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Thomas Choate
Graduate School of Business, Stanford University

John A Weymark
Department of Economics, Vanderbilt University

Alan E. Wiseman
Department of Political Science, Vanderbilt University

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Contact: Thomas Choate - tchoate@stanford.edu, John A Weymark - john.weymark@vanderbilt.edu, Alan E. Wiseman - alan.wiseman@vanderbilt.edu.
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Legislative Bargaining and Partisan Delegation

Thomas Choate[a], **John A. Weymark**[b], **Alan E. Wiseman**[c]

[a]*Graduate School of Business, Stanford University, 655 Knight Way, Stanford, CA 94305, USA. E-mail:* tchoate@stanford.edu
[b]*Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, USA. E-mail:* john.weymark@vanderbilt.edu
[c]*Department of Political Science, Vanderbilt University, PMB #505, 230 Appleton Place, Nashville, TN 37203-5721, USA. E-mail:* alan.wiseman@vanderbilt.edu

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Abstract. We use an extension of the Baron–Ferejohn model of legislative bargaining in which there are three legislators, two of whom have partisan ties, to analyze the division of a fixed political resource in a majoritarian legislature. A legislator's preferences depend on the shares that he and any copartisan receive. We ask if there are circumstances under which a partisan legislator is willing to delegate proposal-making authority to a party leader so as to take advantage of the special proposal rights accorded by the legislature to this office rather than retaining equal-recognition proposal rights for himself. We show that this is the case only if (i) the leader's proposal recognition probability is larger than the probability that one of the partisans is recognized when the legislators act independently, (ii) partisan affiliation is sufficiently strong, and (iii) the legislators are sufficiently impatient. The relevance of this result for Aldrich and Rohde's conditional party government thesis and Krehbiel's First Congressional Parties Paradox are considered.

Keywords. Baron–Ferejohn legislative bargaining; political partisanship; proposal delegation; strong political party.

Legislatures differ widely in their organization and procedures, yet they exhibit certain common features, such as institutional positions that exercise agenda-setting powers (Cox 2006). Examples of such positions include the majority party leadership and legislative committees. As a general rule, legislators with agenda-setting authority can use this power to facilitate the adoption of policies that favor them at the expense of the majority. This raises the question posed by Gilligan and Krehbiel (1987, p. 288): "Why and under what conditions would a majority commit to a process that appears to limit its influence on legislative policy?" In the case that they consider, Gilligan and Krehbiel argue that endowing committees with the power to make proposals that cannot be amended when they are considered by the legislature as whole can be in the interest of the majority if this procedure results in more informed decision-making.

We consider a majority group of legislators who are not organized in a party but exhibit some partisan affinities and consider a related question to the one posed by Gilligan and Krehbiel: Are there conditions under which these partisans would willingly have themselves recognized as a party, giving up their agenda-setting power (or some portion thereof) to a party leader, who then employs that authority to pursue his policy interests—possibly at the expense of some of his copartisans' interests? We consider the possibility of delegation only with respect to proposal rights; a legislator in our model retains his voting rights over policy proposals even if he delegates his agenda-setting rights. In particular, the party leader has no mechanisms at his disposal to enforce party discipline at the voting stage. We assume that the rules of the legislature endow the majority party leader with special agenda-setting power. Hence, our question amounts to asking if there are circumstances under which a group of similarly-minded independent legislators who have equal proposal rights would be willing to delegate these rights to a party leader so as to take advantage of the special proposal rights accorded by the legislature to this postion. We assume that a party leader's proposal rights are fixed by a constitution or by convention and cannot be modified (at least not during the current legislative session). The benchmark from which the benefits of partisan delegation are assessed corresponds to what Cox (2006) calls a "legislative state of nature".

We address our question in a distributive politics context using a simple model in which three legislators, two of whom are copartisans, engage in bargaining over particularistic goods, which we formalize as bargaining over the division of a dollar, as in Baron and Ferejohn (1989). In the Baron–Ferejohn model, legislators only care about their own shares and, hence, there is no partisanship. Following Calvert and Dietz (2005) and Choate et al. (2018), we model partisanship by assuming that a legislator values a distribution of shares by adding a fixed fraction of his copartisan's share of the dollar (if he has a copartisan) to his own share. These other-regarding preferences provide a parsimonious way of modeling preference similarity when there are no ideological differences between legislators, as is the case when the issue being considered is purely distributive. This preference externality could arise, for example, because when partisans share similar interests, they are willing to cooperate in enacting legislation, which enhances their chances of being re-elected.

The equilibrium in the partisan legislative bargaining game with equal recognition probabilities is characterized by Choate et al. (2018). Here, we characterize the bargaining equilibrium that ensues when the two partisans form a party with one of them delegating his proposal rights to his party leader. Using these two characterizations, we identify the conditions under which a partisan strictly prefers to delegate his proposal rights rather than retaining them by remaining as an independent legislator. The rules of the legislature determine what the recognition probabilities are if there is a majority party. In order to make our analysis applicable to different institutional arrangements, we only assume that a majority party leader has at least a 50% chance of being recognized. We show that with our model of Baron–Ferejohn bargaining with partisan affiliations, a partisan is willing to endow a party leader with all of his own proposal-making authority only if (i) the recognition probability of the leader of the majority party is larger than the probability that one of the two partisans is recognized if they serve as independent legislators with no delegation of proposal rights, (ii) partisan affiliation is sufficiently strong, and (iii) the legislators are sufficiently impatient. Moreover, we find that when delegation is preferred, it is due to the resulting larger recognition probability accorded to the majority party leader, given that the leader's proposal is the same as what either partisan would propose if he engaged in independent agenda setting without any de facto leader.

Our model builds most directly on the small literature on legislative bargaining with other-regarding preferences; see Calvert and Dietz (2005), Montero (2007, 2008), and Choate et al. (2018). As is the case here, Čopič (2016) analyzes a Baron–Ferejohn bargaining game in which players have non-equal recognition probabilities. None of these articles, however, allow for the delegation of proposal rights.

Several models of legislative bargaining have been used to analyze the endogenous determination of recognition probabilities. McKelvey and Riezman (1992, 1993), Muthoo and Shepsle (2014), Eguia and Shepsle (2015), and Jeon (2015) consider repeated versions of a divide-the-dollar game in which the recognition probabilities evolve over time. In the case of McKelvey and Riezman (1992, 1993), Muthoo and Shepsle (2014), and Eguia and Shepsle (2015), legislators agree to implement a seniority system whose recognition probabilities depend on relative seniority. In the case of Jeon (2015), a legislator's recognition probability is an increasing function of his previous period's share of the dollar. Diermeier and Feddersen (1998), Diermeier et al. (2015, 2017, 2018), and Diermeier and Vlaicu (2011) consider a variety of legislative bargaining models: distributional and spatial; repeated and one-shot. These models share the feature that there is an organizational stage in which all of the legislators vote on who is to be accorded proposal rights. These rights may or may not be revokable depending on the model being considered. When, as in Diermeier et al. (2017, 2018), policy preferences are not known at the procedural decision stage and the legislators are risk averse, in equilibrium, procedural rights are concentrated in the hands of one or two of the majority party legislators so as to minimize policy choice volatility. In our model, legislators are not risk averse and there is no uncertainty about their preferences. Diermeier et al. (2018) and Diermeier and Vlaicu (2011) show how preference similarity in a spatial model can bias the outcome away from the median ideal point of the legislature towards the median ideal point of the majority party because of the desire of legislators to choose recognition probabilities that attenuate policy uncertainty. In our model, the distribution of shares is not one-dimensional, so there is no natural median alternative.[1]

Our model is designed to shed light on when politically aligned legislators will delegate proposal power so as to take advantage of the special powers accorded by legislative rules to majority party leaders in the simplest setting possible. Proponents of "strong party" theories of lawmaking suggest that parties control the legislative agenda (e.g., Cox and McCubbins 2005; Diermeier and Vlaicu 2011) and/or have the power to make selective use of carrots and sticks in order to induce their members to support the party's policy goals (e.g., Aldrich and Rohde 2001, 2017; Jenkins

[1]There are also models of endogenous proposal rights in which these rights are obtained by the expenditure of resources. See Diermeier et al. (2017) for a discussion of some of these models.

and Monroe 2012; Minozzi and Volden 2013). On the other hand, proponents of "majoritarian" theories of lawmaking note that in the United States Congress (and the U.S. House in particular), nearly all aspects of policy-making and legislative organization are subject to the approval of a simple legislative majority. Because we take the prerogatives of a majority party leader as given, our analysis does not offer an answer to the question of why parties exist in the first place or to why they have various procedural privileges. Nor do we address the question of why a majority party would be empowered with legislative tools that would facilitate outcomes that are inconsistent with the wishes of the legislative majority (Krehbiel 1999). Nevertheless, our findings provide support for a key feature of Aldrich and Rohde's theory of conditional party government (Aldrich and Rohde 2001), namely, that the powers delegated to the party leadership are positively correlated with the degree to which the majority party members share common interests. Our findings also suggest that Krehbiel's First Congressional Parties Paradox (Krehbiel 1999), which states that parties are strong when they are superfluous, does not apply in the circumstances that we consider. We provide the supporting arguments for both of these claims after presenting our formal results.

The Model

Our model of legislative bargaining with partisanship builds on those of Calvert and Dietz (2005) and Choate et al. (2018). There are three legislators who must decide on a *distribution* $x = (x_1, x_2, x_3)$ of a dollar among themselves, where $x_i \geq 0$ for $i = 1, 2, 3$ and $\sum_{i=1}^{3} x_i = 1$. Legislators 1 and 2 are *partisans*. In the period in which agreement on the distribution x is reached, the legislators' utilities are:

$$U^1(x) = x_1 + \alpha x_2, \tag{1}$$

$$U^2(x) = x_2 + \alpha x_1, \tag{2}$$

and

$$U^3(x) = x_3, \tag{3}$$

where $\alpha \in [0, 1)$. Thus, in addition to his own share, a partisan cares for his copartisan's share, but with a weight less than 1. The parameter α can be interpreted as the strength of partisan affiliation. Baron and Ferejohn (1989) suppose that $\alpha = 0$, so each legislator only cares about his own share. Legislators discount future payoffs using a common *discount factor* of $\delta \in [0, 1)$. Because $\delta < 1$, the legislators prefer to receive their shares sooner rather than later.[2]

Bargaining is modeled as an infinite-horizon noncooperative game. In each stage of this game, a legislator is recognized to make a proposal for dividing the dollar. The legislature uses a *closed rule*, so that the proposed distribution is voted on without amendment against the status quo. The bargaining ends if a majority votes in favor a proposal, with the dollar distributed accordingly. If a proposal is defeated, after a one period delay, the stage game is repeated. A *strategy* for a legislator who has any proposal rights has two components. In each period, his *proposal strategy* specifies the proposed distribution should he be recognized, whereas his *voting strategy* indicates

[2]The conditions that characterize when delegation is preferred also apply when there is no discounting ($\delta = 1$), but the description of the equilibrium with delegation is somewhat more complex than that given in Proposition 1 for $\delta < 1$, so, for simplicity, we assume that there is discounting. In their model of partisan legislative bargaining without delegation, Calvert and Dietz (2005) assume that $\delta = 1$.

which distributions he would vote for. If a legislator does not have any proposal rights, he only has a voting strategy. Because voting over distributions is by majority rule, a proposer will only offer shares of the dollar to himself and one other legislator. As in Baron and Ferejohn (1989), attention is restricted to *stationary strategies* in which the proposal and voting strategies are time invariant. Thus, decisions are not contingent on past history.

We consider two legislative bargaining games. In the *partisan legislative bargaining game with equal recognition probabilities*, each legislator is recognized to make a proposal with probability $\frac{1}{3}$. This is the no delegation case. In the *partisan legislative bargaining game with delegation*, the partisan party leader is recognized to make a proposal with probability π, his copartisan is recognized with probability 0, and the nonpartisan is recognized with probability $1-\pi$. Voting rights are not delegated. In the absence of delegation, a proposal is made by a partisan with probability $\frac{2}{3}$. The legislative rules may specify a recognition probability for a majority party leader different from this value. In order for our analysis to apply to a broad range of legislative arrangements, we assume that $\pi \in \left[\frac{1}{2}, 1\right]$ rather than specifying a particular value for it. Requiring π to be at least $\frac{1}{2}$ serves two purposes: (i) it excludes the implausible case in which a majority party does not have as much proposal power as its opposition and (ii) it allows us to set aside the more complicated equilibria that can arise in such cases.

The Bargaining Equilibria

We first consider the legislative bargaining game with delegation. Without loss of generality, we suppose that it is legislator 2 who delegates his proposal rights, but not his voting rights, to legislator 1. Proposition 1 characterizes the stationary subgame perfect equilibrium of the legislative bargaining game with delegation. In a *stationary subgame perfect equilibrium*, (i) each legislator uses a stationary strategy and (ii) the profile of the three legislators' strategies is a Nash equilibrium when restricted to any subgame.

Proposition 1. *For any $\delta \in [0,1)$, if legislator 2 delegates his proposal rights to legislator 1 who is recognized as the proposer in any period with probability $\pi \in \left[\frac{1}{2}, 1\right]$, a set of strategies is a stationary subgame perfect equilibrium if and only if:*

(a) *When legislator 1 is the proposer, he offers* 0 *to his copartisan,* 0 *to the nonpartisan, and proposes for himself to receive* 1.

(b) *When the nonpartisan is the proposer, he offers* 0 *to legislator 1,* $\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$ *to legislator 2, and proposes for himself to receive* $1-\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$.

(c) *For either proposer:*

(i) *legislator 1 votes for any distribution in which he receives utility at least* $\delta\left[\pi+(1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)\right]$*;*

(ii) *legislator 2 votes for any distribution in which he receives utility at least* $\frac{\delta\pi\alpha}{1-(1-\pi)\delta}$*;*

(iii) *the nonpartisan votes for any distribution in which he receives utility at least* $\delta(1-\pi)\left(1-\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)$.

Each of the distributions proposed receives the support of the proposer and legislator 2, and so has the support of a majority.

If $\pi = 1$, legislators 2 and 3 are never recognized to make a proposal. In this case, part (b) does not apply and part (c) is only relevant for voting over a proposal made by legislator 1.

The values in Proposition 1.(c) that specify what each legislator must be offered for his support are their discounted continuation values. If legislator 1 has complete proposal power ($\pi = 1$), then, as expected, legislator 3's continuation value is 0. In the proof of this proposition, we show that when $\pi \neq 1$, legislator 1 must be offered more for his support by legislator 3 than legislator 2 except when $\delta = 0$, in which case they both will accept a zero share. Intuitively, this is the case because the party leader has proposal power and his copartisan does not. Consequently, when the nonpartisan is recognized, he can keep more for himself by making an offer to legislator 2 rather than to legislator 1, and the amount he offers is the minimum amount needed to ensure that the proposal is accepted, thereby ending the bargaining. In contrast, when the majority party leader is recognized, he proposes that the entire dollar be allocated to himself. This proposal is supported by his copartisan in spite of him not receiving any of the dollar because of the positive externalities that he obtains from his party leader acquiring all of the dollar.

When there is no delegation and recognition probabilities are equal, as in Choate et al. (2018), we suppose that the proposal strategies respect the symmetries in the situations of the two partisans, so the strategies are *partisan symmetric*. In the case of the nonpartisan legislator 3, partisan symmetry requires that, if recognized, he offers the same share to each of the partisans with equal probability. Partisan symmetry also requires that if a partisan is recognized, his proposal is the same as what the other partisan would make *mutatis mutandis* had he been recognized. Our equilibrium concept in this case is partisan symmetric stationary subgame perfect equilibrium. In a *partisan symmetric stationary subgame perfect equilibrium*, the equilibrium is a stationary subgame perfect equilibrium with partisan symmetric strategies.[3]

A complete characterization of the equilibrium without delegation is provided by Choate et al. (2018). Here, we summarize the features of a partisan's equilibrium strategy in this equilibrium that are relevant for determining whether the delegation of his proposal rights is beneficial for him. The set of possible values of the strength of partisan affiliation parameter α and the discount factor δ are partitioned into three regions, with the qualitative features of the equilibrium differing between them. There are two α-dependent threshold values of δ, $\underline{\delta}(\alpha) = \frac{6\alpha}{(1+\alpha)(2+\alpha)}$ and $\bar{\delta}(\alpha) = \frac{-3\alpha-\alpha^2+\sqrt{24\alpha+33\alpha^2+6\alpha^3+\alpha^4}}{2(1+\alpha)}$, both of which are increasing in α. Except when $\alpha = 0$, these thresholds differ, with $0 < \underline{\delta}(\alpha) < \bar{\delta}(\alpha) < 1$. For $\delta \in [0, \underline{\delta}(\alpha))$, a partisan proposes to keep all of the dollar for himself when recognized. For $\delta \in \left[\underline{\delta}(\alpha), \bar{\delta}(\alpha)\right)$, a partisan proposer also offers the nonpartisan nothing, but offers his copartisan a positive share. For $\delta \in \left[\bar{\delta}(\alpha), 1\right)$, a partisan proposer offers a positive share to one of the other legislators, with the recipient determined probabilistically. In all three regions, a nonpartisan proposer choses one of the other legislators probabilistically and offers him a share. All proposals receive majority support.

[3]When one of the partisans delegates his proposal rights, they are in asymmetric situations, so partisan symmetry does not apply.

When is Delegation Preferred?

The copartisan of the majority party leader receives nothing if he delegates his proposal rights and his leader is recognized. If he does not delegate, then he keeps some or all of the dollar if recognized and, for some parameter values, a positive amount if the other partisan is recognized. In light of these observations, one naturally wonders whether there are any circumstances in which the two partisans benefit from delegation. We show that there are.

In both of the partisan legislative bargaining games being considered, the stage game is the same in every period and the equilibrium is in stationary strategies. Therefore, in both games, any legislator's expected utility is his undiscounted continuation value.[4] In the game without delegation, the two partisans have the same proposal power and, hence, have the same continuation value. As we have seen, legislator 1's continuation value exceeds (resp. is equal to) that of legislator 2 in the delegation game when $\delta > 0$ (resp. $\delta = 0$), so to determine whether both partisans prefer delegation, we only need to determine if legislator 2 does. In Proposition 2, we characterize the parameter restrictions for delegation to be beneficial for the two partisans.

Proposition 2. *The partisan legislators both prefer the expected equilibrium outcome of the partisan legislative bargaining game with delegation to the expected equilibrium outcome of the partisan legislative bargaining game with equal recognition probabilities if and only if*

$$\pi > \frac{2}{3}, \tag{4}$$

$$\alpha > \frac{1}{3\pi - 1}, \tag{5}$$

and

$$\delta < \frac{2[(3\pi-1)\alpha - 1]}{(1+\alpha)[\pi(2+\alpha)-2]}. \tag{6}$$

Thus, there are conditions such that a partisan prefers to delegate his proposal-making authority to his copartisan. However, for this to be the case, (i) the probability π that the majority party leader is recognized must be larger than the probability that a partisan is recognized when there is no delegation, (ii) the strength of partisan affiliation (α) has to be sufficiently large, and (iii) legislators need to be sufficiently impatient (i.e., δ must be sufficiently small). In particular, in order for a partisan to be willing to delegate his proposal rights, it is necessary that $\alpha > \frac{1}{2}$ and $\delta < \underline{\delta}(\alpha)$. The first inequality follows from (4) and (5). The necessity of the second inequality is established in the proof of Proposition 2.

Hence, for a partisan to prefer to give up his proposal-making authority, it must be the case that by delegating, the probability that a partisan is recognized increases. When $\delta < \underline{\delta}(\alpha)$ and legislator 2 keeps his proposal rights, he receives nothing if legislator 1 is recognized, which is what he receives for the same value of δ if he delegates his proposal power to legislator 1 and the latter is recognized. However, if he keeps his proposal rights and is recognized, he will obtain

[4] As we have noted, the formulas for the continuation values when there is delegation are given in Proposition 1.(c). The functional form for a partisan's continuation value when there is no delegation may be found in the proof of Proposition 2. It depends on whether $\delta \geq \bar{\delta}$ or not.

all of the dollar. Moreover, delegation is only preferable if it reduces the probability that the nonpartisan is recognized, which makes it less likely that legislator 2 will receive any share of the dollar from a nonpartisan proposal. These observations suggest that legislator 2 would only prefer to have the party leader propose on his behalf and keep the entire dollar if recognized if the value of partisan affiliation is sufficiently large so that which of the partisans gets the dollar is less important than that it is allocated to one of them. But, in order for this to be the case, legislators must be sufficiently impatient. With greater impatience, legislator 2 is willing to give up a larger share of the dollar in order to secure policy agreement in the current period. When δ exceeds the bound in (6), his willingness to sacrifice the share that he could obtain if recognized when there are equal recognition probabilities is not sufficient to make delegation worthwhile.

The bound on α in (5) depends on π and the bound on δ in (6) depends on both α and π. Further insight into a partisan's decision about whether to delegate may be obtained by considering how these bounds vary in response to changes in these parameters.

Proposition 3. *The lower bound on α in* (5) *is decreasing in π and the upper bound on δ in* (6) *is increasing in both α and π.*

The more likely that a majority party leader is recognized, the less likely that the nonpartisan is, which makes it less likely that legislator 2 benefits from a nonpartisan proposal. Consequently, he does not need to value his copartisan's share as much, or be so willing to sacrifice future benefits, in order to prefer delegation. Nor does he need to be as impatient if he values his copartisan's share more. Indeed, because the bound on δ approaches 1 as α and π both approach this value, if the value that a partisan places on his copartisan's share and the probability that a majority party leader is recognized are both close to 1, then he prefers to delegate his proposal rights unless he values future benefits almost as much as current ones.

Lessons for Congressional Politics

One of the main features of the theory of conditional party government that Aldrich and Rohde (2001, 2017) have developed to explain the distribution of power in the U.S. Congress is that "[p]arty members will want to give more power to the party leadership when there is greater consensus in the party about what to do with those powers and when it is more important that the leadership have the tools to achieve those goals" (Aldrich and Rohde 2017, p. 34). Our findings provide some support for this thesis. We have found that delegation of proposal power is only in a partisan legislator's interest if his preferences are sufficiently similar to his party leader as measured by the degree of partisan affiliation α. Furthermore, he only wants to delegate when the value to the majority party leader of having proposal power is sufficiently large as measured by the proposal recognition probability π accorded to this position.

Our analysis also contributes to the debate about the rationale for the existence of a "strong" legislative party in a majoritarian legislature when legislators put aside their own interests in favor of party cohesion. Krehbiel's First Congressional Parties Paradox says that "[p]arties are said to be strong exactly when, viewed through a simple spatial model, they are superfluous" (Krehbiel 1999, p. 35). In our distributive politics model, a strong party emerges (i.e, there is delegation) if the legislative rules endow a majority party leader with a proposal recognition probability that is larger than the probability that one of the two partisans is recognized if they act independently, provided that the degree of partisan affiliation is sufficiently strong and legislators are sufficiently impatient.

Thus, there are circumstances in which a strong party is not superfluous. Nevertheless, *conditional* on one of the partisans being recognized to make a proposal, a party is superfluous. The creation of a party with the special proposal rights accorded to its leader is only in the interest of both partisans if they already have such strong bonds between themselves that either of them would consent to his copartisan obtaining the entire dollar. Indeed, even if the majority party could acquire all of the proposal rights (i.e., $\pi = 1$), a partisan would not want to delegate proposal authority to a party leader unless α exceeds $\frac{1}{2}$. If partisan affiliation is strong, the distribution of a given share of the dollar between the partisans is of secondary importance to the probability that one of the partisans is recognized to make a proposal. For this reason, a necessary, but not sufficient, condition for delegation to be in the interest of the partisans is that their degree of partisan affiliation α is so large that a partisan proposer can secure all of the dollar whether or not there is delegation. Thus, a party is essentially "superfluous" for the partisans to achieve their ends *conditional* on one of them being recognized to make a proposal, but only in that limited sense.

Conclusion

A fundamental question in scholarly debates about the role of parties in legislatures revolves around when one would expect legislators to empower a party leader to act on their behalf, even if this power could be used in a way that is counter to their direct interests. We have addressed this question using an extension of the well-studied Baron–Ferejohn model of bargaining over particularistic goods in a majoritarian legislature composed of three legislators who have equal proposal and voting power in the absence of delegation, two of whom have partisan ties, as in Calvert and Dietz (2005) and Choate et al. (2018). In this distributive politics context, we have shown that complete delegation of proposal power to a copartisan is sometimes in the interests of the two partisans and have characterized the conditions under which this is the case.

In order to focus on the role that the special proposal rights conferred on a majority party leader has on the incentives for the delegation of these rights by the individual legislators, we have abstracted from many other important features of legislative bargaining. In future research, it would be of interest to extend our model in order to consider some of them. We conclude by mentioning two of these extensions.

Risk aversion plays a prominent role in the literature on the endogenous choice of proposal rights (e.g., Diermeier et al. 2017, 2018). In our model, delegation reduces the uncertainty over who will be the proposer compared to our benchmark case of equal recognition probabilities. However, because the partisans trade off their shares at a constant rate, this insurance benefit has no value. Risk aversion could be introduced by assuming that this trade-off is nonlinear. We conjecture that risk aversion would make delegation a more attractive option.

We have identified circumstances in which it is in the interest of one of the partisans to delegate proposal rights to his copartisan. However, because the two partisans are identical, we are unable to say who will be the party leader or, with more partisans, who will be the party leadership. A natural way of determining the party leadership is to employ some form of a seniority system. This suggests that it would be worthwhile to consider a repeated game version of our model so that seniority can play a substantive role, as it does in the legislative bargaining models of McKelvey and Riezman (1992, 1993), Muthoo and Shepsle (2014), and Eguia and Shepsle (2015).

Appendix: Proofs

Proof of Proposition 1. We provide the proof for the case in which $\pi \neq 1$. If $\pi = 1$, part (b) does not apply and quite straightforward modifications need to be made to the rest of the proof (e.g., all terms in which $1-\pi$ appear drop out).

We first establish the necessity part of the proof.

(a) Let p be the probability that legislator 1 seeks only the support of legislator 2 by making him an equilibrium offer of y. Similarly, let q be the probability that legislator 3 seeks the support of legislator 2 by making him an offer. Legislator 2's equilibrium continuation value is

$$V^2 = \left[\pi\left(p((1-\alpha)y+\alpha)+(1-p)\alpha(1-\delta V^3)\right)+(1-\pi)\left(q\delta V^2+(1-q)\alpha\delta V^1\right)\right]. \tag{A.1}$$

In (A.1), $(1-\alpha)y+\alpha$ is legislator 2's utility when he receives y. This utility may be greater than the minimum amount δV^2 needed to obtain legislator 2's support if the nonnegativity constraint on y binds.

Because $\delta V^3 \geq 0$, $\alpha \in [0,1)$, and $y \geq 0$,

$$-\alpha\delta V^3 \leq (1-\alpha)y.$$

Adding α to both sides of this inequality, it follows that

$$\alpha(1-\delta V^3) \leq (1-\alpha)y+\alpha. \tag{A.2}$$

Using (A.2) in (A.1), we obtain

$$V^2 \leq \left[\pi\left((1-\alpha)y+\alpha\right)+(1-\pi)\left(q\delta V^2+(1-q)\alpha\delta V^1\right)\right]. \tag{A.3}$$

Next, we show that either $q=1$ or

$$\alpha\delta V^1 \leq \delta V^2. \tag{A.4}$$

Suppose that $q<1$. Then, legislator 3 weakly prefers to make the minimum offer δV^1 needed to obtain legislator 1's support than to make the minimum offer δV^2 needed to obtain legislator 2's support. That is,

$$1-\delta V^1 \geq 1-\delta V^2$$

or, equivalently,

$$\delta V^1 \leq \delta V^2.$$

Multiplying the left hand side of this inequality by α, we obtain (A.4).

Therefore, either by using (A.4) in (A.3) or by setting $q=1$ in the latter inequality, we have

$$V^2 \leq \left[\pi\left((1-\alpha)y+\alpha\right)+(1-\pi)\delta V^2\right].$$

Solving this inequality for V^2 gives the bound

$$V^2 \leq \frac{\pi\left((1-\alpha)y+\alpha\right)}{1-(1-\pi)\delta}. \tag{A.5}$$

We now show that $y = 0$. On the contrary, suppose that $y > 0$. It then follows that y is chosen so that legislator 2's utility is equal to his discounted continuation value. That is,

$$\delta V^2 = (1-\alpha)y + \alpha. \tag{A.6}$$

Because $\alpha \in [0,1)$ and $y > 0$, the right hand side of (A.6) is positive. If $\delta = 0$, we have a contradiction and, hence, $y = 0$. If $\delta \neq 0$, (A.6) implies that $V^2 > 0$. In this case, replacing $(1-\alpha)y + \alpha$ with δV^2 on the right hand side of (A.5) and dividing both sides of the resulting inequality by V^2, we obtain

$$1 \leq \frac{\delta \pi}{1-(1-\pi)\delta},$$

which holds if and only if $\delta \geq 1$. This is a contradiction because δ is assumed to be less than 1. Therefore, we also have $y = 0$ if $\delta \neq 0$.

When $y = 0$, (A.5) implies that

$$\pi\alpha \geq [1-\delta+\pi\delta]V^2 \geq \pi\delta V^2$$

and, hence, that

$$\alpha \geq \delta V^2,$$

which shows that legislator 2's utility is no smaller than his discounted continuation value when the nonnegativity constraint on his share offer binds.

If $\delta V^3 > 0$, legislator 1's utility is 1 if he offers legislator 2 nothing, whereas it is $1 - \delta V^3 < 1$ if he offers legislator 3 the minimum δV^3 needed to get his support. Hence, legislator 1 is strictly better off seeking the support of legislator 2, and so sets $p = 1$. Thus, when $\delta V^3 > 0$, legislator 1 keeps the whole dollar for himself and offers nothing to the other legislators.

If $\delta V^3 = 0$, legislator 1 can obtain the support of legislator 3 by offering him nothing, so in this case as well, legislator 1 keeps all of the dollar, as was to be shown.

(b) If $\delta = 0$, legislator 3 does not need to offer either of the other legislators any share of the dollar to obtain their support. Hence, when $\delta = 0$, legislator 3 keeps the dollar.

Now suppose that $\delta > 0$. Given that $y = 0$, the continuation values for legislators 1 and 2 are respectively

$$V^1 = \pi + (1-\pi)\left(q\alpha\delta V^2 + (1-q)\delta V^1\right) \tag{A.7}$$

and

$$V^2 = \pi\alpha + (1-\pi)\left(q\delta V^2 + (1-q)\alpha\delta V^1\right). \tag{A.8}$$

We show that $V^1 > V^2$. On the contrary, suppose that $V^1 \leq V^2$. If, in fact, $V^1 < V^2$, then it must be that $q = 0$ because legislator 3 can obtain the support of legislator 1 by offering him δV^1 which is less than the amount δV^2 needed to obtain legislator 2's support. Setting $q = 0$ in (A.7) and (A.8), it then follows that

$$V^1 = \pi + (1-\pi)\delta V^1 > \pi\alpha + (1-\pi)\alpha\delta V^1 = \alpha V^1 = V^2$$

because $\alpha < 1$, which contradicts the assumption that $V^1 < V^2$. Hence, it must be the case that $V^1 \geq V^2$.

It remains to show that $V^1 \neq V^2$. On the contrary, suppose that $V^1 = V^2$. Substituting V^1 for V^2 in (A.7) and (A.8), adding the resulting equations, and solving for V^1, we obtain

$$V^1 = \frac{\pi(1+\alpha)}{2-(1-\pi)(1+\alpha)\delta}. \tag{A.9}$$

Equating the right hand sides of (A.7) and (A.8) with V^1 substituted for V^2, we obtain

$$\pi + (1-\pi)\left(q\alpha\delta V^1 + (1-q)\delta V^1\right) = \pi\alpha + (1-\pi)\left(q\delta V^1 + (1-q)\alpha\delta V^1\right)$$

or, equivalently,

$$\pi(1-\alpha) = (1-\pi)(1-\alpha)(2q-1)\delta V^1.$$

The left hand side of the latter equation is positive, so $\delta V^1 \neq 0$. Solving this equation for q, we obtain

$$q = \frac{1}{2}\left[\frac{\pi}{(1-\pi)\delta V^1} + 1\right].$$

Substituting the value of V^1 from (A.9) into this equation and simplifying the resulting right hand side, we find that

$$q = \frac{1}{(1-\pi)\delta(1+\alpha)} - \frac{1}{2} + \frac{1}{2}.$$

Because $\pi \geq \frac{1}{2}$ and both α and δ are less than 1, it then follows that

$$q \geq \frac{2}{\delta(1+\alpha)} > 1,$$

which is not possible. Thus, $V^1 \neq V^2$ and, therefore, $V^1 > V^2$.

Hence, if $\delta > 0$ and $V^1 > V^2$, we have $\delta V^2 < \delta V^1$. Thus, legislator 3 can obtain the support of legislator 2 by offering him less than is needed to obtain legislator 1's support. Therefore, it must be the case that $q = 1$ when $\delta \neq 0$.

From Part (a), we know that if legislator 1 is the proposer, he offers legislator 2 $y = 0$ with probability $p = 1$, so legislator 2's utility is α. Because $q = 1$, it then follows from (A.1) that

$$V^2 = \pi\alpha + (1-\pi)\delta V^2. \tag{A.10}$$

Solving (A.10) for V^2, we obtain

$$V^2 = \frac{\pi\alpha}{1-(1-\pi)\delta}. \tag{A.11}$$

Because $q = 1$, legislator 3 offers legislator 1 nothing, legislator 2 the amount

$$\delta V^2 = \frac{\delta\pi\alpha}{1-(1-\pi)\delta}, \tag{A.12}$$

and keeps the rest the dollar for himself. Note that

$$\delta V^2 \leq \alpha \tag{A.13}$$

because $\delta\pi\alpha \leq \alpha(1-\delta)+\delta\alpha\pi$. Moreover, $\delta V^2 = 0$ when $\delta = 0$, which is the amount that legislator 3 offers legislator 2 when $\delta = 0$.

(c) Reasoning as in the derivation of (A.10), when $\delta > 0$, the continuation values for legislators 1 and 3 are respectively

$$V^1 = \pi + (1-\pi)\alpha\delta V^2 \tag{A.14}$$

and

$$V^3 = (1-\pi)(1-\delta V^2). \tag{A.15}$$

Using (A.11) to eliminate V^2 from (A.14) and (A.15), we obtain

$$V^1 = \pi + (1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)$$

and

$$V^3 = (1-\pi)\left(1-\frac{\pi\delta\alpha}{1-(1-\pi)\delta}\right).$$

Hence,

$$\delta V^1 = \delta\left[\pi + (1-\pi)\alpha\left(\frac{\delta\pi\alpha}{1-(1-\pi)\delta}\right)\right] \tag{A.16}$$

and

$$\delta V^3 = \delta\left[(1-\pi)\left(1-\frac{\pi\delta\alpha}{1-(1-\pi)\delta}\right)\right]. \tag{A.17}$$

The values in (A.16), (A.12), and (A.17) are the minimum utilities needed to secure the support of legislator 1, 2, and 3, respectively, for all values of $\delta \in [0,1)$, not just for $\delta \in (0,1)$.

It remains to confirm that each of the proposals receives the support of its proposer and at least one of the other legislators. When legislator 1 is the proposer, he receives utility 1, which is the maximum possible utility for him, so votes in favor of his proposal. Legislator 2 receives utility α, which by (A.13) is at least as large as his continuation utility δV^2, and so he also votes in favor of this proposal. When legislator 3 is the proposer, he receives utility

$$1-\delta V^2 > \delta(1-\pi)\left(1-\delta V^2\right) = \delta V^3,$$

and so votes in favor of his proposal. Legislator 2 receives utility equal to his continuation value, and so he also votes in favor of this proposal.

This completes the necessity part of the proof.

For the sufficiency part of the proof, we need to show that the strategies described in the proposition are a stationary subgame perfect equilibrium. In other words, we need to show that no legislator wants to deviate unilaterally from these strategies. To do this, we must show that (i) no legislator in his role as a proposer wants to modify the share offered to one of the other legislators in order to receive his support, (ii) no legislator in his role as a proposer wants to modify the probabilities with which he makes offers to the other legislators, and (iii) no legislator wants to deviate from his voting strategy. All of these claims have already been established in demonstrating necessity. Legislator 1 offers legislator 2 nothing and legislator 3 offers him the smallest amount that he will accept, which guarantees each of these proposers a higher payoff than an offer to the other legislator when $\delta > 0$ and the same payoff when $\delta = 0$, so (i) and (ii) hold. The last part of the proof of necessity establishes (iii). □

Proof of Proposition 2. As noted in the discussion preceding the statement of Proposition 2, (i) in both of the the partisan legislative bargaining games being considered, any legislator's expected utility is his undiscounted continuation value and (ii) in the game without delegation, the two partisans have the same continuation value. In the proof of Proposition 1, we show than legislator 1's continuation value exceeds that of legislator 2 in the delegation game when $\delta > 0$. By Proposition 1.(c), their continuation values are both 0 when $\delta = 0$. Therefore, we only need to determine when legislator 2 prefers to delegate. In the game without delegation, the functional form of the expression for this legislator's continuation value depends on whether $\delta \geq \bar{\delta}$, so there are two cases to consider.

Case 1. For $\delta \in [\bar{\delta}(\alpha), 1)$, using the continuation values for legislator 2 in Proposition 1.(a) in Choate et al. (2018) and in Proposition 1.(c) here, legislator 2 prefers delegation if and only if

$$\frac{\pi\alpha}{1-(1-\pi)\delta} > \frac{(1+\alpha)(\alpha+\delta)}{\delta(3+\alpha)}. \tag{A.18}$$

We now prove that this inequality never holds when $\delta \in [\bar{\delta}(\alpha), 1)$. Simple algebra shows that

$$\begin{aligned}
&\pi\alpha\delta(3+\alpha) > (1+\alpha)(\alpha+\delta)[1-\delta+\pi\delta] \\
&\leftrightarrow 2\pi\alpha\delta > (1+\alpha)(\alpha+\delta)(1-\delta)+\pi\delta^2+\pi\alpha\delta^2 \\
&\rightarrow 2\pi\alpha\delta > (1+\alpha)(\alpha+\delta)(1-\delta)+2\pi\alpha\delta^2 \\
&\leftrightarrow 0 > (\alpha+\delta+\alpha^2+\alpha\delta)(1-\delta)+2\pi\alpha\delta(\delta-1) \\
&\leftrightarrow 0 > (\alpha+\delta+\alpha^2+\alpha\delta-2\pi\alpha\delta)(1-\delta) \\
&\rightarrow 0 > (\alpha+\delta+\alpha^2+\alpha\delta-\alpha\delta-\alpha)(1-\delta) \\
&\leftrightarrow 0 > (\delta+\alpha^2)(1-\delta).
\end{aligned}$$

The right hand side of the last inequality is positive, so we have a contradiction.

Case 2. For $\delta \in [0, \bar{\delta}(\alpha))$, using the continuation values for legislator 2 in Proposition 1.(e) in Choate et al. (2018) and in Proposition 1.(c) here, legislator 2 prefers delegation if and only if

$$\frac{\pi\alpha}{1-(1-\pi)\delta} > \frac{2(1+\alpha)}{6-\delta-\alpha\delta}. \tag{A.19}$$

We first show that this inequality holds if and only if (6) is satisfied and then determine the restrictions on α and π needed to ensure that $\alpha \in [0,1)$, $\delta \in [0, \bar{\delta}(\alpha))$, and $\pi \in \left[\frac{1}{2}, 1\right]$.

The inequality in (A.19) holds if and only if

$$\begin{aligned}
&\pi\alpha(6-\delta-\alpha\delta) > 2(1+\alpha)(1-\delta+\pi\delta) \\
&\leftrightarrow 6\pi\alpha-\pi\alpha\delta-\pi\alpha^2\delta > 2(1+\alpha)+2(1+\alpha)\delta(\pi-1) \\
&\leftrightarrow 2[(3\pi-1)\alpha-1] > (1+\alpha)[\pi(2+\alpha)-2]\delta \qquad \text{(A.20)}
\end{aligned}$$

Next, we show that (A.20) is equivalent to (6). We do this by showing that the term that multiplies δ on the right hand side of (A.20) is positive. In order to do this, we first show that the left hand side of (A.20) is less than this term. We have

$$\begin{aligned}
&6\pi\alpha-2(1+\alpha) < (1+\alpha)[\pi(2+\alpha)-2] \qquad \text{(A.21)} \\
&\leftrightarrow 3\pi\alpha < 2\pi+\alpha^2\pi \\
&\leftrightarrow 0 < (\alpha-2)(\alpha-1) \qquad \text{(A.22)}
\end{aligned}$$

The right hand side of (A.22) is positive, so (A.21) holds. Hence, if the right hand side of (A.21) is nonpositive, then the left hand side of (A.21) is negative, in which case we must have $\delta > 1$ in order to satisfy (A.20), which is impossible. Therefore, the right hand side of (A.21) is positive and so (A.20) holds if and only if the bound on δ in (6) is satisfied.

The discount factor δ must be nonnegative. This is only the case if the numerator on the right hand side of (6) is positive because, as we have seen, the denominator is positive. This numerator is positive if and only if the bound on α in (5) is satisfied. By (5), $\alpha < 1$ if and only $\frac{1}{3\pi-1} < 1$, which is equivalent to the restriction on π in (4).

It remains to show that when (4), (5), and (6) hold that $\delta < \bar{\delta}(\alpha)$. We show that, in fact, $\delta < \underline{\delta}(\alpha)$; that is, that

$$\delta < \frac{6\alpha}{(1+\alpha)(2+\alpha)}. \tag{A.23}$$

Using the bound for δ in (6) and recalling that the denominator in this bound is positive, (A.23) holds if and only if

$$\begin{aligned}
\frac{2[(3\pi-1)\alpha-1]}{(1+\alpha)[\pi(2+\alpha)-2]} &< \frac{6\alpha}{(1+\alpha)(2+\alpha)} \\
\leftrightarrow \frac{3\alpha\pi-\alpha-1}{3\alpha} &< \frac{\pi(2+\alpha)-2}{2+\alpha} \\
\leftrightarrow \pi - \frac{\alpha+1}{3\alpha} &< \pi - \frac{2}{2+\alpha} \\
\leftrightarrow \frac{\alpha+1}{3\alpha} &> \frac{2}{2+\alpha} \\
\leftrightarrow \alpha^2 - 3\alpha + 2 &> 0 \\
\leftrightarrow (1-\alpha)(2-\alpha) &> 0.
\end{aligned}$$

The last inequality holds because $\alpha < 1$. Hence, (A.23) is satisfied. □

Proof of Proposition 3. That the lower bound on α in (5) is decreasing in π follows immediately from its functional form. To show the upper bound on δ in (6) is increasing in both α and π requires determining the signs of quite complicated expressions. We do this using *Mathematica*. The *Mathematica* notebook for this part of the proof may be found in the online Supplementary Material.

□

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Online Supplementary Material

Legislative Bargaining and Partisan Delegation

Thomas Choate[a], **John A. Weymark**[b], **Alan E. Wiseman**[c]

[a]*Graduate School of Business, Stanford University, 655 Knight Way, Stanford, CA 94305, USA. E-mail:* `tchoate@stanford.edu`
[b]*Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, USA. E-mail:* `john.weymark@vanderbilt.edu`
[c]*Department of Political Science, Vanderbilt University, PMB #505, 230 Appleton Place, Nashville, TN 37203-5721, USA. E-mail:* `alan.wiseman@vanderbilt.edu`

Here, we provide the *Mathematica* Notebook that was used to help prove Proposition 3.

? D

D[f, x] gives the partial derivative $\partial f / \partial x$.
D[f, {x, n}] gives the multiple derivative $\partial^n f / \partial x^n$.
D[f, x, y, …] differentiates f successively with respect to x, y, ….
D[f, {{x_1, x_2, …}}] for a scalar f gives the vector derivative ($\partial f / \partial x_1$, $\partial f / \partial x_2$, …).
D[f, {$array$}] gives a tensor derivative. >>

? Reduce

Reduce[$expr$, $vars$] reduces the statement $expr$ by solving equations or inequalities for $vars$ and eliminating quantifiers.
Reduce[$expr$, $vars$, dom] does the reduction over
the domain dom. Common choices of dom are Reals, Integers, and Complexes. >>

To differentiate the δ bound in α:

$$\texttt{Simplify}\left[\texttt{D}\left[\frac{2\left(\left(3\,\Pi-1\right)\alpha-1\right)}{\left(1+\alpha\right)\left(\Pi\left(2+\alpha\right)-2\right)},\ \alpha\right]\right]$$

$$\frac{2\,\Pi\left(-5+2\,\alpha+\alpha^2\left(1-3\,\Pi\right)+6\,\Pi\right)}{\left(1+\alpha\right)^2\left(-2+\left(2+\alpha\right)\Pi\right)^2}$$

$$\texttt{Reduce}\left[\left\{\texttt{D}\left[\frac{2\left(\left(3\,\Pi-1\right)\alpha-1\right)}{\left(1+\alpha\right)\left(\Pi\left(2+\alpha\right)-2\right)},\ \alpha\right]>0,\ \frac{1}{3\,\Pi-1}<\alpha<1,\ \frac{2}{3}<\Pi\leq1\right\},\ \alpha\right]$$

$$\frac{2}{3}<\Pi\leq1\ \&\&\ \frac{1}{-1+3\,\Pi}<\alpha<1$$

To differentiate the δ bound in Π:

$$\texttt{Factor}\left[\texttt{D}\left[\frac{2\left(\left(3\,\Pi-1\right)\alpha-1\right)}{\left(1+\alpha\right)\left(\Pi\left(2+\alpha\right)-2\right)},\ \Pi\right]\right]$$

$$\frac{2\left(-2+\alpha\right)\left(-1+\alpha\right)}{\left(1+\alpha\right)\left(-2+2\,\Pi+\alpha\,\Pi\right)^2}$$

$$\texttt{Reduce}\left[\left\{\texttt{D}\left[\frac{2\left(\left(3\,\Pi-1\right)\alpha-1\right)}{\left(1+\alpha\right)\left(\Pi\left(2+\alpha\right)-2\right)},\ \Pi\right]>0,\ \frac{1}{3\,\Pi-1}<\alpha<1,\ \frac{2}{3}<\Pi\leq1\right\},\ \alpha\right]$$

$$\frac{2}{3}<\Pi\leq1\ \&\&\ \frac{1}{-1+3\,\Pi}<\alpha<1$$