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Kevin x.d. Huang
Vanderbilt University

Qinglai Meng
Oregon State University

Jianpo Xue
Renmin University of China

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Contact: Kevin x.d. Huang - kevin.huang@vanderbilt.edu, Qinglai Meng - Qinglai.Meng@oregonstate.edu, Jianpo Xue - jpxue@ruc.edu.cn.
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Money growth targeting and indeterminacy in small open economies*

Kevin X.D. Huang[a]†, Qinglai Meng[b], Jianpo Xue[c]

[a]Department of Economics, Vanderbilt University, Nashville, TN 37235, USA

[b]Department of Economics, Oregon State University, Corvallis, OR 97331, USA

[c]School of Finance, Renmin University of China, Beijing 100872, China

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Abstract

In a closed economy setting a cash-in-advance monetary economy under money growth targeting is prone to self-fulfilling expectations and beliefs-driven fluctuations. This paper shows that such extrinsic instability is less of a problem in a small open economy integrated in the world goods and financial markets. This is because endogenous terms-of-trade movements associated with global goods trade and cross-border capital flows and endogenous international asset price adjustments associated with global financial transaction serve as an endogenous stabilizer to reduce the likelihood of sunspot equilibria. We find that for empirically reasonable parametrization of the small open economy sunspot beliefs are unlikely to become self-fulfilled.

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†Corresponding author. Department of Economics, Vanderbilt University, VU Station B 351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, USA. Tel.: +1 615 936 7271; fax: +1 615 343 8495.

E-mail address: kevin.huang@vanderbilt.edu (K.X.D. Huang).

1 Introduction

In his widely cited study "The World Economy: A Millennial Perspective" Angus Maddison argued that international trade and capital flows were one of the three interactive processes that helped sustain economic advancement over time. Whereas many would agree that the globalization of economic activity and capital movements has brought the world innumerable improvements and opportunities, caution has also been issued to guard against its potential adverse effects, such as those that could make a nation opening to globalization a less stable or equitable place.

The amount of attention paid to the potential destabilizing consequences of globalization due to fundamental disturbances is quite remarkable. Perhaps even more remarkable is the possibility that increased linkages between economies might add to economic instability in these economies for reasons unrelated to economic fundamentals. In particular, it has been known that, for a class of models with production externalities, beliefs-driven extrinsic instability is more likely to arise in a small open economy setting with a perfect world capital market than in an otherwise closed economy setting.[1] The main objective of this paper is to demonstrate that, for a very important class of models, openness to globalization is turned on its head to become a stabilizer for the economy by preventing self-fulfilling expectations and beliefs-driven fluctuations from occurring.

This paper is not the first to showcase openness as a stabilization factor. In a recent contribution, Huang, Meng, and Xue (2017) demonstrate how integration into the world asset and goods markets of an otherwise closed economy helps relieve concerns raised by Schmitt-Grohé and Uribe (1997) about extrinsic uncertainty associated with fiscal policy that relies on income taxes to achieve balanced-budget objective. In this paper, we show how openness to globalization serves as a stabilizer for the class of cash-in-advance (CIA) monetary economies of Lucas and Stokey (1983, 1987), which, in a closed economy framework under money growth targeting such as that called for by Friedman (1969), are well known to be prone to self-fulfilling prophecies (e.g., Woodford 1994; Farmer 1999; Weder 2008a; as well as Matsuyama 1990 under a money-in-the-utility function approach).[2]

The key to comprehending this paper's results is to decipher why extrinsic uncertainty and thus equilibrium indeterminacy can occur in a closed economy setting in the first place. To begin, suppose that

[1]See, for example, Weder (2001), Lahiri (2001), Nishimura and Shimomura (2002a, 2002b), Meng and Velasco (2003, 2004), and Hu and Mino (2013). See, also, Razin, Sadka, and Coury (2003).

[2]One testable implication of these results is that open economies may be less volatile. This is supported by Figure 1 which displays a scatter plot of an inverse relationship between openness and volatility based on OECD data (1978:I-2008:III) for ten developed countries that are classified by Mendoza et al. (1997) as small open economies. Here volatility is measured by the standard deviation of output as in Chen et al. (2014) and openness by the imports of goods and services as a percentage of GDP as is standard in the literature (an admittedly more informative measure pertinent to our results should be obtained by decomposition conditional on sunspot shocks).

households expect the price level to be higher. The complementarity between consumption and real money balances under the CIA constraint then leads the households to contemplate a reduction in consumption. Note that, in order for the sunspot belief to be potentially self-fulling it must be the case that the households also expect real wage to be lower as a result of the higher price: If the households expect a higher real wage (i.e., a greater opportunity cost of leisure) instead, they would cut back on leisure and increase labor supply, but then the increase in output supply due to the increase in labor and the reduction in consumption demand would create an excess supply to depress the price level, invalidating the households' initial expectations. If real wage is expected to stay unchanged, this would similarly pre-empt the sunspot belief. The expectation of a lower real wage, however, would indeed induce the households to substitute away from consumption, towards leisure while decreasing labor supply. That said, this substitution effect is only necessary but not sufficient for rendering the sunspot belief self-fulfilled. This is because a higher price level also exerts a downward pressure on real wealth, and this would induce the households to increase labor supply while curtailing leisure and consumption. If the substitution effect dominates, then the decrease in output resultant from the decrease in labor supply would conform to the contemplated reduction in consumption, fulfilling the households' sunspot expectations of a higher price level. If the income effect dominates, however, then the increased output supply due to increased labor and the reduced consumption demand would create an excess supply to depress the price level, invalidating the households' initial sunspot expectations.

It should be clear that the substitution effect is larger, the larger is the Frisch elasticity of labor supply. For the class of CRRA utility functions, a key parameter that influences the income effect is the relative risk aversion in consumption: A smaller consumption risk aversion implies that a given consumption change would bring about a smaller change in the marginal utility of consumption and hence in the equilibrium shadow value of real income, or conversely, a given change in real income would have a greater effect on consumption (and leisure). In a closed economy setting, the substitution effect dominates the income effect for much of the empirically reasonable parametrization of the model; and, therefore, sunspot expectations can easily become self-fulfilling.

For a small open economy (SOE) integrated in the world goods and financial markets, the income effect is strengthened by endogenous terms-of-trade (TOT) movements associated with global goods trade and cross-border capital flows and endogenous international asset price adjustments associated with global financial transaction. In expectation of a higher domestic price, the households in the SOE would tend to increase their labor supply and thus home production would tend to rise due to a similar income effect presented above for a closed economy. Plus, now, home imports would tend to rise, because foreign goods become relatively cheaper due to the expected improvement in the home TOT, and because the SOE could finance its purchase of the additional imported goods by borrowing in the world asset market. This reinforces the aforementioned income effect to contradict the contemplated reduction in domestic consumption, making the initial sunspot expectations of a higher domestic price less likely to become self-fulfilling.[3] For empirically reasonable parametrization of the SOE model, the strengthened income effect dominates the substitution effect so that self-fulfilling expectations and sunspot equilibria become unlikely.

The remaining of the paper is organized as follows. Section 2 establishes our results in a flexible-price SOE setting, of which the closed economy model of Farmer (1999) can be cast as a limiting case. Section 3 generalizes the results to a sticky-price SOE setting, of which the closed economy model of Weder (2008a) can be cast as a limiting case. These sections focus on showing the stabilization role of trade openness whereas a complete international asset market is postulated. To demonstrate the stabilization role of international financial integration, Section 4 conducts the analysis under the assumption that the SOE has no access to international borrowing or lending and shows that, for any degree of trade openness, sunspot equilibria are more likely to arise in this case of international financial autarky than in the previous case with perfect integration of the SOE in the world capital market. Sections 5 provides some intuition for our results based on analytical expositions. Section 6 extends the analysis to an alternative monetary policy with inflation targeting. Sections 7 uses impulse response functions to get more feel about our results. Section 8 concludes the paper. Proofs of all propositions and other analytical derivations are relegated to the appendix.

2 A small open economy with flexible prices

The global economy consists of Home and Foreign, each producing tradable intermediate goods while populated by identical, infinitely-lived households, with symmetric preferences and technologies. The country sizes of Home and Foreign are n and $1-n$, respectively. In what follows asterisks denote foreign variables, while subscripts H and F denote variables of home and foreign origins, respectively.

Home households consume a composite (C) of domestic (C_H) and imported (C_F) bundles of consumption goods composed of imperfectly substitutable varieties ($\{C_H(i)\}_{i\in[0,n]}$ and $\{C_F(i)\}_{i\in(n,1]}$, respectively) according to,

$$C_t = \left[(1-\chi)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \chi^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

$$C_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\lambda}} \int_0^n C_{H,t}(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}, \quad C_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\lambda}} \int_n^1 C_{F,t}(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}, \tag{2}$$

where $\theta > 0$ measures the elasticity of substitution between Home and Foreign goods, $\chi \equiv (1-n)a$ captures Home households' preferences for imported goods, which is a function of Home's relative size and the degree

[3] We wish to emphasize at the outset that the endogenous terms-of-trade and international asset price adjustments are crucial for our results in the small open economy framework. In a similar small open economy setting under money growth targeting in which all international prices are exogenous and constant for the SOE, Weder (2008b) shows that the indeterminacy condition for the SOE is identical to that for an otherwise closed economy. In such an SOE context, he investigates how allowing the money growth rate to be adjusted endogenously in response to various economic indicators may help pre-empt sunspot equilibria.

of its trade openness $a \in (0,1)$, and $\lambda > 1$ measures the elasticity of substitution between the varieties of goods produced in Home or in Foreign.

Foreign side can be described in an analogous manner. For instance, Foreign aggregate consumption basket is given by,

$$C_t^* = \left[(1-\chi^*)^{\frac{1}{\theta}} C_{F,t}^{*\frac{\theta-1}{\theta}} + \chi^{*\frac{1}{\theta}} C_{H,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\chi^* \equiv na^*$, and $C_{F,t}^*$ and $C_{H,t}^*$ are defined in ways parallel to those for $C_{H,t}$ and $C_{F,t}$.

Since we will focus on a symmetric equilibrium, we assume that the degree of Foreign's trade openness a^* is the same as the degree of Home's trade openness a.

The optimal allocation of expenditures between domestic and imported goods implies the following aggregate demand conditions for Home and Foreign,

$$C_{H,t} = (1-\chi) \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, \quad C_{F,t} = \chi \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, \tag{4}$$

$$C_{F,t}^* = (1-\chi^*) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^*, \quad C_{H,t}^* = \chi^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\theta} C_t^*, \tag{5}$$

along with the following demand schedules for individual goods,

$$C_{H,t}(i) = \frac{1}{n} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\lambda} C_{H,t}, \quad C_{F,t}(i) = \frac{1}{1-n} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\lambda} C_{F,t}, \tag{6}$$

$$C_{F,t}^*(i) = \frac{1}{1-n} \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*} \right)^{-\lambda} C_{F,t}^*, \quad C_{H,t}^*(i) = \frac{1}{n} \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\lambda} C_{H,t}^*, \tag{7}$$

where Home (P_t) and Foreign (P_t^*) consumer price indexes relate to the price sub-indexes for Home-produced goods denominated in Home (P_H) and Foreign (P_H^*) currencies and for Foreign-produced goods denominated in Home (P_F) and Foreign (P_F^*) currencies as follows,

$$P_t = \left[(1-\chi) P_{H,t}^{1-\theta} + \chi P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad P_t^* = \left[(1-\chi^*) P_{F,t}^{*1-\theta} + \chi^* P_{H,t}^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{8}$$

We assume that the law of one price holds so that,

$$P_{H,t} = S_t P_{H,t}^*, \quad P_{F,t} = S_t P_{F,t}^*, \tag{9}$$

where S_t denotes the nominal exchange rate measured by the price of Foreign currency in units of Home currency. The relative price of Foreign goods in terms of Home goods, or, the Home terms of trade, is given by,

$$T_t = \frac{S_t P_{F,t}^*}{P_{H,t}}, \tag{10}$$

and the real exchange rate is measured as,

$$Q_t = \frac{S_t P_t^*}{P_t} = T_t \frac{P_t^*}{P_t} \frac{P_{H,t}}{P_{F,t}^*}. \tag{11}$$

A representative Home household maximizes the expected value of its lifetime utility,

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi \frac{L_t^{1+\gamma}}{1+\gamma} \right), \tag{12}$$

for some $\varphi > 0$, where E is the expectations operator, L_t the household's labor supply, $\beta \in (0, 1)$ a subjective discount factor, $\sigma > 0$ the coefficient of relative risk aversion in consumption, and $\gamma \geq 0$ the inverse of the Frisch labor supply elasticity.

The household carries M_t units of money and B_t units of nominal bonds into a state at date t. Before proceeding to the goods market, the household visits the financial market to purchase state-contingent nominal bonds B_{t+1} that pays one unit of domestic currency at $t + 1$ if a specific state is realized. The household can use this complete set of state-contingent bonds to engage in international borrowing and lending. Denote by $\Omega_{t,t+1}$ the stochastic discount factor from $t + 1$ to t. The price at t of a bond that pays one unit of domestic currency in a specific state at $t + 1$ is equal to $\Omega_{t,t+1}$ times the probability that this specific state will indeed be realized at $t + 1$ conditional on the information available at t. In particular, a bond issued at t that pays one unit of domestic currency in all states at $t+1$ has a nominal value at t equal to $\mathrm{E}_t\Omega_{t,t+1}$, and thus a gross nominal interest rate R_t equal to $(\mathrm{E}_t\Omega_{t,t+1})^{-1}$. The household also receives a lump-sum transfer $M_t^s(\Psi - 1)$ from the government in period t, where M_t^s denotes the money supply and $\Psi > \beta$ the (gross) money growth rate.

The household's budget constraint in period t requires that its consumption expenditure plus asset accumulation do not exceed its income plus transfer during the period, that is,

$$P_tC_t + \mathrm{E}_t\left(\Omega_{t,t+1}B_{t+1}\right) - B_t + M_{t+1} - M_t = P_tw_tL_t + M_t^s(\Psi - 1), \tag{13}$$

where w_t denotes the prevailing real wage rate in period t.

Finally, the household's purchase of the consumption goods is subject to the following cash-in-advance constraint,

$$P_tC_t \leq M_t + M_t^s(\Psi - 1). \tag{14}$$

The first-order conditions for C_t, L_t, M_{t+1}, and B_{t+1} imply,

$$\frac{\beta P_t C_t^{\sigma}}{P_{t+1}C_{t+1}^{\sigma}} = \Omega_{t-1,t}, \tag{15}$$

$$\frac{\varphi L_t^{\gamma} C_t^{\sigma}}{w_t} = \Omega_{t-1,t}, \tag{16}$$

where (15) is the consumption Euler equation and (16) determines the intratemporal tradeoff between consumption and leisure and thus the labor supply. Optimizing behavior implies that the cash-in-advance constraint (14) is binding and appropriate transversality condition is satisfied. Analogous optimality conditions apply to the foreign households.

Combining Equation (15) with its foreign counterpart, and using the definition of the real exchange rate (11), we obtain the following international risk sharing condition,

$$Q_t = q_0 \left(\frac{C_t}{C_t^*} \right)^{\sigma}, \tag{17}$$

where the initial condition is given by $q_0 = Q_0 \left(C_0^*/C_0 \right)^{\sigma}$.

A type i goods is produced according to

$$Y_t(i) = L_t^{\alpha}(i), \tag{18}$$

for some $\alpha \in (0, 1]$, and profit maximization gives rise to,

$$w_t = \frac{P_{H,t}}{P_t} \alpha L_t^{\alpha - 1}. \tag{19}$$

The monetary authority follows money growth targeting,

$$M_{t+1}^s = \Psi M_t^s. \tag{20}$$

Market clearing conditions are $nB_t + (1-n)B_t^* = 0$ for bond, $M_t = M_t^s$ for money, and $Y_t(i) = nC_{H,t}(i) + (1-n)C_{H,t}^*(i)$ for Home-produced goods i, which can be rewritten as,

$$Y_t(i) = \left[\frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\lambda} \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} \left\{ [1 - (1-n)a] C_t + (1-n) a C_t^* Q_t^{\theta} \right\}, \tag{21}$$

using (4)-(7), (9), and (11), along with the definitions of χ and χ^*.

We focus on the case where Home is a small open economy, or, the case in which $n \to 0$. In a symmetric equilibrium $Y_t(i) = Y_t$ and $P_{H,t}(i) = P_{H,t}$, for all i. Thus (21) becomes,

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} \left[(1-a) C_t + a C_t^* Q_t^{\theta} \right], \tag{22}$$

where the CPI index for Foreign in (8) collapses to $P_t^* = P_{F,t}^*$. Consequently, the terms of trade in (10) reduces to,

$$T_t = Q_t \frac{P_t}{P_{H,t}}. \tag{23}$$

For local stability analysis of this SOE, its equilibrium system is log-linearized around a symmetric zero-inflation steady state. In what follows, all hatted variables represent their percentage deviations from the steady state. We shall focus on a perfect foresight equilibrium in our determinacy analysis, so for now we can suppress the expectations operator in expressions to ease exposition.

The log-linearized version of the consumption Euler equation (15) is,

$$\sigma \widehat{C}_{t+1} + \widehat{\pi}_{t+1} = \sigma \widehat{C}_t + \widehat{R}_{t-1}, \tag{24}$$

where $\widehat{\pi}_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t$ is Home CPI inflation rate from t to $t+1$. The cash-in-advance constraint (14) and money growth targeting (20) together imply,

$$\widehat{C}_{t+1} + \widehat{\pi}_{t+1} = \widehat{C}_t. \tag{25}$$

The log-linearized version of the market clearing condition (21) is,

$$\widehat{Y}_t = a\theta(2-a)\widehat{T}_t + (1-a)\widehat{C}_t. \tag{26}$$

Combining (17) and (23) gives rise to,

$$\sigma\widehat{C}_t = (1-a)\widehat{T}_t. \tag{27}$$

Finally, (16), (18), and (19) imply the following aggregate supply condition,

$$\left(\frac{1-\alpha+\gamma}{\alpha}\right)\widehat{Y}_t + \sigma\widehat{C}_t = -a\widehat{T}_t - \widehat{R}_{t-1}. \tag{28}$$

Let $\underline{\sigma} \equiv 1 + (1+\gamma)/\alpha$. It is worth noting that $\underline{\sigma} \geq 2$ for all admissible values of γ and α, where the equality holds if and only if $\gamma = 0$ and $\alpha = 1$. Proposition 1 below summarizes the paper's first main analytical result.

Proposition 1. *In the small open economy with flexible prices, there exists a unique value $\bar{a}^f \in (0,1)$, such that the equilibrium is indeterminate if and only if*

$$\text{(i)} \;\; \sigma > \underline{\sigma} \quad \text{and} \quad \text{(ii)} \;\; a < \bar{a}^f. \tag{29}$$

The dynamic system exhibits saddle-path stability if and only if $\sigma \leq \underline{\sigma}$ or $a > \bar{a}^f$.

A closed economy setting can be cast as a limiting case where $a \to 0$. We then have the following corollary of Proposition 1, which generalizes the result in Farmer (1999).

Corollary 1. *In a closed economy with flexible prices, the necessary and sufficient condition for equilibrium indeterminacy is*

$$\sigma > \underline{\sigma}. \tag{30}$$

The necessary and sufficient condition for equilibrium determinacy is $\sigma < \underline{\sigma}$.

Corollary 1 generalizes the closed economy result in Farmer (1999). Since $\underline{\sigma}$ is monotone in γ, the inverse of the Frisch elasticity of labor supply, and σ negatively affects the income effect, (30) says that the substitution effect must be greater than the income effect in order for equilibrium to be indeterminate, just as discussed in the introduction. When α is set to 1, and γ to 0 (i.e., an infinite labor supply elasticity which is often used in the macroeconomic literature), $\underline{\sigma}$ equals 2, and (30) collapses to $\sigma > 2$. This is the necessary and sufficient condition for indeterminacy presented in Farmer (1999). Even when γ is set to 1 (i.e., a unitary labor supply elasticity which is also commonly used in the macroeconomic literature), the necessary and sufficient condition for indeterminacy is only tightened up to $\sigma > 3$. This is to say that indeterminacy can easily arise from an empirically reasonable parametrization of the closed economy given the broad spectrum of empirical estimates about σ that can range from 0 to 30 (see Huang et al. 2009 and Huang and Meng 2014 for some discussions of this).

According to Proposition 1, indeterminacy is less likely to arise in a small open economy. As (29) shows, for equilibrium to become indeterminate, not only must the condition for indeterminacy in the closed economy be satisfied, but trade openness of the economy must be smaller than some threshold value $\bar{a}^f$. If trade openness of the economy is greater than the threshold value, then the equilibrium is determinate regardless of other features of the economy.

Figure 2 gives a quantitative feel about this result. In generating the figure, the parameter α is set at 1 to ensure a constant returns to scale production function, and β at 0.99 to be consistent with a steady-state annualized real interest rate of 4 percent, as one period in our model corresponds to one quarter of a year. The figure consists of four rows and three columns, to cover a wide range of values for γ and θ, respectively. As discussed in Corsetti et al. (2008) and Engel and Wang (2011), empirical estimates of the trade price elasticity (θ) range from 0.1 to 2, with a number of studies estimating a value around 1 (e.g., Heathcote and Perri 2002; Bergin 2006). As summarized in Huang and Meng (2012, 2014), many empirical estimates of the Frisch elasticity of labor supply ($1/\gamma$) that are relevant for macroeconomic studies lie between 0.5 and 2, whereas indivisibility in labor à là Hansen (1985) and Rogerson (1988) is also often considered in studies on indeterminacy (e.g., Carlstrom and Fuerst 2005; Benhabib and Eusepi 2005; Kurozumi and Van Zandweghe 2008). We therefore examine all values for γ between 0 and 2. Each of the twelve panels in the figure then displays the determinacy (stability) and indeterminacy (instability) regions with various degrees of trade openness a (horizontal axis) and relative risk aversion in consumption σ (vertical axis).

Our numerical exercises across all panels of the figure everlastingly confirm the analytical results in Proposition 1 and Corollary 1. In each panel the division between the stability and the instability regions is determined by the minimal degree of trade openness that guarantees saddle-path stability as a function of relative risk aversion in consumption. As the figure illustrates, while it calls for a greater degree of trade openness to stabilize the economy as we increase σ from its value at the vertical intercept, this effect of σ on the required magnitude of a for stabilization diminishes quickly. As is also clear from the figure, when we increase the trade price elasticity θ or decrease the labor supply elasticity $1/\gamma$, the stability region gets enlarged and the instability region shrinks.[4]

[4]There is one exception to this statement: with an infinite elasticity of labor supply, trade price elasticity has no implications for stability or instability of this economy with a constant returns to scale production function.

To put this into perspective, we examine the implications of our results for the sixteen small open economies presented in Table 1, where each country's trade openness is measured by the share of the country's imports to its GDP as reported by the World Bank World Development Indicators.[5] It is clear that for countries like Belgium, Bulgaria, Czech Republic, Hungary, Ireland, Netherlands, and Switzerland, indeterminacy can never arise simply because these countries are sufficiently open to the global economy. For an economy like Australia, whose trade openness is the smallest among all countries on the list, indeterminacy may still occur, but only for large labor supply elasticity, small trade price elasticity, and large relative risk aversion in consumption. For parameter configurations typically used in the macroeconomic literature, for instance, with a unitary labor supply elasticity, a unitary trade price elasticity, and a relative risk aversion in consumption of no greater than ten, indeterminacy becomes impossible even for Australia, let alone countries that are more open to the global goods market.

3 A small open economy with sticky prices

We consider in this section the implications of sticky prices. In particular, we assume that prices are set in a staggered fashion à là Calvo (1983). The log-linearized aggregate supply condition is modified as,

$$\widehat{\pi}_t^H = \beta \widehat{\pi}_{t+1}^H + \kappa \widehat{mc}_t, \tag{31}$$

where $\widehat{\pi}_t^H \equiv \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$ denotes domestic-price inflation, and the real marginal cost $\widehat{mc}_t$ is given by,

$$\widehat{mc}_t = (\underline{\sigma} - 2)\,\widehat{Y}_t + \sigma \widehat{C}_t + a\widehat{T}_t + \widehat{R}_{t-1}. \tag{32}$$

The parameter $\kappa \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi}\frac{1}{1+\theta(1-\alpha)/\alpha} > 0$ denotes the real marginal cost elasticity of inflation, where the parameter $\psi \in (0,1)$ measures the degree of price stickiness. The log-linearized version of (8) yields the following expression for consumer-price inflation,

$$\widehat{\pi}_t = \widehat{\pi}_t^H + a\left(\widehat{T}_t - \widehat{T}_{t-1}\right). \tag{33}$$

The remaining features of the model are the same as those in Section 2.

We now summarize the paper's second main analytical result in the proposition below.

Proposition 2. *In the small open economy with sticky prices, there exists a unique value $\bar{a}^s \in (0,1)$, such that the equilibrium is indeterminate if and only if*

$$\text{(i)}\;\; \sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa} \quad \text{and} \quad \text{(ii)}\;\; a < \bar{a}^s. \tag{34}$$

[5] It is worth noting that eight out of the sixteen countries listed in Table 1, including Australia, Austria, Canada, Denmark, Netherlands, Spain, Sweden, and Switzerland, are also among the developed countries classified by Mendoza et al. (1997) as advanced small open economies.

The dynamic system exhibits saddle-path stability if and only if $\sigma \leq \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$ *or* $a > \bar{a}^s$.

A closed economy setting can be cast as a limiting case where $a \to 0$. We then have the following corollary of Proposition 2, which generalizes the result in Weder (2008a).

Corollary 2. *In a closed economy with sticky prices, the necessary and sufficient condition for equilibrium indeterminacy is*

$$\sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa}. \tag{35}$$

The necessary and sufficient condition for equilibrium determinacy is $\sigma < \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$.

Corollary 2 generalizes the closed economy result of Weder (2008a). Since κ is negatively related to ψ, the hazard rate of nominal price adjustment, with the case that $\kappa \to \infty$ ($\psi \to 0$) corresponding to the limiting case of flexible prices, the presence of nominal price rigidity alleviates the severity of the indeterminacy problem, as is captured by the last term in (35). That said, given the wide range of empirically plausible values for σ , indeterminacy can still easily arise from an empirically reasonable parametrization of the closed economy even with sticky prices. This is an essential point made by Weder (2008a).

But, once again, as is established by Proposition 2, indeterminacy is less likely to arise in a small open economy. Just as (34) demonstrates, similarly as in the flexible-price case, for equilibrium to become indeterminate, not only must the condition for indeterminacy in the closed economy with sticky prices be satisfied, but trade openness of the economy must be smaller than some threshold value $\bar{a}^s$; if trade openness of the economy is greater than $\bar{a}^s$, then the equilibrium is determinate regardless of other features of the economy.

To put this result into a quantitative perspective, Figure 3 is similarly generated as Figure 2, but for the sticky price model. In generating the figure, the hazard rate of nominal price adjustment ψ is set to 0.33, as is consistent with the evidence in Christiano et al. (2005), among others, but our results are robust to other choices of ψ, while values of the other parameters are as described in the previous section when Figure 2 is generated.

While confirming the results in Proposition 2 and Corollary 2, Figure 3 also illustrates similar effects on (in)stability of trade openness, trade price elasticity, relative risk aversion in consumption, and labor supply elasticity as discussed in the previous section. The key message emanating from comparing Figure 3 to Figure 2 is that the presence of nominal price rigidity always enlarges the stability region and shrinks the instability region. As a result, indeterminacy can never emerge in any of the small open economies presented in Table 1, regardless of how we choose the values of model parameters within their empirically plausible ranges.

4 The role of international financial integration

To help isolate the stabilization role of trade openness for a small open economy we have maintained in Sections 2 and 3 the assumption of a complete world asset market. To demonstrate the stabilization role of international financial integration, in this section we repeat the analysis in these previous sections but under the assumption that the SOE has no access to international borrowing or lending, while all other features and parametrization of the previous models are preserved. As we will show below, for any degree of trade openness, sunspot equilibria are more likely to emerge in this case of international financial autarky than in the previous case with perfect integration of the SOE in the global capital market.

Absent international borrowing and lending, Home household's budget constraint (13) is modified as

$$P_tC_t + M_{t+1} - M_t = P_tw_tL_t + M_t^s(\Psi_t - 1). \tag{36}$$

Utility maximization then implies

$$\beta \mathrm{E}_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}} = \frac{\varphi L_t^{\gamma}}{P_t w_t}. \tag{37}$$

With some algebra, we can derive in equilibrium the log-linearized version of the budget constraint as,

$$a\widehat{T}_t + \widehat{C}_t = \widehat{Y}_t. \tag{38}$$

To sharpen the result and simplify exposition, in this section we restrict attention to the case with $\theta \geq 1$. Not only is this an empirically plausible range of the trade price elasticity, but focusing on this empirically justifiable range greatly facilitates analytical derivations of this section's results. In particular, this practically reasonable parameter restriction allows us to obtain closed form results under international financial autarky for the SOE that are otherwise parallel to Propositions 1 and 2 established in Sections 2 and 3 where the SOE has access to a complete world asset market for international borrowing and lending.[6] We can show that, with either flexible or sticky prices, the necessary and sufficient conditions for (in)determinacy feature an identical threshold value of the relative risk aversion in consumption, but a greater critical value of the degree of trade openness, under international financial autarky than with a complete world asset market. As we show in the appendix, the fact that the solutions in this section are of the closed forms is crucial for establishing these clear-cut comparisons, which lead immediately to the following key result of this section.

Proposition 3. *For any degree of trade openness of the small open economy, indeterminacy is less likely to arise if the SOE is fully integrated in the global capital market than if it is excluded from it.*

Figure 4 presents a quantitative illustration of the result in Proposition 3 under the same parametrization as in plotting Figures 2 and 3. As the figure demonstrates, the stabilization role of openness to world financial market can be quantitatively significant regardless of whether prices are flexible or sticky.

[6] We could have presented these close form results under international financial autarky in two additional propositions in the forms that are otherwise parallel to Propositions 1 and 2. We choose not to do so in order to conserve space.

5 Some intuition based on analytical expositions

The take-home message from our analysis above is that opening a cash-in-advance monetary economy under money growth targeting to international goods and asset markets can help pre-empt sunspot equilibria. We now provide some intuition for this result based on analytical expositions. For much of the point to be made, it is sufficient to use a flexible price setting.

With some algebra, we can show that the dynamic equation that governs the local (in)stability property of a flexible price economy takes the form of a first-order linear difference equation in $\widehat{C}_t$,

$$\widehat{C}_{t+1} = -\rho \widehat{C}_t, \tag{39}$$

where the key parameter ρ depends on whether the economy is closed or open to globalization. Note that, since $\widehat{C}_t$ is a jump variable, the above system is indeterminate if it has a stable root, but exhibits saddle-path stability if it has an explosive root.

We begin by considering a closed economy setting. Denote by ρ^c the ρ in (39) for the closed economy. It is easy to show that $\rho^c = (\underline{\sigma} - 1)/(\sigma - 1)$. Given that $\underline{\sigma} \geq 2$, we can verify that the system for the closed economy has a stable root and thus is indeterminate if and only if $\sigma > \underline{\sigma}$. Recall that this is the classic condition for the substitution effect to dominate the income effect in a flexible price closed economy setting established in Corollary 1 (also see the discussion subsequent to the corollary in Section 2), which implies here $\rho^c \in (0, 1)$. Given that ρ^c is below the critical value 1, the income effect is dominated by the substitution effect so sunspot expectations can become self-fulfilled and extrinsic uncertainty arises.

We now show that opening to globalization may help stabilize this otherwise indeterminate economy by pulling ρ above the critical value 1 so that the income effect becomes dominant over the substitution effect. Denote by ρ^o the ρ in (39) for a small open economy that is fully integrated in the international asset market as considered in Section 2. Using (24)-(28) we can show that $\rho^o = \rho^c + \mathcal{I}(a)$, where

$$\mathcal{I}(a) \equiv \frac{a\sigma}{\sigma - 1} \left[\frac{1 + (\underline{\sigma} - 2)\theta}{1 - a} + \frac{(\underline{\sigma} - 2)(\sigma\theta - 1)}{\sigma} \right]. \tag{40}$$

The central message then is that, even when $\rho^c \in (0, 1)$, we can have ρ^o greater than 1 provided that the degree of trade openness is sufficiently large, since in this case, as can be shown with some algebra, $\mathcal{I}(a)$ is strictly increasing with a, for all $a \in [0, 1)$, with $\mathcal{I}(0) = 0$ being its minimal value that is achieved at $a = 0$. Proposition 1 in Section 2 guarantees the existence of a unique threshold value $\bar{a}^f \in (0, 1)$ above which any degree of trade openness a will result in a greater than unit ρ^o and therefore bring with it a dominant income effect to stabilize the small open economy. We can also show, by applying the Implicit Function Theorem to $\rho^o(\bar{a}^f; \theta, \gamma, \sigma) = 1$, and using the fact that ρ^o like $\mathcal{I}$ is an increasing function of a while also verifying that $\partial\rho^o(\bar{a}^f)/\partial\theta \geq 0$ (where "=" holds if $\underline{\sigma} = 2$) , $\partial\rho^o(\bar{a}^f)/\partial\gamma > 0$, and $\partial\rho^o(\bar{a}^f)/\partial\sigma < 0$, that the minimal degree of trade openness required for ensuring saddle-path stability, that is, $\bar{a}^f$, is decreasing in θ (or invariant with θ if $\underline{\sigma} = 2$ - see also Footnote 4) and γ, but increasing in σ, just as illustrated by Figure 2.

Similar stabilization role of trade openness can also hold for a small open economy that is excluded from international borrowing and lending. This can be demonstrated in a most clean-cut manner by restricting attention to the case with $\theta \geq 1$. Denote by ρ^o_{aut} the ρ in (39) for such an SOE. With some algebra, similarly as in using (24)-(28) above, but with (27) replaced by (38), we can show that $\rho^o_{aut} = \rho^c + \mathcal{I}_{aut}(a)$, where $\mathcal{I}_{aut}(a) \equiv \rho^c a/[(2-a)\theta - 1]$. Then, once again, even when $\rho^c \in (0,1)$, we may have ρ^o_{aut} greater than 1 if the degree of trade openness is sufficiently large, since in this case, as we can show with some algebra, $\mathcal{I}_{aut}(a)$ is strictly increasing in a for all $a \in [0,1)$ with $\mathcal{I}_{aut}(0) = 0$ being its minimal value that is achieved at $a = 0$. Moreover, the comparison of this case with international financial autarky against the above case of complete world asset market also illustrates the stabilization role of global financial integration, since we can verify that $\mathcal{I}(a) > \mathcal{I}_{aut}(a)$ for all $a \in (0,1)$. This is to say that, for any positive degree of trade openness, a perfect integration in the world capital market from international financial autarky will bring with it additional income effect to help stabilize the small open economy. This is a reminiscence of Proposition 3 established in Section 4 for a flexible price setting, and it is also a key message delivered by the first column of Figure 4.

The stabilization role of openness to global goods and financial markets in a sticky price setting can be similarly demonstrated. Furthermore, openness has an additional implication in a sticky price environment. As we have shown in the previous sections, and as has been known at least since Weder (2008a), the presence of price rigidity alleviates the severity of the indeterminacy problem in a closed economy cash-in-advance framework with money growth targeting. We here find that openness to globalization increases the effective degree of price rigidity and thus renders sunspot equilibria less likely to occur. We can illustrate this by substituting equations (24)-(27) into (32) to rewrite the Phillips curve relation for the sticky price model presented in Section 3 as,

$$\widetilde{\kappa}\widehat{mc}_t = \beta \mathrm{E}_t \widehat{C}_{t+1} - (1+\beta)\,\widehat{C}_t + \widehat{C}_{t-1},$$

where $\widetilde{\kappa} = \kappa \frac{1-a}{1-a+a\sigma}$. Here κ and $\widetilde{\kappa}$ are the inverse measures of the degrees of nominal price rigidity in the closed economy and in the small open economy, respectively. We can verify that $\widetilde{\kappa} < \kappa$ for all $a \in (0,1)$ and that $\widetilde{\kappa}$ strictly decreases with a for all $a \in [0,1)$. Hence, a larger degree of trade openness implies a greater effective degree of price rigidity. A direct consequence of this is that, when the closed economy counterpart is indeterminate, the minimal degree of trade openness required for ensuring saddle-path stability for the small open economy is smaller with sticky prices than with flexible prices. This is a key message from Figure 3 in comparison against Figure 2. This is to say that the unique threshold value $\bar{a}^s$ established in Proposition 2 in Section 3 for the sticky price setting lies between 0 and $\bar{a}^f$. In fact, with some algebra, we can show that $\bar{a}^s$ must satisfy $2(1+\beta)/[(\sigma-1)\widetilde{\kappa}(\bar{a}^s;\theta,\sigma)] + \rho^o(\bar{a}^s;\theta,\gamma,\sigma) - 1 = 0$, which implies $\rho^o(\bar{a}^s) < 1$ so $\bar{a}^s < \bar{a}^f$. By similarly applying the implicit function approach for the flexible price setting above, and using the fact that $\widetilde{\kappa}$ is a decreasing function of a while also verifying that $\partial\widetilde{\kappa}(\bar{a}^s)/\partial\theta \leq 0$ (where "=" holds if $\alpha = 1$), $\partial\widetilde{\kappa}(\bar{a}^s)/\partial\gamma = 0$, and $\partial[(\sigma - 1)\widetilde{\kappa}(\bar{a}^s)]/\partial\sigma > 0$, we can also show that $\bar{a}^s$ is decreasing in θ (or invariant with θ if $\underline{\sigma} = 2$) and γ, but increasing in σ, just as Figure 3 illustrates. Also following the above approach in demonstrating the stabilization role of global financial integration for the flexible price setting, we can similarly show how, for any positive degree of trade openness, a perfect integration in global capital market from international financial autarky will bring with it additional income effect to help stabilize the small open economy with sticky prices, just as established by Proposition 3 and illustrated by the second column of Figure 4.

6 An alternative monetary policy

In the previous sections monetary policy with money growth targeting has been considered following much of the literature on cash-in-advance monetary economies that motivated the current study in the first place (e.g., Woodford 1994; Farmer 1999; Weder 2008a; and Matsuyama 1990 under a money-in-the-utility function approach). In this section, we extend the analysis to an alternative monetary policy with inflation targeting,

$$\pi_t = \overline{\pi}, \tag{41}$$

where, without loss of generality, the inflation target $\overline{\pi}$ is set to the model's zero steady-state inflation rate.[7] In order to conserve space we shall focus here on showing the stabilization role of trade openness under inflation targeting (41) while maintaining the assumption of a complete international asset market made in Sections 2 and 3.[8] Thus, other than replacing money growth targeting (20) with inflation targeting (41), all other features of the models remain identical to those in Sections 2 and 3.

Propositions 4 and 5 below summarize the main analytical results of this section.

Proposition 4. *In the small open economy with inflation targeting and flexible prices, there exists a unique value* $\overline{a}^f_{IT} \in (0, 1)$*, such that the equilibrium is indeterminate if and only if*

$$\text{(i) } \sigma > \underline{\sigma} - 2 \text{ and (ii) } a < \overline{a}^f_{IT}. \tag{42}$$

The dynamic system exhibits saddle-path stability if and only if $\sigma \leq \underline{\sigma} - 2$ *or* $a > \overline{a}^f_{IT}$*.*

[7] Similar results would obtain when the inflation target and steady-state inflation rate are set to a nonzero level provided that proper indexation in the Calvo price setting is invoked in the sticky price model.

[8] In our other analysis with inflation targeting (41) unreported here we have found a similar stabilization role of trade openness under international financial autarky. These results can be stated in forms parallel to Propositions 4 and 5 and they are available upon request from the authors.

Proposition 5. *In the small open economy with inflation targeting and sticky prices, there exists a unique value* $\overline{a}^s_{IT} \in (0,1)$*, such that the equilibrium is indeterminate if and only if*

$$\text{(i) } \sigma > \underline{\sigma} - 2 \text{ and (ii) } a < \overline{a}^s_{IT}. \tag{43}$$

The dynamic system exhibits saddle-path stability if and only if $\sigma \leq \underline{\sigma} - 2$ *or* $a > \overline{a}^s_{IT}$.

Once again, a closed economy setting can be cast as a limiting case where $a \to 0$. We then have the following corollary of Propositions 4 and 5.

Corollary 3. *In a closed economy with inflation targeting, the necessary and sufficient condition for equilibrium indeterminacy is*

$$\sigma > \underline{\sigma} - 2. \tag{44}$$

The necessary and sufficient condition for equilibrium determinacy is $\sigma < \underline{\sigma} - 2$.

While these results parallel those obtained in Sections 2 and 3, several remarks follow immediately.

First, as Corollary 3 makes clear, in a closed economy setting with inflation targeting, the (in)determinacy condition is the same regardless of whether prices are flexible or sticky. This stands in contrast to the closed economy setting with money growth targeting where the presence of nominal price rigidity alleviates the severity of the indeterminacy problem (as is illustrated by the contrast between Corollaries 1 and 2). The reason for this nominal rigidity irrelevance is that monetary policy with inflation targeting (41) effectively mutes price dispersion.

Second, as can be seen from comparing Corollaries 1, 2, and 3, in a closed economy setting, sunspot equilibria are more likely to arise under inflation targeting than under money growth targeting, whereas the contrast is more striking with sticky prices than with flexible prices.

Last, and also most important, as Propositions 4 and 5 make clear, opening to the global goods market helps isolate the economy under inflation targeting from sunspot fluctuations, regardless of whether prices are flexible or sticky. This is to say that the stabilization role of trade openness demonstrated in Sections 2 and 3 under money growth targeting also holds under inflation targeting.

Figures 5 and 6, which are similarly generated as Figures 2 and 3 under identical parametrization, but with money growth targeting (20) replaced with inflation targeting (41), give a quantitative feel about these results presented in Propositions 4 and 5 and Corollary 3. In particular, the figures illustrate similar effects on (in)stability of trade openness, trade price elasticity, relative risk aversion in consumption, and labor supply elasticity as discussed in Sections 2 and 3. It is also worth noting that, under inflation targeting, although the presence of nominal price rigidity does not affect the severity of the indeterminacy problem in a closed economy as already noted above (as reflected by the equal intercepts in each pair of the corresponding panels in Figures 5 and 6), it always enlarges the stability region and shrinks the instability region in a small open economy. Everything else equal, it calls for a smaller degree of trade openness of the SOE to pre-empt sunspot equilibria under sticky prices than under flexible prices.

7 Impulse responses

We now use impulse responses to help get more feel about our analytical results obtained above. We shall consider throughout this section a fundamental preference shock: similarly as in Wen (1998), Benhabib and Wen (2004), and Huang and Meng (2012), period utility derived from date-t consumption is now specified as $(c_t - \mu_t)^{1-\sigma} / (1-\sigma)$, where μ_t is a shock to consumption that generates the urge to consume, and where we postulate a stationary AR(1) log-normal process for this fundamental shock,

$$\log(\mu_t) = (1 - \rho_\mu)\log(\mu) + \rho_\mu \log(\mu_{t-1}) + \varepsilon_t, \tag{45}$$

where ε_t is a white-noise innovation with a finite standard deviation σ_μ. In the log-linearized form, (45) can be expressed as $\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_t$. We shall impose the restriction that $\mathrm{E_t}\widehat{\mu}_{t+1} = \rho_\mu \widehat{\mu}_t$, since agents know the probability distribution of this fundamental shock so the forecast error on this shock coincides with the true underlying disturbance.

We introduce an endogenous forecast error η_t as $\eta_t = \widehat{C}_t - \mathrm{E}_{t-1}\widehat{C}_t$ and we denote $\xi_t = \mathrm{E}_t\widehat{C}_{t+1}$. The log-linearized equilibrium system can then be cast into the following generic form,

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \Psi\varepsilon_t + \Pi\eta_t, \tag{46}$$

where $x_t = \left[\xi_t,\, \widehat{C}_t,\, \widehat{\mu}_t\right]'$, $\Psi = [0,\, 0,\, 1]'$, $\Pi = [0,\, 1,\, 0]'$, but the two 3×3 matrices Γ_0 and Γ_1 depend on the configurations of monetary policies, price settings, and integrations of the SOE in world capital market, with the details specified in the appendix. Regardless of the configurations, the 3×3 matrix Γ_0 is invertible in all of our numerical exercises carried out in this section so the system can be inverted into

$$x_t = \Gamma_0^{-1}\Gamma_1 x_{t-1} + \Gamma_0^{-1}\Psi\varepsilon_t + \Gamma_0^{-1}\Pi\eta_t. \tag{47}$$

When the system is determinate, the endogenous forecast error η_t is completely pinned down by the fundamental shock ε_t so there is no room for sunspot to affect either η_t or other endogenous variables.

When the system is indeterminate, there is room for sunspot to affect the endogenous forecast error η_t and other endogenous variables. We consider in this case a reduced-form sunspot shock, $\varepsilon_{s,t} \to i.i.d.\left(0, \sigma_s^2\right)$. Under indeterminacy, the forecast errors η_t can be influenced by both the fundamental shock ε_t and the sunspot shock $\varepsilon_{s,t}$.

To illustrate the effect of indeterminacy on equilibrium dynamics, we present the impulse responses with the various configurations of the economy to the fundamental shock under determinacy, and to both the fundamental shock and the sunspot shock under indeterminacy. Across all of these configurations, we set $\sigma = 8$, $\theta = 1$, and $\gamma = 1$, whereas values of the other parameters are the same as those invoked earlier in the paper. We choose the steady state value of μ_t such that $\mu/C = 0.1$ in the steady state, and we set $\rho_\mu = 0.9$, and $\sigma_\mu = \sigma_s = 1$, while innovations in fundamental and in sunspot are assumed to be orthogonal to each other, as in Benhabib and Wen (2004) and Huang and Meng (2012). Under indeterminacy, when considering the impulse responses to one type of shock, we turn off the other type of shock by setting the corresponding variance to zero; also, when considering the impulse responses to a sunspot shock, we set the steady state ratio μ/c to zero. This is the standard approach followed in the literature on indeterminacy when examining the impulse responses to various shocks (e.g., Benhabib and Wen 2004; Huang and Meng 2012). In each configuration we present the impulse responses under determinacy against the impulse responses under indeterminacy by considering two neighboring values of the openness parameter across the borderline.

Figures 7-10 display the impulse responses of consumption, terms of trade, and real exchange rate to the consumption shock under determinacy, and to both the consumption shock (solid line) and the sunspot shock (dashed line) under indeterminacy for each of the eight configurations of monetary policies, price settings, and integrations of the SOE in global financial market. A common pattern emerges in the contrast between the impulse responses under indeterminacy and under determinacy: Across all configurations of the economy persistent cyclical fluctuations emerge in the impulse responses to either the fundamental shock or the i.i.d. sunspot shock under indeterminacy, whereas no such endogenous cycle ever exists in any of the impulse responses to the exogenously persistent consumption shock under determinacy. This suggests that sunspot expectations may hold a key for the propagation mechanism of the economy over some welfare-reducing fluctuations. In light of the nuanced difference in the two neighboring values of the openness parameter that produce these contrasting results, opening to globalization may go a long way to help insolate the economy from such beliefs-driven extrinsic uncertainty.

8 Concluding remarks

We have generalized the (in)determinacy results in Farmer (1999) and Weder (2008a) for a cash-in-advance monetary economy under money growth targeting to a small open economy framework. This generalization is important, as it shows how opening to globalization may help pre-empt self-fulfilling expectations and beliefs-driven fluctuations unrelated to economic fundamentals. We have shown that much of the result also holds under an alternative monetary policy with inflation targeting. Our insight on terms-of-trade movements associated with global goods trade as an endogenous stabilizer in reducing the likelihood of sunspot equilibria in our model also reveals that it may be worthwhile to revisit the class of models with production externalities, examined by Weder (2001), Lahiri (2001), Nishimura and Shimomura (2002a, 2002b), Meng and Velasco

(2003, 2004), and Hu and Mino (2013), among others, in which beliefs-driven extrinsic instability is more likely to emerge in a small open economy setting than in an otherwise closed economy setting, but from which endogenous terms-of-trade effects are abstracted. We intend to leave such inquiry to future research.

A Appendix

In this appendix we sketch our proofs of Propositions 1-5 and provide some technical details for Section 7.

Proof of Proposition 1. Substituting out all variables other than consumption in (24)-(28), we can collapse the system into a first-order linear difference equation in $\widehat{C}_t$,

$$\widehat{C}_{t+1} = \frac{1}{1-\sigma}\left\{1 + \frac{a\sigma}{1-a} + (\underline{\sigma}-2)\left[\frac{(2-a)a\sigma\theta}{1-a} + (1-a)\right]\right\}\widehat{C}_t.$$

Since $\widehat{C}_t$ is a jump variable, the above system is indeterminate if it has a stable root, but exhibits saddle-path stability (determinacy) if it has an explosive root.

Recall that $\underline{\sigma} \geq 2$. It is then clear that the system has an explosive root for all $\sigma \leq 2$. For $\sigma > 2$, the system has an explosive root if $f(a) < 0$ but a stable root if $f(a) > 0$, where

$$f(a) \equiv (\underline{\sigma}-2)(\sigma\theta-1)a^2 - 2[\sigma - 1 + (\underline{\sigma}-2)(\sigma\theta-1)]a + \sigma - \underline{\sigma}. \tag{A.1}$$

Note that $f(1) = -\sigma[1 + (\underline{\sigma}-2)\theta] < 0$ and $f(0) = \sigma - \underline{\sigma}$. We now break into cases.

The case with $\underline{\sigma} = 2$

We have $f(a) = -2(\sigma-1)a + \sigma - 2$, which is positive if $a < 0.5(\sigma-2)/(\sigma-1)$ but negative if $a > 0.5(\sigma-2)/(\sigma-1)$.

The case with $\underline{\sigma} > 2$

If $\sigma\theta = 1$, then $f(a) = -2(\sigma-1)a + \sigma - \underline{\sigma}$. For $\sigma \leq \underline{\sigma}$, we have $f(a) < 0$ for all $a \in (0,1)$. For $\sigma > \underline{\sigma}$, $f(a)$ is positive if $a < 0.5(\sigma-\underline{\sigma})/(\sigma-1)$ but negative if $a > 0.5(\sigma-\underline{\sigma})/(\sigma-1)$.

If $\sigma\theta > 1$, then the quadratic function in (A.1) is convex. If $\sigma \leq \underline{\sigma}$, then $f(0) \leq 0$, so $f(a) < 0$ for all $a \in (0,1)$. If $\sigma > \underline{\sigma}$, then $f(0) > 0$, so there exists a unique $\bar{a}^f \in (0,1)$ such that $f(a)$ is positive if $a < \bar{a}^f$ but negative if $a > \bar{a}^f$.

If $\sigma\theta < 1$, then the quadratic function in (A.1) is concave. If $\sigma > \underline{\sigma}$, then $f(0) > 0$, so there exists a unique $\bar{a}^f \in (0,1)$ such that $f(a)$ is positive if $a < \bar{a}^f$ but negative if $a > \bar{a}^f$. If $\sigma \leq \underline{\sigma}$, then $f(0) \leq 0$, and there are three possibilities in this case: first, $f(a)$ is monotone increasing in $a \in (0,1)$; second, $f(a)$ is monotone decreasing in $a \in (0,1)$; third, $f(a)$ reaches its stationary point (maxima) within $(0,1)$. In both the first and the second cases, we have $f(a) < 0$ for all $a \in (0,1)$. In the third case, denote by $\tilde{a}^f \in (0,1)$ the stationary point. We can verify that $1 - \tilde{a}^f = (\sigma-1)/[(\underline{\sigma}-2)(1-\sigma\theta)]$, which should also lies within $(0,1)$. Denote by Δ^f the term under the square-root operator in the quadratic formula for solving the quadratic equation $f(a) = 0$. With some algebra, we can show that,

$$\Delta^f = -4\left[(\underline{\sigma}-2)(1-\sigma\theta)\right]^2\left[\tilde{a}^f(1-\tilde{a}^f) + \frac{(\underline{\sigma}-2)\sigma\theta+1}{(\underline{\sigma}-2)(1-\sigma\theta)}\right] < 0.$$

This is to say that, in the third case, we have $f(a) < 0$ for all $a \in (0,1)$ as well. **Q.E.D.**

Proof of Proposition 2. The log-linearized model given by equations (24)-(27) and (31)-(33) can be reduced to the following two-dimensional system,

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \quad \mathbf{z}_t = \left[\widehat{C}_t \; \widehat{C}_{t-1}\right]', \tag{A.2}$$

$$\mathbf{A} \equiv \begin{bmatrix} \frac{\kappa(\underline{\sigma}-2)\left[\frac{(2-a)a\sigma\theta}{1-a}+(1-a)\right]+(1+\kappa+\beta)\left(1+\frac{a\sigma}{1-a}\right)}{\beta\left(1+\frac{a\sigma}{1-a}\right)+\kappa(1-\sigma)} & -\frac{\left(1+\frac{a\sigma}{1-a}\right)}{\beta\left(1+\frac{a\sigma}{1-a}\right)+\kappa(1-\sigma)} \\ 1 & 0 \end{bmatrix}.$$

Since the system has one predetermined variable ($\widehat{C}_{t-1}$) and one jump variable ($\widehat{C}_t$), it exhibits instability if and only if the two eigenvalues of $\mathbf{A}$ are both stable, whereas it is saddle-path stable if and only if one of the two eigenvalues is stable and the other explosive. The system has no equilibrium solutions that converge to the steady state if and only if the two eigenvalues are both explosive.

The determinant and trace of $\mathbf{A}$ are given by, respectively,

$$\det \mathbf{A} = \frac{1+\frac{a\sigma}{1-a}}{\beta\left(1+\frac{a\sigma}{1-a}\right)+\kappa(1-\sigma)}, \tag{A.3}$$

$$\mathrm{tr}\mathbf{A} = \frac{\kappa\left(\underline{\sigma}-2\right)\left[\frac{(2-a)a\sigma\theta}{1-a}+(1-a)\right]+(1+\kappa+\beta)\left(1+\frac{a\sigma}{1-a}\right)}{\beta\left(1+\frac{a\sigma}{1-a}\right)+\kappa(1-\sigma)}. \tag{A.4}$$

Both eigenvalues of $\mathbf{A}$ are stable if and only if

$$|\mathrm{tr}\mathbf{A}| < 1+\det\mathbf{A} \quad \text{and} \quad |\det\mathbf{A}| < 1. \tag{A.5}$$

Both eigenvalues of $\mathbf{A}$ are explosive if and only if

$$|\mathrm{tr}\mathbf{A}| < \frac{|\det\mathbf{A}|}{\det\mathbf{A}} + |\det\mathbf{A}| \quad \text{and} \quad |\det\mathbf{A}| > 1. \tag{A.6}$$

In light of (A.5) and (A.6), the necessary and sufficient condition for one of the two eigenvalues of $\mathbf{A}$ to be stable and the other explosive is $|\mathrm{tr}\mathbf{A}| > 1+\det\mathbf{A}$ if $\det\mathbf{A} > -1$, but $|\mathrm{tr}\mathbf{A}| > -(1+\det\mathbf{A})$ if $\det\mathbf{A} < -1$, or, in a compact form,

$$|\mathrm{tr}\mathbf{A}| > |1+\det\mathbf{A}|. \tag{A.7}$$

Inspecting (A.3) and (A.4) reveals that, in order for the first inequality in (A.5) to hold, it must be that $\det\mathbf{A} < 0$. We can then show that, (A.5) holds if and only if $F(a) > 0$, where

$$F(a) \equiv (\underline{\sigma}-2)(\sigma\theta-1)a^2 - 2[(\sigma-1)(1+\frac{1+\beta}{\kappa}) + (\underline{\sigma}-2)(\sigma\theta-1)]a + \sigma - \underline{\sigma} - \frac{2(1+\beta)}{\kappa}. \tag{A.8}$$

On the other hand, with some algebra, we can show that (A.7) holds if and only if $F(a) < 0$.

We first note that $F(1) = -\sigma[1 + \frac{2(1+\beta)}{\kappa} + (\underline{\sigma} - 2)\theta] < 0$ and $F(0) = \sigma - \underline{\sigma} - \frac{2(1+\beta)}{\kappa}$. We can now break into cases.

The case with $\underline{\sigma} = 2$

We have $F(a) = -2(\sigma - 1)(1 + \frac{1+\beta}{\kappa})a + \sigma - 2 - \frac{2(1+\beta)}{\kappa}$, which is negative for all $\sigma \le 2 + \frac{2(1+\beta)}{\kappa}$ and all $a \in (0, 1)$. We can verify that, for $\sigma > 2 + \frac{2(1+\beta)}{\kappa}$, $F(a)$ is positive if $a < \bar{a}^s$ but negative if $a > \bar{a}^s$, where,

$$\bar{a}^s \equiv \frac{0.5}{1 + \frac{1+\beta}{\kappa}} \left[1 - \frac{1 + \frac{2(1+\beta)}{\kappa}}{\sigma - 1} \right] \in \left(0, \frac{0.5}{1 + \frac{1+\beta}{\kappa}} \right).$$

The case with $\underline{\sigma} > 2$

If $\sigma\theta = 1$, then $F(a) = -2(\sigma - 1)(1 + \frac{1+\beta}{\kappa})a + \sigma - \underline{\sigma} - \frac{2(1+\beta)}{\kappa}$, which is negative for all $\sigma \le \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$ and all $a \in (0, 1)$. We can verify that, for $\sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, $F(a)$ is positive if $a < \bar{a}^s$ but negative if $a > \bar{a}^s$, where,

$$\bar{a}^s \equiv \frac{0.5}{1 + \frac{1+\beta}{\kappa}} \left[1 - \frac{\underline{\sigma} - 1 + \frac{2(1+\beta)}{\kappa}}{\sigma - 1} \right] \in \left(0, \frac{0.5}{1 + \frac{1+\beta}{\kappa}} \right).$$

If $\sigma\theta > 1$, then the quadratic function in (A.8) is convex. If $\sigma \le \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, then $F(0) \le 0$, so $F(a) < 0$ for all $a \in (0, 1)$. If $\sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, then $F(0) > 0$, so there exists a unique $\bar{a}^s \in (0, 1)$ such that $F(a)$ is positive if $a < \bar{a}^s$ but negative if $a > \bar{a}^s$.

If $\sigma\theta < 1$, then the quadratic function in (A.8) is concave. If $\sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, then $F(0) > 0$, so there exists a unique $\bar{a}^s \in (0, 1)$ such that $F(a)$ is positive if $a < \bar{a}^s$ but negative if $a > \bar{a}^s$. If $\sigma \le \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, then $F(0) \le 0$. For $0 < \sigma \le 1$, we can show that $F'(a) = 2[(\underline{\sigma}-2)(1-\sigma\theta)(1-a)+(1-\sigma)(1+\frac{1+\beta}{\kappa})] > 0$, so $F(a) < 0$ for all $a \in (0, 1)$. For $1 < \sigma \le \underline{\sigma} + \frac{2(1+\beta)}{\kappa}$, there are three possibilities: first, $F(a)$ is monotone increasing in $a \in (0, 1)$; second, $F(a)$ is monotone decreasing in $a \in (0, 1)$; third, $F(a)$ reaches its stationary point (maxima) within $(0, 1)$. In both the first and the second cases, we have $F(a) < 0$ for all $a \in (0, 1)$. In the third case, denote by $\tilde{a}^s \in (0, 1)$ the stationary point. We can verify that $1-\tilde{a}^s = (\sigma-1)(1+\frac{1+\beta}{\kappa})/[(\underline{\sigma}-2)(1-\sigma\theta)]$, which should also lies within $(0, 1)$. Denote by Δ^s the term under the square-root operator in the quadratic formula for solving the quadratic equation $F(a) = 0$. With some algebra, we can show that,

$$\Delta^s = -4 \left[(\underline{\sigma} - 2)(1 - \sigma\theta) \right]^2 \left[\tilde{a}^s(1 - \tilde{a}^s) + \frac{(\underline{\sigma} - 2)\sigma\theta + 1 + (\sigma + 1)\frac{1+\beta}{\kappa}}{(\underline{\sigma} - 2)(1 - \sigma\theta)} \right] < 0.$$

This is to say that, in the third case, we have $F(a) < 0$ for all $a \in (0, 1)$ as well. **Q.E.D.**

Proof of Proposition 3. The proofs of the proposition for the flexible-price and the sticky-price settings are similar to the Proofs of Propositions 1 and 2, respectively, but with (27) replaced by (38) throughout. We now provide the proof of the proposition for each of the two cases in sequel.

With flexible prices

By a similar substitution approach as in the Proof of Proposition 1, we derive a dynamic system in $\widehat{C}_t$,

$$\widehat{C}_{t+1} = \frac{\underline{\sigma} - 1}{1 - \sigma} \left[1 + \frac{a}{\theta (2 - a) - 1} \right] \widehat{C}_t.$$

Since $\widehat{C}_t$ is a jump variable, the above system is indeterminate (determinate) if and only if it has a stable (explosive) root. Given $\theta \geq 1$, we have $a / [\theta (2 - a) - 1] > 0$ for all $a \in (0, 1)$. Since $\underline{\sigma} \geq 2$, the system has an explosive root and thus is determinate if $\sigma \in [0, 1)$.

If $\sigma > 1$, then we can verify that the system has a stable root and is thus indeterminate if and only if

$$\sigma > \underline{\sigma} \quad \text{and} \quad a < \overline{a}^f_{aut} \equiv \frac{(\sigma - \underline{\sigma})(2\theta - 1)}{(\sigma - \underline{\sigma})\theta + \underline{\sigma} - 1}. \tag{A.9}$$

Otherwise, the system has an explosive root and is thus determinate.

Note that the first condition in (A.9) is identical to the first condition in (29) of Proposition 1, which is the necessary and sufficient condition for equilibrium indeterminacy presented by (30) in Corollary 1 for a closed economy with flexible prices. We now show that, conditional on this first condition, the threshold value in the second condition in (A.9), $\overline{a}^f_{aut}$, is greater than the threshold value in the second condition in (29), $\overline{a}^f$. This is to say that the instability (stability) region is smaller (greater) if the small open economy is fully integrated in the global capital market than if it is excluded from it.

To complete this proof, recall the function $f(a)$ in (A.1) in the Proof of Proposition 1 above, while we note that $f(1) < 0$, and $f(0) > 0$ given $\sigma > \underline{\sigma}$, and that $\overline{a}^f \in (0, 1)$ satisfies $f\left(\overline{a}^f\right) = 0$. It is then clear that $f(a) \gtrless 0$ implies $a \lessgtr \overline{a}^f$. Now, since

$$f\left(\overline{a}^f_{aut}\right) = -\left\{ \left[\tfrac{(\underline{\sigma}-2)(\theta\sigma-1)}{(\sigma-\underline{\sigma})\theta+\underline{\sigma}-1} + \tfrac{\theta-1}{2\theta-1} \right] (\sigma + \underline{\sigma} - 2) + (\sigma - 1) \right\} \overline{a}^f_{aut} < 0,$$

we conclude that $\overline{a}^f_{aut} > \overline{a}^f$.

With sticky prices

Using a similar substitution approach as in the Proof of Proposition 2, we reduce the system into a form similar to (A.2), with the determinant and trace of $\mathbf{A}$ given by,

$$\det \mathbf{A} = \left[1 + \frac{a}{(2 - a)\theta - 1} \right] \Lambda^{-1},$$

$$\text{tr}\mathbf{A} = 1 + \det \mathbf{A} + \left\{ (\underline{\sigma} - 2) \left[\frac{(2 - a) a\theta}{(2 - a)\theta - 1} + (1 - a) \right] + \frac{a}{(2 - a)\theta - 1} + \sigma \right\} \kappa \Lambda^{-1}.$$

where

$$\Lambda = \beta \left[1 + \frac{a}{(2 - a)\theta - 1} \right] + \kappa (1 - \sigma).$$

If $\sigma \leq 1 + \beta/\kappa$, then $\Lambda > 0$, and so $\det \mathbf{A} > \mathbf{0}$ and $\text{tr}\mathbf{A} > 1 + \det \mathbf{A}$, implying that the system is determinate in light of (A.7).

If $\sigma > 1+\beta/\kappa$ and $a > \tilde{a}^s_{aut} \equiv (2\theta-1)/\{\theta+[\frac{\kappa}{\beta}(\sigma-1)-1]^{-1}\}$, then $\Lambda > 0$ and the system is determinate.

If $\sigma > 1+\beta/\kappa$ and $a < \tilde{a}^s_{aut}$, then $\Lambda < 0$ and $\det \mathbf{A} < \mathbf{0}$. In this case, (A.7) is equivalent to requiring

$$H(a) \equiv (2\theta-1)\left[\sigma - \left(\underline{\sigma} + 2\frac{1+\beta}{\kappa}\right)\right] - \left\{\sigma - 1 + \left[\sigma - \left(\underline{\sigma} + 2\frac{1+\beta}{\kappa}\right)\right](\theta-1)\right\} a < 0,$$

while (A.5) holds if and only if $H(a) > 0$. We can verify that, if $\sigma \le \underline{\sigma} + 2(1+\beta)/\kappa$, then $H(a) < 0$ for all $a \in (0,1)$ and the system is determinate. If $\sigma > \underline{\sigma} + 2(1+\beta)/\kappa$, then we can show that

$$\overline{a}^s_{aut} \equiv \frac{2\theta-1}{\theta + \left(\underline{\sigma} + 2\frac{1+\beta}{\kappa} - 1\right)\left[\sigma - \left(\underline{\sigma} + 2\frac{1+\beta}{\kappa}\right)\right]^{-1}} < \tilde{a}^s_{aut},$$

and that $H(a)$ is negative for $a > \overline{a}^s_{aut}$ but positive for $a < \overline{a}^s_{aut}$.

Summarizing the above analysis we conclude that the equilibrium is indeterminate if and only if

$$\sigma > \underline{\sigma} + \frac{2(1+\beta)}{\kappa} \quad \text{and} \quad a < \overline{a}^s_{aut}. \tag{A.10}$$

Otherwise, the dynamic system exhibits saddle-path stability.

Note that the first condition in (A.10) is identical to the first condition in (34) of Proposition 2, which is the necessary and sufficient condition for equilibrium indeterminacy presented by (35) in Corollary 2 for a closed economy with sticky prices. We now show that, conditional on this first condition, the threshold value in the second condition in (A.10), $\overline{a}^s_{aut}$, is greater than the threshold value in the second condition in (34), $\overline{a}^s$. This is to say that the instability (stability) region is smaller (greater) if the small open economy is fully integrated in the global capital market than if it is excluded from it.

To complete this proof, recall the function $F(a)$ in (A.8) in the Proof of Proposition 2 above, while we note that $F(1) < 0$, and $F(0) > 0$ given $\sigma > \underline{\sigma} + 2(1+\beta)/\kappa$, and that $\overline{a}^s \in (0,1)$ satisfies $F(\overline{a}^s) = 0$. It is then clear that $F(a) \gtrless 0$ implies $a \lessgtr \overline{a}^s$. Now, since

$$F(\overline{a}^s_{aut}) = -\left\{\left[\frac{(\underline{\sigma}-2)(\sigma\theta-1)}{\sigma-1+\left(\sigma-\underline{\sigma}-2\frac{1+\beta}{\kappa}\right)(\theta-1)} + \frac{\theta-1}{2\theta-1}\right]\left(\sigma+\underline{\sigma}-2+2\frac{1+\beta}{\kappa}\right) + (\sigma-1)\right\}\overline{a}^s_{aut} < 0,$$

we conclude that $\overline{a}^s_{aut} > \overline{a}^s$. **Q.E.D.**

Proof of Proposition 4. Replacing money growth targeting (20) with inflation targeting (41) and substituting out all variables other than consumption in (24), (26)-(28), we can collapse the system into a first-order linear difference equation in $\widehat{C}_t$,

$$\widehat{C}_{t+1} = \frac{1}{\sigma}\left\{\frac{a\sigma}{1-a} + (\underline{\sigma}-2)\left[\frac{(2-a)a\theta}{1-a}\sigma + (1-a)\right]\right\}\widehat{C}_t.$$

Following the same logic as in the Proof of Proposition 1, the system is indeterminate (determinate) if and only if it has a stable (explosive) root.

Recall that $\underline{\sigma} \geq 2$. It is straightforward to show that the system has an explosive root if $g(a) < 0$ but a stable root if $g(a) > 0$, where

$$g(a) \equiv (\underline{\sigma} - 2)(\sigma\theta - 1)a^2 - 2[\sigma + (\underline{\sigma} - 2)(\theta\sigma - 1)]a + \sigma - \underline{\sigma} + 2. \tag{A.11}$$

Note that $g(1) = -(\underline{\sigma} - 2)\theta\sigma < 0$ and $g(0) = \sigma + 2 - \underline{\sigma}$. We now break into cases.

The case with $\underline{\sigma} = 2$

We have $g(a) = (1 - 2a)\sigma$, which is positive if $a < 0.5$ but negative if $a > 0.5$.

The case with $\underline{\sigma} > 2$

If $\sigma\theta = 1$, then $g(a) = -2\sigma a + \sigma - \underline{\sigma} + 2$. For $\sigma \leq \underline{\sigma} - 2$, we have $g(a) < 0$ for all $a \in (0, 1)$. For $\sigma > \underline{\sigma} - 2$, $g(a)$ is positive if $a < 0.5(\sigma - \underline{\sigma} + 2)/\sigma$ but negative if $a > 0.5(\sigma - \underline{\sigma} + 2)/\sigma$.

If $\sigma\theta > 1$, then the quadratic function in (A.11) is convex. If $\sigma \leq \underline{\sigma} - 2$, then $g(0) \leq 0$, so $g(a) < 0$ for all $a \in (0, 1)$. If $\sigma > \underline{\sigma} - 2$, then $g(0) > 0$, so there exists a unique $\overline{a}^f_{IT} \in (0, 1)$ such that $g(a)$ is positive if $a < \overline{a}^f_{IT}$ but negative if $a > \overline{a}^f_{IT}$.

If $\sigma\theta < 1$, then the quadratic function in (A.11) is concave. If $\sigma > \underline{\sigma} - 2$, then $g(0) > 0$, so there exists a unique $\overline{a}^f_{IT} \in (0, 1)$ such that $g(a)$ is positive if $a < \overline{a}^f_{IT}$ but negative if $a > \overline{a}^f_{IT}$. If $\sigma \leq \underline{\sigma} - 2$, then $g(0) \leq 0$, and there are three possibilities in this case: first, $g(a)$ is monotone increasing in $a \in (0, 1)$; second, $g(a)$ is monotone decreasing in $a \in (0, 1)$; third, $g(a)$ reaches its stationary point (maxima) within $(0, 1)$. In both the first and the second cases, we have $g(a) < 0$ for all $a \in (0, 1)$. In the third case, denote by $\tilde{a}^f_{IT} \in (0, 1)$ the stationary point. We can verify that $1 - \tilde{a}^f_{IT} = \sigma / [(\underline{\sigma} - 2)(1 - \sigma\theta)]$, which should also lie within $(0, 1)$, and the maximum value of g, denoted as $g_{\max}$, equals

$$g_{\max} = -\left[\left(1 - \tilde{a}^f_{IT}\right)\tilde{a}^f_{IT}(\underline{\sigma} - 2)(1 - \sigma\theta) + (\underline{\sigma} - 2)\sigma\theta\right] < 0.$$

This is to say that, in the third case, we have $g(a) < 0$ for all $a \in (0, 1)$ as well. **Q.E.D.**

Proof of Proposition 5. Replacing money growth targeting (20) with inflation targeting (41) and keeping all the other equilibrium conditions the same as in the Proof of Proposition 2 of Section 3, we obtain a similar two-dimensional system as (A.2) while the determinant and trace of $\mathbf{A}$ are given by, respectively,

$$\det \mathbf{A} = \frac{1}{\beta - \frac{1-a}{a}\kappa},$$

$$\text{tr}\mathbf{A} = 1 + \det \mathbf{A} + \frac{1 + \frac{1-a}{\sigma}(\underline{\sigma} - 2)\left[\frac{(2-a)a\theta}{1-a}\sigma + (1 - a)\right]}{\beta - \frac{1-a}{a}\kappa}\frac{\kappa}{a}.$$

Applying the conditions for two stable roots, we can show that (A.5) holds if and only if $G(a) > 0$ where

$$G(a) \equiv (\underline{\sigma} - 2)(\sigma\theta - 1)a^2 - 2\left[\sigma\left(1 + \frac{1 + \beta}{\kappa}\right) + (\underline{\sigma} - 2)(\sigma\theta - 1)\right]a + \sigma - \underline{\sigma} + 2. \tag{A.12}$$

On the other hand, with some algebra, we can show that (A.7) holds if and only if $G(a) < 0$. Note that $G(1) = -\left[1 + 2\frac{1+\beta}{\kappa} + (\underline{\sigma} - 2)\theta\right]\sigma < 0$ and $G(0) = \sigma - \underline{\sigma} + 2$. We now break into cases.

The case with $\underline{\sigma} = 2$

We have $G(a) = \left[1 - 2\left(1 + \frac{1+\beta}{\kappa}\right)a\right]\sigma$, which is positive if $a < \overline{a}^s_{IT}$ but negative if $a > \overline{a}^s_{IT}$, where

$$\overline{a}^s_{IT} \equiv \frac{0.5}{1 + \frac{1+\beta}{\kappa}} \in (0, 0.5).$$

The case with $\underline{\sigma} > 2$

If $\sigma\theta = 1$, then $g(a) = -2\left(1 + \frac{1+\beta}{\kappa}\right)\sigma a + \sigma - \underline{\sigma} + 2$. For $\sigma \leq \underline{\sigma} - 2$, we have $G(a) < 0$ for all $a \in (0, 1)$. For $\sigma > \underline{\sigma} - 2$, $G(a)$ is positive if $a < \overline{a}^s_{IT}$ but negative if $a > \overline{a}^s_{IT}$, where

$$\overline{a}^s_{IT} \equiv \frac{0.5}{1 + \frac{1+\beta}{\kappa}}\left(1 - \frac{\underline{\sigma} - 2}{\sigma}\right) \in \left(0, \frac{0.5}{1 + \frac{1+\beta}{\kappa}}\right).$$

If $\sigma\theta > 1$, then the quadratic function in (A.12) is convex. If $\sigma \leq \underline{\sigma} - 2$, then $G(0) \leq 0$, so $G(a) < 0$ for all $a \in (0, 1)$. If $\sigma > \underline{\sigma} - 2$, then $G(0) > 0$, so there exists a unique $\overline{a}^s_{IT} \in (0, 1)$ such that $G(a)$ is positive if $a < \overline{a}^s_{IT}$ but negative if $a > \overline{a}^s_{IT}$.

If $\sigma\theta < 1$, then the quadratic function in (A.12) is concave. If $\sigma > \underline{\sigma} - 2$, then $G(0) > 0$, so there exists a unique $\overline{a}^s_{IT} \in (0, 1)$ such that $G(a)$ is positive if $a < \overline{a}^s_{IT}$ but negative if $a > \overline{a}^s_{IT}$. If $\sigma \leq \underline{\sigma} - 2$, then $G(0) \leq 0$, and there are three possibilities in this case: first, $G(a)$ is monotone increasing in $a \in (0, 1)$; second, $G(a)$ is monotone decreasing in $a \in (0, 1)$; third, $G(a)$ reaches its stationary point (maxima) within $(0, 1)$. In both the first and the second cases, we have $G(a) < 0$ for all $a \in (0, 1)$. In the third case, denote by $\tilde{a}^s_{IT} \in (0, 1)$ the stationary point. We can verify that $1 - \tilde{a}^s_{IT} = [1 + (1 + \beta)/\kappa]\sigma / [(\underline{\sigma} - 2)(1 - \sigma\theta)]$, which should also lie within $(0, 1)$, and the maximum value of G, denoted as $G_{\max}$, equals

$$G_{\max} = -\left[(1 - \tilde{a}^s_{IT})\tilde{a}^s_{IT}(\underline{\sigma} - 2)(1 - \sigma\theta) + \frac{1+\beta}{\kappa}\sigma + (\underline{\sigma} - 2)\sigma\theta\right] < 0.$$

This is to say that, in the third case, we have $G(a) < 0$ for all $a \in (0, 1)$ as well. **Q.E.D.**

Technical details for Section 7. We first specify the two 3×3 matrices Γ_0 and Γ_1 in (46) across the eight configurations of monetary policies, price settings, and integrations of the SOE in world capital market.

With **FLEXIBLE PRICES**, Γ_0 and Γ_1 take the following generic forms,

$$\Gamma_0 = \begin{bmatrix} c_1 & c_2 & c_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}.$$

With money growth targeting and perfect world capital market

$$c_1 = \sigma\mu_c - 1,$$

$$c_2 = 1 + a\frac{\sigma\mu_c}{1-a} + (\underline{\sigma} - 2)\left[\frac{a\theta(2-a)}{1-a}\sigma\mu_c + 1 - a\right],$$

$$c_3 = \left\{\rho_\mu + [(\underline{\sigma} - 2)a\theta(2-a) + a]\frac{1}{1-a}\right\}\sigma(1-\mu_c).$$

The system is indeterminate if $a = 0.19$ and determinate if $a = 0.2$.

With inflation targeting and perfect world capital market

$$c_1 = \sigma\mu_c - 1,$$

$$c_2 = 1 + a\frac{\sigma\mu_c}{1-a} + (\underline{\sigma} - 2)\left[\frac{a\theta(2-a)}{1-a}\sigma\mu_c + 1 - a\right],$$

$$c_3 = \left\{\rho_\mu + [(\underline{\sigma} - 2)a\theta(2-a) + a]\frac{1}{1-a}\right\}\sigma(1-\mu_c).$$

The system is indeterminate if $a = 0.24$ and determinate if $a = 0.25$.

With money growth targeting and international financial autarky

$$c_1 = \sigma\mu_c - 1, \quad c_2 = (\underline{\sigma} - 1)\left[\frac{a}{\theta(2-a) - 1} + 1\right], \quad c_3 = \sigma(1-\mu_c)\rho_\mu.$$

The system is indeterminate if $a = 0.74$ and determinate if $a = 0.75$.

With inflation targeting and international financial autarky

$$c_1 = \sigma\mu_c, \quad c_2 = (\underline{\sigma} - 1)\left[\frac{a}{\theta(2-a) - 1} + 1\right] - 1, \quad c_3 = \sigma(1-\mu_c)\rho_\mu.$$

The system is indeterminate if $a = 0.79$ and determinate if $a = 0.8$.

With **STICKY PRICES**, Γ_0 and Γ_1 take the following generic forms,

$$\Gamma_0 = \begin{bmatrix} b_1 & b_2 & b_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 0 & b_4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}.$$

With money growth targeting and perfect world capital market

$$b_1 = \sigma\mu_c - 1 - \frac{\beta}{\kappa}\left(1 + \frac{a\sigma}{1-a}\mu_c\right),$$

$$b_2 = (\underline{\sigma} - 2)\left[\frac{a\sigma\theta(2-a)}{1-a}\mu_c + (1-a)\right] + \frac{1+\kappa+\beta}{\kappa}\left(1 + \frac{a\sigma}{1-a}\mu_c\right),$$

$$b_3 = -\left\{\rho_\mu \left(1 - \frac{a\beta}{\kappa}\frac{1}{1-a}\right) + \left[(\underline{\sigma} - 2)\,\theta\,(2-a) + \frac{1+\kappa+\beta}{\kappa}\right]\frac{a}{1-a}\right\}\sigma\,(1-\mu_c)\,,$$

$$b_4 = \frac{1}{\kappa}\left(1 + \frac{a\sigma}{1-a}\mu_c\right).$$

The system is indeterminate if $a = 0.05$ and determinate if $a = 0.06$.

With inflation targeting and perfect world capital market

$$b_1 = \left(1 - \frac{\beta a}{\kappa}\frac{1}{1-a}\right)\sigma\mu_c,$$

$$b_2 = (\underline{\sigma} - 2)\left[\frac{a\sigma\theta\,(2-a)}{1-a}\mu_c + (1-a)\right] + \frac{1+\kappa+\beta}{\kappa}\frac{a\sigma}{1-a}\mu_c,$$

$$b_3 = -\left\{\rho_\mu \left(1 - \frac{a\beta}{\kappa}\frac{1}{1-a}\right) + \left[(\underline{\sigma} - 2)\,\theta\,(2-a) + \frac{1+\kappa+\beta}{\kappa}\right]\frac{a}{1-a}\right\}\sigma\,(1-\mu_c)\,,$$

$$b_4 = \frac{1}{\kappa}\left(1 + \frac{a\sigma}{1-a}\mu_c\right).$$

The system is indeterminate if $a = 0.13$ and determinate if $a = 0.14$.

With money growth targeting and international financial autarky

$$b_1 = \sigma\mu_c - 1 - \frac{\beta}{\kappa}\left[\frac{a}{\theta\,(2-a) - 1} + 1\right],$$

$$b_2 = \left(\underline{\sigma} - 1 + \frac{1+\beta}{\kappa}\right)\left[\frac{a}{\theta\,(2-a) - 1} + 1\right],$$

$$b_3 = -\sigma\,(1-\mu_c)\,\rho_\mu\ , \quad b_4 = \frac{1}{\kappa}\left[\frac{a}{\theta\,(2-a) - 1} + 1\right].$$

The system is indeterminate if $a = 0.38$ and determinate if $a = 0.39$.

With inflation targeting and international financial autarky

$$b_1 = \sigma\mu_c - \frac{\beta a}{\kappa}\frac{1}{\theta\,(2-a) - 1},$$

$$b_2 = \underline{\sigma} - 2 + \left(\underline{\sigma} - 1 + \frac{1+\beta}{\kappa}\right)\frac{a}{\theta\,(2-a) - 1},$$

$$b_3 = -\sigma\,(1-\mu_c)\,\rho_\mu\ , \quad b_4 = \frac{1}{\kappa}\frac{a}{\theta\,(2-a) - 1}.$$

The system is indeterminate if $a = 0.62$ and determinate if $a = 0.63$.

Other endogenous variables

We can solve for terms of trade after obtaining consumption,

$$\widehat{T}_t = \frac{\sigma\mu_c}{1-a}\widehat{C}_t + \frac{\sigma\,(1-\mu_c)}{1-a}\widehat{\mu}_t, \textbf{ with perfect world capital market;}$$

but

$$\widehat{T}_t = \frac{1}{\theta\left(2-a\right)-1}\widehat{C}_t, \textbf{ under international financial autarky.}$$

Then other endogenous variables can be solved from consumption and terms of trade,

$$\widehat{L}_t = \frac{a\theta\left(2-a\right)}{\alpha}\widehat{T}_t + \frac{1-a}{\alpha}\widehat{C}_t,$$

$$\widehat{Q}_t = \left(1-a\right)\widehat{T}_t.$$

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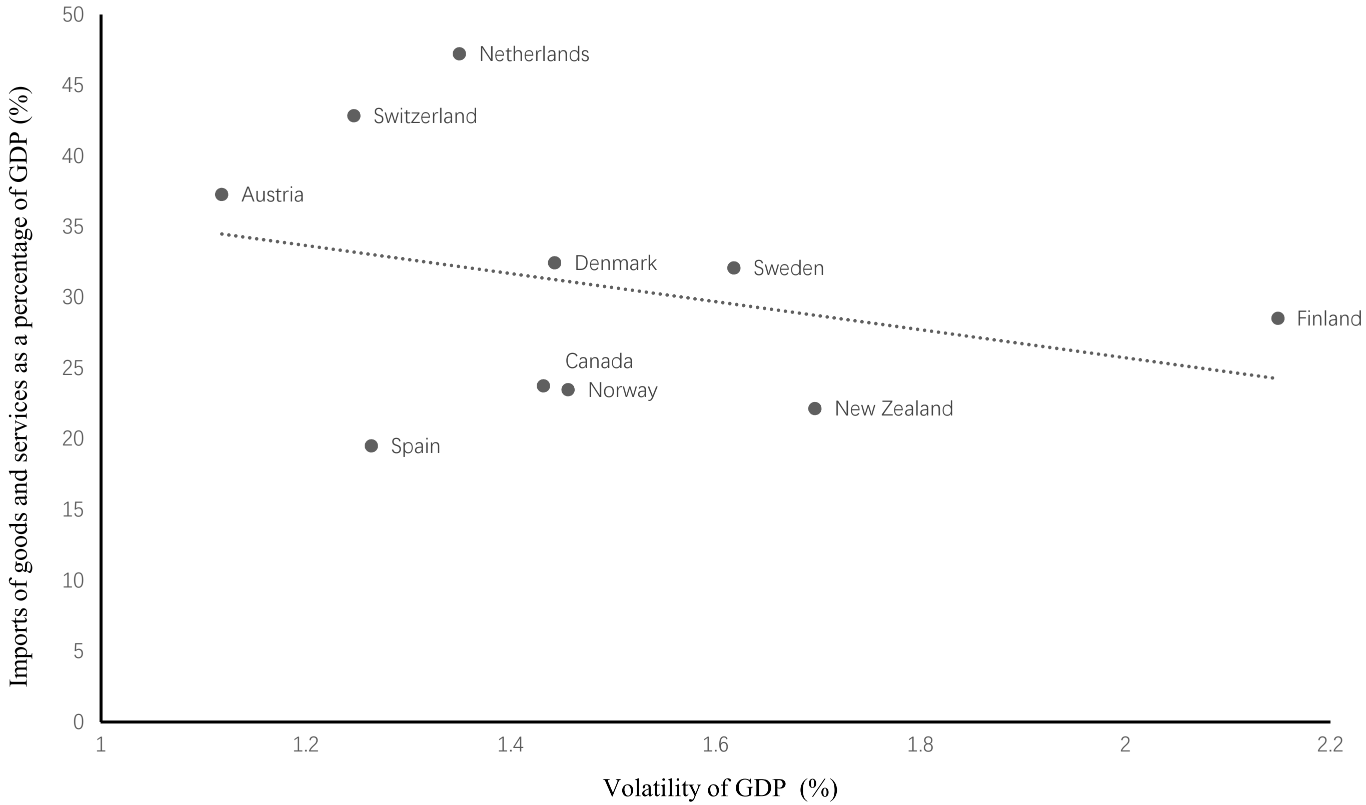
Table 1: Imports of goods and services
as a percentage of GDP (2000-2014)

Australia	21.2	Hungary	72.7
Austria	46.3	Ireland	78.4
Belgium	72.9	South Korea	40.6
Bulgaria	57.2	Netherlands	61.7
Canada	33.2	Poland	39.4
Croatia	43.3	Spain	29.2
Czech Republic	60.4	Sweden	39.3
Denmark	43.8	Switzerland	49.7

Source: World Development Indicators, the World Bank



Note: Volatility is measured by the standard deviation of output as in Chen et al. (2014) and openness by the imports of goods and services as a percentage of GDP as is standard in the literature based on OECD data (1978:I-2008:III) for ten developed countries that are classified by Mendoza et al. (1997) as small open economies.

Fig. 1. Relation between volatility and openness.

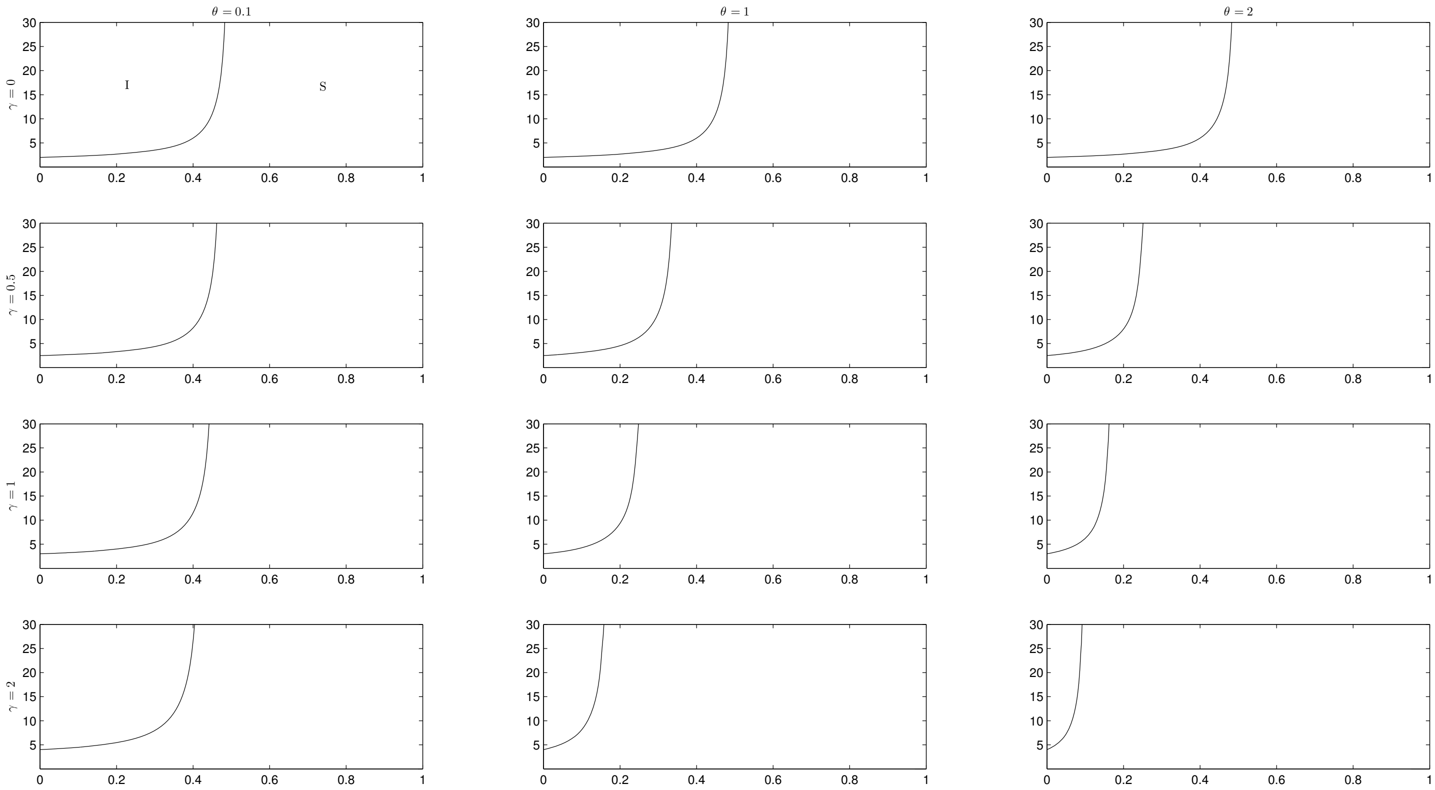


Fig. 2. Minimal degree of trade openness a (horizontal axis) that guarantees saddle-path stability as a function of relative risk aversion in consumption σ (vertical axis) in the flexible price model. S - Stability, I - Instability.

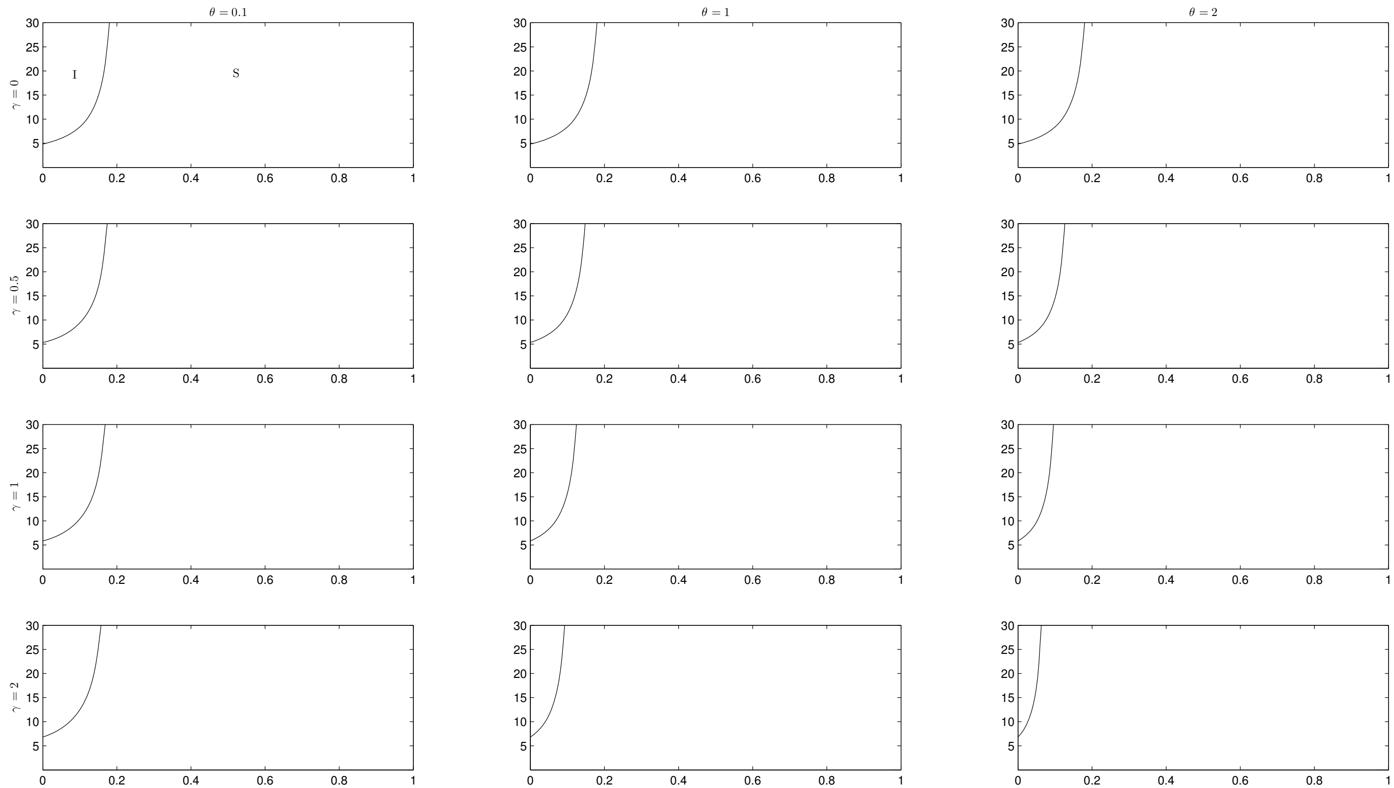


Fig. 3. Minimal degree of trade openness a (horizontal axis) that guarantees saddle-path stability as a function of relative risk aversion in consumption σ (vertical axis) in the sticky price model. S - Stability, I - Instability.

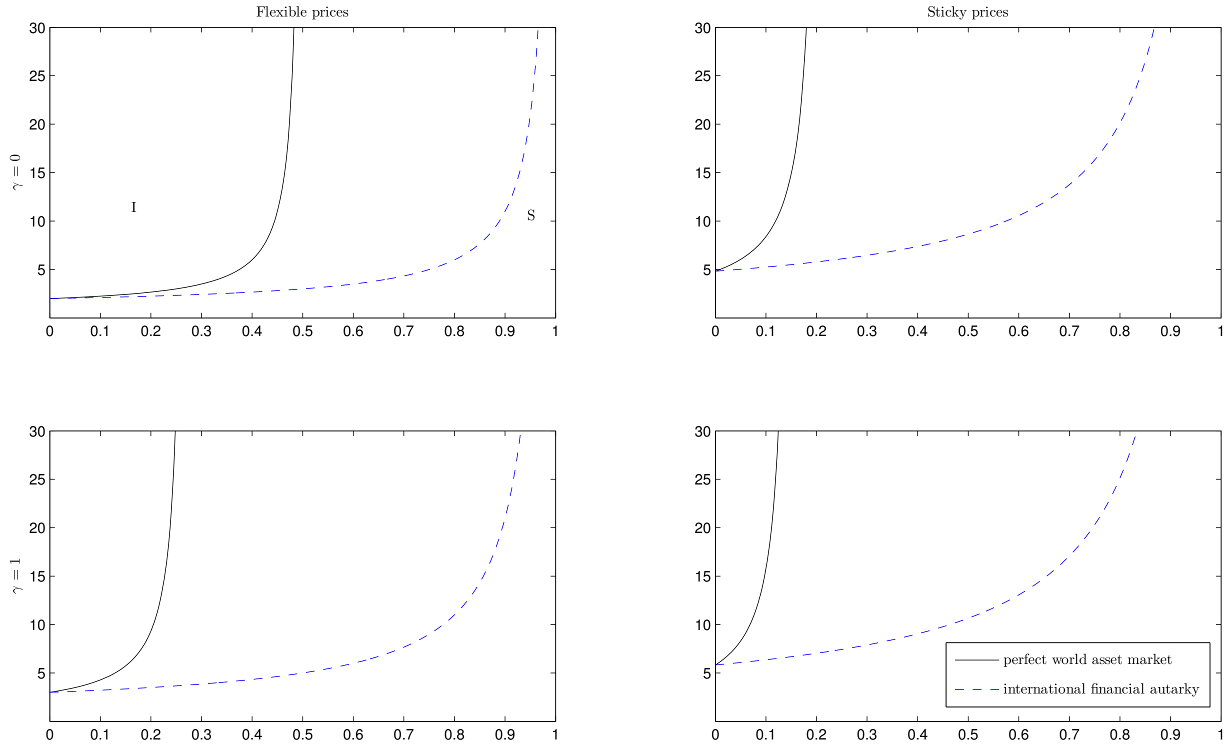


Fig. 4. Minimal degree of trade openness a (horizontal axis) that guarantees saddle-path stability as a function of relative risk aversion in consumption σ (vertical axis) with perfect world asset market (black solid line) and under international financial autarky (blue dashed line). S - Stability, I - Instability.

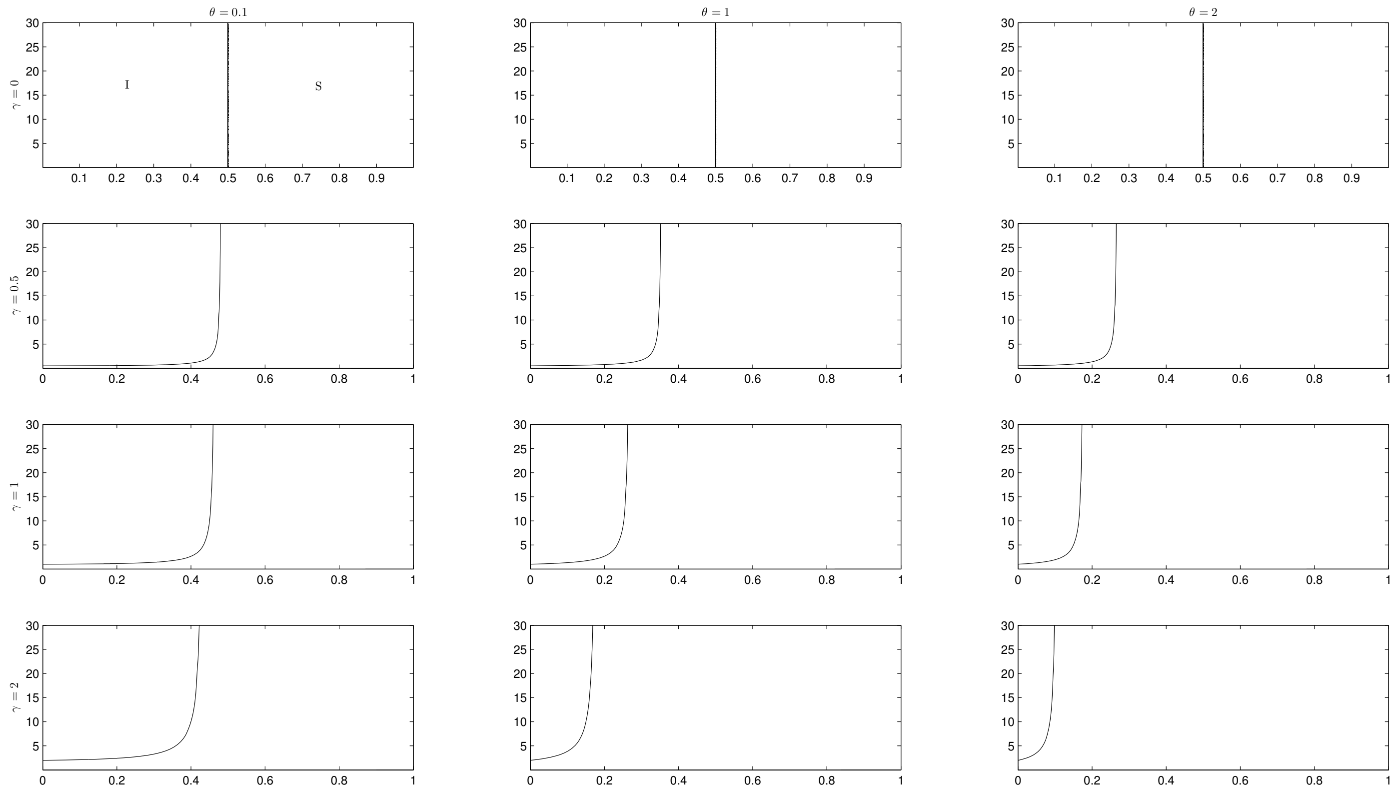


Fig. 5. Minimal degree of trade openness a (horizontal axis) that guarantees saddle-path stability as a function of relative risk aversion in consumption σ (vertical axis) under inflation targeting and flexible prices with perfect world asset market. S - Stability, I - Instability.

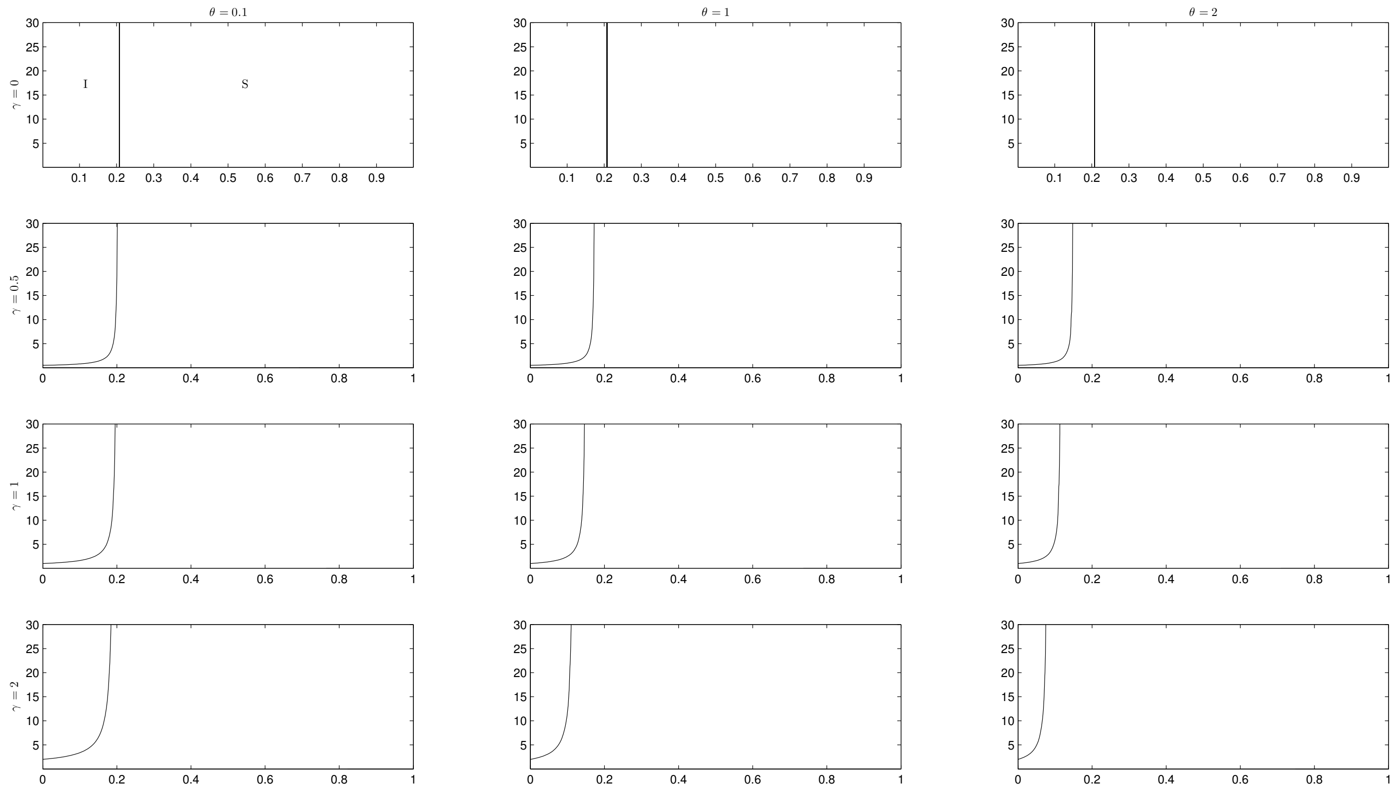


Fig. 6. Minimal degree of trade openness a (horizontal axis) that guarantees saddle-path stability as a function of relative risk aversion in consumption σ (vertical axis) under inflation targeting and sticky prices with perfect world asset market. S - Stability, I - Instability.

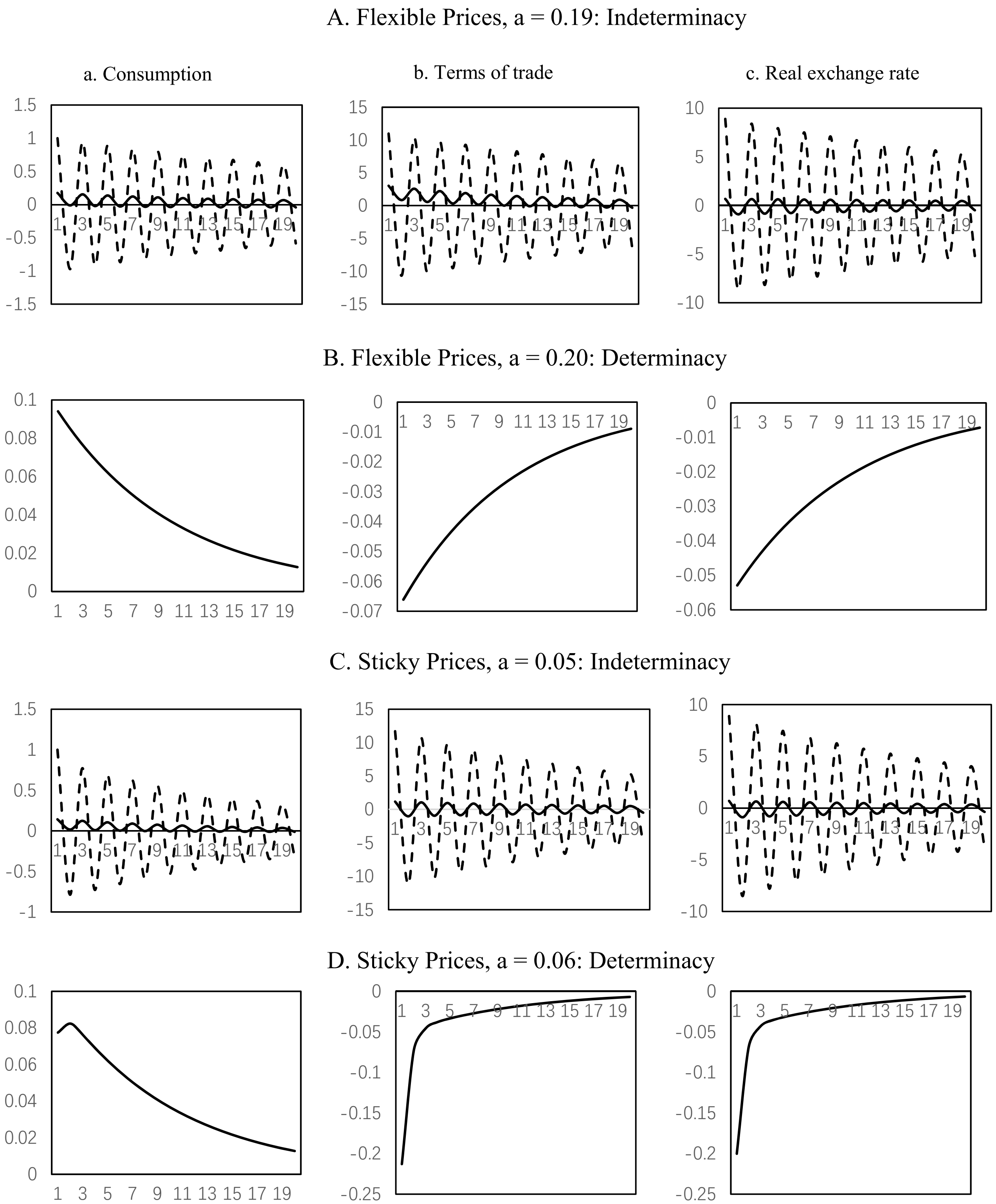


Fig. 7. Impulse responses to a consumption shock (solid line) or a sunspot shock (dashed line) in the small open economy with money growth targeting and perfect world capital market.

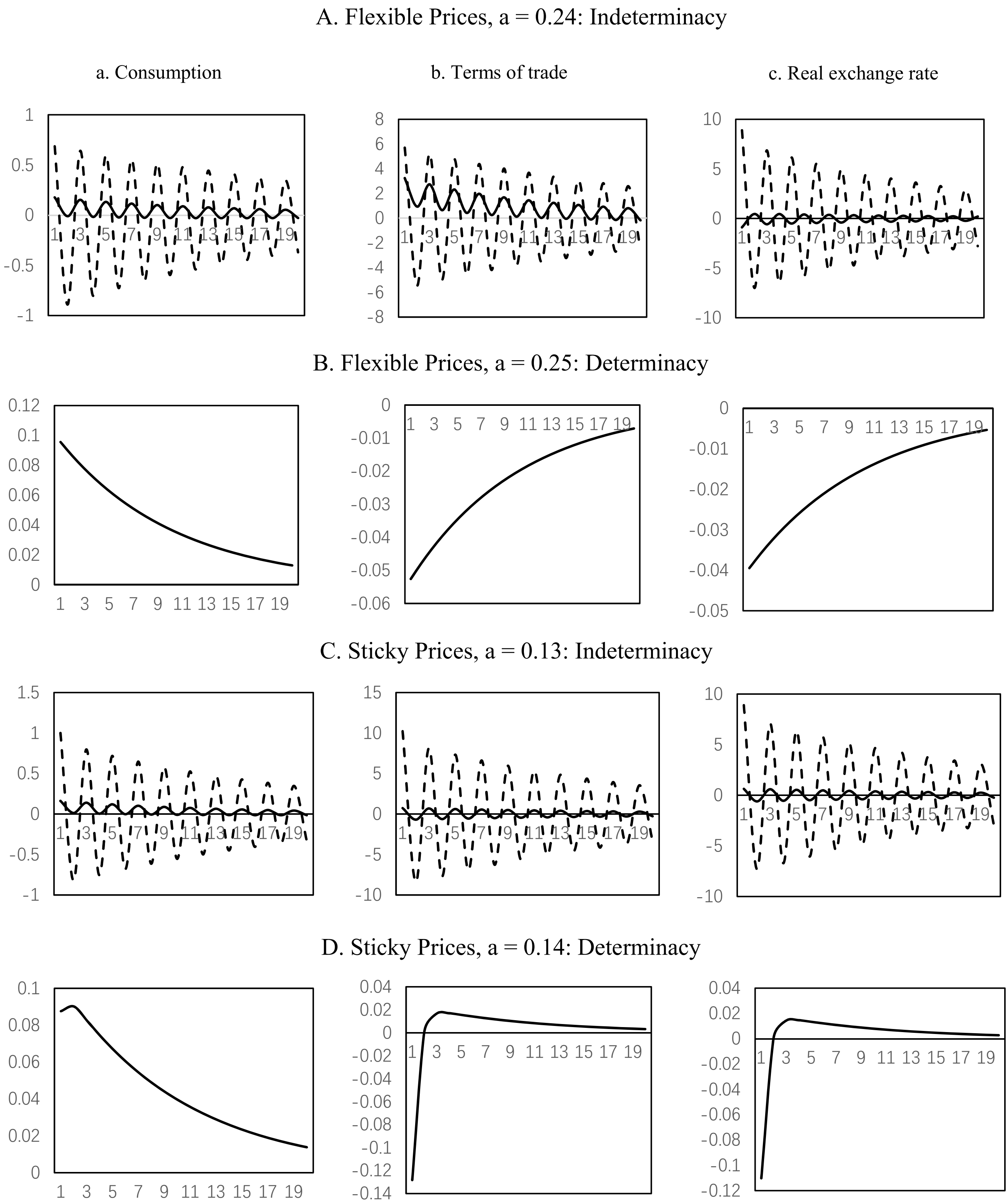


Fig. 8. Impulse responses to a consumption shock (solid line) or a sunspot shock (dashed line) in the small open economy and inflation targeting and perfect world capital market.

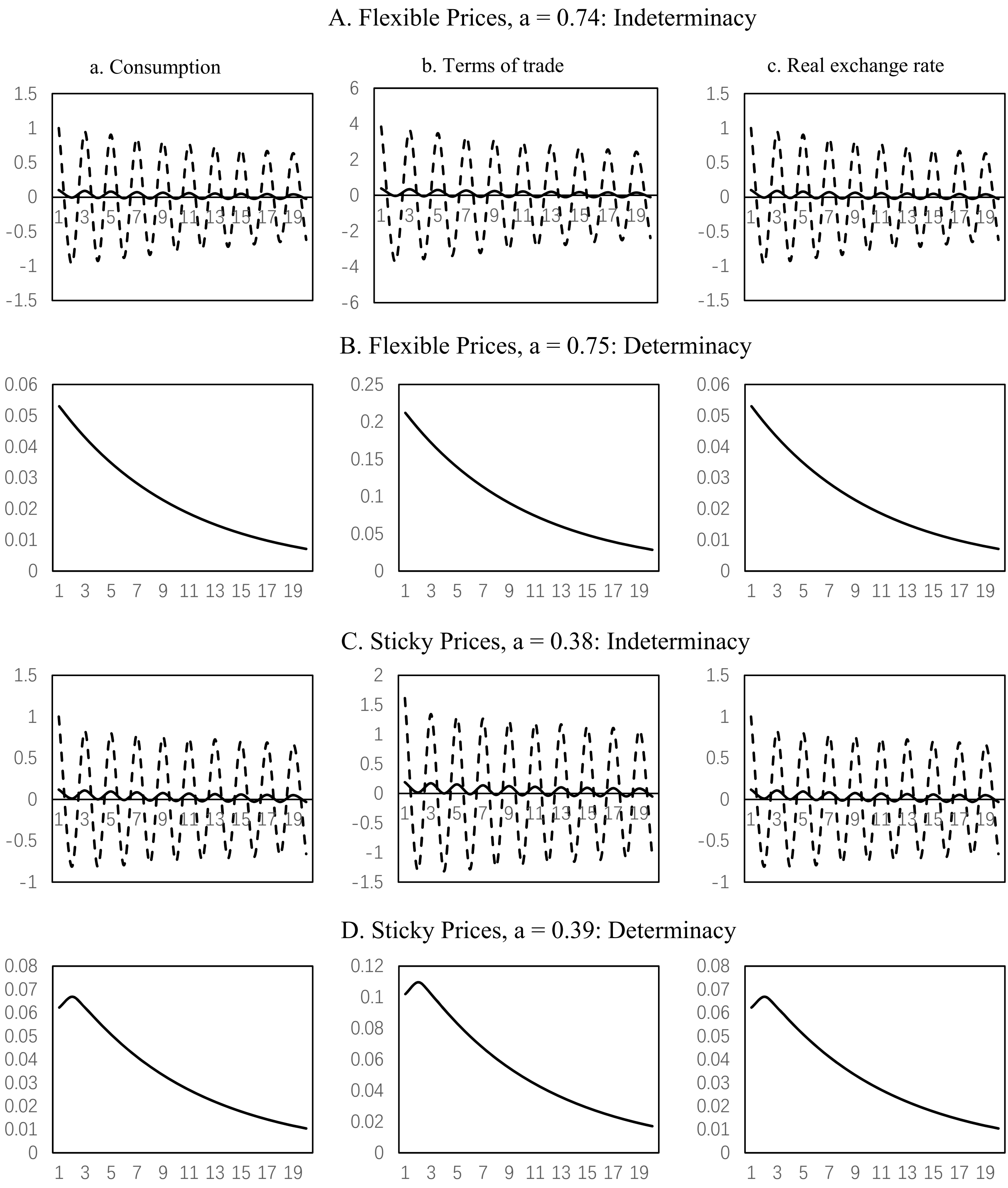


Fig. 9. Impulse responses to a consumption shock (solid line) or a sunspot shock (dashed line) in the small open economy with money growth targeting and international financial autarky.

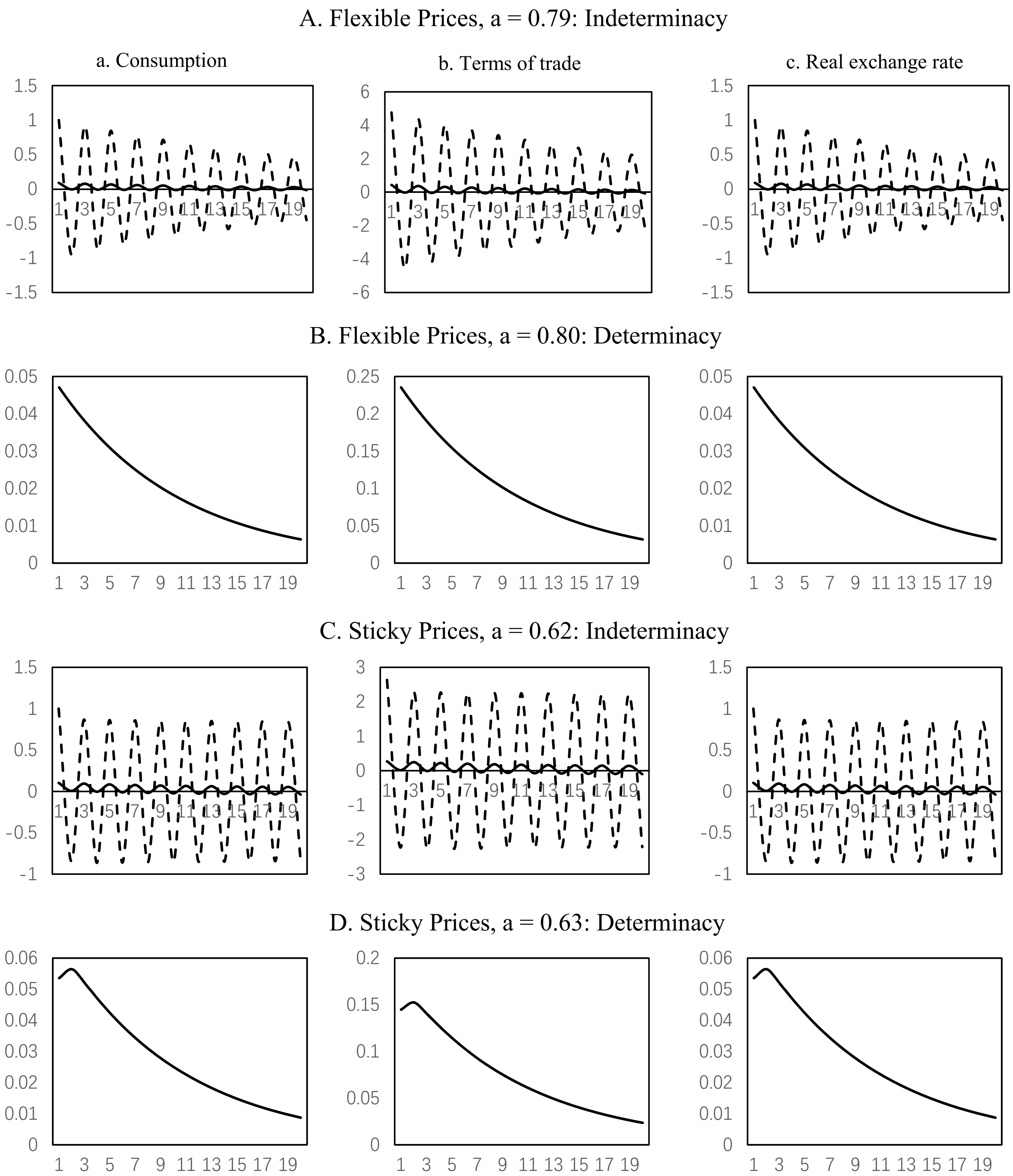


Fig. 10. Impulse responses to a consumption shock (solid line) or a sunspot shock (dashed line) in the small open economy with inflation targeting and international financial autarky.