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## Sticky-Wage Models and Knowledge Capital: A Note

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### Abstract

We present a sticky-wage model with two types of labors: while a worker's labor contributes to current production, a researcher's work helps develop new ideas to add to a firm's knowledge capital that enhances its productivity for many periods. The long-lived effect of knowledge capital on productivity is analogous to the long-lasting effect of consumer durables on utility in the sticky-price model of Barsky, House and Kimball (2007). We show, however, that the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in BHK's sticky-price model is completely switched in our sticky-wage model.

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# Sticky-Wage Models and Knowledge Capital: A Note\*

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## 1 Introduction

In an influential paper, Barsky, House and Kimball (2007, hereafter BHK) emphasize the dominant role of the pricing of durable goods in generating monetary non-neutrality in a New Keynesian model with both durable and nondurable goods. They also provide an intriguing example of monetary neutrality when prices of durables are flexible and those of nondurables are sticky. This note points out that their model framework provides another perverse prediction of the neutrality of money in a very important class of New Keynesian models, which we will elaborate using a sticky-wage model with two types of labors.

As emphasized by Galí (2011), the wage-setting block in the New Keynesian model plays a central role in determining the response of the economy to monetary shocks. As such, the sticky-wage approach has been considered as a workhorse in the New Keynesian literature along with the sticky-price approach.<sup>1</sup> In sticky-wage models, wages are set by households in a way symmetric to how prices are set by firms in sticky-price models (e.g., Erceg, Henderson and Levin 2000). Likewise, we turn BHK's New Keynesian model on its head, replacing their two types of goods with our two types of labors.

In BHK's sticky-price model, firms set prices for two types of goods: once produced, one is immediately consumed entirely, while the other adds to its stock that yields utility over time and wears out only gradually.

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<sup>1</sup>Huang and Liu (2002), Huang, Liu and Phaneuf (2004), Christiano, Eichenbaum and Evans (2005) and many others find that wage rigidity is at least as important as price rigidity for explaining the empirical regularities in the U.S. economy.

In our sticky-wage model, households set wages for two types of labors: while a worker's labor immediately contributes to current production, a researcher's work helps develop new ideas to add to a firm's knowledge capital that enhances its productivity for many periods and becomes obsolete only gradually.<sup>2</sup> The long-lived effect of knowledge capital on productivity in our sticky-wage model is therefore analogous to the long-lasting effect of consumer durables on utility in BHK's sticky-price model.

We show, however, that the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in BHK's sticky-price model is completely switched in our sticky-wage model: the model now critically depends on stickiness in wages of workers rather than of researchers to generate monetary non-neutrality. We also provide a monetary neutrality example comparable to what BHK called the instructive limiting case, but with sticky researchers' wages and flexible workers' wages. We in fact show that, if workers' wages are flexible, output response to monetary shocks is essentially null or substantially muted regardless of researchers' wage stickiness. To generate a prediction similar to what is predicted by the standard New Keynesian model, it is workers' not researchers' wages that need to be sticky. Therefore, the pricing of workers' labor plays a dominant role in determining the response of aggregate output to monetary shocks. This conclusion holds quite generally regardless of other details of the model.

The researchers in our model develop new knowledge based on existing ones and so can be thought of as more skilled or knowledgeable than the workers. While there is only sparse direct evidence on the relative stickiness in wages for skilled versus unskilled labors, relevant empirical studies seem to suggest that wages of unskilled labors tend to be flexible compared to wages of skilled labors even though the latter can be fairly sticky.<sup>3</sup> A larger body of evidence also suggests that unskilled labors are subject to a higher turnover rate than skilled labors and wages of new hires tend to be flexible compared to wages of job stayers.<sup>4</sup> This note's results then pose a challenge to wage rigidity as a key monetary transmission mechanism in light of

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<sup>2</sup>This structure differs from Carlstrom and Fuerst (2010) who simply added sticky wages in the BHK model without introducing two types of labors.

<sup>3</sup>See, for example, Campbell (1997), Kahn (1997), Du Caju, Fuss and Wintr (2007), and Babecký, Du Caju, Kosma, Lawless, Messina and Rõõm (2010).

<sup>4</sup>See Beaudry and DiNardo (1991), Shin (1994), Bewley (1998), Fehr and Goette (2005), Babecký, Du Caju, Kosma, Lawless, Messina and Rõõm (2010), Daly, Hobijn and Wiles (2012), Haefke, Sonntag and van Rens (2013), Kudlyak (2014), Barattieri, Basu and Gottschalk (2014), and Basu and House (2016), among others. Pissarides (2009) and Kudlyak (2010) contain some useful overviews.

these empirical findings, although the recent empirical study by Gertler, Huckfeldt and Trigari (2016) argues that wages for new workers may not be flexible compared to wages for job stayers.<sup>5</sup> Hence future research on the heterogeneity in wage stickiness across labors should be a priority at least as high as that on the heterogeneity in price stickiness across goods.

## 2 Our sticky-wage model

The model features a continuum of households indexed by  $i \in [0, 1]$ , each consisting of a worker and a researcher, and a continuum of firms indexed by  $j \in [0, 1]$ , each producing a differentiated good. The labor skills of the workers are imperfectly substitutable, and so are the labor skills of the researchers. There is a government conducting monetary policy.

At any date  $t$ , the objective of household  $i \in [0, 1]$  is to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s(i)) - V_W(N_{W,s}(i)) - V_R(N_{R,s}(i))], \quad (1)$$

where  $\mathbb{E}_t$  is the conditional expectations operator,  $\beta \in (0, 1)$  is a subjective discount factor,  $N_{W,s}(i)$  and  $N_{R,s}(i)$  are its worker's and researcher's labors, respectively, and  $C_s(i) = \left[ \int_0^1 C_s(i, j)^{(\varepsilon_P - 1)/\varepsilon_P} dj \right]^{\varepsilon_P/(\varepsilon_P - 1)}$  is a consumption basket of the differentiated goods with  $\varepsilon_P > 1$ . The functions  $U$ ,  $V_W$  and  $V_R$  are strictly increasing and twice continuously differentiable, with concave  $U$  and convex  $V_W$  and  $V_R$ . Its budget constraint in period  $t$  is

$$\int_0^1 P_t(j) C_t(i, j) dj \leq W_{W,t}(i) N_{W,t}(i) + W_{R,t}(i) N_{R,t}(i) - \mathbb{E}_t [D_{t,t+1} B_{t+1}(i)] + B_t(i) + \Pi_t(i), \quad (2)$$

where  $P_t(j)$  is good  $j$ 's price,  $W_{W,t}(i)$  and  $W_{R,t}(i)$  are nominal wages of worker  $i$  and researcher  $i$ , respectively,  $D_{t,t+1}$  is the stochastic discount factor from date  $t + 1$  to  $t$ ,  $B_{t+1}(i)$  is a random quantity representing household  $i$ 's holdings of one-period state-contingent nominal bonds in period  $t$ , and  $\Pi_t(i)$  is household  $i$ 's claim to firms' profits.

Utility maximization gives rise to household  $i$ 's demand,  $C_t(i, j) = [P_t(j)/P_t]^{-\varepsilon_P} C_t(i)$ , where  $P_t =$

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<sup>5</sup>See also the interpretation by Hines, Hoynes and Krueger (2001) on the findings by Solon, Barsky and Parker (1994).

$[\int_0^1 P_t(j)^{1-\varepsilon_P} dj]^{1/(1-\varepsilon_P)}$ . The total demand for firm  $j$ 's good is given by  $Y_t(j) = \int_0^1 C_t(i, j) di = [P_t(j)/P_t]^{-\varepsilon_P} Y_t$ , where  $Y_t = \int_0^1 C_t(i) di$ .

Good  $j \in [0, 1]$  is produced using workers' labor inputs and firm  $j$ 's knowledge capital according to

$$Y_t(j) = F(N_{W,t}(j), K_t(j)), \quad (3)$$

where  $N_{W,t}(j) = \left[ \int_0^1 N_{W,t}(i, j)^{(\varepsilon_W-1)/\varepsilon_W} di \right]^{\varepsilon_W/(\varepsilon_W-1)}$  is a composite of the differentiated workers' labor, and  $K_t(j)$  is firm  $j$ 's knowledge capital that satisfies the following law of motion

$$K_t(j) = (1 - \delta)K_{t-1}(j) + G(N_{R,t}(j), K_{t-1}), \quad (4)$$

where  $N_{R,t}(j) = \left[ \int_0^1 N_{R,t}(i, j)^{(\varepsilon_R-1)/\varepsilon_R} di \right]^{\varepsilon_R/(\varepsilon_R-1)}$  is a composite of the differentiated researchers' labor, and  $K_{t-1}$  is a stock of aggregate knowledge capital, with  $\varepsilon_W, \varepsilon_R > 1$ . The functions  $F$  and  $G$  are homogenous of degree one, strictly increasing, concave, and continuously differentiable. The dependence of  $G$  on  $K_{t-1}$  captures a spillover effect of existing stock of knowledge in the economy in helping accumulate firm-specific knowledge capital,<sup>6</sup> which becomes obsolete at a rate  $\delta \in (0, 1)$ , and whose law of motion is in the spirit of the seminal works by Romer (1990) and Grossman and Helpman (1991), among others. We consider a small positive value of  $\delta$  to ensure the stationarity of  $K_t(j)$ .<sup>7</sup>

Cost minimization gives rise to firm  $j$ 's demands for worker  $i$  and researcher  $i$ ,

$$N_{W,t}(i, j) = \left[ \frac{W_{W,t}(i)}{W_{W,t}} \right]^{-\varepsilon_W} N_{W,t}(j) \quad \text{and} \quad N_{R,t}(i, j) = \left[ \frac{W_{R,t}(i)}{W_{R,t}} \right]^{-\varepsilon_R} N_{R,t}(j),$$

where  $W_{W,t} = [\int_0^1 W_{W,t}(i)^{1-\varepsilon_W} di]^{1/(1-\varepsilon_W)}$  and  $W_{R,t} = [\int_0^1 W_{R,t}(i)^{1-\varepsilon_R} di]^{1/(1-\varepsilon_R)}$ . The total demand for

<sup>6</sup>Knowledge capital accumulated by individual firms enhances the the economy-wide knowledge stock that in turn may help each firm develop new idea to add on its own knowledge capital. Hence, firm-specific knowledge capital as modeled here can be thought of as a measure of intangible assets (e.g., patents, copyrights, trademarks and trade names, blueprints or building designs, engineering drawings, organizational expenses, as defined in the Compustat database) that provides a basis for product differentiation in the spirit of Chamberlinian monopolistic competition model formulated by Dixit and Stiglitz (1977) and adopted in the New Keynesian literature. In particular, while not a pure public good, knowledge capital may be only partially excludable or non-rival and represents a cost independent from the level of output (i.e., Romer 1986, 1990; Arrow 1999). In a similar vein, it may also be reinterpreted as a form of organizational capital in the spirit of Beaudry and Devereux (1995), in the sense that its accumulation is an alternative rather than a complement to production.

<sup>7</sup>See, for example, Comin and Gertler (2006) for an introduction of the possibility that knowledge becomes obsolete.

labor is given by

$$N_{W,t}(i) = \int_0^1 N_{W,t}(i,j) dj = \left[ \frac{W_{W,t}(i)}{W_{W,t}} \right]^{-\varepsilon_W} N_{W,t} \quad \text{and} \quad N_{R,t}(i) = \int_0^1 N_{R,t}(i,j) dj = \left[ \frac{W_{R,t}(i)}{W_{R,t}} \right]^{-\varepsilon_R} N_{R,t},$$

where  $N_{W,t} = \int_0^1 N_{W,t}(j) dj$ , and  $N_{R,t} = \int_0^1 N_{R,t}(j) dj$ . Because households are indifferent about working at different firms, wages  $W_{W,t}(i)$  and  $W_{R,t}(i)$  are independent of  $j$ .

While households are price takers in the goods market, workers (researchers) are monopolistic competitors in the labor market for workers (researchers), where they set their wages  $W_{W,t}(i)$  ( $W_{R,t}(i)$ ) in a staggered fashion with hazard rate  $\theta_W$  ( $\theta_R$ ) of unable to adjusting wages, taking as given the labor demand schedules. At date  $t$ , if worker (researcher)  $i$  gets the chance to reset its wage, then it will choose the wage to satisfy

$$W_{h,t}(i) = \frac{\varepsilon_h}{\varepsilon_h - 1} \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_h)^{s-t} V'_h([W_{h,t}(i)/W_{h,s}]^{-\varepsilon_h} N_{h,s}) W_{h,s}^{\varepsilon_h} N_{h,s}}{\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_h)^{s-t} U'(C_s(i)) W_{h,s}^{\varepsilon_h} N_{h,s} / P_s}, \quad h = W, R. \quad (5)$$

While wage takers in the labor markets, firms are monopolistic competitors in the goods market where they set prices for their products in a synchronized fashion taking the goods demand schedules as given. At any date  $t$ , firm  $j$ 's problem is to choose  $\{P_s(j), N_{W,s}(j), N_{R,s}(j), K_s(j)\}_{s \geq t}$  to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} D_{t,s} [P_s(j) Y_s(j) - W_{W,s} N_{W,s}(j) - W_{R,s} N_{R,s}(j)] \quad (6)$$

subject to (3), (4), and the demand schedule for good  $j$ , where  $D_{t,s} = \prod_{\tau=1}^{s-t} D_{t+\tau-1, t+\tau}$  denotes the  $s$ -period stochastic discount factor from  $s$  to  $t$ , for all  $s > t$ , with  $D_{t,t} \equiv 1$ , and we have taken into account the solution to the embodied cost minimization problem.

Analogous to BHK's sticky-price model in which households must use cash to purchase goods (whose prices are sticky), we here in our sticky-wage model assume that firms instead of households hold money in the economy since they face a cash-in-advance (CIA) constraint for the payment of labors.<sup>8</sup> Hence, money demand is introduced here via  $M_t = W_{W,t} N_{W,t} + W_{R,t} N_{R,t}$ .<sup>9</sup> The money supply  $M_t^s$  grows at a rate  $e^{\xi_t}$ ,  $M_t^s = e^{\xi_t} M_{t-1}^s$ , where  $\xi_t$  is a white-noise process with zero mean and a finite variance.

<sup>8</sup>In describing the model, we did not explicitly introduce the CIA constraint into the firm problem in order to conserve space. In an appendix, which is available upon request from the authors, we assume that firms must make wage payment in advance by money. That model with firms' money demand yields identical first-order conditions described in the main text in this note.

<sup>9</sup>See also Schmitt-Grohé and Uribe (2006, 2007) who assume the CIA constraint on the wage bill of firms.

### 3 Results

In this section we first contrast the results in our sticky-wage model and those in BHK’s sticky-price model by simulations. We next demonstrate the robustness of our results to alternative choices of parameter values.

We then provide our intuition for the results.

#### 3.1 Parametrization and simulations

In simulating our model, we closely follow the baseline parametrization in BHK. The subjective discount factor  $\beta$  is set to ensure that the annual risk-free rate equals 2 percent. The depreciation of the stock of knowledge is 5 percent per year. The utility function is parameterized as  $U(C_t(i)) = C_t(i)^{1-\sigma} / (1-\sigma)$ ,  $V_h(N_{h,t}(i)) = N_{h,t}(i)^{1+\psi_h} / (1+\psi_h)$ , where  $\sigma = \psi_h = 1$ , for  $h = W, R$ . The parameter  $\varepsilon_P$  is set to 11 as in BHK, while  $\varepsilon_W$  and  $\varepsilon_R$  are set to 5, consistent with the previous studies on sticky-wage models (e.g., Erceg, Henderson and Levin 2000; Huang and Liu 2002). The technology is parameterized by  $F(N_{W,t}(j), K_t(j)) = [N_{W,t}(j)]^{1-\alpha} [K_t(j)]^\alpha$  and  $G(N_{R,t}(j), K_{t-1}) = [N_{R,t}(j)]^{1-\lambda} K_{t-1}^\lambda$ . Here,  $\alpha$  and  $\lambda$  are both set to 0.5, due to the lack of a broad consensus on the knowledge input share and on the degree of knowledge spillovers. But our results are robust to the choices of these parameter values. Finally, to facilitate the comparison between BHK’s sticky-price model and our sticky-wage model, we set the degree of wage stickiness so that impulse responses of output from the standard sticky-wage model match those from the standard sticky-price model.<sup>10</sup>

We first demonstrate the money neutrality result of our sticky-wage model and compare it to the money neutrality result from BHK’s sticky-price model.<sup>11</sup> As Figure 1 shows, our sticky-wage model generates the near neutrality of money comparable to BHK’s sticky-price model.<sup>12</sup> The solid lines in the figure plot impulse responses of output to a one percent increase in the money supply in the sticky-price model (left panel) and the sticky-wage model (right panel), while for comparison the dashed lines display the impulse

<sup>10</sup>The standard models are the sticky-price model with only nondurables and the sticky-wage model with only workers’ labor. We match the slopes of the New Keynesian Phillips curves in the two standard models to facilitate the comparison.

<sup>11</sup>For the sake of compatibility with our sticky-wage model, here physical capital is abstracted from BHK’s sticky-price model.

<sup>12</sup>In all figures of this note, simulations are based on a period of 100th of a year, as in BHK. For convenience, quarters are marked on the horizontal axes.

responses from the standard sticky-price and sticky-wage models (which by construction are identical across the two standard models). The solid line in the left panel reproduces BHK's simulation results under the assumption that prices of durables are flexible and prices of nondurables are sticky. The solid line in the right panel presents the impulse response of output in our sticky-wage model, but with sticky researchers' wages and flexible workers' wages. It is worth noting that the neutrality result is even more striking in our sticky-wage model than in BHK's sticky-price model, as in response to the money supply increase output barely moves from the steady state in our model, but it rises above the steady state by about 0.06 percent in BHK's model over the horizon in Figure 1.

However, what is more important to emphasize is the fact that the near neutrality of money in our sticky-wage model is generated with the exactly opposite configuration of nominal rigidities. While the long-lived effect of knowledge capital on productivity in our sticky-wage model is analogous to the long-lasting effect of consumer durables on utility in BHK's sticky-price model, the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in BHK's model is completely switched in our model: in BHK's model monetary neutrality occurs with flexible durable prices even if nondurable prices are sticky, whereas in our model neutrality occurs with flexible workers' wages even if researchers' wages are sticky.

The contrast between our sticky-wage model and BHK's sticky-price model goes beyond the above two instructive cases. For example, to generate monetary non-neutrality, BHK's model depends on stickiness in prices of durables rather than of nondurables, whereas our model depends on stickiness in wages of workers rather than of researchers. To put this into perspective, Figure 2 plots the impulse response of output in our model with sticky workers' wages and flexible researchers' wages (dashed lines) against that with flexible workers' wages and sticky researchers' wages (solid lines) which is the money neutrality case studied above. As the figure illustrates, when workers' wages are sticky, output rises above the steady state by 0.49 percent on impact of the monetary shock and it takes more than a year for it to go back to the steady state. As we will show below, significant monetary non-neutrality exists with sticky workers' wages, regardless of whether



researchers' wages are sticky or flexible.

The above contrasts between our sticky-wage model and BHK's sticky-price model actually hold broadly. At the very general level, it is the pricing of durable good in BHK's model, but the pricing of workers' labor in ours, that plays a dominant role in shaping aggregate dynamics following a monetary shock.<sup>13</sup>

### 3.2 Robustness checks

We here check the robustness of the results from our sticky-wage model to alternative values of parameters that govern the degrees of relative risk aversion, knowledge spillovers, and researchers' wage stickiness.

**Degree of relative risk aversion** We first show that our results are robust to the degree of relative risk aversion  $\sigma$ , which is set to 1 in the baseline calibration in Section 3.1. Under our CRRA specification of the utility function, the inverse of  $\sigma$  corresponds to the elasticity of intertemporal substitution in consumption. Hence, intuitively, a lower value of  $\sigma$  can weaken households' consumption smoothing motive, which may then result in greater fluctuations in aggregate demand following monetary shocks so as to magnify the real effects and thus make the near neutrality result harder to obtain.

Figure 3 plots the impulse responses of output to a one percent increase in the money supply for a wide range of values of  $\sigma$  under the two alternative configurations of nominal rigidities: sticky researchers' wages and flexible workers' wages (left panel) versus sticky workers' wages and flexible researchers' wages (right panel). As the figure shows, when  $\sigma$  declines from 5 to 1, and then to 0.01, the real effect of the monetary injection indeed becomes greater and greater under both configurations. However, the three IRFs in the left panel remain virtually indistinguishable from each others so money remains nearly neutral for all the values of  $\sigma$ . While the upward shifts of the IRFs in the right panel are more visible, the effects are moderate.

**Degree of knowledge spillovers** We next show that our results are robust to the degree of knowledge spillovers  $\lambda$ , which is set to 0.5 in the baseline calibration. Figure 4 displays the impulse responses of output

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<sup>13</sup>In verifying this general conclusion we have done many more robustness checks than reported below, including incorporation of sticky prices into our model. These additional checks are not presented in this note due to the space constraint, but they are available upon request from the authors.

for a wide range of values of  $\lambda$ , also under the two alternative configurations of wage rigidities. As can be seen from the figure, when  $\lambda$  declines from 0.8 to 0.4, and then to 0, that is, when the nature of knowledge capital changes from being largely non-rival to being completely excludable, the IRFs in the left panel of the figure edge up only marginally so money remains nearly neutral, while the IRFs in the right panel are virtually identical for the wide-ranging values of  $\lambda$ .

**Degree of researchers' wage stickiness** Recall that in the baseline calibration we set the degree of wage stickiness so that impulse responses of output from the standard sticky-wage model match those from the standard sticky-price model (see also Footnote 10). We now show that our basic conclusions in this paper are robust to the degree of wage stickiness, in particular, stickiness in researchers' wages.

Figure 5 plots output responses with varying average durations of researchers' wages, from 1 to 8 quarters, under flexible (left panel) and sticky (right panel) workers' wages. As the left panel of the figure shows, when workers' wages are flexible, increases in researchers' wage stickiness only marginally strengthen the real effects of monetary shocks, with output responses remaining uniformly small so money remaining nearly neutral. In contrast, as the right panel shows, when workers' wages are sticky (with an average duration of 4 quarters) there is significant monetary non-neutrality regardless of whether researchers' wages are sticky or flexible.

### 3.3 Intuition

To understand the intuition for our near neutrality result, it is helpful to examine the following log-linearized equilibrium condition, which equates the gap between the marginal product of workers' labor and its marginal rate of substitutions for consumption, to workers' desired wage markups (denoted as  $\mu_{W,t}$ ),

$$\left( \frac{F_{NN}N_W}{F_N} - \frac{V''_W N_W}{V'_W} + \frac{U''C}{U'} \frac{F_N N_W}{C} \right) \hat{N}_{W,t} = - \left( \frac{F_{NK}K}{F_N} + \frac{U''C}{U'} \frac{F_K K}{C} \right) \hat{K}_t + \hat{\mu}_{W,t}, \quad (7)$$

where  $F_N \equiv \partial F(N_W, K) / \partial N_W$ ,  $F_K \equiv \partial F(N_W, K) / \partial K$ ,  $F_{NN} \equiv \partial^2 F(N_W, K) / \partial N_W^2$ , and  $F_{NK} \equiv \partial^2 F(N_W, K) / (\partial N_W \partial K)$ , and where a variable with no time index denotes the steady-state value of that variable and a variable with a hat represents the log-deviation of that variable from its steady-state value.

Here we have applied the market clearing condition for the composite good,  $C_t = F(N_{W,t}, K_t)$ .

The effect of a monetary shock on aggregate output depends on how much it moves workers' labor input,  $\hat{N}_{W,t}$  on the left side of (7), and knowledge capital,  $\hat{K}_t$  on the right side of (7). It is worth noting that our formulation of the production technology in (3) and (4) allows newly accumulated knowledge capital to become productive immediately. Indeed, in this instructive case with flexible workers' wages and sticky researchers' wages, the former increases faster than the latter in response to a monetary injection. This is both because some researchers cannot adjust wages and because the desired wage markups for adjusting researchers are counter-cyclical. The resulting decline in real wages for researchers incentivizes firms to employ more researchers to accumulate their knowledge capitals. In fact, given the high durability of knowledge capital, such incentives can be very strong; however, the CIA constraint prevents firms from raising researchers' labor to an extreme level. As shown by Figure 6 (solid line), the increase in researchers' labor input is more than 1.5 percent on impact of a one percent increase in the money supply; yet, this is still not dramatic enough to create a sizable increment in the stock of knowledge capital (the maximal increase of which is no more than 0.02 percent and it takes time to materialize). The slow moving knowledge capital also implies a muted response in workers' labor, since workers' desired wage markups are unresponsive, that is,  $\hat{\mu}_{W,t}$  on the right side of (7) stays at zero for all  $t$ . This is why the near neutrality result holds in this case independent of other details of our model.

This is related to another point of comparison between our neutrality result and that of BHK, for which a negative comovement between labor inputs for producing nondurable and durable goods is critical. Inspecting (7) reveals that the coefficient of  $\hat{N}_{W,t}$  is negative whereas that of  $\hat{K}_t$  can be negative, zero, or positive.<sup>14</sup> In consequence, our neutrality result doesn't hinge upon any specific comovement patterns between workers' labor and knowledge capital or researchers' labor. To see this more clearly, we invoke the functional forms for  $U$ ,  $V_W$ , and  $F$  postulated in our numerical simulations. Then (7) becomes,

$$-[\alpha + \psi_W + \sigma(1 - \alpha)]\hat{N}_{W,t} = -\alpha(1 - \sigma)\hat{K}_t + \hat{\mu}_{W,t}, \quad (8)$$

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<sup>14</sup>The validity of our analysis and basic conclusion doesn't depend on how we parameterize preferences or technology. In order to make the utility-maximization and profit-maximization problems well defined, it is sufficient to assume  $U' > 0$ ,  $U'' \leq 0$ ,  $V'_W > 0$ ,  $V''_W \geq 0$ ,  $F_N > 0$ ,  $F_K \geq 0$ ,  $F_{NK} \geq 0$  and  $F_{NN} \leq 0$ .

which, in this instructive case with flexible workers' wages (thus  $\hat{\mu}_{W,t} = 0$  for all  $t$ ), reduces to,

$$-[\alpha + \psi_W + \sigma(1 - \alpha)]\hat{N}_{W,t} = -\alpha(1 - \sigma)\hat{K}_t. \quad (9)$$

By virtue of (9), the correlation of workers' labor with knowledge capital (thus with researchers' labor as well) is negative, zero, or positive, if  $\sigma$  is greater than, equal to, or smaller than unity. But, in all of these cases, money remains nearly neutral, because knowledge capital is slow moving so the sign of the correlation does not exert any quantitatively significant impact on the real effect of money.<sup>15</sup>

The intuition for our monetary non-neutrality result under the opposite configuration of nominal rigidities, that is to say, with sticky workers' wages and flexible researchers' wages, is more straightforward to explain. Following a monetary injection in this case, the former increases more slowly than the latter. This is both because some workers cannot adjust wages and because their desired wage markups for adjusting workers are counter-cyclical. The resulting decline in real wages for workers incentivizes firms to employ more workers to expand their production of goods. In fact, they prefer to spending all of their available liquidity in the face of the CIA constraint to increase workers' labor input but not researchers', so the stock of knowledge capital stays much as is. As shown by Figure 6 (dashed line), the increase in workers' labor input is about one percent on impact of a one percent increase in the money supply, and this generates a 0.49 percent increase in aggregate output on impact and it takes more than a year for it to go back to the steady state.<sup>16</sup> Such monetary non-neutrality result is robust regardless of whether researchers' wages are flexible or sticky.<sup>17</sup> This, together with the above illustration, explains why in our model whether money is neutral or not hinges upon whether workers' wages are flexible or sticky, but has little to do with rigidity in researchers' wages or lack thereof.

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<sup>15</sup>The near neutrality result for the zero-correlation case (with the baseline calibration of  $\sigma = 1$ ) is what is reported in Figures 1, 2, 3 and 6, in which workers' labor input is invariant to a monetary shock, where Figure 3 also report the near neutrality results under negative and positive correlations between the two types of labor inputs (obtained by setting  $\sigma$  to 5 and 0.01 respectively).

<sup>16</sup>This monetary non-neutrality result is earlier reported in Figures 2 and 3, and is obtained under the baseline calibration of equal shares of knowledge capital and workers' labor in the production of goods, that is, with  $\alpha = 1 - \alpha = 0.5$ . We have verified that, when  $\alpha$  increases, the real effect of money decreases, with monetary neutrality re-established in the limiting case with  $\alpha = 1$ . These additional results under different values of  $\alpha$  are not reported here in order to conserve space, but they are available upon request from the authors.

<sup>17</sup>This is in fact the second set of the sensitivity results reported in Figure 5.

This brings us to a final point of comparison between our neutrality result and that of BHK, which relies on the near constancy of the shadow value for durable good, in addition to the negative comovement between labor inputs for producing durable and nondurable goods. Let  $\gamma_t$  denote the Lagrange multiplier for (4), that is, the shadow value for knowledge capital in our model. We can show that,

$$\gamma_t = \frac{F_K(N_{W,t}, K_t)}{\mu_P} + \beta(1 - \delta)\mathbb{E}_t(\gamma_{t+1}), \quad (10)$$

where  $\mu_P \equiv \varepsilon_P / (\varepsilon_P - 1)$  denotes firms' desired price markups. According to (10), how much  $\gamma_t$  responds to a monetary shock depends on whether workers' wages are flexible or sticky. With flexible workers' wages,  $\gamma_t$  is nearly constant, and this is also the case with near monetary neutrality. With sticky workers' wages,  $\gamma_t$  can vary significantly in response to a monetary shock, and this is also the case with monetary non-neutrality. This is to say that the near constancy of the shadow value for knowledge capital and the near neutrality of money must either both fail or both hold. Which of these two possibilities would take force again depends on rigidity or lack thereof in workers' wages rather than in researchers' wages.

## 4 Concluding remarks

The importance of durable goods for economic fluctuations driven by technology and sunspot shocks has long been studied in real models of the business cycle (e.g., Baxter 1996; Weder 1998). In a more recent paper, Barsky, House and Kimball (2007) show that the pricing of durable goods plays a dominant role, whereas the pricing of nondurable goods is immaterial, in determining the real effects of monetary shocks in a New Keynesian sticky-price model. Specifically, money can be neutral or have real effects depending on whether prices of durable goods' are sticky or flexible, but independent of rigidity or lack thereof in nondurable goods' prices. After concluding that "durables are the most important element in sticky price models," they urge that "researchers must devote more effort to empirical investigation of the pricing of these goods."

The recent development in the New Keynesian literature has assigned a central role to the sticky-wage approach, along with the sticky-price approach. There has also been an increased interest in a broader context in studying labor market frictions, as opposed to goods market frictions, as a transmission mechanism in real

and monetary business cycles. Against this large background, we have turned BHK's sticky-price model on its head, replacing their two types of goods with our two types of labors: while a worker's labor immediately contributes to current production, a researcher's work helps develop new ideas to add to a firm's knowledge capital that enhances its productivity for many periods and becomes obsolete only gradually. The long-lived effect of knowledge capital on productivity in our sticky-wage model is thus analogous to the long-lasting effect of consumer durables on utility in BHK's sticky-price model. We have used our model to study how the pricing of the two types of labors may affect the real effects of monetary shocks.

We have shown that the relative role of the pricing of the two production inputs analogous to consumption durables and nondurables in BHK's sticky-price model is completely switched in our sticky-wage model: whether money is neutral or not hinges upon rigidity or lack thereof in workers' wages rather than in researchers' wages. At the very general level, it is the pricing of workers' not researchers' labor that plays a dominant role in shaping aggregate dynamics following a monetary shock. We have demonstrated that this conclusion holds quite generally regardless of other details of our model. Our results in this note suggest that future research on the heterogeneity in wage stickiness across labors should be a priority at least as high as that on the heterogeneity in price stickiness across goods.

## References

- Arrow, K. (1999). Knowledge as a factor of production. Keynote address, World Bank Annual Conference on Development Economics.
- Babecký, J., Du Caju, P., Kosma, T., Lawless, M., Messina, J., and Rõõm, T. (2010). Downward nominal and real wage rigidity: Survey evidence from european firms. *Scandinavian Journal of Economics*, 112(4):884–910.
- Barattieri, A., Basu, S., and Gottschalk, P. (2014). Some evidence on the importance of sticky wages. *American Economic Journal: Macroeconomics*, 6(1):70–101.
- Barsky, R. B., House, C. L., and Kimball, M. S. (2007). Sticky-price models and durable goods. *American Economic Review*, 97(3):984–998.
- Basu, S. and House, C. L. (2016). Allocative and remitted wages: New facts and challenges for Keynesian models. Working Paper 22279, National Bureau of Economic Research.
- Baxter, M. (1996). Are consumer durables important for business cycles? *Review of Economics and Statistics*, 78(1):147–155.
- Beaudry, P. and Devereux, M. B. (1995). Towards an endogenous propagation theory of business cycles. Mimeo, University of British Columbia.
- Beaudry, P. and DiNardo, J. (1991). The effect of implicit contracts on the movement of wages over the business cycle: Evidence from micro data. *Journal of Political Economy*, 99(4):665–688.
- Bewley, T. F. (1998). Why not cut pay? *European Economic Review*, 42(3-5):459–490.
- Campbell III, C. M. (1997). The variation in wage rigidity by occupation and union status in the us. *Oxford Bulletin of Economics and Statistics*, 59(1):133–147.
- Carlstrom, C. T. and Fuerst, T. S. (2010). Nominal rigidities, residential investment, and adjustment costs. *Macroeconomic Dynamics*, 14(01):136–148.
- Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.
- Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.
- Daly, M. C., Hobijn, B., and Wiles, T. S. (2011). Dissecting aggregate real wage fluctuations: Individual wage growth and the composition effect. Working paper, Federal Reserve Bank of San Francisco.
- Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297–308.
- Du Caju, P., Fuss, C., and Wintr, L. (2007). Downward wage rigidity for different workers and firms: An evaluation for Belgium using the IWFP procedure. Working Paper Series 0840, European Central Bank.
- Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281 – 313.

- Fehr, E. and Goette, L. (2005). Robustness and real consequences of nominal wage rigidity. *Journal of Monetary Economics*, 52(4):779 – 804.
- Galí, J. (2011). The return of the wage Phillips curve. *Journal of the European Economic Association*, 9(3):436–461.
- Gertler, M., Huckfeldt, C., and Trigari, A. (2016). Unemployment fluctuations, match quality, and the wage cyclicalities of new hires. Working Paper 22341, National Bureau of Economic Research.
- Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *The Review of Economic Studies*, 58(1):43.
- Haefke, C., Sonntag, M., and van Rens, T. (2013). Wage rigidity and job creation. *Journal of Monetary Economics*, 60(8):887 – 899.
- Hines, Jr., J. R., Hoynes, H. W., and Krueger, A. B. (2001). Another look at whether a rising tide lifts all boats. Working Paper 8412, National Bureau of Economic Research.
- Huang, K. X. and Liu, Z. (2002). Staggered price-setting, staggered wage-setting, and business cycle persistence. *Journal of Monetary Economics*, 49(2):405–433.
- Huang, K. X. D., Liu, Z., and Phaneuf, L. (2004). Why does the cyclical behavior of real wages change over time? *American Economic Review*, 94(4):836–856.
- Kahn, S. (1997). Evidence of nominal wage stickiness from microdata. *American Economic Review*, 87(5):993–1008.
- Kudlyak, M. (2010). Are wages rigid over the business cycle? *Economic Quarterly*, 96:179–199.
- Kudlyak, M. (2014). The cyclicalities of the user cost of labor. *Journal of Monetary Economics*, 68:53 – 67.
- Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369.
- Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037.
- Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.
- Schmitt-Grohé, S. and Uribe, M. (2006). Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model. In *NBER Macroeconomics Annual 2005, Volume 20*, NBER Chapters, pages 383–462. National Bureau of Economic Research, Inc.
- Schmitt-Grohé, S. and Uribe, M. (2007). Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model. In Mishkin, F. S. and Schmidt-Hebbel, K., editors, *Monetary Policy under Inflation Targeting*, volume 11 of *Central Banking, Analysis, and Economic Policies Book Series*, chapter 5, pages 125–186. Central Bank of Chile.
- Shin, D. (1994). Cyclicalities of real wages among young men. *Economics Letters*, 46(2):137 – 142.
- Solon, G., Barsky, R., and Parker, J. A. (1994). Measuring the cyclicalities of real wages: How important is composition bias. *Quarterly Journal of Economics*, 109(1):1–25.



Weder, M. (1998). Fickle consumers, durable goods, and business cycles. *Journal of Economic Theory*, 81(1):37 – 57.

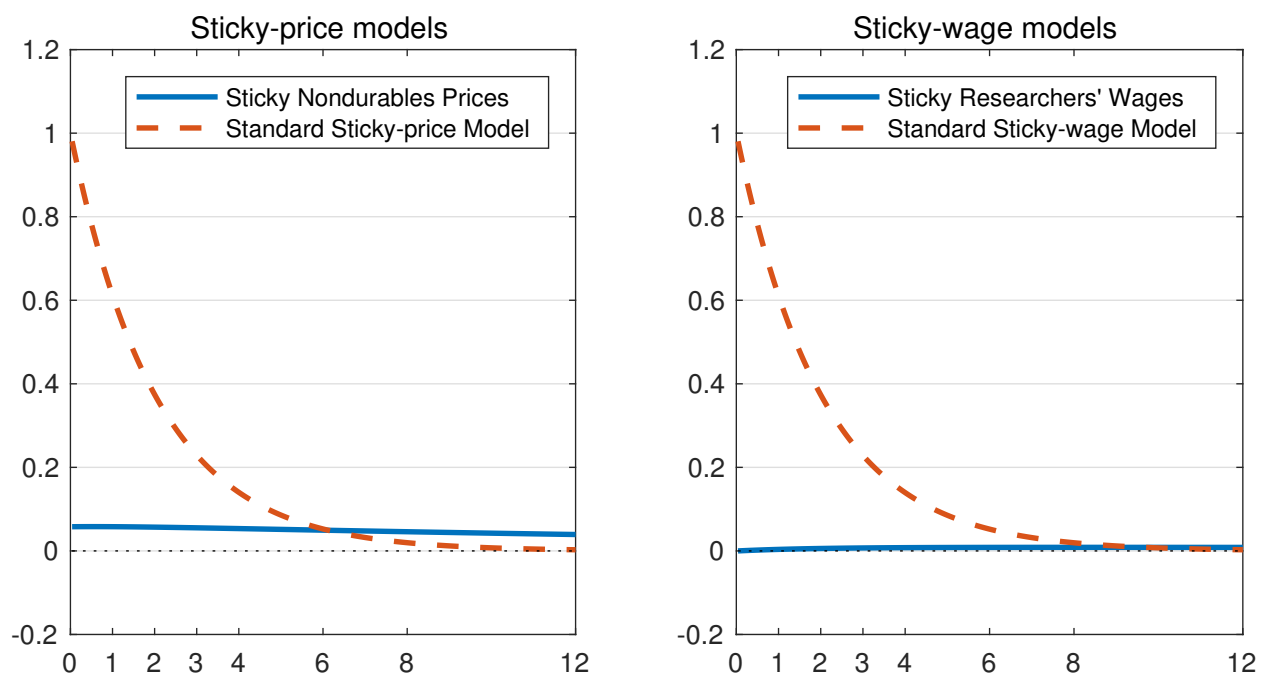


Figure 1: The Near Neutrality of Money in Sticky-price and Sticky-wage Models

Notes: The solid line on the left panel corresponds to the near money neutrality result in BHK's model with sticky nondurable and flexible durable prices. The solid line on the right panel shows responses of output to a one percent increase in the money supply in our model with sticky researchers' wages and flexible workers' wages. The dashed lines represent the responses of output in the standard sticky-price model (left panel) and those in the standard sticky-wage model (right panel). Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.

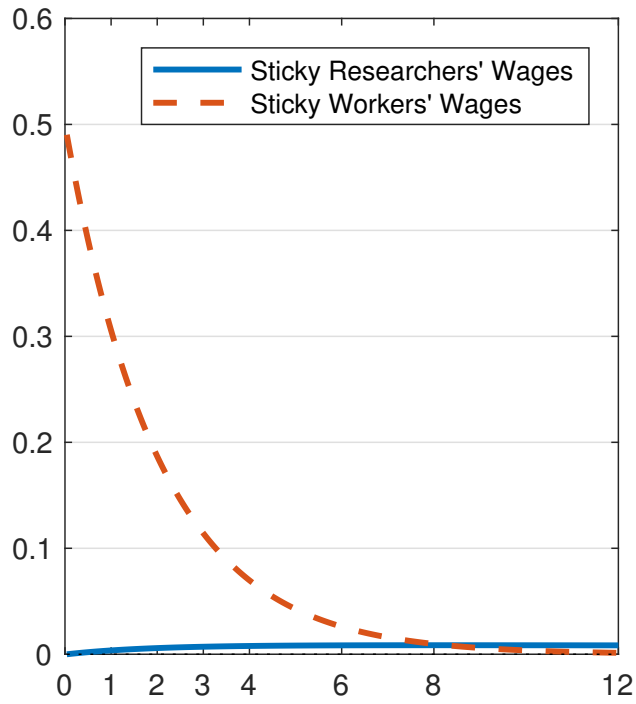


Figure 2: Output Responses to a One Percent Increase in the Money Supply

Notes: The solid line represents responses of output to a one percent increase in the money supply when workers' wages are flexible and researchers' wages are sticky. The dashed line shows responses with sticky workers' wages and flexible researchers' wages. Vertical axis measures percentage deviations from the steady state. Time in quarters is on horizontal axis.

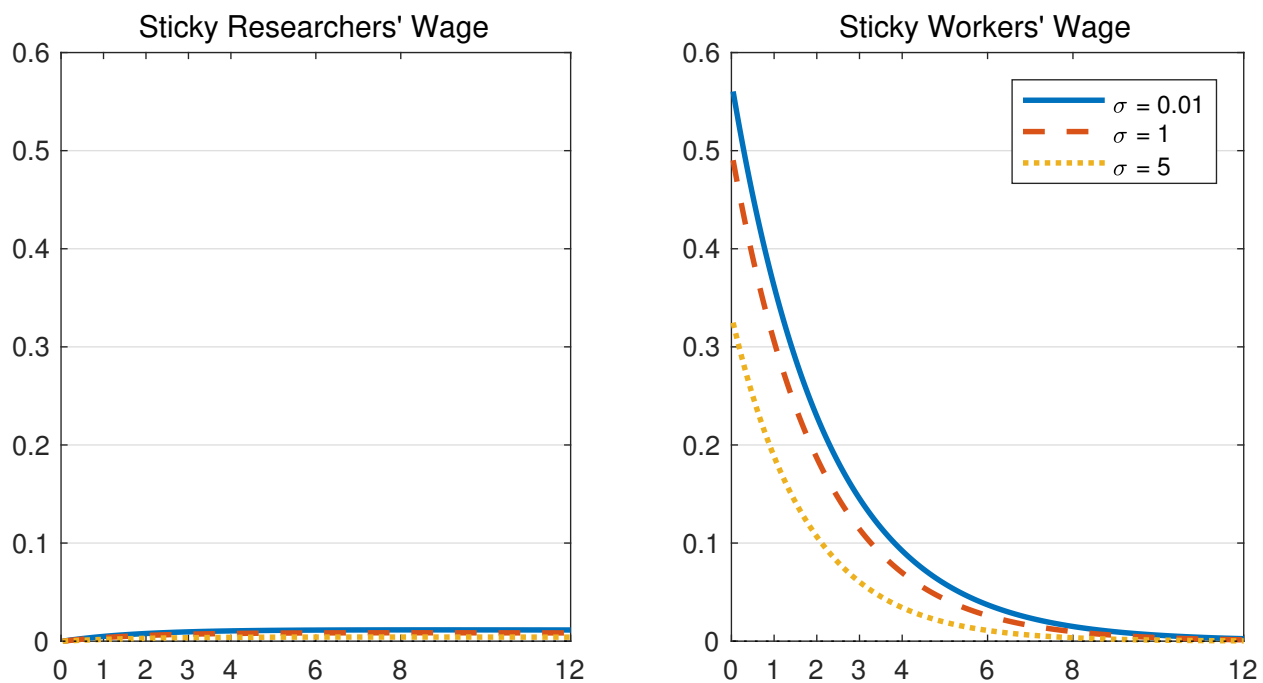


Figure 3: Output Responses with Different Degrees of Relative Risk Aversion

Notes: The left panel shows responses of output to a one percent increase in the money supply when workers' wages are flexible, but researchers' wages are sticky. The right panel displays those when workers' wages are sticky but researchers' wages are flexible. Lines in each panel are for different values of  $\sigma$ . Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.

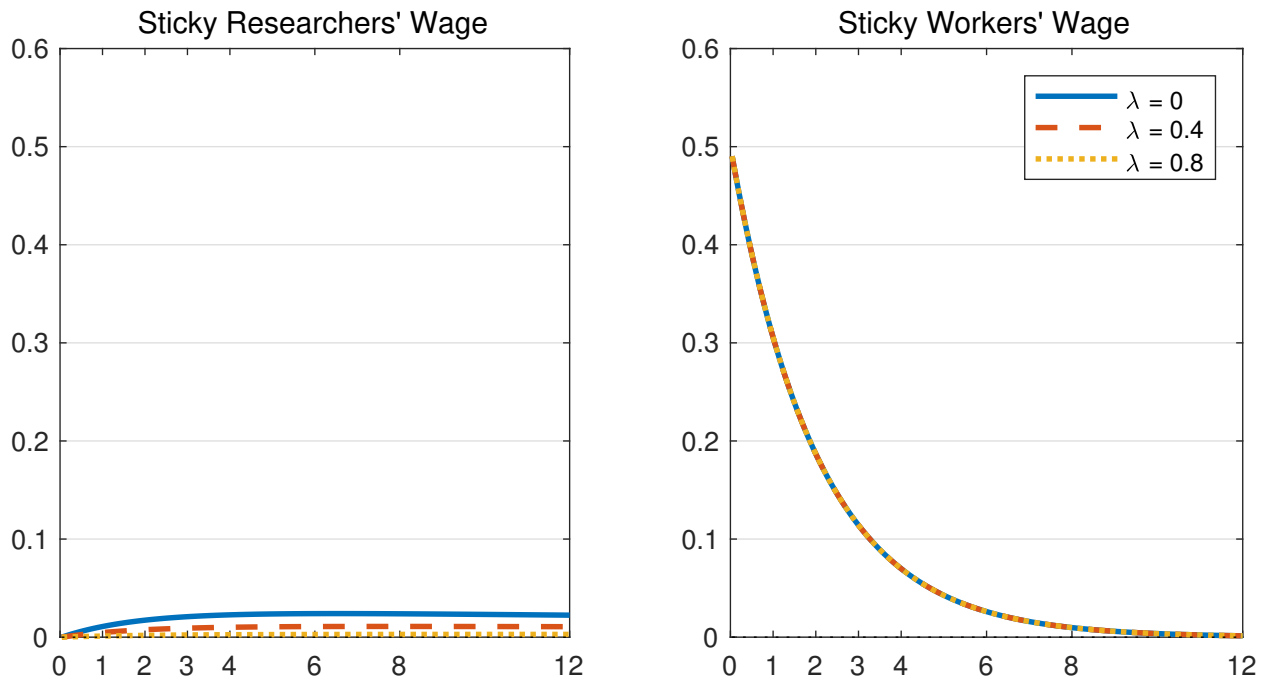


Figure 4: Output Responses with Different Degrees of Knowledge Spillovers

Notes: The left panel shows responses of output to a one percent increase in the money supply when workers' wages are flexible, but researchers' wages are sticky. The right panel displays those when workers' wages are sticky but researchers' wages are flexible. Lines in each panel are for different values of  $\lambda$ . Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.

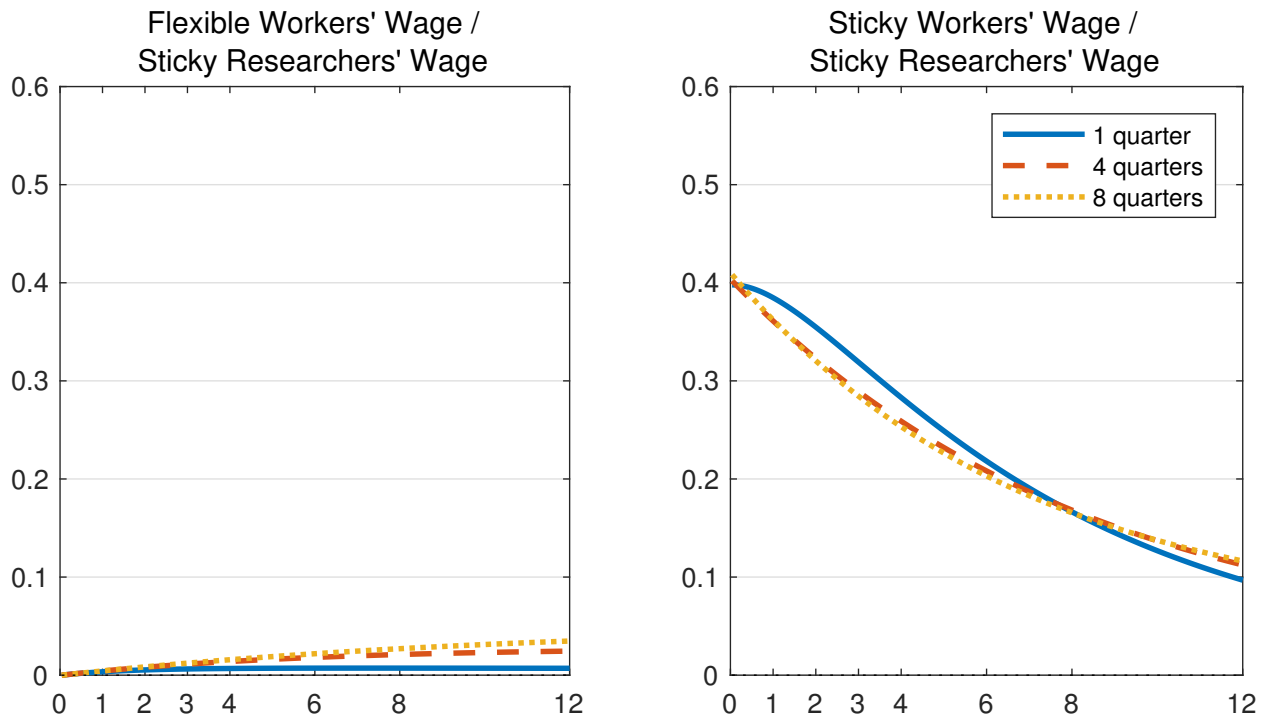


Figure 5: Output Responses with Different Degrees of Researchers' Wage Stickiness

Notes: The left panel shows responses of output to a one percent increase in the money supply when workers' wages are flexible, but the stickiness of researchers' wages varies across lines. The right panel displays those when the average duration for workers' wage changes is four quarters. Solid, dashed, and dotted lines in each panel represent the average duration between researchers' wage changes being one quarter, four quarters, and eight quarters, respectively. Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.

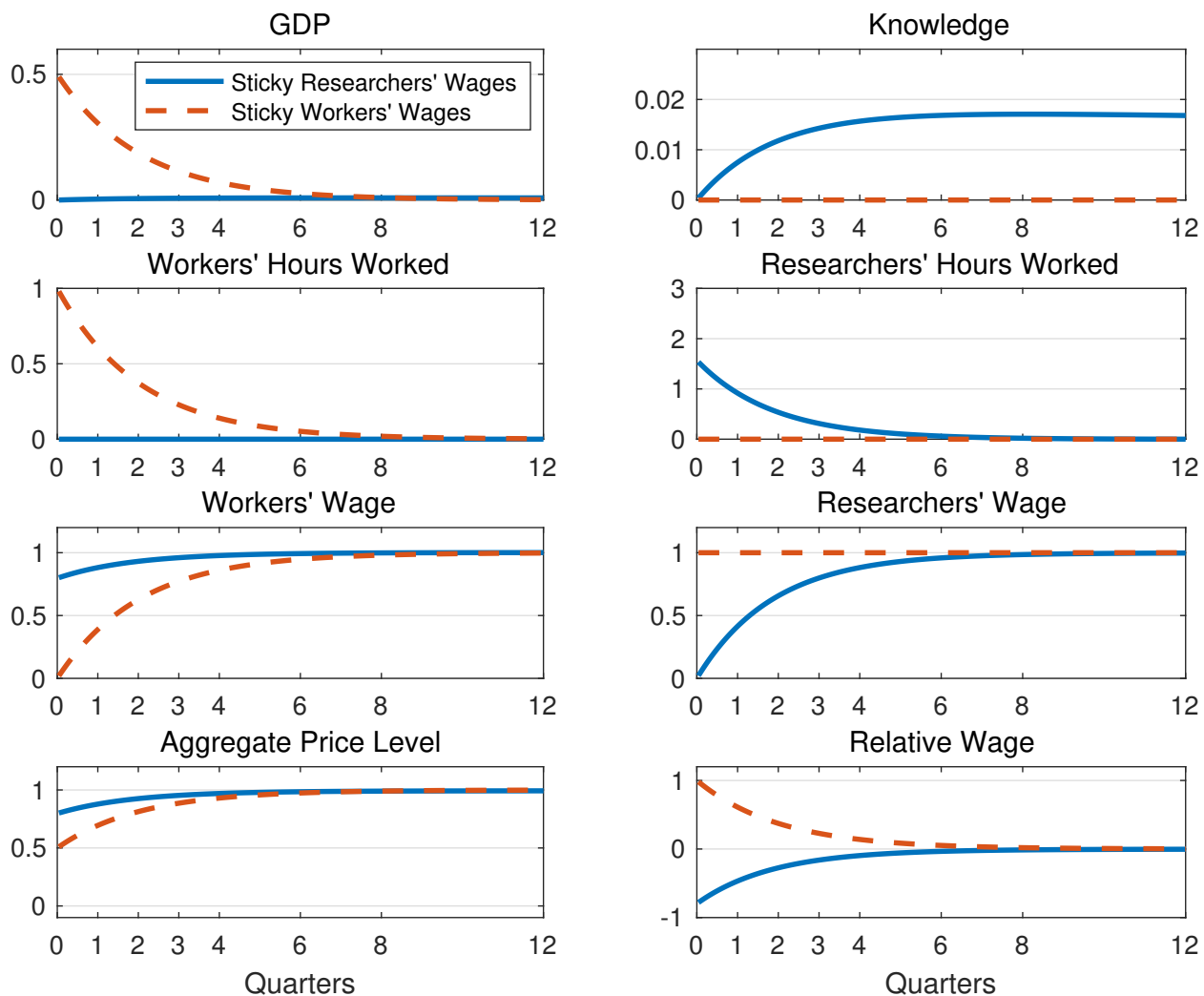


Figure 6: Responses to a One Percent Increase in the Money Supply

Notes: Each panel shows responses of corresponding variables in our sticky-wage model to a one percent increase in the money supply under two alternative configurations on wage stickiness. Solid lines represent the case when workers' wages are flexible and researchers' wages are sticky. Dashed lines show responses with sticky workers' wages and flexible researchers' wages. Vertical axes measure percentage deviations from the steady state. Time in quarters is on horizontal axes.