E C O N O M I C S B U L L E T I N

RDin Markets with Network Externalities

Caroline Boivin
University of Sherbrooke

Désiré Vencatachellum
HEC−Montreal, Université de Montréal

Abstract

We study how network externalities affect research and development (Rinvestments by a non−cooperative duopoly that offers compatible products. We find that multiple RDequilibria may arise when network externalities are non linear in the number of consumers. The lowest RDequilibrium corresponds to the case where network externalities are absent. However, even in the presence of network externalities, firms may be trapped in a low−RDequilibrium where output, and therefore consumers' valuation of the network size, is low. We derive the conditions under which the highest−RD equilibrium Pareto dominates.

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1 Introduction

Network externalities arise when a consumer's utility depends on the number of users of a good or a service (Katz and Shapiro 1985, Farrell and Saloner 1986). Although a characteristic of many markets, network externalities are particularly important in technology-intensive markets. Telephone, fax or e-mail exhibit direct positive network externalities because a larger network increases communication possibilities. Furthermore, larger networks are also appealing to suppliers of complementary goods and services. In such cases, consumers indirectly benefit from an expansion of the market (Berg 1988, Chou and Shy 1990, Church and Gandal 1993). Technology-intensive markets, e.g. telecommunications, are also characterized by their high R&D intensity. Our objective is to investigate how network externalities affect firms' R&D.

To our knowledge, Kristiansen and Thum (1997) are unique in the examination of the impact of network externalities on R&D. They consider a duopoly where firms offer differentiated, but compatible, products. The authors distinguish between commercial and mass users, but where the willingness to pay depends on the number of mass users. They also assume that one firm cannot improve its technology, while the other firm's technology for the mass market may be improved if it performs R&D. They find that network externalities may lead firms to either under- or over- invest in R&D compared to the social optimum. In a related paper, Kristiansen (1998) shows that network externalities may induce firms to introduce technologies earlier than in their absence.

We consider a duopoly with compatible products as in Kristiansen and Thum (1997). However, we depart from their analysis on two main accounts. First, although most theoretical analysis on network externalities assume that a consumer's utility is a linear increasing function of the number of users (Conner 1995, Kristiansen and Thum 1997), this may not be consistent with empirical evidence. In fact, consumer utility may be independent of the market size if there are few users because of indirect externalities. Providers of complementary goods and services enter a market only if potential sales are sufficiently high to cover costs. Moreover, as acknowledged by Katz and Shapiro (1985), after some point, additional users may have a negligible impact on a consumer's utility, as for instance in the stereo market (Postrel 1990, p.176). Between these two thresholds, consumer utility may increase more or less rapidly with the number of users. A more realistic representation of network externalities is similar to the diffusion process: a slow start followed by an acceleration portion, and then deceleration and possibly a decrease.

This note focusses on compatible technologies because there appears to be a consensus that incompatible technologies are less likely to coexist in the presence of network externalities (Farrell and Saloner 1986, Besen and Farrell 1994). Indeed, it is difficult for a new incompatible technology to get a foothold in a market where another technology already has a substantial installed base because few consumers will adopt the new incompatible technology. There are also regulatory standards mandated by governments to meet health, safety, or environmental objectives. In high-technology sectors, however, most standards are voluntary. For instance, Kodak convinced the major players in the photographic industry to jointly develop the specifications of the Advanced Photo System. For these reasons, we assume that firms use the same technology, but contrary to Kristiansen and Thum (1997) that each can invest in R&D to reduce its unit cost of production.

We show that multiple R&D equilibria may arise when a product exhibits network externalities. As firms behave a la Cournot on the goods and services market, and invest in cost-reducing R&D, more R&D is associated with a lower price. In this case, consumers' welfare is highest when firms are located on the highest R&D equilibrium. Interestingly, the lowest R&D equilibrium occurs when no network externalities arise. At this equilibrium, the unit cost of production is high and few consumers are attracted on the market as the network size is too small. Thus, although a product may exhibit network externalities, firms' myopic behavior may preclude them from appearing at all. However, R&D investment is never smaller with network externalities than when they are absent. This result arises because each unit of R&D has an extra effect on a firm's profits through the fall in the cost of production which attracts new consumers and as a result increases the willingness to pay of all consumers. Finally, we derive a sufficient condition for a firm's equilibrium profits to be increasing in R&D, in which case the equilibrium with the highest R&D Pareto dominates.

The remainder of this note is organized as follows. We use the stylized facts of markets with network externalities to set up the model economy in section 2. Next, we solve for each firm's equilibrium R&D investment in section 3 and establish the conditions under which multiple equilibria may arise. Finally we discuss our results in section 4.

2 The Model Economy

Consider an economy populated by a large number of consumers and two firms which produce a homogeneous good which potentially exhibits network externalities. The timing of events is as follows. First, as in Katz and Shapiro (1985), consumers maximize their surplus and form expectations about the network size Q^e. Consumers' expectations determine their willingness to pay for the market size, $v(Q^e)$, where $v : \Re_+ \longrightarrow \Re_+$ which may be a non monotonous function. For instance, the benefits from network size may equal zero when there are few users, increase after a threshold and then decrease after a ceiling because of congestion effects. Second, firms choose their R&D expenditure. Third, firms behave a la Cournot and decide how much to produce. Finally all markets clear.

As in Katz and Shapiro (1985), we augment the standard linear inverse demand function to account for network externalities:

$$p = a - bQ + v(Q^e). \tag{1}$$

where p is the price of the product, a and b are parameters and Q is aggregate output. We assume $v' < b$ for the inverse demand curve to be downward sloping. The remainder of the model follows d'Aspremont and Jacquemin (1988). There are two firms, each sells the good at price p, and have access to the same technology. Each firm i performs x_i units of R&D to lower its unit cost of production. R&D yields positive technical externalities for the other firm by reducing its unit cost of production. Firm i's unit cost of production is given by:

$$c_i \equiv c(x_i, x_j) = A - x_i - \beta x_j, \tag{2}$$

where i, j equal to 1 or 2, $i \neq j$, A is a positive parameter, and $\beta \in [0, 1]$ captures technical spillovers. The cost for firm i to do x_i units of R&D is $\gamma x_i^2/2$, where γ is a positive parameter.

Using (1) and (2), firm i's gross profits before R&D costs is given by:

$$\pi(q_i, q_j; Q^e) = (a + v(Q^e) - b(q_i + q_j) - c_i)q_i. \tag{3}$$

for $i, j = 1, 2$, $i \neq j$. Each firm i chooses q_i which maximizes (3), taking Q^e and q_j as given. Thus:

$$3bq_i \quad = \quad a + v(Q^e) - 2c_i + c_j, \tag{4}$$

for $i, j = 1, 2$ and $i \neq j$. Next, we use (2) to substitute for c_i and c_j in (4), solve for q_i as a function of R&D which we substitute in (3), and take into account R&D costs to obtain firm i's net profits:

$$\bar{\pi}(x_i, x_j; Q^e) = \frac{1}{9b}\left[\tilde{a} + v(Q^e) + (2 - \beta)x_i + (2\beta - 1)x_j\right]^2 - \frac{\gamma}{2}x_i^2. \tag{5}$$

for $i, j = 1, 2$, $i \neq j$, and where $\tilde{a} = a - A$.[1]

Firm i chooses its R&D investment which maximizes (5) while taking the other firm's decision as given. Focusing on symmetric R&D, i.e. $x_1 = x_2 = x$, we obtain:

$$x = [\tilde{a} + v(Q^e)]/r, \tag{6}$$

where $r = [4.5b\gamma - (1 + \beta)(2 - \beta)]/(2 - \beta)$. Aggregate supply is implicitly obtained by summing (4) over i, substituting c_i by the right hand side of (2) and imposing symmetry:

$$3bQ - 2v(Q^e) \quad = \quad 2\tilde{a} + 2(1 + \beta)x. \tag{7}$$

If there are no network externalities ($v(Q^e) = 0$) then (7) yields a closed form solution for aggregate output as a function of a firm's R&D investment. When there are network externalities, and depending on the v function, such a closed form solution may not exist.

[1]The same parameter restrictions as for the non cooperative case in d'Aspremont and Jacquemin (1988) apply here: (i) $0 < A < a$, (ii) $A \geq x_i + \beta x_j$, for $i, j = 1, 2$, $i \neq j$, and (iii) $9b\gamma > 2(2 - \beta)^2$.

3 Equilibrium Analysis

We now define and derive the conditions under which an equilibrium exists.

Definition 1 *A symmetric fulfilled expectations Nash equilibrium is such that* $Q^e = Q$, *and* $x_1 = x_2 = x$, *each firm maximizes its profits by taking its rival's decisions as given and follows the specified timing of events.*

An equilibrium as in definition 1 verifies simultaneously (6) and (7). From (7) evaluated at $Q^e = Q$, using the implicit function theorem, and when the demand curve is downward sloping, there exists a unique differentiable function h such that $Q = h(x)$.[2] Evaluate (6) at $Q^e = Q = h(x)$ gives:

$$v[h(x)] - rx + \tilde{a} = 0 \tag{8}$$

We allow network externalities to be positive over part of the domain of $v[\cdot]$ and derive the conditions for the existence of an equilibrium. We then investigate the possibility of multiple equilibria. To find the solution(s) to (8), we define a function $F : [0, A/(1+\beta)] \longrightarrow \Re$ such that:

$$F(x) = v[h(x)] - rx + \tilde{a}. \tag{9}$$

The lower bound of the domain of F follows from non-negative R&D and the upper bound derives from non-negative unit cost of production. We next study the roots of (9).

Proposition. 1 (Existence) *A symmetric fulfilled expectations equilibrium exists if:*

$$v\left[h\left(\frac{A}{1+\beta}\right)\right] < \frac{rA}{1+\beta} - \tilde{a}. \tag{10}$$

Condition (10) states that the network externalities must not exceed a maximum level for an interior equilibrium to exist. Otherwise, the marginal benefits from an additional unit of R&D would always exceed the marginal costs. An appropriate choice of parameters guarantees that (10) holds.

Note that (10) is a *sufficient* condition for an equilibrium to exist. An equilibrium, however, may exist even when (10) is violated. Consider for example a convex network externality function which violates (10) but where there exists a level of R&D x'' such that $v[h(x'')] < (rx'' - \tilde{a})$. In this case an equilibrium exists. Moreover, if in equilibrium one could

[2] For the implicit theorem to hold $3b > 2v'$, which is met for a downward sloping demand curve ($v' < b$).

explicitly solve for Q as a function of x from (7), then (10) would be an explicit function. For example if as in Kristiansen and Thum (1997) $v(Q) = \phi Q$, with $\phi \in \Re_+$, (10) becomes:

$$\phi < \frac{3b(rA + \tilde{a}(1+\beta))}{2a(1+\beta) + 3(rA + \tilde{a}(1+\beta))} \tag{11}$$

which is an explicit condition on the slope of the network externalities function. In other words the marginal benefits from an increase in market size must not be too high for an equilibrium to exist.

Proposition. 2 (Uniqueness) *If (10) holds and v is a concave function, the fulfilled symmetric expectations equilibrium is unique.*

Corollary. 1 (No network externalities) *When network externalities never arise, $v[\cdot] = 0$ for all x, (8) has a unique solution and is that obtained by d'Aspremont and Jacquemin (1988):*

$$x^{dj} = \tilde{a}/r. \tag{12}$$

Proposition. 3 (R&D investment) *If the network-externalities function is not decreasing then:*

- i A firm never invests less in R&D than when there are no network externalities.
- ii If there exists a level of R&D $\tilde{x}$ which is less than x^{dj} and such that $v[h(\tilde{x})] > 0$, a firm always invests more in the presence of network externalities than otherwise.

The intuition behind Proposition 3 can be understood with the help of Figure 1 which gives the solution(s) to (8) as the intersection(s) between the potentially non-linear function $v[h(x)]$ and the straight line $(rx - \tilde{a})$. The network-externalities function in Figure 1 is such that a consumer derives benefits from the network size only above a minimum number of consumers. That is, $v[h(x)] = 0$ for all $x \leq \underline{x}$, while for $x > \underline{x}$, a consumer's utility increases in the number of consumers but at a decreasing rate. Figure 1 provides an example where these two functions intersect more than once such that there are multiple R&D equilibria.

In Figure 1, at the x^{dj} equilibrium the number of users has no effect on a consumer's utility. Thus, although the product may exhibit network externalities, R&D investment is insufficient for the unit cost of production to be low enough to attract enough consumers. Firms, anticipating a small market, do not deem it profitable to invest more in R&D. Hence, in this example there are three R&D equilibria, $x^{dj} < x' < x^{\dagger}$. One equilibrium, x', is unstable while the other two are stable, and the lowest R&D equilibrium coincides with what would have been obtained if we had assumed no network externalities. Hence, firms may be trapped at x^{dj} although $x^{\dagger}$ may Pareto dominate the other equilibria.

Note that in Figure 1 an *exogenous* increase of the demand for the product, i.e. an increase in a, shifts the $(rx - \tilde{a})$ line to the right. Consequently, if that exogenous increase is sufficiently high, we may achieve a new unique equilibrium to the right of $x^{\dagger}$ with network externalities. However, such an equilibrium may not be obtained if (i) the exogenous increase is small or (ii) the network externalities function is not concave.

We now investigate which equilibrium is the most profitable by determining a condition under which a firm's (symmetric) equilibrium profits are increasing in R&D.

Proposition. 4 *Assume non-negative network externalities,* $v' \geq 0$ *for all* x, *and*

$$4.5b\gamma < (1+\beta)^2. \tag{13}$$

then the highest optimal R&D *is the most profitable one.*

Note that (13) violates the second order condition for the concavity of the *joint* profits function in d'Aspremont and Jacquemin (1988).[3] However, our framework could still be used to investigate the impact of R&D joint ventures on firms' profits in the presence of network externalities. The sufficient condition (13) would have to be relaxed and we would have to resort to numerical solutions to rank the different equilibrium R&D. We do not pursue this avenue here because (i) this note does not tackle R&D joint ventures, and (ii) it would require that we use a specific network externalities function.

4 Conclusion

This paper investigates the impact of network externalities on simultaneous investment in cost-reducing R&D. We augment d'Aspremont and Jacquemin (1988) R&D framework by Katz and Shapiro (1985) analysis to account for network externalities in consumer utility. First we show that firms invest at least as much in R&D than in the absence of network externalities. Second, when network externalities are non linear, multiple R&D equilibria are possible, and firms may be trapped in a low-R&D equilibrium although a higher-R&D equilibrium may be more profitable.

[3]See footnote 7 in d'Aspremont and Jacquemin (1988). Note also that the second order condition for a firm's profits function to be concave, $(2-\beta) < 4.5b\gamma$, and (13) imply that $\beta > 0.5$.

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A Appendix

A.1 Proof of Proposition 1 As $F(0) = \tilde{a}$, which is positive by assumption, a *sufficient* condition for a real root to exist is that $F\left(\frac{A}{1+\beta}\right)$ be negative, hence (10).

A.2 Proof of Proposition 2 If (10) holds and v is strictly concave, (9) has one root, hence (8) has a unique fixed-point.

A.3 Proof of Proposition 3 The proof is in two parts. Let $\hat{x}$ denote an equilibrium R&D and assume that $\hat{x} < x^{dj}$.

i. By (8), if $\hat{x} < x^{dj}$, $v[h(\hat{x})] + \tilde{a} < \tilde{a}, \Rightarrow v[h(\hat{x})] < 0$, hence a contradiction.

ii. By (8), $r(\hat{x} - x^{dj}) = v[h(\hat{x})]$, which is non-negative by [i]. Hence, $\hat{x} \geq x^{dj}$.

A.4 Proof of Proposition 4 Using (5), a firm's (symmetric) equilibrium profits equals:

$$\bar{\pi}(x, x, h(x)) = \frac{1}{9b}\left(\tilde{a} + (1+\beta)\,x + v[h(x)]\right)^2 - \frac{\gamma}{2}x^2. \tag{14}$$

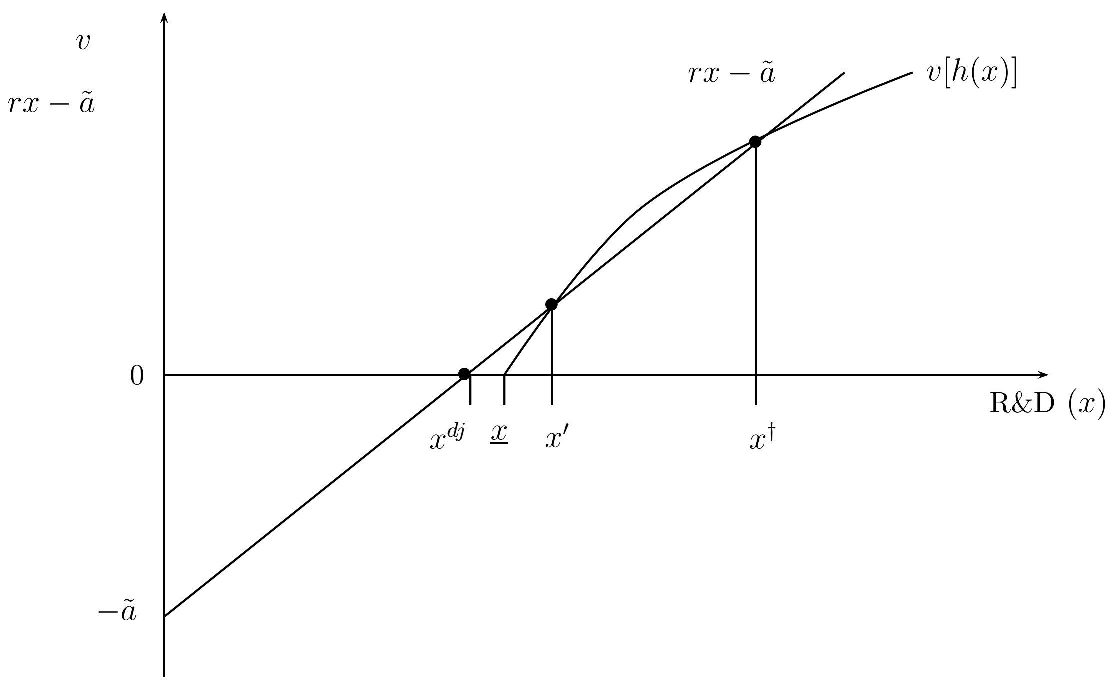
Differentiating (14) with respect to x, $\frac{\partial \bar{\pi}}{\partial x} > 0$ if and only if:

$$v'h' > \frac{[4.5b\gamma - (1+\beta)^2]x - \tilde{a}(1+\beta) - (1+\beta)v[h(x)]}{\tilde{a} + (1+\beta)x + v[h(x)]} \tag{15}$$

for all x. As $h'(x) = \frac{1+\beta}{3b-2v'}$ is positive given that the demand curve is assumed to be downward sloping, then assume $v' \geq 0$ for all x, and use (15) to obtain (13).

A.5 Figure

Figure 1: Equilibrium R&D Investment(s)



The equilibrium R&D investments are: x^{dj}, x' and $x^{\dagger}$; x^{dj} is d'Aspremont and Jacquemin (1988) solution; $\underline{x}$ is a threshold level of R&D below which the network externalities function equals 0.