

# **CREDIT SEARCH WITH HETEROGENEOUS BORROWERS**

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**Working Paper No. 00-W32R**

August 2000

Revised January 2002

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## **ABSTRACT**

What happens when liquidity increases in credit markets and more funds are channeled from borrowers to lenders? We examine this question in a general equilibrium search model where borrowers (firms) and lenders (households) are either brought together via random matching or assortive financial matchmaking and where the composition of heterogeneous borrowers adjusts to satisfy equilibrium entry conditions. We find that enhanced liquidity causes entry by all borrowers and tends to benefit low quality borrowers disproportionately. However, liquid credit markets may or may not be associated with higher social output. The result is determined by whether the effect of higher market participation outweighs that of lower average quality. The net effect depends crucially on the source of the liquidity shock (credit search efficacy, productivity, or entry barriers).

JEL Classification Numbers: C78, G21

Keywords: Credit Search, Assortive Financial Matchmaking, Market Participation Vs. Composition Effects

Acknowledgement: A significant part of this research was completed while the second author was at Princeton and Atlanta Fed and the third author at Kobe and Atlanta Fed. The authors acknowledge valuable comments and suggestions by seminar participants at these and the authors' own institutes as well as at the Econometric Society Winter Meetings in New York, the Midwest Economic Theory Conference at Purdue, the Midwest Macroeconomics Conference in Pittsburgh and the Society for Economic Dynamics Conference in Sardinia (Italy). Especially, we have benefitted from Abhijit Banerjee, Ben Bernanke, Gary Biglaiser, Peter Diamond, Boyan Jovanovic, Derek Laing, Jennifer Reinganum, Koji Shimomura, Neil Wallace and Ruilin Zhou. Any remaining errors are our own responsibility.

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## 1. Introduction

Entrepreneurs, especially new entrants and owners of small businesses, are constantly seeking for funds to enable their production activities.<sup>1</sup> Lenders, particularly individual investors/brokers and less established financial institutes, devote significant resources to identifying viable borrowers to ensure high expected returns to their loanable funds. Just how the loan contracts are negotiated between credit partners and how the composition of borrowers of different types and their matching rates are determined in equilibrium? A thorough answer to these questions may shed light for better understanding of the divergent and sometimes surprising credit market outcomes as observed in reality. Therefore, this paper attempts to develop a model of bilateral credit search to address these important but largely unexplored issues.

In his pivotal work, Diamond (1990) studies search in credit markets with pairwise meeting and matching of *ex ante* identical borrowers and *ex ante* identical lenders.<sup>2</sup> Our paper extends Diamond's framework in three significant aspects. First, we consider heterogeneous borrowers who have different risk, productivity and cost structure associated with investment projects.<sup>3</sup> This enables us to examine, in addition to a "market participation" effect of credit search, a "composition" effect when unrestricted entry of heterogeneous borrowers is determined in equilibrium. Second, we allow the matching probabilities

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<sup>1</sup> For instance, Evans and Jovanovic (1989) find many such entrepreneurs in the U.S. have been denied for obtaining a loan and continued searching for funding. Blanchflower and Oswald (1998) report that in interview surveys, potential entrepreneurs find raising capital to be their principal problem.

<sup>2</sup> While Townsend (1978) compares bilateral and centralized trading and shows the existence of an efficiency-enhancing role for middlemen, the matching process by which trading agents are paired is not explicitly modeled. Den Haan, Ramey, and Watson (1999) consider matching between borrowers and lenders, but focus on the possibility of long-term credit relationships.

<sup>3</sup> These types of heterogeneity are realistic, as observed and discussed by Gertler and Gilchrist (1994) and many others.

and hence the extend of the “tightness” of the credit market to be determined endogenously.<sup>4</sup> In so doing, we can investigate how the tightness of the credit market is affected by search and entry frictions as well as the degree of borrower heterogeneity. Third, in addition to the benchmark model with a random matching technology, we construct an alternative framework with “assortive” matching. Thus, we are able to compare more adequately credit search outcomes with those obtained in the middleman literature.<sup>5</sup>

We develop a general equilibrium model where search frictions arise because seeking out trades and bargaining over contracts takes time and entry frictions arise because market participation for borrowing investors is costly. For illustrative purposes, we only consider productive credit. That is, households would like to lend a part of their endowment to firms who need to finance their productive activity. Firms differ according to their productivity, riskiness, and cost structure, and these attributes are common knowledge. Hence, there does not exist an adverse-selection or a moral hazard problem and the resulting simplicity enables us to highlight the role of search and entry frictions in credit markets.

In the benchmark case, borrowers and lenders are brought together by a random matching technology similar to Diamond (1990) and Rubinstein and Wolinsky (1987). Once a borrower and a lender are brought together, a loan contract is mediated. The contract determines the loan rate paid by firms by dividing the net surplus of the match according to the agents’ bargaining power. As a consequence of search and entry frictions, equilibrium is reflected by the number of active and inactive credit relationships. Equilibrium is also reflected by credit market tightness as measured by the share of unmatched projects from the credit applicant pool, which we interpret as frictional capital unemployment.

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<sup>4</sup> A labor-market search framework with endogenous matching probabilities has been constructed previously; see, for example, Laing, Palivos and Wang (1995).

<sup>5</sup> The middleman literature focuses on the emergence and equilibrium pattern of intermediated trade. Rubinstein and Wolinsky (1987) consider random matching under complete information, where middlemen emerge and capture a percentage of the matching surplus. Biglaiser (1993) constructs a bargaining model with asymmetric information about quality and correlated valuations, where middlemen reduce adverse selection related inefficiencies. Yavas (1994) assumes that matching between demanders/suppliers and middlemen is immediate, thus giving rise to active middlemen when random matching is too ineffective.

In addition to the benchmark case of random matching, we consider the alternative of assortive matching. In this case, financial matchmakers can grant higher priority to loans made to high-type firms, thereby playing an active role in alleviating credit-market frictions and improving the performance of loanable funds.

We prove the existence and the uniqueness of a nondegenerate steady-state search equilibrium with endogenous entry. Then we analyze the effects of shocks to credit markets and firms' profitability and show that the comparative statics depend crucially on the response of endogenous matching rates and differential entry. Generally, any shock that enhances matching rates causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, we show that low quality firms benefit disproportionately and the average quality of firms falls (unless the shock also raises the relative profitability of low quality firms). Thus, liquid credit markets may or may not be associated with high output and welfare, depending on whether the market participation effect outweighs the composition effect on average quality. Welfare may fall when the composition effect is large, so that it is possible to have outcomes in a complete information framework that resemble results found in models with asymmetric information (cf. Biglaiser and Friedman, 1999).

Our analysis demonstrates that the balance of market participation and composition effects depends on the source of increased market liquidity. Firm profitability shocks (or changes in productivity and entry costs) that enhance aggregate liquidity usually have strong market participation effects. By contrast, credit-market shocks such as enhanced matching efficacy increase market liquidity and participation, but because of a strong composition effect output and welfare may rise or fall. When the credit-market shock is due to lower contract quit rates, the outcome is enhanced liquidity and strong market participation effects. Because lower contract quit rates tend to lengthen contractual relationships (that are not guaranteed with random contract renegotiation), long-term relationships are positively related to output and loan rates. We also show that ex ante loan rate spreads depend only on differential profitability shocks, but that realized credit spreads also depend on offsetting market participation and

composition effects. Finally, under assortive matching, we find that both the loan rate to the high-type and the interest rate differential between the high and the low-type are higher, as are the fraction of high-type firms and the aggregate output.

## 2. The Basic Model

Time is continuous. The economy is populated with a continuum of identical lenders (or households) of unit mass and a continuum of identical borrowers (or firms) of mass  $I$ . In the benchmark framework, lenders and borrowers are brought together via an anonymous random matching technology and upon a successful match, bilateral credit arrangements can be made in a way similar to Diamond (1990). In contrast to his paper, however, we determine endogenously the contact rates as well as the mass of firms by unrestricted entry with a setup (or entry) cost,  $v_o$ . The terms of credit arrangements (or loan contracts) are determined by a symmetric Nash bargain between lenders and borrowers. In particular, this structure enables us to highlight the importance of search and entry frictions for credit arrangements.

Each household is endowed with an apple tree that generates flow income normalized to one unit. What they do with their income depends on the environment. Since there is a large number of borrowers and lenders, the probability of rematches in a random environment is zero. Thus, no lender would grant the validity of IOUs from borrowers. In the state of autarchy, each household consumes a flow value of one. By contrast, such flow income can earn positive returns in the presence of credit arrangements with firms which have access to the production technology and hence are able to convert households' saving into productive uses to yield a gross rate of return of  $R > 1$ .

Upon paying the entry cost, firms enter the loanable funds market and search for loanable funds to implement the investment project.<sup>6</sup> When matched with a household, a firm produces flow output  $A$  with probability  $p$  and zero with probability  $(1-p)$ .

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<sup>6</sup>We do not explicitly consider making households have entry costs like firms. Because we fix the mass of households, allowing households an entry choice implies that all households either enter the credit market or stay away.

Since the loanable funds market features spatially separated borrowers and lenders, pairwise meetings are not instantaneous. Assuming Poisson arrivals, we designate  $\mu$  as the household's contact rate (or flow probability of meeting a firm) and  $\eta$  as the firm's contact rate (or flow probability of meeting a household). For finite contact rates, individual households or firms may either be matched or unmatched. Let  $H$  be the mass of searching and unmatched households and  $F$  be the searching firms. The mass of matched households is denoted by  $S$  and by construction the mass of matched household equals the mass of matched firms. By fixing the mass of household to unity, we have:  $S = 1-H$ .

Unmatched households consume their flow endowment or, should lending prove profitable (i.e.,  $R > 1$ ), they proceed to search for a match in the loanable funds market. Once matched with a firm, they lend their endowment and consume the returns of their savings,  $R$ . For simplicity, the length of the lending contract is assumed fixed at  $1/\delta$ . Thus, the flow probability of separation of a matched lender-borrower pair is  $\delta$  and the repayment can be computed as  $R/\delta$ . Upon separation, households and firms go back to the anonymous matching process in the absence of long-term enduring relationships.

We now formalize the flow value associated with searching and matched households of type  $i=1,2$ . Denote  $J_u$  as the value associated with an unmatched household and  $J_m$  as the value associated with a household matched with a firm. We then have:

$$rJ_u = 1 + \mu(J_m - J_u) \quad (1a)$$

$$rJ_m = pR + \delta(J_u - J_m) \quad (1b)$$

Equation (1a) says that the flow value associated with an unmatched household is the sum of the flow rate of consumption of the endowment good and net values gained from being matched with a firm ( $J_m - J_u$ ) which arrives at rate  $\mu$ . Equation (1b) says that the flow value of a household matched with a firm is the sum of the expected returns to the match generated from the loan contract  $R$  and the net value of

terminating the lending contract to re-enter the unmatched state. Notably, should one assume there is a delay in receiving the repayment, say, at the time of contract termination, (1b) should be modified to

$rJ_m = p\delta R + \delta(J_u - J_m)$  and  $\delta R$  becomes the appropriate measure of the gross interest rate. This entire analysis would go through with any substantive changes.

Similarly for firms, let  $\Pi_u$  and  $\Pi_m$  denote, respectively, the unmatched and matched value associated with a firm. These asset values can be specified as:

$$r\Pi_u = \eta(\Pi_m - \Pi_u) \quad (2a)$$

$$r\Pi_m = p(A - R) + \delta(\Pi_u - \Pi_m) \quad (2b)$$

Equation (2a) gives the flow value of an unmatched firm of each type as the product of the rate by which they contact searching households,  $\eta$ , and the net value of becoming matched ( $\Pi_m - \Pi_u$ ). Equation (2b) specifies the flow value of a matched firm of each type as the sum of the net expected productivity of the investment project made possible by the loan contract, less the interest costs, and the net value of terminating the lending contract.

Using (1b) and (2b), the potential (ex ante) gains that accrues from a successful match become:

$$J_m - J_u = \frac{pR - rJ_u}{r + \delta} \quad (3a)$$

$$\Pi_m - \Pi_u = \frac{p(A - R) - r\Pi_u}{r + \delta} \quad (3b)$$

Since both borrowers and lenders are atomistic, they will take their threat points, the market-determined unmatched value, as parametric in the process of bargaining.<sup>7</sup> Consider a cooperative Nash bargain which

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<sup>7</sup>For a detailed description of the atomistic bargaining process, see Pissarides (1984) and related work cited in Laing, Palivos and Wang (1995).

gives a share  $\beta$  of the matched surplus to households and  $1-\beta$  to firms. Bargaining entails solving

$\max_R (J_m - J_u)^\beta (\Pi_m - \Pi_u)^{1-\beta}$  subject to (3a) and (3b), taking both  $J_u$  and  $\Pi_u$  as given. Thus, the bargaining outcome must satisfy the following first-order condition:

$$\frac{\Pi_m - \Pi_u}{1-\beta} = \frac{J_m - J_u}{\beta} \quad (4)$$

Since firms make entry decision based on the expected bargaining outcome, we must solve the term of the credit arrangement  $R$  with  $\Pi_u$  as parametric but can substitute (1a) into (3a) to eliminate  $J_u$ . Thus, the Nash bargaining interest offer function is given by:

$$pR - 1 = \frac{\beta(r+\delta+\mu)(PA - 1) - r\Pi_u}{r + \delta + \beta\mu} \quad (5)$$

Thus, the interest offer is increasing in the household contact rate,  $\mu$ , and the productivity,  $A$ , and decreasing in the unmatched value of firms,  $\Pi_u$ , and the separation (or quit) rate,  $\delta$ .

Steady-state matching in the loanable funds market requires that the flow of firms of both types seeking loanable funds equals the flow of households providing loanable funds, which implies:

$$\mu H = \eta F = m_0 \tilde{M}(H, F) \quad (6)$$

where  $\tilde{M}(, )$  is the random funds matching function that satisfies the following properties: strictly increasing and strictly concave in each argument, homogeneity of degree one, standard Inada conditions and boundary conditions [i.e.,  $\tilde{M}(0, ) = \tilde{M}(, 0) = 0$ ]. Dividing through by the second argument in the matching function and substituting for  $H/F$  yields:

$$\eta = m_0 M\left(\frac{\eta}{\mu}\right) \quad (7)$$

This relationship describes  $\eta$  as a negative function of  $\mu$  which is a ‘‘Beveridge curve’’ (or BC) for the loanable funds market. Under the above-specified properties, we have:

**Lemma 1:** (Beveridge Curve) *The Beveridge curve is downward-sloping in  $(\mu, \eta)$ -space, convex to the origin, asymptotes to both axes, and shifts away from the origin as the matching parameter,  $m_0$ , increases.*

Moreover, since the household population in steady state is fixed at unity, the inflow of households entering the loanable funds market to search for projects (after having been separated from other projects) must equal the outflow from the market, or

$$\delta S = \mu H \quad (8)$$

As there is unrestricted entry of firms of both types, subject to entry costs, the unmatched values of firms of each type  $i$  will be driven down to their respective entry cost:<sup>8</sup>

$$\Pi_u = v_0 \quad (9)$$

Substituting (3b) into (2a) and combining the resulting expressions with the last expression yields the zero-profit (ZP) condition:

$$\eta^{ZP} = \frac{r v_0 (r + \delta)}{p(A - R) - r v_0} \quad (10)$$

Straightforward differentiation implies:

**Lemma 2:** (Unrestricted Entry) *The firm contact rate of type  $i$  that satisfies the zero profit condition is increasing in the own-type entry cost,  $v_0$ , the interest offer,  $R$ , and the quit rate,  $\delta$ ; it is decreasing in the own-type expected productivity,  $pA$ .*

The underlying intuition is clear-cut once we keep in mind that zero profit requires a negative relationship between the net gains of firms accrued from a successful match and the firm contact rates. As net gains rise, more firms tend to participate in the credit market (to restore zero profit), which lowers the probability that an individual firm will locate a household.

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<sup>8</sup>This, of course, requires that the ex-ante population of each type of firm,  $I^i$ , is sufficiently large.

Consider,

**Definition:** A *steady-state equilibrium* is a tuple  $\{R, \mu, \eta, H, S, F, \Pi_u\}$  that satisfies: (i) Nash bargaining, (5); (ii) steady-state matching and separation, (6), (7) and (8); (iii) free entry and zero profit, (9) and (10); and, (iv) population identity,  $S + H = 1$ .

Note that the free entry conditions immediately pin down  $\Pi_u$  at  $v_o$ , whereas (5) gives  $R$  as an increasing function of  $\mu$ . The later relationship can be substituted into (10) to yield an upward-sloping ZP locus in  $(\mu, \eta)$  space, which together with (7) determines the equilibrium contact rates in the steady state,  $\{\mu^*, \eta^*\}$ . Straightforward comparative-static analysis using Figure 1 shows that the equilibrium household contact rates depends positively on matching efficacy ( $m_o$ ) and productivity ( $A$ ), whereas the effects of the entry cost ( $v_o$ ) and the quit rate ( $\delta$ ) on  $\mu^*$  are ambiguous.

One can then use these equilibrium contact rates with (6), (8) and (9) to solve for the equilibrium masses of firms and households:

$$H^* = \frac{\delta}{\delta + \mu^*} \quad (11a)$$

$$S^* = 1 - H^* = \frac{\mu^*}{\delta + \mu^*} \quad (11b)$$

$$F^* = \frac{\delta \mu^*}{(\delta + \mu^*) \eta^*} \quad (11c)$$

These three equations imply that the mass of searching households is negatively related to the household contact rate. Also, the mass of matched household-firm pairs depends positively on the household contact rate. Moreover, the mass of searching firms is increasing in the household contact rate but decreasing in the firm contact rate. Finally, it is useful to point out that equation (11b) is crucial because it pins down the equilibrium value of social output,  $Y^* = S^*A$ , which is increasing in the productivity and the household contact rate but decreasing in the quit rate. In summary, we have:

**Proposition 1:** (Equilibrium with Homogeneous Borrowers) *Provided that  $(pA-1) - rv_o > 0$ , a unique nondegenerate steady-state equilibrium with homogeneous borrowers exists, where the interest rate, the household contact rate and the social output are all increasing in matching efficacy and productivity.*

### 3. From Homogeneous to Heterogenous Borrowers

There are two types of firms indexed by  $i$  that are distinguished by their riskiness and productivity and by their mass  $I^i$ . While a firm's type is known to all, the number of firms of various types is endogenously determined by unrestricted entry with differential costs. Without loss of generality, let type 1 firm have access to a low-risk, low-return investment project whereas the type 2 firm has access to a high-risk, high-return project. Membership in the population set  $I^i$  is determined by a random lottery. Thus, it is necessary to modify the notation defined in the previous section with a superscript  $i$  identifying firm types. Let  $N^i$  ( $N^1 + N^2 = 1$ ) denote the (endogenous) fraction of type  $i$  firms entering the loanable funds market, which need not be the same as the ex ante population share  $I^i$ . Thus,  $N^i F$  represents the population of type  $i$  firms searching for funds.

Specifically, we assume throughout the paper that the high-type pays a higher entry fee, that is,  $v_0^2 > v_0^1$ . Moreover, we assume that the high-type firms are more productive both in absolute terms and on average, but face a lower success rate:

- (A1) (Productivity)  $A^2 > A^1$
- (A2) (Success Rate)  $p^2 < p^1$
- (A3) (Expected Productivity)  $p^2 A^2 > p^1 A^1 > 1$ .

With heterogenous borrowers, all the value functions except that for unmatched households remain valid (with superscript  $i$  added to all relevant variables):

$$rJ_u = 1 + \mu^1(J_m^1 - J_u) + \mu^2(J_m^2 - J_u) \quad (12)$$

Importantly, the household contact rates with different types of firms must be proportional to the relative masses under this random matching framework, implying:  $\mu^i = \mu N^i$  for  $i = 1, 2$ . Nash bargaining now implies, analogous to (5):

$$p^i [A^i - R^i] - r \Pi_u^i = \frac{1-\beta}{\beta} \frac{r+\delta}{r+\delta+\mu} \left\{ (p^i R^{i-1}) + \frac{\mu N^j}{r+\delta} [(p^i R^i - 1) - (p^j R^j - 1)] \right\} \quad (13)$$

This expression clearly illustrates that the outcomes of both types of firms are interdependent because households' threat points depend on the expected returns of both types .

Total differentiation of (13) produces a characterization of the “interest offer function” below:

**Proposition 2:** (Interest Offer) *The interest offer function  $R^i(\mu, N^i; A^i, \Pi_u^i, \delta)$  is increasing in the household contact rate,  $\mu$ , and the own-type productivity,  $A^i$ , but decreasing in the fraction of low-type firms in the loanable funds market,  $N^l$ , the own-type unmatched value of firms,  $\Pi_u^i$ , and the quit rate,  $\delta$ .*

While most of the results are straightforward, that concerning the endogenous composition of firms deserves further comments. From equation (12), an increase in the fraction of type 1 firms entering into the loanable funds market lowers the unmatched value of households ( $J_u$ ) which is their bargaining threat point. As a consequence of the reduced bargaining power of households, the interest offer decreases.

Next, we note that all the steady-state conditions also remain valid, whereas the zero profit conditions for type- $i$  firms ( $i = 1, 2$ ) can be written as:

$$\eta^{ZPi} = \frac{r v_0^i (r+\delta)}{p^i [A^i - R^i] - r v_0^i} \quad (14)$$

which yields identical properties to those described in Lemma 2.

We have already define the most important features of steady-state equilibrium in the homogeneous case. With heterogeneous borrowers, the definition needs to be modified with types added and the endogenous composition  $N^i$  be determined by  $\eta^{ZPi} = \eta$  and  $N^1 + N^2 = 1$ , which the formal states that

each type of firms must face the same contact rate in the anonymous random matching environment.

Although the population masses can still be solved recursively as specified in (11a)-(11c), other key equations all involve the composition variable,  $N^1$ . The remainder of this section will be devoted to using two bargaining equations, two zero profit conditions (with identical  $\eta$ ) and the Beveridge curve to jointly solve for, two interest rates and two contact rates, in addition to  $N^1$ .

For notational convenience, let  $B \equiv [(1-\beta)/\beta](r+\delta)/(r+\delta+\mu)$ ,  $\bar{a}^i \equiv p^i A^i - r v_0^i$ ,  $a^i \equiv \bar{a}^i + B$ ,  $u^i \equiv N^i u$ , and  $u \equiv \mu/(r+\delta) = u^1 + u^2$  where  $dB/d\mu < 0$ ,  $du^1/dN^1 > 0$ ,  $du^2/dN^1 < 0$  and  $du/d\mu > 0$ . Then we can rewrite the Nash bargaining conditions (13) to give:

$$a^i = (1+B+Bu^j)p^i R^i - Bu^j p^j R^j \quad i=1,2, \quad i \neq j$$

This system yields the solution

$$p^i R^i = \frac{\bar{a}^i + B}{1+B} + \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu}(\bar{a}^j - \bar{a}^i)N^j \quad (15)$$

To ensure sensible results, we impose:

**Condition D:** (Net production gain and entry cost differentials)  $p^2 A^2 - p^1 A^1 > r(v_0^2 - v_0^1) > 0$ .

Moreover, to allow low type firms to operate actively, we need  $p^1 R^1 > 1$ , which is given by

$$\bar{a}^1 > 1, \quad \text{or},^9$$

**Condition F:** (Active Operation)  $p^1 A^1 - 1 > r v_0^1$ .

Obviously, given Condition D, Condition F is stronger than Assumption (A3) and it is sufficient but not necessary for active operation. Conditions D and F imply  $\bar{a}^2 > \bar{a}^1 > 1$  and both are sufficient to guarantee active operation for both high and low type firms (i.e.,  $J_m^i > J_u$ ).

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<sup>9</sup> This is clear by rewriting (15) as:  $p^1 R^1 - 1 = \frac{\bar{a}^1 - 1}{1+B} + \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu}(1-N^1)(\bar{a}^2 - \bar{a}^1)$ .

In order to work out the comparative statics, it is useful to note:

**Lemma 3:** (Expected Interest Rates) *Under Assumptions (A1) and (A3), we have the following: (i)*

*$\partial p^1 R^1 / \partial N^1 = \partial p^2 R^2 / \partial N^1 < 0 < \partial p^1 R^1 / \partial N^2 = \partial p^2 R^2 / \partial N^2$ ; (ii)  $\partial p^1 R^1 / \partial \mu = \partial p^2 R^2 / \partial \mu > 0$ ; (iii)  $\partial p^1 R^1 / \partial \delta = \partial p^2 R^2 / \partial \delta < 0$ ; and, (iv)  $\partial p^i R^i / \partial v_0^i < 0$  for  $i = 1, 2$ .*

**Proof:** See Appendix. ■

An increase in  $A^i$  increases the net gains to a match for both the household and firm. However, it raises the net gains to the firm by a greater proportion and hence an increase in the expected returns to the household is required to satisfy the bargaining rule. An increase in the productivity of type  $j \neq i$  firms increases the threat point of households matched to a type  $i$  firm and hence raises the expected interest payment required from the match. Moreover, a change in the proportion of types of firms in the market,  $N^i$ , impacts both the threat point of households and firms. An increase in  $N^1$  lowers the threat point of a matched low type firm and tends to increase the expected returns to the household implied by the bargaining solution. However, it also lowers the threat point of households matched to low type firms by lowering their unmatched value  $J_u$ . Because this latter effect dominates,  $\partial R^1 / \partial N^1 < 0$ . An increase in  $N^1$  also lowers  $J_m$  but increases the threat point of high type firms, both effects leading to  $\partial R^2 / \partial N^1 < 0$ . Similarly, an increase in the fraction of high type firms,  $N^2$ , strengthens the relative bargaining position of households matched with both high and low type firms so that  $\partial R^1 / \partial N^2 > 0$  and  $\partial R^2 / \partial N^2 > 0$ . Finally, entry costs effects are entirely symmetric and opposite to the effects of changes in productivity.

We can compute from (15) the expected interest spread between high and low type firms and the actual interest rate spread as:

$$p^2 R^2 - p^1 R^1 = \beta (\bar{a}^2 - \bar{a}^1) \quad (16)$$

$$R^2 - R^1 = \beta \left( \frac{\bar{a}^2}{p^2} - \frac{\bar{a}^1}{p^1} \right) + \frac{1-\beta}{r+\delta+\beta\mu} [\beta\mu(N^1\bar{a}^1 + N^2\bar{a}^2) + r + \delta] \left( \frac{1}{p^2} - \frac{1}{p^1} \right) \quad (17)$$

where  $\lim_{m_0 \rightarrow \infty, v_0^i \rightarrow 0} R^2 - R^1 = \beta(A^2 - A^1) + (1 - \beta)(1/p^2 - 1/p^1)(N^1 p^1 A^1 + N^2 p^2 A^2 - 1) > 0$ , depending on the

productivity as well as risk differential. We can show:

**Proposition 3:** (Interest Rate Spreads) *Under Assumption (A1) and Condition D, both the expected ( $p^2 R^2 - p^1 R^1$ ) and the actual ( $R^2 - R^1$ ) interest rate spreads are positive. The expected interest rate spread is driven by the expected profitability differential ( $\bar{a}^2 - \bar{a}^1$ ). While the actual interest rate spread in a frictionless economy (with  $m_0 \rightarrow \infty$  and  $v_0^i \rightarrow 0$ ) is determined by productivity as well as risk differentials, such a spread is smaller in an economy with search and entry frictions.*

**Proof:** See Appendix. ■

Condition D says that a positive expected and actual interest spread between high and low types requires that the productivity differential between high and low types be sufficiently large relative to the flow entry cost differential so that  $\bar{a}^2 > \bar{a}^1$ . Hence, interest paid type 2 always exceeds that of type 1 in both expectations and realization. In general, both the ex ante and ex post interest rate spreads depend positively on the expected net productivity differential. The difference between the two is that the actual or realized interest rate spread (which may be referred to as the credit spread) also depends on the composition of firms (decreasing in the fraction of low quality firms) and the household contact rate (positively). Both the composition and contact-rate effects on the actual interest rate spread diminishes as firm's bargaining power ( $1 - \beta$ ) decreases. In a frictionless economy where intermediated matching is instantaneous as in Yavas (1994) and firm entry is costless (i.e.,  $m_0 \rightarrow \infty$  and  $v_0^i \rightarrow 0$ ), productivity and risk differentials pin down the actual interest rate spread. When matching is not instantaneous as considered by Rubinstein and Wolinsky (1987) and when there are entry frictions, such a spread becomes smaller due to the composition effect caused by heterogeneity in entry costs and success rates.

Given the properties of the interest rate function considered above, we now consider determination

of steady-state equilibrium. From Proposition 2, we can write  $R^i = R^i(\mu, \eta, N^1)$  where  $R_\mu^i > 0$ ,  $R_\delta^i < 0$ ,  $R_{N^1}^i < 0$ . Given the interest functions  $R^i$ , steady-state  $\{\mu^*, \eta^*, N^{1*}\}$  thus satisfy (7) and (14).

**Lemma 4:** (Equilibrium Zero-Profit Trace) *Both the  $ZP^1$  and  $ZP^2$  loci described by (14) are downward sloping in  $(N^1, \eta)$ -space with  $-d\eta^*/dN^{1*}/_{ZP^1} > -d\eta^*/dN^{1*}/_{ZP^2}$ . Furthermore, there exists a unique and upward sloping equilibrium zero-profit trace  $EZ$  in  $(N^1, \eta)$ -space,  $\eta = \eta^Z(N^1)$  that satisfies (14) for a given  $\mu$  such that there is a  $N_{min}^1 > 0$  yielding  $\eta^Z(N_{min}^1) = 0$  and  $\infty > \eta^{max} \equiv \eta^Z(1) > 0$ .*

**Proof:** See Appendix. ■

The  $ZP^1$  and  $ZP^2$  loci and the  $EZ$  trace in  $(N^1, \eta)$ -space are shown in Figure 2. How these curves relate to the Beveridge Curve in  $(\mu, \eta)$ -space is also shown. These relationships together pin down the steady-state equilibrium  $\{\mu^*, \eta^*, N^{1*}\}$ .

Satisfying both  $ZP$  conditions in (14) would necessarily imply:

$$D \equiv \frac{p^2[A^2 - R^2]}{rv_0^2} - \frac{p^1[A^1 - R^1]}{rv_0^1} = 0$$

The term  $D$  measures the expected net surplus differential between high and low type firms, or, in short, the firm surplus differential. The firm surplus differential is less than the profitability differential because high type firms must pay a larger entry cost to get it.<sup>10</sup> When  $D$  is positive, the share of low type firms must rise to drive the firm surplus differential back down to equilibrium. To see this, when  $N^1$  rises for a given  $\mu^*$ , households are more likely to contact low type firms. This weakens the ability of high type

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<sup>10</sup>Note that  $D$  is less than the net profitability differential  $((1-\beta)(\bar{a}^2 - \bar{a}^1)/(rv_0^2))$  because entry costs differ. This can be seen by using  $p^2R^2 - p^1R^1 = \beta(\bar{a}^2 - \bar{a}^1)$  to rewrite  $D$  as

$$D = \left\{ (1-\beta)(\bar{a}^2 - \bar{a}^1) - \left[ p^1(A^1 - R^1) - rv_0^1 \right] (v_0^2 - v_0^1)/v_0^1 \right\} / (rv_0^2).$$

Also,  $N^1$  affects  $D$  (through  $R^1$ ) only when relative entry costs differ. Finally, note that

$$\frac{dD}{d\delta} = \frac{v_0^2 - v_0^1}{rv_0^1 v_0^2} \frac{dp^i R^i}{d\delta} < 0 < \frac{dD}{d\mu} = \frac{v_0^2 - v_0^1}{rv_0^1 v_0^2} \frac{dp^i R^i}{d\mu}.$$

firms to extract a higher surplus from households and  $D$  falls. Thus, any changes resulting in  $D > 0$  would require an increase in  $N^1$  to restore zero profit. For example, an increase in  $\mu^*$  or a decrease in  $\delta$  will ultimately increase  $N^1$ . These arguments are useful for understanding the comparative statics derived in Section 5.

The above arguments lead to the following theorem:

**Theorem:** (Existence and Uniqueness) *Under Assumptions (A1) and (A2) and Conditions D and F, there exists a unique, non-degenerate steady-state equilibrium with full information if the expected production gains are sufficiently high such that  $\eta^* \in (0, \eta^{\max})$ .*

**Proof:** The existence and uniqueness will be proved in two steps. First, we claim that the BC and EZ loci uniquely determine steady-state  $\{\mu^*, \eta^*, N^{1*}\}$ . It is clear from the proof of Lemma 3 and expression (14) that as long as the expected production gains are sufficiently high such that  $\eta^* \in (0, \eta^{\max})$ ,  $N^{1*}$  is bounded in the interval  $(0, 1]$ . Then as the determinant of the pre-multiplied matrix of system (7) and (14) is strictly positive, the implicit function theorem implies unique determination of steady-state  $\{\mu^*, \eta^*, N^{1*}\}$ . Thus, for a given pair  $\{\mu, \eta\}$  satisfying (BC), there exists a unique pair  $\{\eta, N^1\}$  which satisfies (EZ). Once we obtain the equilibrium matching rates  $\mu^*, \eta^*$  and fraction of low to high type firms,  $N^{1*}$ , we can use (11a)-(11c) to solve for the equilibrium masses  $\{H^*, S^*, F^*\}$  and so  $F^{1*} = N^1 F^*$  and  $F^{2*} = (1 - N^1) F^*$ . Since (11a)-(11c) are all well-defined monotone functions, the determination of these masses is also unique. ■

#### 4. Comparative Statics under Differential Entry of Heterogeneous Borrowers

We are now prepared to characterize the steady-state equilibrium with heterogeneous borrowers. In addition to examining the determinants of the equilibrium matching rates, the composition and the mass of the matched firms, and the gross rates of interest, we are also interested in the percentage of unmatched projects, the aggregate output of the matched firms, and welfare.

Specifically, from (17) we note that  $S^*$  the equilibrium number of matches is positively related to

$\mu^*/\delta$ . This term reflects the markets' liquidity because it is also equal to the aggregate share of household funds that is channeled to firms. The size of the credit market is measured by  $S^*+F^*$  which adds market participants that are matched to market participants that are unmatched and still searching. Then define  $U^* = F^*/(F^*+S^*)$  as the share of unmatched projects in the credit applicant pool which includes all entrants. This term can be regarded as the "capital-unemployment rate," which measures the tightness of the credit market much like the unemployment rate in the labor market. Since  $F^*=S^*(\delta/\eta^*)$ , we find that  $U^* = 1/(1+(\eta^*/\delta))$  and thus, our measure of capital-unemployment depends on search and entry frictions solely through the factor  $\delta/\eta^*$ .

Next, we compute social output, based on the steady-state masses of matched firms,  $S^*N^i$  ( $i = 1,2$ ):

$$Y^* = S^* [N^{1*}A^1 + (1-N^{1*})A^2] \quad (18)$$

The aggregate output measure can be decomposed into two components. One component,  $S^*$ , reflects aggregate matches and enhanced market liquidity (and enhanced market participation). Another component, the square bracket term, reflects the composition of output and can be interpreted as the average output over all matched firms. Because the two components need not always move in the same direction, the comparative statics with respect to the responses of interest rates and social output are generally ambiguous.

Tedious but straightforward comparative-static analysis yields:

**Proposition 4:** (Credit Market Shocks) *Under the circumstances described in the Theorem, the effects of matching efficiency ( $m_0$ ) and separation rate ( $\delta$ ) on steady-state  $\{\eta^*, \mu^*, N^{l*}, R^{i*}, S^*, U^*, Y^*, J_u^*\}$  are given by:*

- (i) *An improvement in bank's matching productivity generates more matches and leads to a greater fraction of low-type firms.*
- (ii) *An increase in the contract quit rate will raise the firm contact rate and reduce the capital unemployment rate but lower household contact rates and market liquidity. Also, the share of low*

*type firms will fall. When the market participation effect dominates the composition effect, interest rates and output fall; otherwise, the effect is uncertain.*

**Proof:** See Appendix. ■

Table 1 summarizes the comparative statics results for the complete information equilibrium.<sup>11</sup> First, we discuss what happens when bank matching efficiency increases. Intuitively, an increase in bank's matching productivity increases the contact rate for households  $\mu^*$  and encourages firm's entry. The overall level of matchmaking activity increases (as captured by a rise in  $S^*$ ) because of higher household contact rates. From Proposition 3 we know that a rise in the household contact rate raises the interest offer to each firm by an equal amount. This tends to increase the expected net surplus differential ( $D$ ), so there is a disproportionate entry by low type firms as indicated by an increase in  $N^{1*}$ .

This composition effect puts downward pressure on loan rates and is sufficiently strong that they return to where they originally were. Thus, there is no net effect on loan rates and on the firm matching rate  $\eta^*$ . Because the firm matching rate is unchanged, the capital unemployment rate is unaffected. Recall from the discussion of Proposition 1 that  $N^{1*}$  and  $\mu^*$  have opposing effects on the unmatched value of households. Because of the presence of the composition effect, an increase in matching efficiency creates two offsetting forces on output. More matchmaking means higher output because of greater market participation, but this effect is offset by the fact that there are relatively more low type firms in the economy.

Next we ask what happens following an increase in the contract quit rate  $\delta$ , which could arguably be interpreted as an exogenous increase in resistance towards long-term enduring financial relationships. An increase in  $\delta$  reduces the unmatched value of all firms relative to their entry cost, lowers their threat points, and induces firm exit. Thus, firm contact rates of surviving firms rise by the zero-profit condition

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<sup>11</sup>While Figure 2 is useful in illustrating the uniqueness of steady state equilibrium, the comparative statics cannot be easily captured graphically since there are significant feedback effects between the matching rates and the fraction of firm types in the market.

which reinforces the negative direct effect of contract quits on capital unemployment. From the Beveridge curve relationship, household contact rates fall which causes a reduction in market liquidity and a fall in the overall level of matchmaking,  $S^*$ . While fewer productive firms means output falls, output could rise if the average productivity of the remaining firms rises. Average productivity is determined by the firm composition effect. As explained previously, low types enter relative to high types when the firm surplus differential (going to high type) firms is excessive. Contract quits have a negative direct effect on loan rates and a negative indirect effect on loan rates because household contact rates are reduced. Because lower loan rates reduce the surplus differential,  $N^l$  falls. The net effect on output and welfare balances a negative effect from reduced market liquidity with a positive composition effect from an increase in the average productivity of (remaining) firms. Under the assumption that the market participation effect dominates (or production normality) and output fall with a rise in contract quits. Interestingly, our results suggest that long-term relationships (i.e., lower contract quit rates) increase aggregate output and raise loan interest rates to all firms (disproportionately more for high quality firms).

In order to derive sensible comparative dynamic results for entry cost shocks, we assume,

**Condition Q:** (Credit-market Matching Efficacy) There exists a  $m_0$  such that

$$Q \equiv r \frac{r+\delta}{\eta} + r(1-\beta) \left(1 - \frac{\mu}{r+\delta+\beta\mu}\right) \gg 0$$

The restriction on credit-market matching efficacy is met by imposing a sufficient condition on  $m_0$  such that  $\mu < (r+\delta)/(1-\beta)$ . Intuitively, it restricts loanable funds matching not too frictionless to cause entry barriers to affect the interest rates positively.

We can show:

**Proposition 5:** (Firm Profitability Shocks) *Under the circumstances described in the Theorem, the effects of productivity ( $A^i$ ) and entry costs ( $v_0^i$ ) on steady-state  $\{\eta^*, \mu^*, N^l, R^i, S^*, U^*, Y^*, J_u^*\}$  are given by:*

- (i) *Productivity and entry cost shocks that raise the profitability of high types increase household contact rates, market liquidity, and the share of low type firms, but lower firm contact rates which raises the capital unemployment rate. Productivity and entry cost shocks that raise the profitability of low types will have opposite effects on these variables.*
- (ii) *When the market participation effect dominates the composition effect, productivity shocks raise loan rates and output. Cost shocks tend to have the opposite effect (except for output where the outcome is open), if the market participation effect dominates the composition effect and credit-market matching efficacy is not too high to violate Condition Q.*

**Proof:** See Appendix. ■

To understand the effects of shocks that increase firm profitability  $\bar{a}^i$  (due to either an increase in  $A^i$  or a reduction in  $v_o^i$ ), recall that there are two mechanisms at work. First, when  $\bar{a}^1$  (or  $\bar{a}^2$ ) rises,  $\bar{a}^1 - p^1 R^1$  (or  $\bar{a}^2 - p^2 R^2$ ) rises less (or more) than proportionately because of differences in net expected productivity.<sup>12</sup> Zero-profit thus requires  $\eta$  to rise (or fall) which causes the capital unemployment rate to fall (rise). The Beveridge Curve translates the change of  $\eta$  into a fall (or rise) of  $\mu$  so that both market liquidity and market participation fall (rise). Following the discussion of Proposition 1, the direct effect of an increase in  $\bar{a}^1$  (or  $\bar{a}^2$ ) raises loan rates, but the indirect effect of lower (or higher)  $\mu$  causes them to fall (or rise). Loan rates rise when  $\bar{a}^1$  or  $\bar{a}^2$  rises, where production normality guarantees that indirect effects are not too large when the profitability shock benefits high types. Second,  $N^1$  rises whenever shocks cause the firm surplus differential to be excessive. From Propositions 1 and 3, an increase in  $\bar{a}^1$  leads to higher

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<sup>12</sup>A critical relationship for understanding how the zero profit conditions respond to shocks is given by  $\bar{a}^i - p^i R^i = \frac{B}{1+B}(\bar{a}^i - 1) - \frac{(1-\beta)\beta\mu}{r+\delta+\beta\mu} N^j(\bar{a}^j - \bar{a}^i)$ , where the second term of the righthand side is negative (or positive) for  $i = 1$  (or 2).

profitability for low type firms ( $A^1-R^1$ ) and higher loan rate for high types ( $R^2$ ) - the latter results in lower profitability for high type firms. As a consequence, the firm surplus differential decreases, implying a fall in  $N^1$  so as to restore zero profit. By similar arguments, an increase in  $\bar{a}^2$  gives rise to a higher  $N^1$ . As before, the effect of profitability shocks on output balances market participation effects and average productivity effects, whereby the latter is a sum of individual productivity effects and the composition effect. Productivity shocks tend to enhance average productivity directly, while their indirect effects tend to be offsetting. Similar results apply to changes in entry costs.

Overall, any shock that enhances matching rates causes aggregate liquidity to rise. While an increase in liquidity increases market participation by all firms, low quality firms enter disproportionately and the average quality of firms falls (unless the shock raises the profitability of low quality firms). Thus, liquid markets may or may not be associated with high output and welfare, depending on whether the composition effect on average quality outweighs the effect on market participation. Profitability shocks (to productivity or entry costs) that benefit high type firms will enhance aggregate liquidity but create a negative composition effect. By assuming production normality, positive productivity shocks are generally associated with higher output and welfare. Positive credit market shocks will increase market liquidity and market participation, but because of a strong composition effect social output may rise or fall.

Finally, one may also contrast the case of heterogeneous borrowers with that of homogeneous borrowers. It is clear that with heterogeneous borrowers, one can discuss interest rate differentials as well as the presence of the composition effects. The latter gives rise of a result similar to adverse selection without informational asymmetry. Moreover, due to differential entry, an improvement in credit-market matching efficacy no longer generates an unambiguously positive effect on social output. The distinctive effects of composition versus market participation provide a fertile ground for future calibration and empirical studies.

## 5. Assortive Financial Matchmaking

Up to now, we have assumed that despite full knowledge in firm types, matching are random. One may now ponder if there is a role for a “financial matchmaker” to play in improving the performance of the loanable funds market. Specifically, we focus on the case where a financial matchmaker can undertake assortive matching, providing priority to high-type firms available in the credit market for obtaining funds supplied by households. In order for such an active role, it is necessary to assume that there are excess demand for funds (i.e.,  $H < F$ ). Moreover, we assume that funds supply exceeds funds demand by high-type firms, since otherwise assortive matching would yield a corner solution with only high-type granted with loans. This latter restriction requires  $N^2F < H$ , which together with  $H < F$  implies  $N^1F > (H - N^2F)$ .

Under these regularity conditions, all high-type firms receive loans and the mass of the low-type granted with loans is  $H - N^2F$ . Therefore, assortive matching can be thought of as matching in two segmented markets, yet lenders do not undertake directed search *ex ante*. For notational convenience, define the tightness of the loanable funds market as  $\tau = F/H > 1$ . Reinterpreting the contact rates as the arrival rates of the opportunities for funds demands/supplies. We thus have the following relationships:  $\mu^1 = \mu(H - N^2F)/H = [1 - \tau(1 - N^1)]$ ,  $\mu^2 = \mu(N^2F)/H = \tau(1 - N^1)$ , and  $\eta^1 = \eta^2 = \eta$ , where the last expression indicates that arrivals of opportunities are non-discriminating despite matches are assortive. Hence, all the analyses in Sections 3 and 4 remain valid with modifications of the household contact rates as above. Rather than going through the entire analysis, it is therefore sufficient to simply focus on the main difference of assortive financial matchmaking from random matching.

By examining the contact rates, we find  $\mu^1 = [1 - \tau(1 - N^1)] < N^1$  and  $\mu^2 = \tau N^2 > N^2$ . That is, under assortive matching, lenders are more likely to meet with high-type borrowers. Two immediate consequences arise. First, as a result of relative improvement of contact with high-type firms, the loan rate to the high-type is higher, as is the interest rate differential between the high- and the low-type. Second, due to assortive matching, more high-type firms are granted with loans and hence social output increases unambiguously. This may serve to highlight the funds-allocation role of financial matchmakers,

complementing the monitoring function of intermediaries emphasized by previous studies.

## 6. Summary and Extensions

This paper has studied the terms of credit arrangements and the resultant social output within a dynamic general equilibrium search framework with heterogeneous borrowers, where the contact rates as well as the masses of searchers are all endogenously determined. The analysis identifies new channels through which decentralized matching and assortive financial matchmaking may affect the size and quality of loanable funds flows. Specifically, our results suggest that shocks that increase credit market liquidity also lead to increased market participation by firms and a composition effect whereby the participation of low quality firms rises disproportionately. However, more liquid markets only increase output and welfare when the market participation effect dominates the composition effect. This generally is the case when shocks enhance the profitability of firms or when they make contracts less fragile (and financial relationships longer lasting). By contrast, structural shocks that make matching more efficient may have large composition effects. We also demonstrate that the extent of search frictions plays an important role in the determination of market loan rates and loan rate spreads.

Two interesting extensions come to mind. First, one could allow financial matchmakers to, in addition to quality-sorting, choose matchmaking effort by maximizing their output net of a real resource cost (in units of the flow rate of matches). Credit market outcomes under such financial matchmakers can then be compared with those discussed in Section 5 and those under the middlemen framework developed by Rubinstein and Wolinsky (1987). Second, one could introduce asymmetric information about the firm's type. This could be accomplished in two different ways that depend on the timing of firms' actions. When firms make their investment project selection (high or low type) *prior* to bank loan approval, the adverse selection problem may exist as in the middleman theory developed by Biglaiser (1993). Alternatively, when firms select projects *ex post*, the moral hazard problem may arise. In either case, financial contracts must be incentive compatible. Equilibrium credit rationing may be present. This additional source of

capital unemployment will then compound the frictional capital unemployment considered in this paper.

Figure 1: Steady-State Search Equilibrium with Homogeneous Borrowers

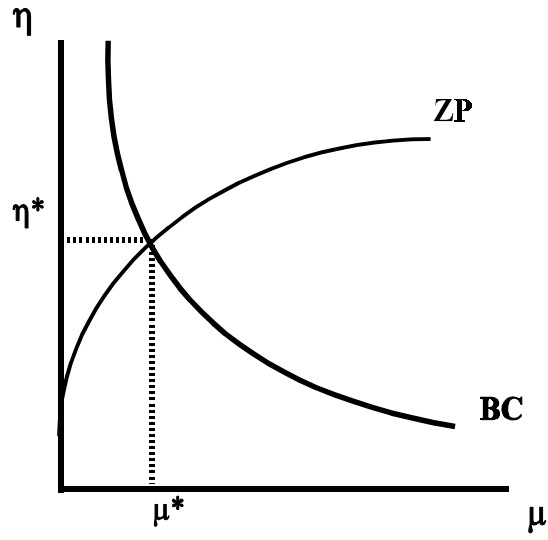
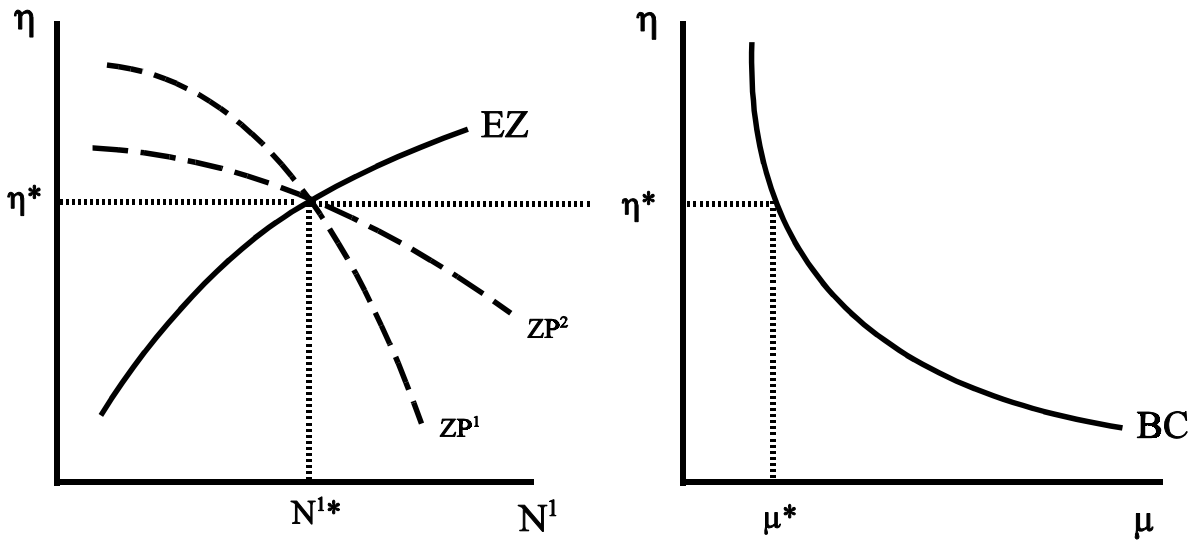


Figure 2: Steady-State Search Equilibrium with Heterogenous Borrowers



**Table 1: Summary of Comparative Statics**

Effect of	Credit Market Shocks		Firm Profitability Shocks			
	Credit Matching Efficacy $m_0$	Contract-Quit Rate $\delta$	Type 1 Productivity $A^1$	Type 2 Productivity $A^2$	Type 1 Entry Costs <sup>1</sup> $v_0^1$	Type 2 Entry Costs <sup>1</sup> $v_0^2$
Effect on						
<b>1. Contact Rates and Population Masses</b>						
$\eta^*$	0	+	+	-	-	+
$U^*$	0	-	-	+	+	-
$\mu^*$	+	-	-	+	+	-
$S^*$	+	-	-	+	+	-
<b>2. Composition of Low-Type Borrowing Firms</b>						
$N^{1*}$	+	-	-	+	+	-
<b>3. Interest Rates and Differential</b>						
$R^{i*}$	0	- <sup>2</sup>	+	+ <sup>2</sup>	-	- <sup>2</sup>
$R^2 - R^1$	0	- <sup>2</sup>	?	+ <sup>2</sup>	?	- <sup>2</sup>
<b>4. Social Output</b>						
$Y^*$	?	- <sup>2</sup>	+ <sup>2</sup>	+ <sup>2</sup>	?	?

Note 1: To sign some of the effects of entry costs we require that  $m_0$  is sufficiently small to satisfy Condition Q.

Note 2: Assume that the composition effects are comparatively small.

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## Appendix

This Appendix contains proofs of Lemma 3 and 4 and Propositions 3-5 in the paper.

### Proof of Lemma 3:

(i) Differentiating (15) with respect to  $N^i$  gives

$$\frac{\partial(p^i R^i)}{\partial N^i} = - \left[ \frac{\beta(1-\beta)\mu}{r+\delta+\beta\mu} \right] (\bar{a}^j - \bar{a}^i) = \frac{\partial(p^j R^j)}{\partial N^i}$$

For  $i = 1$  we have  $\partial p^1 R^1 / \partial N^1 = \partial p^2 R^2 / \partial N^1 < 0$  and for  $i = 2$  we have  $\partial p^1 R^1 / \partial N^2 = \partial p^2 R^2 / \partial N^2 > 0$ .

(ii) Differentiating (15) with respect to  $\mu$  gives

$$\frac{\partial(p^i R^i)}{\partial \mu} = \frac{\beta(1-\beta)(r+\delta)}{(r+\delta+\beta\mu)^2} [(1-N^i)(\bar{a}^j - \bar{a}^i) - (1-\bar{a}^i)] = \frac{\partial(p^j R^j)}{\partial \mu} > 0 \text{ for all } i, j$$

(iii) Notice that  $\partial B / \partial \delta > 0$ , and given  $\bar{a}^i > 1$ , the first term in (15) is strictly decreasing in  $\delta$ . For  $i = 1$ , it is clear that the second term in (15) is also strictly decreasing in  $\delta$ , implying  $\partial p^1 R^1 / \partial \delta < 0$ .

Thus, manipulating (15), we have  $p^2 R^2 = p^1 R^1 + \beta(\bar{a}^2 - \bar{a}^1)$ , implying  $\partial p^2 R^2 / \partial \delta = \partial p^1 R^1 / \partial \delta$ .

(iv) Since  $\partial p^i R^i / \partial v_0^i \propto -\partial p^i R^i / \partial A^i < 0$ , the result is immediate.

### Proof of Proposition 2:

From (16), we can derive  $R^2 - R^1 = \frac{\beta}{p^2} (\bar{a}^2 - \bar{a}^1) + \frac{p^1 - p^2}{p^1 p^2} p^1 R^1$ . Thus both spreads are positive under

Condition D that ensures  $\bar{a}^2 - \bar{a}^1 > 0$ . Utilizing Proposition 2, we can see implies that  $R^2 - R^1$  rises with  $\mu$  and  $\bar{a}^2$ , falls with  $N^1$  and  $\delta$ , may rise or fall with  $\bar{a}^1$ , and is immune to  $m_0$ . Moreover, from (17) we obtain:

$$R^2 - R^1 = \lim_{\mu \rightarrow \infty, v_0^i \rightarrow 0} (R^2 - R^1) - \beta r \left( \frac{v_0^2}{p^2} - \frac{v_0^1}{p^1} \right) - (1-\beta) \left( \frac{1}{p^2} - \frac{1}{p^1} \right) \Theta$$

where  $\Theta = \frac{r+\delta}{r+\delta+\beta\mu} (N^1 p^1 A^1 + N^2 p^2 A^2 - 1) + \frac{\beta\mu}{r+\delta+\beta\mu} (N^1 v_0^1 + N^2 v_0^2)$  is a weighted sum of aggregate net outputs and aggregate entry costs, which is positive under (A3) and Condition F. Thus, given (A2), the actual interest rate spread is smaller than that in the absence of search and entry frictions.

**Proof of Lemma 4:**

From Proposition 3, it is immediate that ZP1 and ZP2 are downward sloping in  $(\eta, N^1)$  space. Differentiating (14) gives

$$\left. \frac{d\eta^*}{dN_1^*} \right|_{zp1} = \frac{d\eta^{zp1}}{dp^1 R^1} \frac{dp^1 R^1}{dN_1} = -\frac{(\eta^*)^2}{rv_0^1(r+\delta)} \frac{\beta(1-\beta)}{r+\delta+\beta\mu} \mu(\bar{a}^2 - \bar{a}^1) < 0$$

$$\left. \frac{d\eta^*}{dN_1^*} \right|_{zp2} = \frac{d\eta^{zp2}}{dp^2 R^2} \frac{dp^2 R^2}{dN_1} = -\frac{(\eta^*)^2}{rv_0^2(r+\delta)} \frac{\beta(1-\beta)}{r+\delta+\beta\mu} \mu(\bar{a}^2 - \bar{a}^1) < 0$$

since  $v_0^1 < v_0^2$ , we have that the pair  $\{\eta^*, N^{1*}\}$  satisfying (14) given  $\mu$  occur where  $|d\eta^*/dN^{1*}|_{zp1} > |d\eta^*/dN^{1*}|_{zp2}$ . Since both locus' are downward sloping, this pair is unique. To characterize the (EZ) locus, equate ZP1 and ZP2 from (14):

$$v_0^2 p^1 [R^1(N^1, \mu) - 1] - v_0^1 p^2 [R^2(N^1, \mu) - 1] = v_0^2 p^1 (A^1 - 1) - v_0^1 p^2 (A^2 - 1) \quad (P1)$$

Notice that from Proposition 3,  $\partial p^i R^i / \partial \mu = \partial p^i R^i / \partial \mu > 0$  and  $\partial p^i R^i / \partial N^1 = \partial p^i R^i / \partial N^1 < 0$ . Consider now that  $\mu$  increases. Since  $p^i R^i$  is higher (for  $i = 1, 2$ ), (14) implies  $\eta$  must be higher. However, this changes the LHS of (P1) away from the RHS: the LHS increases (decreases) iff  $v_0^1 p^2 - v_0^2 p^1 < (>) 0$ . In either case,  $N$  must rise to restore the equality in (P1), implying  $d\eta/dN^1|_{EZ} > 0$ .

To characterize the limit points of the EZ locus, consider the case where  $\mu \rightarrow \infty$  which implies  $\eta \rightarrow 0$  from (7). From (15)  $p^i R^i \rightarrow [(1-\beta)N^i(\bar{a}^j - \bar{a}^i) + \bar{a}^i] > 0$ . The LHS of (P1) can be written as

$$LHS = (v_0^1 p^2 - v_0^2 p^1)(1-\beta) + \beta\{(1-\beta)(\bar{a}^2 - \bar{a}^1)[v_0^2 N^1(v_0^2 - v_0^1)] + (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) + (v_0^1 p^2 - v_0^2 p^1)\}$$

Since  $RHS = (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) + (v_0^1 p^2 - v_0^2 p^1)$ , equating LHS with RHS and solving for  $N^1$  yields

$$N^1 = \frac{\beta(\bar{a}^2 - \bar{a}^1)v_0^2 - (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2)}{\beta(\bar{a}^2 - \bar{a}^1)(v_0^2 - v_0^1)}$$

Thus, a condition for  $N^1 \in [0, 1]$  is given by

$$\beta(\bar{a}^2 - \bar{a}^1)v_0^1 < (v_0^2 \bar{a}^1 - v_0^1 \bar{a}^2) \leq \beta(\bar{a}^2 - \bar{a}^1)v_0^2 \quad (P2)$$

Now consider the limiting case where  $\mu \rightarrow 0$  which implies  $\eta \rightarrow \infty$  from (14). From (15)  $p^i R^i \rightarrow 1 + \beta(\bar{a}^i - 1) > 1$ . Thus, there exists an upper bound for  $\eta$  such that  $\sup_{N^1} \eta(N^1) < \infty$ . Furthermore, there exists a finite  $\eta^{\max} < \infty$  at  $N^1 = 1$ .

### Proof of Propositions 4 and 5:

Totally differentiating (7) and (14) yields

$$C \begin{bmatrix} d\eta \\ d\mu \\ dN^1 \end{bmatrix} = \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} dm_0 + \begin{bmatrix} 0 \\ rv_0^1 + \frac{dp^i R^i}{d\delta} \\ rv_0^2 + \frac{dp^i R^i}{d\delta} \end{bmatrix} d\delta + \begin{bmatrix} 0 \\ -p^1 \left( 1 - \frac{dR^1}{dA^1} \right) \\ p^2 \frac{dR^2}{dA^1} \end{bmatrix} dA^1 + \begin{bmatrix} 0 \\ p^1 \frac{dR^1}{dA^2} \\ -p^2 \left( 1 - \frac{dR^2}{dA^2} \right) \end{bmatrix} dA^2$$

$$+ \begin{bmatrix} 0 \\ \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} \\ \frac{dp^2 R^2}{dv_0^1} \end{bmatrix} dv_0^1 + \begin{bmatrix} 0 \\ \frac{dp^1 R^1}{dv_0^2} \\ \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^2 R^2}{dv_0^2} \end{bmatrix} dv_0^2$$

where

$$C = \begin{bmatrix} 1 - \frac{m_0 M'}{\mu} & \frac{m_0 M' \eta}{\mu^2} & 0 \\ \frac{r(r+\delta)v_0^1}{\eta^2} & -\frac{d(p^1 R^1)}{d\mu} & -\frac{d(p^1 R^1)}{dN^1} \\ \frac{r(r+\delta)v_0^2}{\eta^2} & -\frac{d(p^2 R^2)}{d\mu} & -\frac{d(p^2 R^2)}{dN^1} \end{bmatrix} \equiv \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Note that  $C_{22} = C_{32} = -\frac{dp^i R^i}{d\mu} < 0 < -\frac{dp^i R^i}{dN^1} = C_{23} = C_{33}$  and  $C_{31} - C_{21} = \frac{r(r+\delta)}{\eta^2} (v_0^2 - v_0^1) > 0$ .

Thus,  $|C| = -C_{12}(C_{21} - C_{31})C_{33} > 0$ . We also compute comparative static effects:

1. Matching Efficiency:

$$\frac{d\eta^*}{dm_0} = \frac{M}{|C|}(C_{22}C_{33} - C_{23}C_{32}) = 0 \quad , \quad \frac{d\mu^*}{dm_0} = \frac{M}{|C|}(C_{31} - C_{21})C_{33} > 0 \quad ,$$

$$\frac{dN^{1*}}{dm_0} = -\frac{M}{|C|}(C_{31} - C_{21})C_{22} > 0$$

$$\frac{dp^i R^i}{dm_0} = \left( \frac{dp^i R^i}{d\mu} \right) \frac{d\mu}{dm_0} + \left( \frac{dp^i R^i}{dN^1} \right) \frac{dN^1}{dm_0} = C_{i2} \frac{d\mu}{dm_0} + C_{i3} \frac{dN^1}{dm_0} = 0 \quad \text{after substituting terms from above.}$$

Using this last result and (4) and (5), one sees that  $\Pi_m^i - \Pi_u^i$  and  $J_m^i - J_u$  also are independent of  $m_0$ . Then (1b) implies  $J_m^i$  and  $J_u$  are independent of  $m_0$ .

2. Separation Rate:

$$\frac{d\eta^*}{d\delta} = \frac{C_{12}C_{33}}{|C|}r(v_0^2 - v_0^1) > 0 \quad , \quad \frac{d\mu^*}{d\delta} = -\frac{C_{11}C_{33}}{|C|}r(v_0^2 - v_0^1) < 0$$

$$\frac{dN^{1*}}{d\delta} = \frac{1}{|C|} \frac{r(v_0^2 - v_0^1)}{\eta^2} \left( (r+\delta)C_{12} \frac{dp^i R^i}{d\delta} - \eta^2 C_{11} \frac{dp^i R^i}{d\mu} \right) < 0$$

3. Productivity:

$$\frac{d\eta^*}{dA^1} = \frac{C_{12}C_{33}}{|C|} \left( p^1 \left( 1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) > 0 \quad , \quad \frac{d\mu^*}{dA^1} = -\frac{C_{11}C_{33}}{|C|} \left( p^1 \left( 1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) < 0$$

$$\frac{dN^{1*}}{dA^1} = \frac{C_{11}C_{22}}{|C|} \left( p^1 \left( 1 - \frac{dR^1}{dA^1} \right) + p^2 \frac{dR^2}{dA^1} \right) - \frac{C_{12}}{|C|} \left( C_{31} p^1 \left( 1 - \frac{dR^1}{dA^1} \right) + C_{21} p^2 \frac{dR^2}{dA^1} \right) < 0$$

Also, the impact of  $A^2$  is inversely related to that of  $A^1$  :

$$\frac{d\eta^*}{dA^2} = -\frac{C_{12}C_{33}}{|C|} \left( p^2 \left( 1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) < 0 \quad , \quad \frac{d\mu^*}{dA^2} = \frac{C_{11}C_{33}}{|C|} \left( p^2 \left( 1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) > 0$$

$$\frac{dN^{1*}}{dA^2} = -\frac{C_{11}C_{22}}{|C|} \left( p^2 \left( 1 - \frac{dR^2}{dA^2} \right) + p^1 \frac{dR^1}{dA^2} \right) + \frac{C_{12}}{|C|} \left( C_{21} p^2 \left( 1 - \frac{dR^2}{dA^2} \right) + C_{31} p^1 \frac{dR^1}{dA^2} \right) > 0$$

4. Entry Costs: The effects are more difficult to sign:

$$\frac{d\eta^*}{dv_0^1} = -\frac{C_{12}C_{33}}{|C|} \left( \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right), \quad \frac{d\mu^*}{dv_0^1} = \frac{C_{11}C_{33}}{|C|} \left( \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right)$$

$$\frac{dN^{1*}}{dv_0^1} = -\frac{C_{11}C_{22}}{|C|} \left( \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} - \frac{dp^2 R^2}{dv_0^1} \right) + \frac{C_{12}}{|C|} \left( -C_{21} \frac{dp^2 R^2}{dv_0^1} + C_{31} \left[ \frac{r(r+\delta+\eta)}{\eta} + \frac{dp^1 R^1}{dv_0^1} \right] \right)$$

From (15) one obtains  $\frac{dp^i R^i}{dv_0^i} - \frac{dp^j R^j}{dv_0^j} = -r\beta < 0$  and  $\frac{dp^j R^j}{dv_0^i} = -r \frac{\beta\mu(1-\beta)N^i}{r+\delta+\beta\mu} < 0$  which can be substituted into the above relationships. Therefore, if a temporary variable related to the two contact rates is defined by  $Q \equiv r \left( \frac{r+\delta}{\eta} + (1-\beta) \left( 1 - \frac{\mu}{r+\delta+\beta\mu} \right) \right)$ , we can rewrite the above expressions as:

$$\frac{d\eta^*}{dv_0^1} = -\frac{C_{12}C_{33}}{|C|} Q, \quad \frac{d\mu^*}{dv_0^1} = \frac{C_{11}C_{33}}{|C|} Q, \quad \frac{dN^{1*}}{dv_0^1} = \frac{C_{12}C_{31} - C_{11}C_{22}}{|C|} Q + \frac{C_{12}(C_{31} - C_{21})}{|C|} \frac{dp^2 R^2}{dv_0^1}$$

Thus when  $Q > 0$ , it follows that  $\frac{d\eta^*}{dv_0^1} < 0 < \frac{d\mu^*}{dv_0^1}$ . With a strengthening of this “Q-condition” it is also possible to get  $\frac{dN^{1*}}{dv_0^1} > 0$ . Note that these were the qualitative effects that we had solved for originally.

Using the same approach as above yields:

$$\frac{d\eta^*}{dv_0^2} = \frac{C_{12}C_{33}}{|C|} Q, \quad \frac{d\mu^*}{dv_0^2} = -\frac{C_{11}C_{33}}{|C|} Q, \quad \frac{dN^{1*}}{dv_0^2} = \frac{C_{11}C_{22} - C_{12}C_{21}}{|C|} Q + \frac{C_{12}(C_{31} - C_{21})}{|C|} \frac{dp^1 R^1}{dv_0^2}$$

The simple Q-condition results in  $\frac{d\eta^*}{dv_0^2} > 0 > \frac{d\mu^*}{dv_0^2}$ . Also, because  $C_{22} < 0$ ,  $\frac{dN^{1*}}{dv_0^2} < 0$  without having to make any further assumptions beyond the simple Q-condition (in contrast to the assumptions needed to get  $\frac{dN^{1*}}{dv_0^1} > 0$ ).