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### The political economy of social security in a borrowing-constrained economy

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#### *Abstract*

This paper introduces a three-income-class, overlapping-generations model with borrowing constraints. The labor income tax for financing pay-as-you-go social security is determined in a majoritarian voting game played by successive generations. When the interest-rate elasticity of consumption is low, the political equilibrium might be characterized by an equilibrium where the old and the middle-income young individuals form a coalition in favor of a higher tax rate and greater social security, while the low- and the high-income young individuals favor a lower tax rate and less social security. In this equilibrium, the size of social security is decreased by the mean-preserving reduction of a decisive voter's wage if he/she is borrowing constrained.

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# 1 Introduction

Almost all OECD countries have experienced some increase in wage inequality over the past few decades. Standard political economy theory suggests that higher wage inequality results in a larger volume of redistribution as the decisive voter becomes less well-off as wage inequality increases (Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981; Alesina and Rodrik, 1994; Benabou, 1996; Persson and Tabellini, 1994). This prediction still holds when the focus is on intergenerational redistribution, such as pay-as-you-go (PAYG) social security. That is, the intragenerational redistribution component of the PAYG social security system makes this program palatable to low-income young individuals (Conde-Ruiz and Galasso, 2003, 2005; Bethencourt and Galasso, 2008).

The empirical evidence, however, does not necessarily support the above-mentioned theoretical predictions. OECD cross-country data show that the volume of redistribution is negatively correlated with wage inequality (Gottschalk and Smeeding, 1997; Chen and Song, 2009). For example, the United Kingdom and the United States feature higher wage inequality and smaller redistribution whereas some Continental European and Nordic countries display lower wage inequality and larger redistribution. In fact, Pineda and Rodriguez (2006) found a strong negative correlation between redistribution and the share of capital in GDP where the share is considered an indicator of income inequality.

Several theories have been provided to explain the negative correlation between inequality and redistribution. Examples include political bias toward the rich (Benabou, 2000); the prospect of upward mobility by low-income individuals (Quadrini, 1999; Benabou and Ok, 2001; Alesina and La Ferrara, 2005; Arawatari and Ono, 2009); and lobbying and campaign contributions by the rich (Rodriguez, 2004; Campante, 2010). This paper instead focuses on the borrowing constraint, and demonstrates that the negative correlation arises through majority voting when voters' interest-rate elasticity of consumption is low and the decisive voter is borrowing constrained.

For the purpose of analysis, we introduce an overlapping-generations model with heterogeneous individuals and borrowing constraints. In this economy, young workers fall into three income classes: low, middle and high. Because they are not permitted to borrow when young as a result of imperfect financial markets, lower-income individuals are more likely to be borrowing constrained. Young workers then pay a fixed proportion of their labor income to the government and that tax revenue is used for PAYG social security payments from which the old can benefit. The tax rate and thus the size of social security payments are determined in a majoritarian voting game played by the young and the old. Voters cast a ballot over the labor income tax, which finances social security.

In the above-mentioned environment, we consider the voting behavior of each type of individual. The preferences of the old are identical across all types of individuals because

they owe no tax burden and receive the same level of social security benefit; they prefer the tax rate that attains the top of the Laffer curve. In contrast, the preferences of the young depend on their income class because the tax burden differs across the classes of income. In particular, the key factors contributing to their preferences are the borrowing constraint and the interest-rate elasticity of consumption.

To understand the roles of these two factors, we consider the following two opposing effects of a reduction in wage on the preferred tax rate of the young. First, given a tax rate, a reduction in wage lowers the tax burden and thus raises a benefit-to-burden ratio. This gives the young an incentive to choose a higher tax rate: this is a positive effect on the preferred tax rate. Second, a reduction in wage decreases disposable income and thus consumption level when young. This gives the young an incentive to choose a lower tax rate from the viewpoint of keeping utility of consumption: this is a negative effect on the preferred tax rate.

When a young individual is borrowing unconstrained, he/she can reallocate income freely across periods. Because of this reallocation of income, the negative effect is compensated for by the positive effect regardless of the degree of interest-rate elasticity. Therefore, a reduction in wage results in a higher preferred tax rate when he/she is borrowing unconstrained. However, when the individual is borrowing constrained, the reallocation is restricted because of the borrowing constraint. Under this situation, the borrowing-constrained individual attaches a large weight to the utility loss of a wage reduction. This might lead to a situation where the negative effect overcomes the positive effect, which results in a lower preferred tax rate in response to a wage reduction.

Which effect overcomes the other depends on the degree of interest-rate elasticity. A lower elasticity implies a stronger incentive for individuals to smooth consumption across periods. Because of this incentive, the borrowing-constrained individual attaches a larger weight to the negative effect on youthful consumption as the interest-rate elasticity becomes lower. That is, the borrowing-constrained individual prefers a lower tax rate as his/her wage becomes lower when interest-rate elasticity is low.

Given this property, we show that when the decisive voter is borrowing constrained and the interest-rate elasticity of consumption is low, there is an equilibrium where the middle-income young prefer a higher tax rate than the low-income young. In this equilibrium, the old and the middle-income young form a coalition in favor of a high tax rate and higher social security while the low- and the high-income young favor a low tax rate and less social security. This equilibrium is reminiscent of the ends-against-the-middle equilibrium demonstrated by Epple and Romano (1996).

Based on the characterization of political equilibrium, we investigate how the size of social security is altered in response to the mean-preserving spread of income distribution. In particular, we focus on the mean-preserving reduction of the decisive voter's wage and

show that it creates an inverse U-shaped relationship between the decisive voter's wage and the size of social security when interest-rate elasticity is low. The mean-preserving reduction of the decisive voter's wage results in an increase in the size of social security when his/her wage is above the critical level and thus he/she is borrowing unconstrained. In contrast, it results in a decrease in the size of social security when his/her wage is below the critical level and thus he/she is borrowing constrained. Therefore, interest-rate elasticity and the borrowing constraint are the key factors creating the negative correlation between wage inequality and the size of social security.

The analysis and results of this paper are closely related to the literature on redistributive politics in the presence of borrowing constraints (Casamatta, Cremer and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Conde-Ruiz and Profeta, 2007; Cremer, De Donder, Maldonado and Pestieau, 2007). In particular, these studies show that under certain conditions, a voter prefers less social security as he/she becomes poorer when he/she saves nothing. This result therefore suggests that there might be a negative correlation between inequality and the size of social security when a decisive voter is borrowing constrained.

There are two issues left unresolved from the previous studies. First, their focus was limited to the case of a low interest-rate elasticity of consumption (Casamatta, Cremer and Pestieau, 2000; Conde-Ruiz and Profeta, 2007), or the case of a log-linear utility function (Bellettini and Berti Ceroni, 2007). Cremer, De Donder, Maldonado and Pestieau (2007) considered both high and low elasticity cases. However, the interaction between saving and redistribution was abstracted away in their study; individuals save nothing when young regardless of the size of social security because they are assumed to be myopic and to consume all their labor income when young. Thus, the question remains, whether the negative relationship between inequality and the size of social security still holds true in the presence of the interaction between saving and redistribution when the elasticity is high. The current paper shows that the answer is no: the negative relationship holds if and only if elasticity is low and the decisive voter is borrowing constrained.

Second, the effect of income distribution spread on social security was not analyzed in Casamatta, Cremer and Pestieau (2000), Conde-Ruiz and Profeta (2007) and Cremer, De Donder, Maldonado and Pestieau (2007). Instead, they investigated the effect of the redistributive (i.e., Beveridgean) factor in social security (Casamatta, Cremer and Pestieau, 2000); population aging (Conde-Ruiz and Profeta, 2007); and the proportion of myopic agents in a society (Cremer, De Donder, Maldonado and Pestieau, 2007). Bellettini and Berti Ceroni (2007) investigated the income distribution effect, but their analysis was limited to the case of a log-linear utility function and a productivity-enhancing type of public expenditure. In contrast, the current paper considers the effect of inequality on the size of redistribution under the assumption of a general utility function and a pay-as-you-go social security.

The organization of this paper is as follows. Section 2 introduces the model and characterizes the economic equilibrium. Section 3 develops the political system, introducing the equilibrium concept of the voting game, and demonstrates the voting behavior of each individual. Section 4 characterizes the political equilibrium. Section 5 examines the effect of a mean-preserving spread of income distribution on the size of social security. Section 6 provides some concluding remarks. Proofs of Propositions are given in Section 7.

## 2 The Economic Environment

Consider a discrete time economy where time is denoted by  $t = 0, 1, 2 \dots$ . The economy is made up of overlapping generations of individuals, each of whom lives two periods: youth and old age. The size of a generation born in period  $t$ , called generation  $t$ , is denoted by  $N_t$ . Population grows at a constant rate  $n > 0$ :  $N_{t+1} = (1 + n)N_t$  for all  $t \geq 0$ . Within each generation, there are three types of individuals according to labor productivity: low, middle and high ( $j = L, M, H$ ), whose proportions are respectively  $\rho^L, \rho^M$  and  $\rho^H$ , where  $\sum_j \rho^j = 1$  and  $\rho^j$  satisfies the following assumption.

**Assumption 1.**  $\rho^j > n/\{2(1 + n)\}$ ,  $j = L, M, H$ .

Assumption 1 ensures that a young individual who prefers the highest tax rate among the young becomes the decisive voter. To understand the argument stemming from Assumption 1, consider first the preferences of the old. As we will explain below, the old choose a higher tax rate than any young individual because they bear no tax burden; the tax burden when young is viewed as a sunk cost for the old. In addition, the old have the same preferences over policy because they benefit from the same social security.

Next, consider the preferences of the young. Suppose that a type- $k$  ( $k = L, M$  or  $H$ ) prefers the highest tax rate among the young. When the young and the old vote, the sum of the type- $k$  young and the old is given by  $N_t \rho^k + N_{t-1}$ , which is greater than half of the population in period  $t$ ,  $(N_t + N_{t-1})/2$ , under the assumption of  $\rho^k > n/\{2(1 + n)\}$ . This implies that the decisive voter becomes an old or type- $k$  young voter. However, an old voter cannot become the decisive voter because there are fewer old voters than young under the assumption of  $n > 0$ . Therefore, the type- $k$  young individual, who prefers the tax rate to be highest, becomes the decisive voter. Figure 1 provides an example of preferences over the tax rate.

[Figure 1 about here.]

## 2.1 Individuals

Each individual is assumed to receive utility from private consumption. The utility function of a type- $j$  young individual in period  $t$  is specified by:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(c_{t+1}^{oj})^{1-\sigma} - 1}{1-\sigma},$$

where  $c_t^{yj}$  is consumption when young,  $c_{t+1}^{oj}$  is consumption in old age,  $\beta \in (0, 1]$  is the discount factor and  $\sigma (> 0)$  is the inverse of the elasticity of consumption with respect to the interest rate. A lower  $1/\sigma$  implies a lower interest-rate elasticity of consumption.

Each individual works when young and retires in old age. Wage income is related to labor productivity. The wage of a type- $j$  individual is given by  $w^j$  ( $j = L, M, H$ ), where  $w^j$  is constant over time and  $w^L < w^M < w^H$ . The average wage is denoted by  $\bar{w} \equiv \rho^L w^L + \rho^M w^M + \rho^H w^H \in (w^L, w^H)$ .

Type- $j$ 's individual budget constraints when young and in old age are given by, respectively:

$$\begin{aligned} c_t^{yj} + s_t^j &\leq (1 - \tau_t)w^j, \\ c_{t+1}^{oj} &\leq R s_t^j + b_{t+1}, \end{aligned}$$

where  $s_t^j$  is saving,  $\tau_t$  is the income tax rate in period  $t$ ,  $R$  is the gross interest rate and  $b_{t+1}$  is the per capita social security benefit in old age. We impose the restriction of nonnegative savings, that is:

$$s_t^j \geq 0.$$

This rules out the possibility of borrowing when young against future social security benefits (Diamond and Hausman, 1984; Conde-Ruiz and Profeta, 2007).

We assume that the economy is dynamically efficient.

**Assumption 2:**  $R \geq 1 + n$ .

The assumption implies that the rate of return from social security is lower than the private rate of return from saving. Low- and middle-income individuals may have an incentive to support this inferior system of intertemporal resource reallocation. This is because the current social security system involves an intragenerational redistribution component that transfers resources from the high to the low and the middle.

We also assume that (i) the interest rate is exogenous and (ii) each individual receives the same amount of old age social security benefits regardless of contributions in their youth. The first assumption abstracts away the general equilibrium effect via the interest rate investigated by, for example, Cooley and Soares (1999) and Boldrin and Rustichini

(2000). However, this simplification enables us to more simply demonstrate the analytical solution of the model. The second assumption abstracts from the choice of social security systems (for example, Bismarckian vs. Beveridgean) analyzed by, for example, Borck (2007), Conde-Ruiz and Profeta (2007) and Cremer et al. (2007). We adopt our second assumption to concentrate on the role of the borrowing constraint in the political determination of social security.

The representative type- $j$  young individual maximizes his/her utility subject to their budget constraint and the restriction of nonnegative saving. When  $s_t^j > 0$ , the first-order condition for an interior solution is  $\beta R(c_{t+1}^{oj})^{-\sigma} = (c_t^{yj})^{-\sigma}$ ; this determines an interior solution of saving by a type- $j$  individual:

$$s_t^j = \frac{(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma} + R} \cdot [(1 - \tau_t)w^j - b_{t+1}/(\beta R)^{1/\sigma}].$$

By taking the borrowing constraint into account, the saving function of a type- $j$  individual becomes:

$$s_t^j = \max \left\{ 0, \frac{(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma} + R} \cdot \left[ (1 - \tau_t)w^j - \frac{b_{t+1}}{(\beta R)^{1/\sigma}} \right] \right\}. \quad (1)$$

## 2.2 The Government

In each period, the government collects tax revenue from the young by imposing an income tax. Following the convention in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (see, for example, Casamatta, Cremer and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Cremer et al., 2007). The actual tax revenue is therefore given by  $(1 - \tau_t)\tau_t(\rho^L w^L + \rho^M w^M + \rho^H w^H) = (1 - \tau_t)\tau_t \bar{w}$ , where the term  $(1 - \tau_t)$  is the distortionary factor. The assumption of distortionary taxation is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The government uses the tax revenue for PAYG social security payments. PAYG social security is an intergenerational transfer from the young to the old within a period. The budget constraint is  $N_t(1 - \tau_t)\tau_t \bar{w} = N_{t-1}b_t$ . The per capita social security benefit in period  $t$ ,  $b_t$ , is given by:

$$b_t = (1 + n)(1 - \tau_t)\tau_t \bar{w}. \quad (2)$$

We hereafter focus on a constant sequence of taxes,  $\tau_t = \tau_{t+1} = \tau$  for all  $t$ , because we will assume once-and-for-all voting as described in Section 3.

## 2.3 The Economic Equilibrium

We define the economic equilibrium as follows.

**Definition 1.** For a given tax rate,  $\tau$ , an *economic equilibrium* is a sequence of allocations,  $\{c_t^{yj}, c_t^{oj}, s_t^j\}_{t=0, \dots, \infty}^{j=L, M, H}$  with the initial condition  $s_0^j (j = L, M, H)$ , such that: (i) in every period a type- $j$  individual maximizes his/her utility subject to the budget constraints and the nonnegativity constraint of saving, (ii) the social security budget is balanced every period, and (iii) the goods market clears every period.

The saving function (1) and the government budget constraint (2) imply that there is a critical rate of tax such that:

$$s_t^j \geq 0 \Leftrightarrow \tau \leq \hat{\tau}(w^j) \equiv \frac{(\beta R)^{1/\sigma}}{(1+n)\bar{w}} w^j. \quad (3)$$

Figure 2 illustrates the relationship between savings and the tax rate for each type of individual. A type- $j$  individual chooses positive saving when the tax is below the critical rate. However, when the tax is above the critical rate, a type- $j$  individual faces a borrowing constraint and can save nothing in youth. The critical rate of tax is higher when the wage income is larger because, given a tax rate common to all types of individuals, a higher productivity individual receives a higher level of disposable income.

[Figure 2 about here.]

With the saving function (1) and the private and government budget constraints, we can find that the consumption functions of a type- $j$  individual when young and in old age are given by, respectively:

$$c_t^{yj} = \begin{cases} \frac{R}{(\beta R)^{1/\sigma} + R} \left[ (1-\tau)w^j + \frac{(1+n)(1-\tau)\tau\bar{w}}{R} \right] & \text{if } \tau < \hat{\tau}(w^j), \\ (1-\tau)w^j & \text{if } \tau \geq \hat{\tau}(w^j), \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} \frac{R(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma} + R} \left[ (1-\tau)w^j + \frac{(1+n)(1-\tau)\tau\bar{w}}{R} \right] & \text{if } \tau < \hat{\tau}(w^j), \\ (1+n)(1-\tau)\tau\bar{w} & \text{if } \tau \geq \hat{\tau}(w^j). \end{cases}$$

We use the above-mentioned consumption functions to obtain an indirect utility function of a type- $j$  young individual:

$$V^{yj} = \begin{cases} V_{s>0}^{yj} & \text{if } \tau < \hat{\tau}(w^j), \\ V_{s=0}^{yj} & \text{if } \tau \geq \hat{\tau}(w^j), \end{cases} \quad (4)$$

where:

$$V_{s>0}^{yj} \equiv \frac{1}{1-\sigma} \phi \left[ (1-\tau)w^j + \frac{(1+n)(1-\tau)\tau\bar{w}}{R} \right]^{1-\sigma} - \frac{1+\beta}{1-\sigma}, \quad (5)$$

$$V_{s=0}^{yj} \equiv \frac{[(1-\tau)w^j]^{1-\sigma}}{1-\sigma} + \beta \frac{[(1+n)(1-\tau)\tau\bar{w}]^{1-\sigma}}{1-\sigma} - \frac{1+\beta}{1-\sigma}. \quad (6)$$

The parameter  $\phi$ , observed in Eq. (5), is defined by:

$$\phi \equiv \left( \frac{R}{(\beta R)^{1/\sigma} + R} \right)^{1-\sigma} \cdot [1 + \beta \cdot (\beta R)^{1/\sigma-1}].$$

$V_{s>0}^{y,j}$  denotes the indirect utility of a type- $j$  young individual when he/she saves some portion of his/her income, and  $V_{s=0}^{y,j}$  denotes the indirect utility when he/she is faced with a borrowing constraint and saves nothing. The term in square brackets in (5) represents the lifetime income; the first and the second terms on the right-hand side in (6) represent the utilities of consumption when young and in old age, respectively; and the constant term,  $(1 + \beta)/(1 - \sigma)$ , observed in (5) and (6), summarizes the parameters unrelated to political decisions on taxes.

For an old type- $j$  individual in period  $t$ , the indirect utility function is:

$$V^{o,j} = \begin{cases} V_{s>0}^{oj} \equiv \frac{1}{1-\sigma} [Rs_{t-1}^j + (1+n)(1-\tau)\tau\bar{w}]^{1-\sigma} - \frac{1}{1-\sigma} & \text{if } s_{t-1}^j > 0, \\ V_{s=0}^{oj} \equiv \frac{1}{1-\sigma} [(1+n)(1-\tau)\tau\bar{w}]^{1-\sigma} - \frac{1}{1-\sigma} & \text{if } s_{t-1}^j = 0. \end{cases} \quad (7)$$

Old individuals want to maximize the social security benefit,  $(1+n)(1-\tau)\tau\bar{w}$ , regardless of their saving status because their saving when young is predetermined and has no critical effect on voting behavior.

### 3 The Political Institution and Voting

The tax rate  $\tau$  is determined by individuals through a political process of majoritarian voting. Elections take place every period and all individuals alive, both young and old, cast a ballot over  $\tau$ . Individual preferences over  $\tau$  are represented by the indirect utility functions at Eqs (4) and (7) for the young and the old, respectively. Every individual has zero mass, and thus no individual vote can change the outcome of the election. We thus assume individuals vote sincerely.

This majoritarian voting game is intrinsically dynamic because it describes the interaction among successive generations. To deal with this feature, we assume full commitment, i.e., once-and-for-all voting. That is, voters determine the constant sequence of the parameters:  $\tau_t = \tau_{t+1} = \tau$  for all  $t$  (see, for example, Casamatta, Cremer and Pestieau, 2000; Conde-Ruiz and Profeta, 2007). We can view the full commitment solution as the solution including intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium (see, for example, Conde-Ruiz and Galasso, 2003, 2005).

Given the stationary environment, the current model presents a static voting game. Therefore, the median voter theorem can be applied to the voting game. To find the

voting equilibrium, we need to show that preferences of voters over  $\tau$  are single peaked. The preferences of old voters are single peaked because the objective of the old is to maximize the size of social security benefit specified by the Laffer curve  $(1 - \tau)\tau$ . Their preferred rate of tax is thus given by:

$$\tau^{oj} = \frac{1}{2} \text{ for all } j.$$

Next, we consider the preferences of the young. To show that the preferences of the young are single peaked, we should note the following three properties of their indirect utility functions. First,  $\partial^2 V_{s>0}^{y,j} / \partial \tau^2 < 0$  and  $\partial^2 V_{s=0}^{y,j} / \partial \tau^2 < 0$  hold: that is,  $V_{s>0}^{y,j}$  and  $V_{s=0}^{y,j}$  are single peaked. Second, the indirect utility  $V^{y,j}$  of a type- $j$  young individual is continuous at  $\tau = \hat{\tau}(w^j)$ :

$$V_{s>0}^{y,j} \Big|_{\tau=\hat{\tau}(w^j)} = V_{s=0}^{y,j} \Big|_{\tau=\hat{\tau}(w^j)}.$$

Third, the slope of  $V_{s>0}^{y,j}$  is equivalent to that of  $V_{s=0}^{y,j}$  at  $\tau = \hat{\tau}(w^j)$ :

$$\frac{\partial V_{s>0}^{y,j}}{\partial \tau} \Big|_{\tau=\hat{\tau}(w^j)} = \frac{\partial V_{s=0}^{y,j}}{\partial \tau} \Big|_{\tau=\hat{\tau}(w^j)}.$$

The detail of the calculation is given in Appendix 7.1.

Given these results, the indirect utility of a type- $j$  young individual,  $V^{y,j}$ , is illustrated as in Figure 3:  $V^{y,j}$  has a unique local maximum.  $V^{y,j}$  attains the top of the Laffer curve at  $\tau = \arg \max V_{s>0}^{y,j}$  if  $\arg \max V_{s>0}^{y,j} < \hat{\tau}(w^j)$ . In contrast,  $V^{y,j}$  attains the top of the Laffer curve at  $\tau = \arg \max V_{s=0}^{y,j}$  if  $\arg \max V_{s=0}^{y,j} \geq \hat{\tau}(w^j)$ .

In what follows, we will investigate the taxes preferred by the borrowing-unconstrained (Subsection 3.1) and borrowing-constrained young (Subsection 3.2), respectively. Then we summarize the properties of the preferences of the young in Subsection 3.3.

[Figure 3 about here.]

### 3.1 Voting by the Borrowing-unconstrained Young

Suppose that a young individual is borrowing unconstrained. He/she chooses  $\tau$  to maximize  $V_{s>0}^{y,j}$  in (4). The first derivative of  $V_{s>0}^{y,j}$  with respect to  $\tau$  is given by:

$$\frac{\partial V_{s>0}^{y,j}}{\partial \tau} = \phi \cdot \left[ (1 - \tau)w^j + \frac{(1 + n)(1 - \tau)\tau\bar{w}}{R} \right]^{-\sigma} \cdot \left\{ -w^j + \frac{(1 + n)(1 - 2\tau)\bar{w}}{R} \right\},$$

where the first term within the braces shows the marginal social security burden, and the second term shows the marginal social security benefit. The borrowing-unconstrained

young individual chooses  $\tau$  that balances the marginal cost and benefit in terms of utility. That is, he/she chooses  $\tau$  that attains  $\partial V_{s>0}^{y,j}/\partial\tau = 0$ , which is equivalent to:

$$\tau = \frac{(1+n)\bar{w} - Rw^j}{2(1+n)\bar{w}}. \quad (8)$$

The borrowing-unconstrained young individual prefers a lower tax rate as he/she earns a higher wage; this is consistent with the standard result in the literature of political economy of redistribution (for example, Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981). The result holds regardless of the degree of interest-rate elasticity  $1/\sigma$ . This is because the objective for the borrowing-unconstrained individual is to maximize the lifetime income, which is independent of  $1/\sigma$ .

### 3.2 Voting by the Borrowing-constrained Young

Alternatively, suppose that a young individual is borrowing constrained. He/she chooses  $\tau$  to maximize  $V_{s=0}^{y,j}$  in (4). The first derivative of  $V_{s=0}^{y,j}$  with respect to  $\tau$  is:

$$\frac{\partial V_{s=0}^{y,j}}{\partial\tau} = (1-\tau)^{-\sigma} \cdot [(-1)(w^j)^{1-\sigma} + \beta \{(1+n)\bar{w}\}^{1-\sigma} (\tau)^{-\sigma} (1-2\tau)],$$

where the first term in the square brackets shows the marginal social security burden and the second term shows the marginal social security benefit. The borrowing-constrained young individual chooses  $\tau$  that balances the marginal cost and benefit in terms of utility. That is, he/she chooses  $\tau$  that attains  $\partial V_{s=0}^{y,j}/\partial\tau = 0$ , which is equivalent to:

$$\frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^j)^{1-\sigma} (\tau)^\sigma = (1+n)\bar{w}(1-2\tau). \quad (9)$$

Eq. (9) indicates that the preferences of the borrowing-constrained young depend on the degree of interest-rate elasticity of consumption denoted by  $1/\sigma$ . The left-hand side of Eq. (9) is increasing in  $w^j$  if the interest-rate elasticity is high such that  $1/\sigma > 1$  ( $\sigma < 1$ ); it is decreasing in  $w^j$  if interest-rate elasticity is sufficiently low that  $1/\sigma < 1$  ( $\sigma > 1$ ). The right-hand side is independent of  $w^j$ . Therefore, a reduction of  $w^j$  raises type- $j$ 's preferred tax rate if  $1/\sigma > 1$  whereas it lowers his/her preferred tax rate if  $1/\sigma < 1$ .

### 3.3 Properties of the Preferences of the Young

The results established in Subsections 3.1 and 3.2 are summarized as follows. A young individual prefers a higher tax rate as his/her wage becomes lower when he/she is borrowing unconstrained. When a young individual is borrowing constrained, the results also

hold true if interest-rate elasticity is sufficiently high that  $1/\sigma > 1$ . However, if elasticity is sufficiently low that  $1/\sigma < 1$ , the opposite result holds: a borrowing-constrained young individual prefers a lower tax rate as his/her wage becomes lower. In other words, a young individual prefers a lower tax rate as he/she becomes poorer if and only if he/she is borrowing constrained and the interest-rate elasticity of consumption is low enough that  $1/\sigma < 1$ .

To understand the mechanism behind the above-mentioned result, we consider the following two opposing effects of a reduction in wages on the preferred tax rate. First, given a tax rate, a decrease in the wage reduces the tax burden and thus raises a benefit-to-burden ratio. This gives a young individual an incentive to choose a higher tax rate: this is a positive effect on the preferred tax rate. Second, a decrease in wage reduces disposable income and thus consumption level when young. This gives a young individual an incentive to choose a lower tax rate from the viewpoint of maintaining the utility of consumption: this is a negative effect on the preferred tax rate.

When a young individual is borrowing unconstrained, he/she can reallocate income freely across periods. Because of this intertemporal reallocation of income, the negative effect is compensated by the positive effect regardless of the degree of interest-rate elasticity. Therefore, a decrease in the wage results in a higher preferred tax rate when an individual is borrowing unconstrained.

However, when an individual is borrowing constrained, the positive effect does not necessarily overcome the negative one. The borrowing-constrained individual wants to consume more when young, but his/her demand is restricted by the borrowing constraint. Under this situation, the borrowing-constrained individual attaches a large weight to the utility loss of a reduction in their wage. This might lead to a situation where the negative effect overcomes the positive effect, which results in a lower preferred tax rate in response to a wage reduction.

Which effect outweighs the other depends on the degree of interest-rate elasticity. A lower elasticity results in a smaller change of consumption in response to a change in the interest rate. In other words, a lower elasticity implies a stronger incentive for individuals to smooth consumption across periods. Because of this incentive, the borrowing-constrained individual attaches a larger weight to the negative effect on youthful consumption as the interest-rate elasticity becomes lower. Therefore, the borrowing-constrained individual prefers a lower tax rate as his/her wage becomes lower when the interest-rate elasticity is low enough that  $1/\sigma < 1$ .

## 4 The Political Equilibrium

The previous section analyzed the voting behavior of an individual classified according to the status of saving. Given that preferences are single peaked over  $\tau$ , we now consider the political determination of  $\tau$ . The decisive voter over  $\tau$  belongs to the young generation because (i) the population size of the young is larger than that of the old; and (ii) all types of young individuals choose lower tax rates than the old.

To determine the type of a decisive voter over  $\tau$ , we consider the preferred tax rate of a type- $j$  young individual in the following way. When  $\tau < \hat{\tau}(w^j)$ , he/she saves part of his/her income, and his/her preference for  $\tau$  follows (8). When  $\tau \geq \hat{\tau}(w^j)$ , he/she saves nothing and his/her preference over  $\tau$  follows (9). From (8) and (9), the preferred tax rate of the type- $j$  young individual satisfies the following condition:

$$\underbrace{(1+n)\bar{w}(1-2\tau)}_{y(\tau;\bar{w})} = \underbrace{\begin{cases} Rw^j & \text{if } \tau < \hat{\tau}(w^j) \equiv \frac{(\beta R)^{1/\sigma}}{(1+n)\bar{w}} w^j \\ \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^j)^{1-\sigma} (\tau)^\sigma & \text{if } \tau \geq \hat{\tau}(w^j) \end{cases}}_{z^j(\tau;w^j,\bar{w})} \quad (10)$$

where  $y(\tau; \bar{w})$  and  $z^j(\tau; w^j, \bar{w})$  denote marginal benefit and loss of social security in terms of utility, respectively.

Hereafter, we focus on the parameter  $\sigma$  representing the inverse of the interest-rate elasticity of consumption and consider two cases separately: a high elasticity ( $1/\sigma \geq 1$  in Subsection 4.1) and a low elasticity ( $1/\sigma < 1$  in Subsection 4.2). We adopt this classification because the order of preferences for the tax rate depends critically on the degree of interest-rate elasticity. For the case of  $1/\sigma \geq 1$ , a lower-income young individual prefers a higher tax rate. However, for the case of  $1/\sigma < 1$ , a lower-income young individual may prefer a lower tax rate. For each case, we will show the existence and uniqueness of an equilibrium of the voting game and find conditions that determine the decisive voter.

We impose the following assumption to abstract away a trivial equilibrium.

**Assumption 3:**  $(1+n)\bar{w} > Rw^L$ .

Assumption 3 ensures that a type- $L$  individual definitely prefers a positive tax rate when he/she is borrowing unconstrained (see Eq. (8)). Otherwise, he/she may choose  $\tau = 0$ ; the type- $M$  and type- $H$  individuals also choose  $\tau = 0$ . The equilibrium results in no taxation and thus no social security. We impose Assumption 3 in order to rule out this trivial equilibrium.

## 4.1 High Interest-rate Elasticity of Consumption ( $1/\sigma \geq 1$ )

Figure 4 illustrates the condition (10) that determines the preferred tax rate of a type- $j$  young ( $j = L, M, H$ ) individual for  $1/\sigma \geq 1$ . The left-hand side of (10), denoted by  $y(\tau; \bar{w})$ , is decreasing in  $\tau$  and is independent of the type of young individual. In contrast, the right-hand side of (10), denoted by  $z^j(\tau; w^j, \bar{w})$ , is nondecreasing in  $\tau$ , dependent on the type of young individual and characterized by  $z^H(\tau; w^H, \bar{w}) \geq z^M(\tau; w^M, \bar{w}) \geq z^L(\tau; w^L, \bar{w})$ , where an equality holds if and only if  $\sigma = 1$  and  $\tau \geq \hat{\tau}(w^j)$ . The kink point of  $\tau = \hat{\tau}(w^j)$  implies that a type- $j$  young individual can save part of his/her income if  $\tau < \hat{\tau}(w^j)$  and nothing if  $\tau \geq \hat{\tau}(w^j)$ .

[Figure 4 about here.]

The crossing point of  $y(\tau; \bar{w})$  and  $z^j(\tau; w^j, \bar{w})$  determines the preferred tax rate for a type- $j$  young individual. We see from Figure 4 that a lower-income young individual prefers a higher tax rate:  $\tau^{yH} < \tau^{yM} < \tau^{yL}$ , where  $\tau^{yj}$  ( $j = L, M, H$ ) denotes the preferred tax rate of a type- $j$  young individual. Therefore, we obtain the following result.

**Proposition 1.** *Suppose that  $1/\sigma \geq 1$  holds. There exists a unique equilibrium of the voting game such that the decisive voter over  $\tau$  is a type- $L$  young individual. He/she is*

- (i) *borrowing unconstrained if  $w^L \in \left( \frac{(1+n)\bar{w}}{R+2(\beta R)^{1/\sigma}}, \frac{(1+n)\bar{w}}{R} \right)$ ;*
- (ii) *borrowing constrained if  $w^L \in \left( 0, \frac{(1+n)\bar{w}}{R+2(\beta R)^{1/\sigma}} \right]$ .*

**Proof.** See Appendix 7.2.

Figure 4 illustrates two cases. Panel (a) depicts the case where the wage of type- $L$  individuals is above the critical level  $(1+n)\bar{w}/(R+2(\beta R)^{1/\sigma})$  such that they can save part of their income when young for future consumption. In this case, the equilibrium tax rate represented by the crossing point of  $y(\tau; \bar{w})$  and  $z^L(\tau; w^L, \bar{w})$  is below the critical rate,  $\hat{\tau}(w^L)$ . In contrast, panel (b) illustrates the case where the wage of type- $L$  individuals is below the critical level  $(1+n)\bar{w}/(R+2(\beta R)^{1/\sigma})$  such that they can save nothing in their youth. The equilibrium tax rate is above the critical rate,  $\hat{\tau}(w^L)$ .

## 4.2 Low Interest-rate Elasticity of Consumption ( $1/\sigma < 1$ )

Next, consider low interest-rate elasticity, such that  $1/\sigma < 1$ . The decisive voter over  $\tau$  may differ from the previous case; that is, the order of preferred tax rates may change depending on the distribution of income and other exogenous parameters. To determine the decisive voter over  $\tau$ , let us recall the condition (10) that determines the tax rate

preferred by a type- $j$  young individual. The graphs of (10) for  $j = L, M, H$  when  $1/\sigma < 1$  are illustrated in Figure 5.

The main difference from the previous case is that  $z^L(\tau; w^L, \bar{w})$  and  $z^M(\tau; w^M, \bar{w})$  (or  $z^M(\tau; w^M, \bar{w})$  and  $z^H(\tau; w^H, \bar{w})$ ) cross at some tax rate  $\tau \in (0, 1/2)$ . This is because in an economy with  $1/\sigma < 1$ , the slope of  $z^j(\tau; w^j, \bar{w})$  becomes steeper the lower the wage level becomes when a type- $j$  individual is borrowing constrained. Thus, there are two critical values of  $\tau$ ,  $\tilde{\tau}^{LM} \in (\hat{\tau}(w^L), \hat{\tau}(w^M))$  and  $\tilde{\tau}^{MH} \in (\hat{\tau}(w^M), \hat{\tau}(w^H))$ , such that  $z^L(\tau; w^L, \bar{w})$  and  $z^M(\tau; w^M, \bar{w})$  cross at  $\tau = \tilde{\tau}^{LM}$  and  $z^M(\tau; w^M, \bar{w})$  and  $z^H(\tau; w^H, \bar{w})$  cross at  $\tau = \tilde{\tau}^{MH}$ . By direct calculation, we obtain:

$$\tilde{\tau}^{LM} \equiv \frac{1}{(1+n)\bar{w}} \cdot \left( \frac{\beta R w^M}{(w^L)^{1-\sigma}} \right)^{\frac{1}{\sigma}} \quad \text{and} \quad \tilde{\tau}^{MH} \equiv \frac{1}{(1+n)\bar{w}} \cdot \left( \frac{\beta R w^H}{(w^M)^{1-\sigma}} \right)^{\frac{1}{\sigma}},$$

where  $\hat{\tau}(w^L) < \tilde{\tau}^{LM} < \hat{\tau}(w^M) < \tilde{\tau}^{MH} < \hat{\tau}(w^H)$  (see Figure 5). The derivation of  $\tilde{\tau}^{LM}$  and  $\tilde{\tau}^{MH}$  is given in Appendix 7.3.

[Figure 5 about here.]

The tax rate preferred by a type- $j$  young individual is determined by the crossing point of  $y(\tau; \bar{w})$  and  $z^j(\tau; w^j, \bar{w})$  of (10). Given the assumption of demographic structure (Assumption 1), the decisive voter over  $\tau$  is the one who prefers the highest tax rate among young individuals. Based on the illustration in Figure 5, the politically determined tax rate is now implicitly given by:

$$y(\tau; \bar{w}) = \tilde{z}(\tau; w^L, w^M, w^H), \quad (11)$$

where:

$$\tilde{z}(\tau; w^L, w^M, w^H) \equiv \begin{cases} R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^L)^{1-\sigma} (\tau)^\sigma & \text{if } \hat{\tau}(w^L) \leq \tau \leq \tilde{\tau}^{LM} \\ R w^M & \text{if } \tilde{\tau}^{LM} < \tau < \hat{\tau}(w^M) \\ \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^M)^{1-\sigma} (\tau)^\sigma & \text{if } \hat{\tau}(w^M) \leq \tau \leq \tilde{\tau}^{MH} \\ R w^H & \text{if } \tilde{\tau}^{MH} < \tau < \hat{\tau}(w^H) \\ \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^H)^{1-\sigma} (\tau)^\sigma & \text{if } \hat{\tau}(w^H) \leq \tau. \end{cases}$$

The graph of the function  $\tilde{z}$  is depicted in the bold solid curve in Figure 5. Solving (11) for  $\tau$  determines the equilibrium tax rate.

**Proposition 2.** *Suppose that  $1/\sigma < 1$  holds. There exists a unique equilibrium of the voting game such that the decisive voter over  $\tau$  is:*

(i) a borrowing-unconstrained type- $L$  young individual if

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} < w^L;$$

(ii) a borrowing-constrained type- $L$  young individual if

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} \geq w^L \text{ and } (1+n)\bar{w} \leq 2\frac{(\beta R)^{1/\sigma}(w^M)^{1/\sigma}}{(w^L)^{(1-\sigma)/\sigma}} + Rw^M;$$

(iii) a borrowing-unconstrained type- $M$  young individual if

$$(1+n)\bar{w} > 2\frac{(\beta R)^{1/\sigma}(w^M)^{1/\sigma}}{(w^L)^{(1-\sigma)/\sigma}} + Rw^M \text{ and } \frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} < w^M;$$

(iv) a borrowing-constrained type- $M$  young individual if

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} \geq w^M.$$

**Proof.** See Appendix 7.4.

Figure 6 illustrates the conditions in statements (i)–(iv) of Proposition 2 in a  $w^L - w^M$  space, given  $\bar{w}$ . The area marked by (i) (= (i), (ii), (iii), (iv)) corresponds to the condition in statement (i) of Proposition 2. To understand the result of Proposition 2, suppose first that the wage of the type- $L$  individual is above the critical level  $(1+n)\bar{w}/\{2(\beta R)^{1/\sigma} + R\}$ : the pair of  $w^L$  and  $w^M$  is set within the area marked by (i) in Figure 6. The type- $L$  individual can save a part of his/her wage income for old-age consumption. He/she is borrowing unconstrained, and thus the other two types of individuals are also borrowing unconstrained. In this case, the type- $L$  individual prefers the highest tax rate among the young, and he/she becomes the decisive voter.

[Figure 6 about here.]

Alternatively, suppose that the wage of the type- $L$  agent is below the critical level. To start the consideration in this case, assume that the pair of  $w^L$  and  $w^M$  is set within the area marked by (iv) in Figure 6. In this situation, both type- $L$  and type- $M$  individuals are borrowing constrained because of their low wage levels. As demonstrated in Section 3, a borrowing-constrained individual prefers a lower tax rate as his/her wage becomes lower when the interest-rate elasticity is low enough that  $1/\sigma < 1$ . Therefore, in the area marked by (iv), the type- $M$  individual prefers a higher tax rate than the type- $L$  individual; the type- $M$  individual becomes the decisive voter. The economy is featured by an equilibrium,

which is reminiscent of the ends-against-the-middle equilibrium demonstrated by Epple and Romano (1996).

Next, consider an increase in type- $M$ 's wage  $w^M$ , leaving  $\bar{w}$  and  $w^L$  unchanged. The tax rate preferred by the type- $L$  individual remains unchanged; the type- $H$ 's wage,  $w^H$ , is adjusted to keep the mean constant. Under this situation, the tax rate preferred by the type- $M$  individual is increased by an increase in his/her wage  $w^M$  because he/she is borrowing constrained and the interest-rate elasticity of consumption is below one. However, when the level of  $w^M$  is increased such that the pair of  $w^L$  and  $w^M$  is set within the area marked by (iii), a further increase in  $w^M$  leads the type- $M$  individual to prefer a lower tax rate because he/she is now borrowing unconstrained.

When  $w^M$  is sufficiently high that the pair of  $w^L$  and  $w^M$  is set within the area marked by (ii), the tax rate preferred by the type- $M$  individual becomes lower than that of the type- $L$  individual. The decisive voter switches from the type- $M$  individual to the type- $L$  individual. Therefore, in the area marked by (ii), the economy is characterized by an equilibrium where the type- $L$  individual, who is the poorest among the young, becomes the decisive voter.

Our consideration so far suggests that key factors for the existence of an equilibrium reminiscent of Epple and Romano (1996)'s ends-against-the-middle equilibrium are the borrowing constraint and a low interest-rate elasticity of consumption. When an individual is borrowing constrained, a lower interest-rate elasticity provides him/her an incentive to choose a lower tax rate, as explained in Subsection 3.3. Because of this effect, the borrowing-constrained type- $L$  individual prefers a lower tax rate than the type- $M$  individual.

## 5 Effects of Income Inequality on Policy

Given the characterization of the political equilibrium in Section 4, we wish to investigate how the tax rate ( $\tau$ ) changes in response to the spread of income distribution. In particular, we consider a mean-preserving reduction of the decisive voter's wage where a reduction in the decisive voter's wage,  $w^j$ , is associated with an increase in  $w^H$ , keeping the mean  $\bar{w}$  unchanged. The aim of the analysis is to compare two groups of countries with similar average income levels but different levels of income inequality.

### Proposition 3.

- (i) *In an economy with  $1/\sigma \geq 1$  where the decisive voter is a type- $L$  young individual, the equilibrium tax rate is nondecreased by the mean-preserving reduction of  $w^L$ .*

- (ii) *In an economy with  $1/\sigma < 1$  where the decisive voter is a type- $j$  ( $j = L$  or  $M$ ) young individual, a mean-preserving reduction of the decisive voter's wage  $w^j$  locally produces an inverse U-shaped relationship between  $w^j$  and the tax rate  $\tau$  around  $\tau = \hat{\tau}(w^j)$ .*

**Proof.** See Appendix 7.5.

We can find the mechanism behind the result in Proposition 3 by utilizing the analysis and the result in Section 3. Proposition 3 states that if the interest-rate elasticity is high enough that  $1/\sigma \geq 1$ , there is a monotone relationship between the decisive voter's wage and his/her preferred tax rate. As demonstrated in Section 3, a reduction in wage results in a higher preferred tax rate regardless of status of saving if  $1/\sigma > 1$ . The decisive voter then prefers a higher tax rate as he/she becomes poorer (see Panel (a) of Figure 7). If  $1/\sigma = 1$ , the preferred tax rate is the same for all types of individuals.

[Figure 7 about here.]

When elasticity is low enough that  $1/\sigma < 1$ , a monotone relationship no longer holds. A negative relationship between the preferred tax and wage still holds true when the decisive voter is borrowing unconstrained. However, once his/her wage falls below the critical level that changes his/her status from unconstrained to constrained, he/she prefers a lower tax rate as he/she becomes poorer, as demonstrated in Section 3 (see Panel (b) of Figure 7). There is then an inverse U-shaped relationship between the decisive voter's wage and the preferred tax rate around the critical wage, as depicted in Panel (c) of Figure 7.

### **Empirical Implications**

As mentioned in Section 1, standard political economy theory suggests a positive correlation between inequality and social security; higher inequality results in greater redistribution. However, cross-country data do not necessarily support this prediction. For example, Gottschalk and Smeeding (1997) and Chen and Song (2009) find a negative correlation between wage inequality and social security; put differently, countries with smaller earnings inequality have, on average, greater social security as a percentage of GDP. For example, the United Kingdom and the United States have high income inequality, a low tax rate and a low level of social security benefits. In contrast, some Continental European and Nordic countries feature low income inequality, a high tax rate and a high level of social security benefits.

In the current framework, the negative correlation arises only in the equilibrium where the following two conditions hold: (i) the interest-rate elasticity of consumption is low and (ii) the decisive voter is borrowing constrained. When one of the conditions fails to hold, the economy displays a positive correlation between income inequality and the tax

rate. This is inconsistent with the empirical evidence. Therefore, our analysis suggests that these factors are the key to explaining cross-country differences in income inequality, tax rates and the size of social security payments.

## 6 Conclusion

How is wage inequality related to the size of social security? This paper develops a political economy model that responds to this question. Two features are crucial to our analysis and results: the interest-rate elasticity of consumption and the borrowing constraint. These features derive an equilibrium where old and middle-income young individuals form a coalition in favor of large social security payments while low- and high-income young individuals favor smaller social security payments. In addition, a mean-preserving reduction of a decisive voter's wage produces an inverse U-shaped relationship between inequality and social security. In particular, the decisive voter chooses a smaller size of social security in response to a reduction in wage when he/she is borrowing constrained and the interest-rate elasticity of consumption is low.

The negative correlation between inequality and social security we obtain appears to fit the available empirical evidence. This correlation arises only in an equilibrium where the above-mentioned conditions (the borrowing constraint and low interest-rate elasticity of consumption) are effective for the decisive voter. When one of the conditions fails to hold, the economy displays a positive, rather than negative, correlation between inequality and social security. Thus, our analysis suggests that the two conditions play a key role in explaining cross-country differences in inequality and social security.

## 7 Appendix

### 7.1 Single-peakedness of Preferences

The proof proceeds as follows. First, we show that both  $V_{s>0}^{y,j}$  and  $V_{s=0}^{y,j}$  are single peaked over  $\tau$ . Then we demonstrate that  $\partial V_{s>0}^{y,j}/\partial\tau = \partial V_{s=0}^{y,j}/\partial\tau$  and  $V_{s>0}^{y,j} = V_{s=0}^{y,j}$  hold at  $\tau = \hat{\tau}(w^j)$ , implying that  $V^{y,j}$  has a unique local maximum over the whole range of  $\tau$  and thus that  $V^{y,j}$  is single peaked over  $\tau$ .

The first and the second derivatives of  $V_{s>0}^{y,j}$  and  $V_{s=0}^{y,j}$  with respect to  $\tau$  are:

$$\begin{aligned} \frac{\partial V_{s>0}^{y,j}}{\partial\tau} &= \phi \cdot \left[ (1-\tau)w^j + \frac{(1+n)(1-\tau)\tau\bar{w}}{R} \right]^{-\sigma} \cdot \left\{ -w^j + \frac{(1+n)(1-2\tau)\bar{w}}{R} \right\}; \\ \frac{\partial^2 V_{s>0}^{y,j}}{\partial\tau^2} &= \phi \cdot (1-\tau)^{-\sigma-1} \cdot \left( w^j + \frac{(1+n)\tau\bar{w}}{R} \right)^{-\sigma-1} \\ &\quad \times \left[ (-\sigma) \left\{ -w^j + \frac{(1+n)(1-2\tau)\bar{w}}{R} \right\}^2 + (-2)(1-\tau) \left( w^j + \frac{(1+n)\tau\bar{w}}{R} \right) \frac{(1+n)\bar{w}}{R} \right] \\ &< 0; \\ \frac{\partial V_{s=0}^{y,j}}{\partial\tau} &= (1-\tau)^{-\sigma} \cdot [(-1)(w^j)^{1-\sigma} + \beta \{(1+n)\bar{w}\}^{1-\sigma} (\tau)^{-\sigma} (1-2\tau)]; \\ \frac{\partial^2 V_{s=0}^{y,j}}{\partial\tau^2} &= (-\sigma)(1-\tau)^{-\sigma-1} (w^j)^{1-\sigma} \\ &\quad + (-1)\beta \{(1+n)\bar{w}\}^{1-\sigma} \{(1-\tau)\tau\}^{-\sigma-1} [\sigma(1-2\tau)^2 + 2(1-\tau)\tau] \\ &< 0. \end{aligned}$$

$V_{s>0}^{y,j}$  and  $V_{s=0}^{y,j}$  are single peaked over  $\tau$  because the second derivatives are negative.

Next, we show that  $\partial V_{s>0}^{y,j}/\partial\tau = \partial V_{s=0}^{y,j}/\partial\tau$  and  $V_{s>0}^{y,j} = V_{s=0}^{y,j}$  hold at  $\tau = \hat{\tau}(w^j)$ . By direct calculation, we have:

$$\begin{aligned} \left. \frac{\partial V_{s>0}^{y,j}}{\partial\tau} \right|_{\tau=\hat{\tau}(w^j)} &= \left. \frac{\partial V_{s=0}^{y,j}}{\partial\tau} \right|_{\tau=\hat{\tau}(w^j)} \\ &= (1-\hat{\tau}(w^j))^{-\sigma} (w^j)^{-\sigma} \frac{1}{R} [(1+n)\bar{w} - (R + 2(\beta R)^{1/\sigma}) w^j], \\ V_{s>0}^{y,j} \Big|_{\tau=\hat{\tau}(w^j)} &= V_{s=0}^{y,j} \Big|_{\tau=\hat{\tau}(w^j)} \\ &= \frac{1}{1-\sigma} (1-\hat{\tau}(w^j))^{1-\sigma} (w^j)^{1-\sigma} \left[ 1 + \beta (\beta R)^{(1-\sigma)/\sigma} \right] - \frac{1+\beta}{1-\sigma}. \end{aligned}$$

With this result and the single-peakedness of  $V_{s>0}^{y,j}$  and  $V_{s=0}^{y,j}$  over  $\tau$ , we can conclude that  $V^{y,j}$  has a unique local maximum with respect to  $\tau$  over the whole range of  $\tau$ , as illustrated in Figure 3. In particular,  $V^{y,j}$  is maximized at  $\tau = \arg \max V_{s>0}^{y,j}$  if  $\tau < \hat{\tau}(w^j)$ ;

it is maximized at  $\tau = \arg \max_{s=0} V_{s=0}^{y,j}$  if  $\tau \geq \hat{\tau}(w^j)$ . ■

## 7.2 Proof of Proposition 1

As shown in the text, when  $1/\sigma \geq 1$ , the decisive voter is a type- $L$  individual and his/her preferred tax rate satisfies (10) for  $j = L$ . The functions  $y(\tau; \bar{w})$  and  $z^j(\tau; w^j, \bar{w})$  defined in (10) have the following properties:

$$\begin{aligned} \partial y(\tau; \bar{w})/\partial \tau &< 0, & y(0; \bar{w}) &= (1+n)\bar{w}, & y(1/2; \bar{w}) &= 0, \\ \partial z^j(\tau; w^j, \bar{w})/\partial \tau &\geq 0, & z^j(0; w^j, \bar{w}) &= R w^j, \\ z^L(1/2; w^L, \bar{w}) &> 0. \end{aligned}$$

These properties indicate that there exists a unique  $\tau \in (0, 1/2)$  that satisfies  $y(\tau; \bar{w}) = z^L(\tau; w^L, \bar{w})$  under Assumption 3.

As demonstrated in Figure 4, the tax rate satisfying  $y(\tau; \bar{w}) = z^L(\tau; w^L, \bar{w})$  is below  $\hat{\tau}(w^L)$  if  $R w^L > y(\hat{\tau}(w^L); \bar{w}) \equiv (1+n)\bar{w}(1 - 2\hat{\tau}(w^L))$ ; that is, if  $w^L > (1+n)\bar{w}/\{R + 2(\beta R)^{1/\sigma}\}$ . With Assumption 3, the type- $L$  individual is borrowing unconstrained if  $w^L \in ((1+n)\bar{w}/\{R + 2(\beta R)^{1/\sigma}\}, (1+n)\bar{w}/R)$ . In contrast, the type- $L$  individual is borrowing constrained if  $w^L \in (0, (1+n)\bar{w}/\{R + 2(\beta R)^{1/\sigma}\})$ . ■

## 7.3 The Derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$

The derivation of  $\tilde{\tau}^{LM}$  is as follows. For the range of  $(\hat{\tau}(w^L), \hat{\tau}(w^M))$ , the right-hand side of (10), denoted by  $z^j(\tau; w^j, \bar{w})$ , is given by:

$$z^j(\tau; w^j, \bar{w}) = \begin{cases} z^L(\tau; w^L, \bar{w}) = \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^L)^{1-\sigma} (\tau)^\sigma & \text{for } j = L \\ z^M(\tau; w^M, \bar{w}) = R w^M & \text{for } j = M. \end{cases}$$

$z^L(\tau; w^L, \bar{w}) < z^M(\tau; w^M, \bar{w})$  holds at  $\tau = \hat{\tau}(w^L)$ ;  $z^L(\tau; w^L, \bar{w}) > z^M(\tau; w^M, \bar{w})$  holds at  $\tau = \hat{\tau}(w^M)$ . Thus, there exists a unique  $\tau$ , denoted by  $\tilde{\tau}^{LM} \in (\hat{\tau}(w^L), \hat{\tau}(w^M))$ , that satisfies  $z^L(\tau; w^L, \bar{w}) = z^M(\tau; w^M, \bar{w})$  because  $z^L(\tau; w^L, \bar{w})$  is continuous and strictly increasing in  $\tau$  whereas  $z^M(\tau; w^M, \bar{w})$  is independent of  $\tau$ . We can derive  $\tilde{\tau}^{LM}$  by solving  $\{(1+n)\bar{w}\}^\sigma (w^L)^{1-\sigma} (\tau)^\sigma / \beta = R w^M$  for  $\tau$ .

By the same token, the tax rate that satisfies  $z^M(\tau; w^M, \bar{w}) = z^H(\tau; w^H, \bar{w})$  for the range of  $(\hat{\tau}(w^M), \hat{\tau}(w^H))$  is derived by solving  $\{(1+n)\bar{w}\}^\sigma (w^M)^{1-\sigma} (\tau)^\sigma / \beta = R w^H$  for  $\tau$ . The solution is denoted by  $\tilde{\tau}^{MH}$ . ■

## 7.4 Proof of Proposition 2

As shown in the text, when  $1/\sigma < 1$ , the decisive voter's preferred tax rate satisfies  $y(\tau; \bar{w}) = \tilde{z}(\tau; w^L, w^M, w^H)$ . The functions  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  have the following properties:

$$\begin{aligned} \partial y(\tau; \bar{w}) / \partial \tau &< 0, \\ y(0; \bar{w}) &= (1+n)\bar{w}, \\ y(1/2; \bar{w}) &= 0, \\ \partial \tilde{z}(\tau; w^L, w^M, w^H) / \partial \tau &\geq 0, \\ \tilde{z}(0; w^L, w^M, w^H) &= R w^L, \\ \tilde{z}(1/2; w^L, w^M, w^H) &\in [R w^L, \infty). \end{aligned}$$

These properties indicate that under Assumption 3 ( $(1+n)\bar{w} > R w^L$ ) there exists a unique  $\tau \in (0, 1/2)$  satisfying  $y(\tau; \bar{w}) = \tilde{z}(\tau; w^L, w^M, w^H)$ .

The condition for the determination of the decisive voter over  $\tau$  and his/her status of borrowing can be found with the help of Figure 5.

(i) An unconstrained, type- $L$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[0, \hat{\tau}(w^L)]$ ; that is, if  $y(\tau; \bar{w}) < R w^L$  holds at  $\tau = \hat{\tau}(w^L)$ . This condition is written as:

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} < w^L.$$

(ii) A constrained type- $L$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[\hat{\tau}(w^L), \tilde{\tau}^{LM}]$ ; that is, if  $y(\tau; \bar{w}) \geq R w^L$  at  $\tau = \hat{\tau}(w^L)$  and  $y(\tau; \bar{w}) \leq R w^M$  at  $\tau = \tilde{\tau}^{LM}$ . The two conditions are rewritten as:

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} \geq w^L \text{ and } (1+n)\bar{w} \leq 2 \frac{(\beta R)^{1/\sigma} (w^M)^{1/\sigma}}{(w^L)^{(1-\sigma)/\sigma}} + R w^M.$$

(iii) A constrained, type- $M$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[\tilde{\tau}^{LM}, \hat{\tau}(w^M)]$ ; that is, if  $y(\tau; \bar{w}) > R w^M$  at  $\tau = \tilde{\tau}^{LM}$  and  $y(\tau; \bar{w}) < R w^M$  at  $\tau = \hat{\tau}(w^M)$ . The two conditions are rewritten as:

$$(1+n)\bar{w} > 2 \frac{(\beta R)^{1/\sigma} (w^M)^{1/\sigma}}{(w^L)^{(1-\sigma)/\sigma}} + R w^M \text{ and } \frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} < w^M.$$

(iv) A constrained, type- $M$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[\hat{\tau}(w^M), \tilde{\tau}^{MH}]$ ; that is, if  $y(\tau; \bar{w}) \geq R w^M$  at

$\tau = \hat{\tau}(w^M)$  and  $y(\tau; \bar{w}) \leq R w^H$  at  $\tau = \tilde{\tau}^{MH}$ . The two conditions are rewritten as:

$$\frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} \geq w^M \quad \text{and} \quad (1+n)\bar{w} - R w^H \leq 2 \frac{(\beta R)^{1/\sigma} (w^H)^{1/\sigma}}{(w^M)^{(1-\sigma)/\sigma}}.$$

The second condition always holds because the left-hand side is negative under Assumption 2 whereas the right-hand side is positive. Therefore, the constrained, type- $M$  young individual becomes the decisive voter if the first condition holds.

(v) An unconstrained, type- $H$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[\tilde{\tau}^{MH}, \hat{\tau}(w^H)]$ ; that is, if  $y(\tau; \bar{w}) > R w^H$  at  $\tau = \tilde{\tau}^{MH}$  and  $y(\tau; \bar{w}) < R w^H$  at  $\tau = \hat{\tau}(w^H)$ . The two conditions are rewritten as:

$$(1+n)\bar{w} - R w^H > 2 \frac{(\beta R)^{1/\sigma} (w^H)^{1/\sigma}}{(w^M)^{(1-\sigma)/\sigma}} \quad \text{and} \quad \frac{(1+n)\bar{w}}{2(\beta R)^{1/\sigma} + R} < w^H.$$

The first condition fails to hold because the left-hand side is negative under Assumption 2 whereas the right-hand side is positive. Thus, there exists no equilibrium where the decisive voter is a type- $H$ , unconstrained young individual.

(vi) A constrained, type- $H$  young individual becomes the decisive voter if  $y(\tau; \bar{w})$  and  $\tilde{z}(\tau; w^L, w^M, w^H)$  cross within the range of  $[\hat{\tau}(w^H), 1/2]$ ; that is, if  $y(\tau; \bar{w}) \geq R w^H$  at  $\tau = \hat{\tau}(w^H)$ . The condition is rewritten as:

$$(1+n)\bar{w} - R w^H \geq 2(\beta R)^{1/\sigma} w^H.$$

This condition fails to hold under Assumption 2. Thus, there exists no equilibrium where the decisive voter is a type- $H$ , constrained young individual. ■

## 7.5 Proof of Proposition 3

Suppose that the decisive voter's wage  $w^j$  is initially given such that his/her preferred tax rate is  $\tau = \hat{\tau}(w^j)$ . We denote  $\hat{w}^j$  as the wage rate that makes a type- $j$  young individual choose  $\tau = \hat{\tau}(w^j)$ . Then, around  $w^j = \hat{w}^j$ , there exists a positive real number  $\varepsilon$  such that the equilibrium tax rate satisfies the following condition:

$$\underbrace{(1+n)\bar{w}(1-2\tau)}_{y(\tau; \bar{w})} = z^j(\tau; w^j, \bar{w}) = \begin{cases} \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^j)^{1-\sigma} (\tau)^\sigma & \text{for } w^j \in (\hat{w}^j - \varepsilon, \hat{w}^j), \\ R w^j & \text{for } w^j \in [\hat{w}^j, \hat{w}^j + \varepsilon). \end{cases}$$

Within the range  $(\hat{w}^j - \varepsilon, \hat{w}^j)$ , the equilibrium tax rate satisfies

$$(1+n)\bar{w}(1-2\tau) = \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^j)^{1-\sigma} (\tau)^\sigma.$$

Differentiating this equation with respect to  $\tau$  and  $w^j$ , we obtain:

$$(-2)(1+n)\bar{w} \cdot d\tau = \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} [(1-\sigma)(w^j)^{-\sigma} (\tau)^\sigma \cdot dw^j + \sigma(w^j)^{1-\sigma} (\tau)^{\sigma-1} \cdot d\tau],$$

that is,

$$\begin{aligned} \frac{d\tau}{dw^j} &= \frac{(\sigma-1) \frac{\{(1+n)\bar{w}\}^\sigma}{\beta} (w^j)^{-\sigma} (\tau)^\sigma}{\frac{\{(1+n)\bar{w}\}^\sigma}{\beta} \sigma (w^j)^{1-\sigma} (\tau)^{\sigma-1} + 2(1+n)\bar{w}} \\ &\geq 0 \text{ if and only if } 1/\sigma \leq 1. \end{aligned}$$

In contrast, within the range  $(\hat{w}^j, \hat{w}^j + \varepsilon)$ , the equilibrium tax rate satisfies  $(1+n)\bar{w}(1-2\tau) = Rw^j$ ; that is,

$$\tau = \frac{1}{2} - \frac{Rw^j}{2(1+n)\bar{w}}.$$

A reduction of the decisive voter's wage leads to an increase in the tax rate. ■

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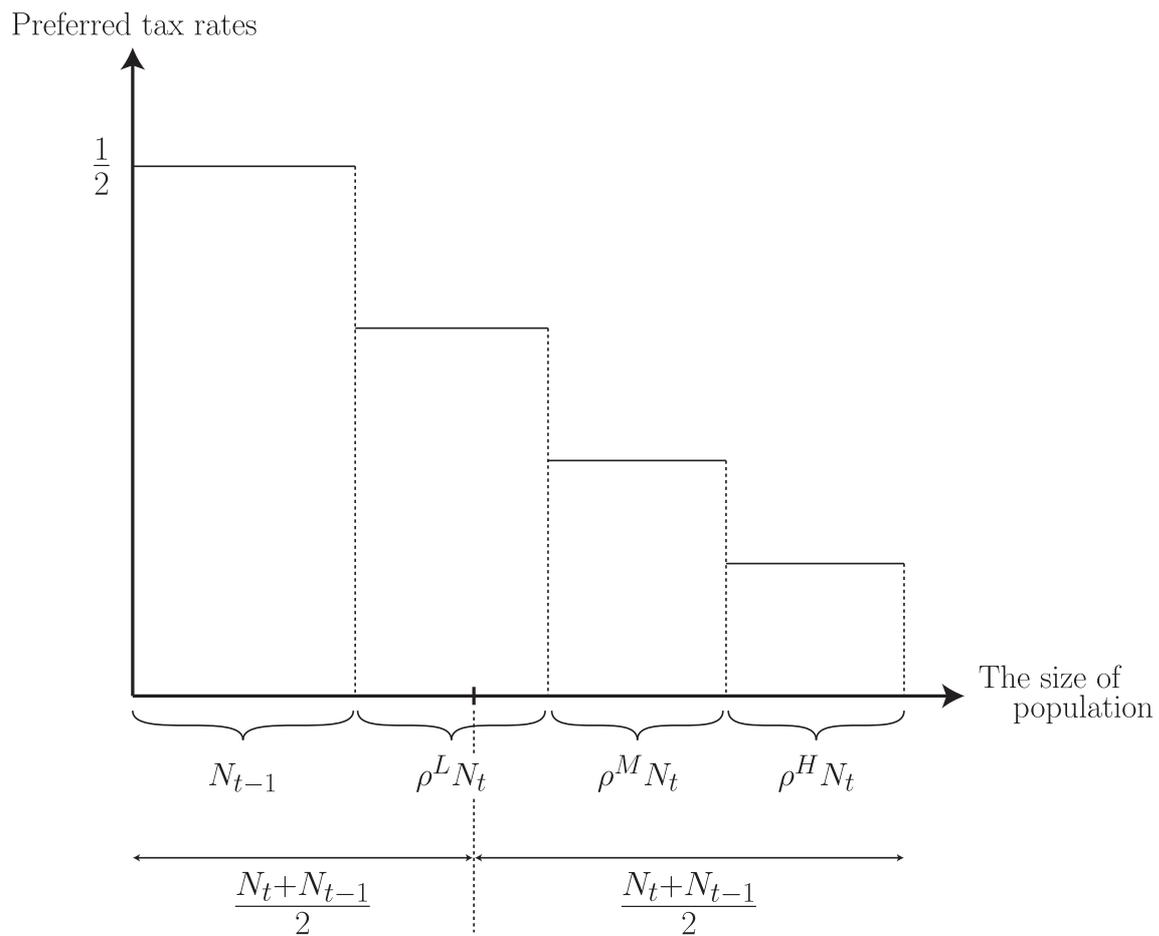


Figure 1: This figure illustrates an example of the tax rates preferred by the old and the young. In this example, a type- $L$  young individual becomes a decisive voter.

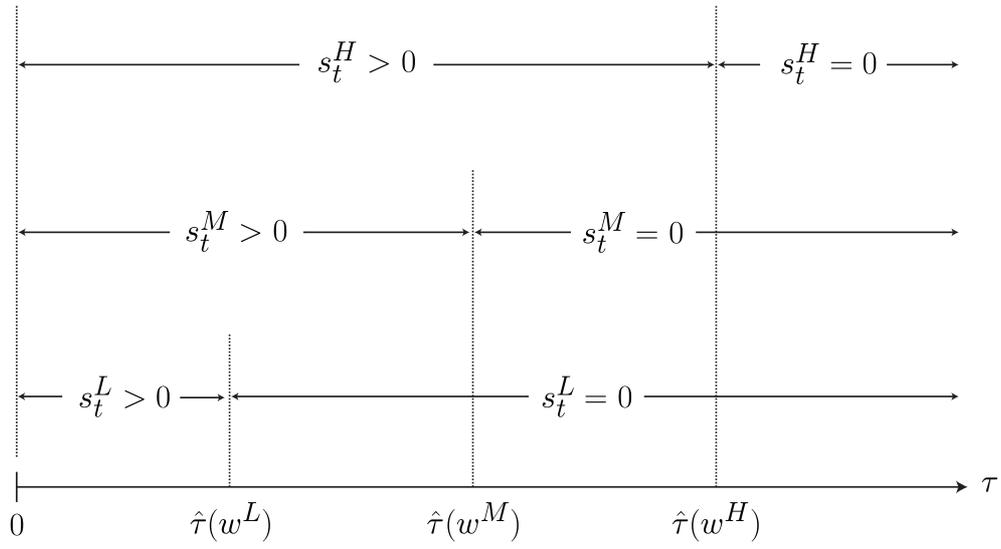


Figure 2: The relation between savings and the tax rate for each type of individual.

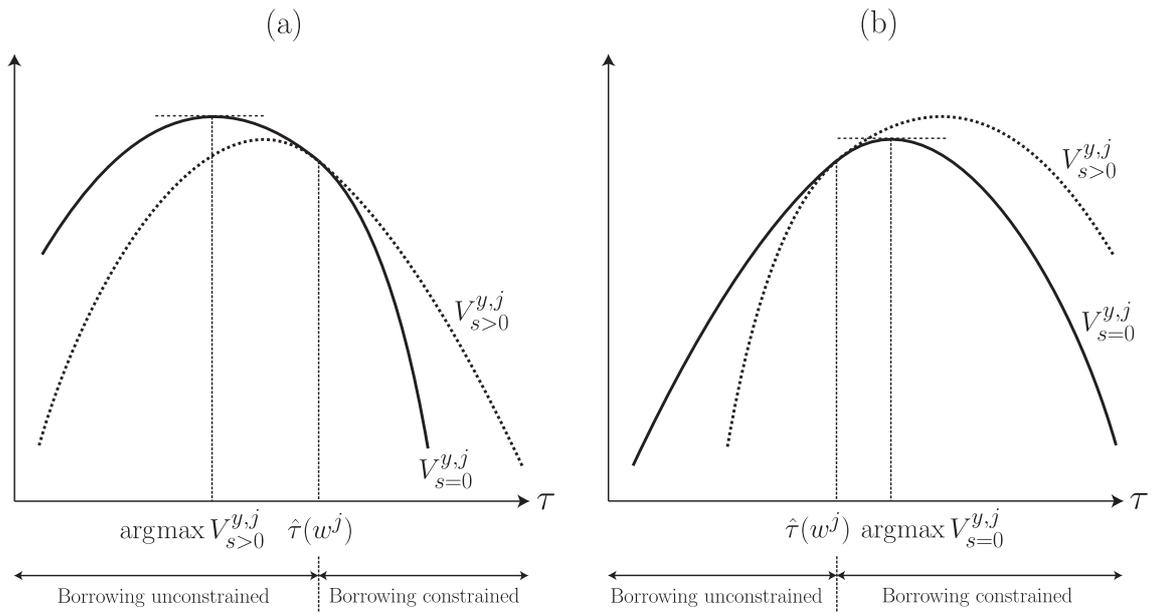


Figure 3: The solid curve depicts  $V^{y,j}$ . Panel (a) depicts the case of  $\text{argmax}_{s>0} V_{s>0}^{y,j} < \hat{\tau}(w^j)$ . Panel (b) depicts the case of  $\text{argmax}_{s>0} V_{s>0}^{y,j} \geq \hat{\tau}(w^j)$ .

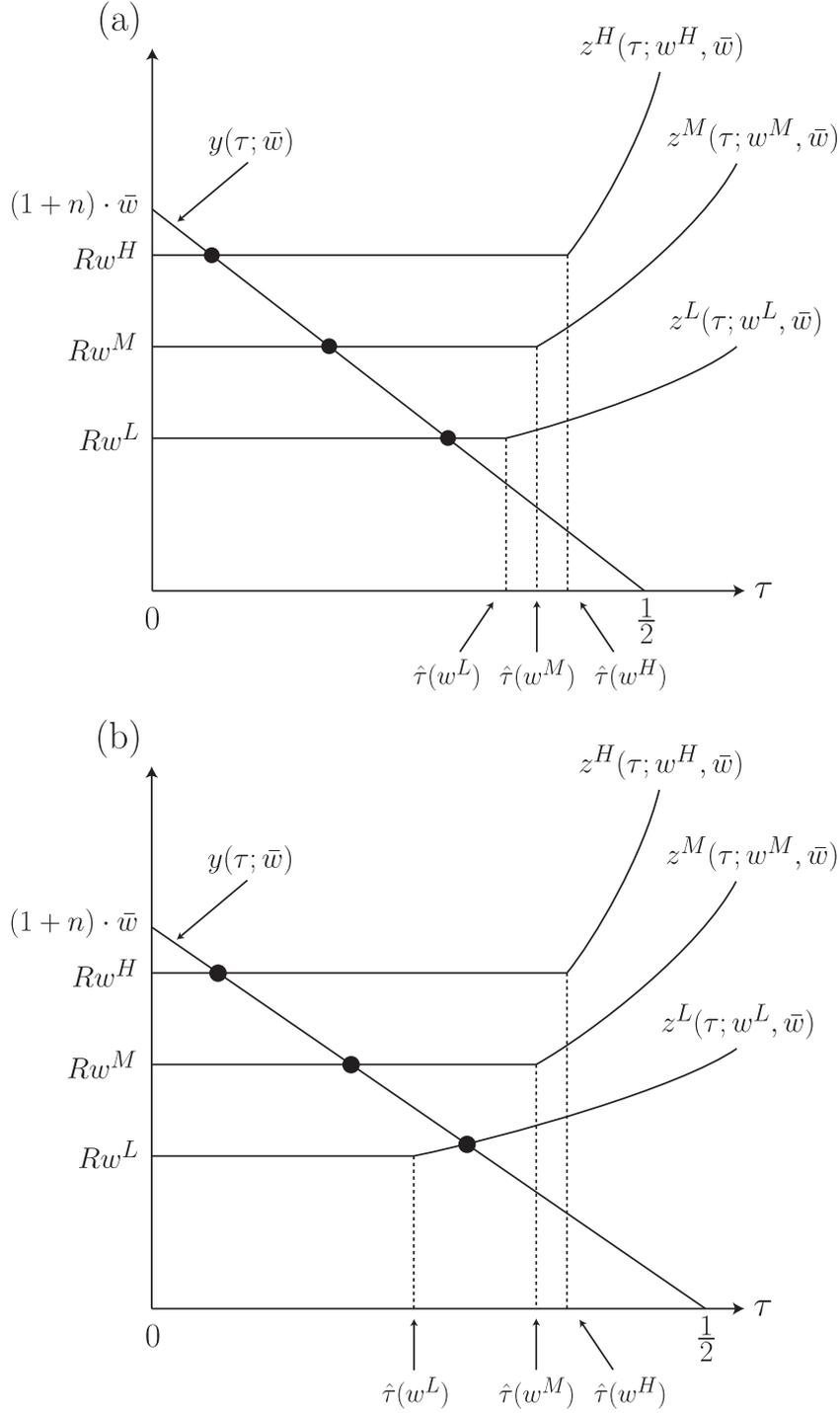


Figure 4: Panel (a) depicts the case of  $w^L \in \left( \frac{(1+n)\bar{w}}{R+2(\beta R)^{1/\sigma}}, \frac{(1+n)\bar{w}}{R} \right)$ : the type- $L$  young agent is borrowing-unconstrained. Panel (b) depicts the case of  $w^L \in \left( 0, \frac{(1+n)\bar{w}}{R+2(\beta R)^{1/\sigma}} \right]$ : the type- $L$  young agent is borrowing constrained.

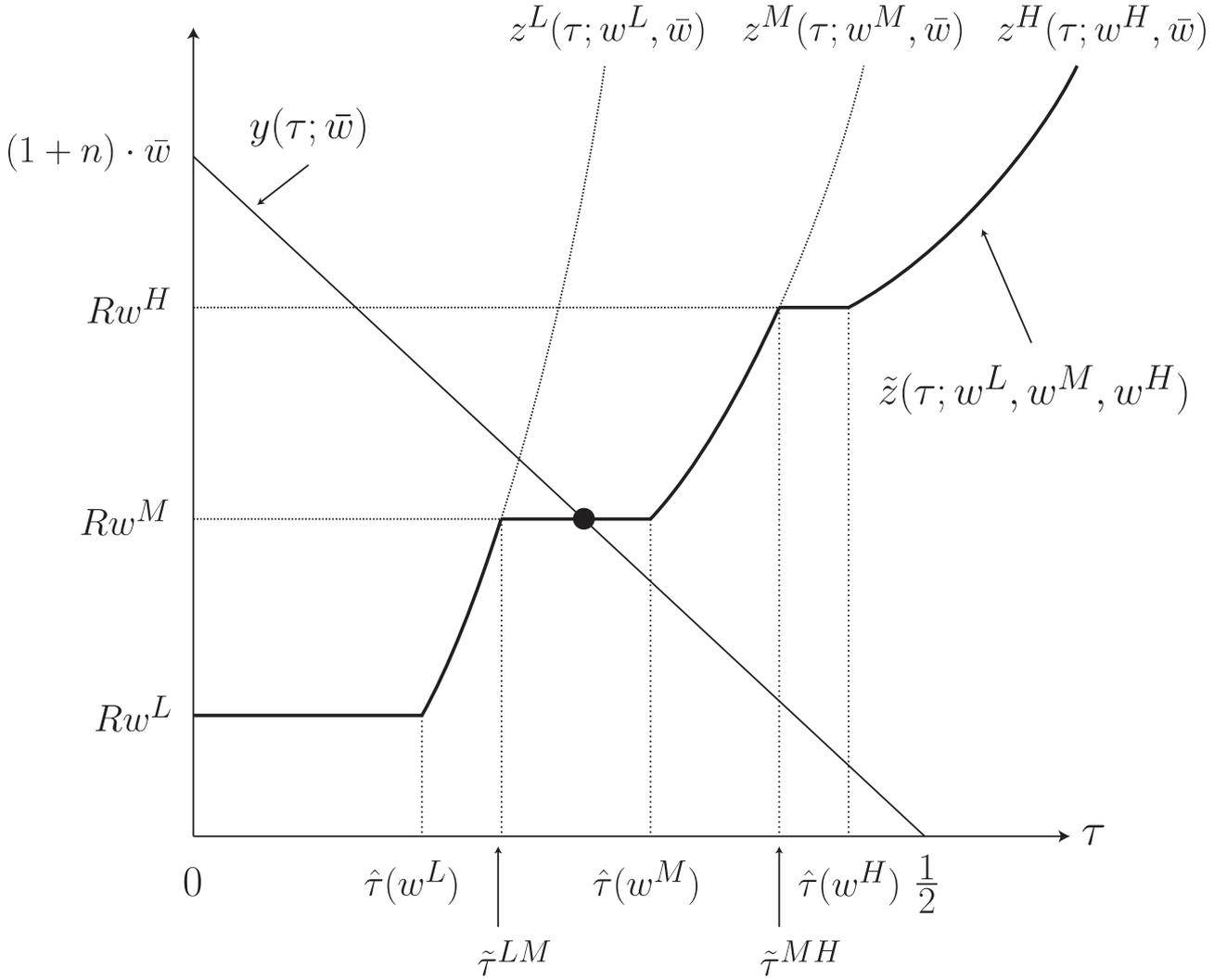


Figure 5: The determination of the tax rate in the case of  $1/\sigma < 1$ . The figure illustrates the case where the decisive voter is a type- $M$  young individual. The solid curve is the graph of  $y(\tau; \bar{w})$ ; the bold solid curve is the graph of  $\tilde{z}(\tau; w^L, w^M, w^H)$  in Eq. (11).

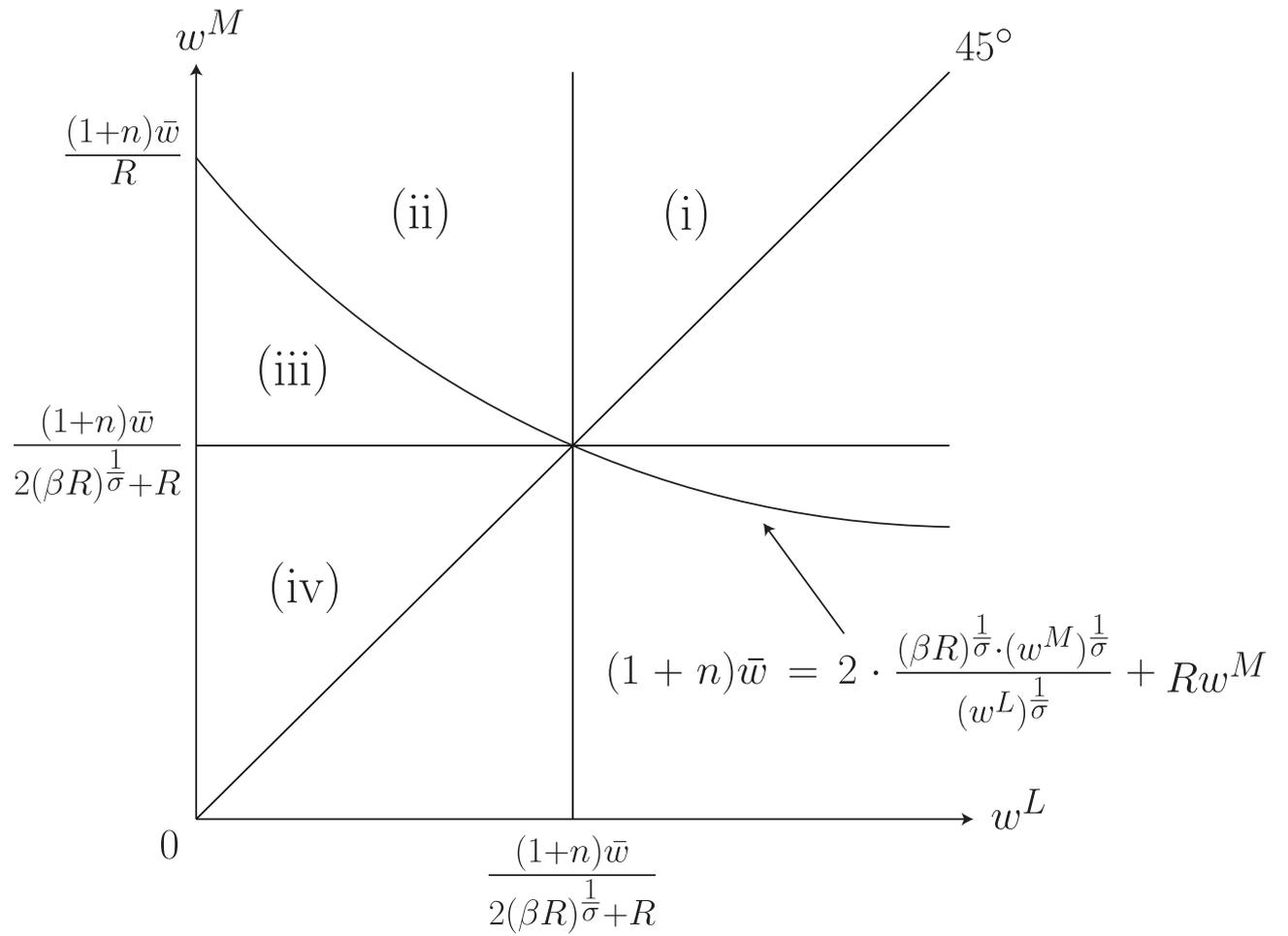


Figure 6: The figure illustrates the conditions in the statements (i) - (iv) of Proposition 2 in a  $w^L - w^M$  space, given  $\bar{w}$ .

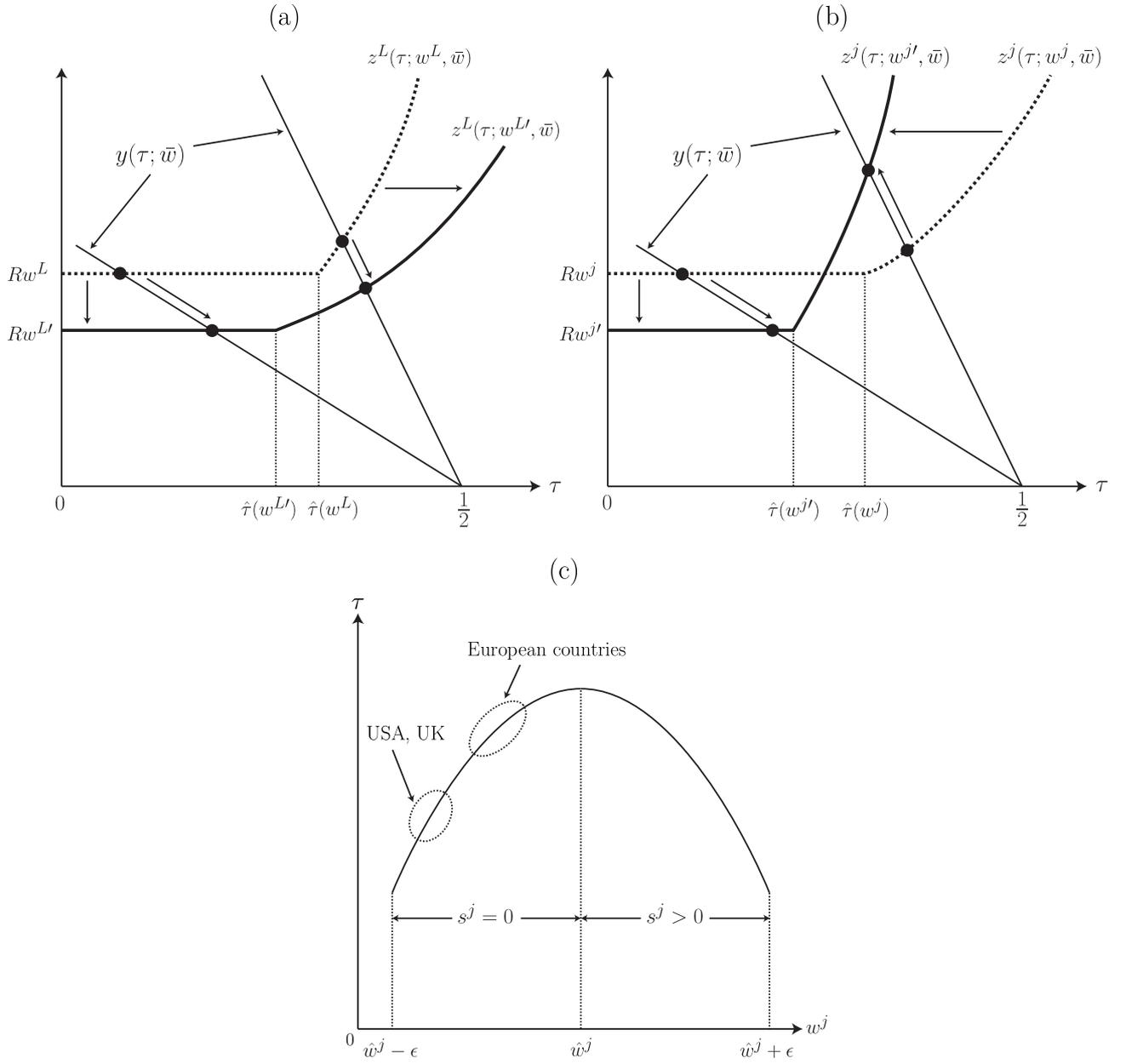


Figure 7: The effect of a mean-preserving reduction of the decisive voter's wage on the tax rate. Panel (a) illustrates the case of  $1/\sigma \geq 1$ . Panel (b) illustrates the case of  $1/\sigma < 1$ . Panel (c) illustrates the relation between the decisive voter's wage and the equilibrium tax rate in the case of  $1/\sigma < 1$ .