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An Analysis of the Importance of Both Destruction and Creation to Economic Growth

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A growth model is studied in which the destruction (or exit) decision is decoupled from the creative (or research) decision. In contrast with the existing literature, the approach adopted here emphasizes that these important decisions are made by different agents, but they ultimately influence each other. As such, the destruction decision is just as important as that of creation, and in the model if destruction ceases, then so will growth. Any distortion introduced into one of these decisions will then inevitably affect the other as well. It is then possible to characterize endogenous features of the equilibrium such as the number of workers and firms, the determinants of income mobility, income inequality (Gini Coefficient), the growth rate, the lifespan of a firm, and the effect of various taxes or distortions. A planning problem is also studied, and it is shown that a multitude of factors may yield an optimum exit decision that is different from the equilibrium decision rule. This may mean that the equilibrium can give rise either too high or low a level of innovation, but also the destruction or exit rate may also be too high or low. It is then shown that a non-linear tax/subsidy scheme, which alters the research and exit decisions, may improve welfare, relative to the equilibrium level. The model also yields welfare benefits/costs that are considerably different from what one might normally expect.

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An Analysis of the Importance of Both Destruction and Creation to Economic Growth

Gregory W. Huffman[*]

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A growth model is studied in which the destruction (or exit) decision is decoupled from the creative (or research) decision. In contrast with the existing literature, the approach adopted here emphasizes that these important decisions are made by different agents, but they ultimately influence each other. As such, the destruction decision is just as important as that of creation, and in the model if destruction ceases, then so will growth. Any distortion introduced into one of these decisions will then inevitably affect the other as well. It is then possible to characterize endogenous features of the equilibrium such as the number of workers and firms, the determinants of income mobility, income inequality (Gini Coefficient), the growth rate, the lifespan of a firm, and the effect of various taxes or distortions. A planning problem is also studied, and it is shown that a multitude of factors may yield an optimum exit decision that is different from the equilibrium decision rule. This may mean that the equilibrium can give rise either too high or low a level of innovation, but also the destruction or exit rate may also be too high or low. It is then shown that a non-linear tax/subsidy scheme, which alters the research and exit decisions, may improve welfare, relative to the equilibrium level. The model also yields welfare benefits/costs that are considerably different from what one might normally expect.

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Appendix to "An Analysis of the Importance of Both Destruction and Creation to Economic Growth"

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July, 2018

1 An Analysis of the Welfare Function

1.1 Calculating the Discounted Value of Output

In the steady-state, at any date there are N workers who are earning the normalized wage of NA_w. Additionally, the normalized value of profits of all firms can be shown to equal

$$\begin{aligned}\int_{\underline{\theta}}^{1} \pi(\theta) f(\theta) d\theta &= \int_{\underline{\theta}}^{1} A_\pi (\theta)^{\frac{1}{1-\alpha}} \left(\frac{1}{\theta}\right) d\theta \\ &= A_\pi (1-\alpha)\left[1 - (\underline{\theta})^{\frac{1}{1-\alpha}}\right].\end{aligned}$$

Since both wages and profits grow at the rate of g, the discounted value of wages plus profit, or total output can be shown to be equal to

$$\frac{NA_w}{r-g} + \frac{A_\pi (1-\alpha)\left[1 - (\underline{\theta})^{\frac{1}{1-\alpha}}\right]}{r-g}. \tag{1}$$

Since all agents have linear preferences, it would seem that if there were no externalities, that the welfare function would be closely related to this last expression. Nevertheless, the analysis below will show that the aggregate welfare function is not the same as equation (1).

1.2 Calculating the Equal-Weight Welfare Function

From the previous analysis, Let W denote the value function of a worker, and $V(\theta)$ denote the value function of a firm-owner with relative technology θ. It is then straightforward to verify that the "equal-weighted" welfare function for this economy can be written in the following manner:

$$\begin{aligned}V' &= \bar{n}W + \int_{\underline{\theta}}^{1} V(\theta) f_\theta(\theta) d\theta \qquad (2)\\ &= \bar{n}W + (1-\alpha) v_1 \left[1 - \left(\underline{\theta}^{\frac{1}{1-\alpha}}\right)\right] - \left(\frac{g}{r-g}\right) v_1 \left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)} \\ &\quad + \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] W\end{aligned}$$

and from following equation for the welfare of a worker

$$W\left[r - g + \mu(z^*)\right] = A_w - h(z^*) + \mu(z^*)(V(1)) \tag{3}$$

*I would like to thank Rick Bond for encouraging me to pursue this analysis.

and also the following value function for a new firm-owner

$$\begin{aligned} V &= v_1 + v_2 \\ &= v_1 \left[1 - (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] (\underline{\theta})^{(1/g)(r-g)} + W (\underline{\theta})^{(1/g)(r-g)}. \end{aligned}$$

Now it will be convenient, in what is written below, to just remove the term $h(z^*)$ from equation (3).[1] With this in mind, equation (3) can now be written as

$$W[r - g + \mu(z^*)] = A_w + \mu(z^*) \left[v_1 \left[1 - (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] (\underline{\theta})^{(1/g)(r-g)} + W (\underline{\theta})^{(1/g)(r-g)}\right]$$

or, equivalently

$$W\left[r - g + \mu(z^*)\left(1 - (\underline{\theta})^{(1/g)(r-g)}\right)\right] = A_w + \mu(z^*) v_1 \left[1 - (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] (\underline{\theta})^{(1/g)(r-g)}. \tag{4}$$

Equation (2) can now be written as

$$\begin{aligned} V' &= W\left[N + \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{(1/g)(r-g)}\right]\right] \\ &\quad + v_1 \left[(1-\alpha)\left[1 - \left(\underline{\theta}^{\frac{1}{1-\alpha}}\right)\right] - \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] \end{aligned}$$

or

$$V' = \xi_2 W + \xi_1 v_1 \tag{5}$$

where

$$\xi_1 = (1-\alpha)\left[1 - (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] - \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right] (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}$$

and

$$\xi_2 = N + \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] > 0. \tag{6}$$

But substituting equation (4) to substitute for W into equation (5) we have

$$V' = \xi_3 A_w + \xi_4 A_\pi \tag{7}$$

where

$$\xi_3 = \frac{\xi_2}{\left[r - g + \mu(z^*)\left(1 - (\underline{\theta})^{(1/g)(r-g)}\right)\right]} > 0 \tag{8}$$

and

$$\xi_4 = \left[\xi_1 + \xi_2 \frac{\mu(z^*)\left[1 - (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] (\underline{\theta})^{(1/g)(r-g)}}{\left[r - g + \mu(z^*)\left(1 - (\underline{\theta})^{(1/g)(r-g)}\right)\right]}\right] \left[r + \left(\frac{g\alpha}{1-\alpha}\right)\right]^{-1}. \tag{9}$$

2 Now Match Up Coefficients

It is now appropriate to match up the coefficients on the two problems. That is, we wish to compare equations (1) and (7), and compare the coefficients on A_w and A_π in these two expressions.

[1] To recapture the exact nature of equations below, just replace A_w with $(A_w - h(z^*))$.

2.1 Labor

There are N workers, and each unit of labor earns A_w wages, which is discounted at the rate of r, and which grows at the rate of g. Therefore the coefficient on A_w in equation (1) is

$$\frac{N}{r-g}. \tag{10}$$

The coefficient A_w in equation (7) is ξ_3, given by equations (8) and (6), which can be written as

$$\frac{N+\left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{\left[r-g+\mu\left(z^{*}\right)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]}. \tag{11}$$

For equations (10) and (11) to be equal, it must be that

$$N+\left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]=\frac{N\left[r-g+\mu\left(z^{*}\right)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]}{r-g},$$

which implies

$$(r-g)\,N+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]=N\left[r-g+\mu\left(z^{*}\right)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$

or, after cancelling terms

$$g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]=N\mu\left(z^{*}\right)\left[\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$

which holds, since

$$g=N\mu\left(z^{*}\right).$$

Therefore, the coefficients on A_w in equations (1) and (7) do indeed match up.

2.2 And Now Profits

Next, it is necessary to see if the coefficients on the term A_π in equations (1) and (7) match up. This means comparing ξ_4 with the following:

$$\frac{(1-\alpha)\left[1-(\underline{\theta})^{\frac{1}{1-\alpha}}\right]}{r-g}. \tag{12}$$

Now this must match up with ξ_4 from equation (9). Now the easiest way to proceed with the following operations (and reversing these steps later):

1. Multiply ξ_4 in equation (9) by $\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]$
2. Multiply the result by $(r-g)$
3. Multiply the result by $\left[r-g+\mu\left(z^{*}\right)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$

This results in the following expression

$$(r-g)\left[\xi_1\left[r-g+\mu\left(z^{*}\right)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]+\xi_2\mu\left(z^{*}\right)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]. \tag{13}$$

Using the definition of ξ_1 we have

$$(r-g)(1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-(r-g)\left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+(r-g)\left[\xi_2\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right].$$

Now using the definition of ξ_2

$$(r-g)(1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+(r-g)\left[N\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$+(r-g)\left[\left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$

or, after cancelling the $(r-g)$ term

$$(r-g)(1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+(r-g)\left[N\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$+\left[g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right].$$

Now using the fact that $g=N\mu(z^*)$ yields

$$(r-g)(1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+(r-g)\left[g\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}.$$

Now expanding the first term yields

$$(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+(r-g)\left[g\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}.$$

Now, expanding the fourth term

$$(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$-g\left[(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}-(\underline{\theta})^{\left(\frac{r}{g}+\frac{\alpha}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$
$$+rg\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$-g^2\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right]$$
$$+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}$$

and cancelling a couple of terms produces

$$\begin{aligned}
&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&+g\left[(\underline{\theta})^{\left(\frac{r}{g}+\frac{\alpha}{1-\alpha}\right)}\right][r-g] \\
&+g\left[(\underline{\theta})^{\left(\frac{r}{g}+\frac{\alpha}{1-\alpha}\right)}\right]\left[\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&+rg\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right] \\
&-g^2\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right] \\
&+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)(\underline{\theta})^{(1/g)(r-g)} \\
&-g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}(\underline{\theta})^{(1/g)(r-g)},
\end{aligned}$$

and cancelling the 4th and 8th terms yields:

$$\begin{aligned}
&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&+g\left[(\underline{\theta})^{\left(\frac{r}{g}+\frac{\alpha}{1-\alpha}\right)}\right][r-g] \\
&+rg\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right] \\
&-g^2\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right] \\
&+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)(\underline{\theta})^{(1/g)(r-g)}.
\end{aligned}$$

Now cancelling the 3rd term, and part of the 4th terms produces the following

$$\begin{aligned}
&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g^2\left[(\underline{\theta})^{\left(\frac{r}{g}+\frac{\alpha}{1-\alpha}\right)}\right] \\
&+rg\left[(\underline{\theta})^{(1/g)(r-g)}\right] \\
&-g^2\left[\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right](\underline{\theta})^{(1/g)(r-g)}\right] \\
&+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)(\underline{\theta})^{(1/g)(r-g)},
\end{aligned}$$

and cancelling the 3rd term, and part of the 5th produces

$$\begin{aligned}
&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&+rg\left[(\underline{\theta})^{(1/g)(r-g)}\right] \\
&-g^2\left[(\underline{\theta})^{(1/g)(r-g)}\right] \\
&+g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\mu(z^*)(\underline{\theta})^{(1/g)(r-g)}.
\end{aligned}$$

Now grouping the last 3 terms yields

$$\begin{aligned}
&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&-g\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right] \\
&+g\left[(\underline{\theta})^{(1/g)(r-g)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right],
\end{aligned}$$

and grouping the last two terms produces

$$\begin{aligned}&(r(1-\alpha)+g\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]\\&-g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right].\end{aligned} \tag{14}$$

Now reversing steps 1, 2, and 3 by now dividing the previous equation by the following:

$$\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right](r-g)\left[r-g+\mu(z^*)\left(1-(\underline{\theta})^{(1/g)(r-g)}\right)\right]$$

yields the following value for the term ξ_4 :

$$\frac{(1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]}{(r-g)}-\frac{g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{(r-g)\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}.$$

But this does not match up with the coefficient of A_π in equation (1).

2.3 Summary

So this analysis has shown that the welfare function, given by equation (2), can be rewritten in the following alternative format:

$$\frac{NA_w}{r-g}+\frac{A_\pi(1-\alpha)\left[1-(\underline{\theta})^{\frac{1}{1-\alpha}}\right]}{r-g}-\frac{A_\pi g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{(r-g)\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}, \tag{15}$$

which is not exactly the same as equation (1).

There is another term. So where does the following term come from:?

$$\frac{A_\pi g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{(r-g)\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}.$$

3 Interpretation:

The interpretation of this stuff is as follows. Consider the extra term in question,

$$\frac{A_\pi g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}. \tag{16}$$

New inventions or discoveries occur at the rate of

$$\mu N=g,$$

and these occur continuously. What happens when a discovery occurs? Each discovery produces *potential* flow of profits (if the firm operates forever) equal to

$$\frac{A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}.$$

However, this flow will be disrupted after an amount of time T has elapsed. Any event that occurs T periods in the future, adjusting for growth, has a discounted value equal to

$$e^{-(r-g)T}=(\underline{\theta})^{(1/g)(r-g)}. \tag{17}$$

When this occurs, a new invention occurs after T periods, which displaces today's discovery, this has a *net* present value of

$$\frac{A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}-\left(\frac{A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}\right)(\underline{\theta})^{(1/g)(r-g)}$$

which is equation (16). Since this "loss" is occurring at each instant, the discounted value of this is then

$$\frac{A_\pi g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right](r-g)}.$$

Another way to think of this is as follows: Consider an innovation, at date $t=0$, of a new firm that produces a flow of profits $\{\pi(\theta_s)\}_{s=0}^{\infty}$. The value of these profits is calculated as follows:

$$Q_0=\int_0^\infty e^{-rs}\pi(\theta_s)\,ds=\frac{A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]} \tag{18}$$

where $\theta_0=1$. Now, this firm will be mothballed after T periods, and replaced by a new firm. The value of the *new firm's profits*, discounted back to date $t=0$, can be shown to be

$$Q_T=e^{gT}\int_T^\infty e^{-rs}\pi(\theta_{s-T})\,ds=\frac{e^{-(r-g)T}A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}.$$

It can be shown that in equilibrium equation (17) must hold, and so this last equation can be written as

$$Q_T=e^{gT}\int_T^\infty e^{-rs}\pi(\theta_{s-T})\,ds=(\underline{\theta})^{(1/g)(r-g)}\left[\frac{A_\pi}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}\right] \tag{19}$$

Hence, the *net cost* of an innovation can then be calculated as the difference between equation (18) and (19):

$$Q_0-Q_T=\frac{A_\pi\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right]}.$$

Remember that these prices or values grow at the rate of g over time. Therefore, the discounted value of having this occur at each instant is then calculated as

$$\int_0^\infty e^{-(r-g)s}\left[Q_s-Q_{T+s}\right]ds=\frac{A_\pi g\left[1-(\underline{\theta})^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right](r-g)}.$$

Remark 1 *Note that this term is zero when $g=0$, when there are no new firms being created, this term is zero. Next, assume g is constant. Then this term is zero when $N=0$ (because then $A_\pi=0$), and also when $N=1$ (because then $\underline{\theta}=1$). For $N\in(0,1)$ this expression is positive, and has an "inverted -U" shape. Taking into account the negative sign that appears in front of this term in equation (15), this would imply that the last term is strictly convex in N.*

Remark 2 *Additionally, it should be noted that in equation (16), when $N=0$ new and old firms are essentially worthless, (since $A_\pi=0$), and so having a new firm replace an old one is just replacing one worthless asset with another one. When $N=1$ new firms are turning over at a very high rate - basically they are shutting down just a moment after they begin initial production, and so the productivity of old firms is almost identical to that of new firms.*[2] *Therefore,*

[2] And labor is turning over between firms at an arbitrarily high rate as well.

4 Calculations:

Now what its the effect on welfare of changing N? According to the above calculations, discounted welfare of the individuals can be written as

$$\frac{NA_w}{r-g}+\frac{A_\pi\left(1-\alpha\right)\left[1-\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)}\right]}{r-g}-\frac{A_\pi g\left[1-\left(\underline{\theta}\right)^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right](r-g)}.$$

or

$$\frac{1}{r-g}\left[NA_w+A_\pi\left(1-\alpha\right)\left[1-\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)}\right]\right]-\frac{A_\pi g\left[1-\left(\underline{\theta}\right)^{(1/g)(r-g)}\right]}{\left[r+\left(\frac{g\alpha}{1-\alpha}\right)\right](r-g)} \tag{20}$$

Now, first focus on the first term, It can be shown that

$$\frac{dA_\pi}{dN}=\alpha\left(1-\alpha\right)^{1-\alpha}N^{\alpha-1}\left[1-\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)}\right]^{-\alpha-1}\left[\left(\frac{N}{1-\alpha}\right)\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}+\left(1-\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}\right)\right]>0$$

while

$$\frac{dA_w}{dN}=-\alpha\left(1-\alpha\right)^{1-\alpha}N^{\alpha-2}\left[1-\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)}\right]^{-\alpha}\left[\left(1-\alpha\right)\left(1-\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}\right)+N\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}\right].<0 \tag{21}$$

Therefore, these two equations imply that

$$N\left(\frac{dA_w}{dN}\right)+\left(1-\alpha\right)\left[1-\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)}\right]\left(\frac{dA_\pi}{dN}\right)=0.$$

This means that when we take the derivative wrt N in the factor price terms in the first term of equation (20), we get zero. Then we are only left with calculating the derivative wrt N for the last term in equation (20). Taking the derivative of

$$-A_\pi g\left[1-\left(\underline{\theta}\right)^{(1/g)(r-g)}\right] \tag{22}$$

with respect to N yields

$$-g\left[1-\left(\underline{\theta}\right)^{(1/g)(r-g)}\right]\frac{dA_\pi}{dN}+A_\pi g\left[\left(\frac{r-g}{g}\right)\left(\underline{\theta}\right)^{(1/g)(r-g)}\right],$$

and this expression is of indeterminate sign. The reason is fairly clear. In equation (22), holding g constant, when $N\nearrow 1\Longrightarrow\underline{\theta}\nearrow 1$, which means the expression in (22) approaches zero. Alternatively, when $N\searrow 0\Longrightarrow A_\pi\searrow 0$, which again implies that the expression in (22) approaches zero. The latter case is one in which there no creation or destruction because the payoff to innovation is too low. In a sense, this is a traditional stagnant economy. The former case is the opposite: when $N\nearrow 1$ each firm exits for an infinitesimally short period of time, as it is immediately "destroyed" or replaced by another firm that is almost identical to it. Hence there is very little difference or disparity between the new and the old firm.

5 Why, Why Why?

Why would there be this negative externality, which appears to be produced by too little N, or destruction? This could happen for several reasons. First, changing N has two effects here. It alters the payoffs to labor and firms (A_w, and A_π). However, it also changes the level of destruction by changing the lifetime over which firms are operational. This changes the discounted level of profits produced by any innovation, and therefore changes the return to innovation.

Secondly, in the model there are really two activities, research (z), and employment (n).But there is only one relative price, which is the wage (w). The research activity is really not priced in the market, although it does have a private reward. But as described above, this reward is not necessarily socially optimal. If the wage is changed, this will alter the incentive work, which will change (N), but this will also change the reward to innovation. In other words, this single price will then determine the level of two activities. One might state that the profit (A_π) is also a price, but this is not independent of the wage, and in fact, in setting the wage, the profit function is determined as a residual.

1 Introduction

An novel growth model is studied in which there are autonomous, endogenous processes for both the creation and destruction of technologies. These processes are separate in that they are the result of decisions made by different agents, although both influenced by equilibrium market forces. While in much of the existing literature the destructive process is a (regrettable) consequence, or secondary effect, of the innovative activity, here the destructive process is of *equal* importance to that of innovation, and if the former were to cease, then so would the latter. This model will permit the study of how these separate decisions interact to produce an equilibrium growth rate, and allocations for individuals. This also permits the study of why each of these decisions may not be made optimally, and what policies could be employed to increase welfare.

Important contributions to the literature on economic growth have been made by the study of models that capture the notion of "Creative Destruction". However, in many of these models the "creative" mechanism is actually indistinct from the "destructive" mechanism, in that they are really the same process. It is often the case that in these models, when one new good (or technology) is introduced, another must necessarily be eliminated. It would then appear that such a model does not capture the true nature of the "destructive process" in market economies, wherein products or firms are purged due to the change in factor or product prices, which ultimately reduce the profitability of older technologies.

As an example, consider the novel growth model of Aghion and Howitt [2], which employs a framework in which there are innovations in the technology for producing an intermediate good, which is then used to produce a final good. In their benchmark model, innovators are given a monopoly (or patent on their good), and this monopoly lasts until some other producer develops a lower-cost technology. The old incumbent is then displaced, or eliminated from the market. In this sense, the creative and destructive channels are really indistinguishable. There are many other papers which have a similar linkage between the entry and exit of firms or technologies, such as that of Grossman and Helpman [7], or Klette and Kortum [9].[1] However, this approach does not capture the notion that these entry and exit decisions are generally made by *different* agents or firms, and that one person's (or firm's) innovation does not necessarily compel the incumbent to leave. Furthermore, it is important to understand and model the exit decision properly because the exit decision must inevitably influence the innovation decisions, and vice versa.[2]

[1] Helpman and Grossman study a model in which the incumbents are not necessarily driven out of the market completely, but instead they are pushed to making zero profits. Aghion and Howitt also consider this case. In this instance, there are at most two participants in the market, so it is not quite a monopoly. But once again, in these frameworks the innovative and destructive processes are essentially the indistinguishable. In the paper by Klette and Kortum, firms can produce a multitude of goods, but if another firm successfully innovates in producing an existing good, then the incumbent automatically loses the right or ability to produce that good. Once again, the incumbent must exit the market when another firm innovates.

[2] There are other papers in which incumbent firms exit an industry, while newer firms enter. For example, Luttmer [13] presents a model that is used to characterize the size distribution of firms. In his paper, firms face exogenous variations in productivity, which eventually leads to exit from the market when they can no longer cover their costs. However, Luttmer does not study many of the issues addressed here, such as why the exit decision may not be made in a socially optimal manner, or how this decision affects the incentives for innovation, or how government policies might alter this decision to achieve a better outcome.

In this paper, there will be separate endogenous creation and destruction processes.[3] The development of new technologies is influenced by expected future destruction or exit, while destruction is influenced by expected future innovation and the change in factor prices. The development of new technologies alone does not necessitate the destruction of older ones, since the latter can be employed forever. However, in equilibrium the development channel makes older technologies more costly to operate, and therefore reduces the incentive to keep them operational. Therefore, the number of operational technologies (or firms) will be determined endogenously. In addition, the separate destruction or exit decision by an incumbent is characterized as an optimal-stopping problem, and is then the result of that firm-owner behaving optimally. In this way, the exit decision is not mandated merely by the entry of a new firm.

The uncoupling of the creative (or innovative) and destructive (or exit) decisions is also important because it is then possible to build these autonomous decisions into a planning problem, and to compare these separate optimization conditions that result from such a problem with those that might arise from an equilibrium. It is then possible to assess why there might be too much, or too little innovation, as well as whether there is the proper degree of destruction of older technologies.

In much of the existing literature, it seems that the creative or innovation activity is viewed as beneficial, while the destructive process is seen as an unfortunate by-product of innovation. However, by separating the creative and destructive processes, it is possible to show that these activities, though interrelated, have a more complex relationship. In this analysis it will be shown that the Creative forces have both a negative and a positive consequence, while the same can be said for the Destructive process as well. The Creative Process has a natural positive impact because it results in more productive technologies, both currently and in the future. However, it also has a *negative* consequence because it raises the cost of resource inputs to existing firms and makes these existing technologies less profitable. Similarly, the Destructive Process has a negative effect because it results in older firms shutting down, and resources moving on to existing firms. Nevertheless, this process also has a *positive* effect because it results in reduced growth of resource factor prices, which in turn makes existing firms more profitable. This raises the incentives to innovation, which raises the future growth rate.

So why might the equilibrium level of destruction, or firm retirement, be suboptimal, even in an environment in which mobile factors are paid their marginal products? Just as there can be a difference between the private and social returns to innovation, say, because of the intertemporal spillover, there can also be a difference between the private and social returns to shutting down a firm or technology. That is, a firm-owner who decides that it is best for him to shut down his firm (or, alternatively, to keep it operational) may not be making the socially optimal decision. Furthermore, since these creation and destruction decisions undoubtedly influence each other, if some factor (or wedge) should be introduced

[3] It may be worthwhile before proceeding to establish the terminology that will be employed. In the context of the present discussion, the term "destruction" refers to the voluntary shutdown of a firm due to low productivity, or the voluntary withdrawal of a product from production due to low profitability. That is, the destruction is a result of market forces. What is *not* meant by this term is the shutdown of a firm or the termination of production due to government intervention or regulation, or of a competing firm encourage government authorities to target a firm.

into one of these problems, then this will distort the other decision as well. If one takes for granted the idea that the innovation or research decision may not be made optimally, perhaps because of some intertemporal spillover or some other externality, then the exit or destruction decision will not be made optimally, since these decisions will inevitably affect each other.

The model studied here has other novel features. It is frequently stated criticism of representative agent models that they are reticent on such topics as economic inequality and income mobility. However, in this model, not only is there inequality, but there is even a specific formula that can be shown to characterize the Lorenz curve, and the Gini Coefficient. This means that it is possible to see if the equilibrium yields to too much inequality, relative to some optimum. Additionally, it is possible to assess how certain government policies, or parameter changes influence the level of inequality. The model has predictions for many relationships that are generated endogenously. In particular, the model is able to reproduce the "Great Gatsby Curve," showing that there is a positive relationship between the level of income inequality, and the measure of income mobility.

The model presented below is also innovative for several other reasons. First, in contrast to many other extant models, this one does not rely on market power (i.e. such as monopolists) to generate innovation or growth. In fact, there will be a continuum of technologies or firms, and these will sell output and hire labor in a competitive market. Therefore, any distortions in the model will not result from non-competitive forces. Secondly, in many existing models the presence of an intertemporal spillover (or externality) will imply that there will be too little innovation or growth. In contrast, the model studied below *will* have an intertemporal spillover, but nevertheless this economy may produce either too high a level of innovation or growth. Third, it is shown that there can be a multitude of factors that will distort the private destructions decision away from the social optimum. Fourth, by severing the direct linkage between the creative and destructive decisions, this permits the study of how government policies might influence these processes separately. For example, it is possible to study the impact of a policy that subsidizes the creation of new technologies, while simultaneously taxing the destruction of old technologies. Such a policy would seem impossible to study within the context of most extant models.

The remainder of the paper is organized as follows. In the next section the basic structure of the model is described. This means characterizing the optimization problems of the workers and firm-owners, and how they interact in an equilibrium. In Section 2 the structure of the model is described, together with the optimization problems faced by the different agents, and the equilibrium conditions. In Section 3 some features of the equilibrium are illustrated, such as the calculation of the Gini coefficient, and the characterization of the degree of income mobility, as well as some financial features. In Section 4 it is shown how the model could be used to understand why an economy might undergo a long period of either reduced or explosive growth, followed by another period that is quite different. Section 5 studies a version of a social planner's problem which maximizes the welfare of individuals, subject to the resource constraints. It is shown that there are a multitude of reasons why the equilibrium decision rules, for both the creative and destructive decisions, will not coincide with those that result from solving the planner's problem. In Section 6 it is shown that there could be gains from introducing a simple linear tax scheme. Section 7 shows that there can be considerable welfare gains from introducing a non-linear, productivity-dependent tax/subsidy scheme, which alters the incentives for innovation. Section 8 shows that the model yields welfare cost/benefit calculations that are quite different from those of other models. Section 9 explores some implications the model may have for immigration policy.

2 Description of the Model

There is some generality in the nature of this economy, and in the procedure for characterizing the equilibrium, but in order to study the qualitative properties of an equilibrium it is useful to use some specific functional forms.

Time is assumed to be continuous, and there is no aggregate uncertainty. There are a continuum of agents and the population size of the economy will be normalized to unity. In the steady-state there will be N agents who will be workers, and $(1-N)$ who will be termed firm-owners or managers. As will be shown below, the equilibrium level of N will be determined by the model itself, since the agents get to choose whether they want to work, or manage a firm. In this model there will be a dynamic evolution of agents from workers to business (or firm) owners, and this movement will accompany and be related to the growth rate. Workers will supply one unit of labor, and the managers will use their unit of time to manage the firm. It will be instructive to focus first on the problem faced by the typical firm-owner.

2.1 The (static) problem of the firm

Each firm-owner has access to a production function $\lambda\left(n_t^{\alpha}\right)$, $\alpha \in (0,1)$, for producing the generic consumption good, with labor as an input. The variable λ denotes the technology parameter for a particular firm-owner, which is fixed while this firm is in operation. It will be convenient to suppose that at any date t, there is a firm with the leading, or best technology, which will be labelled $\bar{\lambda}_t$. It will be supposed that there is a distribution of technologies, which will be denoted $G(\lambda)$, which is defined over some interval $\Lambda_t \equiv \left[\underline{\lambda}_t, \overline{\lambda}_t\right]$.

The firm-owner can hire labor in a competitive market at a price of w_t, and this price *will* change over time. The owner of a firm maximizes profits, which are written as follows:

$$\pi_t = \max_{n_t} \left\{\lambda\left(n_t^{\alpha}\right) - w_t n_t\right\}.$$

Here $w_t n_t$ represents the wage bill. The profit-maximizing condition is then

$$\lambda\alpha\left(n_t^{\alpha-1}\right) = w_t.$$

The demand for labor by this firm is written as

$$n_t = \left(\frac{\lambda\alpha}{w_t}\right)^{\frac{1}{1-\alpha}}.$$

The indirect profit function is then written as

$$\pi_t = (\lambda)^{\frac{1}{1-\alpha}}(\alpha)^{\frac{\alpha}{1-\alpha}}(w_t)^{\frac{\alpha}{\alpha-1}}(1-\alpha). \quad (1)$$

Also, note that for a particular firm since the technology parameter λ is fixed, and so the following relationship must hold:

$$\frac{\dot{\pi}}{\pi} = \frac{\alpha}{\alpha - 1}\left(\frac{\dot{w}}{w}\right) \tag{2}$$

It will be seen that if this economy is growing at a constant rate, the wage will then exhibit growth at this rate, which in turn will imply that the profitability of each firm will be falling. The profit will continue to fall until the firm shuts down, at which time profit drops to zero.

It must be that the quantity of labor available equals the quantity demanded by all firms. Note again that N is the amount of labor available. Then let $G(\lambda_t)$ denote the distribution of technologies in period t. Then the equilibrium condition for labor must be

$$N = \int_{\Lambda_t} \left(\frac{\lambda_t \alpha}{w_t}\right)^{\frac{1}{1-\alpha}} dG(\lambda_t) \tag{3}$$

so the date t wage is determined as follows:

$$w_t^{\frac{1}{1-\alpha}} = \frac{1}{N}\int_{\Lambda_t} (\lambda_t \alpha)^{\frac{1}{1-\alpha}} dG(\lambda_t). \tag{4}$$

Note that the wage is *homogeneous of degree one in all* λ_t. That is, if all the technologies of all firms in the economy were to be scaled up by some factor, then this would also be the case for the wage as well. The equilibrium below will be one in which $\bar{\lambda}_t$ is proportional to $\underline{\lambda}_t$, and in this case $(\dot{w}/w) = \left(\dot{\bar{\lambda}}_t/\bar{\lambda}_t\right)$.

2.2 The Distribution of Technologies

It will be convenient to put more structure on the distribution of the technologies of the firms. Therefore, it will be assumed that the distribution $G(\lambda)$ will be a truncated reciprocal distribution, over the interval Λ_t.[4] The support of Λ_t will be changing over time. This assumption about the structure and distribution of technologies is convenient because the leading technology $\left(\bar{\lambda}_t\right)$ then becomes a sufficient statistic, which embodies all the information about the distribution. Henceforth, we will let $\theta_t = \left(\lambda/\bar{\lambda}_t\right)$ denote the "relative technology" of a particular firm, which possesses technology parameter λ, when the best, or *frontier*, technology is $\bar{\lambda}_t$ at that date. That is, this measures how far this firm is from the technological frontier. Obviously θ_t ranges between $\underline{\theta} = \left(\underline{\lambda}_t/\bar{\lambda}_t\right)$ and unity. Since the distribution $G(\lambda)$ is assumed to be a truncated reciprocal distribution, it follows that the distribution of θ_t will be $f_\theta = (1/\theta)$, over the interval $[\underline{\theta}, 1]$.

Since there are $1 - N$ firms, and their relative technologies are distributed with density $f_\theta = (1/\theta)$, over the interval $[\underline{\theta}, 1]$, it then follows that

$$1 - N = \int_{\underline{\theta}}^{1} \left(\frac{1}{\theta}\right) d\theta = -\ln\left(\underline{\theta}\right). \tag{5}$$

[4] It should be noted that the reciprocal distribution is what the Pareto distribution converges to as the latter's shape parameter approaches zero. The truncated reciprocal distribution has the convenient property that, as you raise the lower and upper limit by the same proportion, the density on the overlapping section is unchanged. In other words, the mass lost on the left side exactly equals the mass gained on the right side.

Since N can range from zero to unity, it follows that $\underline{\theta}$ can range from e^{-1} to unity.

Also, note that along a balanced growth path it will be the case that the frontier technology $\bar{\lambda}_t$ will grow at some rate g. It follows that for a firm with a fixed technology λ, it must be that

$$\frac{\dot{\theta}}{\theta} = \frac{-\dot{\bar{\lambda}}_t}{\bar{\lambda}_t} = -g. \tag{6}$$

2.3 Workers and Firm-Owners

It will be assumed that all individuals are risk-neutral, and so merely wish to consume their income. Their preferences are assumed to be a function of the discounted stream of consumption $(c_t,\, t \geq 0)$ [5]

$$\int_0^\infty e^{-rt} \left[c_t - h\left(z_t, \bar{\lambda}_t\right)\right] dt, \tag{7}$$

where r is the rate of time preference.[6] At any date there are two types of individuals. There are workers, who supply their unit of labor inelastically which means that they earn the market wage, which is the consumed $c_t = w_t$.[7] Additionally, there are firm-owners, or managers, who use their time to manage their firm. These firms hire labor at the market wage, in order to maximize profit (π_t). The firm-owner has proprietary ownership over his technology (λ), and so owners of inferior technologies cannot costlessly upgrade or steal superior technologies.

Workers are also permitted to use some additional time (z_t) to attempt to discover a new technology, which may eventually permit them to become a firm-owner, or manager. It seems appropriate to identify this as time spent in the pursuit of research or innovation. This activity is successful with some probability $\mu(\cdot)$, but also has disutility $-h\left(z_t, \bar{\lambda}_t\right)$.[8] This is the basis of the "creative process" in the economy. A worker who is successful in inventing a new technology suddenly posses the frontier technology $(\bar{\lambda}_t)$, but this is at the frontier only momentarily. Firm-owners cannot engage in this activity, and so for them

[5] Alternatively, it could be assumed that at any date t, the individual has preferences over the consumption goods which deliver a range of *services*. At date t this range is defined over the interval $\left[0, \bar{\lambda}_t\right]$. Individuals then care about the services yielded by those commodities, and preferences are defined as follows: $\int_0^\infty e^{-\rho t} \left(\int_0^{\bar{\lambda}_t} c_{s,t} ds\right) dt$. As time progresses, some older goods are no longer produced, while some new goods are introduced. An innovation in λ could then be interpreted as a new technology for producing a new good. Each commodity then has the same production function $c_{s,t} = n_{s,s}^{\alpha}$, and so it will be optimal to devote more labor to the production of more advanced technologies or commodities. This approach is similar to that employed by Grossman and Helpman [5].

[6] The use of linear preferences makes the analysis simple in that the person basically is interested in his net income. The analysis could also be conducted for any of the CRRA or logarithmic preferences as well, with a suitable modification of the $h(\cdot)$ function, since the value functions can also be characterized for these preferences. One advantage of the present approach is that it is simpler than using a more complicated set of preferences. Additionally, another difference is that for linear preferences, when solving the planning problem, the planner derives no benefit in merely redistributing income from one set of agents to the other.

[7] The reader will realize that there is nothing intrinsic to the model that necessarily means that this factor must be "labor". It could alternatively be given any other name. It is merely important that there be some factor of production, which is in limited supply, that is owned by individuals, which is *mobile* across firms or technologies, and that this factor be priced and allocated through a competitive market.

[8] The rationale for having this function depend up on $\bar{\lambda}_t$ is that as the leading technology rises, the benefits of innovation are increased, but so are the costs.

$z = 0$ (and $h(0, \lambda) = 0$). One could interpret this "research sector"as being an informal, or non-market, sector within which all innovation conducted. For example, it could be that workers supply their work for a wage, and then they come home and spend some extra time, labelled $h\left(z_t, \bar{\lambda}_t\right)$ engaged in puttering around informally, and there is some prospect this activity will turn out something very profitable.[9] It will be assumed that the amount of effort expended by an agent in discovering a new technology (z) cannot be observed by other agents, and so it is not possible to engage in contracts contingent on the amount of effort (z), or the outcome from such effort. The effect this innovative process is fully internalized by the individual.

One can imagine a multitude of factors that might influence the function $\mu(\cdot)$. Obviously first is that it should be an increasing function of the level of z, and so frequently below the shorthand notation of $\mu(z)$ will be used. However, one could also envisage that this might be a function of the economy-wide level of technology as well. More will be said below regarding the importance of the function $\mu(z_t)$ in explaining a multitude of phenomenon in Section 4 below.

It will be assumed that firm-owners spend all their effort to manage their firm, and cannot upgrade their technology parameter (λ). Firm-owners always have the option of disposing of their technology (i.e. shutting down their firm) and becoming a worker at the market wage.[10] This will be part of the "destruction process" of older technologies. However, only workers are assumed to have the opportunity to develop or invent a new technology. This requires effort or disutility. When new technologies or firms are developed, this raises the demand for labor which increases the equilibrium wage. This increases the costs and reduces the profits of existing firms. At some juncture an owner of an older firm will find his profit to be sufficiently low that he will elect to shut down the firm, and to become a laborer. At this point he can begin to seek to obtain a new technology, which will give rise to a new firm in the future. There will then be a churning of workers and firms as this economy grow.

2.3.1 The Optimization Problem for a Worker

All workers are identical, irrespective of their previous history, or how long they have been unemployed. Therefore, they will all devote the same amount of effort (z) in obtaining an idea or new technology (λ) which might become productive. As mentioned above, the effort

[9] This is not entirely ad-hoc, as it has its motivation in economic history. Many of the most historic inventions were produced by individuals who were not employed in research labs, or universities, but instead were people tinkering around in their spare time, and ultimately made historic discoveries. For example, the Wright brothers were merely two capable mechanics who had bicycle shop but who, in their spare time, loved to play around with things that might fly. This is also (or perhaps especially) true of the electronic revolution over the past century. Issacson [8] describes the multitude of inventions that have given rise to electronic, computer, internet, and IT revolutions. In his book, Issacson repeatedly refers to people making or discovering things in their garage in their spare time. The word "garage" seems to arise recurrently in this narrative, especially so when talking about the history of Silicon Valley. Reading this narative one gets the impression that most of the discoveries were made by people, many of whom would never graduate college, working long hours in their garages, and that the company offices or laboratories were merely places where the inventors went to the next day to brief others on the progress of their research effort.

[10] All workers and firm-owners always have the option of using one of their old technologies to re-start an old firm. However, for reasons that will become clear, this is an option that they will never utilize.

that they expend in discovering a new technology is not observable by others. All workers receive the flow of wages, labelled w_t.

It is assumed that workers have discoveries that arrive according to a Poisson distribution. Let $\mu(\cdot)$ be the probability of locating such a technology. For now, it will be convenient to assume that $\mu(z)$, is merely a function of z.

At each instant the flow of utility for a worker is the wage (w_t) net of research effort expended $(h(z, \bar{\lambda}_t))$. It will be convenient to let $W(w_t)$ denote the value function of such a worker. Additionally, he receives the increased value of the job $(\dot{W}(w_t))$, plus with some probability (μ) he acquires a new technology so that he switches to running a firm, instead of being a worker. Each worker takes the wage w_t, and the leading technology $(\bar{\lambda}_t)$ as given while expecting to receive a new technology $(\overline{\lambda}_t)$ for himself, should his research effort be successful. Therefore, the dynamic programming problem of worker is then written as following Hamilton-Jacobi-Bellman equation:[11]

$$rW(w_t) = \max_z \left\{ w_t - h\left(z, \bar{\lambda}_t\right) + \dot{W}(w_t) + \mu(z) \cdot E_t\left[V(\lambda_t, w_t) - W(w_t)\right] \right\}. \tag{8}$$

The optimization condition, for an interior optimum, is written as follows:

$$h_1\left(z, \bar{\lambda}_t\right) = \mu'(z) E_t\left[V(\lambda_{t+1}, w_{t+1}) - W(w_t)\right] > 0. \tag{9}$$

This is the condition that determines the equilibrium amount of innovation (z). The right side of equation (9) is the relative benefit from engaging in research or innovation (z), while the left side is the marginal cost. Clearly, the greater is the benefit, as expressed by $(V - W)$, the greater will be the amount innovation. But this reward $(V - W)$ also reflects the amount of inequality in payoffs to the different agents. It then follows that the amount of innovation is then likely to be linked to the degree of income inequality in the model. Policies that are instituted to reduce inequality are then likely to reduce innovation. To the extent that equilibrium innovation is too low, such policies are then likely to reduce welfare.

If it can be shown that equations (8) and (9) imply that if w_t, $h\left(z, \bar{\lambda}_t\right)$, and $V(\lambda_{t+1})$, are all homogeneous of degree 1 in all λ, then so will $W(w_t)$, and and $\dot{W}(w_t)$. Therefore, it will be convenient to let

$$h\left(z, \bar{\lambda}_t\right) = h(z)\,\bar{\lambda}_t,$$

where $h(\cdot)$ is strictly convex and differentiable. This means that the utility cost of research becomes greater as $\bar{\lambda}_t$ increases.[12] However, this assumption also implies that it now makes both the right and left side of equation (8) homogeneous of degree one in all λ, and this in turn makes both sides of equation (9) also homogeneous of degree one in all λ. This feature will be exploited below.

[11] An alternative, but roughly equivalent formulation, is to assume that the individual gets to consume his wage, less some fraction (z) of this wage income that is spent on research. Since the market wage is going to be proportional to the leading technology $(\overline{\lambda}_t)$, this means that the cost of research rises as $\overline{\lambda}_t$ rises. Consumption of the individual is then $w_t(1 - z)$. This approach would not affect many of the qualitative features of the model, but would affect some quantitative results. For example, under this last interpretation the individual would be using his after-tax income to engage in research, and so a labor tax would likely reduce innovation.

[12] Under the formulation suggested in the prior footnote this latter assumption would not be necessary, since research effort (z) would be proportional to the wage, which is homogeneous of degree one in all of the operational technologies.

2.3.2 The Optimization Problem for the Owner of a Firm

Begin by temporarily using the shorthand notation of $V(\lambda, w_t)$ for the value function for a firm-holder who has access to a fixed (i.e. unchanging) technology λ, when the market wage is w_t. At each instant the owner of a firm, with technology λ, receives a flow of profit of $\pi_t(\lambda, w_t)$ given by equation (1). Additionally, if he wishes to stay as a non-laborer and run the firm, he gets the *change in the value of the firm* $(\dot{V}(\cdot))$, but otherwise he can shut down the firm, and become a worker. The value function for a firm-owner (i.e. which has a *fixed* technology $\lambda \in \Lambda_t$), where the wage w_t is changing over time, is then written as follows:

$$rV(\lambda, w_t) = \pi_t(\lambda, w_t) + \dot{V}(\lambda, w_t). \tag{10}$$

As this economy grows, the value function for a worker $(W(w_t))$ will be rising, because the wage will be increasing. From equation (2), for a fixed technology $V(\lambda, w_t)$ may be *falling* over time. Hence, it must be that $V(\lambda_t, w_t) \geq W(w_t)$, and as soon as this equation holds with equality, the individual will shut down the firm and become a worker. Hence equation (10) can then be written in the following abbreviated notation:

$$rV_t = \max\left\{\pi_t + \dot{V}, rW_t\right\}. \tag{11}$$

This last equation characterizes the optimal stopping problem faced by a firm-owner, who must decide when to shut down his firm.[13] Suppose that this shutdown date is denoted T. Then the solution to this equation is given by the following expression:

$$V_t = \int_t^T e^{-r(s-t)}\pi_s ds + e^{-r(T-t)}W_T, \tag{12}$$

Here, the value (V) is actually the discounted value of the profit of the firm, plus an American put option. The put option entitles the holder of the firm to sell it (i.e. ownership of the profits), or really dispose of it, at any date for the value W_T. This equation satisfies the value matching condition $(V_T = W_T)$ that insures that the welfare of a firm-owner is equal to that of a worker, when the former decides to become a worker.

It is shown in Appendix A that this expression also satisfies the smooth-pasting condition which would imply that $\dot{V}_T = \dot{W}_T$. The optimal shutdown, or exit date (T) of the firm is chosen optimally in equation (12), and this condition is also developed in Appendix A.

A sample path for the value functions for an individual is illustrated in Figure 1. Here the individual begins as a worker, and then at a random date he obtains a new frontier technology, and his value function jumps upward, but then falls and converges to the value function for a worker, at which he then switches (shutters his firm) to become a worker again. Then the process repeats itself at random times in the future.

2.3.3 Characterizing the Steady-State Equilibrium

It will be convenient to characterize the steady-state behavior of the model, in which there is a balanced growth rate. From equation (4) it can be shown after some algebra that the

[13] This analysis bears some distant similarities to that of Lucas [11] in that in both models the marginal manager should be indifferent between being a manager, or being a worker. However the analysis here is more explicitly dynamic.

wage can be written as follows:

$$w_t = A_w \lambda_t,$$

where

$$A_w = \alpha \left[\frac{1}{N} \int_{\underline{\theta}}^{1} (\theta)^{\frac{1}{1-\alpha}} f_\theta(\theta) \, d\theta \right]^{1-\alpha}. \tag{13}$$

Aghion and Howitt term A_w the "productivity-adjusted wage". Similarly, for a firm with *relative technology* $\theta_t = (\lambda/\bar{\lambda}_t) \in (0,1]$, using equations (4) and (1) it is possible to show that profit can be written as

$$\pi_t(\theta_t) = A_\pi \bar{\lambda}_t \left(\lambda/\bar{\lambda}_t\right)^{\frac{1}{1-\alpha}} = A_\pi \bar{\lambda}_t (\theta_t)^{\frac{1}{1-\alpha}},$$

where

$$A_\pi = (1-\alpha) \left[\frac{1}{N} \int_{\underline{\theta}}^{1} (\theta_t)^{\frac{1}{1-\alpha}} f_\theta(\theta) \, d\theta \right]^{-\alpha}. \tag{14}$$

It seems natural to refer to $A_\pi =$ as the "productivity-adjusted profit" for a firm at the technological frontier (i.e. $\theta = 1$). Similarly $A_\pi (\theta)^{\frac{1}{1-\alpha}}$ would be the "productivity-adjusted profit" for a firm with relative technology θ.

As mentioned above, the value functions, and the distribution of the firm productivities will be completely characterized by the leading or frontier technology at any date $(\bar{\lambda}_t)$ The wage and the profit of all firms will be homogeneous of degree one in $(\bar{\lambda}_t)$ In Appendix A it is shown that since equation (12) is homogeneous in $(\bar{\lambda}_t)$ it is possible to re-write it as $\bar{\lambda}_t V(\theta_t)$, where the function $V(\cdot)$ is given as follows:

$$V(\theta) = v_1 (\theta)^{\frac{1}{1-\alpha}} + v_2 (\theta)^{-(r/g)+1} \tag{15}$$

where

$$v_1 = \frac{A_\pi}{r + \left(\frac{\alpha g}{1-\alpha}\right)} \tag{16}$$

$$v_2 = \left[W - v_1 \left[(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)} \right] \right] (\underline{\theta})^{(1/g)(r-g)} > 0. \tag{17}$$

The first term in equation (15) represents the discounted value of the firm's profits, if the firm is operational forever. Since θ is falling over time, this term is also falling over time.[14] The second term of this equation (involving v_2) reflects the fact that at some future date, when $\theta_T = \underline{\theta}$, it is advantageous for the firm-owner to shut down his firm, and elect to

[14] One might find it unpalatable that the value of each firm is falling over time. This is obviously a natural result of the simplifying assumption that new firms have the highest relative productivity, and so existing firms are being surpassed in this dimension. This is reminiscent of the structure of the Pissarides and Mortensen [14] model in which new employees (or matches) are relatively the "best" or most productive – and it is all downhill from there, until finally the match is dissolved. But secondly, in the present study it would be possible to imagine an alternative (but perhaps much more analytically complicated) structure in which existing firms have random shocks to their productivity, but that they are eventually surpassed by enough high-productivity entrants that they desire to shut down. This would be a much more complicated model than the present one, and the simpler approach captures the necessary features that older technologies necessarily die out.

become a worker. Since the exponent $\left(\frac{-r+g}{g}\right)$ in this expression is negative, this term is rising over time as θ falls.

It is straightforward to check that $V(\underline{\theta}) = W$, and it is a little more work to verify that the smooth pasting condition ($\dot{V}_T = \dot{W}_T$) is satisfied.

The equation describing the worker's value function (8) can be written as

$$rW(w_t) = \left\{ A_w \bar{\lambda}_t - h(z^*)\bar{\lambda}_t + \dot{W}(w_t) + \mu(z^*) E_t \left[\bar{\lambda}_t V(1) - W(w_t) \right] \right\}, \tag{18}$$

where z^* is the optimally-chosen value of research. Note that equations (11) and (18) are homogeneous of degree one in $\bar{\lambda}_t$.[15] Equivalently, if all the operational technologies in the economy were to be scaled up by some positive factor, then so would be the value functions, since this would also scale up the wage, as well as all of the profit functions. Also, the worker knows that in the event of obtaining an innovation, it will be right on the technological frontier ($\bar{\lambda}_t$). As a result of the homogeneity, note that $\frac{\dot{W}}{W} = g$. Henceforth, the value functions for the worker and the firm-owner will be written as $\bar{\lambda}_t W$, and $\bar{\lambda}_t V(\theta)$, respectively.

Therefore dividing equation (18) by $\bar{\lambda}_t$, allows this to be written as follows:

$$rW = A_w - h(z^*) + Wg + \mu(z^*)\left[V(1) - W\right], \tag{19}$$

where the latter equation has used the fact that an agent who discovers a frontier technology immediately has technology $\bar{\lambda}_t$.

It is shown in Appendix A that the solution to the optimal stopping problem faced by a firm with an existing relative technology θ, is given by

$$A_\pi (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)} = (r-g)W. \tag{20}$$

The left side of this expression is the instantaneous productivity-adjusted profit for a firm with relative technology ($\theta = \underline{\theta}$). The right side is the instantaneous return that the person would get from becoming a worker. The presence of the growth rate term (g) reflects the fact that the wage of the worker is growing at this rate. This equation implies that for a firm with fixed technology λ, this firm will shut down or exit when the frontier technology $\bar{\lambda}_t$ reaches the point where the following holds

$$\lambda = \bar{\lambda}_t \left(\frac{(r-g)W}{A_\pi} \right)^{1-\alpha}. \tag{21}$$

This means that a firm manager with technology parameter λ, (or technology $\underline{\theta}$ relative to the frontier) would be indifferent between being a firm-owner or a worker at that instant. Since the frontier technology ($\bar{\lambda}_t$) is continuously increasing, the firm-owner would then switch to being a worker at that point. Prior to this shutdown, or exit date, the left side of equation (21) is greater than the right side.

The condition for optimal research is then given by

$$h'(z) = \mu'(z)\left[V - W\right] \tag{22}$$

[15] Here $V(\lambda_t, w_t)$, from equation (17) has been replaced with $\bar{\lambda}_t V(1)$.

Henceforth, z^* will denote the solution to this last equation It is then straightforward to use equation (19) to calculate that

$$W = \frac{A_w - h(z^*) + \mu(z^*) V(1)}{[r - g + \mu(z^*)]}. \tag{23}$$

It should be clear that the value functions of the two types of agents are interdependent. Factors that influence one of the programming problems will then influence the other. For example, a change in, say, the tax on wages, would then undoubtedly affect both value functions, and then also impinge on both optimization conditions, which are influenced by the size of these value functions.

Lastly, it is necessary to specify the determinants of the growth rate of the economy. It would seem proper that this would be a function of the total amount of research undertaken in the economy. That is to say, it is a function of the number of people engaged in research (i.e. workers) and the rate at which they acquire the capability to become firm-owners, which is a function of the effort they expend on innovation. Therefore, it is consistent with the assumption that the technologies are distributed as truncated reciprocal, that the growth rate will then be characterized in the following functional form:

$$g = N\mu(z). \tag{24}$$

This last equation is important in that the growth rate is a function not just of the amount of research effort expended by each worker, but also by the fraction of the population engaged in this activity. Therefore, in response to some change in the environment, it is possible for per-person research effort (z) to fall, but for the growth rate to rise, if N also rises. Note also that, from equation (5), the values of N and $\underline{\theta}$ are closely linked, and the latter is really the measure of firm destruction, or a measure of how quickly firms will shut down. Therefore, equation (24) indicates that both the creation or innovation (z) and destruction (N) contribute to economic growth. Furthermore, if either $z = 0$, or $N = 0$, then growth will cease, so that the absence of destruction will lead a halt in growth.

It should also be noted that employment for a firm with relative technology θ is proportional to $\theta^{\frac{1}{1-\alpha}}$. This in turn implies that employment across relative technologies is distributed as a truncated Kumaraswamy distribution over the interval $[\underline{\theta}, 1]$.

2.3.4 Summary of the Equilibrium Conditions

The parameters of the economy are then r , α, μ, and the functional form for the functions $h(\cdot)$, and $\delta(\cdot)$. The conditions characterizing he equilibrium are then given as follows. Equation (5) gives the relationship between N, and $\underline{\theta}$:

$$1 - N = -\ln(\underline{\theta}). \tag{25}$$

Equations (13) and (14) define how productivity-adjusted wages and profits are determined:

$$A_w = \alpha \left[\left(\frac{1-\alpha}{N} \right) \left(1 - \underline{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{1-\alpha} \tag{26}$$

$$A_\pi = (1-\alpha) \left[\left(\frac{1-\alpha}{N} \right) \left(1 - \underline{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{-\alpha}. \tag{27}$$

Equation (20) gives the solution to the exit decision of an existing firm

$$A_{\pi}\left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)} = (r-g)\, W. \tag{28}$$

Equation (22) gives the solution to the optimal research problem

$$h'(z) = \mu'(z)\left[V - W\right]. \tag{29}$$

Lastly, equations (15) - (17) characterize the value functions for the firm-owner and the worker:

$$V(1) = \frac{A_{\pi}}{r + \left(\frac{\alpha g}{1-\alpha}\right)}\left[1 - \left(\underline{\theta}\right)^{\left(\frac{r}{g}+\left(\frac{\alpha}{1-\alpha}\right)\right)}\right] + W\left(\underline{\theta}\right)^{(1/g)(r-g)} \tag{30}$$

$$W = \frac{A_w - h(z^*) + \mu(z^*)\, V(1)}{\left[r - g + \mu(z^*)\right]}. \tag{31}$$

Lastly, there is also the equation determining the growth rate

$$g = N\mu(z). \tag{32}$$

There are 8 equations in the eight unknowns: $A_w, A_{\pi}, V, W, \underline{\theta},\ g, z, N$. The general equilibrium structure of the model means that the growth rate (g), the level of innovation (z), and the rate of destruction ($\underline{\theta}$ or N), are determined jointly with the wages for workers and the profit for firms. This simultaneous structure makes it difficult to arrive at any analytical results when studying this system of equations. Therefore, it is necessary to study this equilibrium through numerical methods.

Before proceeding it seems appropriate to pause to note what the assumption of a reciprocal distribution for the technologies (λ) is buying here. In short, this assumption simplifies the formulae in equations (26) and (27). Additionally, it provides a convenient association, through equation (25), between the number of people operating firms, and the rate of firm destruction. Lastly, it simplifies equation (31), by way of equation (18), because the expected value of the value function ($V(\cdot)$) for a person who discovers a new frontier technology is then roughly equivalent to the leading technology at that moment.

In an equilibrium, it may be that the growth rate is very low, but it should still be positive, as is shown in the following result:

Proposition 1 *If $h(0) = 0$, and $h'(0) = 0$. then in an equilibrium $g > 0$.*

Proof. *Suppose $g = 0$. Then either $z = 0$, or $N = 0$. Suppose the former is true. Then equation (29) implies that $V(1) = W$. Equation (30) implies that $\underline{\theta} = 1$, which implies that $N = 1$. But this implies that $W = \frac{A_w}{r} < \infty$, while $\lim_{N \nearrow 1}\left(V = \frac{A_{\pi}}{r}\right) = +\infty$, which is a contradiction. Similarly, if $N = 0$, and $\underline{\theta} = e^{-1}$,this would imply that $V < +\infty$,while $W = +\infty$, because $A_w = +\infty$ This means that many firm-owners could improve their utility by shutting down their firms and becoming workers. But this necessitates having $N > 0$.* ■

3 Characterizing the Equilibrium

Before proceeding to study the quantitative behavior of this economy, it might be useful to characterize some features of it. For some experiments it is difficult to obtain analytical results because the general equilibrium nature of the economy produces some complicated feedback effects through the eight equations. Therefore, for some experiments, numerical methods will be used.

The following form will be used for the $h(\cdot)$ function

$$h(z) = \gamma \frac{z^{1+\omega}}{1+\omega} \tag{33}$$

where $\gamma, \omega > 0$. Much of the analysis below is only used to illustrate some of the features of the model, and is not intended to mimic any particular economy. Unless stated otherwise, the following parameter values will be used for the benchmark economy: $r = .07$, $\alpha = .65$, $\mu = .1$, $\gamma = 0.38$, $\omega = 1.0$.

These values produce a resulting equilibrium growth rate of 3%. Some of these parameters (e.g. r, α) have usual justifications. For others, it is not clear how to clear how to arrive at an appropriate value. For example, normally the value of $(1/\omega)$ might be thought of as related to the labor elasticity, but some reflection would reveal that this is not the case here for several reasons. First, there is no intensive margin of employment. Secondly, the choice of z is not an employment decision, and in fact it is the opposite: The choice of z reflects the agent's *desire to exit the labor force*, and to manage a firm.[16]

In general it is problematic to use such an explicit model to attempt to mimic an actual economy because models with linear preferences frequently give implausible results. In particular, the linear preferences imply an infinite intertemporal elasticity of substitution, and this in turn can imply an implausibly large change in the growth rate in response to a change in the after tax return to capital. Consequently, for some experiments it is conceivable that minute changes in a tax rate can then yield infinite, or undefined, utility.

3.1 Calculating the Lifespan of a Firm

It is of interest to calculate the lifespan of a firm. Since there is no aggregate uncertainty and, from the perspective of a firm-owner, there is no uncertainty at all in its operation, each firm will have the same lifespan, because each faces the same optimal-stopping problem. Now for a firm with a fixed value of λ, when the best technology is $\overline{\lambda}_t$, it is the case that $\theta_t = \lambda/\overline{\lambda}_t$. Using equation (6), this implies that θ_t starts out at 1, and falls to $\underline{\theta}$, and so the lifespan of a firm $(\hat{T})$ must satisfy the following:

$$e^{-g\hat{T}} = \underline{\theta}.$$

[16] Additionally, it is natural to suppose that the parameter α represents "labor's share" of income. However, as mentioned above (see footnote 7), a literal interpretation of this as labor may not be appropriate, and instead it may represent any resources that are mobile across alternative technologies. To the extent that resources are not mobile across various firms or industries, the parameter α may have to take on a much lower value.

This in turn implies that the length of time that a firm is operational is calculated as follows:

$$\hat{T} = \frac{-\ln(\underline{\theta})}{g} = \frac{1-N}{g}. \tag{34}$$

However, g, N and $\underline{\theta}$ are all functions of the parameters, and the policy variables in the economy.

3.2 Calculating Income Mobility

In most general equilibrium, representative agent models, it is impossible to establish any predictions about income mobility. This is not the case here. Some reflection would indicate that income mobility will be related to the level of growth. This is shown in the following.

Proposition 2 *The average time it takes the worker to cycle through from initially becoming a worker, to becoming a firm-owner, and finally shutting it down, is*

$$T = \frac{1}{g}. \tag{35}$$

Proof. Let us use the short-hand notation of $\mu(z) = \mu z$. First, let us establish the expected waiting time for each worker to find an innovation. In this case the probability distribution over waiting a length of time s for an innovation is written as

$$F(s) = 1 - e^{-\mu z s}.$$

For a worker, the expected time to an innovation is then written as

$$E(s) = \int_0^\infty s\mu z e^{-\mu z s} ds = 1/\mu z.$$

Equation (34) shows the average amount of time a worker spends in the workforce. Adding these two quantities together delivers the average amount of time an agent will spend in the two activities:

$$T = \frac{-\ln(\underline{\theta})}{g} + \left(\frac{1}{\mu z}\right).$$

Using equation (5), we then obtain equation (35). ■

The degree of income mobility can then be measured as the inverse of the average time to cycle through these two activities:

$$\frac{1}{T} = g. \tag{36}$$

3.3 Calculating the Lorenz Curve and the Gini Coefficient

It has been a long-standing research issue to investigate the relationship between the level of income inequality and the corresponding growth rate. For example, Greenwood and Jovanovic [4] provide a model in which increased inequality may accompany higher growth. But there is mixed evidence on this topic. Fortunately, the model presented here can be used to study this issue, since both inequality and growth are determined by the equilibrium of the model.

It is straightforward to verify that aggregate output in this economy is

$$Y_t = A_w \overline{\lambda}_t N + A_\pi \overline{\lambda}_t \left[\int_{\underline{\theta}}^{1} \left(\theta^{\frac{1}{1-\alpha}} \right) f_\theta \left(\theta \right) d\theta \right] = \overline{\lambda}_t \Omega,$$

where

$$\Omega = A_w N + A_\pi \left[1 - \left(\underline{\theta} \right)^{\frac{1}{1-\alpha}} \right].$$

It is then possible to see that the fraction of the aggregate income earned by the poorest $v \cdot 100$ percent of the population would be

$$L\left(v\right) = \begin{cases} \frac{vA_w}{\Omega} & \text{for } v \in [0, N] \\ \frac{NA_w + A_\pi \left[\int_{\underline{\theta}}^{v^*} \left(\theta^{\frac{1}{1-\alpha}} \right) f_\theta(\theta) d\theta \right]}{\Omega} & \text{where } v^* = e^{v-1}, \text{ for } v \in [N, 1] \end{cases}$$

where v^* is related to v through the relationship between $\underline{\theta}$ and N. It is not straightforward to see how various parameters influence this equation. This is because the parameters influence the endogenous variables, such as N, $\underline{\theta}$, A_w,and A_w.

It is then straightforward to then calculate the Gini Coefficient from this expression. Figure 2 shows some illustrations of Lorenz curves for this economy. The initial (or left) portion of the curve is straight because all workers get paid the same wage which is proportional to total output. However, firms have different levels of profit, and so the Lorenz curve is "curved" toward the right. The "kink" takes place right at N. In Figure 2 one curve corresponds to having no taxes or transfers, while the other has a profit tax of 30%, with the resulting revenue distributed in a lump-sum manner. Introducing the tax reduces inequality as well as the growth rate. Also, note that introducing the tax raises the value of N, which means that the level of business destruction also rises.

For many parameter values that were studied, it turned out that the Gini coefficients tend to be decreasing in the profit tax. However, the relationship between inequality and labor taxation is more complicated. An example of this is shown in Figure 3, for the benchmark model. In this case, the Gini coefficient is shown as a function of the tax rate, for both the labor and profit, and revenue is given back to individuals as a lump-sum transfer. As can be seen, it appears that inequality is decreasing in *both* taxes for this economy. Raising the profit tax reduces inequality because this amounts to taking revenue from the richer agents, and redistributing it to the poorer individuals. As the labor tax increases from zero, inequality is partly increased because of the net transfer from workers to firm-owners. However, the general equilibrium effects dictate that N will fall, which means that business destruction falls. Essentially, an increase in the labor tax increases the incentive for workers to engage in research, and also makes firm-owners want to keep their firms operating for longer. This implies that there will be fewer workers and more firms in equilibrium, and this can result in marginally lower income inequality.

As indicated earlier, there must be some degree of inequality, as reflected in the size of $(V - W)$, for individuals to engage in the research activity (z). There are other models in which greater inequality may accompany higher growth (see, for example, Greenwood and Jovanovic [4]. However, equation (29) shows that in the present model it is absolutely vital for growth that there be inequality. It does not follow, however, that welfare need be strictly increasing in inequality as well.

3.3.1 An Aside on the "Great Gatsby" Curve

This model produces both a growth rate, a measure of income mobility, and a measure of inequality, which depend on various parameters. There seem to be few general equilibrium models that are useful for studying this relationship between these phenomena. It is also noteworthy that Krueger [10] has reviewed the data from various countries, and has found that there may be a positive relationship between the level of inequality (Gini) and the intergenerational earnings elasticity. Until now it has not been clear if relationship could be explained within the context of an equilibrium model.

One way to explain this phenomenon with the benchmark model is to consider otherwise identical economies, but which have different values of the labor tax. This results in variations in the level of inequality, and growth, as well as many other variables. Figure 4 shows the relationship between the resulting levels of the Gini coefficient, and the degree of income mobility, defined by equation (36). This example is not intended to mimic exactly the relationship observed in the data, but the point is to show that it is possible to produce such a negative relationship from an equilibrium model. In this model it is also possible to let other parameters vary so that this relationship is quite different from that shown in the figure.

3.4 Growth and Taxation

It has been recognized that in the US there seems to be very little relationship between the growth rate, and various measures of income taxation (see, for example, Stokey and Rebelo [18]). It is then somewhat of a test of any model to see if it can replicate this (non) relationship. Therefore, consider the benchmark model without taxes, in which the growth rate is 3.0%. If an income tax (i.e. on both labor and profit) of 30% is introduced, with the resulting revenue distributed in a lump-sum manner, the growth rate is only reduced to 2.46%. This is a reduction that is sufficiently small that it is unlikely to be detected in the data.[17] An imposition of the profit income tax alone has a similarly negligible impact.

3.5 Financial Implications

It is possible to establish some features about the value, and the rate of return on a firm in this environment. A firm here is really an asset that yields a flow of profit to the owner. It is possible to calculate the value of this asset, which will then be the value of the stream of profits, discounted at the constant rate r. Suppose that an agent invents a new frontier technology $\left(\overline{\lambda}_t\right)$ at date t. This produces a flow of profit which is $\pi_{t+s}(\theta) = \overline{\lambda}_{t+s} A_\pi (\theta_{t+s})^{\frac{1}{1-\alpha}}$. This flow falls over time at the rate of $\left(\frac{\alpha g}{1-\alpha}\right)$, and is characterized as follows:

$$\pi_{t+s} = \overline{\lambda}_t A_\pi \left[e^{\left(\frac{\alpha}{\alpha-1}\right) gs} \right].$$

[17] These reductions in the growth rate are of a similar magnitude, whether the government revenue is destroyed, or given back to individuals in a lump-sum manner.

This continues until the firm shuts down at date $t+\hat{T}$. In general, the value in period $t+s$, of a technology developed in period in period t, which then has $\hat{T}-s$ periods until shutdown, is then calculated as follows:

$$P_{t+s} = \frac{\left(\overline{\lambda}_t A_\pi\right)\left(1-\left[e^{-\left[r+\left(\frac{\alpha}{1-\alpha}\right)g\right]\left(\hat{T}-s\right)}\right]\right)}{r+\left(\frac{\alpha}{1-\alpha}\right)g.}.$$

This is not the same price that would hold if the profit stream were to prevail forever. The "destruction effect" is evident in the negative term in the numerator, which insures that $P_{t+\hat{T}}=0$. It is also implicit in that the finite horizon for the profit also indirectly affects the values of g and A_π.

It is of interest to look into the behavior of the price-dividend ratio (or P_t/π_t). Figure 5 shows that this ratio approaches zero as the age of the firm approaches $\left(t+\hat{T}\right)$. This means that the model predicts that younger firms will have higher price-dividend ratios than will otherwise identical older firms. This comports with the observation that indeed younger firms indeed do tend to have higher ratios.[18] Note as well that although profit falls as the firm ages, it does not converge to zero, and approaches its terminal date. The reason for this is that as the firm's age approaches $\hat{T}$, its employment approaches $\left(\frac{\alpha\theta}{A_w}\right)^{\frac{1}{1-\alpha}}$.

It is possible to conduct some other interesting experiments here as well. For example, Figure 5 also shows what happens to this ratio as the tax on profits is raised from zero to 40%, and revenue distributed in a lump-sum manner. In this instance, although the lifespan of a firm ($\hat{T}$) increases, and the (before-tax) price-dividend ratio rises as well. The lifespan rises because although the number of operational firms (as measured by $(1-N)$) falls, the growth rate falls even more. Altering the tax rate, or any other policy parameter, changes not only the growth rate, but also changes the value of A_π.

The same figure also shows what happens if *both* taxes are raised to 40%. In this instance the price-dividend ratio rises even more , as does the lifespan of a firm. This results from the fact that the number of firms increases an infinitesimal amount, while the growth rate falls from 3% to 2.26%.

3.6 Political Economy Considerations

It is natural to wonder about the political-economy considerations of the model. In many models in which there are both workers and firm-owners, these types of agents can have radically different preferences over the tax rates. Workers, may then wish for the lowest possible labor tax, and the highest transfers to themselves. Other agents may seek something close to the opposite. However, in this model the agents are not so bifurcated in their preferences, since agents will evolve from one state to the other. Therefore, it is of interest to see how preferences of agents are a function of government taxes and transfers.

Consider the benchmark version of the model, with parameters specified above, but where the government levies a labor tax. The revenue is given back to agents in a lump sum manner, with each individual getting the same amount. This is shown in Figure 6. For

[18] Although the typical view is that younger firms tend to have higher price-dividend ratios because they have higher expected growth opportunities.

the range of taxes under consideration, firm-owners prefer a slightly higher labor tax, while workers prefer a lower one. The curve represents the “average”, or equally-weighted welfare function (see equation (37) below). Generally, it seems the preferences of the workers and the firm-owners are single-peaked in the tax rate. For the benchmark economy ($\gamma = 0.38$), welfare is maximized with a labor tax of 30%. In this environment, the median voter is also a worker. These workers prefer to have a labor tax since they also inevitably benefit from the higher growth that results from this policy.

For this configuration of parameters for the economy, growth is very important in the calculation of welfare, and for this reason all agents prefer a negative tax rate on profits. This is not a generic result, since there are parameter values for which the welfare function might be maximized with a positive profit, or income tax.

Varying the parameters of the economy will obviously change these results. This is illustrated in Figure 6, where the equally-weighted welfare functions for several other values of the parameter γ.[19] Increasing this parameter essentially makes research more costly to workers, and therefore raises the cost of increased growth. When γ is raised from 0.38 to 1.0, average welfare is now maximized at a tax rate of 15%, instead of 30%. When γ is lowered further to 0.20, the tax rate that maximizes welfare now rises to 44.4%.

3.7 Factors Influencing Firm Destruction

An innovative feature of this model is that gives rise to an endogenous level of firm exit, or destruction. It is then instructive to investigate how various factors influence this exit rate. First of all, it is essential to determine how to measure this feature. One way to do this is to let “N” denote a ordinal measure of destruction, since the higher is this variable, the fewer will be the number of firms. An alternative measure of destruction is the average time a new firm will spend being operational. This is given by the variable $\hat{T} = \frac{1-N}{g}$ (equation (34)). It would seem that the inverse of $\hat{T}$ would be a candidate measure of the rate of destruction.

Next, it is necessary to vary some feature of the model to see how this influences the level of destruction. Varying the tax rates seems like a natural candidate, since increases in this policy parameter will be change welfare, as well as the growth rate. Figure 7a shows how both N and $\left(1/\hat{T}\right)$ vary in the steady-state, as the labor tax rate (τ_n) changes, for benchmark level, and the resulting revenue is distributed in a lump-sum manner.[20] As can be seen, increases in the tax rate lead to lower levels of N, and higher levels of $\hat{T}$, both of which indicate a lower level of business exit. Increased labor taxation results in more operational firms, and these firms produce for a longer period of time.

Next, Figure 7b shows how both N and $\hat{T}$ vary in the steady-state, as the tax rate on profit (τ_π) changes, for benchmark economy. This example illustrates that these different measures of business destruction do not always move in the same direction. In this case, raising the profit tax results a higher level of both N and $\hat{T}$. In other words, it results in fewer firms, but also a lower growth rate. Since the latter effect overwhelms the former, the value of $\hat{T}$ rises.

[19] These curves have been normalized so that they each have a value of zero when the tax rate is zero.

[20] In this figure the values of both N and $(1/\hat{T})$ are normalized to unity when the tax rate is zero.

This result may be important for another reason. It seems to be an interesting but open question as to whether there is a "cleansing effect" of recessions, in that a recession may have a beneficial effect of reducing the economy of low-productivity firms. To the extent that comparative dynamics exercises should be taken seriously, an increase in the tax rate on profit will reduce the growth rate, and so could have a similar observed effect to that of a recession, since the growth rate falls. Suppose one were to take the level of 'N' as the measure of business destruction, since as N rises the number of firms falls. Figure 7b suggests that the rate of business destruction could then increase, as some of the low-productivity firms that were operating under the benchmark economy, now would shut down earlier. However, it is not clear that this should be interpreted as a cleansing effect.

In contrast, in Figure 7a, by raising the labor tax, which causes the growth rate to rise, this lowers the rate of destruction. Through this channel there would seem to be a negative relationship between the rate of growth and the rate of business destruction.

4 A Digression on Growth Traps and Surges

It is an ongoing endeavor to try to understand why different economies or countries grow at different rates, or seem to be stuck with low levels of productivity. Some economies have to be innovative in producing new technologies or products, while others can be content to import or copy foreign technologies, which are superior to their domestic counterparts. Parente and Prescott [15] study models which can speak to this topic. The present model can also be used to address this issue as well.

Here the functions $h(z, \bar{\lambda}_t)$ and $\mu(\cdot)$ are important, as these functions are the mechanisms through which agents obtain better technologies. That is, in an alternative setting the process of growth could be generated through research and development, or alternatively it may be through copying or obtaining a foreign technology. Whatever form this takes, the functions $h(z, \bar{\lambda}_t)$ and $\mu(\cdot)$ would characterize the relative costs and benefits to the domestic agents.

There are several cases to consider. First suppose, as is done earlier that $\mu(\cdot) = \mu \cdot z$, where $\mu > 0$. If μ is close to zero, then this economy will have difficulty attaining significant sustained growth. Obviously a higher level of this parameter would mean higher potential levels of growth. Of course, there are other general equilibrium effects to consider, such as the number of workers and firms, as well as the exit decision, which will also influence the growth rate. Additionally, one might also consider a case in which $\mu(\cdot) = z \cdot g\left(\bar{\lambda}_t\right)$, so that the probability of obtaining a new technology evolves as the economy grows. Consider the "S-shaped" function for $g\left(\bar{\lambda}_t\right)$ shown in Figure 8. In this case the economy might grow at a very slow pace at first, but then after attaining some threshold level, it proceeds to grow at a much higher rate. Two other $g(\cdot)$ functions are drawn in the figure: one (g^2) would certainly produce higher persistent growth than the other (g^1). One could imagine various formulations of this function that could help explain various growth puzzles.

5 Characterizing a Social Optimum

It is important to investigate why the equilibrium growth path could produce a measure of welfare that might not be optimal. To do this, it would be necessary to establish what problem a social planner might face. Here we will set up such an artificial optimization problem that will help to clarify the different channels through which decisions will influence the welfare of agents. This analysis is intended merely to explore the many effects that decisions can have on welfare of agents, but not to fully characterize a social optimum.

Imagine an initial state of the world in which all of the agents have some technology λ, which is drawn from a reciprocal distribution with an initial upper bound of $\overline{\lambda}_0$. From an ex-ante perspective, agents are all identical, and do not know which location in the distribution that they will occupy. That is, there is a "veil of ignorance" which makes all individuals identical before economic activity (and growth) begins. Then, let the economy proceed from this point, with either the market outcomes studied above, or alternatively under the solution to the planning problem, studied below. In any case, there is no need to have a transition from one steady state to another.

The planner would then maximize the expected welfare of these agents. Along the growth path there will then be N agents who will be workers with value functions denoted W. There will also be $(1-N)$ agents who will have value functions reflecting the relative technology that they own. That is, for an agent who owns a firm with relative technology θ, their value function will then be denoted by $V(\theta)$. The planner will then seek to maximize the following welfare or objective function

$$V^* = NW + \int_{\underline{\theta}}^{1} V(\theta)\, f_\theta(\theta)\, d\theta \tag{37}$$

or, using equations (15) through (17)

$$\begin{aligned} V^* \;=\; & NW + (1-\alpha)\, v_1 \left[1 - \left(\underline{\theta}^{\frac{1}{1-\alpha}}\right)\right] - \left(\frac{g}{r-g}\right) v_1 \left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] (\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)} \\ & + \left(\frac{g}{r-g}\right)\left[1 - (\underline{\theta})^{(1/g)(r-g)}\right] W \end{aligned} \tag{38}$$

One way to solve for a social optimum is to let the social planner choose both N, and z, which is to say that the planner will choose the level of destruction and creation. Choosing the value of employment (N) is the same as choosing the degree of firm destruction $(\underline{\theta})$, through equation (25). The planner would solve this problem subject to the constraints given in equations (25), (26), (27), (30) (31), and (32). Equations (28) and (29) would not be part of the planner's problem, since these equations constitute the solution to problem faced individual agents.[21]

[21] This is a rather artificial planning problem because we are letting the planner choose z, whereas in Secton 2 it was stated that individual's choice of z could not be observed by others. In Section 7 below it will be shown how distortions might be partially overcome, and welfare improved even if the individual's choice of z cannot be observed. This is done in more detail in the technical appendix.

5.1 Optimal Research

The optimization condition determining the equilibrium amount of research effort is given by equation (29). The corresponding condition for the solution to the planner's problem would be derived by maximizing equation (37), with respect to z, subject to the relevant constraints. Such a condition might loosely be characterized as follows:

$$N\left[-h'(z^*) + \mu'(z^*)(V-W)\right] + \underbrace{\left(\frac{\partial V^*}{\partial g}\frac{\partial g}{\partial z}\right)}_{+} + \underbrace{\xi_1\left(\frac{\partial v_1}{\partial g}\frac{\partial g}{\partial z}\right)}_{?} + \underbrace{\xi_2\left(\frac{\partial W}{\partial g}\frac{\partial g}{\partial z}\right)}_{?} \tag{39}$$

where

$$\xi_1 = (1-\alpha)\left[1-(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right] - \left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{\left(\frac{r-g}{g}\right)}\right](\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}$$

and

$$\xi_2 = N + \left(\frac{g}{r-g}\right)\left[1-(\underline{\theta})^{(1/g)(r-g)}\right] > 0.$$

Now if we ignore the last three terms in equation (39), then setting the first term equal to zero effectively gives us equation (29), which is the equilibrium condition. It is the other three remaining terms in equation (39) that are omitted from the equilibrium condition. These terms reflect the intertemporal externality, in that the private research decision ignores how current research would influence the welfare of either workers (W) or firm-owners (V), through the growth rate. As the discussion from the previous section indicates, the sum of these three terms could be either positive or negative, and so it might be that the equilibrium of such an economy might generate too much, or too little innovation. If both the "V" and "W" functions in Figure 1 are strictly increasing in the growth rate then a marginal increase in the growth rate will increase welfare, and so the sum of the last two terms in equation (39) will be positive, providing the fall in N is not too pronounced.

The second term in equation (39) reflects the fact that the managers of older firms benefit from increased growth because it means reaching the shutdown date for his firm earlier, at which point they benefit from higher wage growth when he becomes a worker. After some work it can be shown that this expression is positive.

The last two terms in equation (39) are really composed of two effects. First is the fact that increased research raises the growth rate, which *reduces* the value ($V(\theta)$) of an existing firm by raising its costs (i.e. wages). This is closely related to what Aghion and Howitt term the "business stealing effect". In many models the individual innovator is unable to capture the benefits from future innovations that build on his initial innovation. But here the owner of a high-productivity firm may be hurt by this effect, because future innovations raise the growth rate, and this can reduce the value of his initial innovation. However, for owners of low-productivity firms, who are soon due to shut down their firm and become workers, raising the growth rate has a positive effect on their welfare. It is clear, from equation (16), that the effect of research on a new firm can be negative $\left(\frac{\partial v_1}{\partial g} < 0\right)$. However, since ξ_1 could be either positive or negative, it is unclear whether this effect on aggregate welfare is positive or negative.

The last term in equation (39) shows that increasing the value of z also affects the value of being a worker (W), and in principle this effect can be positive or negative. The positive effect results from the increased future wage growth. But the negative effect results from the fact that higher growth can result in a lower value of a firm (V), and each worker plans on being firm-owners at a future date.

In equilibrium the individual agents do not take into account the last three terms in equation (39). Since the sum of these terms could be positive or negative, it is not clear if there will be too much or too little research activity in equilibrium. This is noteworthy because in almost all models in which there is an intertemporal spillover, there will be too little innovation in equilibrium. But the analysis of this model suggests that the presence of such an intertemporal spillover need not guarantee that there is too little innovation.

5.2 Optimal Destruction

Next, it is important to characterize how a change in N can influence the objection function, given by equation (37). Roughly speaking, a change in N influences welfare through three channels: i) directly through the value of N and ($\underline{\theta}$), ii) indirectly through the growth rate (g), iii) indirectly through the factor prices A_w and A_π. The planner would consider all these effects, and so the comprehensive optimization condition for equation for choosing N can then be characterized by the following seven terms:

$$\left[W - \left(\frac{A_\pi}{r-g}\right)(\underline{\theta})^{\frac{1}{1-\alpha}}\right]\left[1-(\underline{\theta})^{(1/g)(r-g)}\right] + \underbrace{\left(\frac{\partial V^*}{\partial g}\frac{\partial g}{\partial N}\right)}_{+} \tag{40}$$

$$+\underbrace{\xi_1\left(\frac{\partial v_1}{\partial g}\frac{\partial g}{\partial N}\right)}_{?} + \underbrace{\xi_2\left(\frac{\partial W}{\partial g}\frac{\partial g}{\partial N}\right)}_{?} + \underbrace{\xi_2\left(\frac{\partial W}{\partial A_w}\frac{\partial A_w}{\partial N}\right)}_{-} + \underbrace{\xi_1\left(\frac{\partial v_1}{\partial A_\pi}\frac{\partial A_\pi}{\partial N}\right)}_{?} + \underbrace{N\left(\frac{\partial W}{\partial V}\frac{\partial V}{\partial A_\pi}\frac{\partial A_\pi}{\partial N}\right)}_{+},$$

The first term results from treating W, v_1, v_2, and g as constants, using equation (25) to replace ($\underline{\theta}$), and then taking the derivative with respect to N. The second through fourth terms are the indirect effects on the growth rate, and the fifth through seventh terms operate through the factor prices.

In an equilibrium of this economy, only the first term would be set to zero (from equation (28)) since this individual's optimization condition in an equilibrium. This expression measures the relative welfare of being a worker, as opposed to running a marginal firm, and thereby captures the equilibrium exit decision.

The second term in equation (40) reflects the fact that a manager of an older firm benefits from increased growth because it means reaching the shutdown date for his firm earlier, at which point he benefits from higher wage growth when he becomes a worker. This expression is positive.

The third and fourth terms are the intertemporal growth effects, and these illustrate how changing the *destruction rate* will influence the future growth rate. This is a different type of intertemporal spillover than is usually considered.

The third term measures how an increase in N would change the welfare of firm-owners by *increasing* growth rate (through equation (32)). Once again this effect could be positive or negative. The reason that higher growth might reduce this value is because the higher is the growth rate in the economy, the more rapidly the value of profits of existing firms might fall, and hence the quicker they exit as well. This is truly where the "creation through destruction" effect is present. Increasing the number of people engaged in research (i.e. increasing N) here alters the overall incentive to engage in research, because the resulting effect on growth changes the future value of the firms, which influencing their exit decision.

The fourth term reflects the fact that changing N would influence the welfare of workers through growth rate. Again, this term could be either positive or negative. The positive effect arises because the increased growth raises the growth rate of wages, and so raises the welfare of workers. The potential negative impact arises because eventually the worker will eventually become a firm-owner, and according to the logic of the previous paragraph, increased growth may reduce the value of being a firm-owner.

It should be noted again that if both the value functions V and W, shown in Figure 1, are strictly increasing, then the sum of the third and fourth terms will be positive: that is, a marginal increase in the growth rate alone will increase the welfare of all agents.

The last three terms of equation (40) are the non-dynamic (or quasi-intratemporal) effects of changing the destruction rate.

The fifth term captures the idea that increasing N reduces the welfare of workers for two reasons: first it increases the supply of workers, but it also lowers the number of firms employing workers. Both effects reduce the equilibrium wage. This term is negative, and therefore illustrates that increased business destruction can reduce welfare.

Then, there is the sixth expression. Here increasing N increases the welfare of firm-owners for two reasons: it increases the supply of workers, but it also lowers the number of firms employing workers. Essentially, quicker destruction of older firms increases the stock of labor, which makes existing firms more valuable. Although the term $\left(\frac{\partial v_1}{\partial A_\pi}\right)$ is positive, it is not clear if the sixth term is positive because ξ_1 is of indeterminate sign. Therefore, increased business destruction could raise or lower welfare. The reason for the indeterminate sign is because the firm-owner will eventually become a worker, and at point they may be harmed by the increase in N.

Lastly, there is the seventh expression. This term reflects the fact that higher labor force raises the return or payoff to owning a firm, which each worker will hope to do at some future date.

One might expect that since preferences (equation (7)) is linear in consumption, that the equilibrium welfare function (equation (38)) would also equal the discounted value of output. It is shown in the technical appendix that this is not the case. The welfare function is lower precisely by an amount that is equal to the net value of the creation-destruction process. That is, this difference is equal to the value of firms that are being destroyed minus the value of firms created, at each instant of time.

The notion of destruction is usually thought of as having only negative consequences, but equation (40) shows that this is not necessarily the case. Several of these terms can potentially be positive, which shows that there certainly can be some *positive* effects associated with increased destruction. In particular, increased destruction of firms in this setting frees up resources (labor) for the more productive firms, and this can be socially beneficial. It also can mean that there are more agents engaged in innovation which can result in higher growth.

Since the second through fourth terms in equation (40) characterize the growth effects, one might be tempted to refer to these as the growth or intertemporal effects, in contrast with the fifth through seventh terms, which might be termed static or intratemporal effects.

Applying such terminology would be misleading. The fifth through seventh terms also have *indirect* growth effects, since these reflect the incentives for engaging in research, which ultimately influence the growth rate. Conversely, although the second through fourth terms indeed influence the resulting growth rate, and the growth rate in turn influences the intratemporal trade-offs that are present in the remaining terms.

This analysis is further complicated for another reason. Let us suppose, hypothetically, that in an equilibrium the last seven terms in equation (40) summed to zero. One might then be tempted to conclude that this would mean that an optimum level of destruction would prevail because this condition would then amount to having equation (28) hold. But this would be false. The reason is that for the conditions from the planning problem to coincide with the market equilibrium conditions, it is necessary that *both* equation (40) coincide with equation (28), and *also* for equation (39) to coincide with equation (29). If either of these fails to hold, then *both* the equilibrium levels of creation and destruction will not coincide with the levels for the planning problem, because of the interdependency of these problems.

It is natural to ask why the equilibrium conditions might not deliver the appropriate amount of destruction (through N or $\underline{\theta}$), that is suggested in equation (40). Another way of seeing the avenues through which this works is to recognize that the second through fourth terms in equation (40) are proportional to the second through fourth terms in equation (39). Since these terms operate through altering the growth rate, it would seem that these terms collectively represent the intertemporal spillover.

However, the fifth through seventh terms in equation (40) do not directly influence the growth rate, but affect the instantaneous payoffs to workers and firm-owners. So just from a static or instantaneous perspective, why might the social benefit from shutting down a firm (and becoming a worker) be different from the private benefit? To gain some perspective, consider an experiment in which the wage and the growth rate are held constant. In this instance, for a marginal firm, the cost to society of shutting down this firm and having the manger become a worker is the profit of the firm, less the wage paid to the new worker:[22]

$$\pi\left(\underline{\theta}\right) - w_t = A_\pi \left(\underline{\theta}\right)^{\left(\frac{1}{1-\alpha}\right)} - A_w. \tag{41}$$

In general, this term could be positive or negative.[23] However, from the equilibrium studied above it is possible to establish more. From equations (19) and (20) it follows that

$$\pi\left(\underline{\theta}\right) - w_t = -h\left(z^*\right) + \mu\left(z^*\right)\left[V\left(1\right) - W\right].$$

Given the form of the function described in equation (33), it is straightforward to verify that in equilibrium the right side of this equation is positive. Hence the social cost to shutting down a marginal firm is positive in equilibrium.

[22] Since at the given wage, the labor released by the shuttered firm is able to earn their wage elsewhere. Therefore the loss to society is the output produced by the firm minus the old wage bill, plus the new wage earned by the firm-owner who becomes a worker.

[23] When $N \nearrow 1 \Longrightarrow \underline{\theta} \nearrow 1$, and this term in is strictly positive. This is because there are very few firms (and lots of workers), and so the cost of shutting down a marginal firm is positive. On the other hand, if $N \searrow 0 \Longrightarrow \underline{\theta} \searrow e^{-1}$, and this term approaches minus infinity. In this instance there are lots of firms and few workers. Shutting down a marginal firm is then a large negative number, which is to say that there is a big social benefit to doing so.

At this point a comparison of this model with that of Aghion and Howitt is in order. Aghion and Howitt describe four distortions which are present in their model. Two of these occur because of the presence of monopolists, which do not exist here. Of the remaining two, one of these is due to the "intertemporal spillover", which is present in this model, in equation (39). The remaining distortion in their model is what they term the "appropriability" effect, which is present in the patent-race literature. Usually this last distortion reflects the fact that the return to investing in a new technology would be the profit that results in equilibrium, whereas in the social optimum the return must be total output (profit plus wages). In the present model this effect is located in a peculiar place. Here, ignoring the growth effects, the social benefit to discovering a new technology is encapsulated by V. But the *net* social benefit is $\left(V - V\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}\right)$, which reflects the fact that while a new firm is created, another is shut down. This social benefit can also be re-written as

$$(V - W) + \left(W - V\left(\underline{\theta}\right)^{\frac{1}{1-\alpha}}\right).$$

The first term is the benefit that accrues to the new firm-owner, who ceases to be a worker, and this is present in equations (22), (29), (39). The second term is the benefit that accrues to the individual who shutters his low-productivity firm and becomes a worker, and this is present in equations (28) and (40).[24] But in equilibrium this last term is zero because of the value matching condition, and so all of the *social* benefit to innovation accrues to the innovator. Therefore, it appears that the "appropriability" effect is not present in this model.

In summary then, of the four distortions in Aghion and Howitt's paper, three of them are absent here. The remaining one is "intertemporal spillover", which reflects the fact that future innovators benefit from past innovations. Additionally, there seem to be several distortions in the current model that are not present in that of Aghion and Howitt.

6 The Model with Linear Taxation and Lump-Sum Transfers

It is of interest to see how the model changes if there is government taxation, and to see if there are government policies that may improve welfare. Let us consider this specifically within the context of the economy described in Section 3, but where government uses linear taxation to finance transfers. Let τ_n and τ_π denote the constant tax rates on labor income and profits, respectively. The latter tax is proportional to the profit of the firm. The government revenue from the labor tax is then equal to the tax rate multiplied by the quantity of labor income ($w_t N$). Normalizing or dividing this by the leading technology then yields this quantity of labor tax revenue equal to $A_w N \tau_n$.

Similarly, suppose each firm is taxed at the rate of τ_π. Then, for a firm with relative technology θ, its normalized profit is then $\pi\left(\theta\right) = A_\pi \left(\theta\right)^{\frac{1}{1-\alpha}}$. It is then straightforward to establish that normalized total revenue from the profit tax would be $\tau_\pi A_\pi \left(1-\alpha\right)\left[1 - \underline{\theta}^{\frac{1}{1-\alpha}}\right]$.

It should be noted that this exercise takes some re-working of the analysis of Section 2 because the presence of the transfer changes equations (15), (20), and (23).

[24] Or see equation (49) in Appendix A.

Now suppose that all government revenue is given back to individuals in a lump-sum manner. The size of this transfer is then calculated as

$$\kappa = \tau_n A_w N + \tau_\pi A_\pi (1-\alpha) \left[1 - \underline{\theta}^{\frac{1}{1-\alpha}}\right].$$

It is then possible to calculate how welfare (as given by equation (37)), inequality, or growth vary as these simple tax rates (τ_n, τ_π) change. For the benchmark economy, growth is so important that welfare is strictly decreasing in the profit tax (τ_π) over any reasonable range.

Panels (a) and (b) of Figure 9 present the results from a simple experiment for the benchmark economy. In this case the labor tax rate is varied from zero to 60%, while the profit tax is set to zero. The resulting revenue, whether positive or negative, is given equally to *all* agents in a lump sum manner. As the figure shows, for the benchmark economy welfare is maximized by having a labor tax of 30%. This policy of transferring revenue from workers to firm-owners raises the growth rate, and the number of firms. This policy also raises inequality as well. For some parameterization it is the case that welfare is maximized with a negative labor tax, and a positive profit tax, which would then suggest that the equilibrium (with no taxes) would too little inequality.

Panels (c) and (d) of Figure 9 show the same features for an identical economy, except that $\gamma = 1.0$. In this case welfare is maximized by by having a tax rate on labor of $\tau_n = 15\%$. In this case growth is relatively more expensive, and therefore, less desirable.

Of course, by varying some of the parameters of the economy, it is possible to raise welfare by using either a negative or a positive income tax, or even a profit tax. However, it frequently appears to be the case that the Gini coefficient and the growth rate are negatively related, so that higher growth is accompanied by higher inequality.

7 Welfare Improvements Through Productivity-Dependent Government Taxation and Transfers

The previous sections indicate that the equilibrium of the model may not result in decisions that would maximize aggregate welfare. It is possible to imagine a set of non-linear, or state-dependent taxes and transfers, that may raise welfare welfare. This section develops a specific class of such policies. To do this, consider a problem which treats all workers identically, and denote their productivity-adjusted consumption productivity-adjusted consumption as c_w. Next, let $c_\pi(\theta)$ denote the productivity adjusted consumption of a firm-owner who owns a firm with relative productivity θ. That is, whereas in Section 2 an individual who owned a firm with relative productivity (θ) would consume profit $\pi(\theta) = A_\pi \left(\theta^{\frac{1}{1-\alpha}}\right)$, here we will let the firm-owner consume $c_\pi(\theta)$ instead. Then the consumption of the workers and firm-owners will not necessarily equal their equilibrium counterparts. Furthermore, for computational convenience suppose that the growth rate of consumption of all firm-owners is denoted

$$\frac{\dot{c}_\pi(\theta)}{c_\pi(\theta)} = \hat{g}$$

and it will be assumed that this is constant. Now the analysis of this model can mimic much of Section 2. The value function for the worker, which is the counterpart to equations

(31) is given as follows

$$W = \frac{c_w - h(z^*) + \mu(z^*)\hat{V}(1)}{[r - g + \mu(z^*)]}.$$

The value function for the firm-owner with relative productivity $\hat{V}(\theta)$ is calculated in Appendix B.

Now consider the planning problem of choosing a consumption level (c_w), and a function ($c_\pi(\theta)$) to maximize the analog to equation (37), which is the objective function

$$\max_{c_w, c_\pi(\theta)} \left\{ NW + \int_{\underline{\theta}}^{1} \hat{V}(\theta) f_\theta(\theta) d\theta \right\}. \tag{42}$$

Next equations (28) and (29) must also hold here as well:

$$\hat{V}(\underline{\theta}) = W$$

$$h'(z) = \mu'(z)\left[\hat{V}(1) - W\right].$$

Then, to insure that all output is consumed, it must be the case that total resource constraint is satisfied:

$$Nc_w + \int_{\underline{\theta}}^{1} c_\pi(\theta) f_\theta(\theta) d\theta = \frac{NA_w}{\alpha} = A_\pi \left[1 - \underline{\theta}^{\frac{1}{1-\alpha}}\right],$$

where the equalities define aggregate output, and A_w and A_π are given by equations (26) and (27) respectively.

Lastly, equations (25) and (32) must also hold. This ensures that this new allocations can be compared with those of the previous sections.

The solution to this planners problem can then be decentralized because there will be an implied system of taxes or subsidies that result. The implied tax rate on labor would

$$\tau_w = 1 - \frac{c_w}{A_w},$$

while the (non-linear) tax or subsidy on profits will be

$$\tau_\pi(\theta) = 1 - \frac{c_\pi(\theta)}{A_\pi\left(\theta^{\frac{1}{1-\alpha}}\right)}.$$

For the government to balance its budget, it must then also be the case that

$$0 = \tau_n A_w N + \int_{\underline{\theta}}^{1} \tau_\pi(\theta) A_\pi \left(\theta^{\frac{\alpha}{1-\alpha}}\right) d\theta.$$

A full explanation of how this problem is solved is presented in Appendix B.

To see what the resulting implied tax rates might look like, it is best to consider a couple of parameterized examples. Throughout this analysis, the measure of aggregate welfare will be given by equation (37).

The parameter values used henceforth will be the same as in the benchmark with the exception that the growth equation (32) will now by determined as $g = \delta(zN)^{1/4}$, where δ is chosen so as to imply a value for μ equal to the benchmark value of 0.10.[25]

Example 3 Consider the parameterization of the benchmark economy described in Section 3. Figure 10 shows the welfare function, as a function of the level of employment (N), and the level of research (z), for various paths of the tax and subsidy policies. The marker '■' shows the point at which the equilibrium is located (with $\tau_n = \tau_\pi(\theta) = 0$), while the marker labelled '*' shows where welfare is maximized with the system of non-linear taxes. As can be seen, relative to the equilibrium, the welfare-maximizing outcome with taxes results in more research effort, and slightly more employment. This result means that in equilibrium there is not enough business destruction. In other words, firms are not shutting soon enough. It is possible to remedy this.

Figure 11 shows the tax and subsidy policy, of the sort described above, that results in a higher level of welfare for this economy. In this case it is best to have the government tax labor at a rate of 13%, and then use this revenue to subsidize firms according to the schedule in Figure 11. This schedule shows that the highest productivity firms should be *subsidized* at rate of 57.6%, while the lowest productivity firms be taxed at a rate of 11.5%. This is obviously a shift of resources from the workers, and owners of low-productivity firms (who will soon become workers), to the owners of high-productivity firms. The benchmark model had a growth rate of 3%, while the optimum resulting from this problem produces a growth rate of 3.27%.

To understand why this policy improves welfare it is best to note that Figure 11 implies that, to increase welfare relative to the equilibrium level, research effort and employment need to both be increased. This can certainly be done by shifting resources from the workers to the firms, with a larger subsidy given to the high-productivity firms. As the firms age, however, this subsidy is curtailed until it eventually becomes a tax. Since the reward to being a new firm-owner is so high, this raises the level of research. But taxing owners of low productivity firms will raise the level of destruction (or N).

It is of interest to assess how big the welfare improvement would be from such a policy. Relative to the benchmark, the increase in utility from the tax/subsidy policy is an increase of 1.9%. Since utility is linear in consumption, it seems appropriate to view this as equivalent to an increase of 1.6% in initial consumption for all agents.

Example 4 Now consider the very same parameterization as in the previous example, but now let $\mu = .05$. In this case the equilibrium growth rate is 1.36%. However the solution to the problem of maximizing welfare with the system of non-linear taxes, described above, results in a growth rate of 1.30%, so the equilibrium growth rate is too high. In this example it seems that there is too much of research, and also too much employment (or firm destruction) in equilibrium.

[25] This formulation implies that there is an externality in the "search" or innovation process, so that the more innovation that takes place, either by higher levels of z or N, the lower the level of μ each individual will face. This is equivalent to a search externality in the search literature. This formulation is adopted because, given the linear form of the preferences (equation (7)), and decision rules, the objective function (equation (38)) then tends be rather linear, and that maximizing welfare can result corner solutions which might entail solutions such as $N \to 1$. Adopting this alternative formulation for the growth rate then adds enough curvature to the objective function to produce interesting, non-trivial results.

Figure 12 shows the implied tax and subsidy policies that result from this constrained planning problem. In this case the planner would impose a subsidy, or negative labor tax, of 3.24%. The tax on firms, shown in the figure ranges from -5.2% on the owners of the low productivity firms, to a tax, of 14.2% on the owners of the high productivity firms. As can be seen in the figure, this tax scheme is not linear, and has a slightly concave feature. Such a tax scheme would certainly reduce the amount of research effort, since the benefit of being a firm-owner is reduced. Similarly, the subsidy to low-productivity firms helps raise the overall number of firms, and so lower the level of firm destruction (N).

The increase in welfare as a result of this system of taxes and subsidies, relative to the equilibrium is 0.25%.

These examples are instructive for several reasons. First, suppose the welfare-enhancing tax policies resulting from this example were imposed on such an economy. An independent observer of this economy would see that the government is certainly imposing a distortional tax/transfer policy between firms that certainly looks like the government is "picking winners and losers".[26] Not only that, but this policy would *reduce* the growth rate. All of this is true, but it results from the government trying to maximize welfare, which is what these policies are intended to do. The reason this policy improves welfare is that the planner is recognizing that the level of research, as well as the rate of firm exit (or destruction) is something that needs to be altered, and this is an important economic variable to be considered.

Additionally, this last example illustrates other novel features. In nearly all models with intertemporal spillovers for research, the optimal policy is to subsidize research to take advantage of this externality. However, in this last example there is such a spillover, but nevertheless it is welfare-enhancing to *reduce research*. What is missing from other models in the existing literature is that they do not have a destruction (or firm-exit) decision that is both endogenous and autonomous. In this last example the planner is using this feature, but reducing the amount of destruction, and to some extent this offsets the reduction in research (through equation (32)), and changes the incentive to engage in research. This example shows that by ignoring the endogenous exit behavior of firms, or omitting the destruction feature, much of the existing literature is ignoring an important feature that contributes to the incentives for innovation and growth.

8 A Note on the Welfare Benefits of Increased Growth

Another noteworthy feature of this model is how welfare costs are created. With linear preferences, it is easy to establish that the consumption equivalent benefit from raising the growth rate from g to g', when the rate of time preference is r, is determined by the following equation

$$\frac{g' - g}{r - g'}. \tag{43}$$

So, in the first example, when $r = .07$, and the growth rate rises from 3% to 3.27%, equation (43) would imply a welfare benefit of 7.24%. But, as mentioned above, the welfare benefit

[26] But since such a government policy is known in advance, it no more constitutes "picking winners and losers" than does a progressive or regressive tax code.

is actually only 1.6% - which is substantially lower. In the second example, the growth rate *falls* from 1.36% to 1.3%. Equation (43) would imply a welfare *cost* of 1.1%, but in actuality welfare *rises* by .25%. How could these welfare measures by so different? The reason lies in recognizing that the change in the growth rate affects different agents in different ways. An increase in the growth rate increases the welfare of workers, but it *reduces* the welfare of firm-owners, by increasing the rate at which their profit or consumption *falls*. Furthermore, the increase in the welfare of the workers is mitigated by the fact that they also wish to be a firm-owner in the future, and an increase in the growth rate would reduce the welfare benefit of being a firm-owner.[27] Lastly, changing the value of N also changes the composition of the welfare function (equation (38)), which then affects these calculations.

We have become accustomed to assuming that there can be substantial welfare benefits from raising the growth rate.[28] These examples show that these benefits may be much different than previously thought.

9 Implications for Immigration Policy

It is interesting to note that this model may have implications for immigration policy. Consider a policy a "guest worker" program wherein the government admits some workers into the economy, and these workers can only work - they cannot obtain and run a firm. These workers then receive wage income which can be consumed (or repatriated to their home country; it really does not matter). In this case this would raise the supply of labor (N) in equation (3). This will result in a lower level of wages for all workers, which will lower the value function for a worker (W). This will raise the return to innovation, and result in a higher level of z. As long as this does not result in too big a drop in the number of domestic workers, this can then raise the growth rate. On the other hand, if the number of domestic workers does indeed fall, then it is conceivable that the growth rate would also fall.

10 Final Remarks

It is an accepted fact of life that a growing economy is organic in nature, and exhibits a continual birth and mortality of products and technologies. Yet most studies of economic growth fail to model the decisions that give rise to these phenomena, and therefore cannot assess whether these decisions are made optimally.

Integral to the study of optimal growth is the determination of the optimal incentives for agents to seek innovations of new technologies. Some of these incentives reflect the ability

[27] It might seem that this discussion is ignoring the effect that research effort is having on disutility through the $h(\cdot)$ function in equation (7). It turns out that this effect is of a secondary or minimal size. In fact, for the first example, the allocations from the planning problem results in reduced total research because there are fewer workers. This factor increases total welfare. Despite this the welfare gain is much lower than the gain resulting from equation (43).

[28] And these benefits from increased growth are typically largest for the case of linear utility, considered here. The reason is that with concave utility the welfare benefits are reduced by the diminishing marginal utility, or equivalently, the higher interest rate. If the examples here indicate that the welfare benefits in the case with linear utility have previously been overstated, then it would seem that these benefits in the case of concave utility have also been exaggerated.

for innovators to capture some of the market share, or resources of older incumbents. This frequently means that the innovation process leads to the eventual termination of older technologies. It can then be a mistaken step of logic to conclude that the destruction of older technologies is an unfortunate by-product of innovation. The analysis presented above shows why this is not the case, and instead both the creation and the destruction effects have mutually beneficial *and* detrimental effects. The study of optimal growth, and the development of the optimal incentives to obtain this growth rate, must weigh the different impacts of these decisions. This paper has used a relatively simple model to attempt to describe these channels. It was shown that the design of welfare-enhancing policies must take into consideration not just the appropriate amount of innovation, or entry of new firms, but also the apposite rate of destruction, or exit of old firms. By disregarding this last feature, existing models of growth are ignoring an important observation, and consequently this may result in suggesting policies that are far from optimal.

Much of the existing literature focuses on developing the proper incentives for innovation alone, which would then result in an optimal growth rate. What this literature ignores, and what the analysis of this paper shows, is that it is equally important to provide the proper incentives for the optimal retirement or exit of older firms or technologies, since the exit and innovation decisions are interrelated. This analysis also suggests that the ideal government policy in this model may be quite different from that is most existing growth models. For example, there may be good reasons for imposing tax or subsidies that depend on the productivity (or profit) of the firm, in order to provide the correct incentives for innovation or exit. Also, the presence of an intertemporal spillover need not necessarily imply that there is too little innovation (and growth) in equilibrium.

Characterizing optimal government policies in such an economy can be a very tricky business, since designing the proper incentive scheme in an environment with interrelated decisions can be a delicate matter. As has been shown, in this environment there can be considerable welfare gains from adopting certain government policies. But unless the policy maker knows the exact parameters and structure of the environment, there is a potential of implementing the wrong incentives. Furthermore, there can also be considerable *welfare losses* from implementing the wrong policies.

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11 Appendix A

Here it is shown that equations (15) - (17) characterize the solution to equation (12). First, note that since $\frac{d}{dt}\log\left(\overline{\lambda}_t\right) = g$, and $\pi\left(\theta_t\right) = A_\pi\left(\theta_t\right)^{\frac{1}{1-\alpha}}$. As described in the text, since the profit functions, the wage function, and the function $h(z, \overline{\lambda}_t)$ are all homogenous of degree one in $\overline{\lambda}_t$, it follows that the value functions for the optimization problems will then be homogeneous as well. It follows that the value function (equation (12)) for a firm-owner with relative technology (θ_t) can be written as follows

$$V\left(\theta_t\right)\left(\bar{\lambda}_t\right) = \int_t^T e^{-r(s-t)}\left(\left(\bar{\lambda}_t\right)e^{g(s-t)}\right)A_\pi\left(\theta_s\right)^{\frac{1}{1-\alpha}}ds + e^{-r(T-t)}W\left(\bar{\lambda}_t e^{g(T-t)}\right), \qquad (44)$$

and so dividing by $\bar{\lambda}_t$ results in

$$V(\theta_t) = \int_t^T e^{-r(s-t)} \left(e^{g(s-t)}\right) A_\pi (\theta_s)^{\frac{1}{1-\alpha}} ds + e^{-r(T-t)} W \left(e^{g(T-t)}\right). \tag{45}$$

Since $\left(\frac{\dot{\theta}}{\theta}\right) = -g$, this last expression can be written as

$$V(\theta_t) = \frac{A_\pi (\theta_t)^{\frac{1}{1-\alpha}}}{r + \left(\frac{\alpha g}{1-\alpha}\right)} \left[1 - e^{-(T-t)\left(r + \left(\frac{\alpha g}{1-\alpha}\right)\right)}\right] + W \left(e^{(-r+g)(T-t)}\right), \tag{46}$$

where T is the exit date for the firm. By choosing this date T optimally, this yields the following exit condition:

$$A_\pi (\theta_T)^{\frac{1}{1-\alpha}} = (r-g) W. \tag{47}$$

From equation (6), since the remaining lifetime of a firm with relative technology (θ_t) must satisfy

$$T - t = \frac{\ln(\theta_t) - \ln(\underline{\theta})}{g}.$$

In general for a firm with relative technology (θ) equation (46) must then satisfy

$$V(\theta) = \frac{A_\pi}{r + \left(\frac{\alpha g}{1-\alpha}\right)} \left[(\theta)^{\frac{1}{1-\alpha}}\right] + W \left(\frac{\theta}{\underline{\theta}}\right)^{-(1/g)(r-g)} - \frac{A_\pi}{r + \left(\frac{\alpha g}{1-\alpha}\right)} \left[(\underline{\theta})^{\left(\frac{r}{g} + \left(\frac{\alpha}{1-\alpha}\right)\right)}\right] (\theta)^{-(r/g)+1} \tag{48}$$

which then can be written as

$$V(\theta) = v_1 (\theta)^{\frac{1}{1-\alpha}} + v_2 (\theta)^{-(r/g)+1}$$

where for any $\theta \in [\underline{\theta}, 1]$

$$\begin{aligned} v_1 &= \frac{A_\pi}{r + \left(\frac{\alpha g}{1-\alpha}\right)} \\ v_2 &= \left[W - v_1 \left[(\underline{\theta})^{\left(\frac{1}{1-\alpha}\right)}\right]\right] (\underline{\theta})^{(1/g)(r-g)} > 0. \end{aligned}$$

It is easy to see from equation (48) that the following *value matching condition* must hold

$$V(\underline{\theta}) = W. \tag{49}$$

The optimal exit condition (equation (47)) can also derived by choosing the optimal value of $(\underline{\theta})$. Maximizing the value function in equation (48) with respect to $(\underline{\theta})$ also leads to the condition in equation (47).

The *smooth-pasting* condition is derived by taking the derivative of equation (45) with respect to t, and evaluate the result at $t = T$.[29] Then, using equation (47) yields the fact that $\dot{V}_T = \dot{W}_T$. The latter functions are the technology-normalized value functions (i.e.

[29] Equivalently, one could take the derivative of equation (48) with respect to θ, and evaluate the result at $\theta = \underline{\theta}$. See Stokey [17] for an explanation of the necessity of this condition.

divided by $\overline{\lambda}_t$). Figure (1) may be a little deceptive because this figure shows the actual value functions: $\overline{\lambda}_t V(\theta)$, and $\overline{\lambda}_t W$, rather than $V(\theta)$, and W. These functions are growing at the rate of g, when the two functions equal each other.

Another way to characterize the value function of a new firm-owner is to note that since $\left(\frac{\dot{\theta}}{\theta}\right) = -g$, for the case of a firm-owner with a new technology at $t = 0$, equation (45) can be written as follows:

$$\begin{aligned} V(1) &= \int_0^\infty e^{-rs}\left(e^{gs}\right) A_\pi \left(\theta_s\right)^{\frac{1}{1-\alpha}} ds + e^{-rT} W\left(e^{-gT}\right) - \int_T^\infty e^{-rs}\left(e^{gs}\right) A_\pi \left(\theta_s\right)^{\frac{1}{1-\alpha}} ds \\ &= \int_0^\infty e^{-rs}\left(e^{gs}\right) A_\pi \left(e^{-gs}\right)^{\frac{1}{1-\alpha}} ds + e^{-rT} W\left(e^{-gT}\right) - \left(\theta_T\right)^{\frac{1}{1-\alpha}} \int_T^\infty e^{-rs}\left(e^{gs}\right) A_\pi \left(e^{-gs}\right)^{\frac{1}{1-\alpha}} ds \\ &= \int_0^\infty e^{-rs} A_\pi \left(e^{gs}\right)^{\frac{-\alpha}{1-\alpha}} ds + e^{-rT} W\left(e^{-gT}\right) - e^{-rT}\left(e^{gT}\right)\left(\theta_T\right)^{\frac{1}{1-\alpha}} \int_0^\infty e^{-rs} A_\pi \left(e^{gs}\right)^{\frac{-\alpha}{1-\alpha}} ds \\ &= \frac{A_\pi}{r + \left(\frac{\alpha g}{1-\alpha}\right)} + e^{-(r-g)T}\left[W - \frac{A_\pi \left(\theta_T\right)^{\frac{1}{1-\alpha}}}{r + \left(\frac{\alpha g}{1-\alpha}\right)}\right]. \end{aligned}$$

The first term on the right side of this last expression is the discounted value of profits from running the firm *forever*, given that the profits are falling at the rate of $\left(\frac{\alpha g}{1-\alpha}\right)$. Next, the term $e^{-(r-g)T}$, reflects that fact that at some future date T, which is chosen optimally, the firm will be shut down. At that date the firm will have relative technology denoted by θ_T. By shutting down the firm at that date the firm-owner will be giving up a future profit stream, the value of which is $\frac{A_\pi(\theta_T)^{\frac{1}{1-\alpha}}}{r+\left(\frac{\alpha g}{1-\alpha}\right)}$. But the firm-owner benefit from switching to becoming a worker because the value of doing so exceeds that of keeping the firm operational forever (i.e. $W > A_\pi \left(\theta_T\right)^{\frac{1}{1-\alpha}}$).

12 Appendix B

In this section a description of the welfare-enhancing consumption streams is provided.

In this case define the productivity-adjusted consumption level for a firm-owner with relative technology (θ) as follows:

$$c_\pi(\theta) = \Phi\left(\theta^{\frac{1+\varphi}{1-\alpha}}\right) A_\pi, \tag{50}$$

where Φ and φ are constants to be determined below. This formulation then implies a growth rate of

$$\frac{\dot{c}_\pi(\theta)}{c_\pi(\theta)} \equiv \hat{g} = \left(\frac{1+\varphi}{1-\alpha}\right)\frac{\dot{\theta}}{\theta}.$$

In the basic version of the model $\Phi = 1$, and $\varphi = 0$. With this modification of the model in mind, following the logic of the basic model, the value function, the productivity-adjusted value function for a firm-owner with relative technology (θ) as

$$V(\theta) = v_1(\theta)^{\frac{1+\varphi}{1-\alpha}} + v_2(\theta)^{-(r/g)+1},$$

where

$$v_1 = \frac{\Phi A_\pi}{r + \left(\frac{\alpha \hat{g}}{1-\alpha}\right)}$$

$$v_2 = \left[W - v_1 \left[(\underline{\theta})^{\left(\frac{1+\varphi}{1-\alpha}\right)}\right]\right] (\underline{\theta})^{(1/g)(r-g)} > 0.$$

Letting c_w denote the productivity-adjusted consumption of a worker, the value function for such a worker is given by

$$W = \frac{c_w - h(z^*) + \mu(z^*) \left(\hat{V}(1)\right)}{[r - g + \mu(z^*)]}.$$

The optimal cut-off or shutdown condition is modified to now be

$$(\underline{\theta})^{\frac{1+\varphi}{1-\alpha}} = \frac{(r-g) W}{A_\pi}.$$

The optimal research or innovation condition is unchanged from the benchmark model:

$$h'(z) = \mu'(z) \left[\hat{V}(1) - W\right].$$

The government must choose the level of c_w, and the path or function $c_\pi(\theta)$, for $\theta \in [\underline{\theta}, 1]$, subject to the resource constraint:

$$N c_w + \int_{\underline{\theta}}^{1} c_\pi(\theta) f_\theta(\theta) \, d\theta = \frac{N A_w}{\alpha} = A_\pi \left[1 - \underline{\theta}^{\frac{1}{1-\alpha}}\right].$$

in order to maximize welfare (42). However, given the formulation in equation (50), choosing a path for $c_\pi(\theta)$ is really synonymous with choosing the parameters Φ and φ. Therefore, the welfare-maximizing solution is derived by conducting a parameter search for these two parameters. By permitting values of Φ both above and below unity, and values of φ that range from -.50 to +.80, permits either a regressive or progressive tax regime. Different parameter values then give rise to alternative values for the endogenous variables, such as the equilibrium growth rate, profit, et cetera.

If $\varphi = 0$, and $0 < \Phi < 1$, then this amounts to a constant or flat tax on profits. If $\varphi < 0$ (resp.$\varphi > 0$) then the tax is progressive (regressive), because it increases in income, or falls as relative productivity (θ) falls.

Figure 1

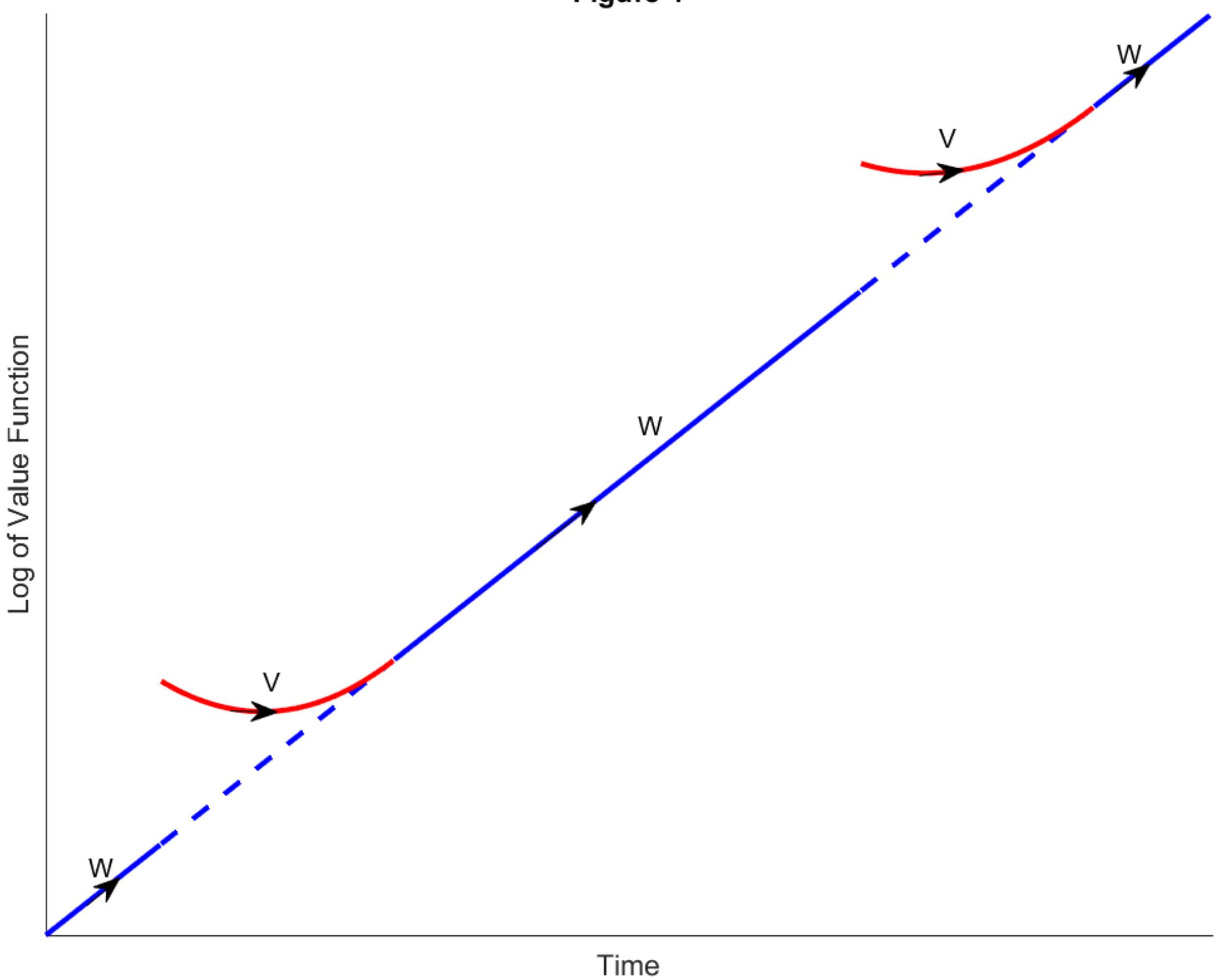
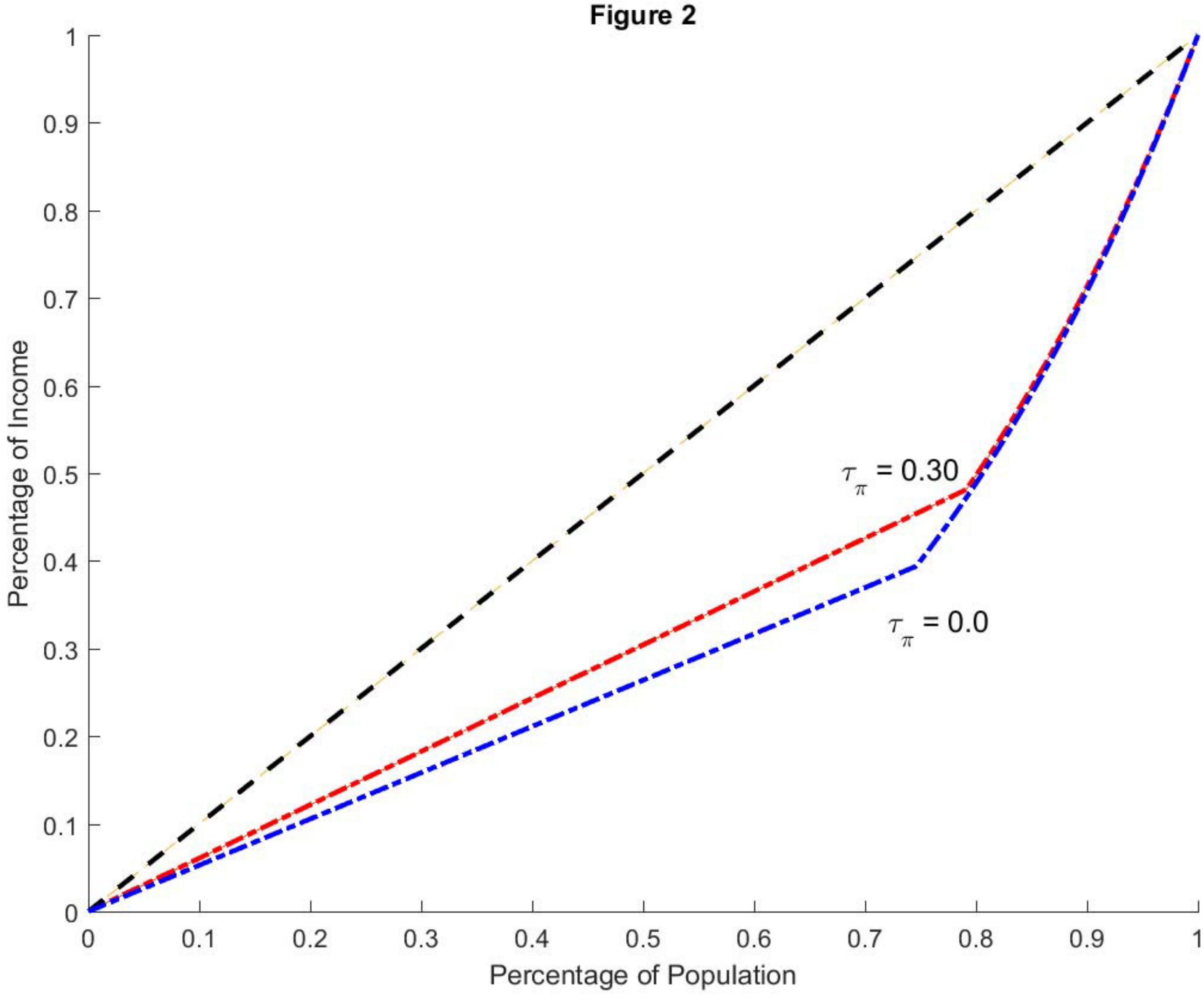
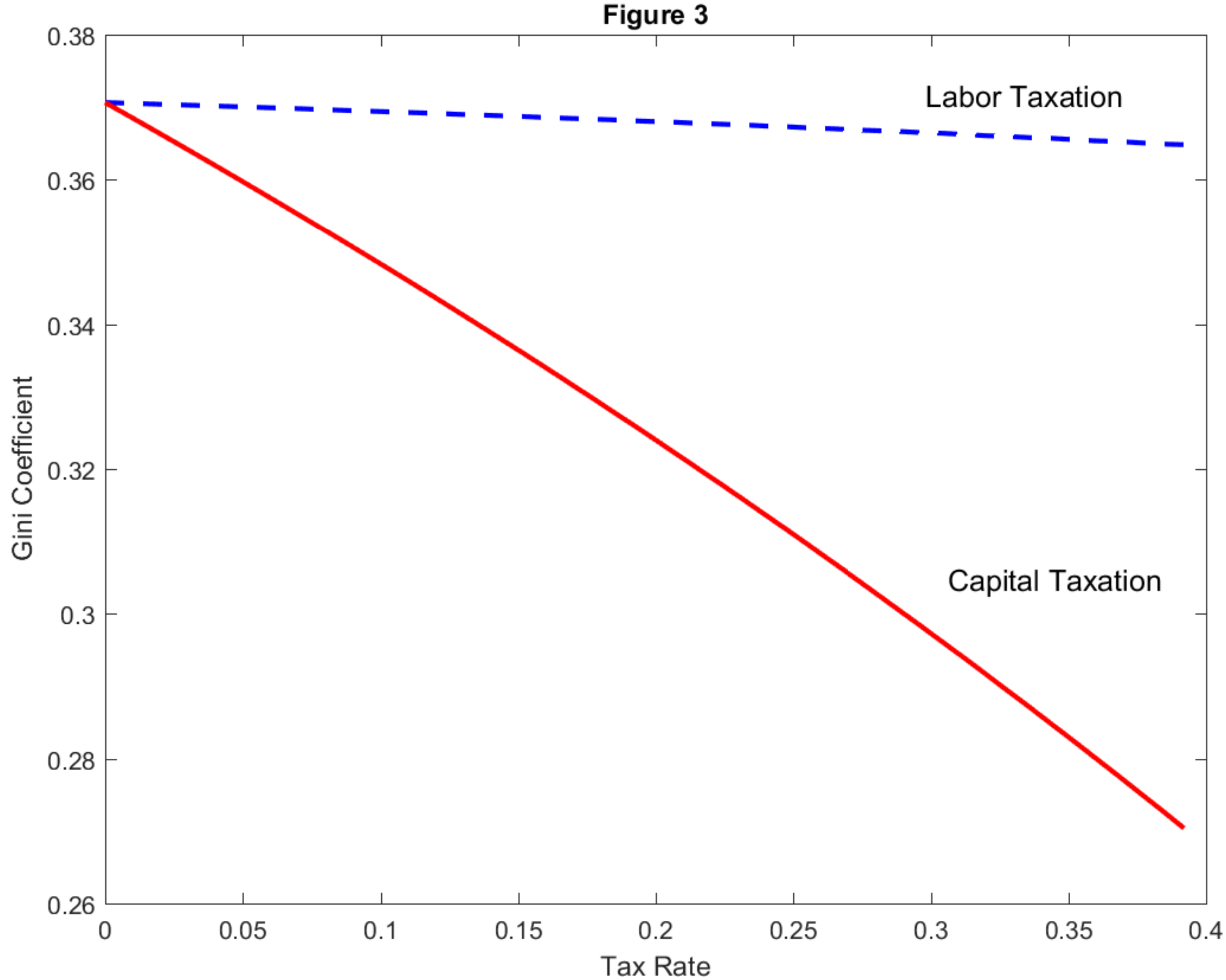


Figure 2



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Percentage of Income
τ_π = 0.30
τ_π = 0.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Percentage of Population

Figure 3



Labor Taxation
Capital Taxation
Gini Coefficient
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Tax Rate

Figure 4

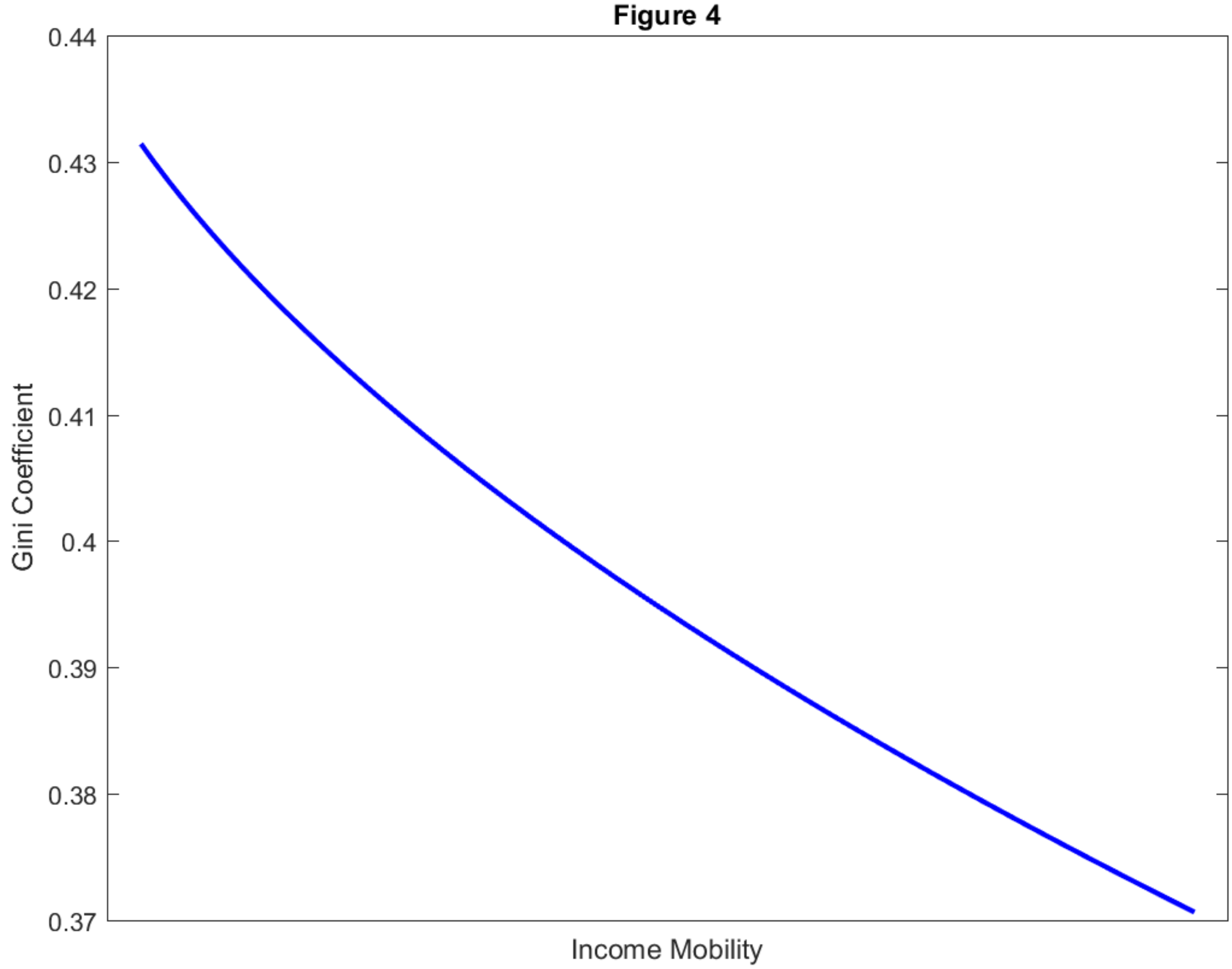
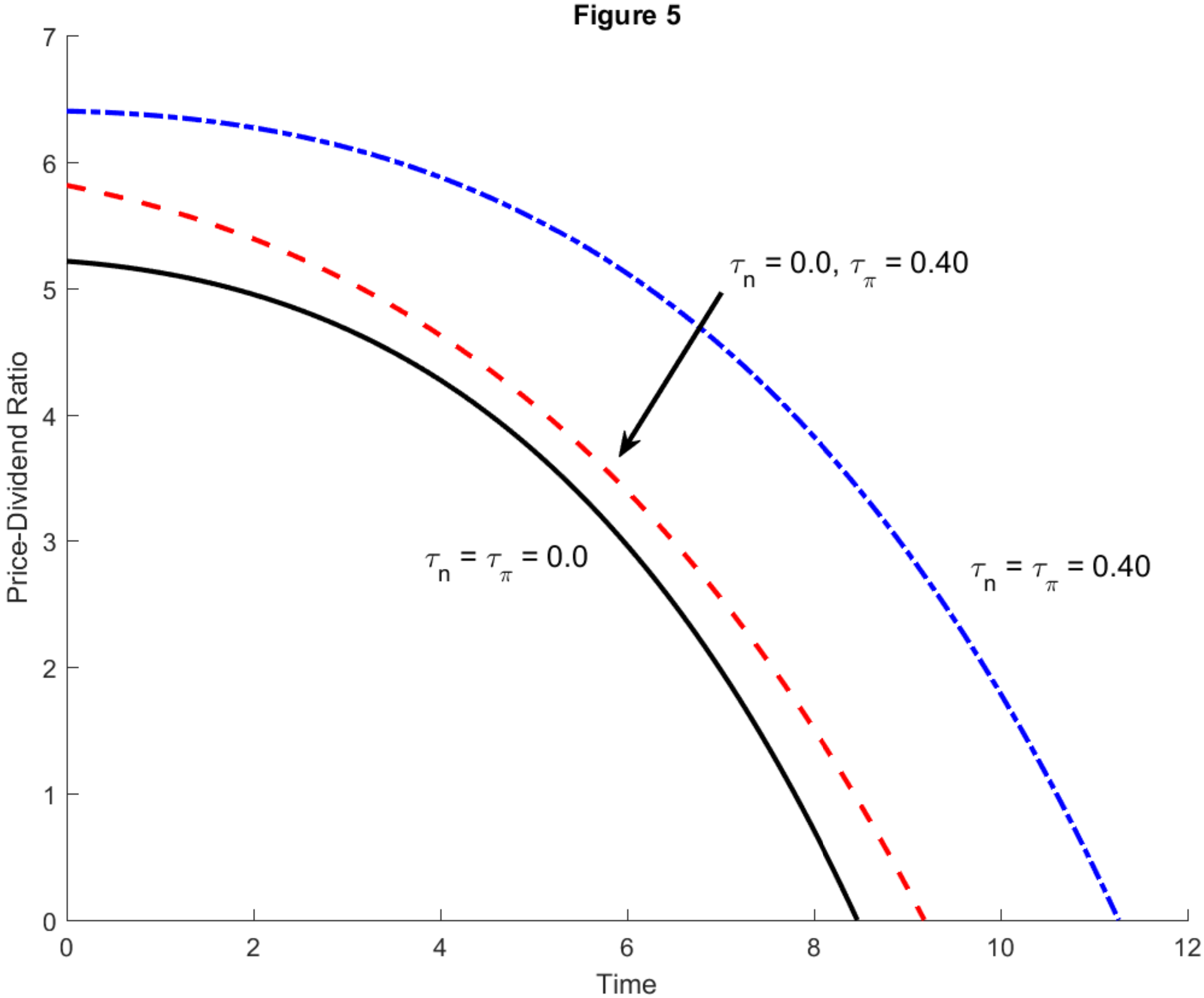

0.44
0.43
0.42
0.41
0.4
0.39
0.38
0.37
Gini Coefficient
Income Mobility

Figure 5



τn = 0.0, τπ = 0.40
τn = τπ = 0.0
τn = τπ = 0.40
Price-Dividend Ratio
Time
0
1
2
3
4
5
6
7
0
2
4
6
8
10
12

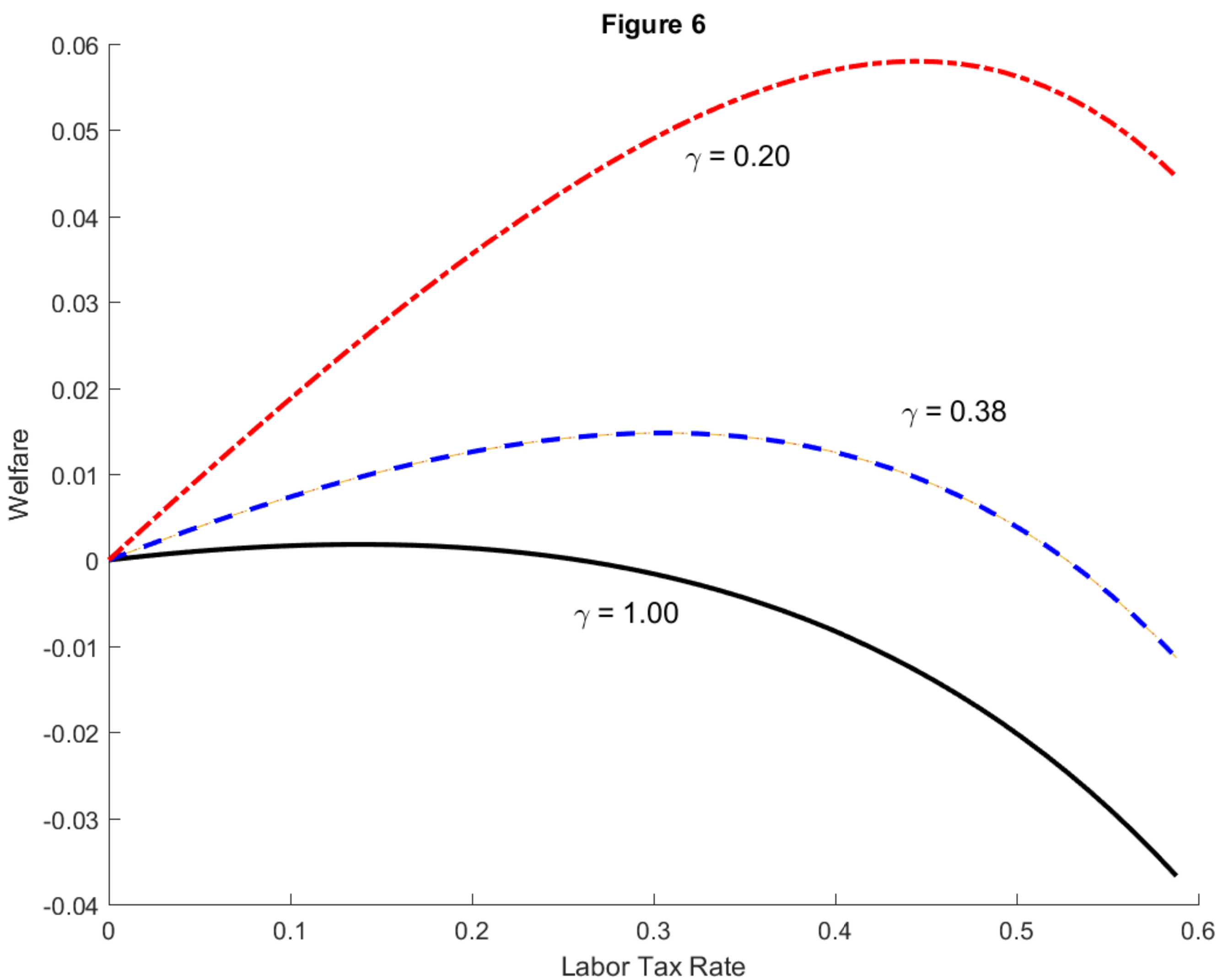


Figure 6
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
Welfare
γ = 0.20
γ = 0.38
γ = 1.00
0
0.1
0.2
0.3
0.4
0.5
0.6
Labor Tax Rate

Figure 7a

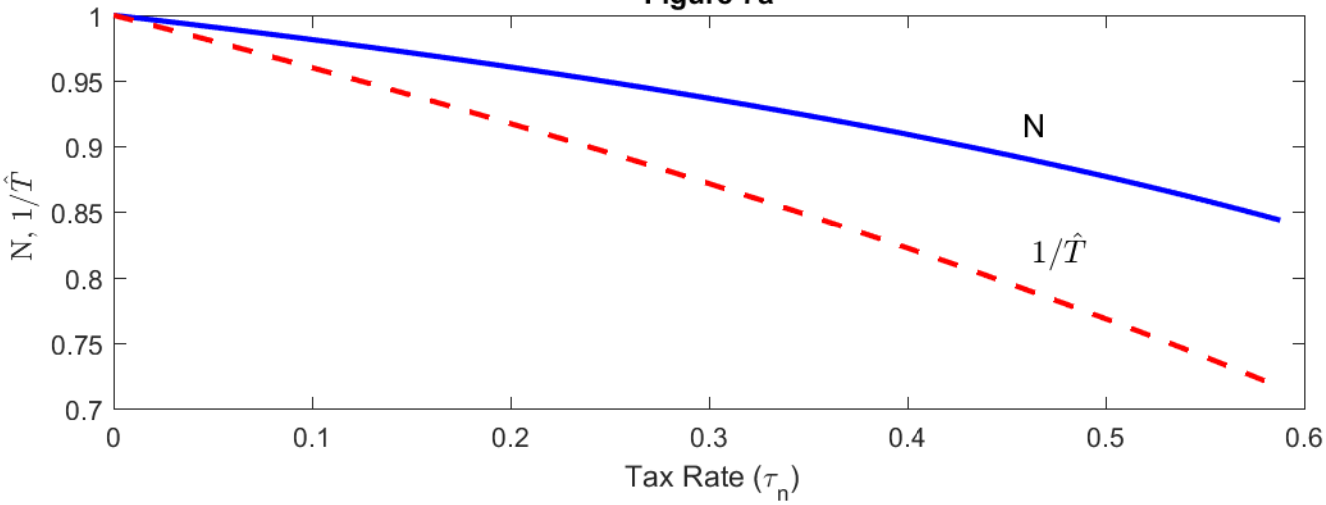


Figure 7b

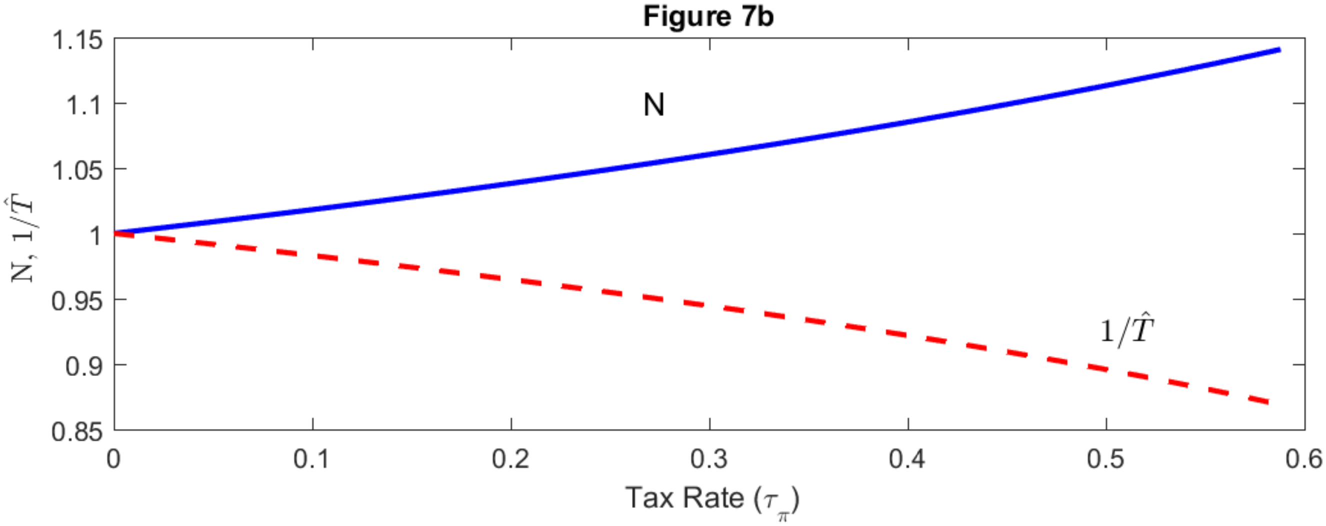
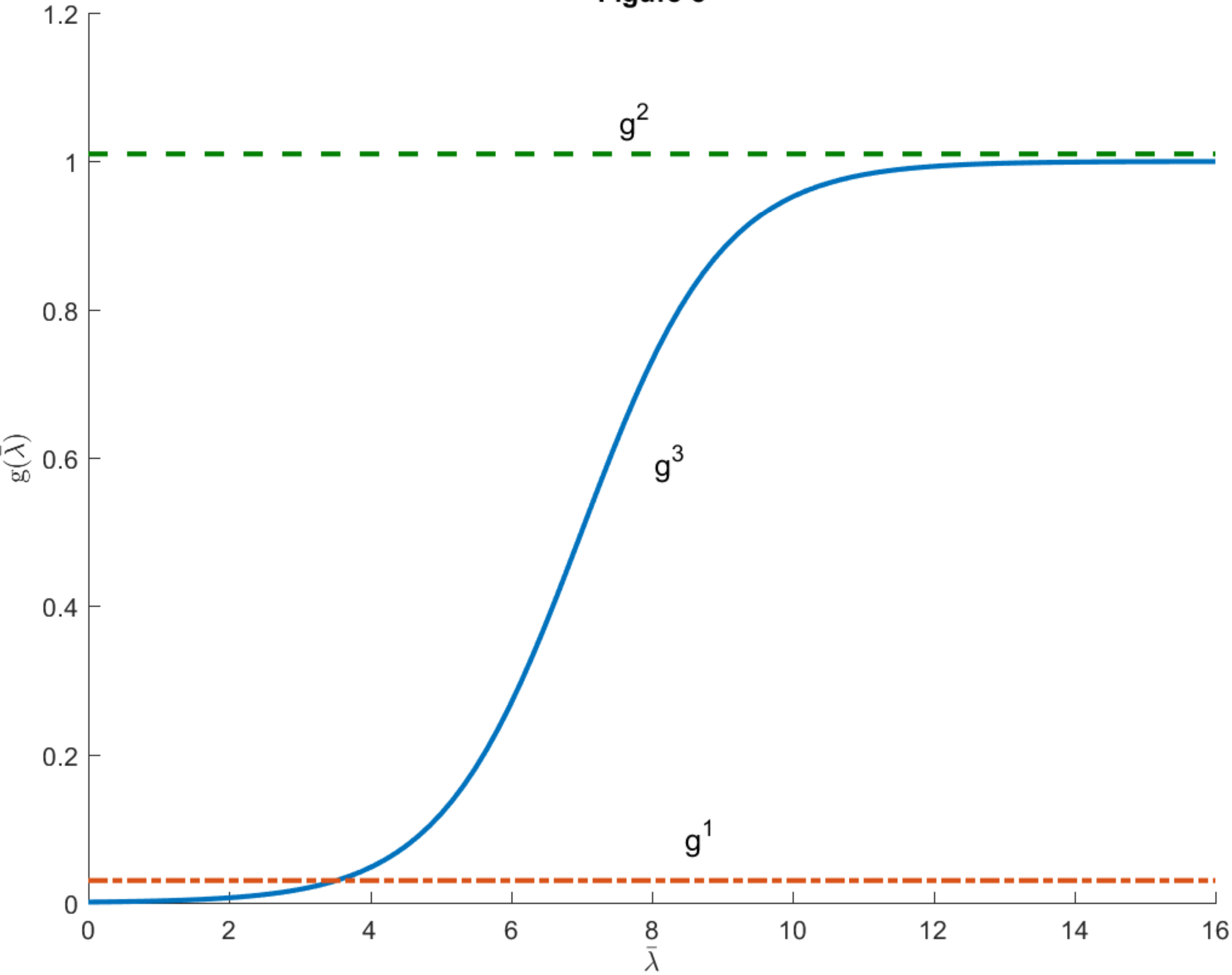


Figure 8



1.2
1
0.8
0.6
0.4
0.2
0
$g(\bar{\lambda})$
0
2
4
6
8
10
12
14
16
$\bar{\lambda}$
g^2
g^3
g^1

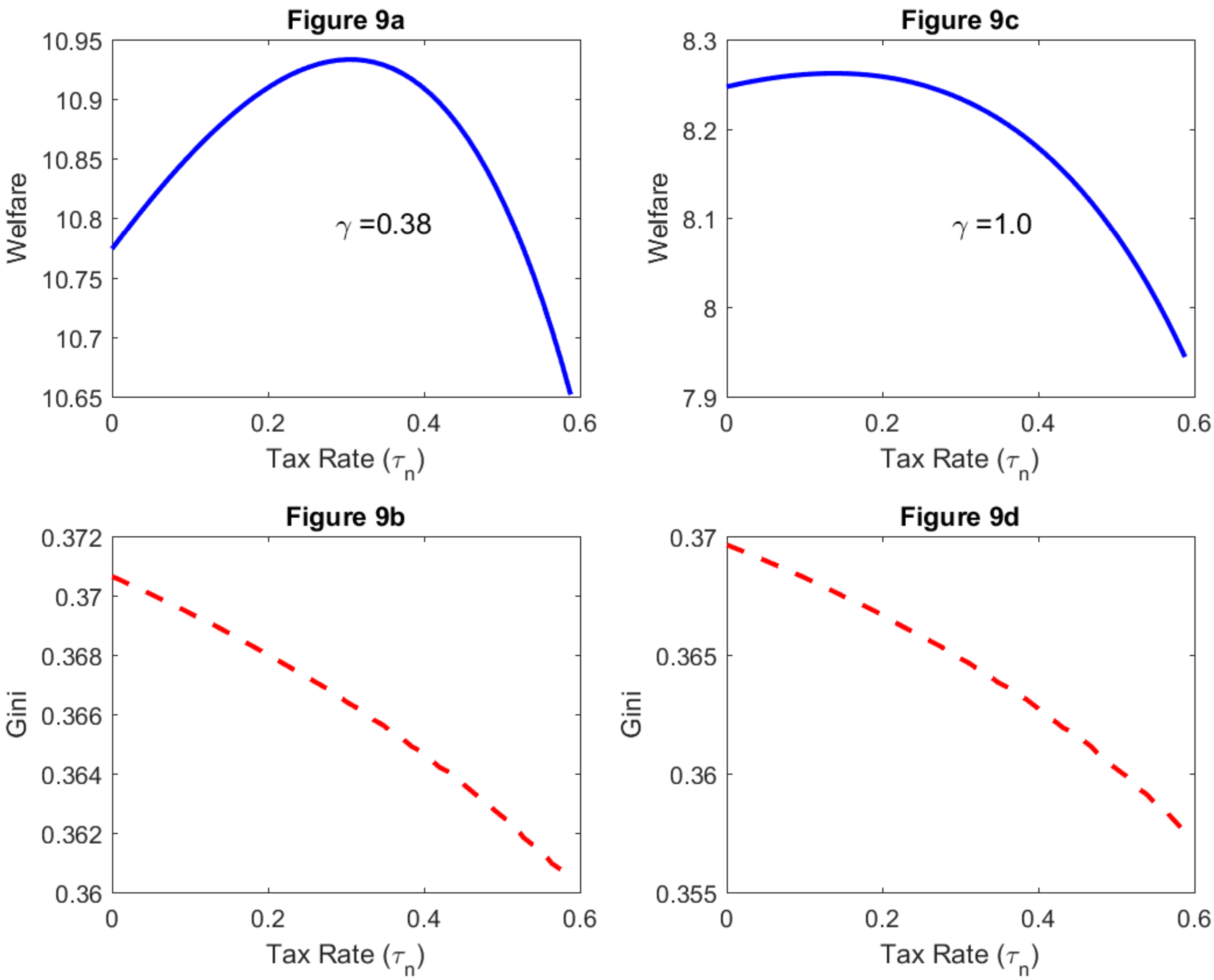


Figure 9a
Welfare
γ =0.38
Tax Rate (τn)
10.95
10.9
10.85
10.8
10.75
10.7
10.65
0
0.2
0.4
0.6
Figure 9c
Welfare
γ =1.0
Tax Rate (τn)
8.3
8.2
8.1
8
7.9
Figure 9b
Gini
Tax Rate (τn)
0.372
0.37
0.368
0.366
0.364
0.362
0.36
Figure 9d
Gini
Tax Rate (τn)
0.37
0.365
0.36
0.355

Figure 10

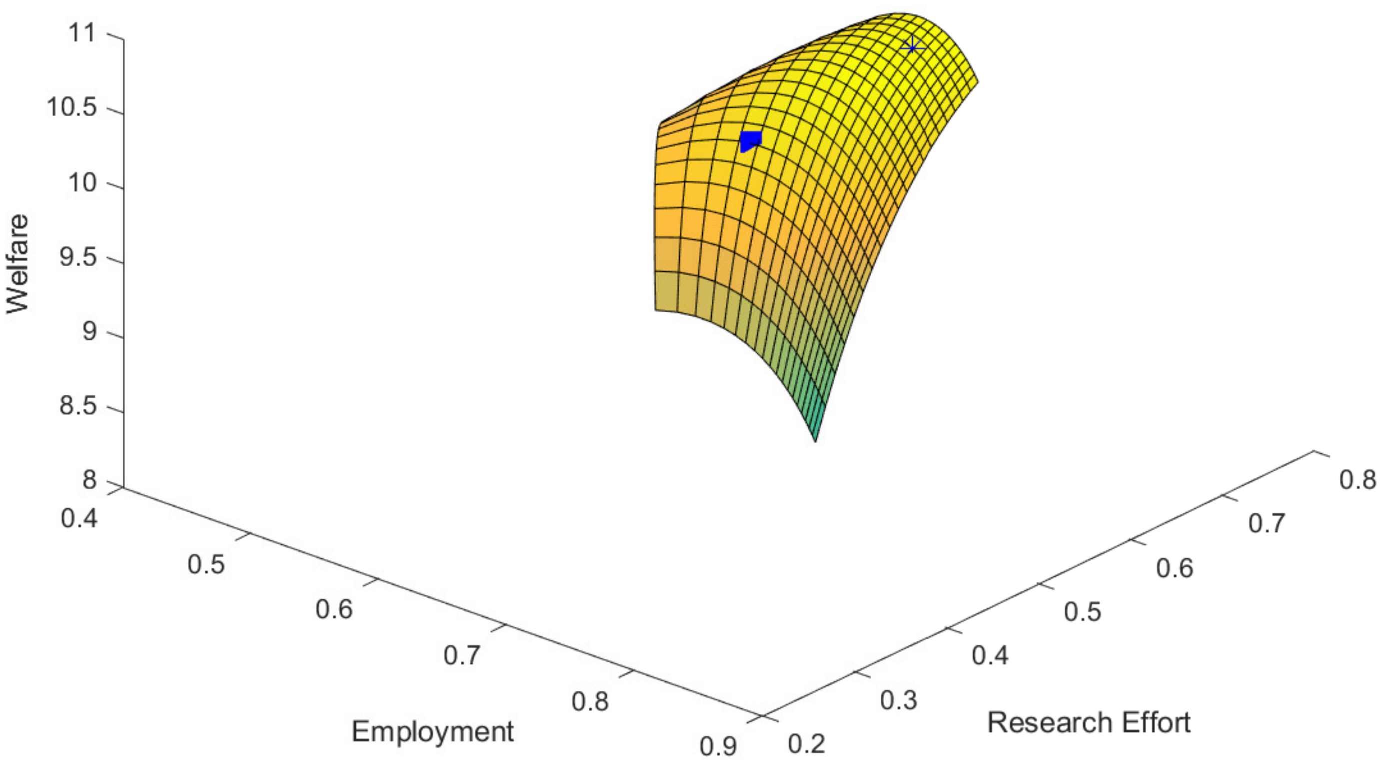

Welfare
11
10.5
10
9.5
9
8.5
8
Employment
0.4
0.5
0.6
0.7
0.8
0.9
Research Effort
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Figure 11

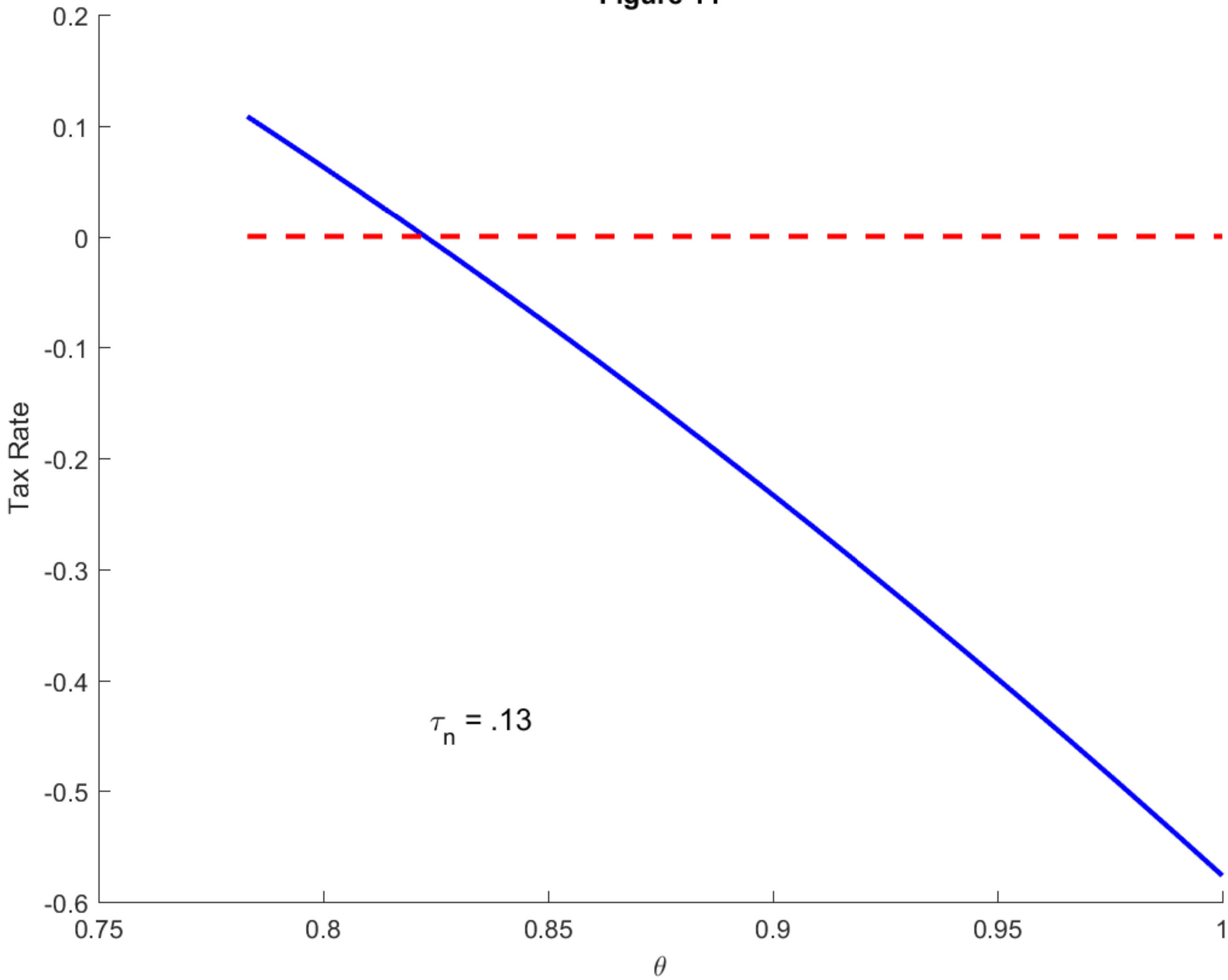


Figure 12