slightly abusing the notation, let $C \in \mathcal{A}_k$ denote any degenerate *lottery* which yields committee C for sure.

A typical domain of preferences on $\Delta\mathcal{A}_k$ is denoted $\mathcal{D}$. For every domain $\mathcal{D}$ in this paper, preferences in $\mathcal{D}$ are orderings that satisfy the expected utility axioms. A (preference) **profile** is an n-tuple $R_N \coloneqq (R_1, \ldots, R_n) \in \mathcal{D}^n$. For any profile R_N and any $i \in N$, $R_{-i} \coloneqq (R_1, \ldots, R_{i-1}, R_{i+1}, \ldots, R_n)$ is the $(n-1)$-tuple that lists the preferences of every player but i. .

A **(selection) mechanism** is a function $M : \mathcal{D}^n \to \Delta\mathcal{A}_k$ that associates a lottery in $\Delta\mathcal{A}_k$ with every profile in $\mathcal{D}^n$. For any $R_N \in \mathcal{D}^n$, $M(R_N)$ is the lottery selected by M when R_N is reported. The **range** of M is the set of $L \in \Delta\mathcal{A}_k$ which can be selected under M; that is,

$$range(M) \coloneqq \{L \in \Delta\mathcal{A}_k \mid M(R_N) = L \text{ for some } R_N \in \mathcal{D}^n\}. \tag{1}$$

For any lottery $L \in \Delta\mathcal{A}_k$ and any committee $C \in \mathcal{A}_k$, C's **selection probability** $L(C)$ is the probability that C is the chosen committee given L. Similarly, for any lottery $L \in \Delta\mathcal{A}_k$ and any candidate $t \in A$, t's **selection probability** $L(t)$ is the probability that t is part of the chosen committee given L. Formally, $L(t) \coloneqq \sum_{\{S \in \mathcal{A}_k | t \in S\}} L(S)$.

The next definition introduces a relatively weak concept of a vetoer. A voter $i \in N$ is a vetoer if for each $t \in A$, voter i can declare a preference $R_i^t \in \mathcal{D}$ which guarantee that candidate t is not part of the chosen committee whatever R_{-i} the other voters report. Formally, given a mechanism M, any voter $i \in N$ is a **vetoer** if for each $t \in A$, there exists $R_i^t \in \mathcal{D}$ with

$$M(R_i^t, R_{-i})(t) = 0 \qquad \text{for all } R_{-i} \in \mathcal{D}^{n-1}. \tag{2}$$

Although a vetoer can veto *any* candidate, a vetoer is only guaranteed the ability to veto *one* candidate at a time. For example, for i to be a vetoer, there does *not* need to be any pair of candidates (t, t') with $t \neq t'$ such that for some $\bar{R}_i \in \mathcal{D}$, $M(\bar{R}_i, R_{-i})(t) = M(\bar{R}_i, R_{-i})(t') = 0$ for all $R_{-i} \in \mathcal{D}^{n-1}$. A selection mechanism M is an $\boldsymbol{r}$**−vetoers mechanism** if there are *at least* r *distinct* vetoers in M.

A selection mechanism M is **strategy-proof** if for all $i \in N$, reporting i's true preference is a dominant strategy; that is, for all $R_i \in \mathcal{D}$

$$M(R_i, R_{-i})\ R_i\ M(R_i', R_{-i}) \qquad \text{for all } R_i' \in \mathcal{D} \text{ and all } R_{-i} \in \mathcal{D}^{n-1}.$$

For any subset $B \subseteq \Delta\mathcal{A}^k$ and any $R \in \mathcal{D}$, the **top set** $top(R, B)$ is the set of best lotteries in B according to R. Formally, for all $B \subseteq \Delta\mathcal{A}_k$ and all $R \in \mathcal{D}$,

$$top(R, B) \coloneqq \{L \in B \mid L\ R\ L' \text{ for all } L' \in B\}.$$

A voter $j \in N$ is a **dictator** for mechanism M if the lottery that M chooses is always in j's top set; that is,

$$M(R_N) \in top\big(R_j, \Delta\mathcal{A}_k\big) \qquad \text{for all } R_N \in \mathcal{D}^n.$$

Finally, for any $i \in N$ and any preference $R_i \in \mathcal{D}$, the **option set** $O_{-i}(R_i)$ is the set of lotteries that M chooses for some report of the preferences of voters in $N\backslash\{i\}$ given that voters i report R_i (Barberà and Peleg, 1990). Formally, for all $R_i \in \mathcal{D}$,

$$O_{-i}(R_i) \coloneqq \left\{L \in \Delta\mathcal{A}_k \mid M(R_i, R_{-i}) = L \text{ for some } R_{-i} \in \mathcal{D}^{n-1}\right\}.$$

Note that i is a dictator if and only if $O_{-i}(R_i) \subseteq top\big(R_i, \Delta\mathcal{A}_k\big)$ for all $R_i \in \mathcal{D}$.

3 Domains of preferences

In selection problems, it is common to assume that preferences are represented by a von Neuman-Morgenstern utility function $u_i : A \to \mathbb{R}$ defined on the set of candidates in the following way:

$$L\ R_i\ L' \Leftrightarrow \sum_{C \in \mathcal{A}_k} L(C) \sum_{t \in C} u_i(t) \geq \sum_{C \in \mathcal{A}_k} L'(C) \sum_{t \in C} u_i(t) \quad \text{for all } L, L' \in \Delta\mathcal{A}_k. \tag{3}$$

The domain $\mathcal{R}_{add}$ is the set of preferences on $\Delta\mathcal{A}_k$ defined by (3) for some utility function u_i on the candidates.

All the impossibilities presented here hold for the domain $\mathcal{R}_{add}$. However, in order to demonstrate the generality of the impossibility theorems, for each result, I identify domain properties for which the result holds. I illustrate these properties using preferences in $\mathcal{R}_{add}$. In a nutshell, the feature of $\mathcal{R}_{add}$ that results in the impossibilities is that some preferences in $\mathcal{R}_{add}$ are close to lexicographic preferences on the candidates' selection probabilities.

4 Preliminary results

This section introduces two propositions that I use repeatedly in the proofs. These propositions follow from results in Le Breton and Weymark (1999).

The first says that given a profile R_N, if M is strategy-proof and if some $i \in N$ has a unique top lottery L in the range of M, then L must be contained in the option set $O_{-i}(R_i)$.

Proposition 1. *Suppose that $M : \mathcal{D}^n \to \Delta\mathcal{A}_k$ is a strategy-proof mechanism. For all $i \in N$ and all $R_i \in \mathcal{D}$, if $top\big(R_i, range(M)\big) = \{L\}$, then $L \in O_{-i}(R_i)$.*

Proof. This proposition is a direct corollary of Le Breton and Weymark (1999, Proposition 3). ■

The second proposition says that if M is strategy-proof and if all voters in $N\backslash\{i\}$ agree on the set of top lotteries B in the option set $O_{-i}(R_i)$, then the chosen lottery must be included in B.

Proposition 2. *Suppose that $M : \mathcal{D}^n \to \Delta\mathcal{A}_k$ is a strategy-proof mechanism. For all $R_N \in \mathcal{D}$ and all $i \in N$, if there exists a nonempty set $B \subseteq O_{-i}(R_i)$ such that $top(R_i, O_{-i}(R_i)) = B$ for all $i \in N\backslash\{i\}$, then $M(R_N) \in B$.*

Proof. This proposition is a direct corollary of Le Breton and Weymark (1999, Proposition 4). ■

5 No sure committee in the range of strategy-proof 2-vetoers mechanisms

In this section, I show that no strategy-proof 2-vetoers mechanism can have in its range a degenerate lottery in which some committee is chosen for sure. (Theorem 1). This impossibility precludes the existence of deterministic strategy-proof 2-vetoers mechanisms (Corollary 1) and severely limits the efficiency of strategy-proof 2-vetoers mechanisms (Corollary 2).

5.1 Main result

I now establish two lemmas that are used in the proof of Theorem 1, Lemmas 1 and 2. Lemma 1 holds on minimin domains. A minimin domain contains sequences of preferences for which the impact of the "worst" candidate on the value of a committee becomes increasingly negative. In such a sequence, voters become increasingly concerned with *minimizing* the selection probability of their "worst" candidate.[6] In addition, for reasons that will become clear in the proof of Lemma 2, preferences in the sequences must have the same most preferred committee, with the "worst" candidate not a member of this committee.

Domain Property 1 (Minimin domain)**.** A domain of preferences $\mathcal{R}$ on the lotteries in $\Delta\mathcal{A}_k$ is **minimin** if for any candidate $t \in A$, for any committee $C \in \mathcal{A}_k$ with $t \notin C$, and for any $\epsilon > 0$, there exists $R^\epsilon \in \mathcal{R}$ such that

(i) $L \; P^\epsilon \; L'$ for all $L, L' \in \Delta\mathcal{A}_k$ for which $L(t) < L'(t) - \epsilon$, and

(ii) $C \; P^\epsilon \; C'$ for all $C' \in \mathcal{A}_k \backslash \{C\}$.

Example 1 ($\mathcal{R}_{add}$ is a minimin domain)**.** For any $t \in A$ and any $C \in \mathcal{A}_k$ with $t \notin C$, consider any preference $R^r \in \mathcal{R}_{add}$ for $r > 0$ defined by

(a) $u^r(t) = -r$,

(b) for all $t' \in A$ with $t' \neq t$, $u^r(t') = c_{t'}$ for some constant $c_{t'} \in \mathbb{R}$, and

(c) for all $a \in C$ and all $b \in A \backslash C$, $c_a > c_b$.

By (a) and (b), for any $\epsilon > 0$, there exists r sufficiently large such that (i) is satisfied in the definition of a minimin domain. Also, (ii) is satisfied by (c).

Lemma 1 shows that, on a minimin domain, if a vetoer $j \in N$ has a sufficiently strong concern for minimizing the selection probability of some $t \in A$, then any lottery L in the option set generated by j must have $L(t)$ arbitrarily small. Otherwise, j would have an incentive to report preferences that veto t, contradicting strategy-proofness.

Lemma 1. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a strategy-proof mechanism and $\mathcal{R}$ is a minimin domain. Let $j \in N$ be a vetoer for M. For any $t \in A$, any $C \in \mathcal{A}_k$ with $t \notin C$, and any $\epsilon > 0$, there exist a preference $R^\epsilon_j \in \mathcal{R}$ such that*

(a) $L(t) < \epsilon$ *for all* $L \in O_{-j}(R^\epsilon_j)$*, and*

[6] Hence, the name "minimin", for "*mini*mizing the selection probability of the candidate whose contribution to the committee is *min*imal".

(b) $C \; P_j^\epsilon \; C'$ *for all* $C' \in \mathcal{A}_k \backslash \{C\}$.

Proof. By assumption, for all $\delta > 0$ there exist a preference $R_j^\delta \in \mathcal{R}$ such that (i) and (ii) are satisfied in the definition of a minimin domain (with δ replacing ϵ in the definition). Also, because j is a vetoer, there exists $R_j^t \in \mathcal{R}$ such that

$$M(R_j^t, R_{-j})(t) = 0 \qquad \text{for all } R_{-j}. \tag{4}$$

If $M(R_j^t, R_{-j}^*)(t) < M(R_j^\delta, R_{-j}^*)(t) - \delta$ for some R_{-j}^*, then by (i) in the definition of a minimin domain, $M(R_j^t, R_{-j}^*) \; P_j^\delta \; M(R_j^\delta, R_{-j}^*)$, contradicting strategy-proofness. Thus, for all R_{-j} we must have $M(R_j^t, R_{-j})(t) \geq M(R_j^\delta, R_{-j})(t) - \delta$. Hence, by (4), $M(R_j^\delta, R_{-j})(t) \leq \delta$ for all R_{-j}. But then, (a) holds for $R_j^\epsilon = R_j^\delta$ whenever $\delta < \epsilon$, and (b) holds by (ii) in the definition of a minimin domain. ■

The next lemma holds on domains that are both minimin and maximax. Maximax is in some sense the inverse of minimin. A domain is maximax if it contains sequences of preferences for which the impact of the "best" candidate on the value of a committee becomes increasingly large. In such a sequence, voters become increasingly concerned with *maximizing* the selection probability of their "best" candidate.[7] In addition, for reasons that will again become clear in the proof of Lemma 2, preferences in the sequences must have the same most preferred committee among the committees that do *not* contain the "best" candidate.

Domain Property 2 (Maximax domain)**.** A domain of preferences $\mathcal{R}$ on the lotteries in $\Delta\mathcal{A}_k$ is **maximax** if for any candidate $t \in A$, for any committee $C \in \mathcal{A}_k$ with $t \notin C$, and for any $\epsilon > 0$, there exists $R^\epsilon \in \mathcal{R}$ such that

(i) $L \; P^\epsilon \; L'$ for all $L, L' \in \Delta\mathcal{A}_k$ such that $L(t) > L'(t) + \epsilon$, and

(ii) $C \; P^\epsilon \; C'$ for all $C' \in \mathcal{A}_k \backslash \{C\}$ with $t \notin C'$.

Example 2 ($\mathcal{R}_{add}$ is a maximax domain)**.** For any $t \in A$ and $C \in \mathcal{A}_k$ with $t \notin C$, consider any preference $R^r \in \mathcal{R}_{add}$ defined by

(a) $u^r(t) = r$,

(b) for all $t' \in A$ with $t' \neq t$, $u^r(t') = c_{t'}$ for some constant $c_{t'} \in \mathbb{R}$, and

(c) for all $a \in C$ and all $b \in A \backslash (C \cup \{t\})$, $c_a > c_b$.

[7] Hence, the name "maximax", for "*maxi*mizing the selection probability of the candidate whose contribution to the committee is *max*imal".

By (a) and (b), for any $\epsilon > 0$, there exists r sufficiently large such that (i) is satisfied in the definition of a maximax domain. Also, (ii) is satisfied by (c).

The next lemma shows that on a domain that is both minimin and maximax, if a strategy-proof mechanism ever selects a sure committee $C \in \mathcal{A}_k$, then C must be chosen whenever any vetoer j likes C best. Informally, suppose that j likes C best and there is a lottery L with $L(t) > 0$ for some $t \notin C$ in the option set generated by j. Because the domain is maximax, there exist preferences for which the inclusion of t in a committee is essential. For any such preferences R^*, some lottery L with $L(t) > 0$ will be chosen when everyone but j reports R^* (by Proposition 2). It is possible to choose such a preference, say R^{**}, so that C is the best committee among the committees that do not contain t. But then, when everybody but j reports R^{**}, j can declare minimin preferences that force t to be chosen with arbitrarily small probability while keeping C as j's best committee. If j does so, C remains in the option set (by Proposition 1) and a lottery that selects C with arbitrarily large probability is chosen instead of L whenever everyone but j reports R^{**} (by Proposition 2), contradicting strategy-proofness.

Lemma 2. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a strategy-proof mechanism with a sure committee $C \in \mathcal{A}_k$ in its range and $\mathcal{R}$ is a minimin and maximax domain. For all $R_j \in \mathcal{R}$, if*

$$C \ P_j \ C' \text{ for all } C' \in \mathcal{A}_k \backslash \{C\}, \tag{5}$$

then

$$L(C) = 1 \qquad \text{for all } L \in O_{-j}(R_j). \tag{6}$$

Proof. Consider any $R_j \in \mathcal{R}$ such that (5) holds. Such an R_j must exist in $\mathcal{R}$ by (ii) in the definition of a minimin domain. In order to derive a contradiction, assume that

$$L^*(C) < 1 \qquad \text{for some } L^* \in O_{-j}(R_j). \tag{7}$$

By the definition of a lottery, $L^*(C) < 1$ implies $L^*(t) = \epsilon + \gamma$ for some $t \in A \backslash C$ and some $\epsilon > 0$ and $\gamma > 0$. By assumption, for all $\delta > 0$, there exists a preference $R^\delta \in \mathcal{R}$ satisfying (i) and (ii) in the definition of a maximax domain, with (a) t as the "best" candidate, (b) C as the best committee not containing t, and (c) δ replacing ϵ in the definition. But then Proposition 2 and $L^* \in O_{-j}(R_j)$ imply

$$M\left(R_j, R^\delta, \ldots, R^\delta\right)(t) \geq \epsilon \qquad \text{for all } \delta < \gamma. \tag{8}$$

This inequality holds because the preference R^δ has a tolerance of $0 < \delta < \gamma$ for a decrease in the selection probability of t. Thus, $M\left(R_j, R^\delta, \ldots, R^\delta\right)(t) < \epsilon$ implies that the lottery selected by M is worse for the preferences R^δ than $L^* \in O_{-j}(R_j)$ because $L^*(t) = \epsilon + \gamma$, contradicting Proposition 2.

By the definition of a lottery and because $t \notin C$, (8) implies

$$M\left(R_j, R^\delta, \ldots, R^\delta\right)(C) < 1. \tag{9}$$

Because $\mathcal{R}$ is a minimin domain, by Lemma 1, there exists a sequence of preferences $\{R_j^r\}_{r=1}^\infty$ in $\mathcal{R}$ such that C is the most preferred committee for all $r > 0$ (see (5)) and

$$\lim_{r\to\infty} M\left(R_j^r, R^\delta, \ldots, R^\delta\right)(t) = 0. \tag{10}$$

Let $L^r := M\left(R_j^r, R^\delta, \ldots, R^\delta\right)$ for all $r > 0$.

We now show that

$$\lim_{r\to\infty} L^r(C) = 1. \tag{11}$$

By Proposition 1, because C is the most preferred committee for R_j^r and because C is in the range, $C \in O_{-j}(R_j^r)$ for all $r > 0$. But then by Proposition 2, we must have

$$L^r \; R^\delta \; C \qquad \text{for all } r > 0.^{8} \tag{12}$$

Let $\widehat{C} \in \mathcal{A}_k$ be (one of) the second most preferred committee(s) according to R^δ among the committees that do not contain t; that is,

$$\widehat{C} \; R^\delta \; C' \qquad \text{for all } C' \in \mathcal{A}_k \backslash \{C\} \text{ with } t \notin C'. \tag{13}$$

Because R^δ satisfies (ii) in the definition of a maximax domain with C as the best committee not containing t, we have

$$C \; P^\delta \; \widehat{C}. \tag{14}$$

We can now rewrite (12) in utility terms as follows:

$$L^r(C)u^\delta(C) + \sum_{\{S\in\mathcal{A}_k | t\in S\}} L^r(S)u^\delta(S) + \sum_{\{S\in\mathcal{A}_k | t\notin S \text{ and } S\neq C\}} L^r(S)u^\delta(S) \geq u^\delta(C).$$

[8] The argument is similar to the one used to prove (8).

By (13), this implies

$$L^r(C)u^\delta(C) + \sum_{\{S\in\mathcal{A}_k|t\in S\}} L^r(S)u^\delta(S) + \left(1 - L^r(C) - \sum_{\{S\in\mathcal{A}_k|t\in S\}} L^r(S)\right) u^\delta(\widehat{C}) \geq u^\delta(C)$$

Finally, because $u^\delta(C) - u^\delta(\widehat{C}) > 0$ by (14), we have

$$L^r(C) \geq \frac{u^\delta(C) - \left(1 - \sum_{\{S\in\mathcal{A}_k|t\in S\}} L^r(S)\right) u^\delta(\widehat{C})}{\left(u^\delta(C) - u^\delta(\widehat{C})\right)} - \frac{\sum_{\{S\in\mathcal{A}_k|t\in S\}} L^r(S)u^\delta(S)}{\left(u^\delta(C) - u^\delta(\widehat{C})\right)}. \tag{15}$$

By (10), $L^r(t)$ tends to 0 as $r \to \infty$, which implies that $\lim_{r\to\infty} \sum_{\{S\in\mathcal{A}_k|t\in S\}} L^r(S) = 0$. Thus, the first term on the right-hand side of (15) tends to 1 as $r \to \infty$. Similarly, the second term on the right-hand side of (15) tends to 0 as $r \to \infty$. Overall, the right-hand side of (15) tends to 1 as $r \to \infty$ and therefore $L^r(C)$ must also tend to 1 as $r \to \infty$, which proves (11).

Together (5), (9) and (11) imply that there exists r sufficiently large such that

$$M\left(R_j^r, R^\delta, \ldots, R^\delta\right)\ P_j\ M\left(R_j, R^\delta, \ldots, R^\delta\right) \qquad \text{for all } \delta < \gamma \tag{16}$$

contradicting strategy-proofness. ■

It is now easy to prove the following theorem.

Theorem 1. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a 2-vetoers mechanism with a sure committee $C \in \mathcal{A}_k$ in its range and $\mathcal{R}$ is a minimin and maximax domain. Then M is not strategy-proof.*

Proof. Let M be a strategy-proof 2-vetoers mechanism with $C \in \mathcal{A}_k$ in its range. Let $j \in N$ be any vetoer and $R_j \in \mathcal{R}$ be any preference with $top(R_j, \Delta\mathcal{A}_k) = C$. By Lemma 2, we have $O_{-j}(R_j) = \{C\}$. Consider any other vetoer $h \in N$. Clearly, h cannot veto any candidate in C whenever j declares R_j, contradicting the assumption that M is a 2-vetoers mechanism. ■

Note that the proof of Theorem 1 does not use the full strength of the 2-vetoers condition. The proof only requires the existence of *one* vetoer j and of some $h \neq j$ with the ability to veto *one* of the candidates in C (where C can be any sure committee in the range of M).

5.2 Applications

The next result is a direct corollary of Theorem 1 for deterministic mechanisms. A mechanism M has a **sure range** if for all $R_N \in \mathcal{D}^n$, $M(R_N) = C$ for some $C \in \mathcal{A}_k$. A mechanism M **only considers the ranking of sure committees** if for all $R_N, R'_N \in \mathcal{D}^n$ that induce the same rankings over sure committees, $M(R_N) = M(R'_N)$. A mechanism M is **deterministic** if it satisfies the two last properties.

Corollary 1. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a deterministic 2-vetoers mechanism and $\mathcal{R}$ is a minimin and maximax domain. Then M is not strategy-proof.*

Proof. A deterministic 2-vetoers mechanism is a 2-vetoers mechanism with a sure committee in its range. Thus, Theorem 1 applies. ■

Note that the sure range condition alone is sufficient to obtain the above impossibility. Corollary 1 might not come as a surprise given the characterization of deterministic strategy-proof mechanisms on $\mathcal{R}_{add}$ in (Barberà et al., 2005, Proposition 2).[9] Corollary 1 holds on much smaller domains than $\mathcal{R}_{add}$ however. The impossibility in Corollary 1 is therefore stronger than the one that could be derived from Barberà et al. (2005, Proposition 2).

Theorem 1 also has negative implications for the efficiency of a strategy-proof 2-vetoers mechanism. Consider the following weakening of Pareto efficiency. A mechanism M satisfies **minimal sure unanimity** if there exists *at least one* committee $C \in \mathcal{A}_k$ for which $top(R_i, \Delta\mathcal{A}_k) = C$ for all $i \in N$ implies $M(R_N) = C$. We then have the following impossibility.

Corollary 2. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a 2-vetoers mechanism that statisfies minimal sure unanimity and $\mathcal{R}$ is a minimin and maximax somain. Then M is not strategy-proof.*

Proof. For any $C \in \mathcal{A}_k$, there are many $R^C_N \in \mathcal{R}$ such that C is the unique best committee for all $i \in N$. Thus, minimal sure unanimity implies $M(R^C_N) = C$, and C is in the range of M. But then by Theorem 1, M cannot be a strategy-proof 2-vetoers mechanism. ■

[9] Barberà et al. (2005, Proposition 2) show that the class of strategy-proof selection mechanisms on $\mathcal{R}_{add}$ is a subset of the mechanisms known as *voting by committees*. If a voting by committees mechanism is a 2-vetoers mechanism, then the two vetoers i and j are in all *winning coalitions* (Barberà et al., 2005). But then when $C^i := top(R_i, range(M)) \neq top(R_j, range(M)) =: C^j$, the chosen committee is $C^* \subseteq C^i \cap C^j$, which implies $\#C^* < k$, a contradiction.

As mentioned in the Introduction, results showing that strategy-proofness is incompatible with unanimity requirements in probabilistic mechanisms date back to Hylland (1980). However, Corollary 1 is independent from the results mentioned in the Introduction. These results either (i) hold on domains that are larger than or independent from the smallest minimin and maximax domain or (ii) rely on unanimity conditions that are stronger than or independent from minimal sure unanimity. Corollary 1 does not imply any of these results either because it uses the 2-vetoers condition.

6 No probability thresholds in strategy-proof 2-vetoers mechanisms

In this section, I show that if M is a strategy-proof 2-vetoers mechanism, then one limit point of the range of M must be a lottery that selects some candidate t with probability zero (Theorem 2). This implies that for every $\epsilon > 0$, there exists a lottery L in the range of M with $0 < L(t) < \epsilon$. Equivalently, if for all $t \in A$ there exists a threshold $\epsilon_t > 0$ such that t is never chosen with a positive probability smaller than ϵ_t, then a 2-vetoers mechanism cannot be strategy-proof.

Theorem 2 may seem innocuous as there is *a priori* no reason to impose such a threshold. It however implies that a large class of mechanisms constructed from extensive game forms violate strategy-proofness (Corollary 3). These extensive game forms include many that are used in practice, notably in jury selection procedures.

6.1 Main result

The proof of Theorem 2 relies on two lemmas, Lemmas 3 and 4. Both lemmas hold on *negative leximin* domains. As with minimin preferences, voters with negative leximin preferences are primarily concerned with minimizing the selection probability of a "worst" candidate. But if the selection probability of the "worst" candidate is fixed, then negative leximin voters become primarily concerned with minimizing the selection probability of the "*second* worst" candidate, and so on.

More precisely, Lemmas 3 and 4 require domains containing preferences that are close to a lexicographic assessment of any lottery for up to $(a-k)$ of the "worst" candidates (recall that a is the number of candidates and k the number of committee members that are to be selected). It is also important that some of these preferences satisfy the defining properties of a maximax domain for some $t \in A$ whenever the selection probability of the candidates these preferences treat in a leximin fashion is unchanged. The importance of the two last requirements will become clear in the proof of Lemma 3.

For any set S, let $\#S$ denote the cardinality of S. For any strict ordering $\succ$ of a finite set S with $s := \#S$, let $\succ_1, \succ_2, \ldots, \succ_s$ denote respectively the best element in S according to $\succ$, the second best element in S according to $\succ$, ..., the worst element in S according to $\succ$.

Domain Property 3 (Negative leximin)**.** A domain of preferences $\mathcal{R}$ on the lotteries in $\Delta\mathcal{A}_k$ is *negative leximin* if for any subset of candidate $X \subset A$ with $x := \#X \leq (a-k)$, any strict ordering $\succ$ of the candidates in X, any $t \in A \backslash X$, and any $\epsilon > 0$, there exists $R^\epsilon \in \mathcal{R}$ such that for all $L, L' \in \Delta\mathcal{A}_k$,

$$\begin{bmatrix} \left[L(\succ_x) < L'(\succ_x) - \epsilon\right] \text{or} \\ \left[L(\succ_x) = L'(\succ_x), L(\succ_{x-1}) < L'(\succ_{x-1}) - \epsilon\right] \text{ or} \\ \vdots \\ \left[L(\succ_x) = L'(\succ_x), \ldots, L(\succ_1) < L'(\succ_1) - \epsilon\right] \end{bmatrix} \Rightarrow [L \; P^\epsilon \; L'] \tag{17}$$

and

$$\begin{bmatrix} \left[L(\succ_x) = L'(\succ_x), \ldots, L(\succ_1) = L'(\succ_1)\right] \text{ and} \\ \left[L(t) > L'(t) + \epsilon\right] \end{bmatrix} \Rightarrow [L \; P^\epsilon \; L']\,. \tag{18}$$

Example 3 ($\mathcal{R}_{add}$ is a negative leximin domain)**.** For any X, any $\succ$, and any $t \in A \backslash X$, consider any preference $R^r \in \mathcal{R}_{add}$ defined by

(a) $u^r(t) = r$,

(b) $u^r(b) = 0$ for all $b \in A \backslash (X \cup \{t\})$, and

(c) $u^r(\succ_h) = -(r^{(h+1)})$, for all $h \in \{1, \ldots, x\}$.

For any $\epsilon > 0$, there exists r sufficiently large such that R^r satisfies both (17) and (18). A formal proof of this claim may be found in the Appendix.

For any candidate $t \in A$, there is a **probability threshold** $\epsilon_t > 0$ **for** t if $L(t) > 0$ implies $L(t) > \epsilon_t$ for all $L \in range(M)$. Lemma 3 below shows that in the presence of probability thresholds, vetoers can generate *singleton* option sets containing any sure committee by reporting appropriate preferences. The proof of Lemma 3 proceeds by induction on $A \backslash C$. Let $j, h \in N$ be two vetoers and C be j's best committee. Informally, the proof shows that if there are probability thresholds for all $t \in A$, then for larger and larger subsets of $A \backslash C$, j can reveal particular preferences which guarantee that no candidate in the subset is ever chosen with positive probability. The lemma then follows by strategy-proofness.

For a single $t \in A \backslash C$, this is true because j is a vetoer. Now consider any $t, t' \in A \backslash C$ with $t \neq t'$. Because the domain is negative leximin, there exists a preference $R_h^{t,t'}$ such that h cares primarily about *min*imizing the selection probability of t, and secondarily about *max*imizing the selection probability of t'. By an argument similar to the one used in Lemma 1, when such a preference is sufficiently extreme (i.e., for ϵ sufficiently small in the definition of a negative leximin domain), the selection probability of t must tend to zero (otherwise h would want to veto t). But because of the threshold assumption, this implies that t's selection probability *is* actually zero for some sufficiently extreme preference. That is, h effectively vetoes t when reporting this extreme preference.

When j has a negative leximin preference $R_j^{t,t'}$ that focuses on *min*imizing the selection probability of both t and t', the option set generated by j cannot contain lotteries in which the selection probability of t' is positive. Otherwise, t' is selected with positive probability when everyone but j reports $R_h^{t,t'}$ (Proposition 2) and j would prefer to veto t' because h vetoes t by reporting $R_h^{t,t'}$.

Thus both vetoers can in fact veto *two* different candidates. Extending the argument by induction, if j has a preference that focuses on *min*imizing the selection probability of t, t' and t'', then the three candidates must be vetoed. Otherwise, whenever h vetoes t and t' while caring about *max*imizing the selection probability of t'' (which is possible by the previous step) and everyone but h and j reports the same preferences as h, j would be better off vetoing t'' than revealing her true preference.

Lemma 3. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a strategy-proof 2-vetoers mechanism and $\mathcal{R}$ is a negative leximin domain. Let $j \in N$ be a vetoer for M. If for all $t \in A$ there exists a probability threshold $\epsilon_t > 0$, then for all $C \in \mathcal{A}_k$ there exists $R_j^* \in \mathcal{R}$ such that $O_{-j}(R_j^*) = \{C\}$.*

Proof. We need to show that there exists $R_j^* \in \mathcal{R}$ such that for all $t \in A \backslash C$

$$L(t) = 0 \qquad \text{for all } L \in O_{-j}(R_j^*). \tag{19}$$

Let $X = A \backslash C$ and $\succ$ be any strict ordering of X. Because $\mathcal{R}$ is a negative leximin domain, for all $\epsilon > 0$ there exists $R_j^\epsilon \in \mathcal{R}$ that ranks lotteries by lexicographic order of the selection probability of candidates in X as defined in (17).[10] We will prove that there exists $\epsilon > 0$ such that R_j^ϵ satisfies (19). The argument is by induction on the elements of X. We provide the two first steps in detail.

[10] Note that $\#C = k$ and, hence, $\#X \leq (a - k)$, in accordance with the definition of a negative leximin domain. Here the choice of $t \in C$ in (18) is irrelevant.

Step 1 : There exists $\epsilon > 0$ such that $L(\succ_x) = 0$ for all $L \in O_{-j}(R_j^\epsilon)$.

By the threshold assumption, it is sufficient to show that for any $\epsilon > 0$,

$$L(\succ_x) \leq \epsilon \qquad \text{for all } L \in O_{-j}(R_j^\epsilon). \tag{20}$$

The claim in Step 1 then follows by choosing ϵ with $0 < \epsilon < \tau_{\succ_x}$, where $\tau_{\succ_x}$ is the threshold for $\succ_x$.

The proof of (20) is similar to the proof of Lemma 1. Recall that because j is a vetoer, there exists $R_j^{\succ_x}$ such that $L(\succ_x) = 0$ for all $L \in O_{-j}(R_j^{\succ_x})$. By strategy-proofness, j can never benefit from declaring $R_j^{\succ_x}$ instead of her true preference R_j^ϵ. In particular, for any $L \in O_{-j}(R_j^\epsilon)$, voter j must weakly prefer L to the worst possible lottery for which $L(\succ_x) = 0$, say $\underline{L}$. By the definition of a negative leximin preference in (17), if $\underline{L}(\succ_x) < L(\succ_x) - \epsilon$ then $\underline{L}(\succ_x)\ P_j^\epsilon\ L(\succ_x) - \epsilon$. Because $L(\succ_x)\ R_j^\epsilon\ \underline{L}(\succ_x) - \epsilon$, we thus have $\underline{L}(\succ_x) \geq L(\succ_x) - \epsilon$. But because $\underline{L}(\succ_x) = 0$, this implies that $\epsilon \geq L(\succ_x)$, the desired result.

Step 2 : There an exists $\epsilon > 0$ such that $L(\succ_x) = L(\succ_{x-1}) = 0$ for all $L \in O_{-j}(R_j^\epsilon)$.

By Step 1, it is sufficient to show that there exists an ϵ with $0 < \epsilon < \tau_{\succ_x}$ such that

$$L(\succ_{x-1}) = 0 \qquad \text{for all } L \in O_{-j}(R_j^\epsilon). \tag{21}$$

Applying the threshold assumption again, (21) holds provided that for all ϵ with $0 < \epsilon < \tau_{\succ_x}$,

$$L(\succ_{x-1}) \leq \epsilon \qquad \text{for all } L \in O_{-j}(R_j^\epsilon). \tag{22}$$

In order to derive a contradiction, assume that there exists an ϵ with $0 < \epsilon < \tau_{\succ_x}$ and some $\gamma > 0$ such that

$$L^\epsilon(\succ_{x-1}) = \epsilon + \gamma \qquad \text{for some } L^\epsilon \in O_{-j}(R_j^\epsilon). \tag{23}$$

Let h be any vetoer with $h \neq j$. Because $\mathcal{R}$ is a negative leximin domain, for all $\delta > 0$ there exists $R_h^\delta \in \mathcal{R}$ satisfying (17) for $X = \{\succ_x\}$, and satisfying (18) for $t = (x-1)$ (where δ replaces ϵ in both (17) and (18)). Because h is a vetoer, the argument in Step 1 applies to h, and for any δ with $0 < \delta < \tau_{\succ_x}$,

$$M(R_h^\delta, R_{-h})(\succ_x) = 0 \qquad \text{for all } R_{-h}. \tag{24}$$

Together, (23) and $0 < \epsilon < \tau_{\succ_x}$ imply

$$L^\epsilon(\succ_{x-1}) = \epsilon + \gamma \text{ and } L^\epsilon(\succ_x) = 0. \tag{25}$$

But then, because $L^\epsilon \in O_{-j}(R_j^\epsilon)$, by Proposition 2,

$$M\left(R_j^\epsilon, R_h^\delta, \ldots, R_h^\delta\right)\ R_h^\delta\ L^\epsilon. \tag{26}$$

Because $\gamma > 0$, there exists δ with $0 < \delta < \min\{\gamma, \tau_{\succ_x}\}$. Observe that by the construction of negative leximin preferences in (17) and by the threshold assumption, for any such δ,

$$L^\epsilon\ P_h^\delta\ L \text{ for any } L \text{ with } L(\succ_x) > 0 \text{ or with } L(\succ_x) = 0 \text{ and } L(\succ_{x-1}) \leq \epsilon. \tag{27}$$

Hence, (26) and (27) imply that

$$M\left(R_j^\epsilon, R_h^\delta, \ldots, R_h^\delta\right)(\succ_x) = 0 \quad \text{and} \quad M\left(R_j^\epsilon, R_h^\delta, \ldots, R_h^\delta\right)(\succ_{x-1}) > \epsilon. \tag{28}$$

Because j is a vetoer, by (24), there exists $R_j^{\succ_{x-1}}$ such that

$$M\left(R_j^{\succ_{x-1}}, R_h^\delta, \ldots, R_h^\delta\right)(\succ_{x-1}) = M\left(R_j^{\succ_{x-1}}, R_h^\delta, \ldots, R_h^\delta\right)(\succ_x) = 0. \tag{29}$$

But then by the construction of a negative leximin preference,

$$M\left(R_j^{\succ_{x-1}}, R_h^\delta, \ldots, R_h^\delta\right)\ P_j^\epsilon\ M\left(R_j^\epsilon, R_h^\delta, \ldots, R_h^\delta\right), \tag{30}$$

contradicting strategy-proofness.

The remaining steps follow the same inductive pattern. ■

Using Lemma 3, we can prove the following result.

Lemma 4. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ a strategy-proof 2-vetoers mechanism and $\mathcal{R}$ is a negative leximin domain. If for all $t \in A$ there exists a probability threshold $\epsilon_t > 0$, then every vetoer is a dictator.*

Proof. Let $j \in N$ be any vetoer and $h \in N$ be any other vetoer. Consider any $R_N \in \mathcal{R}^n$. Because R_j satisfies the expected utility axioms, there exists a sure committee $C \in \mathcal{A}_k$ such that $C \in top(R_j, \Delta\mathcal{A}_k)$. By Lemma 3, there exists R_j^* such that $M(R_j^*, R_{-j}) = C$. If $M(R_N) \notin top(R_j, \Delta\mathcal{A}_k)$, we have $M(R_j^*, R_{-j})\ P_j\ M(R_N)$, contradicting strategy-proofness. Hence, we must have $M(R_N) \in top(R_j, \Delta\mathcal{A}_k)$ and thus j is a dictator. ■

We can now prove the main theorem of this section.

Theorem 2. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a strategy-proof 2-vetoers mechanism and $\mathcal{R}$ is a negative leximin domain. Then there exists $t \in A$ such that, for all $\epsilon > 0$,*

$$0 < L^\epsilon(t) \leq \epsilon \qquad \textit{for some } L^\epsilon \textit{ in the range of } M. \tag{31}$$

Proof. Because M always selects a well-defined lottery over committees, there exists $t \in A$ and $L \in \Delta\mathcal{A}_k$ such that L is in the range of M and $L(t) > 0$. Let

$$\ell := \inf \big\{ p \in (0,1] \;\big|\; p = L^*(t) \text{ for some } L^* \in range(M) \big\}.$$

If $\ell > 0$, then ℓ is a probability threshold for t, and by Lemma 4 every vetoer is a dictator for M. But then, because there are at least two vetoers, there are at least two dictators, which is impossible.[11] Thus, we must have $\ell = 0$ and (31) holds. ■

The fact that there exists a lottery L^* with $L^*(t) = 0$ that is a limit point of the range follows from Theorem 2 by the Bolzano-Weirstrass Theorem (see, e.g., Rudin, 1976, Theorem 3.6(b)).[12]

6.2 Applications

The rest of this section illustrates the usefulness of Theorem 2 by showing how it rules out strategy-proofness for a wide class of mechanisms constructed from sequential procedures. Constructing direct mechanisms from sequential procedures is common in market and mechanism design. A simple example in the case of a selection mechanism is presented below.

Example 4 (Repeatedly veto the worst)**.** Chose two vetoers $j, h \in N$. For any profile of preferences $R_N \in \mathcal{R}^n_{add}$, select a committee by repeating the following two steps until there are only k candidates left.

(i) Remove the worst candidate according to u_j among the candidates in N that have not yet been removed (break ties randomly).

(ii) Remove the worst candidate according to u_k among the candidates in N that have not yet been removed (break ties randomly).

To every $R_N \in \mathcal{R}^n_{add}$, the above algorithm associates a unique lottery in $\Delta\mathcal{A}_k$ and therefore defines a (direct) mechanism $M : \mathcal{R}^n_{add} \to \Delta\mathcal{A}_k$.

[11] Alternatively, if one of the vetoers is a dictator, then the other vetoers cannot always veto every alternative. For example, when a vetoer has a favorite sure committee, a second vetoer cannot veto any candidate in the dictator's favorite sure committee. Hence, the mechanism is not a 2-vetoers mechanism, which again yields a contradiction.

[12] By Theorem 2, there exists a sequence of lotteries $\{L^r\}_{r=1}^{\infty}$ in the range such that for some $t \in A$ we have $L^r(t) > 0$ for all $r > 0$ and $\lim_{r\to\infty} L^r(t) = 0$. By the Bolzano-Weirstrass Theorem, this sequence has a converging subsequence (the sequence is bounded because all lotteries belong to a 2^a-dimensional simplex). Clearly, the limit of that subsequence must be a lottery L^* with $L^*(t) = 0$. Also, L^* is a limit point of the range.

The algorithm in Example 4 can be viewed as an extensive game form in which the strategies of the players have been fixed as a function of their preferences.

In general, let a (selection) **procedure** be an extensive game form Γ in which

(a) the set of players is $I \coloneqq N \cup \{Nature\}$ and

(b) every terminal node is a committee $C \in \mathcal{A}_k$.

For any domain of profiles $\mathcal{D}^n$ and any procedure Γ, a **generalized strategy profile** g associates every preference profile $R_N \in \mathcal{D}^n$ with a strategy profile $g(R_N)$ in the space of strategy profiles of Γ. A mechanism $M^{\Gamma}_{g,\rho}$ is **constructed from** procedure Γ if there exists a generalized strategy profile g and an assignment of probabilities ρ for Nature's moves such that

$$M^{\Gamma}_{g,\rho}(R_N) = \Gamma\big(g(R_N), \rho\big) \qquad \text{for all } R_N \in \mathcal{D}^n, \tag{32}$$

where $\Gamma\big(g(R_N), \rho\big)$ is the lottery resulting from Γ when strategy profile $g(R_N)$ is played and the probabilities associated with Nature's move are ρ.

For example, the mechanism described in Example 4 is constructed from the extensive game form in which two vetoers take turns vetoing candidates, which is similar in spirit to procedures used in Mueller (1978), Moulin (1981) and in jury selection. In Example 4, the generalized strategy is what Moulin (1981) defines as the *prudent* strategy. At each decision node, a vetoer j chooses the action that maximizes his or her utility assuming that all further actions will be chosen in such a way as to minimize her utility.

As in Example 4, procedures used to construct mechanisms are often **finite**, in the sense that they have a finite number of *nodes*. For mechanisms constructed from such procedures, the following is an implication of Theorem 2.

Corollary 3. *Suppose that $M : \mathcal{R}^n \to \Delta\mathcal{A}_k$ is a 2-vetoers mechanism constructed from a finite procedure and $\mathcal{R}$ is a negative leximin domain. Then M is not strategy-proof.*

Proof. Because there is a finite number of nodes in Γ, there is a finite number of strategy profiles in Γ. Because for all $R_N \in \mathcal{R}^n$, $M^{\Gamma}_{g,\rho}(R_N) = \Gamma\big(s_N, \rho\big)$ for some strategy profile s_N, there is also a finite number of lotteries in the range of M. Thus, there must exist probability thresholds for all $t \in A$. But then, Theorem 2 applies because M is a 2-vetoers mechanism. Hence, M cannot be strategy-proof. ■

A special class of finite procedures extensively used in jury selection feature two voters $j, h \in N$ (the prosecutor and the defense) sequentially vetoing candidates (potential jurors) among sets of candidates drawn at random from A (the pool).[13] Corollary 3 shows that any 2-vetoers mechanism constructed from such a procedure cannot be strategy-proof.

Finally, observe that Corollary 3 implies that on a negative leximin domain, no finite procedure exists for which two players can both (a) veto any candidate and (b) have dominant strategies.

makes it impossible for finite procedures in which two players can veto a candidate to have dominant strategies on a negative leximin domain. By a revelation principle argument, if such a procedure Γ has dominant strategies for some choice ρ of nature's move, then there exists a generalized strategy g^* that makes $M^{\Gamma}_{g^*,\rho}$ a 2-vetoers strategy-proof mechanism, contradicting Corollary 3.[14]

7 Concluding remarks

Many open questions remain. One concerns the necessity of the sure range condition in Theorem 1 and of the threshold conditions in Theorem 2. Whether there exists any strategy-proof 2-vetoers mechanism in the absence of these conditions is unknown.

Another question is whether strategy-proof mechanisms exist for weaker veto conditions. In a 2-vetoers mechanism, vetoers are allowed to veto a single candidate, but this candidate can be any candidate. What happens when vetoers can only veto a subset of candidates is another open question.

Finally, Theorems 1 and 2 rely extensively on domains containing preferences that are *arbitrarily* close to lexicographic, maximax, and minimin preferences. How much flexibility can be gained by further constraining the domain of preferences has not been determined.[15] The proofs of Theorems 1 and 2 suggest that any possibility result would depend on a combination of restrictions on the richness of (i) the domain of preferences of the vetoers and (ii) the range of the mechanism.

[13] See the references in the Introduction.

[14] See Van der Linden (2016) for more details and Hylland (1980, Section 4) for similar results.

[15] In this respect, see Dutta et al. (2006) who study the extension of an impossibility result of Hylland (1980) to domains in which utility functions must take values in a discrete utility grid.

Appendix

$\mathcal{R}_{add}$ is a negative leximin domain

It is easy to see that R^r satisfies (18) for all $r > 0$. Let us illustrate the argument for the first part of (17). That is, we want to show that

$$[L(\succ_x) < L'(\succ_x) - \epsilon] \Rightarrow L \ P^r \ L'. \tag{A.1}$$

For any $\epsilon > 0$ and any $L, L' \in \Delta\mathcal{A}_k$ with

$$L(\succ_x) < L'(\succ_x) - \epsilon, \tag{A.2}$$

we need to find some $r_\epsilon > 0$ which guarantees that

$$\sum_{S \in \mathcal{A}_k} L(S) \sum_{t \in S} u^{r_\epsilon}(t) > \sum_{S \in \mathcal{A}_k} L'(S) \sum_{t \in S} u^{r_\epsilon}(t). \tag{A.3}$$

Note that (A.3) is equivalent to

$$\sum_{t \in \{1,\dots,a\}} L(t) u^{r_\epsilon}(t) > \sum_{t \in \{1,\dots,a\}} L'(t) u^{r_\epsilon}(t),$$

which implies that

$$\big(L(\succ_x) - L'(\succ_x)\big) u^{r_\epsilon}(\succ_x) > \sum_{t \in A \setminus \{\succ_x\}} L'(t) u^{r_\epsilon}(t) - \sum_{t \in A \setminus \{\succ_x\}} L(t) u^{r_\epsilon}(t). \tag{A.4}$$

First, consider the left-hand side of (A.4). By the construction of R^r and (A.2),

$$\big(L(\succ_x) - L'(\succ_x)\big) u^{r_\epsilon}(\succ_x) < (-\epsilon)\big(-({r_\epsilon}^{(x+1)})\big) = \epsilon {r_\epsilon}^{(x+1)}. \tag{A.5}$$

Now, consider the first term on the right-hand side of (A.4). Observe that for all $L \in \Delta\mathcal{A}_k$, the sum of the candidates' selection probabilities is $\sum_{t \in \{1,\dots,a\}} L(t) = k$. Thus, by the construction of R^r,

$$\sum_{t \in A \setminus \{\succ_x\}} L'(t) u^{r_\epsilon}(t) \leq r_\epsilon \sum_{t \in A \setminus \{\succ_x\}} L'(t) \leq r_\epsilon k.$$

Finally, consider the second term on the right-hand side of (A.4). We have

$$\left(- \sum_{t \in A \setminus \{\succ_x\}} L(t) u^{r_\epsilon}(t)\right) \leq -\big({r_\epsilon}^{(x)}\big) \left(- \sum_{t \in A \setminus \{\succ_x\}} L(t)\right) \leq {r_\epsilon}^{(x)} k.$$

From the two last displayed inequalities we obtain

$$\sum_{t\in A\setminus\{\succ_x\}} L'(t)u^{r_\epsilon}(t) - \sum_{t\in A\setminus\{\succ_x\}} L(t)u^{r_\epsilon}(t) \leq k\big(r_\epsilon + {r_\epsilon}^{(x)}\big). \tag{A.6}$$

Together, (A.5) and (A.6) imply that (A.4) holds provided

$$\epsilon {r_\epsilon}^{(x+1)} > k\big(r + {r_\epsilon}^{(x)}\big)$$

or, equivalently,

$$\frac{r_\epsilon^x}{1 + r_\epsilon^{(x-1)}} > \frac{k}{\epsilon}. \tag{A.7}$$

Because the left-hand side of (A.7) tends to ∞ as $r_\epsilon \to \infty$, there must exist a r_ϵ sufficiently large such that this inequality holds.

It is relatively straightforward to adapt the above argument to the $(x-1)$ other parts of (17). For example, for

$$\big[L(\succ_x) = L'(\succ_x),\ L(\succ_{x-1}) > L'(\succ_{x-1}) - \epsilon\big] \Rightarrow L\ P^r\ L$$

the argument can be repeated with the second line of (A.4) simplified to

$$\begin{aligned}&\big(L(\succ_{x-1}) - L'(\succ_{x-1})\big)u^{r_\epsilon}(\succ_{x-1})\\ &\qquad > \sum_{t\in A\setminus\{\succ_x,\succ_{x-1}\}} L'(t)u^{r_\epsilon}(t) - \sum_{t\in A\setminus\{\succ_x,\succ_{x-1}\}} L(t)u^{r_\epsilon}(t)\end{aligned}$$

because the terms $L'(\succ_x)u^{r_\epsilon}(\succ_x)$ and $L(\succ_x)u^{r_\epsilon}(\succ_x)$ cancel out.

After repeating this argument for each of the x components of (17), one obtains a set of thresholds $\{r_\epsilon^x, r_\epsilon^{x-1}, \ldots, r_\epsilon^1\}$ for each of the components of (17) to hold. Recall that (18) is satisfied whenever $r > 0$. Because x is finite, $\bar{r}_\epsilon := \max\{0.1, r_\epsilon^x, r_\epsilon^{x-1}, \ldots, r_\epsilon^1\}$ is well defined. Then, because the left-hand side of (A.7) and its counter-parts for the other components of (17) are increasing in r_ϵ, $R^{\bar{r}_\epsilon}$ satisfies (17) and (18), the desired result.

References

Barberà, S., Massó, J., Neme, A., 2005. Voting by committees under constraints. Journal of Economic Theory 122, 185–205.

Barberà, S., Peleg, B., 1990. Strategy-proof voting schemes with continuous preferences. Social Choice and Welfare 7, 31–38.

Barberà, S., Sonnenschein, H., Zhou, L., 1991. Voting by committees. Econometrica 59, 595–609.

Benoît, J.P., 2002. Strategic manipulation in voting games when lotteries and ties are permitted. Journal of Economic Theory 102, 421–436.

Chatterji, S., Roy, S., Sen, A., 2012. The structure of strategy-proof random social choice functions over product domains and lexicographically separable preferences. Journal of Mathematical Economics 48, 353–366.

de Clippel, G., Eliaz, K., Knight, B., 2014. On the selection of arbitrators. American Economic Review 104, 3434–3458.

Dutta, B., Peters, H., Sen, A., 2006. Strategy-proof cardinal decision schemes. Social Choice and Welfare 28, 163–179.

Flanagan, F.X., 2015. Peremptory challenges and jury selection. Journal of Law and Economics 58, 385–416.

Hylland, A., 1980. Strategy proofness of voting procedures with lotteries as outcomes and infinite sets of strategies. Unpublished manuscript, Department of Economics, University of Oslo .

Ju, B.G., 2003. A characterization of strategy-proof voting rules for separable weak orderings. Social Choice and Welfare 21, 469–499.

Le Breton, M., Weymark, J.A., 1999. Strategy-proof social choice with continuous separable preferences. Journal of Mathematical Economics 32, 47–85.

Maskin, E., 1999. Nash equilibrium and welfare optimality. Review of Economic Studies 66, 23–38.

Moulin, H., 1981. Prudence versus sophistication in voting strategy. Journal of Economic Theory 24, 398–412.

Mueller, D.C., 1978. Voting by veto. Journal of Public Economics 10, 57–75.

Nandeibam, S., 2012. The structure of decision schemes with cardinal preferences. Review of Economic Design 17, 205–238.

O'Malley, J.W., 2015. Catholic History for Today's Church: How our Past Illuminates our Present. Rowman and Littlefield, Lanham, MD.

Rudin, W., 1976. Principles of Mathematical Analysis. McGraw-Hill, New York.

Schummer, J., 1999. Strategy-proofness versus efficiency for small domains of preferences over public goods. Economic Theory 13, 709–722.

Sen, A., 1970. The impossibility of a Paretian liberal. Journal of Political Economy 78, 152–157.

Van der Linden, M., 2016. Bounded rationality and the choice of jury selection procedures. Unpublished manuscript, Department of Economics, Vanderbilt University .

Yuval, F., 2002. Sophisticated voting under the sequential voting by veto. Theory and Decision 53, 343–369.