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Foreign exchange reserves as a tool for capital account management

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Abstract

Many recent theoretical papers have argued that countries can insulate themselves from volatile world capital flows by using a variable tax on foreign capital as an instrument of monetary policy. But at the same time many empirical papers have argued that only rarely do we observe these cyclical capital taxes used in practice. In this paper we present a small open economy framework where the central bank can engage in sterilized foreign exchange intervention. When private agents can freely buy and sell foreign bonds, sterilized foreign exchange intervention has no effect. But we analytically prove that when private agents cannot freely buy and sell foreign bonds, that is, under acyclical capital controls, optimal sterilized foreign exchange intervention is equivalent to an optimally chosen tax on foreign capital. Numerical simulations of the model show that a variable capital tax is a reasonable approximation for sterilized foreign exchange intervention under the levels of capital controls observed in many emerging markets.

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1 Introduction

Repeated cycles of capital flows into and out of emerging markets are a fixture of the financially integrated global economy. Rey (2015) and Forbes and Warnock (2012) argue that capital flows into and out of emerging markets are largely driven by global factors. Surges in capital inflows have led to talk of "currency wars" and the danger of overheating in many emerging markets. Likewise, a sudden reversal of capital flows is often the reason behind financial and macroeconomic instability in many emerging markets.

Rey (2015) argues that this cycle of capital inflows and outflows means that the "trilemma" of international finance is actually more of a "dilemma", and that "independent monetary policies are possible if and only if the capital account is managed."[1] In 2012 the IMF argued that active capital controls might be a useful policy instrument to manage the macroeconomic and financial risks associated with large swings in capital inflows and outflows (International Monetary Fund (2012)). This has led to a renewed interest in capital controls as a tool for active capital account management. Jeanne, Subramanian, and Williamson (2012) argue that capital controls "properly designed ... might even be a regular instrument of economic policy" (p. 95).

Farhi and Werning (2012) introduce capital account management measures as a variable tax, τ_t, on the price of foreign bonds.[2] In this model, and others that model capital controls in a similar way, the variable tax affects fluctuations in net capital outflows. This is a separate policy instrument that the policy maker can adjust in real time to achieve some policy

[1] The trilemma has been a feature of the international macroeconomics literature since Mundell (1963). The trilemma states that a country cannot simultaneously maintain a fixed exchange rate, an open capital account, and monetary policy autonomy.

In technical terms, the fact that the combination of a fixed exchange rate and an open capital account lead to the loss of monetary policy autonomy is purely mechanical. When a central bank maintains a fixed exchange rate, the central bank's monetary policy takes the form of a rule stating that the nominal exchange rate is held constant. So for instance, in response to a fall in net capital inflows, the central bank is forced to raise the interest rate to attract capital flows and prevent depreciation.

[2] For other examples of models with this type of variable capital control tax, see Schmitt-Grohe and Uribe (2012b), Schmitt-Grohe and Uribe (2012a) , Costinot, Lorenzoni, and Werning (2014), De Paoli and Lipinska (2013) , Korinek (2013), Farhi and Werning (2014) , Benigno, Chen, Otrok, Rebucci, and Young (2016), Davis and Presno (2017) , Aoki, Benigno, and Kiyotaki (2016), Schmitt-Grohe and Uribe (2016) .

objective like welfare maximization. Theory suggests that these variable capital controls should be cyclical and should vary with booms and busts in net capital flows.

But while these variable capital taxes are intuitive and easy to implement in a model, the empirical evidence that policy makers make use of these variable capital controls is lacking. Eichengreen and Rose (2014) and Fernnandez, Rebucci, and Uribe (2015) find that in practice capital controls are acyclical and do not respond to booms and busts in output, the current account, or the real exchange rate.

Klein (2012) and Forbes, Fratzscher, and Straub (2015) argue that temporary, episodic type capital controls, what Klein refers to as capital control "gates", tend to be ineffective.[3] Moreover, Klein argues that permanent capital controls, what he refers to as capital control "walls", can be effective at deflecting capital inflows (although they may come at a high cost in terms of lost growth, so while they are effective, whether they are beneficial is another story).[4]

We begin this paper with two simple empirical observations. First, emerging markets maintain significant capital controls, while the advanced economies have basically eliminated capital controls. These capital controls are very persistent and unchanging, Klein's "wall-type" capital controls. Second, central bank foreign exchange interventions are frequently observed in emerging market economies but not in advanced economies. While central bank reserve accumulation tends to be small and uncorrelated with other types of capital flows in most advanced economies, reserve accumulation is a large and volatile component of the balance of payments in most emerging markets, and it is highly correlated with capital

[3]Forbes, Fratzscher, Kostka, and Straub (2016) argue that the episodic capital controls that Brazil put in place during and after the recent crisis did have the effect of deflecting capital flows away from Brazil and towards other emerging markets, although they argue that the effect was more likely due to the signaling nature of imposing capital controls and less due to the effect of the controls themselves.

[4]Similarly, Ghosh, Qureshi, Kima, and Zalduendo (2014) find that capital controls can be effective at limiting the frequency and magnitude of capital inflow "surges". In line with the theory of the trilemma, a number of papers show that imposing capital account restrictions leads to a significant increase in monetary policy autonomy, particularly for countries with a fixed exchange rate. See e.g. Shambaugh (2004) , Obstfeld, Shambaugh, and Taylor (2005), Baba and Kokenyne (2011) , Magud, Reinhart, and Rogoff (2011), and Klein and Shambaugh (2015). However in this paper we are entirely concerned with countries with a floating currency and an independent monetary policy.

inflows from abroad, indicating that many emerging market central banks adjust their stock of foreign exchange reserves in tandem with surges and stops in capital inflows in order to smooth fluctuations in the current account.[5] In simple cross-country regressions we show that countries that have greater capital controls tend to be more active in using foreign exchange intervention.

We then construct a small open economy dynamic stochastic general equilibrium (DSGE) model where the central bank can engage in sterilized foreign exchange intervention that explain these results. In the model we introduce cash and a central bank balance sheet into a model of a small open economy. The central bank potentially has two instruments, the size of its balance sheet and the composition of its balance sheet. Both are set optimally to maximize household welfare. The first instrument determines the money supply and thus the interest rate, the second instrument determines the central bank's stock of foreign bonds, which can (potentially) be adjusted to smooth fluctuations in net capital flows.

The effectiveness of sterilized foreign exchange intervention depends on frictions or adjustment costs which determine how easily private agents can buy or sell foreign bonds, as in Gabaix and Maggiori (2015). We show that when private agents can freely buy and sell foreign bonds, sterilized foreign exchange intervention has no effect, and optimal policy with two instruments, both the size and composition of its balance sheet, is equivalent to optimal policy where the central bank only has one instrument, the size of the balance sheet. This finding of the ineffectiveness of sterilized foreign exchange intervention in a frictionless environment echoes that in Obstfeld (1981) and Backus and Kehoe (1989). In our contribution we prove that when private agents cannot freely buy and sell foreign bonds, optimal sterilized foreign exchange intervention is equivalent to the optimally chosen variable capital control tax from Farhi and Werning (2012). We provide analytical proofs of the two sets of

[5] In their revised exchange rate classification system, Ilzetzki, Reinhart, and Rogoff (2019), argue that many countries fall into the managed float or managed peg category, and few countries truly allow the exchange rate to float. Similarly, Ghosh, Ostry, and Chamon (2016) find that emerging market central banks actively use sterilized foreign exchange intervention to smooth the booms and busts in net capital inflows. Bussere, Cheng, Chinn, and Lisack (2015) argue that there is a complementarity between reserve accumulation and capital controls.

equivalence results. The model that we use to prove these equivalence results is very general, and we do not need to specify the functional forms describing the behavior of private agents. These equivalence results hold without recourse to firm side specifications, and hold under any type of frictions, or lack of frictions (nominal, financial, etc.) that we see in today's New Keynesian DSGE literature.

We then build a fully specified model for numerical simulation of the equivalence results. The model is similar to the one in Chang, Liu, and Spiegel (2015).[6] We map the observed levels of capital controls in the data into the adjustment cost parameter in the model that determines how easily private agents can buy or sell foreign bonds. As in Gabaix and Maggiori (2015), capital controls increase the effectiveness of foreign exchange intervention. We show that under levels of capital controls that are observed in many emerging market countries, sterilized foreign exchange intervention can be an effective tool for capital account management. Under these observed levels of capital controls, a model with an optimally chosen variable capital control tax is a reasonable approximation to a more complex model with a central bank balance sheet and sterilized foreign exchange intervention.

Thus in this paper we make a purely positive contribution. We show that under the levels of acyclical "wall-type" capital controls that are observed in many emerging markets, the central bank can use sterilized foreign exchange intervention to stabilize the current account in the face of volatile capital inflows. Our equivalence result provides a justification for the use of a variable capital tax in a model as a capital account management measure even when these variable capital taxes are not observed in practice.

The remainder of this paper is organized as follows. In section 2 we present some simple descriptive statistics describing capital controls and central bank reserve accumulation in both advanced and emerging market economies. The generalized small open economy model

[6] Liu and Spiegel (2015) also compare sterilized foreign exchange intervention with variable capital controls, but in their model, variable capital controls apply only to capital inflows, not total net flows. Thus in their version of variable capital controls, the central bank does not have complete control over net flows (i.e. the current account) and thus they find that even when using variable capital controls, there is room for welfare improvement from sterilized foreign exchange intervention.

where the central bank can engage in sterilized foreign exchange intervention is presented in section 3. The proof of the equivalence between sterilized foreign exchange intervention and a variable capital tax are presented in section 4. The results from a numerical model are presented in section 5. Finally section 6 concludes.

2 Descriptive Statistics on Reserve Accumulation

The index of capital account restrictions from Chinn and Ito (2006) is presented in Figure 1. This figure shows the simple average of the Chinn-Ito index, $KAOpen$, updated through 2015, for a sample of 23 advanced countries and a sample of 66 emerging market and developing countries. We re-normalize the original Chinn-Ito index to a 0-1 scale, where 0 represents no capital account restrictions and 1 represents a closed capital account.

The figure shows that on average, advanced economies have fewer capital account restrictions, and in 1970 the average value of the Chinn-Ito index in the advanced economies was 0.50 and in emerging and developing economies it was around 0.7. The figure also shows that over the past 45 years, there has been a near monotonic decrease in the level of the Chinn-Ito index of capital account restrictions in the advanced economies, and now capital account restrictions in the advanced economies are close to 0. Meanwhile capital account restrictions in the emerging markets increased substantially in the 1980s, before falling in the 1990s and 2000s. Interestingly capital account restrictions have actually increased slightly in the emerging markets since the crisis in 2008.

The current account (CA) is equal to savings minus investment, and is equal to a country's net savings. The capital and financial account (KA, for brevity, henceforth we refer to the capital and financial account by the colloquial term capital account) is equal to capital outflows minus capital inflows: the net purchase of foreign assets by domestic residents (not the central bank) minus the net purchase of domestic assets by foreign residents, and thus measures a country's net foreign asset purchases not including central bank foreign asset purchases.[7] Central bank net purchases have a special line-item in the balance of payments accounts, foreign exchange reserve accumulation (ΔR). This leads to the fundamental balance of payments identity where a country's net savings should equal its net purchases of foreign assets, both by the central bank and the rest of the economy, and thus $CA = KA + \Delta R$.[8]

Some statistics describing the standard deviation of the current account, capital account, reserve accumulation and the components of the capital account (Capital outflows, OF, and Capital inflows, IF, as well as the subcomponents of outflows and inflows, FDI flows, Portfolio flows, and Other flows, where other is mostly bank lending) are presented in Table 1. All capital flows are annual and all are normalized by GDP.

The table presents the average standard deviation in the advanced and emerging market economies over the 1990-2015 period. To ensure that any results are not driven solely by the 2008 crisis and subsequent recession, we also calculate the same statistics over the 1990-2007 period.

Reserve accumulation is the least volatile entry in the balance of payments statistics in the advanced countries, where the standard deviation of reserve accumulation is far smaller than the standard deviations of both the current account and the capital account, and also far smaller than any of the components or subcomponents of the capital account. In contrast

[7] Since the release of Balance of Payments Manual 5 in 1993, the IMF and other international organizations distinguish between the capital account and the financial account when recording international asset purchases. The capital account involves the net purchase of all non-financial assets (e.g. land and natural resource rights, leases and licenses, marketing capital, brand names, and goodwill). The cross-border purchases of non-financial assets (the capital account) is generally very small relative to cross-border purchases of financial assets (the financial account), and thus for brevity, we just refer to all net asset purchases as the capital account.

[8] In this paper, we follow the simple asset/liability approach from BPM6 accounting where a positive net purchase of foreign assets is recorded as positive capital outflows, as in Broner, Didier, Erce, and Schmukler (2013). Under this accounting standard, positive net purchases of foreign assets would result in a positive capital and financial account.

Some papers instead follow the direction of flow principle from BPM5 accounting and consider that positive net purchases of foreign assets by domestic residents is a cash outflow from the domestic country to be recorded with a negative sign, as in Forbes and Warnock (2012). Under this accounting standard, a positive net purchase of foreign assets would be a cash outflow and thus would result in a negative capital and financial account. Thus under BPM5 accounting, the balance of payments identity is commonly written as the current account plus the capital and financial account is equal to the net change in reserves.

reserve accumulation is one of the most volatile entries in the balance of payments statistics in the emerging markets. The standard deviation of reserve accumulation is nearly the same as the standard deviation of the current account and the capital account, and it is greater than the standard deviation of any of the subcomponents of the capital account.

The correlations between reserve accumulation and the components of the capital account are reported in the top half of Table 2. The correlation between reserve accumulation and any of the components of capital outflows is small in both the advanced economies and the emerging markets. However, the table shows that the correlation between reserve accumulation and capital inflows is close to 0 in the advanced economies but close to 0.5 in the emerging markets.

The bottom half of the table reports the correlations between the three components of the balance of payments identity. We have already seen in Table 1 that in the advanced economies, the standard deviation of the net change in official reserves is small. So not surprisingly, the correlation between the current account and the capital account in the advanced economies is close to 1.

The standard deviation of reserve accumulation is high in the emerging markets, and reserve accumulation is positively correlated with capital inflows. As a result, the correlation between reserve accumulation and the capital account is close of -0.5 in the emerging markets. That negative correlation between reserve accumulation and the capital account, combined with the fact that reserve accumulation is relatively volatile in the emerging markets, means that the correlation between the current account and the capital account is only around 0.5 in the emerging markets.

Thus the level of capital controls tends to be higher in emerging markets, and at the same time reserve accumulation is more volatile, and more highly correlated with capital inflows and other items in the balance of payments in emerging markets. In Table 3 we regress the relative standard deviation and correlation of reserve accumulation and other items in the balance of payments on the natural log of the Chinn-Ito capital account restrictions index, *KAOpen*, across the 89 countries in the sample. The capital account restrictions index is potentially endogenous, countries that face very volatile external capital flows may impose high capital account restrictions, so in addition to the OLS results, the table presents two stage least squares (TSLS) results where the Chinn-Ito index is instrumented by a measure of domestic financial sector liberalization from Abiad, Detragiache, and Tressel (2010).[9]

The first four lines of the table regress the relative volatility of reserve accumulation on the measure of capital account restrictions: first the standard deviation of reserve accumulation relative to the standard deviation of the current account, then relative to the standard deviation of the capital account, the standard deviation of capital inflows, and the standard deviation of capital outflows. In nearly every case the coefficient of the capital account restrictions index is positive and significant, indicating that as capital account restrictions increase, the volatility of reserve accumulation relative to the other items in the balance of payments increases.

In the next three rows in the table, the dependent variable is a correlation between two of the three components of the balance of payments identity ($CA = KA + \Delta R$). As expected, given the contrast between advanced and emerging markets in the earlier set of descriptive statistics, the correlation between the current account and the capital account falls as capital account restrictions increase, and the correlation between reserve accumulation and the current account increases. Interestingly there is no relationship between the capital account restrictions and the correlation between reserve accumulation and the capital account.

Finally, in the remaining rows in the table, the dependent variable is the correlation between reserve accumulation and one of the subcomponents of the capital account. As expected, given the descriptive statistics presented earlier, there is little evidence that capital account restrictions affect the correlation between reserve accumulation and capital outflows, or any of the subcomponents of capital outflows. But the coefficients on the regressions

[9] The Abiad et al. measure of financial liberalization contains both a component related to domestic financial liberalization and a component related to international financial liberalization (capital account openness). For an instrument, we use the measure only related to domestic financial liberalization.

involving the correlation between reserve accumulation and capital inflows, or any of the subcomponents of capital inflows, are positive and highly significant.

3 Model

In the model there are two countries, home and foreign. The home country is of size n and the foreign country is of size $1-n$, and we consider the case of the small open economy where $n \to 0$. The home economy is populated by a representative household and a continuum of firms. There is a central bank which sets the domestic money supply, and under different policy scenarios may set a tax rate on foreign borrowing/lending or may alter their stock of foreign exchange reserves.

In this section, we will present the analytical model in its general form without recourse to most of the specific functional forms of the model, to show that our key analytical results hold broadly for a large class of models. In the next section, we will specify a set of functional forms in a particular model for numerical simulations.

3.1 Households

In the small open home economy, the representative household chooses consumption, C_t, real money balances, $\frac{M_t}{P_t}$, labor effort, H_t, and stocks of domestic and foreign currency denominated bonds, B_t and F_t, to maximize expected lifetime utility given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, H_t\right) \tag{1}$$

subject to their sequence of budget constraints:

$$P_tC_t + M_t + B_t + (1-\tau_t)\,S_tF_t + P_t\chi\Omega\left(\hat{F}_t\right) \tag{2}$$
$$= W_tH_t + M_{t-1} + (1+i_{t-1})\,B_{t-1} + \left(1+\Phi_{t-1}i^*_{t-1}\right)S_tF_{t-1} + \Pi_t + \Pi^{cb}_t$$

where U is an increasing, concave function, B_t is the household's stock of domestic currency denominated bonds, F_t is their stock of foreign currency denominated bonds, S_t is the nominal exchange rate (in units of the home currency per units of the foreign currency), P_t is the consumer price level, W_t is the nominal wage rate, i_t is the nominal interest rate on home currency denominated bonds, i^*_t is the nominal interest rate on foreign currency denominated bonds, Π_t represents the profits of domestic firms, τ_t is a tax on the purchase of foreign bonds, and Π^{cb}_t represents the central bank profits. Both firm and central bank profits are returned lump sum to the household. β is the household's discount factor. $\chi\Omega\left(\hat{F}_t\right)$ is an adjustment cost and is a function of the household's holdings of foreign bonds, where $\hat{F}_t = F_t - F_{ss}$ and F_{ss} is the steady state value of F_t. The function $\Omega(\cdot)$ is differentiable and strictly convex and satisfies $\Omega(0) = \Omega'(0) = 0$, where Ω' denotes the first derivative of $\Omega(\cdot)$, as in Schmitt-Grohe and Uribe (2003).

Φ_{t-1} is a risk premium shock that affects home country borrowing in the foreign currency. Neumeyer and Perri (2005) show how important these country-specific risk premium shocks are for explaining business cycle fluctuations in an emerging market economy.[10]

With Λ_t denoted as the marginal utility of household consumption in period t, the first-order conditions of the household's problem with respect to B_t and F_t are:

$$\frac{\Lambda_t}{P_t} = \beta\,(1+i_t)\,\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right) \tag{3}$$

[10] The type of shock is not important for the analytical proofs of equivalence. We consider a shock to the world interest rate, but the same results would hold under other types of shock common in the New Keynesian literature.

$$\left((1-\tau_t)S_t + P_t\chi\Omega'\left(\hat{F}_t\right)\right)\frac{\Lambda_t}{P_t} = \beta\left(1+\Phi_t i_t^*\right)\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}S_{t+1}\right) \tag{4}$$

The variable τ_t is a policy variable. In one scenario we will give the central bank the ability to control this tax τ_t on the purchase of foreign currency denominated bonds. In other policy scenarios we will simply set $\tau_t = 0$. The substitution of equation (3) into equation (4) gives:

$$\frac{1+i_t}{1+\Phi_t i_t^*} = \frac{\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}S_{t+1}\right)}{\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right)\left(\left(1-\tau_t\right)S_t + P_t\chi\Omega'\left(\hat{F}_t\right)\right)} \tag{5}$$

In a linearized model we can ignore the covariance between $\frac{\Lambda_{t+1}}{P_{t+1}}$ and S_{t+1}, and the $\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right)$ terms cancel out of the numerator and denominator, leading to the familiar uncovered interest parity (UIP) condition. We linearize the model later in the numerical results section, but for now, we leave the UIP condition in this more general form.

Home households own home country firms. For now we are agnostic about the firm's production function or any type of frictions the firm faces. Those details are not necessary for the analytical proofs of equivalence. We will fill in those details when discussing the functional forms for the numerical simulation model.

3.2 Monetary Policy

The central bank solves a Ramsey problem to maximize household welfare in (1). We describe that Ramsey problem and the planner's constraints later in this section.

The central bank issues nominal money balances, M_t, and uses it to finance holdings of home currency and foreign currency denominated bonds, B_t^{cb} and F_t^{cb}. It conducts monetary policy by adjusting the stock of money and its stocks of home and foreign currency denominated bonds subject to its balance sheet given by:

$$M_t = B_t^{cb} + S_t F_t^{cb} \tag{6}$$

The central bank potentially has a few monetary policy instruments. One monetary instrument is the nominal money supply. Given the household's first-order conditions with respect to real money balances and domestic currency bond holdings, controlling the nominal money supply is tantamount to controlling the nominal interest rate i_t.[11]

The second instrument has to do with the central bank's ability to control the capital account. Here we consider two scenarios. In both scenarios the second instrument is determined by the solution to a Ramsey problem to maximize household welfare in (1). In the first, the central bank's stock of foreign currency bonds is held fixed at its steady state value, $F_t^{cb} = F_{ss}^{cb}$, but the central bank can adjust the variable tax τ_t on the purchase of foreign currency denominated bonds by private agents. In the second scenario, this variable tax $\tau_t = 0$, but the central bank can adjust the stock of foreign currency bonds on their balance sheet, F_t^{cb}.

The central bank earns a positive interest rate on its assets but pays no interest on its liabilities, so its seigniorage revenue is given by:

$$
\begin{aligned}
&(M_t - M_{t-1}) - \left(B_t^{cb} - (1 + i_{t-1})\, B_{t-1}^{cb}\right) - \left(S_t F_t^{cb} - \left(1 + \Phi_{t-1} i_{t-1}^*\right) S_t F_{t-1}^{cb}\right) \qquad (7) \\
&= i_{t-1} B_{t-1}^{cb} + (S_t - S_{t-1})\, F_{t-1}^{cb} + \Phi_{t-1} i_{t-1}^* S_t F_{t-1}^{cb}
\end{aligned}
$$

In addition if the central bank can charge a variable tax τ_t on purchases of foreign bonds then the revenue from this tax is $-\tau_t S_t F_t$. The central bank also faces an adjustment cost, denoted by $\chi^{cb}\Omega(\hat{F}_t^{cb})$, to adjusting its stock of foreign currency denominated bonds, where $\hat{F}_t^{cb} = F_t^{cb} - F_{ss}^{cb}$. Recall that the function $\Omega(\cdot)$ is differentiable and strictly convex and satisfies $\Omega(0) = \Omega'(0) = 0$.

Thus the central bank profits are given by seigniorage revenue net of any tax revenue and bond adjustment cost:

[11]The substitution of these two first-order conditions yields the demand for real money balances as a function of the home nominal interest rate.

$$\Pi_t^{cb} = i_{t-1} B_{t-1}^{cb} + (S_t - S_{t-1}) F_{t-1}^{cb} + \Phi_{t-1} i_{t-1}^{*} S_t F_{t-1}^{cb} - \tau_t S_t F_t - P_t \chi^{cb} \Omega(\hat{F}_t^{cb}) \tag{8}$$

3.2.1 Bond Market Clearing Conditions

Home currency denominated bonds are held exclusively by home country households and the home country central bank. Thus the market clearing condition for home currency denominated bonds is:

$$B_t + B_t^{cb} = 0 \tag{9}$$

Foreign country bonds are held by home households, the home country central bank, foreign households, and the foreign country central bank:

$$nF_t + nF_t^{cb} + (1-n) F_t^{*} + (1-n) F_t^{cb*} = 0 \tag{10}$$

Note that in the limit where $n \to 0$, this constraint does not link home country household and central bank holdings of foreign currency denominated bonds, F_t and F_t^{cb}, and is not a constraint from the perspective of the home country.

3.3 The balance of payments identity

Central bank profits in equation (8), the central bank's balance sheet in equation (6), and the home currency bond market clearing condition in equation (9) can be substituted into the household's budget constraint in equation (2) to yield the budget constraint for the small open economy:

$$\begin{aligned} & P_t C_t - W_t H_t - \Pi_t + S_t \left(F_t + F_t^{cb}\right) - \left(1 + \Phi_{t-1} i_{t-1}^{*}\right) S_t \left(F_{t-1} + F_{t-1}^{cb}\right) \\ & = -P_t \chi \Omega(\hat{F}_t) - P_t \chi^{cb} \Omega(\hat{F}_t^{cb}) \end{aligned} \tag{11}$$

The first term in this economy-wide budget constraint, $P_tC_t - W_tH_t - \Pi_t$ represents net imports of the home country, or the negative of the trade balance and thus this economy-wide budget constraint can be rewritten as the fundamental balance of payments identity, where the current accounts equal to the capital account plus the net change in central bank foreign exchange reserves, $CA_t = KA_t + \Delta R_t$.

Specifically, the current account is equal to net exports plus net primary interest income (net interest income on foreign currency denominated bonds):

$$CA_t = W_tH_t + \Pi_t - P_tC_t + \Phi_{t-1}i^*_{t-1}S_tF_{t-1} + \Phi_{t-1}i^*_{t-1}S_tF^{cb}_{t-1} - P_t\chi\Omega(\hat{F}_t) - P_t\chi^{cb}\Omega(\hat{F}^{cb}_t)$$

The capital account is equal to the negative change in the household's stock of foreign currency denominated bonds:

$$KA_t = S_tF_t - S_tF_{t-1} \tag{12}$$

And finally the change in the central bank's stock of foreign currency denominated bonds is given by:

$$\Delta R_t = S_tF^{cb}_t - S_tF^{cb}_{t-1} \tag{13}$$

4 Proofs of two equivalence results

The Ramsey policymaker sets policy to maximize household welfare (1) subject to structural constraints that include the economy-wide budget constraint (11), the UIP condition (5), a set of first order conditions from households' utility maximization, a set of first order conditions from firms' profit-maximization and cost-minimization problems, and market clearing conditions.

We present here analytical proofs of our two equivalence results. First we show that, when households can freely buy and sell foreign currency denominated bonds, Ramsey optimal policy when the central bank adjusts its stock of foreign bonds is equivalent to the outcome under Ramsey optimal policy when the central bank does not use foreign exchange reserves as an additional policy instrument. In other words, with an open capital account this additional policy instrument is redundant.

Second, we prove that Ramsey optimal policy when the central bank uses a variable tax rate on private holdings of foreign currency denominated bonds as an additional policy instrument is equivalent to the outcome under Ramsey optimal policy when the central bank instead uses foreign exchange reserves as an additional policy instrument but households cannot buy or sell foreign bonds.

Specifically we consider Ramsey optimal policy under four policy scenarios:

1. In the first, the central bank's only instrument is the money supply. The central bank has no separate instrument for capital account management (i.e., $\tau_t = 0$ and $\hat{F}_t^{cb} = 0$). Households can buy and sell foreign bonds freely (i.e., the capital account is open).

2. In the second, the central bank has two instruments, the money supply and the variable tax rate τ_t on households' holdings of foreign currency denominated bonds.

3. In the third, the central bank has two instruments, the money supply and its stock of foreign currency denominated bonds F_t^{cb}. Households cannot buy or sell foreign bonds (i.e., the capital account is closed so $\hat{F}_t = 0$).

4. In the fourth, the central bank has two instruments, the money supply and its stock of foreign currency denominated bonds F_t^{cb}. Households can buy and sell foreign bonds freely (i.e., the capital account is open).

A crucial first step in our analytical proofs is to note that among the constraints faced by the Ramsey policymaker, the policy instruments τ_t and F_t^{cb} appear in only two conditions, the economy wide budget constraint (11) and the UIP condition (5). Thus these are the only two conditions that may differ across the four policy scenarios. All other conditions in the planner's constraint set are identical across the four scenarios.

4.1 Proof of equivalence between Scenarios 1 and 4

In both Scenario 1 and Scenario 4 the UIP condition (5) takes the following form since in both scenarios $\tau_t = 0$:

$$\frac{1+i_t}{1+\Phi_t i_t^*} = \frac{\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}} S_{t+1}\right)}{\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right)\left(S_t + P_t \chi \Omega'(\hat{F}_t)\right)}. \tag{14}$$

As discussed before, in a linearized model the $\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right)$ terms will cancel out of the numerator and denominator of the right hand side of this expression, leading to the familiar UIP condition. We will do this later in a numerical model, but for now, we leave the UIP condition in the more general form, as our equivalence proofs do not rely on being able to say $\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}} S_{t+1}\right) = \mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right) \mathrm{E}_t\left(S_{t+1}\right)$.

Scenario 1 In this scenario the central bank does not use foreign exchange reserves as an additional policy instrument, so $F_t^{cb} = F_{ss}^{cb}$ and $\hat{F}_t^{cb} = 0$. Together with the property $\Omega\left(0\right) = 0$, this implies that the economy-wide budget constraint (11) simplifies to:

$$\begin{aligned} &P_t C_t - W_t H_t - \Pi_t + S_t\left(F_t + F_{ss}^{cb}\right) - \left(1 + \Phi_{t-1} i_{t-1}^*\right) S_t\left(F_{t-1} + F_{ss}^{cb}\right) \\ &= -P_t \chi \Omega(\hat{F}_t). \end{aligned} \tag{15}$$

Scenario 4 In this scenario the central bank uses foreign exchange reserves F_t^{cb} as an additional policy instrument, so the economy-wide budget constraint (11) remains as:

$$P_tC_t - W_tH_t - \Pi_t + S_t\left(F_t + F_t^{cb}\right) - \left(1 + \Phi_{t-1}i_{t-1}^*\right)S_t\left(F_{t-1} + F_{t-1}^{cb}\right)$$
$$= -P_t\chi\Omega(\hat{F}_t) - P_t\chi^{cb}\Omega(\hat{F}_t^{cb}). \tag{16}$$

For this proof, the presence of χ (for both Scenarios 1 and 4) and χ^{cb} (for Scenario 4) is a standard device for closing the small open economy model. To serve this technical purpose without creating any material frictions or distortions otherwise, it suffices to set χ and χ^{cb} to being infinitesimally small. In fact in the current context an infinitesimally small χ also serves a dual purpose for making the capital account effectively open, which is a condition assumed in both Scenarios 1 and 4.

Hence we present without loss of generality our proof of equivalence between Scenarios 1 and 4 for the case where χ and χ^{cb} go to zero. It then follows that the right hand sides of both (15) and (16) go to zero, and so does the last term in the denominator on the right hand side of (14). As a result, the central bank's and households' holdings of foreign currency denominated bonds become perfect substitutes, and thus the central bank's use of foreign exchange reserves as an additional policy instrument in Scenario 4 does not materially relax (of course neither does it tighten) the constraint faced by the Ramsey policymaker beyond that in Scenario 1. As such, the Ramsey solution would give rise to identical current account and other welfare-relevant macroeconomic variables across Scenarios 1 and 4, and it is only a matter how the (identical) total amount of foreign currency denominated bonds held at home would be distributed between households and the central bank across the two scenarios.

4.2 Proof of equivalence between Scenarios 2 and 3

Scenario 2 In this scenario the central bank uses a capital control gate τ_t as an additional policy instrument, but it does not use foreign exchange reserves as a tool for capital account management so $F_t^{cb} = F_{ss}^{cb}$ and $\hat{F}_t^{cb} = 0$. Given the property $\Omega(0) = 0$, the economy-wide budget constraint (11) reduces to:

$$P_t C_t - W_t H_t - \Pi_t + S_t \left(F_t^{(2)} + F_{ss}^{cb} \right) - \left(1 + \Phi_{t-1} i_{t-1}^* \right) S_t \left(F_{t-1}^{(2)} + F_{ss}^{cb} \right)$$
$$= -P_t \chi \Omega \left(\hat{F}_t^{(2)} \right), \tag{17}$$

while the UIP condition (5) takes the general form:

$$\frac{1 + i_t}{1 + \Phi_t i_t^*} = \frac{\mathrm{E}_t \left(\frac{\Lambda_{t+1}}{P_{t+1}} S_{t+1} \right)}{\mathrm{E}_t \left(\frac{\Lambda_{t+1}}{P_{t+1}} \right) \left((1 - \tau_t) S_t + P_t \chi \Omega' \left(\hat{F}_t^{(2)} \right) \right)}. \tag{18}$$

Scenario 3 In this scenario the central bank uses foreign exchange reserves F_t^{cb} as an additional policy instrument, but households cannot buy or sell foreign currency denominated bonds, $\hat{F}_t = 0$. Together with the property $\Omega(0) = 0$, this implies that the economy-wide budget constraint (11) reduces to:

$$P_t C_t - W_t H_t - \Pi_t + S_t \left(F_{ss} + F_t^{cb(3)} \right) - \left(1 + \Phi_{t-1} i_{t-1}^* \right) S_t \left(F_{ss} + F_{t-1}^{cb(3)} \right)$$
$$= \quad - P_t \chi^{cb} \Omega \left(\hat{F}_t^{cb(3)} \right). \tag{19}$$

Since with a closed capital account households cannot optimally choose their holdings of foreign currency denominated bonds, the Ramsey policymaker does not face as a constraint the UIP condition (5), which is derived by combining the households' first order conditions for their optimal choices of home and foreign bonds.

Therefore, proving the equivalence between Scenarios 2 and 3 is to prove that (17)-(18) in Scenario 2 and (19) in Scenario 3 yield identical constraint sets for the Ramsey policymaker. Note that we have used the superscript (2) in equations (17)-(18) and the superscript (3) in equation (19) to make a distinction between Scenario 2 and Scenario 3 for the households' and central bank's holdings of foreign currency denominated bonds, but not for other variables.

This is the only distinction that needs to be made across these two alternative scenarios given that the two policy regimes are equivalent in terms of generating feasible choices of variables that are relevant for households' welfare.

We can set the households' adjustment cost parameter χ in Scenario 2 and the central bank's adjustment cost parameter χ^{cb} in Scenario 3 equal, say, to some positive number $\tilde{\chi}$ that can be made arbitrarily small and interpret this presence for the technical purpose of closing the small open economy model. The equivalence between Scenarios 2 and 3 certainly holds under this interpretation. But the equivalence between the two scenarios actually holds more generally. In fact it holds even when $\tilde{\chi}$ takes on any positive value. This generality can be important since $\tilde{\chi}$ may also be used to proxy foreign exchange market imperfections faced by the home country, in addition to technically close the small open economy model. Thus our proof of the equivalence between Scenarios 2 and 3 below does not restrict to the case that $\tilde{\chi}$ is an infinitesimally small number.

We take two steps to prove that (17)-(18) in Scenario 2 and (19) in Scenario 3 generate identical constraint sets for the Ramsey policymaker.

First, for any $F_t^{(2)}$ and other variables that satisfy (17)-(18) and other private sector optimality conditions under a variable capital tax rate τ_t in Scenario 2, there exists a corresponding choice of foreign exchange reserves in Scenario 3 given by:

$$\hat{F}_t^{cb(3)} = \hat{F}_t^{(2)}, \tag{20}$$

which yields identical home total holding of foreign currency denominated bonds and other variables that satisfy (19) and other private sector optimality conditions.

Second, for any variables that satisfy (19) and other private sector optimality conditions under a foreign exchange intervention $F_t^{cb(3)}$ in Scenario 3, there exists a corresponding choice of variable capital tax rate τ_t in Scenario 2 given by:

$$\tau_t = 1 + \frac{P_t}{S_t}\tilde{\chi}\Omega'\left(\hat{F}_t^{cb(3)}\right) - \frac{\left(1+\Phi_t i_t^*\right)\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}S_{t+1}\right)}{\left(1+i_t\right)S_t\mathrm{E}_t\left(\frac{\Lambda_{t+1}}{P_{t+1}}\right)}, \tag{21}$$

that via influencing the price of foreign currency denominated bonds would induce households to choose their holding of the foreign bonds equal to:

$$\hat{F}_t^{(2)} = \hat{F}_t^{cb(3)}, \tag{22}$$

which yields identical home total holding of foreign currency denominated bonds and other variables that satisfy (17)-(18) and other private sector optimality conditions.

The above two steps together establish that Scenarios 2 and 3 are isomorphic in terms of generating identical feasible choices of welfare-relevant variables for the Ramsey policymaker. This is to say that a set of welfare-relevant variables is feasible for the Ramsey policymaker in Scenario 2 if and only if it is feasible for the Ramsey policymaker in Scenario 3. This establishes the equivalence of the two scenarios. In particular, the Ramsey solution produces identical home total holding of foreign currency denominated bonds (thus identical current account as well as other welfare-relevant macroeconomic variables) across the two scenarios, although the distribution of the total holding between households and the central bank differs across the two regimes.

It is important to emphasize that the only conditions in the Ramsey policymaker's constraint set that matter for our proof of the equivalence results are the economy-wide budget constraint and the UIP condition, where all other optimality conditions relevant to private sectors are identical across the four policy scenarios. It is irrelevant to the equivalence results how exactly the functional forms describing the behavior of the private agents are specified, because such specifications will be identical across the four scenarios. It should also be stressed that the equivalence results hold without recourse to firm side specifications, and they hold under any types of frictions (nominal, financial, etc.) that we see in today's New Keynesian DSGE literature incorporated into the firm side. This means that our equivalence results apply to a wide range of macroeconomic models that are often used in New Keynesian DSGE literature. The equivalence results hold quite generally.

5 Numerical Results

When presenting numerical results, we first define some specific functional forms to replace the generalized forms in the model presented in the last section and we define the details of the firm side. We then discuss calibration of this numerical model and a mapping between observed levels of capital controls and the adjustment cost parameter χ. We then present impulse responses to a risk premium shock for some key variables in the model. Finally we present the moments of some key variables in the model and show how these change under different policy scenarios and parameter values.

5.1 Functional forms for numerical model

5.1.1 Functional forms for the household

In the numerical results the specific functional form for the household's utility function is given by:

$$U\left(C_t, \frac{M_t}{P_t}, H_t\right) = \ln\left(C_t\right) + \phi_m \ln\left(\frac{M_t}{P_t}\right) - \psi \frac{H_t^{1+\eta}}{1+\eta} \tag{23}$$

The aggregate consumption good is simply the aggregation of domestic and imported goods from individual firms aggregated in the CES function:

$$C_t = \left[\begin{array}{c} v^{\frac{1}{\theta}} \left[\left(\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n C_t^H(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\theta-1}{\theta}} \\ + (1-v)^{\frac{1}{\theta}} \left[\left(\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 C_t^F(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\theta-1}{\theta}} \end{array} \right]^{\frac{\theta}{\theta-1}} \tag{24}$$

where θ is the elasticity of substitution between home and foreign goods, and σ is the elasticity of substitution between goods from different firms within the same country.

From the aggregator function in equation (24), the demand for either the home consumption good from firm i or the foreign consumption good from firm j are given by:

$$\begin{aligned} C_t^H(i) &= v\left(\frac{P_t^H(i)}{P_t^H}\right)^{-\sigma}\left(\frac{P_t^H}{P_t}\right)^{-\theta} C_t \\ C_t^F(j) &= (1-v)\left(\frac{P_t^F(j)}{P_t^F}\right)^{-\sigma}\left(\frac{S_t P_t^F}{P_t}\right)^{-\theta} C_t \end{aligned} \tag{25}$$

and the corresponding demand functions in the foreign economy are given by:

$$\begin{aligned} C_t^{F*}(j) &= v^*\left(\frac{P_t^F(j)}{P_t^F}\right)^{-\sigma}\left(\frac{P_t^F}{P_t^*}\right)^{-\theta} C_t^* \\ C_t^{H*}(i) &= (1-v^*)\left(\frac{P_t^H(i)}{P_t^H}\right)^{-\sigma}\left(\frac{P_t^H}{S_t P_t^*}\right)^{-\theta} C_t^* \end{aligned} \tag{26}$$

where $P_t^H(i)$ is the price set by the home country firm i (in home currency), and $P_t^F(j)$ is the price set by foreign firm j (in the foreign currency). The share of imported goods in the home consumption basket is given by $1-v$ and the share of imported goods in the foreign consumption basket is given by $1-v^*$.

Thus the various price indices are given by:

$$P_t^H = \left(\frac{1}{n}\int_0^n P_t^H(i)^{1-\sigma}\,di\right)^{\frac{1}{1-\sigma}} \tag{27}$$

$$P_t^F = \left(\frac{1}{1-n}\int_n^1 P_t^F(j)^{1-\sigma}\,dj\right)^{\frac{1}{1-\sigma}}$$

$$P_t = \left[v\left(P_t^H\right)^{1-\theta} + (1-v)\left(P_t^F S_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$P_t^* = \left[v^*\left(P_t^F\right)^{1-\theta} + (1-v^*)\left(\frac{P_t^H}{S_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{28}$$

where $1-v=(1-n)\lambda$, $1-\nu^*=n\lambda$, and in the limit as $n\rightarrow 0$, $v=1-\lambda$, $\frac{1-n}{n}(1-v^*)=\lambda$, where λ is the steady-state import share.

5.1.2 Functional forms for the firm

Firm $i\in[0,n]$ produces $Y_t(i)$ of output for the domestic and export markets. The firm's total output is produced by hiring $h_t(i)$ of homogenous labor service from domestic households at nominal wage W_t. Market clearing in the labor market requires that the total demand for labor by firms is equal to the supply of labor from households: $\frac{1}{n}\int_0^n h_t(i)\,di = H_t$. Aggregate firm profits, which are returned lump-sum to households, are given by: $\Pi_t = \frac{1}{n}\int_0^n \Pi_t(i)\,di$.

The firm's output is simply a linear function of its labor input:

$$Y_t(i) = h_t(i) \tag{29}$$

Thus the firm's marginal cost is simply equal to the wage rate and firm profits are given by $\Pi_t(i) = \left(P_t^H(i) - W_t\right)\left(C_{t+\tau}^H(i) + \frac{1-n}{n}C_{t+\tau}^{H*}(i)\right)$.

In period t, the firm will be able to change its price with probability $1-\xi_p$. If allowed to change their price in period t, the firm will set prices to maximize:

$$P_t^H(i) = \arg\max_{P_t^H(i)} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \Lambda_{t+\tau}\Pi_{t+\tau}(i) \tag{30}$$

The firm that can change prices will set its price to:

$$\mathbf{P}_t^H(i) = \frac{\sigma E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \Lambda_{t+\tau} W_{t+\tau}(i) \left(C_{t+\tau}^H(i) + \frac{1-n}{n} C_{t+\tau}^{H*}(i)\right)}{(\sigma-1) E_t \sum_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \Lambda_{t+\tau} \left(C_{t+\tau}^H(i) + \frac{1-n}{n} C_{t+\tau}^{H*}(i)\right)} \tag{31}$$

Firms that can reset prices in period t will all reset to the same level, so $\mathbf{P}_t^H(i) = \mathbf{P}_t^H$. We substitute this optimal price into the price index $P_t^H = \left(\frac{1}{n}\int_0^n P_t^H(i)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$. Since a firm has a probability of $1-\xi_p$ of being able to change their price, then by the law of large numbers in any period $1-\xi_p$ percent of firms will reoptimize prices. Thus the price index can be written as:

$$P_t^H = \left(\xi_p \left(P_{t-1}^H\right)^{1-\sigma} + \left(1-\xi_p\right)\left(\mathbf{P}_t^H\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{32}$$

Thus the aggregate resource constraint for the home economy is given by:

$$H_t = \frac{1}{n}\int_0^n Y_t(i)\, di = \left[(1-\lambda)\left(\frac{P_t^H}{P_t}\right)^{-\theta} C_t + \lambda \left(\frac{P_t^H}{S_t P_t^*}\right)^{-\theta} C_t^*\right] \Delta_t^H \tag{33}$$

where the price dispersion wedge $\Delta_t^H = \frac{1}{n}\int_0^n \left(\frac{P_t^H(i)}{P_t^H}\right)^{-\sigma} di = \left(1-\xi_p\right)\left[\frac{1-\xi_p\left(1+\pi_t^H\right)^{\sigma-1}}{1-\xi_p}\right]^{\frac{\sigma}{\sigma-1}} + \xi_p\left(1+\pi_t^H\right)^\sigma \Delta_{t-1}^H$.

For the simulations the model is driven by a shock to Φ_t, the risk premium on borrowing in the foreign currency. This shock follows an exogenous AR(1) process and is the only source of stochastic fluctuations in the model.

5.1.3 Functional form of the bond adjustment costs

The functional form of the foreign currency denominated bond adjustment cost functions for the household and central bank are given by:

$$\Omega\left(\hat{x}_t\right) = \frac{1}{2}\left(\frac{S_t}{P_t}\right)^2 \left(x_t - x_{ss}\right)^2, \tag{34}$$

for $x_t = F_t$ or F_t^{cb}.

5.1.4 Calibration of the numerical model

The full list of model parameters are presented in Table 4. The first five parameters, the Calvo price stickiness parameter, the discount factor, the labor supply elasticity, the elasticity of substitution across goods from firms in the same country, and the elasticity of substitution between home and foreign goods, are all set to values commonly found in the literature.

The steady-state import share $\lambda = 0.25$. The coefficient on labor in the household's utility function, ψ, is set such that in the steady state, household consumption and labor effort are both equal to one. The coefficient on real money balances in the household's utility function, ϕ_m, is set such that the steady state velocity of money is equal to one.

The only exogenous shock in the model, Φ_t follows an AR(1) process with persistence 0.90. The standard deviation of the shock is normalized to one.

We must assume a steady state composition of the central bank's balance sheet. Assume that 90% of the central bank's assets are home currency denominated bonds and 10% are foreign currency denominated bonds.

In the analytical proofs in the previous section we simply set the bond adjustment costs, χ and χ^{cb}, to a small and positive number to close the model. In these numerical simulations we can use the household's bond adjustment cost to calibrate the openness of the capital account. We assume that the central bank can freely buy and sell foreign bonds, and we set $\chi^{cb} = 10^{-5}$. If households can buy and sell foreign bonds just as freely, $\chi - \chi^{cb} = 0$. As capital account restrictions increase, $\chi - \chi^{cb}$ increases.

It is possible to create a mapping from an observed measure of capital account restrictions, like the Chinn-Ito index used in Section 2, to the model parameter χ. Begin by considering the empirical relationship between the log of the Chinn-Ito index and the standard deviation of reserve accumulation relative to the standard deviation of the current account, $SD(\Delta R)/SD(CA)$, or the empirical relationship between the log of the Chinn-Ito index and the correlation between the current account and the capital account in Table 3. The two-stage-least-squares regression of $SD(\Delta R)/SD(CA)$ on the log of $KAOpen$ yields a constant term of 1.08 and a coefficient of 0.10 and the two-stage-least-squares regression of $Corr(CA, KA)$ on the log of $KAOpen$ yields a constant term of 0.44 and a coefficient of -0.07; the fitted values of $SD(\Delta R)/SD(CA)_{\text{data}}$ or $Corr(CA, KA)_{\text{data}}$ as a function of $KAOpen$ from these estimated parameters are plotted in the top panel of Figure 2.

The next step is to calculate $SD(\Delta R)/SD(CA)_{\text{model}}$ and $Corr(CA, KA)_{\text{model}}$ as a function of the household's bond adjustment cost parameter χ. This is plotted in the second panel in Figure 2.

In the top panel, each value of $KAOpen$ leads to fitted values of $SD(\Delta R)/SD(CA)_{\text{data}}$ and $Corr(CA, KA)_{\text{data}}$. In the middle panel each value of χ leads to values of $SD(\Delta R)/SD(CA)_{\text{model}}$ and $Corr(CA, KA)_{\text{model}}$ in model simulations. For each value of $KAOpen$, we choose the value of χ that minimizes the sum of squared deviations between the values of $SD(\Delta R)/SD(CA)$ or $Corr(CA, KA)$ in the model and the data:

$$\chi(KAOpen) = \arg\min \left[\begin{array}{c} \left(SD(\Delta R)/SD(CA)_{\text{model}} - SD(\Delta R)/SD(CA)_{\text{data}}\right)^2 \\ + \left(Corr(CA, KA)_{\text{model}} - Corr(CA, KA)_{\text{data}}\right)^2 \end{array} \right]$$

The mapping between $KAOpen$ in the data and the parameter χ in the model is shown in the third panel of Figure 2. When $KAOpen$ is close to zero, indicating an open capital account, the corresponding value of χ is close to zero. When $KAOpen$ is close to 1, indicating a closed capital account, the corresponding value of χ is around 0.10, which, under this parameterization is the value of the household bond adjustment cost that would effectively close the capital account.

5.2 Numerical results when capital account is perfectly open or closed

First we will consider results from numerical simulations of the model under the extreme cases of a perfectly open capital account or a perfectly closed capital account. This is akin to the four scenarios presented earlier in the analytical proofs in Section 4. Recall that scenarios 1 and 2 are the two scenarios where the central bank does not engage in sterilized foreign exchange intervention. In scenario 1 the central bank's only instrument is the money supply (and thus the nominal interest rate). In scenario 2 the central bank has two instruments, the money supply and the variable tax τ_t on the purchases of foreign bonds. In these two scenarios, assume that private agents face the same bond adjustment costs as the central bank, $\chi = \chi^{cb}$.

In scenarios 3 and 4 the central bank can engage in sterilized foreign exchange intervention. In scenario 3 the capital account is closed, which in this numerical model is achieved in the limit as $\chi \to \infty$. In scenario 4 the capital account is open, which in this numerical model is achieved in the limit as $\chi \to 0$.

The impulse responses are presented in Figures 3 and 4. These plot the responses to a one standard deviation shock to the risk premium on foreign borrowing (a 100 basis point shock). In the impulse responses, scenario 1 is represented by the RED impulse response, scenario 2 is represented by the BLUE impulse response, scenario 3 is represented by the GREEN impulse response, and scenario 4 is represented by the BLACK impulse response.

The responses of the home country current account, capital account, change in central bank foreign exchange reserves, and the variable tax rate τ_t are presented in Figure 3. The responses for the current account, capital account, and change in reserves measure the deviation from the steady state, in percent of GDP. And thus a 100 basis point increase in the risk premium leads to an increase in the current account of about 5-7% of GDP.

Turning to the first policy scenario, where the central bank has no separate instrument for capital account management (the Red impulse response). The increase in the risk premium on foreign borrowing leads to a fall in net capital inflows (an increase in the capital account), and when the central bank cannot adjust the stock of foreign exchange reserves, the capital account is simply equal to the current account.

In the second policy scenario, the central bank can adjust the variable tax τ_t on foreign borrowing (the Blue impulse response). The increase in the current account (the increase in net savings) in scenario 1 is greater than optimal, so if given a separate instrument to manage the capital account, the central bank will cut τ_t. As seen in the household budget constraint in equation (2), this will raise the price of foreign currency denominated bonds, encouraging households to sell foreign bonds (borrow more from abroad, even at the higher interest rate since the fall in τ will raise the price of foreign bonds, and thus lower their return, partially offsetting the exogenous increase in the interest rate of foreign currency denominated bonds). Given this policy intervention the increase in the capital account (and increase in the current account) is not as severe as it would have been when the central bank has no capital account management instrument.

In the third policy scenario, the variable tax rate $\tau_t = 0$, but the central bank can adjust its stock of foreign currency bonds (the Green impulse response). In this scenario we assume that households cannot buy and sell foreign bonds freely. Here the capital account is equal to zero, and central bank reserve accumulation increases given the higher interest rate on foreign currency denominated bonds. But since all foreign borrowing and lending is an instrument of the central bank, the increase in net saving in the third policy scenario is the welfare maximizing level.

In the fourth policy scenario, again the variable tax rate $\tau_t = 0$ and the central bank can adjust its stock of foreign currency bonds (the Black impulse response). But in this scenario households can buy and sell foreign bonds freely. Here the central bank will increase reserve accumulation following the increase in the foreign interest rate, but households will also increase their stock of foreign bonds. Unlike the third policy scenario, the central bank no longer has full control over the amount of net national saving, and the amount of net saving is higher than optimal in the fourth policy scenario.

Of course, in order to close the model for numerical simulation, in Scenarios 1 and 4, χ needs to be positive, no matter how small it is. Then, given the convexity of the adjustment cost, Jensen's inequality implies that letting the central bank's holding of foreign currency denominated bond to bear some burden of adjustments in response to a shock, that is, Scenario 4, can help reduce total adjustment cost relative to that in Scenario 1, for similar total amount of foreign currency denominated bond held at home. Nevertheless, the reduction is small, so is the effect on the UIP condition (14), given that χ is infinitesimally small. This is to say that the central bank's use of foreign exchange reserves as an additional policy instrument in Scenario 4 only expands a little the Ramsey planner's constraint set relative to that in Scenario 1. As a consequence, the Ramsey solution produces approximately equivalent current account and other welfare-relevant macroeconomic variables across the two scenarios. In the case that χ is a finite positive number, approximate although not exact equivalence holds between Scenarios 2 and 3.

The responses of other macro variables like domestic inflation, the real exchange rate, the nominal interest rate on domestic bonds, and the domestic money supply, under these four policy scenarios are shown in Figure 4. Just as we see in the current account graph in Figure 3, the graph presents the responses from four policy scenarios, but only two lines appear in the graph. This is because the responses from scenarios 1 and 4 and the responses from scenarios 2 and 3 lie on top of one another. Thus in numerical simulations of the model, there is a equivalence between policy scenarios 1 and 4 and between scenarios 2 and 3.

The variance of the current account as a function of χ, the household's foreign bond adjustment cost, is presented in Figure 5. The green line in the figure presents the variance of the current account as a function of χ when the central bank engages in sterilized foreign exchange intervention. When χ is very small, this corresponds to Scenario 4 in the previous section. As $\chi \to \infty$, this corresponds to Scenario 3 in the previous section.

The red dashed line in the figure is a horizontal line representing the variance of the current account when the central bank only has one instrument, and does not have a separate instrument for capital account management (Scenario 1). The blue dashed line in the figure is a horizontal line representing the variance of the current account when the central bank can vary a tax rate on foreign bond holdings as a second instrument for capital account management (Scenario 2).

Not surprisingly, given the equivalence between scenarios 1 and 4 and the equivalence between 2 and 3, when χ is very small the variance of the current account when the central bank engages in sterilized foreign exchange intervention is nearly equal to the variance of the current account when the central bank has no separate instrument for capital account management. As $\chi \to \infty$ and the household's adjustment cost becomes larger, the model approaches one where the capital account is closed, $\hat{F}_t = 0$.

5.3 Numerical results under observed levels of capital account openness

Figure 5 highlights the equivalence between the various scenarios under the extreme cases of $\chi \to 0$ or $\chi \to \infty$, but it is also interesting to note that the model with sterilized foreign exchange intervention produces dynamics similar to the model with a variable tax on foreign bond holdings under more reasonable values of χ. Recall from Figure 1 that the average value of the Chinn-Ito capital account restrictions index, $KAOpen$, in the advanced economies is close to 0 and in the emerging markets is around 0.5. The bottom panel of Figure 2 provides a mapping from the Chinn-Ito index to the parameter χ in the model. The mapping shows that when $\chi = 0.061$, the simulations of the numerical model yield dynamics similar to what we see in the data when $KAOpen = 0.50$. Figure 5 shows that when $\chi = 0.061$ the model with sterilized foreign exchange intervention as the central bank's instrument of capital account management leads to a variance of the current account that is very similar to the variance in the model where the central bank has a variable tax τ_t as an instrument of capital account management.

Figure 6 reproduces the same impulse responses from earlier in Figure 4, except now $\chi \approx 0.061$ in the scenario with sterilized foreign exchange intervention. In Figure 4, when χ was very large, the model with sterilized foreign exchange intervention and the model with the variable capital tax τ_t yielded nearly identical impulse responses. In Figure 6, when the model is calibrated to match the level of capital account restrictions observed in most emerging markets, the two sets of impulse responses are very similar. Thus while the model with a variable capital tax τ_t is identical to sterilized foreign exchange intervention only in the extreme case of when the capital account is closed, it is still a reasonable approximation under more reasonable levels of capital account restrictions.

6 Conclusion

When private agents cannot freely buy and sell foreign bonds, equilibrium under optimal sterilized foreign exchange intervention is equivalent to equilibrium when agents can buy and sell foreign bonds subject to a tax τ_t that is chosen optimally by the central bank. When private agents can freely buy and sell foreign bonds, sterilized foreign exchange intervention has no effect at all.

This explains the descriptive statistics on capital controls and reserve accumulation that were presented in section 2. Emerging markets tend to have higher capital controls, meaning that private agents are less free to buy and sell foreign bonds. Given this, sterilized foreign exchange intervention is a potential policy instrument available to emerging market central banks. Capital account restrictions are much lower, and close to zero in many advanced economies. So sterilized foreign exchange intervention would be ineffective in many advanced economies. This explains why reserve accumulation tends to be high and volatile in the emerging markets, but much smaller and much less volatile in the advanced economies.

At the same time this provides justification for the use of a variable capital tax τ_t in a model as a capital account management measure even when these variable capital controls are not observed in practice. An optimally chosen variable capital tax τ_t simply mimics the dynamics that we would see in a model with a central bank balance sheet and sterilized foreign exchange intervention when households cannot freely buy and sell foreign bonds.

But this neglects some key issues that have to be addressed when discussing optimal sterilized foreign exchange intervention. Foreign exchange intervention faces a key non-linearity, central bank foreign exchange reserves can't be negative.[12] This non-linearity introduces two problems that we abstract from in this paper. In reality there is a limit to how far reserves can fall, and thus a limit to their effectiveness in response to certain types of shocks when reserve balances are low. In this paper we abstracted from this feature of reality.

In addition, this potential ineffectiveness when balances are low leads central banks to hold a precautionary level of foreign exchange reserves. By construction, central bank foreign exchange reserves are held as safe, highly liquid assets, the type of assets that pay a low rate of return. Sterilized foreign exchange intervention involves financing the purchase of these low yielding assets by selling higher yielding domestic bonds. This spread is a real cost to holding a large precautionary stock of foreign exchange reserves. We abstracted from that cost here. We assumed that home and foreign bonds have the same steady state return, and the central bank can adjust its stock of foreign bonds nearly costlessly.

An interesting direction for future research is the normative paper that we intend to be the complement to this positive paper. This direction for further research will take into account these two features arising out of the zero-lower-bound on reserve balances when deriving the optimal stock of central bank foreign exchange reserves and optimal foreign exchange intervention.

[12] Theoretically, reserve balances can be negative when one considers central bank swap lines, like the swap lines that the Federal Reserve established with a number of foreign central banks between 2007 and 2013. Similarly in 2010, China, Japan, South Korea, and the members of the Association of South East Asian Nations established the Chiang Mai Initiative to share a pool of foreign exchange reserves. But apart from these few cases, which are the exceptions that prove the rule, central bank reserve balances face a zero lower bound.

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A Appendix

In this section, we report the structural equations.

A.1 Block of equations for home country households:

Variables: U_t C_t Λ_t w_t m_t C_t^H C_t^F $\tilde{C}_t^H$ π_t f_t b_t Π_t^{cb} r_t

Structural equations

$U_t = \log(C_t) + \phi_m \log(m_t) - \phi_h \frac{H_t^{1+\eta}}{1+\eta}$

$\frac{1}{C_t} = \Lambda_t$

$\Lambda_t = \beta \Lambda_{t+1} (1 + r_t)$

$\psi \frac{H_t^\eta}{C_t^{-1}} = w_t$

$m_t = \phi_m C_t \frac{1+i_t}{i_t}$

$C_t^H = (1-\lambda) \left(p_t^H\right)^{-\theta} C_t$

$C_t^F = \lambda \left(Q_t p_t^{F*}\right)^{-\theta} C_t$

$\tilde{C}_t^H = \lambda \left(p_t^{H*}\right)^{-\theta} C_t^*$

$1 = \left[(1-\lambda)\left(p_t^H\right)^{1-\theta} + \lambda \left(Q_t p_t^{F*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$

$b_t + b_t^{cb} = 0$

$C_t + m_t + b_t + Q_t (1-\tau_t) f_t + \frac{\chi_t}{2} \left(Q_t (f_t - f_{ss})\right)^2 + Q_t \tau_t f_t = p_t^h y_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1}) b_{t-1}}{1+\pi_t} + \left(1 + \Phi_{t-1} i_{t-1}^*\right) \frac{Q_t f_{t-1}}{1+\pi_t^*} + \Pi_t^{cb}$ (note that the tax τ is rebated lump sum)

$\Pi_t^{cb} = i_{t-1} \frac{b_{t-1}^{cb}}{1+\pi_t} + i_{t-1}^* \frac{Q_t f_{t-1}^{cb}}{1+\pi_t^*} + \left(\frac{Q_t}{1+\pi_t^*} - \frac{Q_{t-1}}{1+\pi_t}\right) f_{t-1}^{cb}$

$1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$

A.2 Block of equations for home country firms

Variables: MC_t H_t Y_t Δ_t^H p_t^H K_t^H F_t^H π_t^H π_t^{H*} p_t^{H*} p_t^F π_t^F

Structural equations

$MC_t = w_t$

$Y_t = H_t$

$$Y_t = \left[(1-\lambda)\left(p_t^H\right)^{-\theta} C_t + \lambda \left(\frac{p_t^H}{Q_t}\right)^{-\theta} C_t^*\right] \Delta_t^H$$

$$\Delta_t^H = \left(1-\xi_p\right)\left[\frac{1-\xi_p\left(1+\pi_t^H\right)^{\sigma-1}}{1-\xi_p}\right]^{\frac{\sigma}{\sigma-1}} + \xi_p \left(1+\pi_t^H\right)^{\sigma} \Delta_{t-1}^H$$

$$\frac{K_t^H}{F_t^H} = \left[\frac{1-\xi_p\left(1+\pi_t^H\right)^{\sigma-1}}{\left(1-\xi_p\right)}\right]^{\frac{1}{1-\sigma}}$$

$$K_t^H = C_t^{-1}\mu_t MC_t\left((1-\lambda)\left(p_t^H\right)^{-\theta} C_t + \lambda \left(\frac{p_t^H}{Q_t}\right)^{-\theta} C_t^*\right) + \beta\xi_p\left(1+\pi_{t+1}^H\right)^{\sigma} K_{t+1}^H$$

$$F_t^H = C_t^{-1}p_t^H\left((1-\lambda)\left(p_t^H\right)^{-\theta} C_t + \lambda \left(\frac{p_t^H}{Q_t}\right)^{-\theta} C_t^*\right) + \beta\xi_p\left(1+\pi_{t+1}^H\right)^{\sigma-1} F_{t+1}^H$$

$$\left(1+\pi_t^H\right) = \left(1+\pi_t\right)\frac{p_t^H}{p_{t-1}^H}$$

$$\left(1+\pi_t^{H*}\right) = \left(1+\pi_t^*\right)\frac{p_t^{H*}}{p_{t-1}^{H*}}$$

$$1+\pi_t^F = \left(1+\pi_t\right)\frac{p_t^F}{p_{t-1}^F}$$

$$p_t^{H*} = \frac{p_t^H}{Q_t}$$

$$p_t^F = Q_t p_t^{F*}$$

A.3 Block of equations for the home country central bank

Variables: b_t^{cb} i_t τ_t f_t^{cb}

Policy equations:

Depending on the policy scenario, you may block the equation that $\tau_t = 0$ (so when this is blocked, τ_t is chosen optimally, or you may block the equation that $f_t^{cb} = f_{ss}^{cb}$, if this is blocked, f^{cb} is chosen optimally)

$$m_t = b_t^{cb} + Q_t f_t^{cb}$$

$f_t^{cb} = f_{ss}^{cb}$ (if not Ramsey)

$\tau_t = 0$ (if not Ramsey)

A.4 Block of equations for international linkages

Variable: Q_t ca_t ka_t res_t

Structural equations:

$$\frac{(1+i_t)\left(1+\pi^*_{t+1}\right)}{(1+\Phi_t i^*_t)(1+\pi_{t+1})} = \frac{Q_{t+1}}{Q_t(1-\tau_t+\chi_t Q_t(f_t-f_{ss}))}$$

$$ca_t = p^h_t y_t - C_t + \Phi_{t-1} i^*_{t-1} Q_t \frac{f_{t-1}}{1+\pi^*_t} + i^*_{t-1} Q_t \frac{f^{cb}_{t-1}}{1+\pi^*_t} - \frac{\chi_t}{2}\left(Q_t\left(f_t - f_{ss}\right)\right)^2$$

$$ka_t = -\left(Q_t f_t - Q_t \frac{f_{t-1}}{1+\pi^*_t}\right)$$

$$res_t = Q_t f^{cb}_t - Q_t \frac{f^{cb}_{t-1}}{1+\pi^*_t}$$

A.5 Exogenous Shocks

Variables: Φ_t

Equations:

$$\Phi_t = \left(1-\rho^\Phi\right) + \rho^\Phi \Phi_{t-1} + \varepsilon^\Phi$$

Table 1: Standard deviation of the subcomponents of capital outflows and inflows.

Standard Deviation:	1990-2015		1990-2007	
	Advanced	Emerging	Advanced	Emerging
CA	1.94	3.09	2.00	2.80
KA	2.27	3.35	2.14	2.37
ΔR	0.74	3.15	0.44	3.04
OF	7.71	2.28	6.81	1.89
OF^{FDI}	2.34	0.73	2.33	0.46
OF^{P}	3.17	0.94	2.56	0.85
OF^{Ot}	4.91	1.66	3.71	1.31
IF	7.91	3.45	7.29	2.61
IF^{FDI}	2.07	1.43	2.12	1.32
IF^{P}	3.51	1.63	3.22	1.41
IF^{Ot}	5.25	2.27	3.70	1.98

Notes: All capital flows are normalized by GDP.

Table 2: Correlations between the subcomponents of capital inflows and outflows.

	1990-2015				1990-2007		
	Advanced ΔR	Emerging ΔR			Advanced ΔR	Emerging ΔR	
OF	-0.19	0.01		OF	-0.05	0.13	
OF^{FDI}	0.01	0.08		OF^{FDI}	0.04	0.18	
OF^{PD}	-0.01	0.08		OF^{PD}	-0.07	0.14	
OF^{Ot}	-0.27	-0.10		OF^{Ot}	-0.07	-0.05	
IF	-0.09	0.48		IF	-0.05	0.44	
IF^{FDI}	-0.06	0.20		IF^{FDI}	0.00	0.21	
IF^{PD}	-0.02	0.36		IF^{PD}	-0.08	0.40	
IF^{Ot}	-0.16	0.37		IF^{Ot}	0.00	0.41	
Advanced:				Advanced:			
	CA	KA	ΔR		CA	KA	ΔR
CA	1.00			CA	1.00		
KA	0.81	1.00		KA	0.88	1.00	
ΔR	0.04	-0.21	1.00	ΔR	0.04	-0.08	1.00
Emerging:				Emerging:			
	CA	KA	ΔR		CA	KA	ΔR
CA	1.00			CA	1.00		
KA	0.45	1.00		KA	0.46	1.00	
ΔR	0.42	-0.50	1.00	ΔR	0.49	-0.37	1.00

Notes: All capital flows normalized by GDP.

Table 3: Results from univariate regressions of capital flow moments on Chinn-Ito capital account openness index.

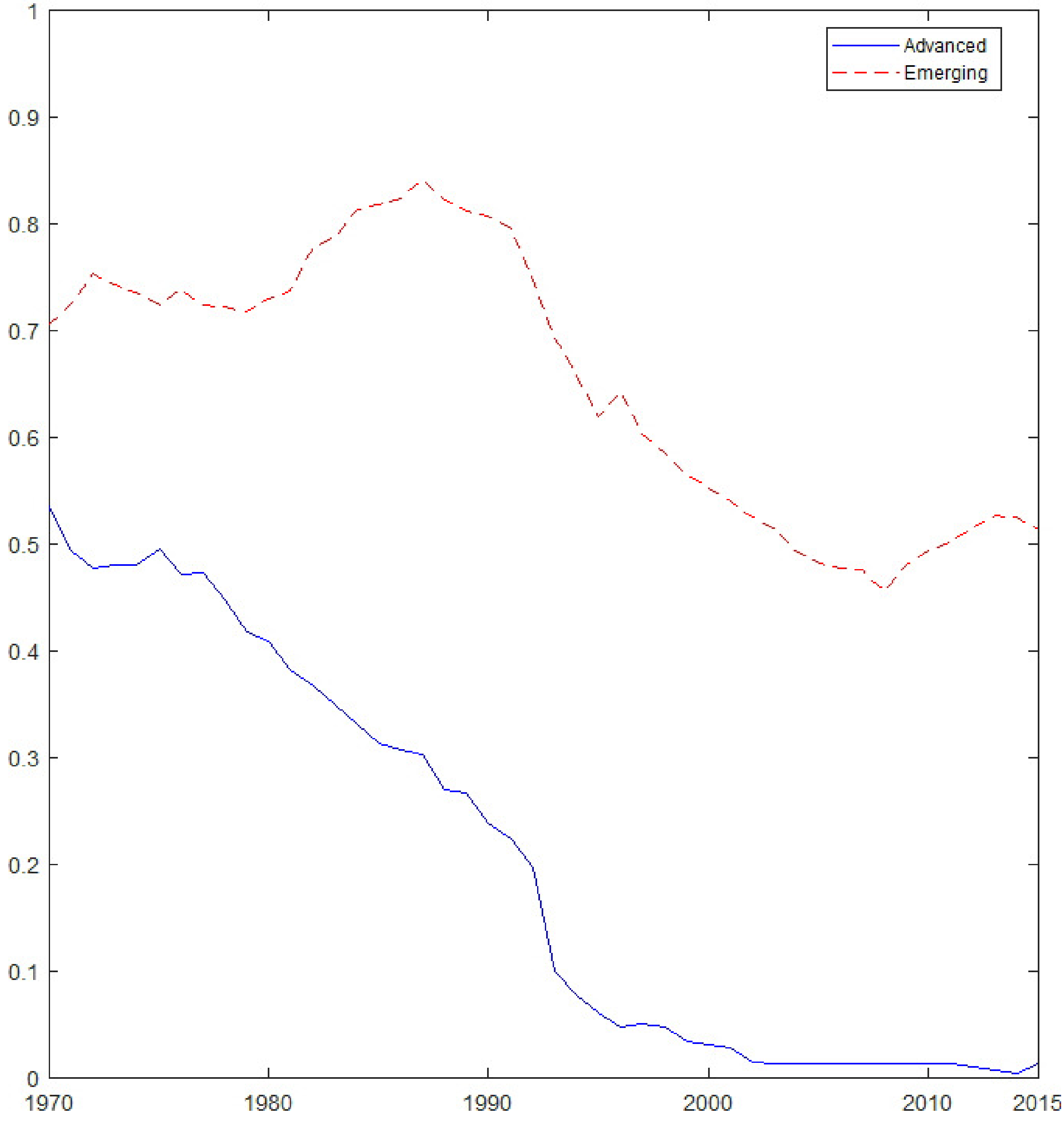
	OLS				2SLS			
	Constant		$\ln(KAOpen)$		Constant		$\ln(KAOpen)$	
Dependent Variable:								
$SD(\Delta R)/SD(CA)$	0.98***	(0.08)	0.04	(0.03)	1.08***	(0.10)	0.10**	(0.04)
$SD(\Delta R)/SD(KA)$	0.81***	(0.06)	0.07***	(0.02)	0.86***	(0.06)	0.10***	(0.03)
$SD(\Delta R)/SD(IF)$	0.90***	(0.07)	0.14***	(0.03)	0.97***	(0.08)	0.18***	(0.03)
$SD(\Delta R)/SD(OF)$	2.20***	(0.29)	0.37***	(0.11)	2.49***	(0.33)	0.54***	(0.14)
$Corr(CA, KA)$	0.50***	(0.04)	-0.04**	(0.02)	0.44***	(0.05)	-0.07***	(0.02)
$Corr(CA, \Delta R)$	0.29***	(0.04)	0.06***	(0.02)	0.33***	(0.05)	0.08***	(0.02)
$Corr(KA, \Delta R)$	-0.39***	(0.04)	0.01	(0.02)	-0.41***	(0.05)	0.00	(0.02)
$Corr(\Delta R, OF)$	0.02	(0.04)	0.03*	(0.01)	0.02	(0.04)	0.03	(0.02)
$Corr(\Delta R, OF^{FDI})$	0.05	(0.04)	0.01	(0.01)	0.03	(0.04)	0.00	(0.02)
$Corr(\Delta R, OF^{P})$	0.03	(0.03)	0.02	(0.01)	0.03	(0.04)	0.01	(0.02)
$Corr(\Delta R, OF^{Ot})$	-0.03	(0.04)	0.02	(0.02)	-0.03	(0.04)	0.02	(0.02)
$Corr(\Delta R, IF)$	0.41***	(0.04)	0.06***	(0.02)	0.43***	(0.05)	0.07***	(0.02)
$Corr(\Delta R, IF^{FDI})$	0.19***	(0.04)	0.03*	(0.02)	0.22***	(0.04)	0.04**	(0.02)
$Corr(\Delta R, IF^{P})$	0.26***	(0.04)	0.04***	(0.01)	0.25***	(0.04)	0.03	(0.02)
$Corr(\Delta R, IF^{Ot})$	0.33***	(0.04)	0.07***	(0.02)	0.37***	(0.05)	0.09***	(0.02)

Notes: Prior to taking the log, we add 0.001 to each observation of $KAOpen$. Coefficient and standard error results from the univarate regression of the dependent variable on a constant term and the Chinn-Ito capital account openness index (where 0 is open, and 1 is a closed capital account). Results are from a cross-sectional regression over 89 countries, where moments are calculated over the 1990-2015 period and the independent variable is the average value of the capital account openness index over the 1990-2015 period. Standard errors in parentheses. All capital flows are normalized by GDP. In the 2SLS regression the Chinn-Ito index is instrumented by a measure of domestic financial reform. ***/**/* denotes significance at the 1/5/10% level.

Table 4: Model parameter values.

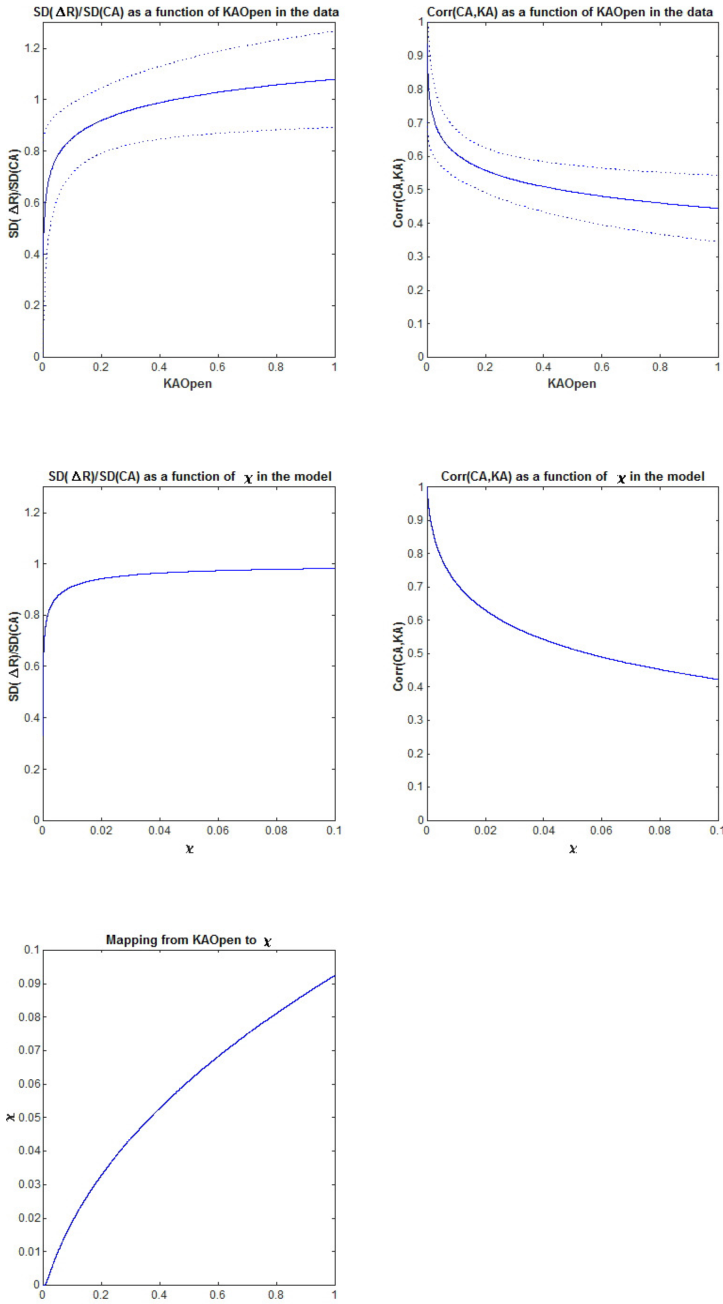
Parameters	Values	Explanation
ξ_p	0.75	Percent of firms that cannot change price in a given period
β	0.99	Subjective discount factor
η	1	Inverse of the Frisch elasticity
σ	10	Elasticity of substitution among differentiated goods from same country
θ	1.5	Elasticity of substitution between home and foreign goods
λ	0.25	Steady-state import share
ψ	$\frac{\sigma-1}{\sigma}$	disutility of labor
ϕ_m	$1-\beta$	coefficient on real money balances in utility
χ^{cb}	10^{-5}	central bank's foreign bond adjustment cost
χ	10^{-5}	household's foreign bond adjustment cost

Figure 1: Unweightd average of the Chinn-Ito index of capital account restrictions across 23 advanced countries and 66 emerging markets..



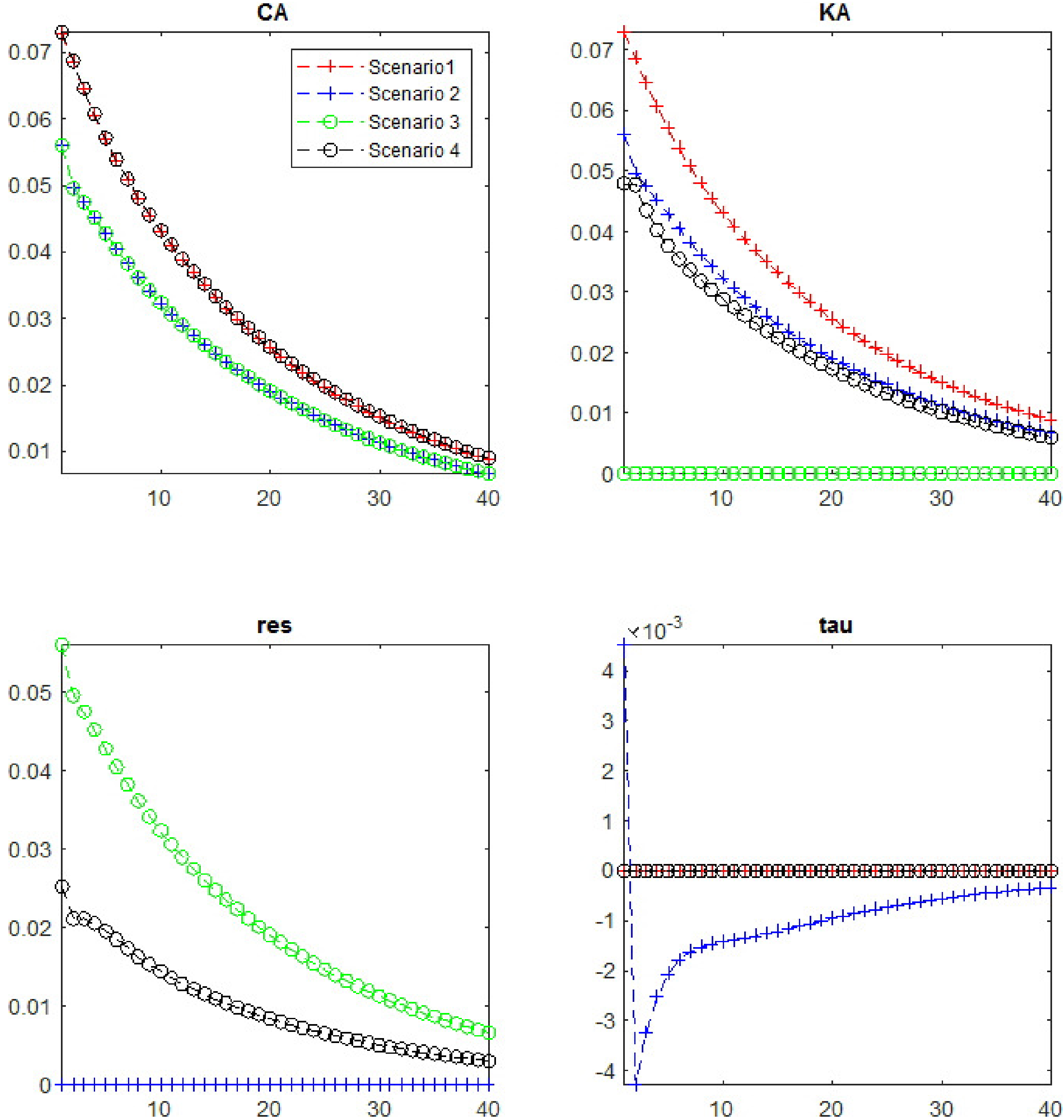
Notes: Simple average of the Chinn-Ito capital account restrictions index, normalized on a 0-1 scale (where 0 represents an open capital account).

Figure 2: The implied mapping between the capital controls index in the data and a bond adjustment cost in the model.



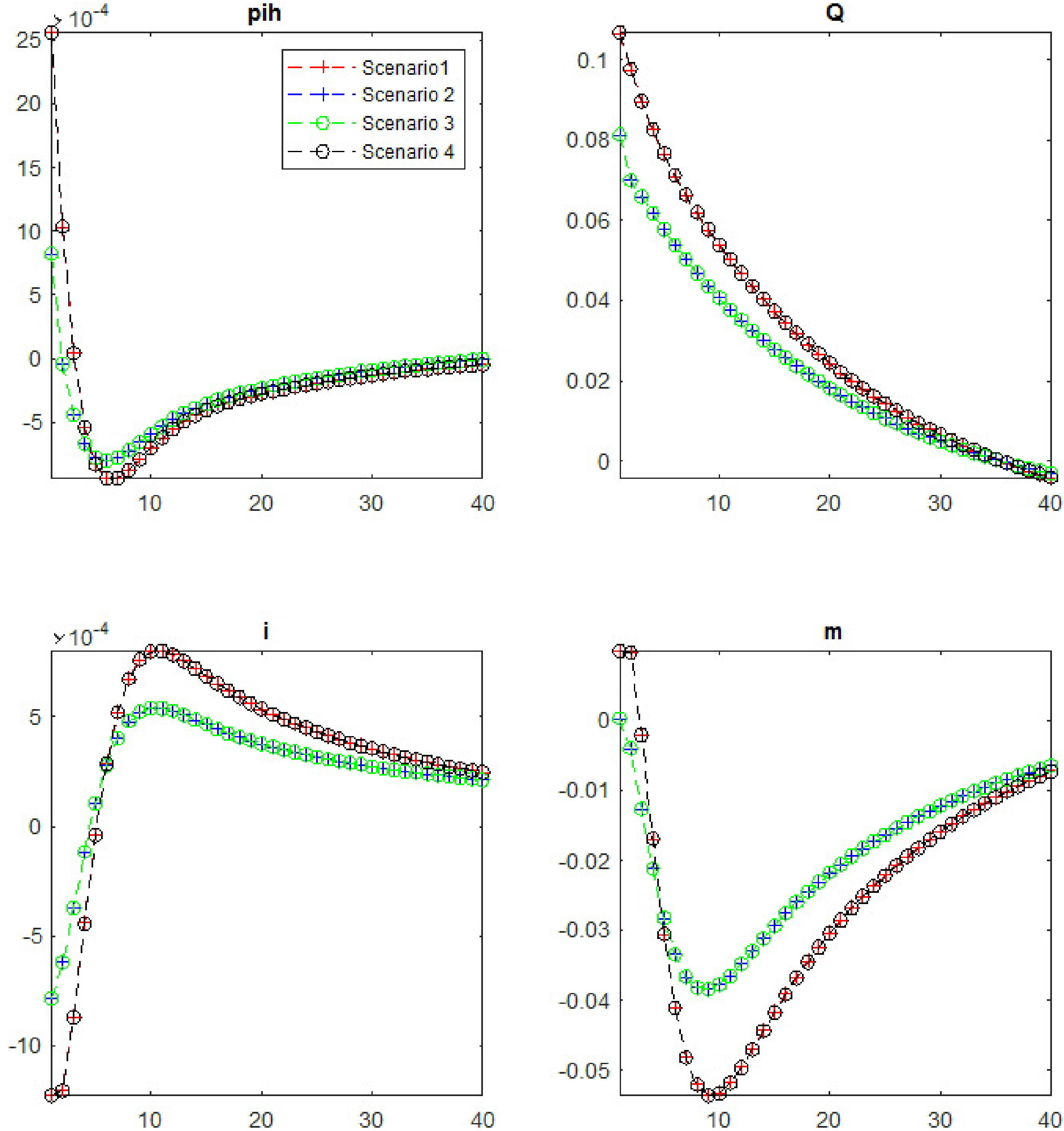
Notes: The top panel graphs are the fitted values from the TSLS estimation on $KAOpen$ shown with 95% confidence bands, the middle panels are from numerical simulations of the model as χ increases. The third graph maps the value of χ for every value of KAOpen which would make the graph of $Corr\,(CA, KA)$ as a function of KAOpen in numerical simulations of the model identical to the fitted value in the first graph.

Figure 3: Responses to a positive risk premium shock.



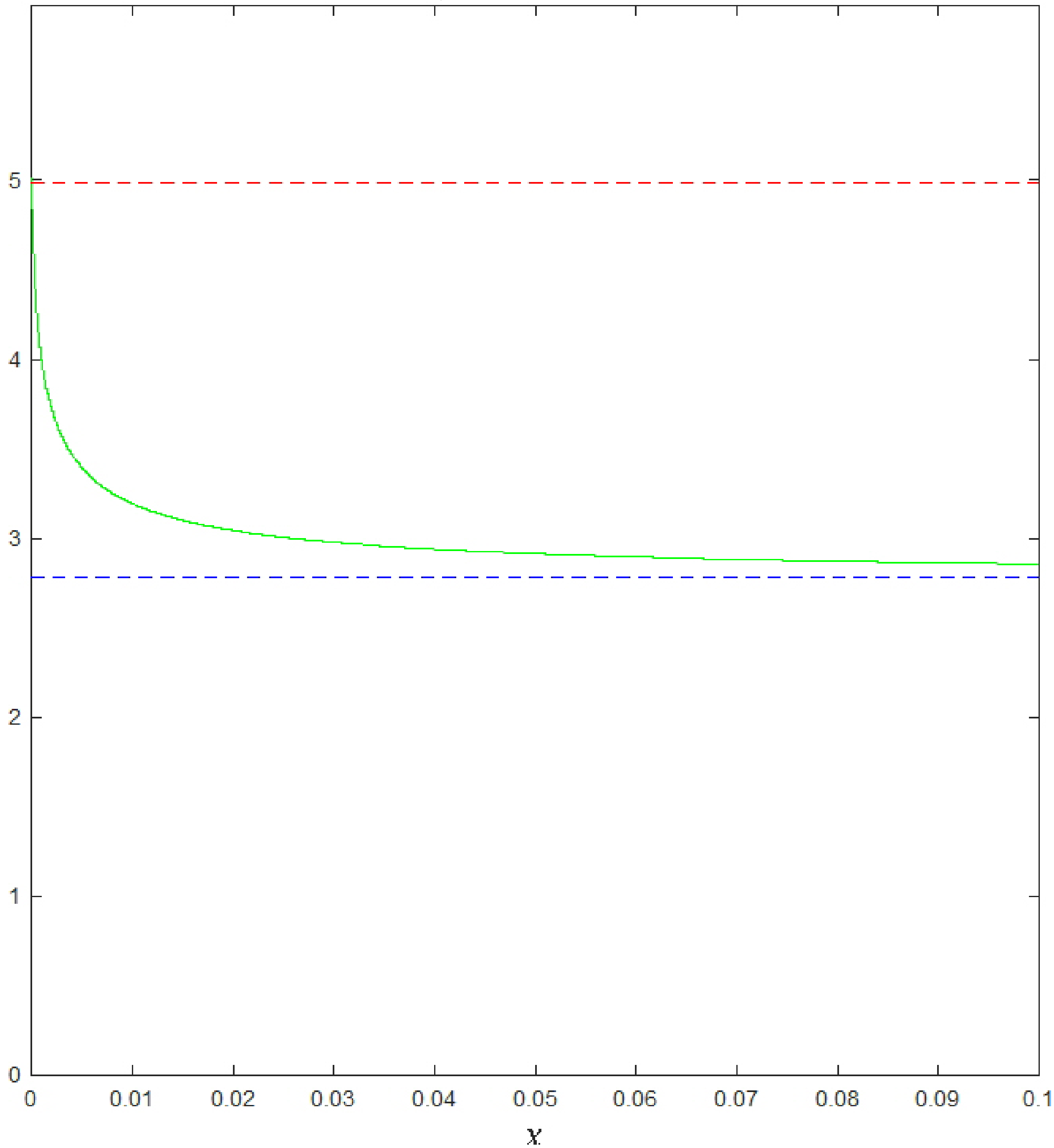
Notes: Red line: The central bank has no capital account management instrument, Blue: The central bank can adjust a variable tax to buying foreign bonds, Green: The central bank can buy foreign bonds, but households cannot, Black: The central bank and households can both buy foreign bonds freely.

Figure 4: Responses to a positive risk premium shock.



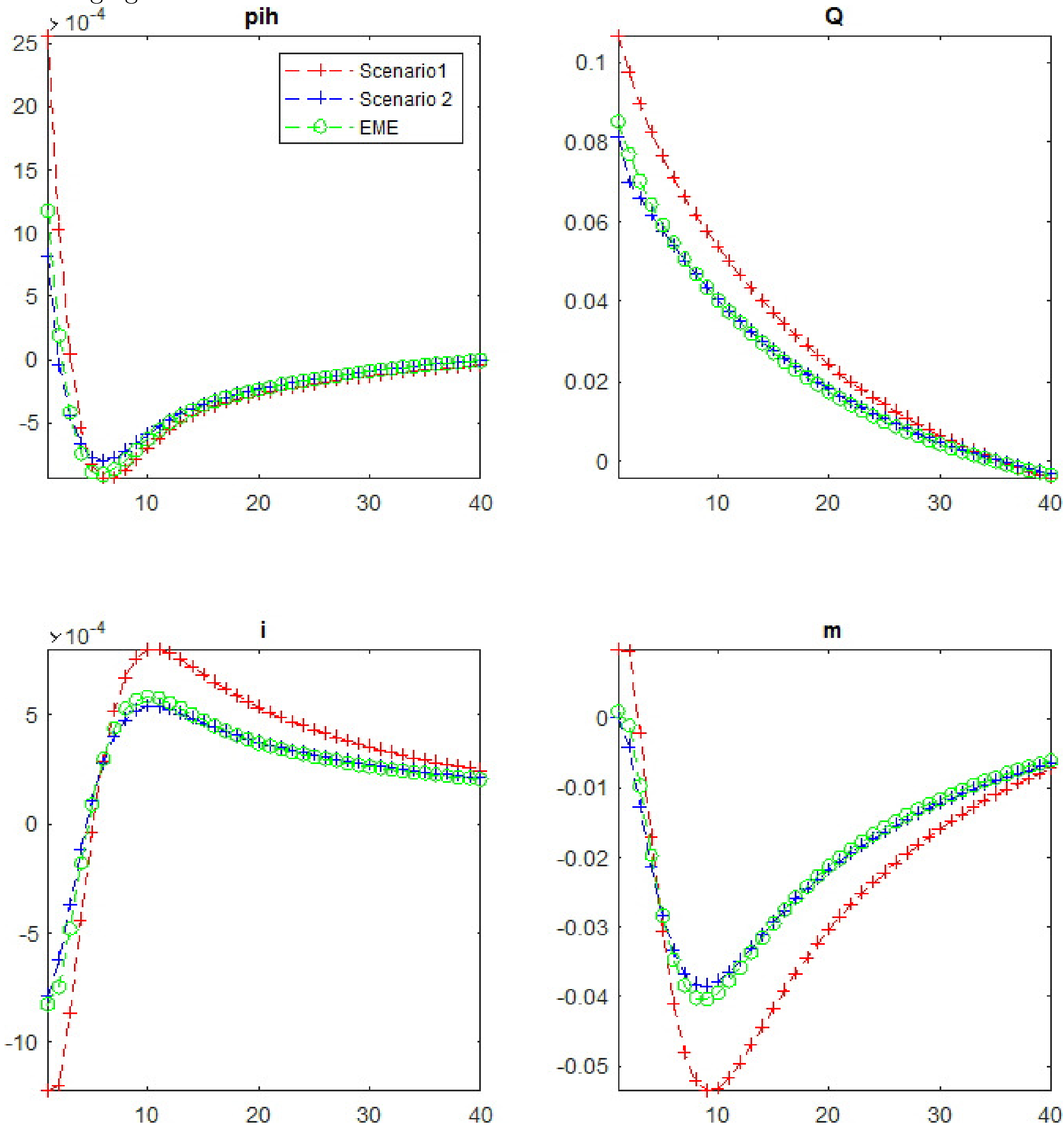
Notes: Red line: The central bank has no capital account management instrument, Blue: The central bank can adjust a variable tax to buying foreign bonds, Green: The central bank can buy foreign bonds, but households cannot, Black: The central bank and households can both buy foreign bonds freely.

Figure 5: Variance of the current account under sterilized foreign exchange intervention, as a function of the household's bond adjustment cost.



Notes: The horizontal red dashed line is the variance when the central bank does not engage in sterilized foreign exchange intervention (Scenario 1). The horizontal blue dashed line is the variance when the central bank can set a variable tax on foreign bond holdings (Scenario 2). The green line is the variance when the central bank can engage in sterilized intervention at a given level of the household's bond adjustment cost (This is Scenario 4 when chi is small, it becomes scenario 3 as chi becomes larger).

Figure 6: Responses to a positive risk premium shock. The scenario with sterilized foreign exchange intervention is calibrated to match observed levels of capital account restrictions in emerging markets.



Notes: Red line: The central bank has no capital account management instrument, Blue: The central bank can adjust a variable tax to buying foreign bonds, Green: The central bank can buy foreign bonds, household faces capital account restrictions similar to those in emerging markets.