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Rare but Long-lasting Liquidity Traps and Fiscal Stimulus*

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November 30, 2019

Abstract

A DSGE model with (i) state-dependent pricing and (ii) history-dependent monetary policy that compensates for lost opportunities of cutting the nominal interest rate due to a binding effective zero lower bound (ZLB) generates rare but long-lasting liquidity traps with *endogenous* transitions between the traps and normal times. Dynamic government spending multipliers (GSMs) are typically above unity in the liquidity traps but are uniformly below unity in normal times. Without (i) or (ii), the model generates only short-lived ZLB events while producing below-unity GSMs irrespective of the state of the economy.

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Keywords: State-dependent pricing, history-dependent monetary policy, zero lower bound, fiscal stimulus, dynamic government spending multipliers

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1 Introduction

The world's three economic catastrophes over the past one hundred years, the Great Depression a century ago, the Japanese Recession during the 1990s-2000s, and, more recently, the Great Recession and its aftermath, are all characterized by extremely long stays of short-term nominal interest rates at their effective lower bounds. The rare but long-lasting liquidity traps have posed challenges to fiscal and monetary policies and are the two most prominent empirical features of ZLB episodes that a satisfactory approach to the zero lower bound problems should account for.

One of the zero lower bound problems is the question of how effective fiscal policy can be in stimulating an economy when conventional monetary policy is at the constraint. With policy rates in major developed economies spending much of the past decade at their own effective lower bounds, the interest in this question intensifies, with a particular focus on the magnitude of government spending multipliers at the ZLB. A related question is how effective fiscal stimulus can be in helping an economy get out of a liquidity trap. To this end, it is crucial for a model that is designed to answer these questions to incorporate *endogenously* long-lasting (and rare) liquidity traps, with the probabilities of entering and exiting these extreme events determined jointly with other endogenous variables, rather than being fixed exogenously.

We make three contributions in this paper. First, we construct a model that jointly produces the two key empirical features of ZLB episodes, with *endogenous* transitions between normal times and rare but long-lasting liquidity traps. Second, we employ the model to compute the dynamic effects on real GDP of a government spending increase within each of the two endogenous states of the economy. Third, we use the model to gauge the effectiveness of fiscal stimulus as a measure for shortening the duration of a deep recession associated with a liquidity trap. We note that the first contribution serves as a foundation for the second and the third.

Our model features state-dependence in firm pricing à la Golosov and Lucas (2007)[1] along with history-dependence in monetary policy in the spirit of the Reifschneider and Williams (2000) rule

[1]For other classic state-dependent pricing models, see Caplin and Leahy (1991), Caplin and Leahy (1997), Dotsey et al. (1999), King and Wolman (2004), Dotsey and King (2005), Midrigan (2007), Gertler and Leahy (2008), John and Wolman (2008), Nakamura and Steinsson (2010), Midrigan (2011), Kryvtsov and Midrigan (2012), and Dotsey and Wolman (2019), among others.

that would make up for lost opportunities to lower policy rate due to a binding ZLB constraint. The model's stochastic mode and cross-sectional distributions are driven by AR(1) shock processes for risk premium, government spending, and idiosyncratic productivity. Unlike previous studies, we do not rely on increasing the persistence or the volatility of risk premium shocks to lengthen a ZLB spell. Doing so would generate an empirically unreasonable distribution of ZLB events, with the vast majority of ZLB instances being extremely short-lived which also occur at too high a frequency, something we do not observe in actuality. And, it does not help accomplish our second and third tasks properly.[2] We instead fix the processes governing the forcing variables in our model to direct estimates from the actual data. This also permits fair comparisons of our baseline model against its variants that modify in one way or another its two defining features.

We find that the presence of these two features together enables our model to generate rare but long-lasting liquidity traps as observed historically, with endogenous transitions between these extreme events and normal times. Variants of the model that replace either state-dependence with time-dependence in firm pricing or the ZLB-compensation rule with a similar history-dependent monetary policy produces only short-lived ZLB events which also occur at too high a frequency.

[Figure 1 Here]

Although the paucity of ZLB episodes experienced historically makes pinpointing their exact empirical distribution with a high degree of confidence a tall order, there is a general consensus on the rarity of such extreme events. On the other hand, earlier studies based on shorter samples typically underestimated their average duration. Including more recent data up to the time of their writing, Dordal-i-Carreras et al. (2016) estimated the average duration of ZLB events to be 14 to 17 quarters for advanced economies. But even these numbers likely are underestimates given that many central banks in advanced economies are still holding the nominal interest rate at the effective lower bounds at the time of our writing (see Table 1 and Figure 1 for illustrations). Based on a long historical sample for the U.S. economy, extended all the way back to 1889, Ramey and

[2] Dordal-i-Carreras et al. (2016) demonstrate unappealing characteristics of this approach in addressing yet another ZLB related problem for monetary policymakers, concerning the optimal inflation target. They show that a regime-switching model with an exogenously fixed switching probability does a better job in addressing the issue under their consideration. For related research on the zero lower bound problems, see also Wolman (2005) and Wolman (2006), among others.

Zubairy (2018) document that the average duration of ZLB episodes can be as long as 52 quarters. Kocherlakota (2018) presents two factors for why we may anticipate even longer stays at the zero lower bound than observed in the recent past. Our model does well in fitting these numbers: the ZLB binds 2.2 percent of the time and the average duration of ZLB events is 46.3 quarters (see Figure 2). This is to say that the policy rate in the model rarely hits the ZLB, but when it does, it tends to stay there for a very long time.

[Table 1 Here]

The two model features are both essential for this success. If state-dependence in firm pricing is replaced by time-dependence à la Calvo (1983) under the same mean frequency of price adjustment as computed from the state-dependent setting, but with all the other model features kept the same as in the baseline, the frequency of ZLB events would more than double, to 4.9 percent, and the average duration of ZLB events would decline drastically, to 7.4 quarters (see Figure 3b). On the other hand, in an otherwise identical setting, but with the ZLB-compensation rule replaced by a similar history-dependent monetary policy, for example, the shadow-rate rule studied by Hills and Nakata (2018) and Wu and Zhang (2019), under the same degree of policy inertia and responsiveness to inflation and output as under the baseline monetary policy, the frequency of ZLB events would more than triple, to 6.9 percent, and the average duration of ZLB events would also decrease dramatically, to around 10 quarters (see Figure 3a). A variant with both of the two modifications made to the model would generate similarly more frequent and much shorter-lived ZLB events compared to the baseline (see Figure 3c).

It is the joint presence of the two baseline features of our model that generates a powerful interaction between monetary policy and private-sector expectations and decisions in delivering an empirically reasonable distribution of ZLB episodes. At the ZLB, the baseline policy rule yields distributed lags over current and past economic conditions going back to the point when the central bank first entered the ongoing ZLB event, so even if the current data may justify a liftoff from the ZLB, the policymaker becomes more patient to do so, because the (weaker) past data still exert positive weights on its desired policy rate. Or, stated in a forward-looking manner, once constrained by the ZLB, weakening current economic conditions will lower future desired policy rates to compensate for all unrealized below-the-bound desired rates accumulated by then, until after the constraint unbinds. This "wait-to-lift-off" desire resembles certain ZLB compensation, like the kinds of "lower for longer" or "more gradual for longer" announcements issued by the U.S. Federal Reserve and other central banks during the Great Recession and its aftermath when the policymakers were constrained at their effective lower bounds.[3] Knowing the central bank is more patient in lifting rates off the ZLB than current data may suggest, firms become more patient in raising prices even when seeing risen demands or improved economic conditions, resulting in larger fractions of non-adjusting firms and thus more sluggish buildups in inflation pressures. Firms' "hold-off-on" price adjustment and the "missing inflation" in turn reinforce the central bank's "wait-to-lift-off" desire, and a feedback loop is formed between the two features of the model to generate a long-lasting ZLB episode. This "missing inflation" result is consistent with the empirical evidence from Dupor and Li (2015).

In contrast to the baseline policy, the shadow-rate rule yields an infinitely distributed lag mode that renders the central bank's desired policy rate dependent on current and all lagged economic conditions going back to the indefinite past, and this is so regardless of the current state of the economy. At the ZLB, the policymaker's desire to lift off the bound is greater than under the baseline rule, given that normal times were the historical norms with presumably stronger economic conditions. Feeding back to firms' expectations and pricing decisions, especially along the extensive margin, this implies a shorter duration of the ZLB event than under the baseline. Away from the ZLB, the central bank's desired policy rate, which now is also the actual rate, is generally lower than under the baseline policy, due mainly to its dependence on the weaker economic conditions experienced in the recent ZLB episode, while under the baseline rule the policy rate now is determined solely by the current data. Interacted with firms' expectations and pricing decisions, especially along the extensive margin, this implies a more frequent occurrence of ZLB events under the shadow-rate rule than under the baseline policy.

On the other hand, if we fix the extensive margin in firms' pricing decisions from outside,

[3]Such resemblance is also noted by Bundick (2015), and Hills and Nakata (2018) who also provide a detailed documentation of empirical evidence from structural and reduced-form estimations (e.g., Gust et al. (2017) and Coibion and Gorodnichenko (2012)) on the kind of history-dependence in monetary policy embedded in our baseline rule or the shadow-rate rule.

by switching from state-dependent to Calvo pricing, then firms being exposed to positive tail probabilities of being stuck to sub-optimal prices for a long time would make greater intensive margin adjustments whenever hit by a chance to reset their prices. At the ZLB, even if the policymaker is patient in lifting rates off the bound in the face of risen demands or improved economic conditions, firms are more eager to raise their prices when having a lucky draw than if they had the freedom to decide when and by how much to make price adjustments. This leads to more rapid buildups in inflation pressures for the policymaker to justify a sooner-than-later liftoff from the bound. As a result, ZLB events are much shorter-lived under Calvo than under state-dependent pricing, even if the central bank is patient in lifting its policy rate off the bound. Away from the ZLB, similar reasoning suggests that when the economy is hit by adverse shocks firms would be more eager to lower their prices in the Calvo setting and in response policy easing would be quicker and greater than in the state-dependent setting. This implies more frequent occurrence of ZLB events with Calvo pricing than in our baseline model.

A key conclusion of the paper is therefore that it is the interaction between state-dependence in firm pricing and the kind of ZLB compensation in monetary policy that empowers our baseline model to generate rare but long-lasting liquidity traps with endogenous transitions between normal times and the extreme events. Generating such endogenous distribution of ZLB events has been challenging in the existing literature, but it is crucial for properly conducting the second and third tasks of this paper. Our model's success in doing so makes it fitting to isolate the dynamic effects of government spending increases conditional on the two endogenous states of the economy, and also permits an accurate assessment of the effectiveness of fiscal stimulus as a measure for shortening the duration of a liquidity trap.

The model's results on government spending multipliers are in line with empirical evidence. Using a long time-series sample, Ramey and Zubairy (2018) find that, while government spending multipliers on average are below unity, whether in regular recessions or in good times, they can be above unity at the zero lower bound. Differentiating deep recessions associated with the liquidity traps from regular ones, Caggiano et al. (2015) also find that government spending multipliers are greater in these extreme events than in other states. Similar in methodology to our approach that uses a global method to solve the fully nonlinear model, these two empirical studies employ nonlinear techniques to allow for non-absorbing ZLB state while carefully isolating the effects of government spending increases conditional on different economic states.[4]

Consistent with the empirical evidence, dynamic output multipliers generated by an additional government spending shock relative to the stochastic mode of our baseline model are uniformly below unity in normal times, with the maximal effect occurring immediately following a fiscal spending increase and decaying monotonically over time, but are modestly above unity in the liquidity traps on impact of a fiscal spending increase, with the maximal effect realized several quarters afterwards (see Figure 5). In contrast, in variants of the baseline model that replace either state-dependence with time-dependence in firm pricing or the ZLB-compensation rule with the shadow-rate rule for monetary policy, dynamic government spending multipliers are uniformly below unity irrespective of the state of the economy – importantly, in each variant of the baseline, and at any given point in time, the two conditional multipliers are strikingly similar to each other, and to the unconditional multiplier as well (see Figure 6).

This similarity manifests the inability of the alternative models to isolate the effects of fiscal spending shocks in the different states of the economy. This inability is due fundamentally to the fact that ZLB events in variants of the model are too short-lived and occur too frequently, so the effects of an extra dollar of fiscal spending in the two economic states are easily mixed up with each other. This also speaks to the importance of our baseline model's success in generating rare but long-lasting ZLB episodes for an accurate evaluation of conditional fiscal spending multipliers.

As explained above, this empirical success of our baseline model also permits an accurate evaluation of the effectiveness of fiscal stimulus in helping an economy get out of a liquidity trap. Using the baseline model we find that the maximal reduction in the average duration of liquidity traps that can be achieved by fiscal stimulus is about 2 years (see Figure 7). The maximal reduction is obtained with a 2.75 standard deviation increase in fiscal spending above the stochastic mode.[5]

[4]For other recent estimates of state-dependent government spending multipliers, see Barro and Redlick (2011), Auerbach and Gorodnichenko (2013), Auerbach and Gorodnichenko (2012), Auerbach and Gorodnichenko (2017), and Fazzari et al. (2015), among others.

[5]While the size of an additional shock relative to the stochastic mode generally matters for results in this paper, variation in result across different sizes of the additional shock is fairly moderate. To conserve space, throughout the paper we only report output multipliers generated by a one standard deviation government spending shock.

Solving our fully nonlinear model globally is important for this calculation.

To our knowledge, the first and the third contributions of this paper are new to the literature, due to our unique modeling approach that generates an endogenous distribution of ZLB episodes matching what is observed from the data. The second contribution of the paper complements a large body of literature on government spending multipliers at the ZLB. Studies in this literature mostly adopt time-dependent pricing models and abstract from history-dependence in monetary policy, where they either assume a fixed length or fixed transition probabilities into and out of a ZLB state or use other exogenous measures to fix the duration of a ZLB event,[6] with some recent works starting to examine the effects of history-dependence in monetary policy on conditional government spending multipliers.[7] Our paper shows why the joint presence of state-dependence in firm pricing and ZLB-compensation in monetary policy can generate an empirically reasonable and endogenous distribution of liquidity traps that are both rare and long-lasting, how this can serve as a precondition for accurately evaluating dynamic government spending multipliers in liquidity traps and in normal times, and what a natural setting this may provide for confidently assessing the effectiveness of fiscal stimulus in helping an economy pull out of a liquidity trap.

The rest of the paper will proceed as follows. In Section 2, we describe the model and show that it can generate endogenously stochastic transitions between normal times and ZLB events in a way that is consistent with the historical observation of rare but long-lasting liquidity traps. In Section 3, we compute conditional dynamic government spending multipliers by isolating the effects of fiscal stimulus in the liquidity traps from those in normal times, where we also compute unconditional multipliers by mixing up the effects of fiscal stimulus across the two economic states. In Section 4, we analyze the effectiveness of fiscal stimulus as a measure for helping the economy get out of a deep recession associated with a liquidity trap. Throughout these sections, we also show that variants of the model absent either state-dependence in firm pricing or ZLB-compensation in monetary policy are able to generate neither a long-lasting ZLB event nor a larger than unitary

[6]See Christiano et al. (2011), Eggertsson (2011), and Woodford (2011) for some influential earlier works. See, also, Braun and Waki (2010), Canzoneri et al. (2011), Corsetti et al. (2012), Braun et al. (2013), Corsetti et al. (2013), Denes et al. (2013), Albertini et al. (2014), Bouakez et al. (2014), Carlstrom et al. (2014), Fernández-Villaverde et al. (2014), Mertens and Ravn (2014), Michaillat (2014), Schmidt et al. (2016), Roullaeu-Pasdeloup (2018), and Sims and Wolff (2018).

[7]See, for example, Cogan et al. (2010), Coenen et al. (2012), Carrillo and Poilly (2013), Aruoba et al. (2018), Erceg and Lindé (2014), Drautzburg and Uhlig (2015), and Hills and Nakata (2018).

government spending multiplier irrespective of the state of the economy.

2 The Model

The economy consists of a representative household and a continuum of firms that produce differentiated intermediate goods indexed by $j \in [0,1]$ and are monopolistic competitors on the markets for their products. At each date t, a representative distributor combines all differentiated goods $\{Y_{jt}\}_{j\in[0,1]}$ into a composite good $Y_t = \left[\int_0^1 Y_{jt}^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}$, where $\theta > 1$ is the elasticity of substitution between the differentiated goods. The distributor takes the prices $\{P_{jt}\}_{j\in[0,1]}$ of the individual goods as given and chooses the bundle of these intermediate goods to minimize the cost of fabricating a given quantity of the composite good. It sells the composite good to the household at its unit cost $P_t = \left(\int_0^1 P_{jt}^{1-\theta} dj\right)^{1/(1-\theta)}$, which is also the price level of the economy. The demand for a type j good is given by $Y_{jt} = (P_{jt}/P_t)^{-\theta} Y_t$. This is a standard framework of monopolistic competition, in which the inverse of the elasticity of substitution between the differentiated goods also measures the market power of the monopolistically competitive firms.

At any date t, the household seeks to maximize

$$\mathrm{E_t} \sum_{s=t}^{\infty} \beta^{s-t} \gamma_s \left(\frac{C_s^{1-\sigma}}{1-\sigma} - \psi \frac{N_s^{1+\varphi}}{1+\varphi} \right), \quad \text{for } \psi > 0,$$

where $\mathrm{E_t}$ denotes the expectations operator conditional on information up to date t, $\beta \in (0,1)$ is a subjective discount factor, C_s and N_s are the household's consumption and labor supply in period s, and σ and φ denote its relative risk aversion in consumption and in labor, respectively, subject to a sequence of budget constraints,

$$P_s C_s + \mathrm{E_s}\left(Q_{s,s+1} D_{s+1}\right) \leq D_s + W_s N_s + \int_0^1 \Pi_{js} dj + T_s,$$

for $s \geq t$, where D_s is the household's holding of nominal bonds at the beginning of period s, $Q_{s,s+1}$ denotes the stochastic discount factor from $s+1$ to s, thus the gross nominal interest rate in period s is $R_s = \mathrm{E_s}(Q_{s,s+1})^{-1}$, W_s is the nominal wage rate, Π_{js} is the profit that the household receives from firm j in period s, and T_s is a lump-sum tax in period s. Here γ_s is a stochastic process that captures risk-premium shocks,

$$\gamma_s = \gamma_{s-1}^{\rho_\gamma} \exp(\epsilon_{\gamma,s}), \;\; \epsilon_{\gamma,s} \sim N(0, \sigma_\gamma), \tag{1}$$

for $\rho_\gamma \in (0,1)$ and a finite σ_γ.

At any date t, intermediate good j is produced using labor as an input according to

$$Y_{jt} = A_{jt} N_{jt},$$

where A_{jt} is a stochastic process that captures idiosyncratic productivity shocks,

$$A_{jt} = A_{jt-1}^{\rho_A} \exp(\epsilon_{A,jt}), \;\; \epsilon_{A,jt} \sim N(0, \sigma_A), \tag{2}$$

for $\rho_A \in (0,1)$ and a finite σ_A, and firm j seeks to maximize the expected present value of its profit stream in the current and all future periods

$$\mathrm{E_t} \sum_{s=t}^{\infty} Q_{t,s} \left[P_{js} Y_{js} - W_s N_{js} - \Gamma_{js} \left(\chi W_s \right) \right],$$

where $Q_{t,s} = \prod_{h=1}^{s-t} Q_{t+h-1,t+h}$ is a s-period stochastic discount factor, from date $s > t$ to date t, with $Q_{t,t} \equiv 1$, consistent with the household's optimization problem described above, and where Γ_{js} is an indicator function that equals 1 if firm j changes its price at date s but 0 otherwise.

Hence, at any date, a firm's pricing decision has two dimensions: whether to adjust its price (the extensive margin), and if so by how much (the intensive margin), whereby an adjusting firm must pay a fixed cost in units of labor that is independent of the adjustment size. This is a standard way of modeling state-dependence in firm pricing (e.g., Golosov and Lucas (2007)).

Denoting its steady-state value by G, real government spending G_t follows an AR(1) process,

$$\frac{G_t}{G} = \left(\frac{G_{t-1}}{G} \right)^{\rho_G} \exp(\epsilon_{G,t}), \;\; \epsilon_{G,t} \sim N(0, \sigma_G), \tag{3}$$

for $\rho_G \in (0,1)$ and a finite σ_G, and it is financed by the lump-sum tax so $P_t G_t = T_t$.

Monetary policy follows an interest-rate feedback rule that involves both the actual and the desired (shadow or notional) nominal interest rates, R_t and $\tilde{R}_t$, respectively,

$$\frac{\tilde{R}_t}{R} = \left(\frac{\tilde{R}_{t-1}}{R_{t-1}}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{b_1} \left(\frac{Y_t}{Y}\right)^{b_2}\right]^{1-\rho_r}, \qquad R_t = \max\left\{\underline{R}, \tilde{R}_t\right\}, \tag{4}$$

for $\rho_r \in (0,1)$, and b_1, $b_2 > 0$, where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate in period t, $\underline{R}$ is an effective lower bound on the actual nominal interest rate, and R, π and Y represent the steady-state values of R_t, π_t and Y_t.

Policy rule (4) captures the notion that the monetary authority is committed to making up for lost opportunities of reducing the actual nominal interest rate due to the binding effective lower bound (e.g., Reifschneider and Williams (2000)). This is a notion that is consistent with recent central bank behaviors, resembling in particular the various forward-guidance statements issued by the U.S. Federal Reserve and other central banks during the Great Recession and its aftermath when the policymakers were constrained at their effective lower bounds.[8]

2.1 Equilibrium

Denote by U the household's period utility, and U_C and U_n marginal utility from consumption and dis-utility from labor, the first order conditions for consumption, labor, and bonds imply

$$w_t = -\frac{U_{N,t}}{U_{C,t}}, \tag{5}$$

where $w_t \equiv W_t/P_t$ denotes the real wage rate in period t, and

$$1 = \beta \mathrm{E_t}\left[\frac{U_{C,t+1}}{U_{C,t}} \frac{\gamma_t R_t}{\pi_{t+1}}\right]. \tag{6}$$

To make its pricing decision, a firm compares the expected profit stream if adjusting its price (net of the adjustment cost) versus if not while, conditioning on adjusting, it needs to compute the optimal amount of price adjustment in order to facilitate the comparison. The decision is

[8]See our more detailed discussion in the Introduction. See, also, Bundick (2015) and Hills and Nakata (2018), and the references therein.

based on the current realization of its idiosyncratic productivity shock as well as expected future productivity shocks, along with the current and expected realizations of other state variables. The conditional distribution of the idiosyncratic productivity shocks has crucial implications for the cross-sectional price distribution since serial correlation in the shocks implies that a current realization of high productivity leads to the expectation of high productivity in the future. This affects not only the decision along the extensive margin but also that along the intensive margin where an adjusting firm currently with a high productivity draw may choose to make a different amount of price adjustment from an adjusting firm currently having a low productivity draw.

A firm makes a pricing decision at the beginning of a period after it observes the past values of all variables, the current realizations of aggregate shocks, its own current-period productivity draw, but not yet the current-period productivity draws of other firms, and thus not yet the price level or other endogenous variables to prevail in the current period. To solve its problem, the firm needs to forecast the current-period endogenous variables (and form expectations about future variables) - the key is to produce a forecast of the current-period inflation rate, $\pi(\Omega)$, where Ω is the information set of the aggregate state on which the forecast is based.

To formulate the optimization problem in a dynamic programming framework, we denote by V^{a} and V^{na} a firm's value of adjusting its price net of the adjustment cost and the value of keeping its price unchanged from the previous period, respectively. Given firm j's current state $S = \{p_{j,-1}, A_j, \Omega\}$, its value function can then be expressed as

$$V(S) = \max\left\{V^{\mathrm{a}}(S), V^{\mathrm{na}}(S)\right\}, \qquad S = \{p_{j,-1}, A_j, \Omega\} \tag{7}$$

$$V^{\mathrm{a}}(S) = \max_{P_j}\{\mathrm{E}[\Pi_j(p_j, A_j, \mathrm{y}, \mathrm{x})|\pi(\Omega)] + \mathrm{E}_{S'|S}[Q'V(S')]\}, \qquad S' = \{p_j, A_j', \Omega'\} \tag{8}$$

$$V^{\mathrm{na}}(S) = \mathrm{E}\left[\Pi_j\left(\frac{p_{j,-1}}{\pi(\Omega)}, A_j, \mathrm{y}, \mathrm{x}\right)|\pi(\Omega)\right] + \mathrm{E}_{S'|S}\left[Q'V(S')\right], \qquad S' = \left\{\frac{p_{j,-1}}{\pi(\Omega)}, A_j', \Omega'\right\} \tag{9}$$

where y and x are vectors of endogenous and exogenous variables relevant to the firm's profit. In the Bellman equations (8) and (9), the first expectation is taken over the endogenous distribution of the current-period inflation rate in the support based on the projected probabilities, and the second expectation is taken over the conditional distributions of the exogenous driving variables that follow the processes (1), (2), and (3). Solving this optimization problem entails finding a pair of value-policy functions, $\{V, f\}$, and an inflation forecast rule, $\{\pi(\Omega)\}$, such that (i) given the inflation forecast rule, the value-policy functions solve the problem described by (7) - (9), and (ii) the inflation forecast rule is self-validating, that is, the inflation forecast matches the actual inflation in a simulated economy under the corresponding value-policy functions.

To complete the characterization of an equilibrium, fiscal and monetary policies are as specified by (3) - (4), and the markets for goods, labor, and bonds clear, that is, $C_t + G_t = Y_t$, $\int_0^1 N_{jt} dj = N_t$, and $D_t = 0$, for all t.

2.2 Calibration

Given that one period in our model corresponds to one quarter of a calendar year, we set $\beta = 0.99$ to be consistent with a steady-state annualized real interest rate of 4 percent. While some studies in the literature suggest that relative risk aversion in consumption, σ, can be as low as 0 or as high as 30, the general consensus is that it lies between 1 and 10, and a value between 1 and 3 is used most frequently in the macroeconomic literature. We set σ to 2, which is in the middle of its empirically reasonable range.

The inverse of φ corresponds to the Frisch elasticity of labor supply, which has been estimated in many empirical studies. The evidence obtained based on different sources of data, frequencies of time, sample periods, aggregation levels, substitution margins, seasonality adjustments, and estimation procedures, suggests that the elasticity can lie between 0.05 and 2. This suggests a value for φ between 0.5 and 20. On the other hand, a value of $\varphi = 0$, corresponding to an infinite labor supply elasticity, is sometimes also used. We set φ to 1, a value well-within its empirically reasonable range, corresponding to a unitary labor supply elasticity.

We set to 1 the parameter ψ that governs the importance of leisure relative to consumption in household preferences. The two parameters governing the risk-premium shock process (1) are chosen as $\rho_\gamma = 0.87$ and $\sigma_\gamma = 0.0023$, consistent with the estimates in Campello et al. (2008) and Amano and Shukayev (2012) based on an ex-ante measure of risk premium implied by microdata. These values are also consistent with recent estimates by Gust et al. (2017).

A reasonable range for θ, the elasticity of substitution between the differentiated goods, is from 4 to 20, in light of existing empirical studies. We set θ to 10, right in the middle of its empirically reasonable range. We set $\chi = 0.07$, to be consistent with the evidence on price adjustment cost as a share of revenue presented in Nevo (2001), Levy et al. (1997) and Zbaracki et al. (2004). The two parameters governing the idiosyncratic productivity shock process (2), ρ_A and σ_A, are chosen to match moments for the distribution of frequency and size of price changes observed in the U.S. retail pricing data (e.g., Nakamura and Steinsson (2010)).

For the parameters governing the monetary policy rule (4), we set $\rho_r = 0.75$, $b_1 = 1.5$ and $b_2 = 0.5$, in light of the empirical estimates by Taylor (1993), Brouwer and Ellis (1998), Kozicki (1999), Clarida et al. (2000), Orphanides and Wieland (2000), Levin et al. (2003), Coenen et al. (2005), and Gust et al. (2017). We set $\pi = 1.02^{1/4}$, consistent with the FOMC's inflation target. We set the effective lower bound on the nominal interest rate, $\underline{R}$, to 25 basis points, consistent with the average 3-month T-bill rate from 2009:Q1 to 2016:Q4. Our results are robust when we set $\underline{R}$ to 10 basis points to be in line with the average 3-month T-bill rate from 2008:Q4 to 2015:Q4.

For the parameters relevant to the government spending process (3), we choose the steady-state value of G such that $G/Y = 0.2$ in the steady state, and we set $\rho_G = 0.439$ and $\sigma_G = 0.0025$, to be consistent with empirical estimates based on aggregate data, as well as the estimates by Crucini and Vu (2019) based on county-level data spanning 2009:Q2 - 2013:Q4, aggregated from the zip code-level data in the *American Recovery and Reinvestment Act* (ARRA) of 2009.[9]

Table 2 summarizes our model calibration.

[Table 2 Here]

2.3 The stochastic mode: low frequency but long duration of the liquidity trap

The stochastic mode of the calibrated model is simulated with all shocks turned on at all times. In generating the stochastic mode, we simulate the model for 1,000 periods, discarding the first 100 periods as burn-ins, to obtain a sub-sample; and, we repeat this process 600 times, totaling

[9] The data were initially collected by William Dupor at the Federal Reserve Bank of St. Louis and supplemented with additional data from Recovery.gov, a federal government repository of the ARRA data.

600,000 periods, or 540,000 periods after discarding all burn-ins, to obtain the full sample.[10]

The mode features endogenously stochastic transitions between normal times and ZLB events with rare but long-lasting liquidity traps, consistent with the historical observation. While the frequency of ZLB events recorded from the stochastic mode is as low as 2.2 percent, the average ZLB duration is as high as 46.3 quarters, falling in the ballpark of the empirical estimates by Ramey and Zubairy (2018) based on long time-series data and the recent experiences in major developed economies such as Japan, the United States, the United Kingdom, and Germany.

[Figure 2 Here]

To visualize the result in a compact way, Figure 2 presents the histogram and kernel density estimation of the distribution of ZLB durations recorded from the stochastic mode.[11] There are several ZLB episodes that last more than 100 quarters, but for scaling reason they are not displayed in Figure 2. As can be seen from the figure, almost all of the ZLB events last more than 10 quarters, with most of them centered around 40 quarters and significantly many distributed between 50 and 100 quarters (plus the few ZLB episodes not displayed in the figure that last even longer). To our knowledge, this paper is the first in the literature that is able to generate endogenously long-lasting and an empirically reasonable distribution of liquidity traps.[12]

2.4 Variants of the model

The two defining features, state-dependence in firm pricing and ZLB-compensation in monetary policy, are both essential for our model's success in generating an endogenous and data-consistent distribution of liquidity traps. Absent either feature, the model would produce only short-lived

[10] The same procedure is followed in generating the stochastic modes of the model's variants considered below.

[11] We apply the Epanechnikov kernel for bandwidth selection in generating Figure 2 and all subsequent figures that contain kernel density estimations.

[12] Existing studies of the zero lower bound usually generate short-lived ZLB events with the average ZLB duration typically less than 10 quarters, unless some exogenous devices, either deterministic or stochastic, are invoked to peg the nominal interest rate to the ZLB for a specified period of time. For previous studies of the ZLB, see, among others, Fernández-Villaverde et al. (2014) and Richter and Throckmorton (2015). Models that rely on increasing the persistence or the volatility of risk premium shocks to lengthen a ZLB spell generates a counterfactual distribution of ZLB events, with the vast majority of ZLB instances being extremely short-lived which also occur at too high a frequency. See Dordal-i-Carreras et al. (2016) for a discussion of the unappealing characteristics of this approach to the ZLB.

ZLB events which also occur at a much higher frequency. To illustrate this point, we examine three variants of the model which are identical to the baseline in all aspects, except –

Variant 1: Monetary policy follows a shadow-rate rule that modifies the ZLB-compensation rule by replacing the lagged actual rate R_{t-1} on the right side of (4) by its steady-state value R;

Variant 2: Time (instead of state) dependence in firm pricing à la Calvo (1983) with an identical mean frequency of price adjustment as computed from the baseline state-dependent pricing;

Variant 3: The two modifications above are made at the same time.

Panels a, b and c of Figure 3 present the histograms and kernel density estimations of the distributions of ZLB durations in Variants 1, 2 and 3, respectively. Recall that the baseline model generates a low frequency (2.2 percent) and long duration (46.3 quarters) of the liquidity traps. If the ZLB-compensation rule is replaced by the shadow-rate rule, the frequency of ZLB events would more than triple, to 6.9 percent, and the average ZLB duration would decrease dramatically, to around 10 quarters (Figure 3a for Variant 1). If state-dependence is replaced by time-dependence in firm pricing, the frequency of ZLB events would more than double, to 4.9 percent, and the average ZLB duration would decline drastically, to only 7.4 quarters (Figure 3b for Variant 2). If both modifications are made to the baseline, the model would generate similarly more frequent and much shorter-lived ZLB events (Figure 3c for Variant 3). To summarize Figure 3, all three variants of the model generate counterfactual distributions of ZLB events, with the vast majority of ZLB instances being very short-lived which also occur at too high a frequency.

[Figure 3 Here]

2.5 Intuition

To see how state-dependence in firm pricing and ZLB-compensation in monetary policy interact to generate endogenously long-lasting and an empirically plausible distribution of liquidity traps, note that, in a liquidity trap, this baseline policy rule yields distributed lags over current and past economic conditions tracing back to the time when the ongoing ZLB event was first entered, so even if the current data may justify a liftoff from the ZLB, the policymaker becomes more patient to do so because the (weaker) past data still exert positive weights on its desired policy rate. Stated in a forward-looking perspective, once constrained by the ZLB, weakening current economic conditions will lower future desired policy rates to compensate for all of the unrealized below-the-bound desired rates accumulated by then, until after the constraint unbinds. Knowing the policymaker is more patient in lifting rates off the ZLB than current data may suggest, firms become more patient in raising prices even when seeing risen demands or improved economic conditions, resulting in larger fractions of non-adjusting firms and thus more sluggish buildups in inflation pressures. Firms' "hold-off-on" price adjustment and the "missing inflation" in turn reinforce the policymaker's "wait-to-lift-off" desire. A feedback loop is then formed between the two defining features of the model to generate a long-lasting liquidity trap.

In contrast to the ZLB-compensation rule, the shadow-rate rule yields an infinitely distributed lag mode that renders the policymaker's desired policy rate dependent on current and all lagged economic conditions going back to the indefinite past, and this is so regardless of the current state of the economy. At the ZLB, the policymaker's desire to lift off the bound is greater than under the baseline policy, given that normal times were the historical norms with presumably stronger economic conditions. Feeding back to firms' expectations and pricing decisions, especially along the extensive margin, this implies a shorter duration of the ZLB event than under the baseline. Away from the ZLB, the policymaker's desired policy rate, which now is also the actual rate, is generally lower than under the baseline policy, due mainly to its dependence on the weaker economic conditions experienced in the recent ZLB episode, while under the baseline rule the policy rate now is determined solely by the current data. Interacted with firms' expectations and pricing decisions, especially along the extensive margin, this implies a more frequent occurrence of ZLB events under the shadow-rate rule than under the baseline policy.

To get a more concrete feel about firms' diminished incentives to adjust prices in a liquidity trap under the ZLB-compensation rule, Figure 4 presents the histogram (panel a) and the kernel estimation of the cumulative distribution function (panel b) of the time-series distribution of the fraction of non-adjusting firms over all ZLB events. For comparison, the figure also plots such estimates under the shadow-rate rule. As the figure shows, in a liquidity trap there is a larger fraction of non-adjusting firms under the ZLB-compensation rule than under the shadow-rate rule.

[Figure 4 Here]

Under time-dependent pricing, whether firms can adjust their prices is exogenously determined. Consequently, firms being exposed to positive tail probabilities of being stuck to sub-optimal prices for a long time would make greater price adjustments whenever hit by a chance to reset prices given the possibility of not being able to adjust prices in the future. At the ZLB, even if the policymaker is patient in lifting the nominal interest rate off the bound in the face of risen demands or improved economic conditions, firms are more eager to raise their prices when the odds are in their favor than if they are allowed to decide on both the timing and the size of price adjustments. This results in more rapid buildups in inflation pressures for the policymaker to justify a sooner-than-later liftoff from the bound. As a result, ZLB events are much shorter-lived under Calvo pricing than under state-dependent pricing, even if the central bank is patient in lifting its policy rate off the bound. In normal times when the economy is hit by adverse shocks, firms would be more eager to lower their prices, and in response policy easing would be quicker and greater in the Calvo setting than in the state-dependent setting. This implies more frequent occurrence of ZLB events with Calvo pricing than in our baseline model.

3 Dynamic Government Spending Multipliers

Denote by $\partial\tilde{\epsilon}_{G,t}$ an additional government spending shock relative to the stochastic mode that is triggered at time t, and $\{\partial\tilde{G}_s\}_{s\geq t}$ and $\{\partial\tilde{Y}_s\}_{s\geq t}$ the resultant deviations of government spending and aggregate output from the stochastic mode in date t and subsequent periods. The corresponding dynamic government spending multipliers in the H periods following the shock are given by

$$\tilde{\Phi}_{t,k} = \frac{\sum_{s=t}^{t+k-1} \partial\tilde{Y}_s/\partial\tilde{\epsilon}_{G,t}}{\sum_{s=t}^{t+k-1} \partial\tilde{G}_s/\partial\tilde{\epsilon}_{G,t}},\ \ k = 1, ...H, \tag{10}$$

where $\tilde{\Phi}_{t,1}$ is the multiplier on impact of the shock. We consider dynamic multipliers generated by one standard deviation government spending shocks to the stochastic mode.

3.1 Conditional multipliers

Our main task here is to compute dynamic government spending multipliers conditional on one of the two endogenous states of the economy. To isolate the effect of fiscal stimulus in a liquidity trap from that in normal time, the two endpoints of the horizon for which (10) is computed are chosen such that the economy remains in the same state from period $t-2$ throughout period $t+H-1$ both in the stochastic mode and after the additional shock to government spending is triggered at time t, while the economy is in a different state in period $t-3$. We allow for a two-period burn-ins time before shocking the stochastic mode so the economy can condition well into the current state, and we set H as long as possible in order to give the economy a better chance to work out its full dynamics within the prevailing state. Our methodology of isolating the effects of government spending shocks in different endogenous states of the economy is in the same spirit of the strategy used by Ramey and Zubairy (2018) and Caggiano et al. (2015) in their empirical studies on conditional government spending multipliers at the ZLB and in normal time.

3.1.1 Multipliers in liquidity traps

Since most of the ZLB events recorded from the stochastic mode of our model last 22 quarters or longer (see Figure 2), we can comfortably choose $H = 20$ when computing dynamic government spending multipliers in the liquidity traps. It is worth reminding that we include only those ZLB episodes that last 22 quarters or longer not only in the stochastic mode but also after the additional government spending shock to the stochastic mode takes place.[13]

We compute (10) for each of these *long-lasting* ZLB events. We then take the average multiplier at each date within the horizon across all of these liquidity-trap episodes. Figure 5 displays these (averaged) multipliers during the 20 quarters following the date (date 1 in the figure) when the additional one standard deviation government spending shock to the stochastic mode is triggered.

As Figure 5 illustrates, dynamic government spending multipliers in the liquidity traps are greater than unity over the entire horizon for which they are computed and plotted in the figure.

[13] A few ZLB events that last 22 quarters or a bit longer in the stochastic mode are excluded from the computation of dynamic government spending multipliers in the liquidity traps because their durations are shortened to being less than 22 quarters by the additional one standard deviation government spending shock to the stochastic mode.

The multiplier on impact of a fiscal spending shock is about 1.17 in magnitude, and it increases over time before leveling off at around 1.7 near the end of the 20-quarter horizon.

[Figure 5 Here]

3.1.2 Multipliers in normal times

Using the same methodology, we compute dynamic government spending multipliers conditional on the economy being in normal times. For comparison, this result is also displayed in Figure 5.

As the figure shows, dynamic government spending multipliers in normal times are uniformly smaller than unity. The multiplier on impact of a fiscal spending shock is only 0.65, and it decreases over time and levels off quickly to slightly below 0.6 during the rest of the 20-quarter horizon.

The above results on conditional government spending multipliers at the ZLB and in normal times are broadly consistent with the empirical evidence presented in Ramey and Zubairy (2018) and Caggiano et al. (2015). While dynamic government spending multipliers are uniformly below unity in normal times, they are typically above unity in the liquidity traps. The contrast between the two conditional multipliers is quantitatively significant. This significant difference manifests the importance of isolating the effects of fiscal stimulus in the two endogenous states of the economy.

3.2 Unconditional multipliers

For the purpose of comparison, we also compute unconditional government spending multipliers by mixing up the effects of fiscal stimulus across the two endogenous states of the economy. To do so, we trigger at each date (except for the last 19 and irrespective of the state of the economy) in the stochastic mode an additional one standard deviation government spending shock and simulate the resultant output multipliers over the subsequent 20 dates (including the date when the additional government spending shock is initiated). We then take the average multiplier at each point in time within the 20-quarter horizon across all of these simulations. The result is displayed in Figure 5 for comparison with the conditional multipliers.

As the figure confirms, the unconditional multiplier lies between the two conditional multipliers at any point in time and is essentially constant at around 0.95 over the entire 20-quarter horizon.

All in all, our results on conditional and unconditional government spending multipliers signifies the importance of isolating the effects of fiscal stimulus in the two endogenous states of the economy. Needless to say, the ability of our model in generating rare but long-lasting liquidity traps is key to such effective isolation.

3.3 Multipliers in variants of the model

As we have shown in Subsection 2.4, variants of the model that abstract either state-dependence in firm pricing or ZLB-compensation in monetary policy from the baseline setting produce only short-lived ZLB instances which also occur at too high a frequency (see Figure 3). This imposes a restriction on how long we can choose H when computing dynamic government spending multipliers at the ZLB in the variants of the model.

Figure 3 suggests that we cannot set H to as long as 20 quarters as we have done for the baseline model, because virtually all ZLB instances in the variants of the model last less than 22 quarters. We instead set H to 5 quarters in order to include the majority of the ZLB events recorded from the stochastic modes of the variants for the computation of conditional multipliers at the ZLB. This is to say that we include all of those ZLB events that last 7 quarters or longer (including the two-period burn-ins time) both in the stochastic modes of the variants and after the additional government spending shocks to their stochastic modes take place.

For the purpose of comparison, we also choose $H = 5$ when computing conditional multipliers in normal times as well as unconditional multipliers in the variants of the model.

Figure 6 displays these conditional and unconditional multipliers over the 5-quarter horizon for the three variants of the model that replace either state-dependence with time-dependence in firm pricing or\and the ZLB-compensation rule with the shadow-rate rule for monetary policy. As is clear from the figure, absent either of the two defining features of the baseline, the model-predicted dynamic government spending multipliers would be uniformly slightly below unity irrespective of the state of the economy. What seems striking from the figure is also that, in each variant of the baseline model, and at any given point in time over the entire 5-quarter horizon, the two conditional multipliers are extremely similar to each other, as well as to the unconditional multiplier.

[Figure 6 Here]

This close similarity is a reflection on the inability of these variants of the model in effectively isolating the effects of government spending shocks in the different states of the economy. This inability is due fundamentally to the fact that the ZLB events in the variants of the model are too short-lived and occur too frequently, and thus the effects of an extra dollar of government spending in the two economic states are easily mixed up with each other. This also speaks to the significance of our baseline model's success in generating rare but long-lasting ZLB episodes for the accurate evaluation of conditional government spending multipliers in the liquidity traps.

As discussed in the introduction of this paper, this success of our baseline model also permits a confident assessment of the effectiveness of fiscal stimulus as a means to pull the economy out of a deep recession associated with a liquidity trap. The following section is devoted to accomplishing this third task of the present paper.

4 How Effective Is Fiscal Stimulus in Shortening a Liquidity Trap?

The endogenous transitions between the long-lasting liquidity traps and normal times featured in the stochastic mode of our baseline model provide a natural setting for examining how effective fiscal stimulus can be in shortening the duration of a liquidity trap.

To this end, we take the endogenous distribution of all of the ZLB episodes recorded from the stochastic mode of the baseline model, and for each ZLB event we initiate an additional government spending shock relative to the stochastic mode in the third period after the economy enters into the current ZLB state and see how much shorter now it lasts than before. We then compute the average reduction in the ZLB duration across all of the ZLB episodes.[14]

It is worth noting that non-linearity in the model's equilibrium dynamics solved by our global method implies that the size of the additional government spending shock to the stochastic mode may matter for an answer to the question posted in this section's title. We therefore conduct the experiment for different sizes of the additional shock. We find that, while shock size does matter,

[14]Since almost every ZLB episode recorded from the stochastic mode of the baseline model lasts longer than 10 quarters, requiring a two-period burn-ins time to set the economy well into the ongoing ZLB state still allows us to include virtually all of the ZLB events in the calculation.

variation in result across different sizes of the shock is quite modest.

Specifically, a one standard deviation shock reduces the average ZLB duration by 7.46 quarters. For a shock half of this size, the average reduction in the ZLB duration is still around 7.36 quarters. On the other hand, a shock doubling this size reduces the average ZLB duration only a bit more, by 7.86 quarters. The maximal reduction is 8.12 quarters, achieved by a 2.75 standard deviation shock. Increasing the size of a shock further does not produce a distinguishably greater reduction in the average ZLB duration.

To summarize the results, fiscal stimulus can shorten a typical liquidity trap by up to two years, reducing the average ZLB duration from 46.3 quarters in the stochastic mode to 38.2 quarters with a 2.75 standard deviation government spending shock to the stochastic mode. This is as much reduction as a fiscal stimulus can get in our baseline model. Figure 7 illustrates these results.

[Figure 7 Here]

5 Conclusion

We have accomplished three tasks in this paper. First, a DSGE model is constructed to generate endogenously stochastic transitions between normal times and rare but long-lasting liquidity traps consistent with historical observation. Second, the model so constructed is used to compute the dynamic effects on GDP of government spending shocks conditional on each of the two endogenous states of the economy, as well as unconditional multipliers. Third, the constructed model is also used to assess the effectiveness of fiscal stimulus as a measure for helping an economy pull out of a deep recession associated with a liquidity trap.

The first accomplishment is new in the literature, which also serves as a starting point for conducting the second and the third tasks effectively. As we have shown in the paper, variants of the model absent either of its two defining features produce only short-lived ZLB instances which also occur too frequently. Consequently, these variants easily mix up the effects of fiscal stimulus across the two economic states – if one were to use these variants to conduct the second task, one would conclude that dynamic government spending multipliers are equally slightly below unity irrespective of the state of the economy. In contrast, the first accomplishment enables our baseline model to effectively isolate the effect of fiscal stimulus in a ZLB state from that in a normal time, leading to the conclusion that dynamic government spending multipliers are uniformly above unity in the liquidity traps, although they are uniformly below unity in normal times, and the differences between the two conditional multipliers are quantitatively significant.

All that said, it is also important to note that while dynamic government spending multipliers in the liquidity traps are uniformly greater than unity, they are only moderately above unity, with an impact multiplier of 1.17 and a maximum multiplier of 1.7. Likewise, and as we have shown in accomplishing the third task, while fiscal stimulus can help an economy pull out of a deep recession associated with a liquidity trap, the effect is quite modest, with a maximal reduction in the average ZLB duration of 2 years – fiscal stimulus can shorten the mean duration of the liquidity traps from 46.3 quarters to 38.2 quarters. Thus one take-home message of this paper is that, given the large welfare cost associated with a liquidity trap documented in the literature, other means should be invoked or combined with the fiscal measure in order to combat the zero lower bound problem more effectively. We intend to leave this inquiry to future research.

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Table 1: Postwar ZLB Experiences in Advanced Economies

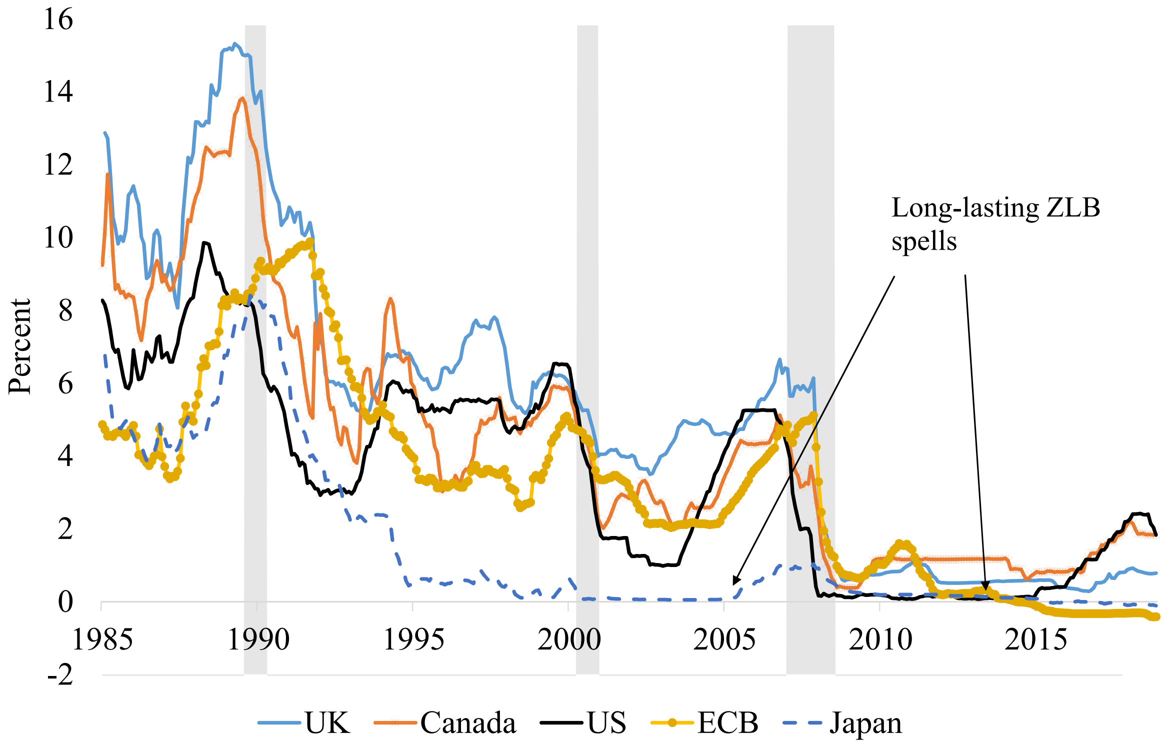
Country	ZLB Episode	Duration (Quarters)
Germany	2009Q2 - 2010Q4	7
Germany	2012Q1 - 2019Q3	34
Japan	1998Q3 - 2006Q4	34
Japan	2008Q4 - 2019Q3	44
United Kingdom	2009Q1 - 2017Q3	34
United States	2008Q4 - 2015Q4	29

Note: This table summarizes the durations (in quarters) of ZLB events in selected advanced economies. Source: Dordal-i-Carreras et al. (2016) with extended data from St. Louis Fed's FRED. As of November 2019, the European Central Bank (Germany) still holds its policy rate steady at its effective lower bound.

Table 2: Calibration

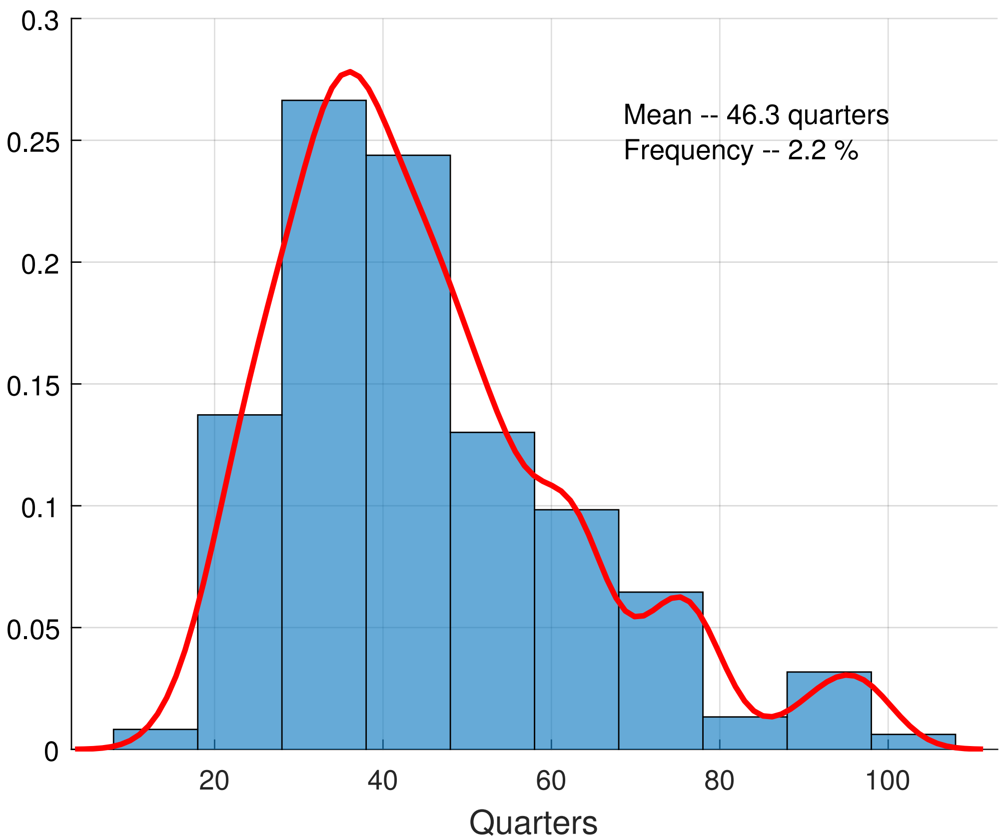
Parameter	Value	Note
β	0.99	Annualized real interest rate $\approx 4\%$
σ	2	Relative risk aversion in consumption
φ	1	Inverse Frisch elasticity of labor supply
ψ	1	Disutility of labor
ρ_γ	0.87	Risk premium shock (persistence)
σ_γ	0.0023	Risk premium shock (std.)
χ	0.07	Menu cost
θ	10	Elasticity of substitution between goods
$[\rho_r; b_1; b_2]$	$[0.75; 1.5; 0.5]$	Monetary policy rule
$\underline{R}$	0.0025	Effective lower bound on nominal interest rate
π	$1.02^{1/4}$	FOMC's inflation target
G/Y	0.2	Government spending-output ratio
ρ_G	0.439	Government spending (persistence)
σ_G	0.0025	Government spending shock (std.)

Figure 1: Duration of ZLB Spells in Data (1985M12-2019M10)



Note: Short-term nominal interest rates in selected advanced economies. German interest rate is used as representative for ECB. Data are from St Louis Fed's FRED. Shaded bars are NBER official recession dates.

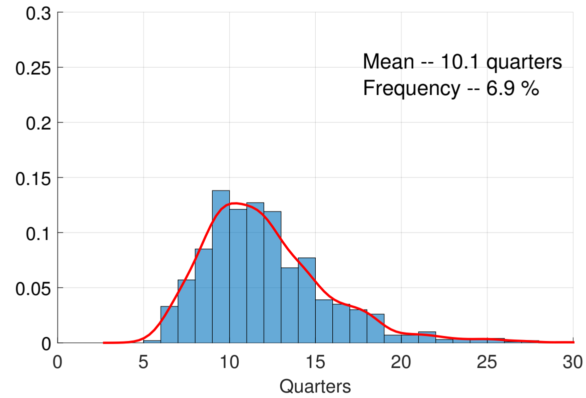
Figure 2: Distribution of ZLB Duration (Model)



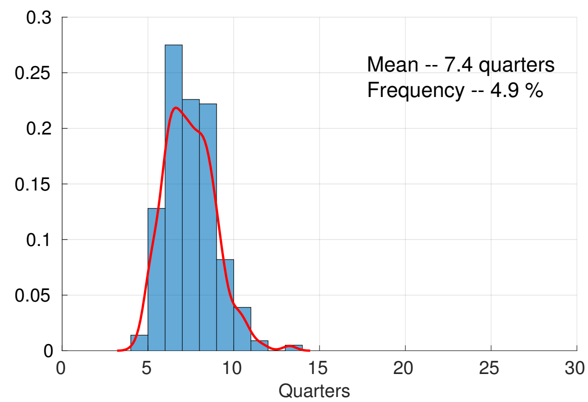
Note: This figure displays the histogram and kernel density estimation of the distribution of ZLB durations in the baseline model. For aesthetic reason, we restrict the plots to ZLB events that last 100 quarters or less; yet, the reported mean duration and frequency of ZLB events are based on the whole simulated sample.

Figure 3: Distribution of ZLB Duration (Variants of Model)

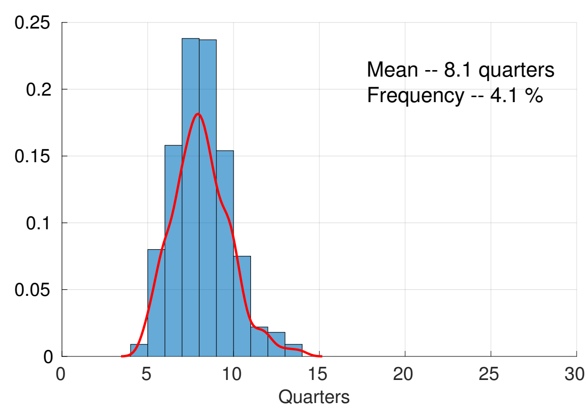
(a) Variant 1 (SDP + Shadow-rate rule)



(b) Variant 2 (TDP + ZLB-compensation rule)



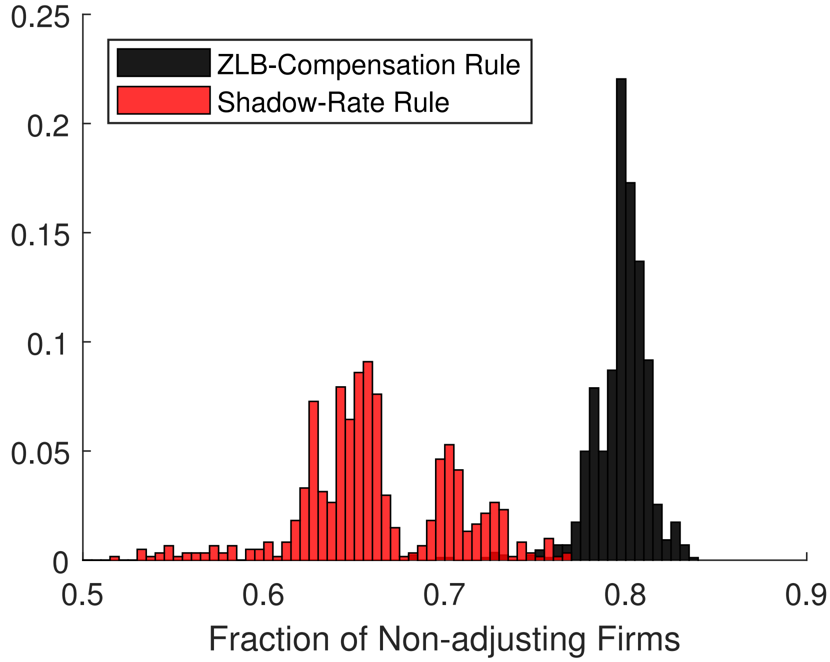
(c) Variant 3 (TDP + Shadow-rate rule)



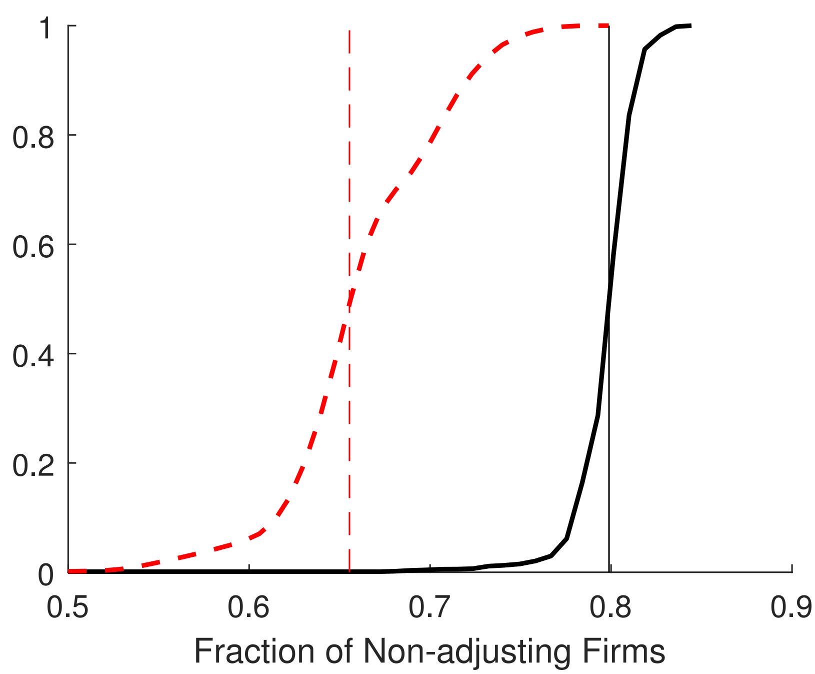
Note: This figure displays the histograms and kernel density estimations of the distributions of ZLB durations in the three variants of the baseline model. For each variant, the whole simulated sample is displayed.

Figure 4: Extensive Margin at ZLB under State-Dependent Pricing

(a) Histograms

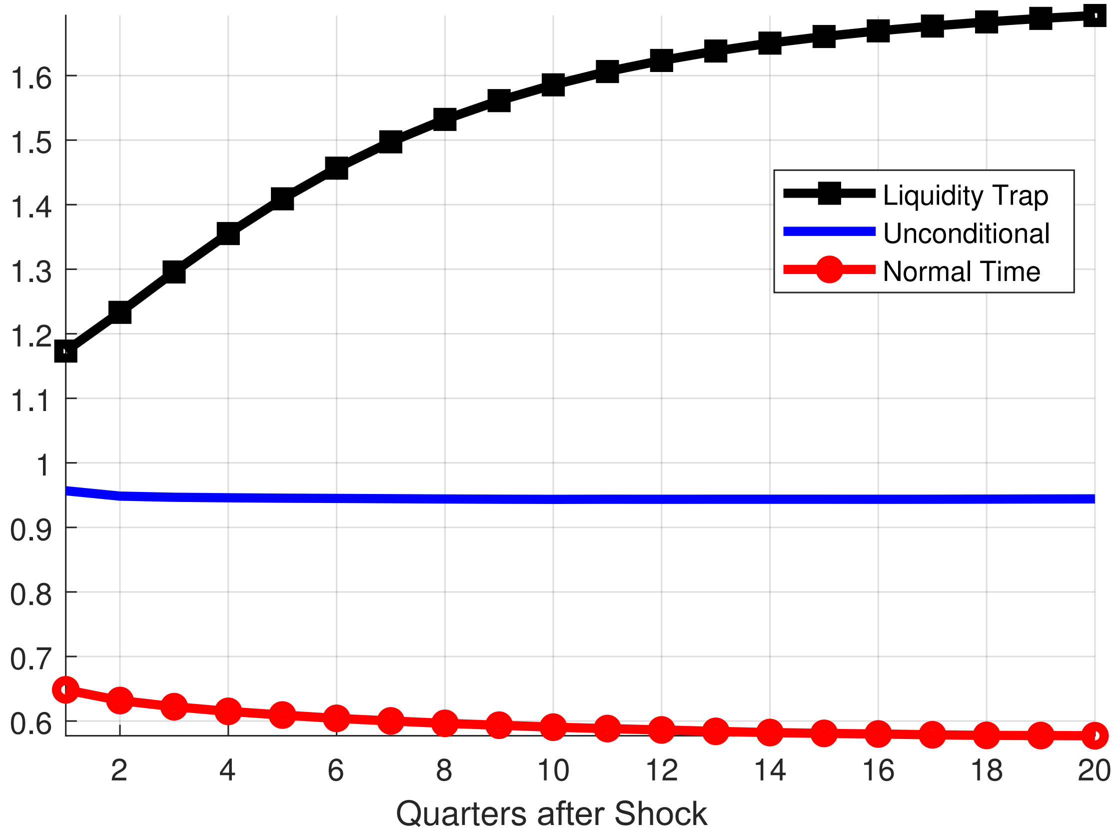


(b) Kernel Estimations of Cumulative Distribution Functions



Note: This figure displays the histograms (panel a) and the kernel estimations of the cumulative distribution functions (panel b) of the time-series distributions of the fractions of non-adjusting firms across all ZLB events under state-dependent pricing with the ZLB-compensation rule (black) and the shadow-rate rule (red). Vertical lines denote the corresponding averages.

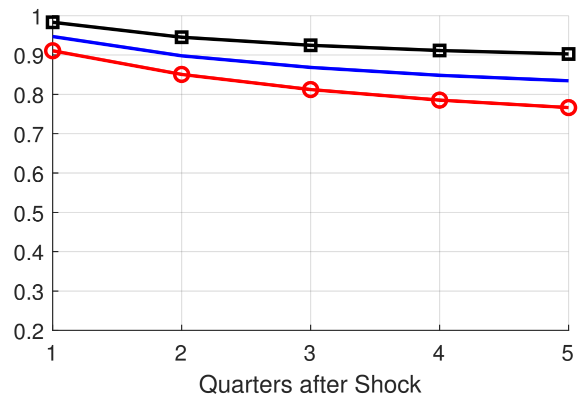
Figure 5: Dynamic Government Spending Multipliers (Model)



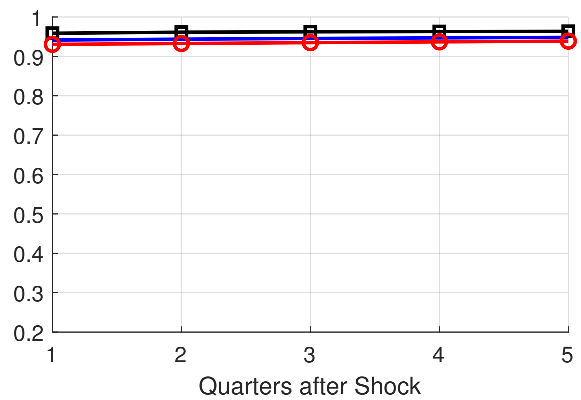
Note: Date 1 is the time when an additional one standard deviation government spending shock relative to the stochastic mode is triggered, which is the third date after the economy enters into the prevailing state (i.e., liquidity trap or normal time) for which the respective conditional multipliers are computed.

Figure 6: Dynamic Government Spending Multipliers (Variants of Model)

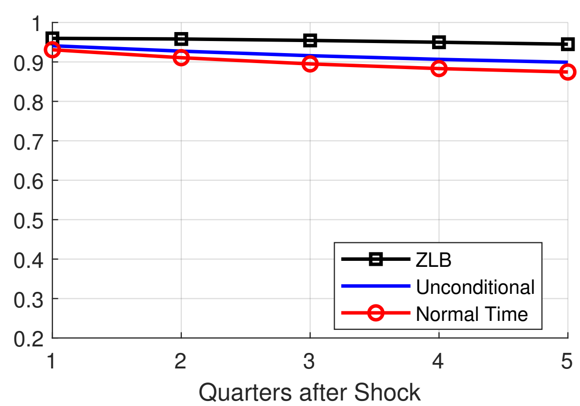
(a) Variant 1 (SDP + Shadow-rate rule)



(b) Variant 2 (TDP + ZLB-compensation rule)

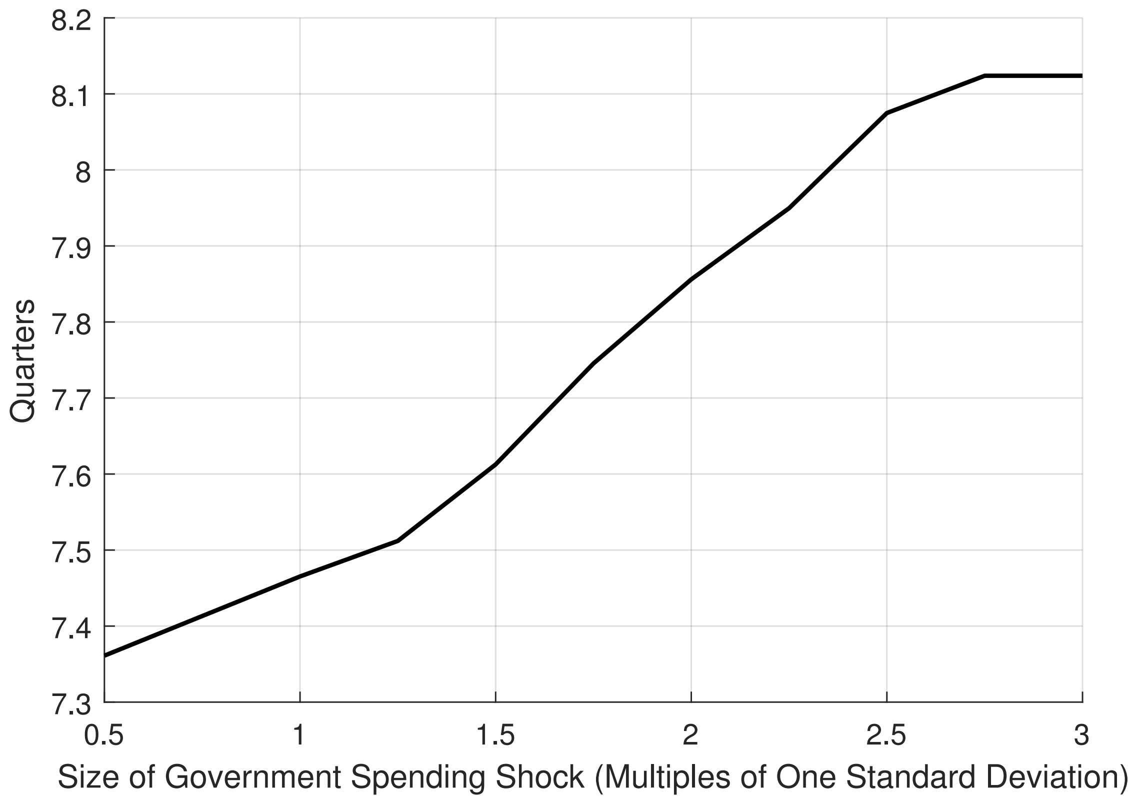


(c) Variant 3 (TDP + Shadow-rate rule)



Note: Date 1 is the time when an additional one standard deviation government spending shock relative to the stochastic mode is triggered, which is the third date after the economy enters into the prevailing state (i.e., liquidity trap or normal time) for which the respective conditional multipliers are computed.

Figure 7: Reduction in ZLB Duration by Fiscal Stimulus (Model)



Note: The x-axis measures the size of an additional government spending shock relative to the stochastic mode (in multiples of one standard deviation), which is triggered at the beginning of the third period after the economy enters into the prevailing liquidity trap, while the y-axis measures the number of quarters shortened by the fiscal stimulus in the duration of the liquidity trap.