We explore the role of economic growth as a cause of reverse trade diversion in an asymmetric three-country Melitz model. A regional trade agreement between countries 1 and 2 decreases country 3’s growth rate and the revenue shares of varieties country 3 exports to countries 1 and 2 in the short run, but increases them in the long run, compared with the old balanced growth path. This is because faster short-run growth in countries 1 and 2 than country 3 starts to increase the members’ market entry costs more than the nonmember, thereby making the latter relatively more competitive.
Trade diversion is reversed in the long run

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Abstract

We explore the role of economic growth as a cause of reverse trade diversion in an asymmetric three-country Melitz model. A regional trade agreement between countries 1 and 2 decreases country 3’s growth rate and the revenue shares of varieties country 3 exports to countries 1 and 2 in the short run, but increases them in the long run, compared with the old balanced growth path. This is because faster short-run growth in countries 1 and 2 than country 3 starts to increase the members’ market entry costs more than the nonmember, thereby making the latter relatively more competitive.

JEL classification: F13; F15; F43

Keywords: Reverse trade diversion; Melitz model; Regional trade agreement; Endogenous growth; Transitional dynamics

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1 Introduction

Trade creation and diversion have been the central concepts in the economics of regional trade agreements (RTAs). Whereas the former brings about mutual benefits for members of an RTA, the latter incurs costs on its nonmembers in terms of decreased exports to the member countries and the resulting welfare losses. To understand the logic, suppose that countries 1 and 2 liberalize their imports only from each other, keeping their external trade barriers on imports from country 3 unchanged. On the one hand, this facilitates the members' trade with each other (i.e., trade creation). On the other hand, since it decreases the members' demands for the nonmember's exports, the world prices and hence the quantities of country 3's exports to countries 1 and 2 decrease (i.e., trade diversion). The logic is so robust that one cannot avoid trade diversion theoretically so far (without additional policy changes to be discussed soon). So, how serious is trade diversion in reality? Recent empirical studies based on the gravity model with trade creation and diversion dummies reveal that trade diversion is not always the case (e.g., Endoh, 1999; Carrère, 2006; Lee and Shin, 2006; Magee, 2008; Acharya et al., 2011). They even find that members of some RTAs do import more from nonmembers than pairs of nonmember countries as the control group, a phenomenon Baldwin (2011) calls “reverse trade diversion”. How can we reconcile the new evidence with theory? The purpose of this paper is to create a new theory to explain reverse trade diversion.

One explanation for reverse trade diversion is “tariff complementarity”: members of an RTA are inclined to reduce their external tariffs following internal trade liberalization. This is because internal trade liberalization causes trade distortions, that is, overimports from members and underimports from nonmembers. Then it will be optimal for members to reduce their external tariffs in order to alleviate the distortions. As a result, nonmembers' exports to members will increase. The tariff complementarity story works for both small-country (e.g., Richardson, 1993) and large-country models (e.g., Bagwell and Staiger, 1999; Bond et al., 2004; Ornelas, 2005; Saggi and Yildiz, 2010). However, the empirical evidence on the hypothesis is mixed: while Bohara et al. (2004) and Estevadeordal et al. (2008) report that industries subject to preferential trade liberalization tend to experience larger cuts in external tariffs than the other industries, Limão (2006) and Karacavaol and Limão (2008) find just the opposite. Given the weak empirical support for tariff complementarity, we have to develop an alternative explanation for reverse trade diversion, where the external tariffs are not set optimally but exogenously given.

In this paper, we explore the role of economic growth as a cause of reverse trade diversion. One of the main purposes of most RTAs is to promote members' economic growth and development. Following this, consider a situation where an RTA between countries 1 and 2 raises their growth rates in the short run. If

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1 In the literature, a trade creation dummy takes the value of one if both the exporter and importer in a pair are members of the same RTA. Its positive coefficient indicates the presence of trade creation. A trade diversion dummy takes the value of one if only one of the exporter and importer in a pair is a member of an RTA. It should have a negative coefficient in the presence of trade diversion.

2 Magee (2008) and Acharya et al. (2011) report the evidence of reverse trade diversion for RTAs including major ones such as AFTA (ASEAN Free Trade Area), EFTA (European Free Trade Association), and NAFTA (North American Free Trade Agreement). Lee and Shin (2006) even find reverse trade diversion from the full panel of 175 countries from 1948 to 1999.

3 Bagwell and Staiger (1999) consider a symmetric three-country, three-(nonnumeraire)-good, partial equilibrium pure exchange model, where country $i$ exports goods $j$ and $k$ but imports good $(i, j, k = 1, 2, 3, j \neq i, k \neq i, j)$. This means that each country imports a homogeneous good from the other two countries. Bond et al. (2004) formulate a three-country, three-good pure exchange model, where country $i$ exports good $i$ but imports goods $j$ and $k(i, j, k = 1, 2, 3, j \neq i, k \neq i, j)$. Unlike Bagwell and Staiger (1999), each country imports two distinct goods. To keep calculations simple, they assume symmetric FTA members. Ornelas (2005) uses a three-country partial equilibrium model with segmented markets and political economy. In each segmented market, firms from the three countries provide a homogeneous good and play a Cournot competition. Saggi and Yildiz (2010) use an asymmetric version of Bagwell and Staiger (1999) to endogenize the formation of trade agreements.

4 Article B of the Treaty on European Union states that its first objective is: “to promote economic and social progress which is balanced and sustainable”. The Preamble of NAFTA states that its members aim to: “PROMOTE sustainable development.”
this relatively raises the market entry costs of their firms in line with the empirical findings of Bollard et al. (2016), country 3 might become relatively more competitive, exporting more to the RTA members and growing faster in the long run. To pursue such a possibility, we start from the Melitz (2003) model, where the market entry costs play a significant role in reallocating resources across heterogeneous firms within an industry. For our present purpose, we need to extend the Melitz model to allow for at least three asymmetric countries possibly growing at different rates during the transition to a balanced growth path (BGP). We build on Naito’s (2017) asymmetric two-country Melitz model of trade and endogenous growth with transitional dynamics, which combines the static asymmetric two-country Melitz models of Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013) with the asymmetric multi-country AK growth model of Acemoglu and Ventura (2002).

Focusing on the three-country case and starting from the symmetric BGP, we examine the effects of an RTA between countries 1 and 2 (i.e., permanent decreases in their import trade costs from each other by the same rate). We obtain two main results analytically. First, the growth rate of country 3 decreases in the short run, but increases in the long run. Second, the revenue shares of varieties country 3 exports to countries 1 and 2 decrease in the short run, but increase in the long run. The intuitions for these results are as follows. In the short run, the RTA encourages the members’ internal exports to each other, which in turn pushes their inefficient firms out of their domestic markets. Since this makes firms in the nonmember country relatively less competitive in their export markets, more efficient firms stop exporting whereas more inefficient firms stay in their domestic market, thereby slowing down the country’s growth. However, the fact that the members start to grow faster than the nonmember pushes up the former’s market entry costs more than the latter. Since this means that the members become relatively less competitive than the nonmember, more efficient firms start exporting whereas more inefficient firms exit in the latter. In the long run, the nonmember country grows faster, and exports more to the member countries, than the old BGP. It is the market entry costs rising with development that is responsible for reverse trade diversion in the long run.

The rest of this paper is organized as follows. Section 2 sets up a general N-country model. Section 3 focuses on the three-country case and derives countries’ growth functions around the symmetric BGP. Section 4 examines the growth effects of trade liberalization in general. Section 5 studies the effects of an RTA on all bilateral revenue shares. Section 6 provides further discussions. Section 7 concludes.

# 2 N-country model

The following model is a multi-country extension of Naito (2017). The world consists of \( N (\geq 2) \) possibly asymmetric countries. In each country \( i = 1, \ldots, N \), a nontradable final good is produced from tradable differentiated intermediate goods under constant returns to scale and perfect competition, and is used for consumption and investment. The intermediate goods are produced from nontradable capital under increasing returns to scale and monopolistic competition. The growth rate of capital is endogenously determined in general equilibrium.

## 2.1 Households

The representative household in country \( i \) maximizes its overall utility \( U_i = \int_0^{\infty} \ln C_{it} \exp(-\rho_i t) dt \), subject to its budget constraint:

\[
p_{it}^Y (C_{it} + K_{it} + \delta_i K_{it}) = r_{it} K_{it}; \quad K_{it} \equiv dK_{it}/dt, \tag{1}
\]
with \( \{p^1_t, r_t\}_{t=0}^{\infty} \) and \( K_{t0} \) given, where \( t \in [0, \infty) \) is time, \( C_t \) is consumption, \( \rho_i \) is the subjective discount rate, \( p^Y_i \) is the price of the final good, \( K_i \) is the supply of capital, \( \delta_i \) is the depreciation rate of capital, and \( r_i \) is the rental rate of capital. The time subscript is omitted whenever no confusion arises. Under the logarithmic instantaneous utility function, it is optimal to keep the consumption/capital ratio constant at \( C_t/K_t = \rho_i \) over time, which means that capital always grows at the same rate as consumption given by the Euler equation:

\[
\dot{K}_t/K_t = \dot{C}_t/C_t = r_t/p^Y_t - \delta_i - \rho_i \equiv \gamma \forall t. \tag{2}
\]

We simply call this “the growth rate of country \( i \).”

### 2.2 Final good firms

The representative final good firm in country \( i \) maximizes its profit \( \Pi^Y_i = p^Y_i Y_i - \int_{\Omega_i} p_i(\omega) x_i(\omega) d\omega \), subject to its production function \( Y_i = (\int_{\Omega_i} x_i(\omega)^\alpha d\omega)^{1/\alpha}; \alpha \equiv (\sigma - 1)/\sigma \in (0, 1) \), with \( p^Y_i \) and \( \{p_i(\omega)\}_{\omega \in \Omega_i} \) given, where \( Y_i \) is the supply of the final good, \( \Omega_i \) is the set of available varieties of intermediate goods, \( p_i(\omega) \) is the demand price of variety \( \omega, x_i(\omega) \) is the demand for variety \( \omega \), and \( \sigma(>1) \) is the elasticity of substitution between any two varieties. As a result of cost minimization, the demand for variety \( \omega \) is derived as:

\[
x_i(\omega) = p_i(\omega)^{-\sigma} P^\sigma_i Y_i; \quad P_i \equiv \left( \int_{\Omega_i} p_i(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}, \tag{3}
\]

where \( P_i \) is the price index of the intermediate goods. Substituting this back into the total cost, the latter is rewritten as \( \int_{\Omega_i} p_i(\omega)x_i(\omega)d\omega = P_i Y_i \equiv E_i \), where \( E_i \) is the minimized cost of producing \( Y_i \) units of the final good, and \( P_i \) also works as the minimized unit cost of the final good. Finally, profit maximization under perfect competition implies that the price of the final good should be equal to its unit cost:

\[
p^Y_i = P_i. \tag{4}
\]

### 2.3 Intermediate good firms

In each period, an entrant in source country \( i \) first pays a fixed initial entry cost \( F^e_{ij} \) to draw its productivity of capital \( \varphi \) from a distribution function \( G_i(\varphi) \) with the corresponding density function \( g_i(\varphi) \). For each possible realization of \( \varphi \), the entrant calculates its profit in destination country \( j \) as \( \pi_{ij}(\varphi) \) net of a fixed market entry cost \( F_{ij} \). If \( \pi_{ij}(\varphi) \geq 0 \), then the entrant actually pays \( F_{ij} \) to sell to market \( j \). Otherwise, the entrant exits from market \( j \) without paying \( F_{ij} \). The free entry condition requires that the fixed initial entry cost be equal to the sum of the expected net profits over all markets. In line with Naito (2017), the fixed entry costs are specified as \( F^e_i = r_i K_i f^e_i \) and \( F_{ij} = r_i K_i f_{ij} \), where \( f^e_i \) and \( f_{ij} \) are exogenous constants. This means that, as country \( i \)’s GDP \( r_i K_i \) grows, it gets more and more difficult for country \( i \)’s entrants to invent a new variety at home and start up a business in each market. This is consistent with Bolland et al. (2016), who find that such entry costs rise with development.

Given \( \varphi \), an intermediate good firm in country \( i \) maximizes \( \pi_{ij}(\varphi) = p^f_{ij}(\varphi)y_{ij}(\varphi) - r_i k_{ij}(\varphi) \), subject to its cost function in terms of capital \( k_{ij}(\varphi) = K_i f_{ij} + y_{ij}(\varphi)/\varphi \), the market-clearing condition for its variety \( y_{ij}(\varphi) = \tau_{ij} x_{ij}(\varphi) \), and the demand function for its variety \( x_{ij}(\varphi) = p_{ij}(\varphi)^{-\sigma} P^\sigma_j E_j = (\tau_{ij} p^f_{ij}(\varphi))^{-\sigma} P^\sigma_j E_j \) from Eq. (3), with \( r_i, K_i, P_j, \) and \( E_j \) given, where \( p^f_{ij}(\varphi) \) is the supply price of the firm’s variety, \( y_{ij}(\varphi) \) is the supply of the firm’s variety, \( k_{ij}(\varphi) \) is the firm’s demand for capital as market
entry and variable costs, \(x_{ij}(\phi)\) is country \(j\)’s demand for the firm’s variety, \(p_{ij}(\phi)\) is country \(j\)’s demand price of the firm’s variety, and \(\tau_{ij} (\geq 1)\) is the iceberg trade cost factor of delivering one unit of a variety from country \(i\) to country \(j\), with \(\tau_i = 1\). We regard \(\tau_{ij}\), country \(j\)’s iceberg import trade cost from country \(i\), as the only policy variables in this paper. We assume away import tariffs and the accompanying tariff revenues because we explore the possibility of reverse trade diversion without relying on the tariff complementarity story based on the optimal tariffs. The profit-maximizing supply price for each source-destination pair \((i, j), i, j = 1, ... , N\) is given by:

\[
(p'_{ij}(\phi) - r_i/\phi) / p_{ij}(\phi) = 1/\sigma \Leftrightarrow p'_{ij}(\phi) = r_i/(\phi \sigma) \forall j.
\]  

(5)

Then the corresponding revenue and profit are calculated as:

\[
e_{ij}(\phi) \equiv p'_{ij}(\phi)g_{ij}(\phi) = (\tau_{ij} r_i)^{1-\sigma} (\alpha \phi P_j)^{\sigma - 1} E_j,
\]

\[
\pi_{ij}(\phi) = e_{ij}(\phi)/\sigma - r_i K_i f_{ij} = (\tau_{ij} r_i)^{1-\sigma} (\alpha \phi P_j)^{\sigma - 1} E_j / \sigma - r_i K_i f_{ij}.
\]

A firm in country \(i\) survives (i.e., makes a nonnegative profit) in country \(j\) if and only if \(\phi \geq \phi_{ij}\), where the productivity cutoff \(\phi_{ij}\) is determined by the following zero cutoff profit condition:

\[
\pi_{ij}(\phi_{ij}) = 0 \Leftrightarrow e_{ij}(\phi_{ij}) = (\tau_{ij} r_i)^{1-\sigma} (\alpha \phi_{ij} P_j)^{\sigma - 1} E_j = \sigma r_i K_i f_{ij}.
\]  

(6)

From Eq. (6), we obtain \(\phi_{ij}/\phi_{ii} = (P_i/P_j)(E_i/E_j)^{1/(\sigma - 1)} \tau_{ij}(f_{ij}/f_{ii})^{1/(\sigma - 1)}\). Following Melitz (2003), it is assumed that the variable and fixed export costs are so high that not all domestic firms in country \(i\) also export to foreign country \(j( \neq i)\):

\[
\phi_{ij}/\phi_{ii} > 1, j \neq i.
\]

Let \(\tilde{\phi}_{ij}(\phi_{ij}) \equiv (\int_{\phi_{ij}}^{\infty} \phi^{\sigma - 1} \mu_{ij}(\phi|\phi_{ij})d\phi)^{1/(\sigma - 1)} \geq 1\) be the aggregate productivity of firms in country \(i\) surviving in country \(j\), where \(\mu_{ij}(\phi|\phi_{ij}) = g_i(\phi)/(1 - G_i(\phi_{ij}))\) is the density of \(\phi\) conditional on survival. Considering that \(e_{ij}(\phi) = (\phi/\phi_{ij})^{\sigma - 1} e_{ij}(\phi_{ij}) = (\phi/\phi_{ij})^{\sigma - 1} \sigma r_i K_i f_{ij}\) from Eq. (6), the (unconditional) expected revenue and profit are given by:

\[
\int_{\phi_{ij}}^{\infty} e_{ij}(\phi) g_i(\phi)d\phi = (1 - G_i(\phi_{ij})) \int_{\phi_{ij}}^{\infty} e_{ij}(\phi) \mu_{ij}(\phi|\phi_{ij})d\phi = (1 - G_i(\phi_{ij})) (h_{ij}(\phi_{ij}) + 1) \sigma r_i K_i f_{ij},
\]  

(7)

\[
\int_{\phi_{ij}}^{\infty} \pi_{ij}(\phi) g_i(\phi)d\phi = (1 - G_i(\phi_{ij})) \int_{\phi_{ij}}^{\infty} \pi_{ij}(\phi) \mu_{ij}(\phi|\phi_{ij})d\phi = H_{ij}(\phi_{ij}) r_i K_i f_{ij};
\]

\[
h_{ij}(\phi_{ij}) \equiv (\tilde{\phi}_{ij}(\phi_{ij})/\phi_{ij})^{\sigma - 1} - 1 > 0,
\]

\[
H_{ij}(\phi_{ij}) \equiv (1 - G_i(\phi_{ij})) h_{ij}(\phi_{ij}) > 0.
\]

The expected profit of a firm in country \(i\) surviving in country \(j\) is expressed as its fixed market entry cost \(r_i K_i f_{ij}\) times the multiplier \(H_{ij}(\phi_{ij})\), which is decreasing in \(\phi_{ij}\) (see Appendix A for proof). This is because an increase in \(\phi_{ij}\) makes it less likely for the firm to survive in market \(j\). The free entry condition is given by \(r_i K_i f_{ij} = \sum_j \int_{\phi_{ij}}^{\infty} \pi_{ij}(\phi) g_i(\phi)d\phi\), which is simplified to:
Eq. (8) implies that, whenever more inefficient firms exit from their domestic market (i.e., $\phi_{ii}$ increases), more efficient firms enter their export markets (i.e., $\phi_{ij}$ decreases for some $j \neq i$). Conversely, whenever more efficient firms enter their export markets (i.e., $\phi_{ij}$ decreases for all $j \neq i$), more inefficient firms exit from their domestic market (i.e., $\phi_{ii}$ increases). In other words, more domestic selection implies more exports, and vice versa.

Finally, let $M^e_i$ represent the mass of entrants in country $i$. Then the mass of entrants in country $i$ surviving in country $j$, or the mass of varieties country $i$ sells to country $j$, is expressed as $M_{ij} = M^e_i (1 - G_i(\phi_{ij}))$.

### 2.4 Markets

The market-clearing conditions for the final good, capital, and intermediate goods are given by, respectively:

\begin{align*}
Y_i = C_i + \dot{K}_i + \delta_i K_i, & \quad i = 1, \ldots, N, \quad (9) \\
K_i = \sum_{j} M_{ij} \int_{0}^{\infty} k_{ij}(\varphi) \mu_{ij}(\varphi | \varphi_{ij}) d\varphi + M^e_i K_i f^e_i, & \quad i = 1, \ldots, N, \quad (10) \\
y_{ij}(\varphi) = r_{ij} x_{ij}(\varphi), & \quad i, j = 1, \ldots, N. \quad (11)
\end{align*}

Summing up the household budget constraint (1), the zero profit condition in the final good sector (4), and the free entry condition in the intermediate good sector (8), we can derive Walras’ law in country $i$:

\[ 0 = p_i^Y (C_i + \dot{K}_i + \delta_i K_i - Y_i) + r_i (\sum_{j} M_{ij} \int_{0}^{\infty} k_{ij}(\varphi) \mu_{ij}(\varphi | \varphi_{ij}) d\varphi + M^e_i K_i f^e_i - K_i) + \sum_{j} M_{ji} \int_{0}^{\infty} p^i_j(\varphi) r_{ji} x_{ji}(\varphi) \mu_{ji}(\varphi | \varphi_{ji}) d\varphi - \sum_{j} M_{ij} \int_{0}^{\infty} p^i_j(\varphi) y_{ij}(\varphi) \mu_{ij}(\varphi | \varphi_{ij}) d\varphi. \]

This has two implications. First, summing it up for all $i$ yields Walras’ law in the world, meaning that any one of the $N(N + 2)$ market-clearing conditions is redundant, and that any one of the $N(N + 2)$ prices can be normalized to unity. Second, substituting Eqs. (9) to (11) into Walras’ law in country $i$, we obtain:

\[ \sum_{j} M_{ij} \int_{0}^{\infty} e_{ij}(\varphi) \mu_{ij}(\varphi | \varphi_{ij}) d\varphi = \sum_{j} M_{ji} \int_{0}^{\infty} e_{ji}(\varphi) \mu_{ji}(\varphi | \varphi_{ji}) d\varphi = E_i, \]

\[ \sum_{j \neq i} M_{ij} \int_{0}^{\infty} e_{ij}(\varphi) \mu_{ij}(\varphi | \varphi_{ij}) d\varphi = \sum_{j \neq i} M_{ji} \int_{0}^{\infty} e_{ji}(\varphi) \mu_{ji}(\varphi | \varphi_{ji}) d\varphi, j \neq i. \]

The first line simply says that country $i$’s total revenue from selling the intermediate goods to all destinations is equal to its total expenditure for buying the intermediate goods from all sources. The second line shows country $i$’s zero balance of trade, which is obtained by subtracting country $i$’s domestic revenue and expenditure from both sides of the first line.
2.5 Dynamic system

Let capital in the last country \( N \) be the numeraire: \( r_N \equiv 1 \), and let \( \kappa_i \equiv K_i/K_N \) be the relative supply of capital in country \( i \) to the last country \( N \). Dividing the zero cutoff profit condition (6) by itself with \( i = j \), and solving the resulting equation for \( \varphi_{ij}/\varphi_{jj} \), we obtain:

\[
\varphi_{ij}/\varphi_{jj} = (v_i/v_j)\tau_{ij}(f_{ij}/f_{jj})^{1/(\sigma-1)}; v_i \equiv (r_i^\kappa_i)^{1/(\sigma-1)}, j \neq i.
\]  

(12)

An increase in \( \varphi_{ij}/\varphi_{jj} \) means that country \( i(j \neq j) \) becomes relatively less competitive in country \( j \) in the sense that relatively fewer firms from the former can survive in the latter. \( \varphi_{ij}/\varphi_{jj} \) is larger: (i) the larger \( \tau_{ij} \) is; (ii) the larger \( r_i \) is relative to \( r_j \); and/or (iii) the larger \( \kappa_i \) is relative to \( \kappa_j \). The most important to the present model is case (iii): faster growth in a country makes that country relatively less competitive by increasing its fixed market entry costs. Combining the relative competitiveness condition (12) with the free entry condition (8), all productivity cutoffs can be solved as:

\[
\varphi_{ij} = \varphi_{ij}(\{r_k\}_{k=1}^N, \{\tau_{kl}\}_{l=1}^N), i, j = 1, ..., N.
\]  

(13)

Together with Eq. (13), the dynamic system is concisely expressed as (see Appendix B for derivations):

\[
\begin{align*}
\sum_{j \neq i} \beta_{ij} r_i \kappa_i &= \sum_{j \neq i} \beta_{ji} r_j \kappa_j, i = 1, ..., N - 1, \\
\kappa_i/\kappa_i &= \gamma_i - \gamma_N, i = 1, ..., N - 1;
\end{align*}
\]

(14)

\[
\beta_{ij} = (H_{ij}(\varphi_{ij}) + 1 - G_i(\varphi_{ij}))f_{ij}/\sum_k(H_{ik}(\varphi_{ik}) + 1 - G_i(\varphi_{ik}))f_{ik},
\]

(16)

\[
\gamma_i = 1/q_i - \delta_i - \rho_i,
\]

(17)

\[
q_i = \left\{ \sum_k M_{ji}[(\tau_{ji} r_j/r_i)/(\alpha \varphi_{ji}(\varphi_{ij}))]^{1-\sigma} \right\}^{1/(1-\sigma)}
\]

(18)

\[
M_{ij} = (1/\sigma)(1 - G_i(\varphi_{ij}))/\sum_k(H_{ik}(\varphi_{ik}) + 1 - G_i(\varphi_{ik}))f_{ik}.
\]

(19)

Eq. (14) is country \( i \)'s zero balance of trade, which determines \( r_i \). Eq. (15) gives the growth rate of \( \kappa_i \), which is just the difference between the growth rates of countries \( i \) and \( N \). In Eq. (14), \( \beta_{ij} \equiv M_{ij} \int_0^\infty e_{ij}(\varphi \mu_{ij}(\varphi \varphi_{ij})d\varphi / \sum_k M_{ik} \int_0^\infty e_{ik}(\varphi \mu_{ik}(\varphi \varphi_{ki})d\varphi \in [0, 1]; \sum_j \beta_{ij} = 1 \), is the revenue share of varieties country \( i \) sells to country \( j \).\(^5\) Eq. (16) says that \( \beta_{ij} \) depends only on country \( i \)'s cutoffs in all markets.\(^6\) Eq. (18) is country \( i \)'s intermediate good price index \( P_i \), from Eq. (3) divided by its rental rate \( r_i \), and \( 1/q_i = r_i/P_i \) is equal to country \( i \)'s gross rate of return to capital \( r_i/p_i^N \) from Eq. (4). This implies that the Euler equation (2) is rewritten as Eq. (17). Eq. (19) means that \( M_{ij} \), the mass of varieties country \( i \) sells to country \( j \), depends only on country \( i \)'s cutoffs in all markets. With the initial condition \( \{\kappa_0\}_{i=1}^N \) and the iceberg trade costs \( \{\tau_{ij}\}_{i,j=1}^N \) given, Eqs. (13), (14), and (15) characterize an equilibrium path \( \{\varphi_{ij}\}_{i,j=1}^N, \{r_i\}_{i=1}^N, \{\kappa_i\}_{i=1}^N \).

\(^5\)The expenditure share of varieties country \( i \) buys from country \( j \) is defined as \( \varsigma_{ij} \equiv M_{ji} \int_0^\infty e_{ji}(\varphi \mu_{ji}(\varphi \varphi_{ij})d\varphi / \sum_j M_{ij} \int_0^\infty e_{ij}(\varphi \mu_{ij}(\varphi \varphi_{ij})d\varphi \in [0, 1]; \sum_j \varsigma_{ij} = 1 \). Although it is not necessarily true that \( \beta_{ij} = \varsigma_{ji} \) in a multi-country setting, country \( i \)'s zero balance of trade implies that \( \sum_j \beta_{ij} = \sum_j \varsigma_{ji} \) or \( \beta_{ji} = \varsigma_{ij} \).

\(^6\)Using Eqs. (7) and (19), \( \varsigma_{ij} \) is rewritten as \( \varsigma_{ij} = \sum_k [(H_{ij}(s_{ij})+1-G_i(s_{ij}))f_{ij}/\sum_j(H_{ij}(s_{ij})+1-G_i(s_{ij}))f_{ij}]r_i \kappa_i \), which cannot be simplified any more. Compared with this, \( \beta_{ij} \) has a much simpler functional form. This is why we express the dynamic system in terms of the revenue shares.
A balanced growth path (BGP) is defined as a path along which all variables grow at constant (including zero) rates. A BGP is determined by Eqs. (13), (14), (15), and $\dot{\kappa}_i / \kappa_i = 0$. We call the common growth rate at a BGP $\gamma_i^* = \ldots = \gamma_N^*$ “the balanced growth rate”, where an asterisk represents a BGP.

2.6 Cutoffs, masses and revenue shares of varieties, and growth rates

Our main variables of interest are $M_{ij}$ (mass of varieties country $i$ sells to country $j$), $\beta_{ij}$ (revenue share of varieties country $i$ sells to country $j$), and $\gamma_i$ (country $i$'s growth rate). Although their expressions (19), (16), and (17), respectively, look quite complicated, the rate or amount of change in each variable can be tied to the rate of change in a single productivity cutoff under a commonly used productivity distribution:

Lemma 1.

1. The amount of change in country $i$’s growth rate is given by:

$$d\gamma_i = (1/q_i) \frac{\hat{\varphi}_{ii}}{\varphi_{ii}} \equiv d\ln \varphi_{ii} \equiv d\varphi_{ii}/\varphi_{ii}. \quad (20)$$

2. Suppose that $\varphi$ is Pareto distributed with a country-specific scale parameter $b_i (> 0)$ and a common shape parameter $\theta (> \sigma - 1)$: $G_i(\varphi) = 1 - (b_i/\varphi)^{\theta} = 1 - b_i^{\theta} \varphi^{-\theta}; \varphi \in [b_i, \infty)$. Then the rates of changes in $M_{ij}$ and $\beta_{ij}$ are given by:

$$\hat{M}_{ij} = -\theta \hat{\varphi}_{ij}, \quad (21)$$
$$\hat{\beta}_{ij} = -\theta \hat{\varphi}_{ij}. \quad (22)$$

Proof. See Appendix C. ■

Eq. (20) results from the zero cutoff profit condition (6) for domestic sales: an increase in $\varphi_{ii}$ means that fewer most productive domestic firms survive in country $i$, who can tolerate a higher gross rate of return to capital $r_i/p_i Y = r_i/P_i = 1/q_i$ in the Euler equation (17). Eqs. (21) and (22) follow from the free entry condition (8): noting that $(H_{ij}(\varphi_{ij}) + 1 - G_i(\varphi_{ij}))/H_{ij}(\varphi_{ij}) = \theta/\sigma - 1 \forall i, j$ under the assumed Pareto distribution, the denominators of Eqs. (19) and (16) are constant. This ensures that, whenever $\varphi_{ij}$ decreases, an entrant from country $i$ earn more expected profit in market $j$, thereby increasing both $M_{ij}$ and $\beta_{ij}$. Finally, Eqs. (20) and (22) imply:

$$d\gamma_i = -[1/(\theta q_i)] \hat{\beta}_{ii}. \quad (23)$$

This is a growth version of the ACR formula in Arkolakis et al. (2012): country $i$ grows faster if and only if it becomes more open in terms of its domestic revenue (and expenditure) share.

3 Three-country model

3.1 General case

To examine the dynamic effects of preferential trade liberalization in the simplest possible setting, we focus on the three-country case: $N = 3$. Eqs. (13), (14), and (15) can be reduced to a two-dimensional autonomous
dynamic system in $\kappa_1$ and $\kappa_2$ as follows: (i) substituting the cutoff function (13), together with Eq. (22) and $v_i = (r_i^\sigma \kappa_i)^{1/(\sigma - 1)}$, into the zero balance of trade condition (14), we solve for $r_1$ and $r_2$ in terms of $\kappa_1, \kappa_2$, and the iceberg trade costs; (ii) substituting the result in step (i) back into $v_i = (r_i^\sigma \kappa_i)^{1/(\sigma - 1)}$, Eqs. (13), and (20), we solve for $\gamma_i$ in terms of $\kappa_1, \kappa_2$, and the iceberg trade costs; (iii) substituting the result in step (ii) into Eq. (15), we obtain a system of differential equations in $\kappa_1$ and $\kappa_2$, with the iceberg trade costs given. The effects of any trade liberalization on the paths of endogenous variables can in principle be characterized in the reduced dynamic system.

Appendix D derives a system of equations in $\hat{r}_1$ and $\hat{r}_2$ in step (i). Unfortunately, we cannot generally determine the sign of any coefficient even at this early stage. To obtain meaningful analytical results, we will evaluate the coefficients at the symmetric BGP in the rest of this paper (see section 6.3 and Appendix I for the case of an asymmetric old BGP).

### 3.2 Growth functions around the symmetric balanced growth path

Suppose that all parameters are symmetric across countries at the old BGP. Specifically, let $\rho_i = \rho, \delta_i = \delta, f_i^* = f^*, b_i = b, \tau_{ij} = 1$ for $j = i$; $\tau_{ij} = \tau(\geq 1)$ for $j \neq i$, and $f_{ij} = f_d$ for $j = i$; $f_{ij} = f_x(> f_d)$ for $j \neq i$. A candidate for an equilibrium at the symmetric BGP is given by $r_1^* = 1, \kappa_1^* = 1$, and $\varphi_{ij}^* = \varphi_x$ for $j \neq i$. From Appendix C and Eq. (16), we obtain:

$$1/q_i^* = (\sigma f_d)^{(1-\sigma)} \alpha \varphi_d \equiv 1/q \forall i,$$

$$\beta_{ij}^* = \varphi_x^{-\theta} f_x/(\varphi_d^{-\theta} f_d + 2 \varphi_x^{-\theta} f_x) \equiv \beta_i, j, j \neq i,$$

$$\beta_i^* = 1 - \sum_{j \neq i} \beta_{ij}^* = 1 - 2 \beta_i.$$

It can be easily verified that the above candidate for an equilibrium satisfies Eqs. (14), (15), and $\dot{\kappa}_i/\kappa_i = 0$, and hence constitutes the symmetric BGP. Moreover, since $\varphi_x/\varphi_d = \tau(f_x/f_d)^{1/(\sigma - 1)} > 1$ from Eq. (12) implies that $(\varphi_x/\varphi_d)^{-\theta} f_x/f_d = \tau^{-\theta}(f_x/f_d)^{-\lambda/(\sigma - 1)} < 1$, we have $\beta < 1/3$.

Starting from the symmetric BGP, the system of equations in $\hat{r}_1$ and $\hat{r}_2$ in step (i) mentioned in section 3.1 can be solved as (see Appendix E for derivations):

$$\hat{r}_1 = \{1/(3a)\} \{3c\hat{\kappa}_1 + \theta(\sigma - 1)\hat{\tau}_{12} - \hat{\tau}_{32} + (2 - 3/\beta)(\hat{\tau}_{13} - \hat{\tau}_{31}) + (1 - 3/\beta)(\hat{\tau}_{23} - \hat{\tau}_{21})\}, \tag{23}$$

$$\hat{r}_2 = \{1/(3a)\} \{3c\hat{\kappa}_2 + \theta(\sigma - 1)\hat{\tau}_{21} - \hat{\tau}_{31} + (2 - 3/\beta)(\hat{\tau}_{23} - \hat{\tau}_{32}) + (1 - 3/\beta)(\hat{\tau}_{13} - \hat{\tau}_{12})\}; \tag{24}$$

$$a \equiv -\{(\sigma - (\sigma - 1))(1 - 3/\beta + \theta \sigma) < 0, c \equiv [\theta - (\sigma - 1)](1 - 3/\beta + \theta) > 0.$$

To interpret Eq. (23), we look at the zero balance of trade condition (14) for country 1: $(1 - \beta_{11})r_1\kappa_1 = \beta_{21}r_2\kappa_2 + \beta_{31}$, together with Eqs. (12) and (22). An increase in $\kappa_1$ increases $v_1 = (r_i^\sigma \kappa_i)^{1/(\sigma - 1)}$, ceteris paribus. Since this discourages exports from country 1 to the other countries but encourages exports the other way around, country 1’s balance of trade tends to be negative. For that to get back to zero, $r_1$ decreases so that country 1 can export more and import less. When country 1 liberalizes its imports from country 2 (i.e., $\tau_{21}$ decreases), country 1’s balance of trade also tends to be negative, implying that $r_1$ should decrease as well. When country 2 liberalizes its imports from country 1 (i.e., $\tau_{12}$ decreases), country 1 tends to run a trade surplus, which should be cleared through an increase in $r_1$. Eq. (24) can be interpreted similarly.
Proceeding to step (ii), the totally differentiated forms of countries’ growth functions are obtained as (see Appendix E for derivations):

\[
\begin{align*}
    d\gamma_1 &= (1/q)(\beta/a)((\sigma - 1)(2\hat{\kappa}_1 - \hat{\kappa}_2) + [\sigma\theta - (\sigma - 1)][(1 - \beta)(\hat{\tau}_{12} + \hat{\tau}_{13}) - \beta(\hat{\tau}_{21} + \hat{\tau}_{32})] + [\sigma\theta(1 - \beta) + (\sigma - 1)\beta](\hat{\tau}_{21} + \hat{\tau}_{31})],
    \\
    d\gamma_2 &= (1/q)(\beta/a)((\sigma - 1)(2\hat{\kappa}_2 - \hat{\kappa}_1) + [\sigma\theta - (\sigma - 1)][(1 - \beta)(\hat{\tau}_{21} + \hat{\tau}_{23}) - \beta(\hat{\tau}_{13} + \hat{\tau}_{31})] + [\sigma\theta(1 - \beta) + (\sigma - 1)\beta](\hat{\tau}_{12} + \hat{\tau}_{32})],
    \\
    d\gamma_3 &= (1/q)(\beta/a)(-(\sigma - 1)(\hat{\kappa}_1 + \hat{\kappa}_2) + [\sigma\theta - (\sigma - 1)][(1 - \beta)(\hat{\tau}_{31} + \hat{\tau}_{32}) - \beta(\hat{\tau}_{12} + \hat{\tau}_{21})] + [\sigma\theta(1 - \beta) + (\sigma - 1)\beta](\hat{\tau}_{13} + \hat{\tau}_{23})].
\end{align*}
\]

Leaving aside the iceberg trade costs until the next section, Eqs. (25) to (27) reveal the convergence mechanism at work. In country 1, for example, an increase in \(\kappa_1\) increases \(v_1 = (r_1^0\kappa_1)^{-1/(\sigma - 1)}\) even if \(r_1\) decreases (see Eq. (E.1)). Since this discourages exports from country 1 to the other countries (see Eq. (12)), more inefficient firms remain in its domestic market (see Eq. (8)), which is bad for its growth (see Eq. (20)). In short, a country’s faster growth from past to present slows down its growth from present to future.

The consequence of the above convergence mechanism becomes apparent in step (iii): substituting Eqs. (25) to (27) with \(\hat{\tau}_{ij} = 0\) into the totally differentiated form of Eq. (15), we obtain:

\[
\begin{align*}
    d(d\ln\kappa_1/dt) &= d\gamma_1 - d\gamma_3 = (1/q)(\beta/a)(\sigma - 1)3\hat{\kappa}_1 = d(d\ln\kappa_1/dt)/d\ln\kappa_1 = (3/q)(\beta/a)(\sigma - 1) < 0, \\
    d(d\ln\kappa_2/dt) &= d\gamma_2 - d\gamma_3 = (1/q)(\beta/a)(\sigma - 1)3\hat{\kappa}_2 = d(d\ln\kappa_2/dt)/d\ln\kappa_2 = (3/q)(\beta/a)(\sigma - 1) < 0.
\end{align*}
\]

This means that our reduced dynamic system is stable: for any initial condition \((\kappa_{10}, \kappa_{20})\) around the symmetric BGP: \((\kappa_1^*, \kappa_2^*) = (1, 1)\), the world economy converges to it in the long run.

4 Growth effects of trade liberalization

4.1 Short-run effects

Suppose that the world economy is originally on the symmetric BGP, and that one country decreases its import trade cost from another country from \(t = 0\) on. The short-run growth effects can be seen by setting \(\hat{\kappa}_1 = \hat{\kappa}_2 = 0\) in Eqs. (25) to (27):

**Proposition 1** Starting from the symmetric BGP, a permanent decrease in the import trade cost of country \(j\) from country \(i(\neq j)\) increases the growth rates of both countries \(i\) and \(j\), but decreases that of country \(k(\neq i, j)\), in the short run.

Suppose that country 1 liberalizes its imports from country 2, that is, \(\tau_{21}\) decreases. This encourages country 2’s exports to country 1, thereby inducing more domestic selection and faster growth for country 2. For the other countries, exporting to country 2 becomes tougher, causing less domestic selection and slower growth. These are the direct effects, with the rental rates as well as the capital stocks given.

In fact, the rental rates are adjusted to clear countries’ balances of trade. Since country 1 tends to run a trade deficit whereas country 2 tends to run a trade surplus, country 1’s capital becomes cheaper whereas
that of country 2 becomes more expensive (see Eqs. (23) and (24)). This makes country 1 relatively more competitive in both of its export markets. The effects are mixed for country 3: the increase in $r_2$ is good for its exporters to country 2, whereas the decrease in $r_1$ is bad for its exporters to country 1. Because of the difference in the indirect effects, country 1 grows faster as well as country 2, whereas country 3 grows more slowly, than the old BGP.

Proposition 1 implies that, in the three-country setting, a regional trade agreement (RTA) (i.e., preferential trade liberalization) between two countries raises growth of the member countries but lowers growth of the nonmember country in the short run. It can be pointed out that the short-run growth effects discussed here correspond to the welfare effects in the static version of our model, where $\dot{K}_i + \delta_i K_i = 0$.

The fact that the growth rate of the nonmember country falls in the short run means that its welfare falls in the static Melitz model in line with the literature on RTAs. However, we cannot predict the direction of change in the nonmember’s welfare in our dynamic model until we evaluate the long-run growth effect.

4.2 Long-run effects

In the long run, $\kappa_1$ and $\kappa_2$ are adjusted so that the growth rates are equalized across countries. Substituting Eqs. (25) to (27) into $0 = d\gamma_i - d\gamma_3$ from Eq. (15), we obtain:

\begin{align*}
\hat{\kappa}_1^* &= -\{1/[3(\sigma - 1)]\} \{(\sigma \theta - (\sigma - 1)(\hat{\tau}_{12} - \hat{\tau}_{32}) + \sigma \theta (\hat{\tau}_{21} - \hat{\tau}_{23}) + (\sigma - 1)(\hat{\tau}_{31} - \hat{\tau}_{13})\}, \\
\hat{\kappa}_2^* &= -\{1/[3(\sigma - 1)]\} \{(\sigma \theta - (\sigma - 1)(\hat{\tau}_{21} - \hat{\tau}_{31}) + \sigma \theta (\hat{\tau}_{12} - \hat{\tau}_{13}) + (\sigma - 1)(\hat{\tau}_{32} - \hat{\tau}_{23})\}.
\end{align*}

Substituting Eqs. (28) and (29) back into Eq. (27), the long-run growth effects are simply given by:

\begin{equation}
d\gamma_1^* = d\gamma_2^* = d\gamma_3^* = -[\beta/(3q)](\hat{\tau}_{12} + \hat{\tau}_{13} + \hat{\tau}_{21} + \hat{\tau}_{23} + \hat{\tau}_{31} + \hat{\tau}_{32}).
\end{equation}

Proposition 2. Starting from the symmetric BGP, a permanent decrease in the import trade cost of country $j$ from country $i (\neq j)$ increases the growth rates of all countries in the long run.

Proposition 2 says that trade liberalization anywhere increases the balanced growth rate. To see what drives this result, we again consider a decrease in $r_{21}$. Since this increases the growth rates of countries 1 and 2 but decreases that of country 3 from Proposition 1, $\kappa_1$ and $\kappa_2$ start to increase (see Eqs. (28) and (29)). This continues to slow down the growth rates of countries 1 and 2 but speed up that of country 3. On the new BGP, not only countries 1 and 2 but also country 3 grow faster than the old BGP. The convergence mechanism working through the fixed market entry costs is responsible for the result.

Proposition 2 has an important implication for the welfare effects of trade liberalization. The fact that an RTA between countries 1 and 2 decreases country 3’s growth rate in the short run but increases that in the long run means that country 3’s new consumption path is lower than the old BGP in the early periods but the former overtakes the latter in a certain period of time. Consequently, unlike the existing literature on RTAs, even the nonmember country gains from the RTA in our dynamic Melitz model as long as its subjective discount rate is sufficiently small.

\footnote{Country $i$’s consumption in period $t$ is given by $C_{it} = \rho_i K_{it} = \rho_i K_{i0} \exp(\int_{s}^{t} \gamma_i ds)$.}
5 Effects of a regional trade agreement on revenue shares

Suppose that, starting from the symmetric BGP, countries 1 and 2 make an RTA: \( \hat{\tau}_{12} = \hat{\tau}_{21} < 0, \hat{\tau}_{13} = \hat{\tau}_{23} = \hat{\tau}_{31} = \hat{\tau}_{32} = 0 \). To examine its effects on revenue shares, Eq. (22) implies that we only have to see the directions of changes in \( \phi_{ij} \), which are analytically determined in Appendix F.

Table 1 summarizes the results. In the short run, we first observe that \( \beta_{21} \) increases but \( \beta_{11} \) and \( \beta_{31} \) decrease, meaning that both trade creation (i.e., \( \beta_{11} \) is replaced by \( \beta_{21} \)) and trade diversion (i.e., \( \beta_{31} \) is replaced by \( \beta_{21} \)) occur in terms of revenue shares in market 1 (the same is true for market 2). A decrease in \( \beta_{11} \) can be understood by remembering the growth version of the ACR formula and Proposition 1. \( \beta_{21} \) increases because the direct export-enhancing effect of the RTA outweighs its indirect effect through tougher competition with more efficient domestic firms in country 1. The last effect also partly decreases \( \beta_{31} \), but the resulting increase in \( r_1 \) (and hence \( v_1 \)) partly increases \( \beta_{31} \) by making country 3 more competitive relative to country 1. Since the former effect is stronger than the latter in the short run, \( \beta_{31} \) decreases as a result. Finally, both \( \beta_{13} \) and \( \beta_{23} \) decrease because increases in \( r_1 \) and \( r_2 \) (and hence \( v_1 \) and \( v_2 \)) make countries 1 and 2 less competitive relative to country 3, which outweighs the counteracting effects through easier competition with less efficient domestic firms in country 3.

In the long run, three out of nine revenue shares, \( \beta_{31}, \beta_{32}, \) and \( \beta_{33} \), move in the opposite directions of their short-run effects. Remarkably, the fact that \( \beta_{31} \) and \( \beta_{32} \) increase as well as \( \beta_{21} \) and \( \beta_{12} \) means that both trade creation and reverse trade diversion occur at the same time. This is because faster growth in countries 1 and 2 than country 3 in the short run starts to increase \( \kappa_1 \) and \( \kappa_2 \) (and hence \( v_1 \) and \( v_2 \)), which intensifies the second positive effect on \( \beta_{31} \) mentioned in the previous paragraph. Here again, the convergence mechanism working through the fixed market entry costs plays a key role.

**Proposition 3** Starting from the symmetric BGP, permanent decreases in the import trade costs of countries \( j \) and \( i(\neq j) \) from each other by the same rate decrease the revenue shares of varieties country \( k(\neq i,j) \) sells to countries \( i \) and \( j \) in the short run, but increase them in the long run.

Proposition 3 provides a new theoretical explanation for recent empirical evidence of reverse trade diversion based on the gravity models (e.g., Lee and Shim, 2006; Magee, 2008; Acharya et al., 2011; Baldwin, 2011). Our result is strong because it is caused only by reductions in internal trade costs, not by additional policy adjustments applied to external trade (e.g., Richardson, 1993; Bagwell and Staiger, 1999; Bond et al., 2004; Ornelas, 2005; Saggi and Yildiz, 2010). By incorporating endogenous growth with capital accumulation and transitional dynamics into an asymmetric Melitz model, we have shown that trade diversion in a static setting is reversed in the long run.

6 Discussions

6.1 Quantitative comparison of short- and long-run effects

We have found that, starting from the symmetric BGP, an RTA between countries 1 and 2 decreases \( \gamma_3, \beta_{31}, \) and \( \beta_{32} \) in the short run, but increases them in the long run. Then, which are larger, the long-run increases or the short-run decreases? Appendix G shows that, for all three variables, the long-run increases are greater than the short-run losses. This implies that the short-run negative effects of the RTA on the nonmember country are quantitatively minor compared with the long-run positive effects. This further supports our argument that the RTA can increase the nonmember’s welfare in our dynamic Melitz model.
6.2 Expenditure shares

Although we discuss the effects of an RTA on revenue shares in section 5, one might also be interested in what happens for \( \zeta_{ij} \equiv M_{ij} \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi) \mu_{ij}(\varphi|\varphi_{ij}) d\varphi / \sum_k M_{kj} \int_{\varphi_{kj}}^{\infty} e_{kj}(\varphi) \mu_{kj}(\varphi|\varphi_{kj}) d\varphi \in [0, 1]; \sum_i \zeta_{ij} = 1 \), the expenditure share of varieties country \( j \) buys from country \( i \). Appendix H shows that, starting from the symmetric BGP, permanent decreases in \( \tau_{21} \) and \( \tau_{12} \) by the same rate: (i) move \( \zeta_{ij} \) in the same direction as \( \beta_{ij} \) for all \( i \) and \( j \) in the short run; (ii) decrease \( \zeta_{31} \) and \( \zeta_{32} \), but increase \( \zeta_{13} \) and \( \zeta_{23} \), in the long run unlike the revenue shares. The most interesting is that, in the long run, an RTA between countries 1 and 2 causes normal trade diversion in terms of the members’ expenditure shares from the nonmember. This implies that we observe reverse trade diversion only in terms of \( \beta_{31} \) and \( \beta_{32} \), the nonmember’s revenue shares to the members. Still, considering that it is the nonmember country that is mostly affected by (normal or reverse) trade diversion, it is more appropriate to use the revenue shares from the nonmember’s perspective as indicators of (normal or reverse) trade diversion.

6.3 Asymmetric old BGP

One might wonder if Propositions 1 to 3 are valid when an old BGP is asymmetric. We can generally say that they are true by continuity as long as the old BGP is sufficiently close to the symmetric one. Then how far from symmetry can it be? Appendix I provides some numerical examples, where Propositions 1 to 3 continue to hold even if the GDP of the largest country is more than seven times as large as the smallest country at the old BGP. This suggests that our analytical results obtained by starting from the symmetric BGP applies to sufficiently asymmetric countries.

7 Concluding remarks

In contrast to the conventional wisdom among trade theorists, in reality RTAs often encourage nonmember countries to export more to member countries. Although such a phenomenon called reverse trade diversion can be theoretically explained by tariff complementarity, the hypothesis is not consistently supported by empirical evidence. By formulating an asymmetric three-country Melitz model of trade and endogenous growth with transitional dynamics, we provide an alternative theoretical explanation for reverse trade diversion. The fact that an RTA benefits the members and harms the nonmember in the short run starts to increase the former’s market entry costs more than the latter. This makes the nonmember relatively more competitive, exporting more to the members and growing faster in the long run. The main message of this paper is that we may not have to worry about trade diversion from a long-run viewpoint.
Appendix A. Elasticity of $H_{ij}(\varphi_{ij})$

Let $h_{ij}(\varphi_{ij}) = (\bar{\varphi}_{ij}(\varphi_{ij})/\varphi_{ij})^{\sigma-1} - 1 = N_{ij}(\varphi_{ij})/\varphi_{ij}^{\sigma-1} - 1$, where $N_{ij}(\varphi_{ij}) \equiv \bar{\varphi}_{ij}(\varphi_{ij})^{\sigma-1} = \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} h_{ij}(\varphi|\varphi_{ij})d\varphi = (\int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} g_{ij}(\varphi)d\varphi)/(1 - G_{i}(\varphi_{ij}))$. Differentiating $N_{ij}(\varphi_{ij})$ gives:

$$N'_{ij} = [\varphi_{ij}^{\sigma-1} g_{ij}/(1 - G_{ij})](-1 + N_{ij}/\varphi_{ij}^{\sigma-1}) = \varphi_{ij}^{\sigma-1} g_{ij}h_{ij}/(1 - G_{ij}) > 0;$$

$$g_{ij} \equiv g_{i}(\varphi_{ij}), G_{ij} \equiv G_{i}(\varphi_{ij}).$$

Using this, $h'_{ij}(\varphi_{ij})$ is calculated as:

$$h'_{ij} = g_{ij}h_{ij}/(1 - G_{ij}) - (h_{ij} + 1)(\sigma - 1)/\varphi_{ij}.$$  

Differentiating $H_{ij}(\varphi_{ij}) = (1 - G_{i}(\varphi_{ij}))h_{ij}(\varphi_{ij})$, and using the above expression, we obtain:

$$H'_{ij} = -(1 - G_{ij})(h_{ij} + 1)(\sigma - 1)/\varphi_{ij} < 0.$$  

Multiplying this by $\varphi_{ij}/H_{ij}(\varphi_{ij})$, the elasticity of $H_{ij}(\varphi_{ij})$ is expressed as:

$$H'_{ij}\varphi_{ij}/H_{ij} = -\eta_{ij} < 0; \eta_{ij} \equiv -d\ln H_{ij}/d\ln \varphi_{ij} \equiv [(h_{ij} + 1)/h_{ij}](\sigma - 1) > \sigma - 1 > 0. \quad (A.1)$$

Appendix B. Derivations of Eqs. (14) to (19)

Using Eqs. (7), (8), and $M_{ij} = [(1 - G_{i}(\varphi_{ij}))/1 - G_{i}(\varphi_{ii})]M_{ii}$, Eq. (10) is solved for $M_{ii}$. Substituting the result back into $M_{ij} = [(1 - G_{i}(\varphi_{ij}))/1 - G_{i}(\varphi_{ii})]M_{ii}$, we obtain Eq. (19). Using Eqs. (7) and (19), the revenue share of varieties country $i$ sells to country $j$: $\beta_{ij} \equiv M_{ij} \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi/\sum_{k} M_{ik} \int_{\varphi_{ik}}^{\infty} e_{ik}(\varphi)\mu_{ik}(\varphi|\varphi_{ik})d\varphi \in [0, 1]; \sum_{j} \beta_{ij} = 1$, is rewritten as Eq. (16). Using $M_{ij} \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi = \beta_{ij} E_{i}$ from the definition of $\beta_{ij}$, and $E_{i} = r_{i}K_{i}$ from Eqs. (1), (4), and (9), country $i$’s zero balance of trade is rewritten as Eq. (14). Eq. (14) for $i = N$ is redundant because it is implied from Eq. (14) for $i = 1, ..., N - 1$.

Using Eqs. (5) and the definition of $\bar{\varphi}_{ij}(\varphi_{ij})$, country $i$’s intermediate good price index:

$$P_{i} = \{\sum_{j} M_{ji} \int_{\varphi_{ij}}^{\infty} p_{ji}(\varphi)^{1-\sigma} \mu_{ji}(\varphi|\varphi_{ji})d\varphi\}^{1/(1-\sigma)}$$  

from Eq. (3), is simplified to:

$$P_{i} = \{\sum_{j} M_{ji} |\tau_{ji}r_{j}/(\alpha\bar{\varphi}_{ji}(\varphi_{ji}))|^{1-\sigma}\}^{1/(1-\sigma)}, \quad (B.1)$$

where $M_{ji}$ is given by Eq. (19). Considering that the simplified price index (B.1) is homogeneous of degree one in $\{\tau_{ji}r_{j}\}_{j=1}^{N}$, country $i$’s gross rate of return to capital $r_{i}/p_{i}^{Y}$ in the Euler equation (2) is rewritten as $r_{i}/p_{i}^{Y} = r_{i}/P_{i} = 1/q_{i}$, where Eq. (4) is used to obtain the first equality, and $q_{i}$ is given by Eq. (18). This immediately implies Eq. (17). Finally, time differentiating $\ln \kappa_{i} = \ln K_{i} - \ln K_{N}$, and using Eq. (17), we obtain Eq. (15).

Appendix C. Proof of Lemma 1

First, considering that $E_{i} = r_{i}K_{i}$ and $r_{i}/P_{i} = 1/q_{i}$, the zero cutoff profit condition (6) for $j = i$ is rewritten as $1/q_{i} = (\sigma f_{ii})^{1/(1-\sigma)}\alpha\varphi_{ii}$, implying that:
Logarithmically differentiating Eq. (12) gives:

$$\hat{q}_i = -\hat{\varphi}_i; \hat{q}_i \equiv d\ln q_i \equiv dq_i/q_i.$$  \hfill (C.1)

Totally differentiating Eq. (17), and using Eq. (C.1), we obtain Eq. (20).

Using Eqs. (16) and (A.1), the logarithmically differentiated form of Eq. (8) is obtained as:

$$0 = \sum_j \beta_{ij} \hat{\varphi}_{ij}. \hfill \text{(C.2)}$$

Logarithmically differentiating Eqs. (19) and (16) gives general expressions for $\hat{M}_{ij}$ and $\hat{\beta}_{ij}$:

$$\hat{M}_{ij} = - \frac{g_{ij} \varphi_{ij}}{1 - G_{ij}} \hat{\varphi}_{ij} - \sum_k \frac{(H'_{ik} - g_{ik}) \hat{\varphi}_{ik}}{H_{ik} + 1 - G_{ik}} \beta_{ik} \hat{\varphi}_{ik},$$

$$\hat{\beta}_{ij} = \frac{(H'_{ij} - g_{ij}) \varphi_{ij}}{H_{ij} + 1 - G_{ij}} \hat{\varphi}_{ij} - \sum_k \frac{(H'_{ik} - g_{ik}) \hat{\varphi}_{ik}}{H_{ik} + 1 - G_{ik}} \beta_{ik} \hat{\varphi}_{ik}.$$  \hfill (D.1)

Under $G_i(\varphi) = 1 - (b_i/\varphi)^{\theta} = 1 - b_i \varphi^{-\theta}; b_i > 0, \theta > \sigma - 1$, we can derive $g_i(\varphi) = \theta b_i \varphi^{-\theta - 1}, 1 - G_i(\varphi_{ij}) = b_i \varphi_i^{\theta}, \mu_{ij}(\varphi|\varphi_{ij}) = \theta \varphi_i^{\theta}, \varphi_{ij}^{\theta - 1}, \hat{\varphi}_{ij}(\varphi_{ij})^{\theta - 1}; \lambda \equiv \theta - (\sigma - 1) > 0$, and $h_{ij}(\varphi_{ij}) = \theta/\lambda - 1$. Noting that $H_{ij}(\varphi_{ij}) = (1 - G_i(\varphi_{ij}))/(\theta/\lambda - 1)$ and $H'_{ij}(\varphi_{ij}) + 1 - G_i(\varphi_{ij}) = (1 - G_i(\varphi_{ij}))\theta/\lambda = H_{ij}(\varphi_{ij})(\theta/\lambda)/(\theta/\lambda - 1)$, we obtain $(H'_{ij} - g_{ij}) \varphi_{ij}/(H_{ij} + 1 - G_{ij}) = -g_{ij} \varphi_{ij}/(1 - G_{ij}) = H'_{ij} \varphi_{ij}/H_{ij} = -\theta \varphi_{ij}, j$. Then $\hat{M}_{ij}$ and $\hat{\beta}_{ij}$ are rewritten as:

$$\hat{M}_{ij} = -\theta \hat{\varphi}_{ij} - \sum_k (-\theta) \beta_{ik} \hat{\varphi}_{ik} = -\theta \hat{\varphi}_{ij} + \theta \sum_k \beta_{ik} \hat{\varphi}_{ik},$$

$$\hat{\beta}_{ij} = -\theta \hat{\varphi}_{ij} - \sum_k (-\theta) \beta_{ik} \hat{\varphi}_{ik} = -\theta \hat{\varphi}_{ij} + \theta \sum_k \beta_{ik} \hat{\varphi}_{ik}.$$  \hfill (D.2)

Using Eq. (C.2), we obtain Eqs. (21) and (22).

**Appendix D. System of equations in $\hat{r}_1$ and $\hat{r}_2$ in the general case**

Logarithmically differentiating Eq. (12) gives:

$$\hat{\varphi}_{ji} - \hat{\varphi}_{ii} = \hat{\varphi}_j = \hat{\varphi}_i + \hat{\varphi}_{ji}, j \neq i. \hfill (D.1)$$

From Eqs. (C.2) and (D.1), we obtain:

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} \hat{\varphi}_{11} \\ \hat{\varphi}_{22} \\ \hat{\varphi}_{33} \end{bmatrix} = \begin{bmatrix} -\beta_{12}(\hat{v}_1 + \hat{v}_2 + \hat{v}_3) - \beta_{13}(\hat{v}_1 + \hat{v}_3) \\ -\beta_{21}(\hat{v}_1 - \hat{v}_2 + \hat{v}_3) - \beta_{23}(\hat{v}_1 + \hat{v}_3) \\ -\beta_{31}(\hat{v}_1 - \hat{v}_3) - \beta_{32}(\hat{v}_2 + \hat{v}_3) \end{bmatrix}. \hfill (D.2)$$

Eq. (D.2) is solved as:
\begin{align}
\hat{\phi}_{11} & = (1/D_3) \{-[\beta_{12}(\beta_{33} - \beta_{23}) + \beta_{13}(\beta_{22} - \beta_{32})]\hat{r}_1 + (\beta_{12}\beta_{33} - \beta_{13}\beta_{32})\hat{r}_2 \\
- (\beta_{22}\beta_{33} - \beta_{23}\beta_{32})(\beta_{12}\hat{r}_{12} + \beta_{13}\hat{r}_{13}) + (\beta_{12}\beta_{33} - \beta_{13}\beta_{32})(\beta_{21}\hat{r}_{21} + \beta_{23}\hat{r}_{23})
+ (\beta_{13}\beta_{22} - \beta_{12}\beta_{23})(\beta_{31}\hat{r}_{31} + \beta_{32}\hat{r}_{32})\}, \\
(\text{D.3}) \\
\hat{\phi}_{22} & = (1/D_3) \{(\beta_{21}\beta_{33} - \beta_{23}\beta_{31})\hat{r}_1 - [\beta_{21}(\beta_{33} - \beta_{13}) + \beta_{23}(\beta_{11} - \beta_{31})]\hat{r}_2 \\
+ (\beta_{21}\beta_{33} - \beta_{23}\beta_{31})(\beta_{12}\hat{r}_{12} + \beta_{13}\hat{r}_{13}) - (\beta_{11}\beta_{33} - \beta_{13}\beta_{31})(\beta_{21}\hat{r}_{21} + \beta_{23}\hat{r}_{23})
+ (\beta_{23}\beta_{31} - \beta_{21}\beta_{33})(\beta_{31}\hat{r}_{31} + \beta_{32}\hat{r}_{32})\}, \\
(\text{D.4}) \\
\hat{\phi}_{33} & = (1/D_3) \{(\beta_{22}\beta_{31} - \beta_{21}\beta_{32})\hat{r}_1 + (\beta_{11}\beta_{32} - \beta_{12}\beta_{31})\hat{r}_2 \\
+ (\beta_{22}\beta_{31} - \beta_{21}\beta_{32})(\beta_{12}\hat{r}_{12} + \beta_{13}\hat{r}_{13}) + (\beta_{11}\beta_{32} - \beta_{12}\beta_{31})(\beta_{21}\hat{r}_{21} + \beta_{23}\hat{r}_{23})
- (\beta_{11}\beta_{22} - \beta_{12}\beta_{21})(\beta_{31}\hat{r}_{31} + \beta_{32}\hat{r}_{32})\}; \\
(\text{D.5}) \\
D_3 \equiv & \begin{vmatrix}
\beta_{11} & \beta_{12} & \beta_{13} \\
\beta_{21} & \beta_{22} & \beta_{23} \\
\beta_{31} & \beta_{32} & \beta_{33}
\end{vmatrix} = \beta_{11}\beta_{22} - \beta_{12}\beta_{21} + \beta_{12}\beta_{31} - \beta_{11}\beta_{32} + \beta_{21}\beta_{32} - \beta_{22}\beta_{31}.
\end{align}

For \(N = 3\), Eq. (14) implies that \((1 - \beta_{11})r_1\kappa_1 = \beta_{21}r_2\kappa_2 + \beta_{31} \text{ and } (1 - \beta_{22})r_2\kappa_2 = \beta_{12}r_1\kappa_1 + \beta_{32}.\) Logarithmically differentiating them gives:

\begin{align}
r_1\kappa_1[1 - \beta_{11}\hat{\beta}_{11} + (1 - \beta_{11})(\hat{\kappa}_1 + \hat{\kappa}_1)] &= \beta_{21}r_2\kappa_2(\hat{\beta}_{21} + \hat{\kappa}_2) + \beta_{31}\hat{\beta}_{31}, \\
r_2\kappa_2[1 - \beta_{22}\hat{\beta}_{22} + (1 - \beta_{22})(\hat{\kappa}_2 + \hat{\kappa}_2)] &= \beta_{12}r_1\kappa_1(\hat{\beta}_{12} + \hat{\kappa}_1) + \beta_{32}\hat{\beta}_{32}.
\end{align}

Substituting \(\hat{\phi}_{ij}\) from Eqs. (D.1) and (D.3) to (D.5) into (22), substituting them into the above equations, and noting that \(\hat{\kappa}_i = (\sigma\hat{r}_i + \hat{\kappa}_i)/(\sigma - 1),\) we obtain:

\begin{align}
a_{11}\hat{r}_1 + a_{12}\hat{r}_2 & = c_{11}\hat{r}_1 + c_{12}\hat{r}_2 + \theta(\sigma - 1)(T_{12}^1\hat{r}_{12} + T_{13}^1\hat{r}_{13} + T_{21}^1\hat{r}_{21} + T_{23}^1\hat{r}_{23} + T_{31}^1\hat{r}_{31} + T_{32}^1\hat{r}_{32}), \\
a_{21}\hat{r}_1 + a_{22}\hat{r}_2 & = c_{21}\hat{r}_1 + c_{22}\hat{r}_2 + \theta(\sigma - 1)(T_{12}^1\hat{r}_{12} + T_{13}^1\hat{r}_{13} + T_{21}^1\hat{r}_{21} + T_{23}^1\hat{r}_{23} + T_{31}^1\hat{r}_{31} + T_{32}^1\hat{r}_{32});
\end{align}

\begin{align}
a_{11} & \equiv r_{1\kappa_1}\{(\sigma\theta - (\sigma - 1))D_3(1 - \beta_{11}) + \sigma\theta\beta_{12}(\beta_{33} - \beta_{23}) + \beta_{13}(\beta_{22} - \beta_{32})\}, \\
a_{12} & \equiv -[\sigma\theta - (\sigma - 1)]r_{2\kappa_2}D_3\beta_{21} + \sigma\theta r_{1\kappa_1}(\beta_{12}\beta_{33} - \beta_{13}\beta_{32}), \\
a_{21} & \equiv -[\sigma\theta - (\sigma - 1)]r_{1\kappa_1}D_3\beta_{12} + \sigma\theta r_{2\kappa_2}(\beta_{21}\beta_{33} - \beta_{23}\beta_{31}), \\
a_{22} & \equiv r_{2\kappa_2}\{(\sigma\theta - (\sigma - 1))D_3(1 - \beta_{22}) + \sigma\theta\beta_{21}(\beta_{33} - \beta_{13}) + \beta_{23}(\beta_{11} - \beta_{31})\},
\end{align}
Appendix E. Derivations of Eqs. (23) to (27)

In step (i), we solve for \( \hat{\tau}_1 \) and \( \hat{\tau}_2 \) in terms of \( \tilde{\kappa}_1, \tilde{\kappa}_2, \) and \( \tilde{\kappa}_{ij} \). Evaluating the coefficients in Eqs. (D.6) and (D.7) at the symmetric BGP gives:

\[
D_3 = (1 - 2\beta)^2 - \beta^2 - (1 - 2\beta)\beta + \beta^2 - (1 - 2\beta)\beta = (1 - 3\beta)^2 > 0.
\]

\[
a_{11} = [\sigma\theta - (\sigma - 1)][(1 - 3\beta)^2\beta + \sigma\theta(1 - 2\beta - \beta) + \beta(1 - 2\beta - \beta)] = -2\beta(1 - 3\beta)a,
\]

\[
a_{12} = -[(\sigma\theta - (\sigma - 1))(1 - 3\beta)^2\beta + \sigma\theta(1 - 2\beta - \beta^2)] = \beta(1 - 3\beta)a,
\]

\[
a_{21} = -[(\sigma\theta - (\sigma - 1))(1 - 3\beta)^2\beta + \sigma\theta(1 - 2\beta - \beta^2)] = a_{12},
\]

\[
a_{22} = [(\sigma\theta - (\sigma - 1))(1 - 3\beta)^2\beta + \sigma\theta(1 - 2\beta - \beta) + \beta(1 - 2\beta - \beta)] = a_{11};
\]

\[
a = -[(\sigma\theta - (\sigma - 1))(1 - 3\beta)^2 + \sigma\theta] < 0, a_{12} < 0, a_{11} = -2a_{12} > 0.
\]

\[
c_{11} = -\{\lambda(1 - 3\beta)^2\beta + \theta[\beta(1 - 2\beta - \beta) + \beta(1 - 2\beta - \beta)]\} = -2\beta(1 - 3\beta)c,
\]

\[
c_{12} = \lambda(1 - 3\beta)^2\beta + \theta[\beta(1 - 2\beta - \beta^2)] = \beta(1 - 3\beta)c,
\]

\[
c_{21} = \lambda(1 - 3\beta)^2\beta + \theta[\beta(1 - 2\beta - \beta^2)] = c_{12},
\]

\[
c_{22} = -\{\lambda(1 - 3\beta)^2\beta + \theta[\beta(1 - 2\beta - \beta) + \beta(1 - 2\beta - \beta)]\} = c_{11};
\]

\[
c = \lambda(1 - 3\beta) + \theta > 0, c_{12} > 0, c_{11} = -2c_{12} < 0.
\]
\[ T_{12}^1 = -[(1-2\beta)^2 - \beta^2] = -\beta(1-3\beta)(1-\beta) < 0, T_{13}^1 = -[(1-2\beta)^2 - \beta^2] = T_{12}^1 < 0, \]
\[ T_{21}^1 = [(1-3\beta)^2 + \beta(1-2\beta) - \beta^2] = \beta(1-3\beta)(1-2\beta) > 0, T_{23}^1 = [\beta(1-2\beta) - \beta^2] = \beta^2(1-3\beta) > 0, \]
\[ T_{31}^1 = [(1-3\beta)^2 + \beta(1-2\beta) - \beta^2] = T_{21}^1 > 0, T_{32}^1 = [\beta(1-2\beta) - \beta^2] = T_{23}^1 > 0. \]

Substituting Eqs. (E.1) and (E.2) back into Eqs. (D.3) to (D.5), and evaluating \( \hat{v}_i \) at the symmetric BGP, we obtain Eqs. (20). Finally, substituting Eqs. (E.3) to (E.5) into Eq. (20), and evaluating \( q_i \) at the symmetric BGP, we obtain Eqs. (25) to (27).
Appendix F. Effects of a regional trade agreement on $\varphi_{ij}$

Short-run effects

Let $\hat{\tau}_{21} = \hat{\tau}_{12} \neq 0$ and $\hat{\tau}_{13} = \hat{\tau}_{23} = \hat{\tau}_{31} = \hat{\tau}_{32} = 0$. Noting that $\hat{\kappa}_1 = \hat{\kappa}_2 = 0$ in the short run, Eqs. (E.1) and (E.2) give:

$$\hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} = (1/a)\sigma\theta\beta < 0,$$
$$\hat{\nu}_2/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} = \hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} < 0.$$

From Eqs. (E.3) to (E.5), we obtain:

$$\hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = (\beta/a)[\sigma\theta - (\sigma - 1)](1 - \beta) + \sigma\theta(1 - \beta) + (\sigma - 1)\beta < 0,$$
$$\hat{\varphi}_{22}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} < 0,$$
$$\hat{\varphi}_{33}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = -(2/\beta/a)[\sigma\theta - (\sigma - 1)]\beta > 0.$$

For $\hat{\varphi}_{ij}, j \neq i$, we combine the above results with Eq. (D.1) to obtain:

$$\hat{\varphi}_{21}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + 1 = -(1/a)((1 - 2\beta)[\sigma\theta - (\sigma - 1)](1 - \beta) + \sigma\theta) + (\sigma - 1)\beta > 0,$$
$$\hat{\varphi}_{31}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} - \hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = (\beta/a)[\sigma\theta - (\sigma - 1)](1 - 2\beta) < 0,$$
$$\hat{\varphi}_{12}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{22}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + 1 = \hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + 1 = \hat{\varphi}_{21}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} > 0,$$
$$\hat{\varphi}_{32}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{22}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} - \hat{\nu}_2/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{11}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} - \hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{31}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} < 0,$$
$$\hat{\varphi}_{13}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{33}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + \hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = (\beta/a)[\sigma\theta(1 - 2\beta) + 2(\sigma - 1)\beta] < 0,$$
$$\hat{\varphi}_{23}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{33}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + \hat{\nu}_2/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{33}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} + \hat{\nu}_1/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} = \hat{\varphi}_{13}/\hat{\tau}_{21}|_{\hat{\varphi}_{12} = \hat{\varphi}_{21}} < 0.$$

Long-run effects

In the long run, additional effects come from changes in $\kappa_1$ and $\kappa_2$ according to Eqs. (28) and (29):

$$\hat{\kappa}_1^*/\hat{\tau}_{21}|_{\hat{\kappa}_{12} = \hat{\kappa}_{21}} = -\{1/[3(\sigma - 1)]\}[\sigma\theta - (\sigma - 1) + \sigma\theta] < 0,$$
$$\hat{\kappa}_2^*/\hat{\tau}_{21}|_{\hat{\kappa}_{12} = \hat{\kappa}_{21}} = \hat{\kappa}_1^*/\hat{\tau}_{21}|_{\hat{\kappa}_{12} = \hat{\kappa}_{21}} < 0.$$

Taking this into account, Eqs. (E.1) to (E.5) and (D.1) give:

$$\hat{\nu}_1^*/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} = -1/3 < 0,$$
$$\hat{\nu}_2^*/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} = \hat{\nu}_1^*/\hat{\tau}_{21}|_{\hat{\nu}_{12} = \hat{\nu}_{21}} < 0.$$
\[
\phi_{11}/\hat{\tau}_{21}\hat{\tau}_{12} = -2\beta/3 < 0, \\
\hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} = \phi_{11}/\hat{\tau}_{21}\hat{\tau}_{12} < 0, \\
\hat{\phi}_{33}/\hat{\tau}_{21}\hat{\tau}_{12} = \phi_{11}/\hat{\tau}_{21}\hat{\tau}_{12} < 0.
\]

\[
\hat{\phi}_{21}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} + 1 = (3 - 2\beta)/3 > 0, \\
\hat{\phi}_{31}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} - \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} = (1 - 2\beta)/3 > 0, \\
\hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} + 1 = \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} + 1 > \hat{\phi}_{21}/\hat{\tau}_{21}\hat{\tau}_{12} > 0, \\
\hat{\phi}_{32}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} - \hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} - \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{31}/\hat{\tau}_{21}\hat{\tau}_{12} > 0, \\
\hat{\phi}_{31}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{31}/\hat{\tau}_{21}\hat{\tau}_{12} + \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} < 0, \\
\hat{\phi}_{32}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{32}/\hat{\tau}_{21}\hat{\tau}_{12} + \hat{\phi}_{22}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{31}/\hat{\tau}_{21}\hat{\tau}_{12} + \hat{\phi}_{11}/\hat{\tau}_{21}\hat{\tau}_{12} = \hat{\phi}_{32}/\hat{\tau}_{21}\hat{\tau}_{12} < 0.
\]

**Appendix G. Quantitative comparison of short- and long-run effects**

From Eqs. (25) to (27) and (30), the difference between the short- and long-run effects of a change in \(\tau_{ij}(j \neq i)\) on \(\gamma_k(k \neq i, j)\) is given by:

\[
|\partial \phi_{ij}/\partial \ln \tau_{ij}| - |\partial \phi_{ik}/\partial \ln \tau_{ij}| = \left[|\beta/(3a)\right] \{a + 3[\sigma \theta - (\sigma - 1)]\}.
\]

Since \(\beta < 1/3 \Leftrightarrow a + 3[\sigma \theta - (\sigma - 1)] \beta < (\sigma - 1) < 0\), we have \(|\partial \phi_{ik}/\partial \ln \tau_{ij}| > |\partial \phi_{ij}/\partial \ln \tau_{ij}|\).

Similarly, from Appendix F and Eq. (22), the difference between the short- and long-run effects of \(\hat{\tau}_{21} \neq \hat{\tau}_{12}\) is calculated as:

\[
|\hat{\beta}_{3j}/\hat{\tau}_{21}\hat{\tau}_{12}| - |\hat{\beta}_{3j}/\hat{\tau}_{21}\hat{\tau}_{12}| = \left[\theta(1 - 2\beta)/(3a)\right] \{a + 3\beta[\sigma \theta - (\sigma - 1)]\}.
\]

Again, \(\beta < 1/3 \Leftrightarrow a + 3[\sigma \theta - (\sigma - 1)] \beta < (\sigma - 1) < 0\) implies that \(|\hat{\beta}_{3j}/\hat{\tau}_{21}\hat{\tau}_{12}| > |\hat{\beta}_{3j}/\hat{\tau}_{21}\hat{\tau}_{12}|\).

**Appendix H. Expenditure shares**

Since the expenditure shares \(\zeta_{ij} = M_{ij} \int_{\phi_{ij}}^{\infty} e_{ij}(\phi) \mu_{ij}(\phi) d\phi / \sum_k M_{kj} \int_{\phi_{kj}}^{\infty} e_{kj}(\phi) \mu_{kj}(\phi) d\phi\), cannot be simplified as Eq. (16), it would be very complicated to calculate \(\zeta_{ij}\) directly. However, once we realize that \(M_{ij} \int_{\phi_{ij}}^{\infty} e_{ij}(\phi) \mu_{ij}(\phi) d\phi = \beta_{ij} E_i = \zeta_{ij} E_j\), it is simply expressed as \(\zeta_{ij} = \beta_{ij} + \hat{\tau}_i + \kappa_i - \hat{\tau}_j - \kappa_j\). This immediately implies that \(\hat{\zeta}_{11} = \beta_{11}, \hat{\zeta}_{22} = \beta_{22},\) and \(\hat{\zeta}_{33} = \beta_{33}\). Moreover, as long as \(\hat{\tau}_{12} = \hat{\tau}_{21}\) starting from the symmetric BGP, we have \(\hat{\tau}_2 = \hat{\tau}_1\) and \(\kappa_2 = \kappa_1\), ensuring that \(\hat{\zeta}_{21} = \beta_{21}\) and \(\hat{\kappa}_{12} = \beta_{12}\). For the remaining four expenditure shares, considering the symmetry between countries 1 and 2, we have to calculate only two expenditure shares, \(\hat{\zeta}_{31} = \beta_{31} - \hat{\tau}_1 - \kappa_1\) and \(\hat{\zeta}_{14} = \beta_{13} + \hat{\tau}_1 + \kappa_1\). We use the results in Appendix F, Eq. (22), and \(\hat{\tau}_i + \kappa_i = [(\sigma - 1)/\sigma](\hat{\tau}_i + \kappa_i)\) from the definition of \(v_i\), to calculate them.
In the short run, where ̂κ_i = 0 and ̂r_i = [(σ - 1)/σ]̂r_i, we obtain:

\[ \hat{\zeta}_{11}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = \hat{\beta}_{11}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} - \hat{r}_1/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} > 0, \]
\[ \hat{\zeta}_{13}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = \hat{\beta}_{13}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} + \hat{r}_1/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = -\theta(\beta/\alpha)(1 - 2\beta)[\sigma \theta - (\sigma - 1)] > 0. \]

This means that all expenditure shares move in the same directions as the revenue shares.

In the long run, using the result that \((\hat{r}_1 + \hat{\kappa}_1)/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = -2\theta/3 < 0\), we obtain:

\[ \hat{\zeta}_{11}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = \hat{\beta}_{11}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} - (\hat{r}_1 + \hat{\kappa}_1)/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = (\theta/3)(1 + 2\beta) > 0, \]
\[ \hat{\zeta}_{13}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = \hat{\beta}_{13}/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} + (\hat{r}_1 + \hat{\kappa}_1)/\hat{r}_{21}|_{\hat{r}_{12}=\hat{r}_{21}} = -\theta(3)(1 - 2\beta) < 0. \]

These expenditure shares move in the opposite directions of the revenue shares.

**Appendix I. Asymmetric old BGP**

The purpose of the following numerical exercises is not to calibrate our model to data, but to see how far from symmetry an old BGP can be for Propositions 1 to 3 to remain valid. To do this, parameters are just borrowed from other work or set arbitrarily: \( \rho_i = 0.02, \delta_i = 0.05 \) (e.g., Acemoglu, 2009), \( \sigma = 4, \theta = 4 \) (e.g., Balistreri et al., 2011), \( f_i^* = 1, f_{ij} = 1 \) for \( j = i \); \( f_{ij} = 1.5 \) for \( j \neq i \), \( K_{30} = 100 \).

We consider the Pareto scale parameters and import trade costs as the sources of asymmetry. To describe the situation where an RTA between countries 1 and 2 affects country 3 most seriously, we suppose that country 3 is the smallest country, followed by country 2, and then country 1. Let \( b_3 = 0.13, \tau_{13} = 0.5 \). If countries 1 and 2 have the same Pareto scale parameters and import trade costs as country 3, then we have the symmetric BGP, where the balanced growth rate of 1.71% is close to 1.68%, the average annual growth rate of the world real GDP per capita during 1967-2016 according to the World Development Indicators.

In the benchmark case (a), \( b_2 \) is 2.5% larger, whereas \( \tau_{21} \) is 2.5% smaller, than country 3’s. Similarly, \( b_1 \) is 5% larger, whereas \( \tau_{11} \) is 5% smaller, than country 3’s. Case (b) doubles the relative parameter differences in case (a). Similarly, case (c) triples the relative parameter differences in case (a). A movement from case (a) to (b) to (c) can be interpreted as order-preserving technological progress in countries 1 and 2.

As stated in section 5, an RTA between countries 1 and 2 is expressed as decreases in \( \tau_{21} \) and \( \tau_{12} \) by the same rate. Let \( \hat{\tau}_{21} = \hat{\tau}_{12} = -0.1 \), implying that the new values of \( \tau_{21} \) and \( \tau_{12} \) are 90% of their old values.

Table 2 summarizes the numerical results. In case (a), we have \( r_1^* \kappa_1^* = 2.24, r_2^* \kappa_2^* = 1.46, \) and \( \gamma_3^* = 1.97% \) at the old BGP. As we move from case (a) to (b) to (c), \( r_1^* \kappa_1^*, r_2^* \kappa_2^* \), and \( \gamma_3^* \) all increase monotonically. In case (c), we have \( r_1^* \kappa_1^* = 7.47 \), meaning that the GDP of the largest country 1 is over seven times as large as the smallest country 3. One of the most striking findings from Table 2 is that, in all three cases, all qualitative results reported in Table 1 continue to hold. In particular, \( \gamma_3, \beta_{31}, \) and \( \beta_{32} \) all decrease in the short run, but increase in the long run, compared with the old BGP. This implies that Propositions 1 to 3 are valid even if the GDP of the largest country is more than seven times as large as the smallest country at the old BGP.

We could consider case (d), where the relative parameter differences are four times as large as those in case (a). Although there still exists an old BGP with \( r_1^* \kappa_1^* = 11.46, r_2^* \kappa_2^* = 2.82, \) and \( \gamma_3^* = 3.00% \), the RTA does not increase \( \beta_{31} \) in the long run, which is the only departure from Table 1. This indicates that there
exist threshold relative parameter differences between case (c) and case (d) until which Propositions 1 to 3 remain valid.
References


Table 1: Effects of a regional trade agreement between countries 1 and 2 around the symmetric BGP

<table>
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<th>$\tau_{21}$ ↓, $\tau_{12}$ ↓</th>
<th>$\gamma_1$</th>
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<th>$\gamma_3$</th>
<th>$\beta_{11}$</th>
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<th>$\beta_{32}$</th>
<th>$\beta_{13}$</th>
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<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↓</td>
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Table 2: Effects of a regional trade agreement between countries 1 and 2 around asymmetric BGP

Note: the other parameter values are set as follows:

- $\rho_i = 0.02, \delta_i = 0.05$ (e.g., Acemoglu, 2009)
- $\sigma = 4, \theta = 4$ (e.g., Balistreri et al., 2011)
- $f_i^j = 1, f_{ij} = 1$ for $j = i; f_{ij} = 1.5$ for $j \neq i$
- $K_{30} = 100$