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Longevity, fertility and Demographic Transition in an OLG model.

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Abstract

The paper investigates the effects of declining mortality on fertility and income in the standard OLG neoclassical growth model under the assumptions of accidental bequests as well as fully annuitised savings. It is shown whether and how different countries may expect increasing or decreasing fertility rates under increasing longevity, and argued that mortality decline may be another explanation of the Demographic Transition process. In particular, the fact that some countries have completed the process while others are entrapped in the second stage may depend on the initial level of mortality as well as on differences in technology and preferences. It is also argued that the third stage may not necessarily occur in some less developed countries even if their mortality rates converge towards those of industrialised countries.

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1. Introduction

The current literature views the search for a unified model able to encompass the entire "long transition process, from thousands of years of Malthusian stagnation through the demographic transition to modem growth, as one of the most significant research challenges facing economists interested in growth and development."(Galor and Weil, 1999, p. 150). A brief historical description of the Demographic Transition (DT) may evidence three periods[1]: (1) an increase in life expectancy, for instance due to advances in medical knowledge; (2) mortality decline associated with economic development and, in the early stages, with rising population growth due to both reduced mortality and increased fertility rate; (3) declining fertility rate when life expectancy and development increase.[2]

The aim of this paper is to study the relationships between adult mortality, fertility and income when children are a consumption good and enter their parents' utility function (e.g. Galor and Weil, 1996)[3] and the duration of life is uncertain. Although this paper does not present a unified model in the sense mentioned above, its value lies in suggesting a simple framework able to capture the main aspects of the Demographic Transition, focusing on the crucial role of the increased longevity.[4]

Hence, we address the following important question: is there a theoretical relationship between DT and development in the simple neoclassical OLG model in which the reduction in adult mortality (the so-called "mortality revolution")[5] plays a crucial role?

A stylised fact is that while early-developed countries have all experienced a DT, developing economies are still those that cannot get out of the high-fertility trap. This raises an intriguing puzzle: why has DT been completed in some countries and not in others despite the common trend towards a worldwide convergence in mortality rates? What causes a country to be persistent in the second phase? To answer such questions, we examine how and whether changes in longevity affect fertility behaviour. Interestingly, we show that short-sighted behaviour is sufficient, i.e. a low preference for future consumption, which is typical of some underdeveloped countries, to prevent the transition to the third phase of DT, even if the mortality rate would tend to approach that of industrialised countries. Therefore our model makes predictions in line with observed cases of completed as well as uncompleted DT, depending on the current level of adult mortality, which still differs between countries (see for example Tab. A1, in Lorentzen et al., 2008, p. 112). Moreover, it argues that the third phase of DT may not be a necessary consequence even in the case of the observed worldwide downward trend of adult mortality with final convergence, especially as regards some less developed countries (LDCs).

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[1] In a nutshell, demographic transition refers to a shift in demographic behaviour from a state of high birth and death rates to a state of low birth and death rates. An equivalent well-known classification (Galor and Weil, 1999; 2000), more focused in macroeconomic terms, assumes three distinct regimes that have characterized the process of economic development: the "Malthusian Regime," the "Post-Malthusian Regime," and the "Modern Growth Regime."

[2] All the more developed countries have entered this third stage of the demographic transition; by contrast, many developing countries only recently began the process. For the sake of precision, a few developed countries have gone on to a fourth stage in which death rates exceed birth rates, and the population declines.

[3] We note that in the literature there have been two other approaches to the endogenisation of the fertility decision into an economic model. The first, in addition to assuming that children are a consumption good, assumes that either the children's utility or the utility of all the future generations enter their parents' utility function (Barro and Becker, 1988; Becker and Barro, 1988). The second assumes that parents are selfish and that children serve only to provide old age support (Ehrlich and Lui, 1991; Raut and Srinivasan, 1994; Chakrabarti, 1999), and thus seems more suited to developing than developed economies.

[4] Fanti and Gori (2007) attempt to provide a further explanation, complementary to those already existing in the literature and also to the present one, focusing on the effects of a unionization of the economies as a cause of an aspect of the DT, namely the emergence of modern fertility behaviour in place of the Malthusian one.

[5] The mortality revolution began in the 1700s in Europe and spread to North America by the mid-1800s when human population grew rapidly, not because the birth rate increased, but because the death rate began to fall (the main causal factors are considered new farming and transportation technology as well as improved public health and living standards owing to increasing economic development). In contrast, in most less developed countries, the mortality revolution did not begin in earnest until after World War II.

The distinctive feature of the introduction of longevity in a standard OLG model is the treatment of the savings of the deceased persons. There are two polar cases, which obviously also embodying intermediate cases: 1) there is a perfect annuity market and all savings are fully annuitized; 2) there is no annuity market, for instance as in Abel (1985), so the savings of a deceased person become an accidental bequest[6] to his/her child. Interestingly, we show that for whatever assumption on the treatment of savings of the deceased, our results hold. To the best of my knowledge, no basic OLG model with a preference for children and exogenous longevity arguments, embodying both assumptions on savings of the deceased, has been studied to explain all these three stages at the same time.[7] The remainder of this paper is organized as follows. Section 2 introduces the general model. Sections 3 and 4 derive the results for the cases of fully annuitized savings and accidental bequests, respectively, and discuss some implications. Section 5 makes some concluding remarks.

2. The general model

2.1. The firms

The representative firm acts competitively. The constant returns to scale production function is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[8] where Y, K and $L = N$ are output, capital and the labour input respectively, $A > 0$ is a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology. The intensive form technology of production may be written as

$$y_t = Ak_t^{\alpha} \tag{1}$$

with $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ being capital and output per-capita respectively. Assuming total depreciation of capital at the end of each period, profit maximisation leads to the following marginal conditions:[9]

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{2}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{3}$$

2.2. Individuals

The life of a representative individual is separated into three periods: childhood, young adulthood and old age. During childhood individuals do not make decisions. Young adulthood is a working period fixed with certainty and old age is a retirement period whose length is uncertain. We assume, for the sake of simplicity, that the individual is either alive or dead at the beginning of the retirement period, with probability *p* and *(1-p)* respectively. The wage (w_t) is used to consume, raise children, and save. The cost of child rearing is *m* for each child, measured in terms of output. The labour supply (net of leisure) is constant and normalised to unity.

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[6] Other major bequest motives are altruism and exchange. While there is no consensus on which bequest motive dominates (see, e.g., Altonji et al., 1997), Hurd (1997) argues that bequests are largely accidental.

[7] Although some papers (e.g. Zhang et al., 2001, Yakita, 2001, Strulik, 2003) embody – generally in an endogenous growth context – exogenous longevity and endogenous fertility, they focused on other issues such as growth and social security and in any case argued a monotonic relationship between longevity and fertility: for instance, Strulik (2003), who does not consider the treatment of the savings of the deceased persons, Zhang et al. (2001), who consider accidental bequests, and Yakita (2001), who considers annuitised savings, state that a rise in longevity always reduces fertility. This paper shows that, even in the textbook OLG model and regardless of whether accidental bequests or a perfect annuity market is considered, the relationship between fertility and longevity is an inverted U-shape. To the best of our knowledge, this result has not so far been pointed out elsewhere.

[8] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and hence it is not included here.

[9] The price of output has been normalised to unity.

The representative agent born at time t chooses saving and number of children to maximise a standard Cobb-Douglas utility function

$$U = (1-\phi)\ln c_t^{\,1} + p\gamma \ln c_{t+1}^{\,2} + \phi \ln n_t \qquad (4)$$

where $c_t^{\,1}$ and $c_{t+1}^{\,2}$ are consumption in the first and second periods, respectively, γ is the subjective discount factor and $(1-\phi)$ and ϕ are the preference toward first-period consumption and the number of children, respectively.

We now further develop the model, distinguishing the two polar cases of 1) a perfect annuity market and 2) unintentional bequests without annuity markets.

3. The model with fully annuitised savings

It is assumed that: i) the private annuity market is competitive and the companies are risk neutral; ii) individuals are willing to invest their assets in such insurance companies, given the hypothesis of absence of bequests, so that savings are fully annuitised.

The budget constraint of the young individual is:

$$c_t^{\,1} = w_t - mn_t - s_t, \qquad (5.1)$$

where s_t are savings. Following a simplified two-period version of the Blanchard (1985) model, as in Yakita (2001), we assume that: 1) the insurance companies exchange a payment to individuals of $\frac{1+r_{t+1}}{p_a} s_t$ for estate s_t accruing to the companies, where p_a is the average probability; 2) the probability p is the same for all individuals. Therefore the budget constraint of individuals when old is

$$c_{t+1}^{\,2} = \frac{1+r_{t+1}}{p} s_t \qquad (5.2)$$

The standard maximisation of the utility function (4), subject to constraints (5.1) and (5.2), leads to the following choices of savings and number of children, respectively:

$$s_t = \frac{p\gamma w_t}{1+p\gamma} \qquad (6)$$

$$n_t = \frac{\phi w_t}{m[1+p\gamma]} \qquad (7)$$

The market-clearing condition in goods as well as in capital markets is expressed as $n_t k_{t+1} = s_t$. Substituting out for n, s and w from eqs. (3), (6) and (7), the market-clearing condition boils down to the following long-run capital per-capita:

$$k^* = p\gamma m / \phi. \qquad (8)$$

3.1. Steady state analysis

What are the longevity effects on the long-run rate of fertility? This simple question gives rise to interesting findings in our basic OLG model. Making use of (3), (7) and (8) the long-run rate of fertility is determined by:

$$n^*(p) = \frac{\phi(1-\alpha)A[p\gamma m/\phi]^{\alpha}}{m[1+p\gamma]}, \qquad (9)$$

and the following propositions holds:

Proposition 1. *The increase in longevity reduces fertility if and only if* $p > \frac{\alpha}{(1-\alpha)\gamma}$;

Proof: The proof straightforwardly derives from:

$$\frac{\partial n^*(p)}{\partial p} \gtrless 0 \quad \Leftrightarrow \quad [\alpha - \gamma p(1-\alpha)] \gtrless 0 \tag{10}$$

The simple observation of the condition (10) reveals that the higher the existing longevity, the more likely increasing longevity reduces the long-run fertility.
Simple manipulations of (10) show that the necessary and sufficient condition such that increasing longevity is accompanied by decreasing fertility may also be expressed in terms of the capital share: $\alpha < \frac{p\gamma}{1+p\gamma}$ or, alternatively, in terms of the thriftiness parameter: $\gamma > \frac{\alpha}{(1-\alpha)p}$. The latter inequalities suggest that the higher the capital share and the lower the "parsimony", the more likely that an increase in longevity increases the long-run fertility as well.

4. The model with accidental bequests

Since agents do not know when they will die, additional unintentional bequests may occur. If an agent dies at the onset of old age (with probability *(1-p)*), his accumulated savings, $(1+r_{t+1})s_t$, are bequeathed in full to his heirs. To maintain the representative agent formulation, bequests

$$b_{t+1} = \frac{(1-p)(1+r_{t+1})s_t}{n_t} \tag{11}$$

are equally divided among all the young individuals.[10]
The representative agent born at time *t* maximises the utility function (4) subject to

$$c^1{}_t = w_t - mn_t - s_t + b_t \tag{12}$$

$$c^2{}_{t+1} = s_t(1+r_{t+1}) \tag{13}$$

taking as given bequests from his/her parents, and constraint (3) encompasses the assumption that bequests are allocated equally across all members of a generation.
The following optimal choices of saving and fertility rate are derived, respectively:

$$s_t = \frac{\gamma p(w_t + b_t)}{1+\gamma p} \tag{14}$$

$$n_t = \frac{\phi(w_t + b_t)}{m(1+\gamma p)} \tag{15}$$

In the next section, we examine the steady-state outcome of this model.

4.1. Steady-state analysis

Making use of (11), (14) and (15) we obtain the steady-state values of bequests,[11] savings and fertility, respectively:

[10] This means that the bequest-dependent wealth distribution is uniform, as in Hubbard and Judd (1987). This assumption allows us to conduct a representative agent analysis and thus to focus more clearly on the effects of changes in expected longevity.
[11] Note that while in the models with exogenous fertility bequests are reduced when longevity increases, bringing about a reduction in young people's income and thus reducing savings and further fertility (see eqs. 14 and 15), in the present model bequests may either increase or decrease with increasing longevity.

$$b = \frac{\gamma p m(1+r)(1-p)}{\phi} \quad (16)$$

$$s = \frac{\gamma p\left[\phi w + m\gamma p(1+r)(1-p)\right]}{\phi(1+\gamma p)} \quad (17)$$

$$n = \frac{\left[\phi w + m\gamma p(1+r)(1-p)\right]}{m(1+\gamma p)} \quad (18)$$

The accumulation equation for per capita capital stock is given by $k_{t+1} = s_t / n_t$. By using (17) and (18) and solving the steady state k^* is obtained:

$$k^* = \frac{p\gamma m}{\phi} \quad (19)$$

By using (2), (3), (18) and (19), the long-run fertility rate is:

$$n = \frac{\phi A(1-\alpha)\left(\frac{p\gamma m}{\phi}\right)^{\alpha} + m\gamma p\alpha A\left(\frac{p\gamma m}{\phi}\right)^{\alpha-1}(1-p)}{m(1+p\gamma)} \quad (20)$$

The effect of longevity on fertility rates is determined by the investigation of the following:

$$\frac{\partial n}{\partial p} = \frac{-\phi A\left[\alpha^2 p(1+\gamma p) - \alpha(1+\gamma p - p) + \gamma p\right]\left(\frac{p\gamma m}{\phi}\right)^{\alpha}}{\left[mp(1+p\gamma)\right]^2} \quad (21)$$

Proposition 2. *The increase in longevity reduces (increases) fertility if, and only if,*

$$p > (<)\, p^{\circ} = \frac{\sqrt{\alpha^4 + 2\alpha^3(1+\gamma) + \alpha^2(1+\gamma^2)2\alpha\gamma(1-\gamma) + \gamma^2} - \alpha^2 - \alpha(1-\gamma) - \gamma}{2\alpha^2\gamma} \quad (22)$$

Proof: The proof straightforwardly derives from:

$$\frac{\partial n}{\partial p} \substack{> \\ <} 0 \quad \Leftrightarrow \quad \left\{-p^2\alpha^2\gamma - p\left[\alpha^2 + \alpha(1-\gamma) + \gamma\right] + \alpha\right\} \substack{> \\ <} 0, \quad (23)$$

and thus, resolving for the unique positive value of longevity, $p°$, it follows that:

$$\frac{\partial n}{\partial p} \substack{> \\ <} 0 \quad \Leftrightarrow \quad p \substack{< \\ >} p^{\circ}.$$

From the observation of (22) or (23) it is easy to see that the lower the thriftiness parameter γ, the more likely $p°$ is larger than one and thus fertility is never reducing with increasing longevity.

5. An illustration of the main results

The following table illustrates the content of propositions 1 and 2 above, for the parameter set *A=2, ϕ=0.27, α=0.25, γ=0.55, m=0.11* (chosen only for illustrative purposes), showing an inverted U-shaped relationship between fertility and longevity, for both the assumptions on the savings of the deceased persons.

Table 1- Numerical results of the relationships between expected longevity and fertility rates under the two alternative treatments of savings of the deceased persons.

Fertility rate (*n*)	*p*=0.25	*p*=0.40	*p*=0.50	*p*=0.60	*p*=0.70	*p*=0.80	*p*=0.90
Annuitised savings case	1.574	1.651	1.671	1.676	1.672	1.663	1.650
Accidental bequests case	1.969	1.981	1.948	1.900	1.8401	1.774	1.704

To shed light on the possible relevance of the "humped" relationship fertility-longevity shown by our model, let us imagine that, for the sake of simplicity, the results about such a relationship depicted in Tab. 1 for the case of annuitised savings, although only based on an illustrative parameter set, hold for all economies. Such results predict that countries with a probability of adult survival less than about 65% would experience an increase in fertility as a consequence of an exogenous reduction in the adult mortality rate. Therefore by observing the behaviour of adult mortality in the world (see for example Tab. A1, in Lorentzen et al., 2008, p. 112, where the adult mortality rate - defined as the probability of a male surviving to age 60, conditional on surviving to age 15 - is reported for 160 countries) we may see that for about 53 countries such a rate is less than 0.35: therefore this would mean that, with the assumed illustrative parameter set, about one third of the world would again be entrapped in the second phase of DT. But our results are again more interesting as regards the puzzle of the completed versus uncompleted DT: if some countries are not sufficiently "thrifty" such as occurs in developing economies, then they could never complete the Demographic Transition even if their longevity approaches – especially thanks to pervasive medical improvements - that of the advanced countries which completed such a transition. Indeed, it is easy to show that for sufficiently low values of the parameter γ the relation of fertility rates to longevity could always be positive (rather than inverted U-shaped). This means that economies with too many parsimonious individuals may always be prevented from entering the third stage of DT. The intuition is as follows. First, while a mortality decline incentives capital accumulation and hence wages, it makes it convenient to increase old age consumption in place of raising children. Therefore, when individuals are sufficiently short-sighted or parsimonious, the result is that the (Malthusian) income effect of higher wages on fertility is greater than the substitution effect between (fewer) children and (more) old age consumption, leading to a positive relationship between fertility and longevity.

5.1. Mortality revolution and the demographic transition process

We note that the model above is able to depict not only the DT in strict sense, but also the three stages of the process of demo-economic development once endowed with the usual relationship between longevity and income, which is commonly thought to have a bi-directional causal nexus: on one side, from eqs. 1, 8 and 19 our model predicts an equilibrium relationship indicated by a monotonic increasing function $y=f(p)$, while, on the other, it is usual to assume the (monotonic or non) increasing function $p=F(y)$ (e.g. Strulik, 2003, Blackburn and Cipriani, 2002, Kalemli and Ozcan, 2002). By using the system consisting in (1), either (9) or (20), either (8) or (19), and $p=F(y)$, it would be a trivial exercise to exemplify the transition, after a shift of the intercept and/or of the slope of the function F,[12] towards the new modern equilibrium through the three stages described in the introduction. A "mortality revolution" owing to an exogenous event rather than to the effect of economic growth seems to be the case of many developing countries; thus the process triggered by a shift in the longevity function, as depicted above, might occur in those countries

[12] Such a shift may be thought of as an exogenous event which permits, *ceteris paribus* for output, a reduction in adult mortality: for instance, among others, cooked instead of raw foods, improvements in medical know-how, climate changes, a reduction in wars and murders, and so on.

which have not so far experienced the DT.[13] We believe that a model as parsimonious as possible, such as that in the present paper, is able to provide, without embarking on additional assumptions, predictions in accord with the historical evidence on demography and development.

6. Conclusions

We investigated the effects of declining mortality on fertility and income in the standard OLG neoclassical growth model, arguing that mortality decline may be another explanation of the demographic transition process. In particular, the present model predicts the economic conditions necessary for the occurrence of the third phase of the DT. For instance, increasing longevity goes hand in hand with decreasing fertility only if the thriftiness is sufficiently high: countries with more "cicadas" than "ants" might not complete the third stage of the DT. It is worth noting that short-sighted behaviour may be correlated with underdevelopment of financial markets as well as insufficient financial education of people.[14] This reinforces the probability that many developing countries are prevented from entering the third stage. These results shed new light on the issue of why some countries may undergo a "mortality revolution" without completing the third stage of DT.

These results complement those obtained by the growing literature on fertility, long-run development of economies and the transition from a pre-industrial to post-industrial world (see, e.g., Nelson, 1956, Becker and Barro, 1988; Kremer, 1993, Raut and Srinivasan, 1994, Tamura, 1996, Galor and Weil, 1996, 2000, Galor, 2005). Given their simplicity, we believe that, by resorting to a, loosely speaking, Occam's razor-type reasoning, they offer a probably simple explanation of the somewhat intricate DT process.

The interest of these results lies in their robustness, in that they hold under the assumptions of accidental bequests and fully annuitised savings, and in the simplicity with which are obtained, that is within a standard OLG model. They also show how and whether different countries may expect increasing or decreasing fertility rates under a persistent trend of reduced adult mortality.

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[13] In fact, in most less developed countries, the mortality revolution followed a different pattern from that in European countries and it did not begin in earnest until after World War II. In particular death rates fell swiftly through the introduction of i) medical and public health technology; ii) antibiotics and immunization (reducing deaths from infectious diseases); and iii) insecticides (helping control malaria). Therefore these changes did not result as an effect of the economic development within countries, but were a result of international foreign aid and thus may be interpreted as an upward shift of the function linking longevity and output, according to the reasoning mentioned in the main text.

[14] As noted for instance by Chakraborty (2004, p. 120) "in poorer societies, when life expectancy is low, individuals discount the future more heavily and are less inclined to save and invest."

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