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Capitalization, Decentralization, and Intergenerational Spillovers in a Tiebout Economy with a Durable Public Good

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Abstract: We consider an overlapping generations model with a Durable Local Public Good (DLPG). We establish a Tiebout Theorem (equilibrium exists and is first best) as well as a Second Welfare Theorem in this dynamic DLPG economy. We define conditions under which local provision of durable public goods results in the full internalization of the intergenerational spillovers that durability entails. In contrast, when durable public goods are provided by the national government, internalization does not take place and underprovision of public goods results. This sets up an institutional tradeoff between national and local provision of public goods that balances the relative strength of intergenerational and interjurisdictional spillovers. (JEL Classification: H4, D9, H0, D7)

Keywords: Durable Local Public Goods, Capitalization and Intergenerational Spillover Effects, Dynamic Tiebout Equilibrium, Welfare Analysis.

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1 Introduction

Many of the goods and services provided by governments are intended for immediate consumption. Examples include health services, food assistance, and redistributional transfers. Others, such as highways, public buildings, and research and development, are forms of public capital intended to last for many years. Providing such Durable Public Goods (DPGs) at efficient levels is difficult not only because of free riding, but also because they produce intergenerational spillovers. Even if all of the agents alive in a given period truthfully revealed their marginal benefits and made the appropriate Lindahl contributions to DPG provision, we would still have the problem of how to make these agents take into account the benefits that future generations receive from the stock of DPGs they leave behind. Clearly, unless this intergenerational spillover is somehow internalized, DPGs will be systematically underprovided.

Tiebout (1956) observed that many public goods, such as fire protection, libraries, and primary education, are provided by cities, counties, and states, instead of national governments. He suggested that this creates a kind of market in which different localities offer varying bundles of taxes and local public goods. Agents evaluate these alternatives, and by choosing their most preferred locations, reveal their willingness to pay for public good. Thus, local provision of public goods and "voting with one's feet" solves the free riding problem. A large literature exploring static coalition formation, optimal sorting of agents by taste, and overcoming free riding though tax/public good bundles offered by competing jurisdictions has since developed. See Conley and Wooders (1998, 2001) for an extensive discussion of this branch of the Tiebout literature.

Of course, many of the public goods provided locally are also durable. Examples include city streets, storm drains, and public school buildings. Although competition between jurisdictions seems to result in efficient provision of nondurable public goods in static models, how this result might be extended to a dynamic economy in order to deal with intergenerational spillovers is largely an open question.

To address this, we consider a simple model with multiple jurisdictions and an overlapping generations demographic structure. We emphasize the role of intergenerational spillovers by assuming that every jurisdiction is identical and every agent has the same taste for public good. Agents go through their life cycle by buying land in a particular jurisdiction when young, and then selling it when old. The generation that lives in a jurisdiction in any given period enjoys the services of the Durable Local Public Good (DLPG) stock they inherit from the previous generation and then, in turn, chooses how much to add to the stock to be inherited by the next generation.

The main contribution of this paper is to establish reasonable conditions under which the value of any DLPG left at the end of a period is fully *capitalized* into the price of land. Such capitalization causes the present generation to internalize the intergenerational spillover and therefore to invest optimally. This result does not depend upon the presence of an outside offer such as undeveloped land at the jurisdictions' periphery in order to pin down land prices.

In contrast, we show that capitalization does not take place when decision making is centralized. This is because when DPG is provided at the national level, all the jurisdictions are identical and the price of housing is not responsive to public investment. In this sense, decentralization is both necessary and sufficient to ensure first best outcomes in the presence of intergenerational spillovers. At a formal level, this paper provides a Tiebout Theorem (equilibrium exists and is first best) as well as a Second Welfare Theorem for a DLPG economy.

Our analysis also has interesting implications for the impact of housing price booms and busts. While competition between jurisdictions pins down the *relative* price between different locations, it says nothing about what the *absolute* prices must be. We show that moderate booms or busts in housing prices can take place without affecting either the equilibrium or efficient levels of DLPG in the jurisdictions. The effect of such booms and busts is simply to redistribute wealth across generations. However, if the absolute price level of housing becomes too high or too low (compared to income), it can become impossible to support the efficient DLPG levels through prices and so the First Welfare Theorem fails. Fortunately, prices seen in the real world seem to fall within the range that supports efficient outcomes. It is nevertheless noteworthy that both relative and absolute prices play a role in achieving socially optimal policies.

This paper attempts to tie together the literature on intergenerational goods and Tiebout economies. In general, intergenerational goods have been treated as private goods that are voluntary transferred either forward or backward across generations in the context of a dynamic and unified (that is a single jurisdiction) economy. The central question of this literature is: assuming present generations are selfish, why should they make such transfers? See, among others, Kotlikoff, Persson, Svensson (1988), Rangel (2003, 2005), Boldrin and Montes (2005), and Hatfield (2008, 2014).[1] There are policy implications for a wide range

[1] It may be possible to induce self interested agents to provide backward intergenerational goods (BIGs) through a game in which all generations play the trigger strategy that they will transfer goods to the currently

of issues including social security, education spending, global warming, and research and development, to name only a few.

Turning now to Tiebout (1956), recall that his main argument was that interjurisdictional competition should lead to efficient provision of public goods. In the early literature especially, the focus was on static coalition formation in economies without land. See, for example, Buchanan (1965), Pauly (1970), McGuire (1974), Berglas (1976), Wooders (1978), and Bewley (1981), or Conley and Wooders (1998) for a survey. Other work has added either divisible land (Rose-Ackerman 1979 and Epple, Filimon and Romer 1984, for example) or indivisible land (Dunz 1985 and Nechyba 1996, for example) to the model. See Konishi (1996) for an excellent discussion of this literature and additional results. The general conclusion of this work is that under some conditions, interjurisdictional competition is sufficient to cause agents to internalize spillovers between agents within the same jurisdiction resulting in optimal contributions to nondurable local public good provision.

From an empirical standpoint, there is a great deal of evidence that capitalization of some type is an important phenomenon. Such studies go back at least as far as the famous paper of Oates (1969), who confirms that both property taxes and spending get capitalized into property values. See Nguyen-Hoang and Yinger (2011) for a recent survey. The correct econometric treatment of this question is quite subtle, however. For example, it is unclear if spending is strongly correlated to public goods quality (especially school quality), and thus, it is not immediate what exactly should be capitalized. See Hanushek (1986), Hayes and Taylor (1998) and Black (1999). How to test for capitalization in a steady state is especially troublesome and we will not attempt to deal with this issue here. We refer the reader to Epple, Zelenitz and Visscher (1978) Yinger (1982, 1995) Brueckner (1982), and Starrett (1997) for enlightening discussions on this topic.

In addition to the papers above, there is a small theoretical literature on capitalization in a static economy. Notable contributions include Wildasin (1979), Stiglitz (1983), Brueckner and Wingler (1984) de Bartolome (1990) and more recently Wildasin and Wilson (1998). For the most part, this work considers the optimality of equilibrium local public good and

old only if the currently old made similar transfers when they were young. As long as these intergenerational transfers grow at least as fast as the interest rate, the agents are best served by not defecting from this strategy. See Rangel (2005) for details. Forward intergenerational goods (FIGs) can also be sustained in equilibrium, but only if they are linked to the provision of BIGs. The problem is that both optimal and nonoptimal levels of BIGs and FIGs can be supported in these games. Thus, although institutions exist that can incentivize selfish generations to make transfers, they do not ensure optimal outcomes.

tax levels, and the response of property values in specific economic contexts. For example, Brueckner and Wingler are concerned about public goods as intermediate inputs, de Bartolome is interested in how peer groups affect the value of school districts, Wildasin looks at how capitalization affects the possibility of risk pooling. It is not immediate how these results might be extended to dynamic economies in which public goods are durable.

The theoretical literature on dynamic Tiebout economies with DLPG is similarly small. The earliest paper of which we are aware is Kotlikoff and Rosenthal (1993) who consider a two period model with two jurisdictions and discover that one should not expect competition to generate efficient provision of such goods.[2] They argue that inefficiency arises because different generations cannot be coordinated at the beginning of time. Glaeser (1996) analyzes a two period, two jurisdiction model with revenue maximizing (Leviathan) local governments making local public good decisions a period before their services are enjoyed under fixed tax rates. He finds that local property taxes provide better incentives for local provisions of public goods than centralized national taxes.[3] The current paper has its origins in a working paper by Conley and Rangel (2001) who treat a two period economy but focus on how intergenerational spillovers might be internalized under different institutional regimes. Finally, in a two period, multiple jurisdiction setting where local governments choose both tax rates and levels of debt, Hatfield (2014) shows that in a land tax regime (comparable to our Lindahl contributions to DLPG provision), the Nash equilibrium under centralization involves no investment in the intergenerational public good by the first generation. On the other hand, while the land tax Nash equilibrium under decentralization yields positive provision, intergenerational good is underprovided relative to the efficient level unless the

[2] It is also worth calling the reader's attention to the literature of dynamic Tiebout models without DLPG or capitalization. See especially Kotlikoff and Raffelhueschen (1991), Glomm and Lagunoff (1999), Benabou (1996), and Brueckner (1997), and more recently Chen, Peng and Wang (2009) and Epple, Romano and Sieg (2012). Schultz and Sjöström (2001) treat a two period, two jurisdiction model with public debt and free mobility. They find that the equilibrium is generally inefficient, but their model does not allow either DLPG or debt to be capitalized into housing prices (also see Schultz and Sjöström, 2004).

[3] Wildasin and Wilson (1996) consider an overlapping generations economy with imperfectly mobile agents but with local public goods that are nondurable. They discover that the capitalization mechanism may not induce efficient provision of local public goods. Similarly, Sprunger and Wilson (1998) consider how the desire of governments to exploit imperfectly mobile households may be expressed when public goods choices are made a period before the goods are consumed. These goods fully depreciate the period they are produced, however, so may have more of a flavor of a standard intergenerational good than of a DLPG.

number of districts goes to infinity. Our paper, in a somewhat more general multiple period overlapping generations framework with multiple jurisdictions, dynamic Tiebout equilibrium under centralization yields zero provision, but with reasonable assumptions, decentralization induces efficient provision even with a finite number of jurisdictions.[4]

2 The Model

Our objective is to show that welfare theorems hold in a competitive DLPG economy, that is, one without transactions costs, market power, strategic behavior, incomplete information, or other distortions. As a result, we choose a straightforward model as a benchmark. There are many interesting ways that this model could be generalized and elaborated, such as allowing for a heterogeneous landscape, divisible land, or agglomerative externalities. We choose not to do so in the current paper in order to increase the transparency of the results.

Consider a simple finite horizon, overlapping generations (OLG) economy with one private consumption good, c, and one DLPG, G. The DLPG is provided by a set of local jurisdictions indexed $j \in \{1, \ldots, J\} \equiv \mathcal{J}$. Each jurisdiction contains L plots of indivisible land.[5]

Time is indexed by $t \in \{1, \ldots, T\} \equiv \mathcal{T}$. In each *ordinary* period, $t \in \{2, \ldots, T-1\} \equiv \mathcal{T}^O$, a generation of two period lived young agents, indexed by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = J \times L$, and endowed with ω units of private good, is born, and a similar set of two period lived old agents sell their land and die. On the other hand, in period $t = 1$, there exists a set of old agents with time index 0 (referred to as the *initial old*) endowed with land only, and in period $t = T$, a final cohort of young agents (referred to as the *terminal young*) are born endowed with ω units of private good but who live only for one period and then die.

There are three main reasons reason for adopting this finite horizon OLG framework. First, it avoids the typical "transfer from infinity" problem often seen in infinite horizon models (Shell 1971) thus permitting us to use quasilinear utility functions which allow for great analytic tractability in characterizing the willingness to pay for the public good (Bergstrom and Cornes 1983). Second, it ensures the validity of using the conventional definition of

[4]In Hatfield (2014), efficient provision with a finite number of jurisdictions can be attained only in the head tax regime. We will return to the issue of efficient provision in Section 5 below after establishing the First Welfare Theorem.

[5]This approach to indivisible land follows Fujita (1985), Dunz (1985) and Nechyba (1996). See also McCallum (1983), Wang (1987), Glomm (1992), and Geanakoplos (2008) for work that introduces land into OLG models.

Pareto optimality without requiring modifications such as forward looking Pareto optimality.[6] Finally, it allows us to compare our results to those obtained in dynamic models of DLPG by Wildasin and Wilson (1996) and Sprunger and Wilson (1998) on an equal footing.

We assume that there is no storage technology for the private good and so the total social endowment in any given period must be divided between investment in DLPG and private good consumption for the young and old agents currently alive.[7]

All agents except the initial old and the terminal young are identical and receive utility from consuming both private good and DLPG in the first period of their lives, but from private good alone obtained from selling their land in the second period. Thus, all agents born between periods 1 and $T-1$ have the following quasilinear utility functions:

$$U(c_{t,t}, c_{t,t+1}, G_t) = c_{t,t} + \beta c_{t,t+1} + V(G_t) \quad \text{for} \quad t = 1, ..., T-1,$$

where V is a strictly increasing and strictly concave C^2 function, $\beta \in (0,1)$ is the exogenous discount factor, and $c_{t,t'}$, is the level of private good consumption for an agent born in period t but consumed in period $t' = t, t+1$. Agents of the initial old and terminal young cohorts have utility functions that account for the timing of their consumption:

$$\begin{aligned} U(c_{0,1}) &= \beta c_{0,1} \\ U(c_{T,T}, G_T) &= c_{T,T} + V(G_T). \end{aligned}$$

We denote additions (or subtractions, in some cases) to the DLPG stock by g and assume one unit of private good produces one unit of DLPG. We assume that DLPG requires one period to build and depreciates over time at a rate of $\delta \in (0,1)$. This implies that DLPG evolves according the following rule:

$$G_t = (1-\delta)(G_{t-1} + g_{t-1}) \text{ for } t = 2, \ldots, T,$$

with an exogenous initial level assumed to be identical across jurisdictions of $G_1 \geq 0$.

Young agents must buy a plot of land from old agents at prices p_t^j and thereafter enjoy the services of the DLPG level that is currently in place. They decide how much private good to consume and how much to add to the existing stock of the DLPG, which in turn, will be

[6]We shall return to both the transfer from infinity and forward looking Pareto optimality issues in Section 7. We will also discuss the difficulty associated with infinite horizon OLG structure and the generality of some of our key results.

[7]This is not essential. See Section 7 for a generalization.

enjoyed by the next generation. In the next period, the now old agents sell their land to the newly born young agents, consume the proceeds, and leave the economy. Old agents do not consume DLPG.

A *feasible allocation* consists of $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ where

$$\begin{aligned}
\mathbf{c} &= (c^1_{0,1}, \ldots, c^I_{0,1}, \ldots, c^i_{t,t+1}, \ldots, c^1_{T-1,T}, \ldots, c^I_{T-1,T}, c^1_{1,1}, \ldots, c^I_{1,1}, \ldots, c^i_{t,t}, \ldots, c^1_{T,T}, \ldots, c^I_{T,T}) \\
\mathbf{g} &= (g^1_1, \ldots, g^J_1, \ldots, g^j_t, \ldots, g^J_T, \ldots, g^J_T) \\
\mathbf{G} &= (G^1_1, \ldots, G^J_1, \ldots, G^j_t, \ldots, G^J_T, \ldots, G^J_T),
\end{aligned}$$

such that:

$$I\omega = \sum_{i \in \mathcal{I}} c^i_{t-1,t} + \sum_{i \in \mathcal{I}} c^i_{t,t} + \sum_{j \in \mathcal{J}} g^j_t \quad \text{for } t \in \mathcal{T} \tag{1}$$

$$G^j_t = (1-\delta)\left(G^j_{t-1} + g^j_{t-1}\right) \quad \text{for } t = 2, \ldots, T \text{ and } G^j_1 = G_1, \text{ for } j \in \mathcal{J}. \tag{2}$$

We will also include nonnegativity constraints on DLPG investment and private good consumption in some of the analysis below:

$$g^j_t \geq 0 \quad \text{for } t \in \mathcal{T} \text{ and } j \in \mathcal{J} \tag{3}$$

$$c^i_{t,t} \geq 0 \text{ and } c^i_{t-1,t} \geq 0 \quad \text{for } t \in \mathcal{T} \text{ and } i \in \mathcal{I}. \tag{4}$$

The reader may notice that we omit describing how agents are allocated to jurisdictions in the definition of feasibility above. Since all agents are identical and there are exactly as many residential locations as agents each period, it will not make any difference. On the planner's side, any mapping of agents to locations gives the same welfare level. On the market side, all agents face the same prices and have the same initial allocations. Thus, equal treatment must prevail. We therefore omit this notational detail in the interest of simplicity.

To summarize, each period t evolves as follows:

Stage 1: Young agents are born with a private good endowment of ω, choose a jurisdiction, j in which to live, purchase a parcel of land from old agents, and enjoy the services of the current DLPG stock G^j_t that they inherit from the previous generation. Simultaneously, old agents sell their land at equilibrium prices, consume the proceeds as $c_{t-1,t}$ and leave the economy. Agents who are old in period 1 simply sell their land and consume the proceeds.

Stage 2: Young agents in each jurisdiction participate in a majority vote over how much to add to (and in some cases, subtract from) the DLPG stock for the next generation.[8] We denote this investment by g_t^j and assume the cost is equally shared over all the agents in the jurisdiction. Note that the investment of the young generation does not affect the level of DLPG that they, themselves, enjoy, but only the levels that are inherited by the young in the next period.[9]

Stage 3: Young agents consume $c_{t,t} = \omega - p_t^j - \frac{g_t^j}{L}$, where p_t^j is the price of a plot of land in jurisdiction j in period t. This is the amount of private good that remains from their endowment after buying land and paying for their share of the new investment in the DLPG stock for future generations.

Stage 4: The current DLPG stock depreciates at a rate δ and generation $t+1 \leq T$ inherits $(1-\delta)(G_t + g_t)$.

3 The Planner's Problem

We take the planner's objective to be maximizing the sum of the discounted utilities of all agents over all periods. We adopt this Benthamite welfare function without considering inequality aversion because it is well linked to Pareto optimality as elaborated in Negishi (1960). Given the concavity of V and the symmetry of agents and jurisdictions, this is equivalent to maximizing the sum of utilities of a representative agent from each period.[10]

We begin by stating the complete planner's problem with nonnegativity constraints:

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[8] We assume that only the young have the franchise. This is because the young realize both the costs of investing in period t and the consequences on housing prices in period $t+1$. In contrast, the old are not responsible for sharing the cost of DLPG investment in period t, and leave the economy before period $t+1$ arrives. Thus, the old are completely indifferent over all political outcomes in period t and so would have no reason to vote even if they had the franchise.

[9] Stage 2 is equivalent to a notion defined later at a more formal level that we call *Political Equilibrium.*

[10] We show that the set of social optima defined in this way is equivalent to the set of Pareto optimal allocations. Of course, one could introduce inequality aversion to the social welfare function, but given the quasilinearity of utility, this formulation of the planner's problem would yield similar necessary conditions for social optimality.

$$\max_{c_{0,1},c_{1,1},\ldots,c_{T-1,T},c_{T,T},g_1,\ldots,g_T,G_2,\ldots,G_T} W \equiv \sum_{t=0}^{T} \beta^{t-1} U_t \tag{5}$$

subject to

$$\begin{aligned}
\omega &= c_{t-1,t} + c_{t,t} + \frac{g_t}{L} \quad \text{for } t \in \mathcal{T} \\
G_t &= (1-\delta)\left(G_{t-1} + g_{t-1}\right) \quad \text{for } t = 2, \ldots, T \\
g_t &\geq 0 \quad \text{for } t \in \mathcal{T} \\
L\omega - g_t &\geq 0 \quad \text{for } t \in \mathcal{T}
\end{aligned}$$

where

$$\begin{aligned}
U_0 &= \beta c_{0,1} \\
U_t &= c_{t,t} + \beta c_{t,t+1} + V(G_t) \text{ for } t = 1, \ldots, T-1 \\
U_T &= c_{T,T} + V(G_T).
\end{aligned}$$

To solve (5), we use the quasilinear property of the utility functions to set up the following Lagrangian:

$$\begin{aligned}
\max_{g_1,\ldots,g_T,G_2,\ldots,G_T} W &= \sum_{t=1}^{T} \beta^{t-1} \left(w - \frac{g_t}{L} + V(G_t) \right) \qquad (6) \\
&+ \sum_{t=1}^{T} \beta^{t-1} \lambda_t \left((1-\delta)\left(G_{t-1} + g_{t-1}\right) - G_t \right) \\
&+ \sum_{t=1}^{T} \beta^{t-1} \theta_t g_t + \sum_{t=1}^{T} \beta^{t-1} \phi_t (L\omega - g_t).
\end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the evolution equation for DLPG, (2), and ϕ_t and θ_t are the multipliers for the two sets of nonnegativity constraints, (3) and (4), respectively.

Remark: In solving for the planner's solution and competitive equilibrium, we will consider two cases: one with, and one without, nonnegativity constraints which require that DLPG investment is irreversible and that agents consume nonnegative levels of private good. Clearly, imposing these constraints makes the solutions more realistic and likely to agree with what we actually observe. However, we will see that they only really affect things in the beginning and ending time periods. In the middle periods at a steady state[11] these constraints do not bind and so do not affect the consumption path. Since the equilibrium and optimal consumption paths agree at such a steady state, we think it is useful to consider welfare theorems *without nonnegativity constraints*, and this will be the central focus of the paper.[12] We will also consider welfare theorems *with nonnegativity constraints* at a less formal level in Section 7. This is both for completeness and because the differences between optimal and equilibrium consumption paths in the initial buildup periods and the final builddown periods are interesting in their own right.

Denote the solution to the planner's problem as g_t^* for $t = 1, \ldots, T-1$ and G_t^* for $t = 2, \ldots, T$. We solve (5) using (6) to derive the socially optimal steady state level of DLPG, G_{ss}, and the socially optimal steady state value of DLPG investment, g_{ss}. Note that this steady state is optimal regardless of whether the nonnegativity constraints are imposed or not.

Lemma 1. *The socially optimal steady state level of DLPG is determined by:*

$$V'(G_{ss}) = \frac{1}{\beta(1-\delta)L} - \frac{1}{L} \tag{7}$$

and the socially optimal steady state value of DLPG investment g_{ss} by:

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta}. \tag{8}$$

Proof. All proofs are relegated to the Appendix. ■

Next we turn to solving the planner's problem without nonnegativity constraints. See Section 7 for a solution to the full problem.

Theorem 1. *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then the solution to the planner's problem becomes:* $g_1^* = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^* = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^* = -G_T^* = -G_{ss}$.

Theorem 1 says that if we ignore the nonnegativity constraints, the planner invests enough in period 1 to get to the optimal steady state DLPG level immediately in period 2. Over all

[11] We refer to this part of the solution to the planner's problem as the *optimal steady state*, and to the DLPG levels and the per period contributions by each jurisdiction needed to maintain this DLPG level as the *optimal steady state DLPG* and *investment levels*, respectively.

[12] We thank anonymous referees for suggesting this direction, which considerably simplifies and streamlines the paper.

of the following ordinary periods $t \in \mathcal{T}^O$, the planner invests enough to keep the DLPG stock at the optimal steady state. In the final period T, the planner cannibalizes the DLPG stock by choosing the largest feasible disinvestment level $g_T^* = -G_{ss}$. Over the ordinary periods on which we focus, we therefore have an interior solution. We show below that this solution to our finite horizon OLG model mimics the infinite horizon steady state equilibrium.

Note that it may or may not be feasible to sustain this steady state level depending on how large the private good endowments of agents are compared to the required periodic DLPG investment. However, if the solution to the unconstrained problem above does not require negative private goods consumption, then it is the same as planner's solution to the constrained problem, at least in the steady state.

4 Dynamic Tiebout Equilibrium

In our economy, agents choose where to live and then vote over how much to add to the current stock of DLPG. The price system will affect both of these decisions and so it is worth spending some time discussing it. At a formal level, the price system specifies the cost of housing for each period, and for every possible level of DLPG in each jurisdiction. We denote prices in each period as follows:

$$p_t(G_t) = (p_t^1(G_t), \ldots, p_t^J(G_t)),$$

where $G_t = (G_t^1, \ldots, G_t^J)$, and a price system by:

$$\mathbf{p} = (p_1(G_1), \ldots, p_T(G_t)).$$

Note that an agent of generation t must consider prices for both periods he is alive. First, he must compare both the level of inherited DLPG and the cost of buying land under period t prices across jurisdictions in order to make an optimal location choice. Second, he must consider the impact on period $t+1$ prices (when he will sell his land) when choosing a level of public investment, g_t^j, to add to the current DLPG stock. In other words, an agent must anticipate the effect on his property values of public investment *both in his own community and in others*. We will see below that without further constraints, commonly held beliefs among agents about the relationship between DLPG levels and property values can generate a wide variety of equilibria. One of the contributions of this paper will be to show that a simple, economically motivated refinement gets rid of all socially suboptimal outcomes.

The reader may object that we are constraining the functional form of the price of land to depend only upon the current state of DLPG by specifying this form. It excludes the possibility that prices might depend on the history of DLPG levels or anticipations of future levels. We have two defenses. First, from an economic standpoint, it really should not matter how the current state evolved. Agents should be indifferent between jurisdictions if they have the exact same DLPG levels. Future levels are determined by future generations, so the current generation has neither certain knowledge of, nor any degree of control over, what they might be (although they may speculate that unborn agents will choose the optimal path). Second, we will demonstrate in the next section that given *free mobility*, defined below, at least relative prices between jurisdictions must depend only on the current state of DLPG. Absolute prices, however, are not pinned down even with this restriction. While this has some interesting implications, it also means that allowing prices to depend on either past or future states does not change the set of equilibrium allocations that prices will support. We therefore choose the more intuitive form for the price functions.

Next we define our equilibrium concept. A feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and a price system $\mathbf{p}$ constitute a *Dynamic Tiebout Equilibrium* if the following two conditions are met:

Free Mobility: *A feasible allocation* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and a price system* $\mathbf{p}$ *satisfy Free Mobility if for all* $t = 1, \ldots, T-1$, *and all* $i \in \mathcal{I}$ *where agent* i *chooses to live in jurisdiction* j, *it holds for all* $\bar{j} \in \mathcal{J}$ *that:*

$$V(G_t^j)+\omega-p_t^j(G_t)-\frac{1}{L}g_t^j+\beta p_{t+1}^j(G_{t+1}) \geq V(G_t^{\bar{j}})+\omega-p_t^{\bar{j}}(G_t)-\frac{1}{L}g_t^{\bar{j}}+\beta p_{t+1}^{\bar{j}}(G_{t+1}),$$

and for T, *and all* $i \in \mathcal{I}$ *where agent* i *chooses to live in jurisdiction* j, *it holds for all* $\bar{j} \in \mathcal{J}$ *that:*

$$V(G_T^j) + \omega - p_T^j(G_T) - \frac{1}{L}g_T^j \geq V(G_T^{\bar{j}}) + \omega - p_T^{\bar{j}}(G_t) - \frac{1}{L}g_T^{\bar{j}}.$$

Political Equilibrium: *A feasible allocation* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and a price system* $\mathbf{p}$ *satisfy Political Equilibrium if for all* $t = 1, \ldots, T-1$, *all* $j \in \mathcal{J}$, *and all* $\bar{g}$,

$$\beta p_{t+1}^j(G_{t+1}) - \frac{g_t^j}{L} \geq \beta p_{t+1}^j(\delta(\bar{g} + G_t^j), G_{t+1}^{-j}) - \frac{\bar{g}}{L},$$

where $G_{t+1}^{-j} \equiv (G_{t+1}^1, \ldots, G_{t+1}^{j-1}, G_{t+1}^{j+1}, \ldots, G_{t+1}^J)$. *In addition, for* T, *and all* $j \in \mathcal{J}$, g_T^j *is the lowest number that is feasible (either* 0 *or* $-G_T$ *depending upon whether the nonnegativity conditions are imposed).*

The Free Mobility assumption requires that the land market clears each period. It says that when agents take land prices, DLPG levels, and the tax contributions they will have to make to maintain the DLPG levels specified in the allocation, all agents are in their most preferred location. Since agents are identical, this is equivalent to stating that the inequalities given in Free Mobility are in fact, equalities.

The Political Equilibrium assumption requires that given prices and DLPG levels, the investment levels, g_t^j, are chosen in a way that balances the cost of higher investment in the current period with the benefit of being able to sell land at higher prices in the next period. Since T is the terminal period, minimal investment is trivially optimal.

In a strict sense, Political Equilibrium is not needed as a separate condition since this is exactly what agents are required to do at stage 2 of each period. We think it is useful to define these political actions formally in order to emphasize the role they play in establishing equilibrium. Note that since we only treat the case of identical agents, majority rule, the Condorcet winner, and unanimity are all equivalent to allowing a representative voter in each jurisdiction choose the level of public good investment. We discuss generalizations below.

Unfortunately, Free Mobility and Political Equilibrium are not sufficient to guarantee that all Dynamic Tiebout Equilibria are Pareto optimal. The next Lemma shows that in general, there will exist price systems that support many nonoptimal equilibria.

Lemma 2. *Consider any arbitrarily chosen steady state level of DLPG, $\bar{G}$. Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then there exists a price system* $\mathbf{p}$ *satisfying Free Mobility and Political Equilibrium which for all $j \in \mathcal{J}$* supports

$$\bar{g}_t^j = \begin{cases} \frac{\bar{G}-(1-\delta)G_1}{1-\delta} & t = 1 \\ \frac{\delta\bar{G}}{1-\delta} & t \in \mathcal{T}^O \\ -G_T^j = -\bar{G} & t = T \end{cases}$$

as part of a Dynamic Tiebout Equilibrium.

In effect, any commonly held set of beliefs about the effect of public investment on land prices that respect the differences in the attractiveness between jurisdictions are self fulfilling prophecies. This multiplicity is similar to the phenomena of sunspot equilibria in macroeconomics (see, for example, Cass and Shell 1983). Sunspots can arise in dynamic models with multiple stages if there are multiple equilibria in the spot markets. In this case, the equilibrium behavior in earlier stages might depend upon the selection of the continuation equilibrium. In our case, the problem is that while equilibria exist, almost all of them are inefficient. For example, if all agents alive at some time t believe that putting a subway in every city and town, no matter how small, will result in the cost of subway construction plus $\$1,000,000$ being added to the price of every plot of land in the country, then all locations will choose to build the subway. In period $t+1$, the land market clears since relative prices are maintained. Thus, the price system described induces agents in period t to overinvest in DLPG. Of course, other equilibrium price systems exist that cause period t agents to underinvest DLPG, or to invest in them efficiently. The point is that there is no reason to expect that the market should induce optimal investment decisions in general.[13]

A closer look at these sunspots, however, shows that they depend on price expectations that may not be very plausible. Sunspots arise only if the choice of DLPG that each jurisdiction makes affects the price in every other jurisdiction. In our example above, it only takes the failure of one small town to build a subway to cause every plot of land in the country to lose $\$1,000,000$ in "extra" value. This seems highly unlikely when the number of jurisdictions is large and each jurisdiction is a tiny part of the economy. It turns out that adding a small refinement to the formation of price expectations that this observation suggests is enough to eliminate all implausible and inefficient equilibria. Formally, the assumption we make is the following:

> **Small Jurisdictions**: *Suppose for any* $t \in \mathcal{T}$, G_t *and* $\bar{G}_t$ *differ only in the amount of DLPG in single jurisdiction* $j \in \mathcal{J}$. *Then there exists a another jurisdiction* $\bar{j} \neq j$ *such that* $p_t^{\bar{j}}(G_t) = p_t^{\bar{j}}(\bar{G}_t)$.

This is a fairly weak assumption. All it says is that if any single jurisdiction changes its DLPG level, there is at least one other jurisdiction in which land prices are unaffected. For example, this implies that if San Diego builds a new airport, the price of housing in Boston should not change, or at any rate, there should be at least one city somewhere in the world that is not affected.

We view Small Jurisdictions as equivalent to requiring that the economy be "competitive" in the sense that all voters believe that their choices have no effect on prices in other jurisdictions (in a very weak sense). Strictly speaking, this assumption is false, just as the assumption that agents are price takers in finite private goods economies. As such, it cannot be an implication of the underlying primitives of the economy. Small Jurisdictions and price taking are essentially behavioral assumptions that imply that agents in a large economy do

[13] Clearly, we could construct similar nonoptimal price systems even if the nonnegativity constraint bound.

not act strategically in markets. We would argue that in a world with many jurisdictions, agents might reasonably believe that the choices they make locally are too insignificant to have a global impact, or, alternatively, that these global impacts are small or difficult to estimate and therefore should be ignored. If these arguments are unpersuasive, then we must accept that there may be a large set of "sunspot" type equilibria in the real world and that there is no particular reason to expect that land prices will capitalize the DLPG investment decisions of voters.

This refinement dramatically reduces the set of allocations that can be supported as Dynamic Tiebout Equilibria. Under Small Jurisdictions we also are able to prove First and Second Welfare Theorems. We begin by characterizing the set of Dynamic Tiebout Equilibria under Small Jurisdictions.

Theorem 2. *Let* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *be a Dynamic Tiebout Equilibrium for an economy satisfying Small Jurisdictions. Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then for all* $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^j = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$, $G_t^j = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^j = -G_T^j = -G_{ss}$.

In proving Theorem 2, we show that Political Equilibrium and Free Mobility imply the following key necessary condition for equilibrium prices for all $t \in \mathcal{T}$, and all $j, \bar{j} \in \mathcal{J}$, which we refer to as the *relative price condition*:

$$p_t^j(G_t) - p_t^{\bar{j}}(G_t) = \left(V(G_t^j) - V(G_t^{\bar{j}})\right) + \frac{1}{L}\left(G_t^j - G_t^{\bar{j}}\right). \tag{9}$$

This shows that even if we included the entire history of DLPG levels in each period and every jurisdiction as arguments in the price function, the only thing that could have an effect on the differences in price between jurisdictions in any period t is the current state of DLPG. Thus, the relative price of land across jurisdictions in any given period depends only on the current state and is pinned by Political Equilibrium and Free Mobility.

5 Welfare Theorems

In this section, we provide welfare theorems for Dynamic Tiebout Equilibrium. We restrict our attention to the case without nonnegativity constraints. The reason is that the welfare theorems relating the unconstrained social optimum and equilibrium allocations are particularly clean and strong. They also provide a very useful benchmark which can be compared to the infinite horizon case. When the nonnegativity constraints are imposed on the problem, on the other hand, the optimal and equilibrium allocations may end up being corner solutions. To complicate matters more, they may not be the same corner solutions, and even if they are, may not be encountered during the same time periods. This turns out to have some interesting economic implications which we explore in Section 7.

Our first step is to show that the set of planner's solutions is identical to the set of Pareto optimal allocations.

Lemma 3. *Assume $G_{ss} \geq G_1$ and suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then a feasible allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, is Pareto efficient if and only if it is also a solution to the planner's problem.*

A First Welfare Theorem follows almost immediately.

Theorem 3. (Strong First Welfare Theorem) *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then if $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a Dynamic Tiebout Equilibrium for an economy satisfying Small Jurisdictions, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ must also be Pareto optimal.*

Recall that in the two period model of Hatfield (2014), the Nash equilibrium with land taxes under decentralization yields the efficient level of local public good provision if the number of districts goes to infinity. In the limit, his economy becomes perfectly competitive. In our multiple period overlapping generations setup, a dynamic Tiebout equilibrium satisfying Small Jurisdictions induces efficient provision even with a finite number of jurisdictions. This is because Small Jurisdictions requires that in large but finite economies, agents behave as if they have no market power. This eliminates inefficient sunspot type equilibria.

A Second Welfare Theorem also holds. In fact, it is possible to implement any equal treatment Pareto optimal allocation solely through the price system without redistributing endowments at all. By equal treatment we mean that agents in a given period get identical levels of private good, although this level may differ across periods. Formally,

> **Equal Treatment in Private Goods** (ET): *A feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ satisfies ET if for all $t \in \mathcal{T}$, and all $i, \bar{\imath} \in \mathcal{I}$, $c^i_{t-1,t} = c^{\bar{\imath}}_{t-1,t} \equiv c_{t-1,t}$ and $c^i_{t,t} = c^{\bar{\imath}}_{t,t} \equiv c_{t,t}$.*

Theorem 4. (Strong Second Welfare Theorem) *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption and that a feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal and satisfies ET. Then there exists a price system $\mathbf{p}$ such that $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a Dynamic Tiebout Equilibrium.*

The prices that support these equilibria take the form:

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t = V(G_t^j) + \frac{G_t^j}{L} + c_{t-1,t} - V(G_{ss}) - \frac{G_{ss}}{L} = c_{t-1,t}$$

for some $K_t \geq 0$. This implies that as long as $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ satisfies the nonnegativity constraints, the supporting prices are nonnegative. If we are willing to relax this and allow negative prices, we would be imposing an economic assumption that says that old agents cannot walk away from land with negative value and would therefore be forced to pay young agents take land off their hands. In this case, however, we could support any feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, equal treatment or not, with prices and a redistribution of initial allocations. To do so, we would need to add a set of individualized transfer constants K_t^i to each agent's endowments where

$$K_t^i = c_{t-1,t}^i - V(G_{ss}) - \frac{G_{ss}}{L},$$

and

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t}{L}. \tag{10}$$

The form of the supporting price is intuitive and is similar to other pricing functions in the literature. For example, Bruekner and Soo (1991) derive a voting equilibrium property price which equals the rental value minus property taxes. In other words, the price is equal to the net capitalized value of a property's services. Without nonnegativity constraints, agents in any period have the option of consuming the existing DLPG stock after they enjoy its services. Thus, the "capitalized" value to each agent of this stock is its per capita level $\frac{G_{ss}}{L}$. As a result, the utility value of living in a jurisdiction to an agent is the "use" value of the inherited DLPG stock, $V(G_{ss})$, plus the per capita "salvage" value of the stock, $\frac{G_{ss}}{L}$ (which agents can either consume as private good, or pass on to the next generation as depreciated DLPG for their use). As shown in Section 4 above, Free Mobility and Political Equilibrium together imply that the relative price between any two locations must take the form (9) for all periods. Thus, aside from a transfer K_t associated with redistribution to support the chosen Pareto optimum, the supporting price must repeat the same form $V(G_t^j) + \frac{G_t^j}{L}$ for all t, where the transfer term would adjust in response to changes in the patterns of DLPG investments over time.

The same logic used to prove Theorem 4 applies here. Making these transfers does not change the relative price condition and so agents would choose the Pareto optimal steady state levels of DLPG in each period. It is easy to verify that these individualized transfers both leave agents with the consumption levels specified in $\mathbf{c}$, and are feasible. The problem is that for some private good allocations, **c**, the transfers might be so large that agents end up with negative endowments after redistribution.

Therefore, if we are willing to allow endowments and prices to be negative, we get a Strong Second Welfare Theorem that says that all Pareto optimal allocations can be supported as Dynamic Tiebout Equilibria for some reallocation of endowments. If we require prices and endowments to be nonnegative, however, we get a somewhat weaker Second Welfare Theorem that says that the set of allocations that can be supported as Dynamic Tiebout Equilibria for some reallocation of endowments is larger than the set of equal treatment Pareto optimal allocations, but smaller than the entire set of Pareto optimal allocations.

The Second Welfare Theorem is also a constructive proof that equilibrium exists. Thus, the two Welfare Theorems together imply that Dynamic Tiebout Equilibria exist and are first best. This means that Tiebout's basic insight, if agents vote with their feet to choose tax/public good combinations, then the outcome will be first best, carries over to overlapping generations economies with a DLPG (at least under the conditions above). Thus, we have a Dynamic Tiebout Theorem.

6 Centralization versus Decentralization

In this section, we compare the performance of centralized and decentralized institutions in the presence of intergenerational spillovers. Previous studies of decentralization have emphasized the role of differences in the taste for public goods. In these papers, decentralization is valuable because it allows agents to sort into jurisdictions populated by agents with similar tastes. Here, we provide a new case for decentralization based *solely* on the capitalization effect.

The model of centralization we use is a straightforward variation of the decentralized one outlined in previous sections. The only difference is that the level of DLPG is chosen in a national election and so is *identical* across jurisdictions.[14] Let G_t denote the common level of DLPG in each jurisdiction in period t. Note that agents have identical tastes and so the

[14]Note that the results in this section would also hold if we had agents in many jurisdictions voting collectively for the national level of a pure public good like defense or research and development. To make direct comparisons to the previous sections clear, however, we set this up as a kind of national vote over grants in aid to local governments to build a common specified level of DLPG such as city streets or school buildings in each.

level of DLPG that agents would like to consume is the same as in the decentralized case and is still agreed upon unanimously. If there is any difference in the outcome of the vote, it is because centralization has distorted the capitalization effect through the price system.

Since the DLPG levels are the same in each jurisdiction (and thus, per capita investment is also the same) it is immediate that a price system $\mathbf{p}$ satisfies Free Mobility if and only if for all $t \in \mathcal{T}$, any $j, \bar{j} \in \mathcal{J}$, and any $G_t \in \Re^1_+$,

$$p_t^j(G_t, \ldots, G_t) = p_t^{\bar{j}}(G_t, \ldots, G_t).$$

Thus, Free Mobility has no bite since we can never have price or DLPG level differences between jurisdictions within a given period. The Small Jurisdictions assumption has no bite either for essentially the same reason. There is no possibility of agents in a single jurisdiction contemplating the effect on local land prices of increasing or decreasing DLPG provision within their own city alone.

This implies that arbitrary sunspots can arise, and anything can be a Dynamic Tiebout Equilibrium under centralization.

Theorem 5. *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption and agents vote in a central election over a common level investment for all jurisdictions each period. Consider an arbitrary path of DLPG levels for each period:* $(\bar{G}_2, \ldots, \bar{G}_T) \in \Re^{T-1}_+$ *(not necessarily a steady state). Then there exists a price system* $\mathbf{p}$ *that satisfies Political Equilibrium and Free Mobility and which supports this path.*

Notice that sunspots can arise if local land prices depend on the national level of DLPG. But why should this be so? Agents have a taste for DLPG, but their taste is not based on how the DLPG interacts with land. Under decentralization, agents bid up the price of jurisdictions with higher levels of DLPG because they want *access* to this DLPG. Under centralization, access is not tied to land because the DLPG is provided at the national level. Thus, the only economic force behind these sunspots are self fulfilling beliefs. Since the plots of land are identical in every jurisdiction *and* DLPG levels are also identical by construction, it might make sense to remove the dependence of land prices on centrally provided DLPG. Formally,

> **No Sunspots**: *For all* $t \in \mathcal{T}$, *all* $j \in \mathcal{J}$, *and all* $G_t \in \Re^1_+$, *prices take the form:* $p_t^j(G_t, \ldots, G_t) = K_t$.

The next Theorem shows that although the no sunspot refinement gets rid of the problem of multiple equilibria, the one that remains is inefficient.

Theorem 6. *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption and agents vote in a central election over common level investment for all jurisdictions each period. Then any price system* **p** *that satisfies Political Equilibrium, Free Mobility and No Sunspots results in zero provision of DLPG in each period.*

7 Extensions

In this section, we extend the basic model by considering nonnegativity constraints on DLPG investments and private good consumption, allowing for heterogeneous preferences for the DLPG, and taking the time horizon to infinity.

7.1 Optimality and Decentralization with Nonnegativity Constraints

We show in Section 5 that if we allow for the possibility of negative investment, private good consumption, and land prices, a strong First Welfare Theorem obtains. In this subsection, we explore how well this result holds up if we impose the more realistic assumption that all of these must be nonnegative.

We begin by giving a full characterization of the solution to the planner's problem with nonnegativity constraints

Theorem 7. *Assume $g_{ss} < L\omega$, and $G_{ss} > G_1$. Then the socially optimal levels of DLPG relate to the socially optimal steady state level in the following manner:*

$$\beta \left(V'(G_{ss}) - V'(G_t) \right) = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t \text{ for } t = 1, \ldots, T-1. \tag{11}$$

Moreover, the solution to the planner's problem is the following:

(i) $g_t^* = L\omega$ *from* $t = 1$ *to some* t' *(note that* t' *may equal* 1 *or* $T-1$*)*

(ii) $L\omega > g_{t'+1}^* \geq 0$

(iii) $g_t^* = g_{ss}$ *for period* $t'+2$ *to some period* $t'' \geq t'$ *or* $g_{t'+2}^* = 0$

(iv) $g_t^* = 0$ *for period* $t''+1$ *to* T*.*

What Theorem 7 says in essence is that the socially optimal plan is to start by investing the entire endowment of private good until the steady state level of DLPG is reached, maintain this level by investing g_{ss} for the next interval of periods, but at some point in time, stop investing entirely and let the DLPG depreciate until the final period.[15] Obviously this is not as good at building directly to the steady state DLPG level in period 1, but given the nonnegativity constraints, the path outlined above is the most efficient one available to the planner.

Unfortunately, this constrained Pareto optimal path may not be supportable is a Dynamic Tiebout Equilibrium in all cases. To understand why, consider the following observation:

Observation 1. No equilibrium prices can support an investment level greater than half of agents' endowment, $\frac{\bar{g}}{L} > \frac{1}{2}\omega$, for two consecutive periods. To see this, consider the land price that would be required to give young agents born in some period, t, living in some jurisdiction, j, the incentive to make such an investment. Clearly, these agents must expect to get at least their investment back so: $p^j_{t+1} \geq \frac{G^j_t \bar{g}}{L} > \frac{1}{2}\omega$.[16] Otherwise, period t agents would be better off investing zero and consuming all of their endowment right away. By doing so, they gain $\frac{\bar{g}}{L}$ today, but only give up $p^j_{t+1} < \frac{\bar{g}}{L}$ tomorrow. The problem with this is that if the generation born in period $t+1$ pays such a high price for land, they are left with only $\omega - p^j_{t+1} < \frac{\bar{g}}{L}$ to invest in DLPG for generation $t+2$. In other words, it is simply infeasible for generation $t+1$ to invest more than half of their endowments in DLPG after paying the required price.

This observation immediately leads to two possible types of failures of the First Welfare Theorem:

- The buildup phase will be longer at a Dynamic Tiebout Equilibrium than under the planner's solution if the optimal plan requires the maximum feasible investment (ω) for some of the initial periods since investment levels can be no more than half of this at a Dynamic Tiebout Equilibrium.

[15] Depending on the economic parameters and the size of T, the first or second interval could be degenerate. However, if T is large enough and $g_{ss} < L\omega$ (which ensures the steady state is feasible without violating the nonnegativity constraints), then both will be nondegenerate. The proof of Theorem 7 also reveals that if T is large enough to make the second (steady state) interval nondegenerate, the number of periods of decumulation (that is, the length of the interval from $t'' + 1$ to T) is independent of T. Finally, note that there may or may not be one transitional period between maximal investment and the steady state interval when investment is something positive, but less than the endowment.

[16] We use the shorthand p^j_{t+1} for $p^j_{t+1}(G^{-j}_{t+1}, \delta(G^j_t + \bar{g})$ to make the argument easier to parse.

- If the Pareto optimal steady state requires that each generation invests more than half of its endowment in DLPG, $\frac{g_{ss}}{L} > \frac{1}{2}\omega$, then the steady state cannot be supported with prices as a Dynamic Tiebout Equilibrium.

Thus, if we impose the nonnegativity constraints, the First Welfare Theorem holds only if the endowment is large enough so that the Pareto efficient steady state DLPG level can be achieved by investing less than half of the endowment in period 1. Alternatively, we could ignore the buildup phase and show that a steady state First Welfare Theorem holds only if the Pareto efficient steady state can be maintained by investing less than half of agents' endowments each period.

Notice we say "only if" in the statements above. This is because, even under the conditions outlined, it may still be the case that some Dynamic Tiebout Equilibria are not Pareto optimal. Recall that without the nonnegativity constraints, the relative land prices in different jurisdictions are pinned down and all such prices support efficient outcomes regardless of their absolute level. This is no longer true with the nonnegativity constraints imposed.

To see this, suppose the Pareto optimal investment level happened to be $\frac{g_{ss}}{L} = \frac{1}{4}\omega$ which yielded a steady state DLPG level of G_{ss}. We can construct a Dynamic Tiebout Equilibrium with suboptimal steady state investment and DLPG levels, $\frac{\bar{g}_{ss}}{L} = \frac{1}{8}\omega$ and $\bar{G}_{ss} < G_{ss}$, as follows. Assume that the discount rate is zero for simplicity and let $p^j_{t+1}(G_{t+1}) = \frac{7}{8}\omega$ if $G^j_{t+1} = \bar{G}_{ss}$, for all $j \in \mathcal{J}$, and follow the relative price condition for all other DLPG levels. Of course, under such prices, it would pay agents to invest the efficient level, $\frac{1}{4}$, if they could. This is because the benefit in higher land prices in period $t+1$ would exceed the cost of the needed private good investment under the relative price condition. Unfortunately, agents run out of private good to invest at $\frac{g^j_t}{L} = \frac{1}{8}\omega$ since they paid $\frac{7}{8}\omega$ to purchase land. The best they can do is to choose the corner solution and invest as much as possible, $\frac{1}{8}\omega$. It follows that prices such as these which are high relative to the optimial investment levels can suppport suboptimal steady states as Dynamic Tiebout Equilibria.

A similar logic holds if the absolute price levels are too low. Consider the same Pareto optimal steady state as above. Suppose that $p^j_t(G_{t+1}) = \frac{1}{10}\omega$ if $G^j_{t+1} = G_{ss}$ for all $j \in \mathcal{J}$, and follows the relative price condition for all other DLPG levels. Notice that agents would be better off investing zero, consuming the extra $\frac{g_{ss}}{L} = \frac{1}{4}\omega$ in period t, and foregoing $p^j_t(G_{ss}) = \frac{1}{10}\omega$ in period $t+1$. Thus, even though we satisfy the relative price condition, agents choose the minimum investment corner solution and this is a Dynamic Tiebout Equilibrium outcome.

We summarize these two corner solutions as follows:

- There exist Dynamic Tiebout Equilibria in which land prices are so high compared to the Pareto efficient investment levels, that even when agents invest all they can given their endowments, equilibrium investment levels are suboptimal.

- There exist Dynamic Tiebout Equilibria in which land prices are so low compared to the Pareto efficient investment levels, that agents choose zero investment in equilibrium.

We see that absolute prices can be too high or too low, but if they are somewhere in the middle, then they may be able to support the optimal steady state. Fortunately, the "normal price" range for housing probably falls within this middle zone. In most places, property taxes are on the scale of .5% to 5% of property values. If we follow the rule of thumb that one can afford a house costing about four times gross income, then even at a 5% local property tax rate, DLPG investments are about 20% of income, well below the 50% sustainability cutoff. On the other hand, prices are high enough relative to these DLPG levels that investing zero would not be optimal for the current generation. Within this middle range, absolute prices can shift up or down in ways that are either anticipated or unanticipated by current and future generations without reducing the efficiency of the current generation's DLPG investment choice. The only effect is that wealth is transferred between generations.

To summarize, while there are many ways for a First Welfare Theorem to fail if we impose the nonnegativity constraints, it is probable that Dynamic Tiebout Equilibria are Pareto optimal in practice (at least in the steady state). Absolute land prices seem to be in the right range in the real world, and it is hard to think of a jurisdiction that demands that agents give up half or more of their endowments to fund DLPG.

7.2 Heterogeneous Agents

The case we build for decentralization in this paper is based purely on the capitalization effect. For this reason, we considered a benchmark in which all agents are identical (and in particular, had homogeneous tastes). In this subsection, we examine whether capitalization continues to provide incentives for agents to invest optimally in DLPG when tastes are heterogeneous. To explore this, we consider the following economy in which agents are completely ordered by their degree of preference for DLPG:

$$
\begin{aligned}
U^i(c_{0,1}) &= \beta c_{0,1} \\
U^i(c_{t,t}, c_{t,t+1}, G_t^j) &= c_{t,t} + \beta c_{t,t+1} + \rho(i)V(G_t^j) \text{ for } \quad t = 1, ..., T-1 \\
U^i(c_{T,T}, G_T^j) &= c_{T,T} + \rho(i)V(G_T^j).
\end{aligned}
$$

where for all $i, \bar{\imath} \in \mathcal{I}$ $\rho(i) > \rho(\bar{\imath})$ if $i > \bar{\imath}$. We will say that agent i has a "higher taste" for DLPG than $\bar{\imath}$.

Taste heterogeneity generates disagreement about the optimal level of DLPG. As a result, political equilibrium becomes nontrivial and we will need to adjust the concept of Dynamic Tiebout Equilibrium accordingly. In seems natural in this context to assume that the investment in DLPG is determined by the *median voter* in each jurisdiction. Note that this does not immediately imply that the median voter chooses his own personally optimal level of DLPG. He must also concern himself with the effect his decision will have on the value of his land in the next period. We will see that this leads to a range of possible outcomes in equilibrium.

To define this a bit more formally, we need some additional notation. Recall that $\mathcal{I}$ denotes set of agents in the entire population. Thus, for any $j \in \mathcal{J}$ let

$$\mathcal{I}^j \subset \mathcal{I}$$

denote the set of agents living in jurisdiction j. Of course, $\{\mathcal{I}^1, \ldots, \mathcal{I}^J\}$ is a partition of $\mathcal{I}$.

Under our assumptions, agents have single peaked preferences in the sense that if they share the cost of DLPG with the other agents in their jurisdiction equally, they have a single most preferred DLPG level and their utility decreases monotonically on either side of this optimum. From this, it is immediate that any the Pareto optimal outcome requires agents to be *completely stratified* in the sense that, for all $j \in \mathcal{J}$ and $i \in \mathcal{I}^j$, it holds for all $\bar{\imath} \in \mathcal{I}^{\bar{j}}$ such that $\bar{j} \neq j$, either $\bar{\imath} > i$ or $\bar{\imath} < i$.

Let (c_{ss}, g_{ss}, G_{ss}) be a stratified Dynamic Tiebout Equilibrium Pareto optimal steady state. Note that these DLPG levels must maximize the average utility of agents within each jurisdiction given the Benthamite social welfare function. We can construct supporting prices as follows. Consider any steady state period t and any price you like for jurisdiction 1 at the steady state DLPG level: $p_t^1(G_{ss})$. To satisfy Free Mobility, we have to choose $p_t^2(G_{ss})$ such that the given $(p_t^1(G_{ss}), G_{ss}^1, g_{ss}^1)$ and $(p_t^2(G_{ss}), G_{ss}^2, g_{ss}^2)$, the highest taste agent in jurisdiction 1 prefers jurisdiction 1 to 2, and the lowest taste agent in jurisdiction 2 prefers jurisdiction 2 to 1. If prices are set in this way, then single peakedness of preferences implies that no other agents in jurisdictions 1 or 2 would prefer to move. This fact also implies that such a price exists since we can start by setting the price in both jurisdictions to be equal, and then raise the price in jurisdiction 2 until the highest taste agent in jurisdiction 1 finds that jurisdiction 2 is too expensive to be an optimal choice. The prices in jurisdictions 3 through J are constructed in the same way with reference to the next lowest jurisdiction. Finally, the prices for all DLPG profiles besides G_{ss} are chosen to respect the relative price condition, where for each jurisdiction in the stratified allocation, the average benefits to members of the jurisdiction are internalized. Given this, Political Equilibrium implies that each jurisdiction chooses investment levels that support the Pareto optimal steady state.

Not surprisingly, complicating the model like this introduces some small distortions. As a result, First and Second Welfare Theorems fail in a strict sense. Other Dynamic Tiebout Equilibria exist that internalize the benefits of the median voter in each jurisdiction, and in fact, any member of each jurisdiction. However, approximate versions continue to hold. For example, if the number of agents and jurisdictions is large, each jurisdiction would only have to accommodate a small fraction of the set heterogeneous agents. Thus, each jurisdiction would only cover a small segment of tastes and agents would be approximately homogeneous within each jurisdictions. As a result, the Dynamic Tiebout Equilibrium and the Pareto/socially optimal investments in DLPG would be approximately the same, and an approximate First Welfare Theorem would be true. A Second Welfare Theorem could be established on similar lines.

The argument above is informal, and intentionally so. The point is to show how the results could be extended to heterogeneous economies. However, showing this formally would lengthen the paper materially.[17] Interested readers can contact the authors for more detailed and formal arguments.

7.3 Infinite Horizon

As we described the base model in Section 2, we argued for the value of using a finite horizon OLG framework to establish welfare theorems and to characterize the equilibrium and optimal levels of DLPG over time. In this subsection, we consider an infinite horizon OLG setting in order to explore the robustness of our results. Specifically, we maintain the assumption that agents live two periods, but note that the *terminal young* no longer exist since time is now unbounded. As a result, the utility functions of agents are:

$$U(c_{0,1}) = \beta c_{0,1}; \quad U(c_{t,t}, c_{t,t+1}, G_t) = c_{t,t} + \beta c_{t,t+1} + V(G_t) \quad \text{for} \quad t \geq 1.$$

We begin by reexamining the planner's problem where the social welfare function is: $W \equiv \sum_{t=0}^{\infty} \beta^{t-1} U_t$. Since resources do not grow and V is strictly concave, there must exist

[17] We thank referees for arguing that adding too much on this topic diverts attention from the main focus of the paper.

$\bar{U} < \infty$ such that $U_t \leq \bar{U}$ $\forall t$. Given $\beta \in (0,1)$, W is therefore bounded. Under quasilinear preferences, we may again write a version of the planner's problem parallel to (6) with T replaced by ∞. Lemma 1 can be established for this problem following the same argument as before. In particular, the socially optimal steady state level of DLPG, G_{ss}, is still determined by (7) and the socially optimal steady state value of DLPG investment g_{ss} by (8). Thus, Theorem 1 continues to be valid. The argument needed to establish Lemma 3 also goes through in the same way and so the Pareto optimal and social optimal allocations are equivalent.

Recall that we adopted a Benthamite welfare function when analyzing the finite horizon case since it is linked well to standard notions of Pareto optimality. However, the possibility of "transfers from infinity" breaks this linkage in the infinite horizon case. This is because linear utility in the private good makes a small transfer of private good from each old generation to each young generation Pareto improving. To fix this, we assume that all agents consume private good only when they are old. This cuts off the direct transfer channel. This means that the utility functions are modified to become:

$$U(c_{0,1}) = \beta c_{0,1}; \quad U(c^j_{t,t+1}, G^j_t) = \beta c^j_{t,t+1} + V\left(G^j_t\right) \quad \text{for} \quad t \geq 1.$$

Of course, transfers may still be made by trading off private good consumption and DLPG investments across periods. To prevent this from happening, we impose a Gaussian Curvature Condition on each agent's indifference surface between private good and DLPG consumption.[18] Basically, this curvature condition requires that the indifference surfaces are never arbitrarily close to being "flat" (i.e., perfect substitutability). As such, it imposes sufficiently strong diminishing marginal rates of substitution ($MRS = \frac{V'\left(G^j_t\right)}{\beta}$), to ensure that for some distant future generation (large t), transfers by reducing DLPG production must lead to a Pareto suboptimal outcome.

With these modifications, the infinite horizon planner's problem is parallel to (6) with T being replaced by ∞. As a consequence, Lemmas 1 and 3 as well as Theorems 1 all remain valid.

Next, we reexamine the dynamic Tiebout equilibrium. Assumption that all agents consume private good only when they are old requires that the decentralized optimization problem be modified as well. Since agents are endowed with ω units of private good only when young but consume it only when old, a kind of "forced saving" via the investment in the DLPG might be necessary, depending on equilibrium land prices. This could force inefficient

[18] See Debreu (1972, pp. 612-613) for definition and illustration of Gaussian curvature of smooth preferences.

investment levels, and is inconsistent with the finite horizon case where any surplus private good in a given period could simply be consumed by the young. Thus, to be consistent with the original economy, we allow for perfect intertemporal borrowing and lending so that agents can fully optimize between the private good and the DLPG. Without loss of generality, let such borrowing/lending be through a risk free bond b_t at a market interest r_{t+1} over the periods from t to $t+1$. The budget constraints therefore become:

$$\begin{aligned} b_t^j + p_t^j &= \omega - \frac{g_t^j}{L} \\ c_{t,t+1}^j &= (1+r_{t+1})b_t^j + p_{t+1}^j, \end{aligned}$$

and the lifetime budget constraint is given by:

$$p_t^j + \frac{c_{t,t+1}^j}{1+r_{t+1}} = y_t^j = \omega - \frac{g_t^j}{L} + \frac{p_{t+1}^j}{1+r_{t+1}}. \tag{12}$$

By substituting out $c_{t,t+1}^j$ and the DLPG evolution condition, the decentralized optimization problem facing an agent with an inherited DLPG level of G_t^j can then be written as:

$$\max_{g_t^j} V\left(G_t^j\right) + \beta\left(1+r_{t+1}\right)\left[\omega - \frac{g_t^j}{L} + \frac{p_{t+1}^j((1-\delta)G_t^j + g_t^j)}{1+r_{t+1}} - p_t^j\left(G_t^j\right)\right] \tag{13}$$

Of course, in equilibrium, Free Mobility and Political Equilibrium (taking T to ∞) must hold. Since all net borrowing/lending across all agents in all jurisdictions must sum to zero in equilibrium, the resource constraint is simply:

$$I\omega = \sum_{i\in\mathcal{I}} c_{t-1,t}^i + \sum_{j\in\mathcal{J}} g_t^j \qquad \text{for} \qquad t \geq 1. \tag{14}$$

It is not surprising that the proofs of Lemma 2 and Theorems 3-5 no longer work because backward induction cannot be used without a finite terminal date T. Nevertheless, we are able to establish decentralized price support for socially optimal allocations under Equal Treatment). In the Appendix, we show that these supporting prices take the form:

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t \tag{15}$$

for some $K_t \geq 0$ for all $j \in \mathcal{J}$. They are consistent with the Dynamic Tiebout Equilibrium prices derived under the finite horizon setting.

Thus, while the stronger welfare theorem results (specifically, First Welfare Theorem results) in the finite horizon setting cannot be reproduced here, the main properties regarding optimal allocations and supporting prices remain valid even under the infinite horizon setup. Also, since K_t is the same across all jurisdictions, the relative price condition (9) continues to hold. Accordingly, the difference between the supporting prices of any two jurisdictions in a given period depends only on the current state of DLPG levels in these jurisdictions.

It should be noted that even if the transfer from infinity problem could somehow be dealt with, it is not obvious how the First Welfare Theorem might be recovered. As elaborated by Geanakoplos (2008), although the presence of a durable good/asset may rectify the incomplete market problem in overlapping generations models (in the sense that different generations cannot trade at all markets in different periods), a generic problem is the *lack of market clearing at infinity*. This requires additional conditions on the intertemporal prices of the endowment/composite goods and the intertemporal prices of land. For example, a possible case to restore efficiency is to have positive interest rates and an asymptotically decreasing land price sequence. Of course, this may limit the responsiveness of prices to changes in DLPG level to an undesirable degree.[19]

8 Conclusions

We have constructed an overlapping generations model with a durable local public good and established a Tiebout Theorem and an equal treatment Second Welfare Theorem. Without the nonnegativity constraints on private good consumption and DLPG investments, we have shown that, given the Small Jurisdiction assumption, a First Welfare Theorem holds which implies that Dynamic Tiebout Equilibria are Pareto optimal. To summarize, under Small Jurisdictions, the following welfare theorems hold for Dynamic Tiebout Equilibrium:

- Full First Welfare Theorem and Second Welfare Theorem: Finite horizon, taste homogeneous agents, without nonnegativity constraints;
- Approximate First Welfare Theorem and Second Welfare Theorem: Finite horizon, taste heterogeneous agents, without nonnegativity constraints;
- Moderate Price First Welfare Theorem and Moderate Public Good Level Second Welfare Theorem: Finite horizon, taste homogeneous agents, with nonnegativity constraints;

[19] A more formal and detailed argument is available from the authors on request.

- Equal treatment Second Welfare Theorem: infinite horizon.

Our main conclusion is that capitalization is indeed an effective mechanism to cause agents to internalize intergenerational spillovers. The effectiveness of this mechanism is, however, limited by the degree to which there are more general spillovers across jurisdictions. The establishment of a Tiebout Theorem for a simple economy with DLPG is largely in contrast to the existing Tiebout literature, which either shows that equilibria exist, or that equilibria are efficient, but typically not both (see Conley and Konishi 1999 for further discussion). Our finding is important because current studies of DLPG generally include economic distortions in various forms (e.g., uncertainties, incomplete information, and market power). Unless we have a baseline case of a competitive economy for which a First Welfare Theorem applies, it is hard to know if the inefficiencies in these models come from the distortions in question, or are simply a result of the underlying economic structure.

If one takes the view, perhaps because of real world frictions, that jurisdictions of fixed size and indivisible land are a reasonable approximation to reality, this paper shows that there is an essential trade off between intergenerational spillovers which can be internalized by competing jurisdictions through capitalization, and interjurisdictional spillovers which may be internalized when agents vote centrally over public goods levels. This suggests the following policies for optimal public good provision:

	Durability	
Rivalry	nondurable	durable
local	by jurisdictions	by jurisdictions
pure	by central government	cannot be provided optimally

(i) Nondurable local public goods should be provided by jurisdictions. Examples include police and fire protection, local services and fireworks displays. This is because of heterogeneous tastes only.

(ii) Durable local public goods should be provided by jurisdictions. Examples include city streets and local infrastructure. This is because of both heterogeneous tastes and intergenerational spillovers.

(iii) Nondurable purely public goods should be provided nationally. This also includes private goods and public services with widespread externalities. Examples include medical care, poverty relief, and research relating to immediate problems like what this year's flu shot should contain. This is because of both interjurisdictional and interpersonal spillovers. Of course, efficiency requires that some sort of mechanism be used to figure out the right levels of public goods and set the correct tax rates.

(iv) Durable purely public goods cannot be provided optimally at any level. Examples include defense, environmental protection, abatement of global warming and most types of pure research. This is because of the conflict between internalizing intergenerational and interjurisdictional spillovers. It is interesting to note that questions of how to deal with goods of this type seem to be at the center of many of the most politically contentious issues today. It may be that there is a kind of continuing crisis surrounding these goods because of the failure of any institution to provide them efficiently.

Finally, we have shown that moderate property value booms and busts, whether anticipated or not, do not affect the result that the value of the existing DLPG stock will be capitalized into local housing prices. This in turn means that agents continue to have the correct incentives to internalize the intergenerational spillovers that are produced by investing in DLPG. However, if these booms or busts raise prices too high or depress them too low in absolute terms relative to income, then this result breaks down. Thus, both relative and absolute prices play a role in generating efficient market outcomes when local public goods are durable.

Appendix

In this appendix, we present all the detailed mathematical proofs of Lemmas and Theorems, as well as detailed elaborations of the extensions. A significant portion of the Appendix is not intended for publication.

Lemma 1. *The socially optimal steady state level of DLPG is determined by:*

$$V'(G_{ss}) = \frac{1}{\beta (1-\delta) L} - \frac{1}{L}$$

and the socially optimal steady state value of DLPG investment g_{ss} by:

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta}.$$

Proof of Lemma 1

Problem (6) gives the following First Order Conditions:

$$\frac{\partial W^*}{\partial g_t} = 0 = -\frac{1}{L} + \theta_t - \phi_t + \delta\lambda_t \quad \text{for } t = 1, \ldots, T \tag{16}$$

$$\frac{\partial W^*}{\partial G_t} = 0 = -\lambda_{t-1} + \beta\delta\lambda_t + \beta V'(G_t) \quad \text{for } t = 2, \ldots, T-1 \tag{17}$$

$$\frac{\partial W^*}{\partial G_T} = 0 = -\lambda_{T-1} + \beta V'(G_T) \tag{18}$$

$$\frac{\partial W^*}{\partial \lambda_t} = (1-\delta)(G_{t-1} + g_{t-1}) - G_t = 0 \text{ for } t = 2, \ldots, T$$

and Kuhn-Tucker Conditions associated with the nonnegativity constraints on g and $L\omega - g$:

$$\begin{aligned}
\theta_t g_t &= 0 \quad \text{for } t \in \mathcal{T} \\
\theta_t &\geq 0 \quad \text{for } t = 1, \ldots, T-1 \\
\phi_t (L\omega - g_t) &= 0 \quad \text{for } t = 1, \ldots, T-1 \\
\phi_t &\geq 0 \quad \text{for } t = 1, \ldots, T-1.
\end{aligned}$$

Rearranging (16), (17) and (18), respectively, we get:

$$\lambda_t = \frac{1}{(1-\delta)} \left(\frac{1}{L} - \theta_t + \phi_t \right) \quad \text{for } t = t \in \mathcal{T} \tag{19}$$

$$\lambda_{t-1} = \beta\delta\lambda_t + \beta V'(G_t) \quad \text{for } t \in \mathcal{T}^O \tag{20}$$

$$\lambda_{T-1} = \beta V'(G_T). \tag{21}$$

Using this, we can characterize the stationary state of the planner's problem. We define an (interior) optimal stationary state the level of DLPG, G_{ss}, that solves the first order conditions of the planner's problem when $\lambda_{t-1} = \lambda_t$ and $\phi_{t-1} = \phi_t = \theta_{t-1} = \theta_t = 0$ for $t \in \mathcal{T}$. Substituting this into equation (19) gives us:

$$\frac{\partial W^*}{\partial g_t} = 0 \Rightarrow \lambda_t = \frac{1}{(1-\delta)\,L}.$$

Since $\lambda_{t-1} = \lambda_t = \frac{1}{(1-\delta)L}$, we can put this into equation (20) to get:

$$\overbrace{\frac{1}{(1-\delta)\,L}}^{\lambda_{t-1}} - \overbrace{\frac{\beta}{L}}^{\beta\delta\lambda_t} = \beta V'(G_t).$$

which yields (7).

Finally, if we have G_{ss} DLPG at the end of a period, $(1-\delta)G_{ss}$ survives into the next period. Thus, to maintain the steady state, $\frac{G_{ss}-(1-\delta)G_{ss}}{1-\delta G_{ss}}$. It immediately follows that

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta}.$$

■

Theorem 1. *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then the solution to the planner's problem becomes:* $g_1^* = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^* = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^* = -G_T^* = -G_{ss}$.

Proof of Theorem 1

Relaxing the nonnegativity constraints allows agents to make negative investments bounded only by the current level of DLPG. In effect, this allows agents the option of consuming the existing stock. Given the timing of periods, agents first enjoy the services of DLPG and only afterward decide how much to add or subtract from the current DLPG stock. This immediately implies that any DLPG remaining at time T should be consumed by setting $g_T^* = -G_T$. It also allows us to state the planner's problem in a very simple way. Imagine for a moment that agents in each period consume all the current stock of DLPG, but afterward invest enough private good to get to the planner's chosen level of DLPG for the next period. Then we can directly incorporate the capital evolution constraint into the problem as follows:

$$\max_{G_2,\ldots,G_T} W = \sum_{t=1}^{T} \beta^{t-1}\left(\omega + \frac{G_t}{L} - \frac{G_{t+1}}{(1-\delta)\,L} + V(G_t)\right).$$

This gives the following First Order Conditions:

$$\frac{\partial W^*}{\partial G_t} = 0 = \frac{1}{L} - \beta\frac{1}{(1-\delta)\,L} + V'(G_t) \text{ for } t = 2,\ldots,T$$

or,

$$\frac{1}{(1-\delta)\,L} - \frac{\beta}{L} = \beta V'(G_t) \text{ for } t = 2, \ldots, T.$$

Since we know that G_{ss} is the solution to this equation, we conclude that the planner jumps to the steady state by investing whatever is necessary in period 1. Thus, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$. The planner then maintains this until period T and so: $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ and $G_t^* = G_{ss}$ for $t \in \mathcal{T}^O$. Finally, in the last period, the planner allows the current stock to be completely consumed and so: $g_T^* = -G_{ss}$ ■

Lemma 2. *Consider any arbitrarily chosen steady state level of DLPG, $\bar{G}$. Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then there exists a price system* **p** *satisfying Free Mobility and Political Equilibrium which for all $j \in \mathcal{J}$* supports

$$\bar{g}_t^j = \begin{cases} \frac{\bar{G}-(1-\delta)G_1}{1-\delta} & t = 1 \\ \frac{\delta \bar{G}}{1-\delta} & t \in \mathcal{T}^O \\ -G_T^j = -\bar{G} & t = T \end{cases}$$

as part of a Dynamic Tiebout Equilibrium.

Proof of Lemma 2

Define prices as follows:

$$p_t^j(G_t) = \begin{cases} V(G_t^j) + \frac{1}{L}G_t^j + K & \text{if } G_t^{\bar{j}} = \bar{G} \text{ for all } \bar{j} \in \mathcal{J} \\ V(G_t^j) + \frac{1}{L}G_t^j & \text{otherwise} \end{cases}$$

where K is a large constant. Consider period T. First suppose that all agents born from period 1 to $T-1$ have followed the plan. This implies that $G_T^j = \bar{G}$ for all $j \in \mathcal{J}$. Working backwards, suppose that all period T agents have chosen a jurisdiction. It is immediate that at stage 2 of the period, these agents determine that it is optimal to set $\bar{g}_T^j = -G_T^j = -\bar{G}$ which is also what is required by Political Equilibrium.

Since

$$p_T^j(\bar{G}) = V(\bar{G}) + \frac{1}{L}\bar{G} + K,$$

the price of land in each jurisdiction is equal across all jurisdictions. Thus, for all $j, \bar{j} \in \mathcal{J}$, the net utility is equal:

$$V(\bar{G}) + \omega - p_T^j(\bar{G}) + \frac{1}{L}\bar{G} = V(\bar{G}) + \omega - p_T^{\bar{j}}(\bar{G}_T) + \frac{1}{L}\bar{G}.$$

It follows that agents are equally well off regardless of where they decide to buy land, and so Free Mobility is satisfied in period T.

Consider any period $t \in \mathcal{T}^O$. Suppose all jurisdictions followed the plan from period 1 to $t-1$. Again, working backwards, suppose that all period t agents have chosen a jurisdiction. If any jurisdiction j decides to deviate from the plan, the price of land for period $t+1$ for both jurisdiction j and all other jurisdictions drops by K. If K is chosen to be large enough, this loss in period $t+1$ consumption is more than enough to offset any potential utility gain from choosing any other investment level. Thus, investing according to plan gives higher utility than any other choice and so the investment decision in period t satisfies the Political Equilibrium requirement.

This implies for all $j, \bar{j} \in \mathcal{J}$, $G_t^j = G_{t+1}^j = G_t^{\bar{j}} = G_{t+1}^{\bar{j}} = \bar{G}$ and therefore $p_t^j(\bar{G}) = p_{t+1}^j(\bar{G}) = p_t^{\bar{j}}(\bar{G}) = p_{t+1}^{\bar{j}}(\bar{G})$. Thus, the utility received by an agent born in period t is equal to

$$V(\bar{G}) + \omega - p_t^j(\bar{G}) - \frac{1}{L}\bar{g} + \beta p_{t+1}(\bar{G})$$

regardless of where he chooses to live and so Free Mobility is satisfied.

Since the same argument holds for period 1, we conclude that for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, and $g_t = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ for $t \in \mathcal{T}^O$, since these are the only investment levels that support the specified DLPG plan. Therefore, $\mathbf{p}$ supports this plan and satisfies Free Mobility and Political Equilibrium, which proves the Theorem. ■

Theorem 2. *Let* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *be a Dynamic Tiebout Equilibrium for an economy satisfying Small Jurisdictions. Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then for all* $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^j = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$, $G_t^j = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^j = -G_T^j = -G_{ss}$.

Proof of Theorem 2

We start in period T. Since T is the last period, $g_T^j = -G_T^j$, $j \in \mathcal{J}$, by Political Equilibrium. By Free Mobility, for all $j, \bar{j} \in \mathcal{J}$,

$$\omega - p_T^j(G_T) + V(G_T^j) + \frac{G_T^j}{L} = \omega - p_T^{\bar{j}}(G_T) + V(G_T^{\bar{j}}) + \frac{G_T^{\bar{j}}}{L}$$

which implies the following about the equilibrium prices system $\mathbf{p}$:

$$p_T^j(G_T) = p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) + \frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}.$$

This is the relative price condition that will constrain agents born in period $T-1$.

Now consider the problem for agents born in period $T-1$. Working backwards, suppose that all agents have chosen a jurisdiction. Consider any particular jurisdiction j and consider what level of DLPG the agents in j would optimally choose to pass on to the next generation T. The implicit maximization problem is the following:

$$\max_{G_T^j} \beta p_T^j(G_T) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j).$$

Substituting the relative price condition gives:

$$\max_{G_T^j} \beta \left(p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) \right) + \beta \left(\frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L} \right) - \frac{1}{(1-\delta)\,L}(G_T^j - G_{T-1}^j).$$

Taking the derivative with respect to G_T^j gives:

$$\beta \frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} + \beta \frac{dV(G_T^j)}{dG_T^j} - \beta \frac{dV(G_T^{\bar{j}})}{dG_T^j} + \frac{\beta}{L} - \frac{1}{(1-\delta)\,L} = 0.$$

The key observation is that $\frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} = 0$ by Small Jurisdictions. Since $\frac{dV(G_T^{\bar{j}})}{G_T^j} = 0$ by construction, the First Order Condition becomes:

$$\beta V'(G_T^j) = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L}.$$

Thus, by Political Equilibrium, $G_T^j = G_{ss}$ for all $j \in \mathcal{J}$. On the other hand, Free Mobility requires:

$$V(G_{T-1}^j) + \omega - p_{T-1}^j(G_{T-1}) - \frac{1}{L} g_{T-1}^j + \beta p_T^j(G_T) = V(G_{T-1}^{\bar{j}}) + \omega - p_{T-1}^{\bar{j}}(G_{T-1}) - \frac{1}{L} g_{T-1}^{\bar{j}} + \beta p_T^{\bar{j}}(G_T).$$

Noting that we have shown that whatever generation $T-2$ leaves to generation $T-1$, generation $T-1$ will adjust investment such that $G_T^j = G_T^{\bar{j}} = G_{ss}$ and so $p_T^j(G_T) = p_T^{\bar{j}}(G_T)$, we can restate this as:

$$V(G_{T-1}^j) + \omega - p_{T-1}^j(G_{T-1}) + \frac{G_{T-1}^j}{L} - \frac{G_{ss}}{(1-\delta)\,L} = V(G_{T-1}^{\bar{j}}) + \omega - p_{T-1}^{\bar{j}}(G_{T-1}) + \frac{G_{T-1}^{\bar{j}}}{L} - \frac{G_{ss}}{(1-\delta)\,L}$$

and solve this to get:

$$p_{T-1}^j(G_{T-1}) = p_{T-1}^{\bar{j}}(G_{T-1}) + V(G_{T-1}^j) - V(G_{T-1}^{\bar{j}}) + \frac{G_{T-1}^j}{L} - \frac{G_{T-1}^{\bar{j}}}{L}$$

which is the relative price condition for generation $T-2$ and is identical in form to the relative pricing equation for generation $T-1$. By the same argument we made above and applying Small Jurisdictions, we conclude:

$$\beta V'(G_{T-1}^j) = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L},$$

and so $G_{T-1}^j = G_{ss}$ for all $j \in \mathcal{J}$.

Now suppose for any $t = 1, \ldots, T-1$, and all $j \in \mathcal{J}$, $G_{t+1}^j = G_{ss}$. Then again,

$$p_t^j(G_t) = p_t^{\bar{j}}(G_t) - V(G_t^j) - V(G_t^{\bar{j}}) + \frac{G_t^j}{L} - \frac{G_t^{\bar{j}}}{L}$$

and so by Small Jurisdictions,

$$\beta V'(G_t^j) = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L}.$$

Thus, by backwards induction, for all $t \in \mathcal{T}^O$, and all $j \in \mathcal{J}$, $G_t^j = G_{ss}$.

Finally, the only levels of investment that support this DLPG plan are for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, and $g_t = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ for $t \in \mathcal{T}^O$, which proves the Theorem. ■

Lemma 3. *Assume $G_{ss} \geq G_1$ and suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then a feasible allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, is Pareto efficient if and only if it is also a solution to the planner's problem.*

Proof of Lemma 3

From Lemma 2, we know that if an allocation solves the planner's problem, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$, and $g_T^j = -G_{ss}$ for all $j \in \mathcal{J}$. Suppose that there existed a feasible plan $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ that Pareto dominated $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. It is immediate that such a Pareto dominant allocation could not be found by altering $\mathbf{c}$ to some other $\hat{\mathbf{c}}$ alone. Utility is linear in private good for all agents, so if any agent gets more, another must necessarily gets less. Thus, the new allocation could not be Pareto dominant. It follows that if $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, it must be that for at least one jurisdiction j for at least one period $t \in \{2, \ldots, T\}$, $\hat{G}_t^j$ is different from G_{ss}.

Note that if residents of any jurisdiction j at some time t invest an extra unit of consumption good in DLPG in period $t-1$, but also receive an extra $1/\beta$ units of consumption good in period t, their net utility would be unchanged. Similarly, they would be just as well off if they invested one less unit of consumption good in period $t-1$ and were given $1/\beta$ fewer units of consumption good in period t

Keeping this in mind, what is the best that the planner can do for the generation born in period t living in jurisdiction j while leaving all other generations and jurisdictions exactly as well off? The planner must solve the following equation for g_j^{Δ}:

$$\max V(\delta(\hat{G}_{t-1}^j + \hat{g}_{t-1}^j + g^{\Delta})) - \frac{g^{\Delta}}{\beta L} + \frac{g^{\Delta}}{\beta L}.$$

In words, the planner chooses an optimal increment or decrement to period $t-1$ investment in jurisdiction j, g^{Δ}, within the constraint of leaving all agents besides those born in period t living in jurisdiction j exactly as well off as they are at allocation $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$. The benefit to each generation t jurisdiction j agent is a result of the services of provided the new level of DLPG: $V(\delta(\hat{G}_{t-1}^j + \hat{g}_{t-1}^j + g^{\Delta}))$. However, these agents must compensate generation $t-1$ jurisdiction j agents $\frac{g^{\Delta}}{\beta L}$ in period t. In addition, they must invest in DLPG to the point that generation $t+1$ jurisdiction j agents inherit $\hat{G}_{t_1}^j$ DLPG. Since the investment changed by g^{Δ} in period $t-t$, each jurisdiction j agent in period t must alter his investment from the planned $\frac{\hat{g}_t^j}{L}$ in period t by $\frac{\delta g^{\Delta}}{\beta L}$, which could be positive or negative. The First Order Condition is the following:

$$\delta V_t' = \frac{1}{\beta L} - \frac{\delta}{L}$$

which gives:

$$\beta V_t' = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L},$$

the same equation that defines G_{ss}. Altering $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ such that $\hat{G}_t^j = G_{ss}$ and making the transfers between generations in jurisdiction j outlined above is therefore a Pareto improvement. Moreover, making the same alteration in DLPG along with compensating transfers for every period and jurisdiction for which the DLPG level in $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ is not G_{SS} is also a Pareto improvement. Denote the feasible allocation resulting from all of these alterations in $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ as $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$. Then $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$ is a feasible allocation that Pareto dominates $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ which in turn Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ by hypothesis. This implies that $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. But $\tilde{G}_t^j = G_{ss}$ for $t = 2, \ldots, T$ and all $j \in \mathcal{J}$, and so $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$ and $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ only differ in the private good allocations $\tilde{\mathbf{c}}$ and $\mathbf{c}$, contradicting the argument made above.

We can also conclude that if an allocation is Pareto efficient, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$, and $g_T^j = -G_{ss}$ for all $j \in \mathcal{J}$. Otherwise we could do the same exercise of altering the DLPG level to G_{ss} along with compensating transfers to find a Pareto dominant allocation. Note that if we start from any feasible allocation and make any set of feasible transfers of private good over agents alive within a given period (that is, any private good consumption levels that satisfy $\sum_i c_{t-1,t}^i + \sum_i c_{t,t}^i = I\omega - Jg_{ss}$ for $t \in \mathcal{T}$), the resulting allocations are Pareto unranked since utility is quasilinear). Therefore, any allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, such that for all $j \in \mathcal{J}$, for $t = 1$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, all $t = 2, \ldots, T-1$, $g_t^j = g_{ss}$, and $G_t^j = G_{ss}$ and for $t = T$, $g_T^j = -G_{ss}$ and $G_T^j = G_{ss}$, is Pareto optimal regardless of $\mathbf{c}$.

Turning to the planner's problem, we see immediately that any division between old and young agents in a given period of what private good remains after optimal investments are made leaves the value of the social welfare function unaffected. Therefore, any allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, such that for all $j \in \mathcal{J}$, for $t = 1$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, all $t = 2, \ldots, T-1$, $g_t^j = g_{ss}$, and $G_t^j = G_{ss}$ and for $t = T$, $g_T^j = -G_{ss}$ and $G_T^j = G_{ss}$, is a solution to the social planner's problem regardless of $\mathbf{c}$.

We conclude that the set of Pareto efficient allocations and the set of solutions to the social planner's problem are identical. ■

Theorem 3. (Strong First Welfare Theorem) *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption. Then if* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *are a Dynamic Tiebout Equilibrium for an economy satisfying Small Jurisdictions,* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *must also be Pareto optimal.*

Proof of Theorem 3

By Theorem 2, if $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a Dynamic Tiebout Equilibrium under these conditions, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$ and $j \in \mathcal{J}$. Then by Lemma 3, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal. ■

Theorem 4. (Strong Second Welfare Theorem) *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption and that a feasible allocation* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *is Pareto optimal and satisfies ET. Then there exists a price system* $\mathbf{p}$ *such that* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *are a Dynamic Tiebout Equilibrium.*

Proof of Theorem 4

By Lemma 3, if a feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal, $G_t^j = G_{ss}$ for $t = 2, \ldots, T$ and all $j \in \mathcal{J}$. Define the price system for all $t \in \mathcal{T}$ and all $j \in \mathcal{J}$ as follows:

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t$$

where K_t is a (positive or negative) constant defined below. We start with period T. Since T is the last period, it is optimal for agents in every jurisdiction to choose $g_T^j = -G_T^j$. By construction of the price system, for all $j, \bar{j} \in \mathcal{J}$,

$$p_T^j(G_T) = p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) + \frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}.$$

Now consider the problem for agents born in period $T-1$. Working backwards, suppose that all agents have chosen a jurisdiction. Consider any particular jurisdiction j and consider what level of DLPG the agents in j would optimally choose to pass on to the next generation T. The implicit maximization problem is the following:

$$\max_{G_T^j} \beta p_T^j(G_T) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j).$$

Substituting the relative price condition gives:

$$\max_{G_T^j} \beta \left(p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}})\right) + \beta \left(\frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}\right) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j).$$

Now take the derivative with respect to G_T^j,

$$\beta \frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} + \beta \frac{dV(G_T^j)}{dG_T^j} - \beta \frac{dV(G_T^{\bar{j}})}{G_T^j} + \frac{\beta}{L} - \frac{1}{(1-\delta)L} = 0.$$

This time, $\frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} = 0$ by construction rather than Small Jurisdictions. Also by construction, $\frac{dV(G_T^{\bar{j}})}{G_T^j} = 0$ and so the First Order Condition becomes:

$$\beta V'(G_T^j) = \frac{1}{(1-\delta)L} - \frac{\beta}{L}.$$

Thus, agents choose $G_T^j = G_{ss}$ for all $j \in \mathcal{J}$ under the price system defined above. This implies:

$$p_{T-1}^j(G_{T-1}) = p_{T-1}^{\bar{j}}(G_{T-1}) - V(G_{T-1}^j) - V(G_{T-1}^{\bar{j}}) + \frac{G_{T-1}^j}{L} - \frac{G_{T-1}^{\bar{j}}}{L}.$$

By the same argument we made above we conclude:

$$\beta V'(G_{T-1}^j) = \frac{1}{(1-\delta)L} - \frac{\beta}{L},$$

and so $G_{T-1}^j = G_{T-1}^{\bar{j}} = G_{ss}$ for $j \in \mathcal{J}$.

Now suppose for any $t \in \{1, \ldots, T-1\}$, and all $j \in \mathcal{J}$, $G_{t+1}^j = G_{ss}$. Then again,

$$p_t^j(G_t) = p_t^{\bar{j}}(G_t) - V(G_t^j) - V(G_t^{\bar{j}}) + \frac{G_t^j}{L} - \frac{G_t^{\bar{j}}}{L}$$

and so,

$$\beta V'(G_t^j) = \frac{1}{(1-\delta)L} - \frac{\beta}{L}.$$

Thus, by backwards induction, for all $t = 2, \ldots, T$, and all $j \in \mathcal{J}$, $G_t^j = G_{ss}$. Note that this result is independent of K_t.

It only remains to find a set of constants K_t for each period to add to land prices that result in each generation consuming c_t, the private good consumption level specified in the feasible equal treatment allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. We can define the constant in each period as follows:

$$K_t = c_{t-1,t} - V(G_{ss}) - \frac{G_{ss}}{L}.$$

Observe that without the constant added to prices, private good consumption levels would have been $\bar{c}_{t-1,t}^i = V(G_{ss}) + \frac{G_{ss}}{L}$ for all $t = 1, \ldots, T$, and all $i \in \mathcal{I}$. Therefore, if an old agent i gets an extra K_t when he sells his land, his consumption becomes $c_{t-1,t}$ the specified equal treatment level. Since by hypothesis, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is feasible, it must be that $c_{t,t} = \omega - \frac{g_{ss}}{L} - c_{t-1,t}$, and so adding the constant to prices also results in young agents getting the specified equal treatment consumption level in each period. We conclude that

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t$$

for all $t \in \mathcal{T}$, $j \in \mathcal{J}$ supports $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ as a Dynamic Tiebout Equilibrium. ■

Theorem 5. *Suppose we impose the nonnegativity constraints on DLPG investments and private good consumption and agents vote in a central election over a common level investment for all jurisdictions each period. Consider an arbitrary path of DLPG levels for each period: $(\bar{G}_2, \ldots, \bar{G}_T) \in \Re_+^{T-1}$ (not necessarily a steady state). Then there exists a price system $\mathbf{p}$ that satisfies Political Equilibrium and Free Mobility and which supports this path.*

Proof of Theorem 5

Define prices as follows:

$$p_t^j(G_t, \ldots, G_t) = \begin{cases} \bar{K} & \text{if } G_t = \bar{G}_t \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{K}$ is a large constant. Consider any period $t \in \mathcal{T}^o$. Working backwards, suppose that all period t agents have chosen a jurisdiction. If the young agents alive in period t vote in favor of an investment level that results in DLPG level next period of $G_{t+1} = \bar{G}_t$, they can sell their land for $\bar{K}$. If they vote for anything else, they get can get some finite increment to the utility by choosing G_{t+1} optimally while accepting a price of zero in the next period for their land. Clearly, it is possible to choose $\bar{K}$ to be large enough to exceed any potential gain from this strategy. Therefore $G_{t+1} = \bar{G}_{t+1}$ is a Political Equilibrium under these prices. In addition, since prices and DLPG levels are the same in every jurisdiction in period t, all jurisdictions are equally attractive and so these prices clear the land market and therefore satisfy Free Mobility. ■

Theorem 6. *Suppose we remove the nonnegativity constraints on DLPG investments and private good consumption, but that agents vote in a central election over a common level investment for all jurisdictions each period. Then any price system $\mathbf{p}$ that satisfies Political Equilibrium, Free Mobility and No Sunspots results in zero provision of DLPG in each period.*

Proof of Theorem 6

Given No Sunspots, the political decision faced by period t voters becomes the following:

$$\max_{g_t} V(\delta G_t) - \frac{g_t}{L} - K_t + \beta K_{t+1}.$$

This implies:

$$\frac{d}{dg_t}\left[V(\delta G_t) - \frac{g_t}{L} - K_t + \beta K_{t+1}\right] = -\frac{1}{L} < 0.$$

In words, investing in DLPG is a pure gift to the next generation and so it is optimal to invest nothing. ■

Theorem 7. *Assume $g_{ss} < L\omega$, and $G_{ss} > G_1$. Then the socially optimal levels of DLPG relate to the socially optimal steady state level in the following manner:*

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t \text{ for } t = 1, \ldots, T-1. \tag{22}$$

Moreover, the solution to the planner's problem is the following:

(i) $g_t^* = L\omega$ *from* $t = 1$ *to some* t' *(note that* t' *may equal* 1 *or* $T-1$*)*

(ii) $L\omega > g_{t'+1}^* \geq 0$

(iii) $g_t^* = g_{ss}$ *for period* $t'+2$ *to some period* $t'' \geq t'$ *or* $g_{t'+2}^* = 0$

(iv) $g_t^* = 0$ *for period* $t''+1$ *to* T.

Proof of Theorem 7

The Kuhn-Tucker conditions immediately imply that for all $t = 1, \ldots, T$ one of the two following things is true:

$$\phi_t \geq 0 \text{ and } \theta_t = 0 \quad \text{if } g_t^* > 0$$

or

$$\phi_t = 0 \text{ and } \theta_t \geq 0 \quad \text{if } g_t^* < L\omega.$$

We will use this fact in the proof below.

Inserting (19) into (20) for λ_{t-1} and λ_t gives the following:

$$\frac{\frac{1}{L} - \theta_{t-1} + \phi_{t-1}}{1-\delta} = \beta\delta\left[\frac{\frac{1}{L} - \theta_t + \phi_t}{1-\delta}\right] + \beta V'(G_t) \text{ for } t = 1, \ldots, T-1.$$

Rearranging and using equation (7) gives:

$$\overbrace{\frac{1}{(1-\delta)L} - \frac{\beta}{L}}^{\beta V'(G_{ss})} = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t + \beta V'(G_t) \text{ for } t = 1, \ldots, T-1.$$

which we can rewrite to obtain the key equation (22) in the Theorem.

Using this, we show a series of simple claims:

Claim (a): For all $t = 1, \ldots, T-1$, if $g_{t-1}^* < L\omega$ and $g_t^* = L\omega$, then $G_t \geq G_{ss}$. Suppose for some $t \in \{1, \ldots, T-1\}$, $g_{t-1}^* < L\omega$ and $g_t^* = L\omega$. Then, $\phi_t \geq 0$, and $\theta_t = 0$ and $\phi_{t-1} = 0$, and $\theta_{t-1} \geq 0$. From equation (22):

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = \frac{\theta_{t-1}}{1-\delta} + \beta\phi_t \geq 0$$

which in turn implies

$$G_t \geq G_{ss}.$$

Claim (b): For all $t = 1, \ldots, T-1$, if $g^*_{t-1} = L\omega$ and $g^*_t < L\omega$, then $G_t \leq G_{ss}$. Suppose for some $t \in \{1, \ldots, T-1\}$, $g^*_{t-1} = L\omega$ and $g^*_t < L\omega$. Then, $\phi_t = 0$, and $\theta_t \geq 0$ and $\phi_{t-1} \geq 0$, and $\theta_t = 0$. From equation (22):

$$\beta \left(V'(G_{ss}) - V'(G_t) \right) = -\beta\theta_t - \frac{\phi_{t-1}}{1-\delta} \leq 0$$

which in turn implies

$$G_t \leq G_{ss}.$$

Claim (c): For all $t = 1, \ldots, T-1$, if $g^*_{t-1} = 0$ and $g^*_t > 0$, then $G_t \geq G_{ss}$. Suppose for some $t \in \{1, \ldots, T-1\}$, $g^*_{t-1} = 0$ and $g^*_t > 0$,. Then, $\phi_t \geq 0$, and $\theta_t = 0$ and $\phi_{t-1} = 0$, and $\theta_{t-1} \geq 0$. From equation (22):

$$\beta \left(V'(G_{ss}) - V'(G_t) \right) = \frac{\theta_{t-1}}{1-\delta} + \beta\phi_t \geq 0$$

which in turn implies:

$$G_t \geq G_{ss}.$$

Claim (d): For all $t = 1, \ldots, T-1$, if $g^*_{t-1} > 0$ and $g^*_t = 0$, then $G_t \leq G_{ss}$. Suppose for some $t \in \{1, \ldots, T-1\}$, $g^*_{t-1} > 0$ and $g^*_t = 0$. Then, $\phi_t = 0$, and $\theta_t \geq 0$ and $\phi_{t-1} \geq 0$, and $\theta_{t-1} = 0$. From equation (22):

$$\beta \left(V'(G_{ss}) - V'(G_t) \right) = -\beta\theta_t - \frac{\phi_{t-1}}{1-\delta} \leq 0$$

which in turn implies

$$G_t \leq G_{ss}.$$

Claim (e): If $g^*_{T-1} > 0$ then $G_T < G_{ss}$. From the first order conditions, we know:

$$\begin{aligned}
\lambda_{T-1} &= \beta V'(G_T); \\
\lambda_{T-1} &= \frac{1}{(1-\delta)\,L} - \frac{\theta_{T-1}}{1-\delta} + \frac{\phi_{T-1}}{1-\delta} \\
\Rightarrow \quad \overbrace{\beta V'(G_T)}^{\lambda_{T-1}} &= \frac{1}{(1-\delta)\,L} - \frac{\theta_{T-1}}{1-\delta} + \frac{\phi_{T-1}}{1-\delta}.
\end{aligned}$$

Suppose first that $g^*_{T-1} = L\omega$. Then $\theta_{T-1} = 0$, and so

$$\frac{1}{(1-\delta)\,L} + \frac{\phi_{T-1}}{1-\delta} = \beta V'(G_T).$$

Remember,

$$\frac{1}{(1-\delta)L} - \frac{\beta}{L} = \beta V'(G_{ss}).$$

So,

$$V'(G_{ss}) < V'(G_T)$$

which implies

$$G_T < G_{ss}.$$

Suppose now that $L\omega > g^*_{T-1} > 0$. Then $\phi_{T-1} = \theta_{T-1} = 0$, and so

$$\frac{1}{(1-\delta)L} = \beta V'(G_T).$$

But

$$\frac{1}{(1-\delta)L} - \frac{\beta}{L} = \beta V'(G_{ss})$$

so again,

$$G_T < G_{ss}.$$

Using these claims, we can derive the following implications:

(I-1) For all $t \in \mathcal{T}^O$, if $g^*_t = L\omega$ then $g^*_{t-1} = L\omega$. Suppose for some $t \in \mathcal{T}^O$, $g^*_{t-1} < L\omega$, but $g^*_t = L\omega$. Then by (a) $G_t \geq G_{ss}$. But since $g^*_t = L\omega > g_{ss}$ we are adding more than the steady state level of investment and so it must be that $G_{t+1} > G_{ss}$ Suppose $t_1 \in \mathcal{T}^O$ and $g^*_{t+1} < L\omega$. Then by (b), $G_{t+1} \leq G_{ss}$, a contradiction. Thus, $g^*_{t+2} = L\omega$. By the same argument, for all $t' \in \mathcal{T}^O$, $g^*_{t'} = L\omega > g_{ss}$ and $G_t > G_t$. In particular, $G_T > G_{ss}$. However, by (e), $G_T < G_{ss}$, a contradiction. It follows that if $g^*_t = L\omega$ then $g^*_{t-1} = L\omega$.

(I-2) For all $t \in \mathcal{T}^O$, if $g^*_{t-1} = 0$ then $g^*_t = 0$. Suppose for some $t \in \mathcal{T}^O$, $g^*_{t-1} = 0$, but $g^*_t > 0$. Then by (c) $G_t \geq G_{ss}$. Consider period $t-2 > 1$. Suppose that $g^*_{t-2} > 0$, then from (d) $G_{t-1} \leq G_{ss}$. This is impossible since nothing was added to the public good stock in period $t-1$, and yet $G_{t-1} \leq G_{ss} \leq G_t$. It follows that $g^*_{t-2} = 0$. Now consider period $t-3 > 1$. Suppose that $g^*_{t-3} > 0$, then by (d) $G_{t-2} \leq G_{ss}$. This is similarly impossible since nothing was added to the public good stock in period $t-2$ or $t-1$, and yet $G_{t-2} \leq G_{ss} \leq G_t$. It follows that $g^*_{t-3} = 0$. By the same argument, for all $t' = 1, \ldots t-1$, $g^*_{t'-1} = 0$ and $G_{t'} \geq G_{ss}$. In particular, $G_1 \geq G_{ss}$. This contradicts the assumption that $G_{ss} > G_1$. It follows that if $g^*_{t-1} = 0$ then $g^*_t = 0$.

(I-3) For all $t \in \mathcal{T}^O$, if $L\omega > g^*_{t-1} > 0$ and $L\omega > g^*_t > 0$, then either $G_t = G_{ss}$ *and* $g^*_t = g_{ss}$ or $g^*_t = 0$. Suppose for some $t \in \mathcal{T}^o$, $L\omega > g^*_{t-1} > 0$ and $L\omega > g^*_t > 0$, Then we are at an interior optimum, and $\phi_t = \theta_t = \phi_{t-1} = 0$, and $\theta_t = 0$. From equation (7)

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = 0$$

which in turn implies:

$$G_t = G_{ss}.$$

Suppose $g^*_{t+1} > 0$. Since $g^*_t > 0$, by the same argument $G_{t+1} = G_{ss}$, which is only possible if $g^*_t = g_{ss}$. It is immediate that for all $k \geq 1$ if $t + k < T$ and $g^*_{t+k} > 0$, then $G_{t+k+1} = G_{ss}$, and $g^*_{t+k} = g_{ss}$. Suppose for some $k \geq 1$, $g^*_{t+1} = 0$. This is possible and by implication (I-2), investment would stay at zero until T. Thus, from some (possibly degenerate) interval from t' to t'', $G_{t+1} = G_{ss}$, and $g^*_t = g_s$. In addition, $g^*_t = 0$ for periods $t'' + 1$ to T.

It is clear that (I-1) directly implies part (i) of the Theorem and (I-2) directly implies part (iv) of the Theorem. To see the remainder, consider period t' as mentioned in the statement of the Theorem. Note that $g^*_{t'+1} < L\omega$ or else we would still be in case (i). Suppose $g^*_{t'+1} = 0$. Then $t' = t''$ and case (ii) is satisfied by assumption, case (iv) obtains in the next period, and case (iii) is vacuous. Finally suppose $L\omega > g^*_{t'+1} > 0$. Then case (ii) is satisfied by assumption and (I-3) implies that the optimal investment level stays at g_{ss} unless and until it drops to zero at some period t'' and stays for the rest of the future, that is, part (iii) of the Theorem obtains. ■

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